Business Economic and Financial Data Project

Interest Rate

Introduction

- My data are U.S. daily interest rates of Treasury Securities with 1-year maturity rate and 3-year maturity rate between January 5, 1962 and December 3, 2021 which are measured in percentage. The series are obtained from the Federal Reserve Bank of St. Louis.

What is the “interest rate”?

- An interest rate is the amount of interest due per period, as a proportion of the amount lent, deposited, or borrowed.

What are the relationships between the data and business, economic or finance ?

- Interest rates have Impacts on wide range of criteria such as:
  - Bond prices and Stock market
  - Inter-bank borrowing
  - International Investments
  - Mortgage rates
  - Savings
  - Inflation
  - etc.

Bond Prices and Interest rates

- When market interest rates rise, prices of fixed-rate bonds fall. This phenomenon is known as interest rate risk.
- So the relationship is opposite.
- Bonds with longer maturities generally have higher interest rate risk than similar bonds with shorter maturities. Therefore, higher maturities have higher fluctuations in interest rates.
- Related information:

  (https://www.sec.gov/files/ib_interestraterisk.pdf)

Inter-bank borrowing and interest rate

- Inter-bank borrowing is essentially a way for banks to quickly raise money. For example a bank may want to finances a major industrial effort but may not have the time to wait for the deposits or interest(on loan payments) to come in. In such cases the bank will quickly raise this amount from other banks at an interest rate equal or higher than the Federal funds rate.

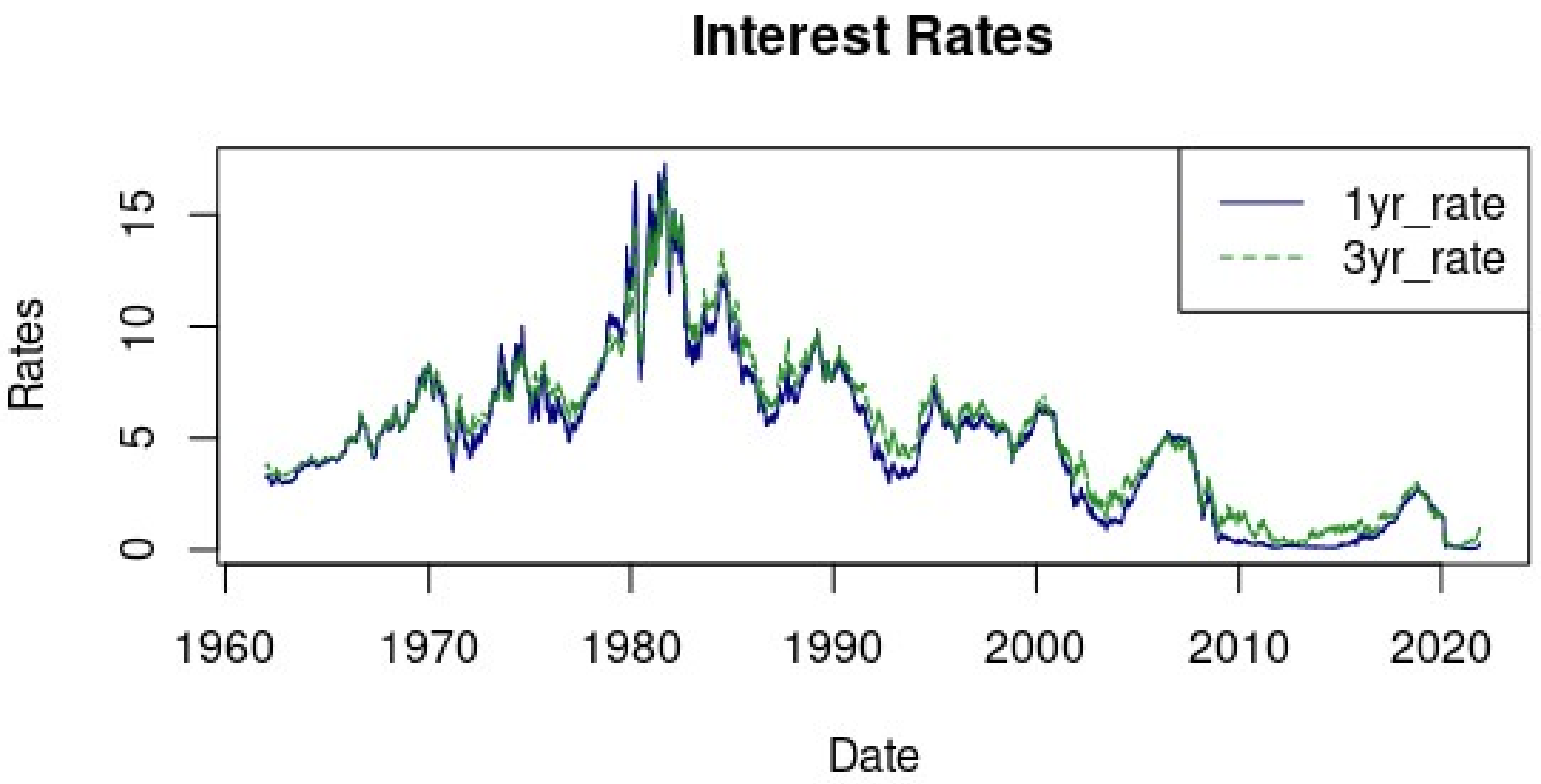
  Therefore the interest rate is used as a regulatory tool to control how freely the U.S economy operates.

International effects

- A low interest rate makes investment in developing countries such as China or Mexico more attractive and vice versa.
- Therefore it affects the value of currency in those countries.

Overview

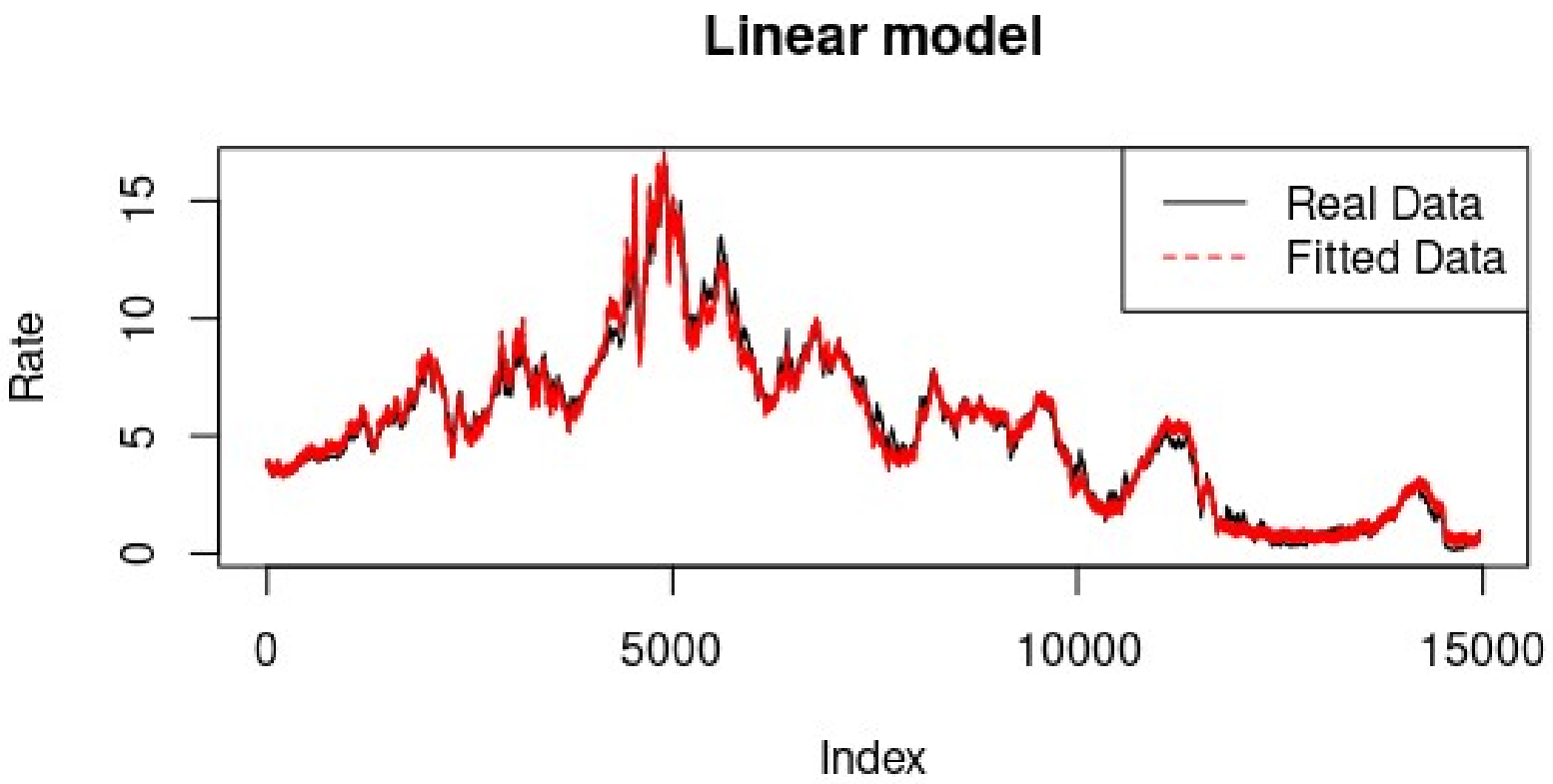
- Basically my goal was to find the best model to describe 3-year maturity rate data with 1-year maturity rate data.
- I tried different models(namely: linear, ARIMA, Generalized additive and Gradient Boosting model)
- I also tried to model the Exchange rate of Yuan of China and Peso of Mexico to show the relationship between interest rate and investment in developing countries.

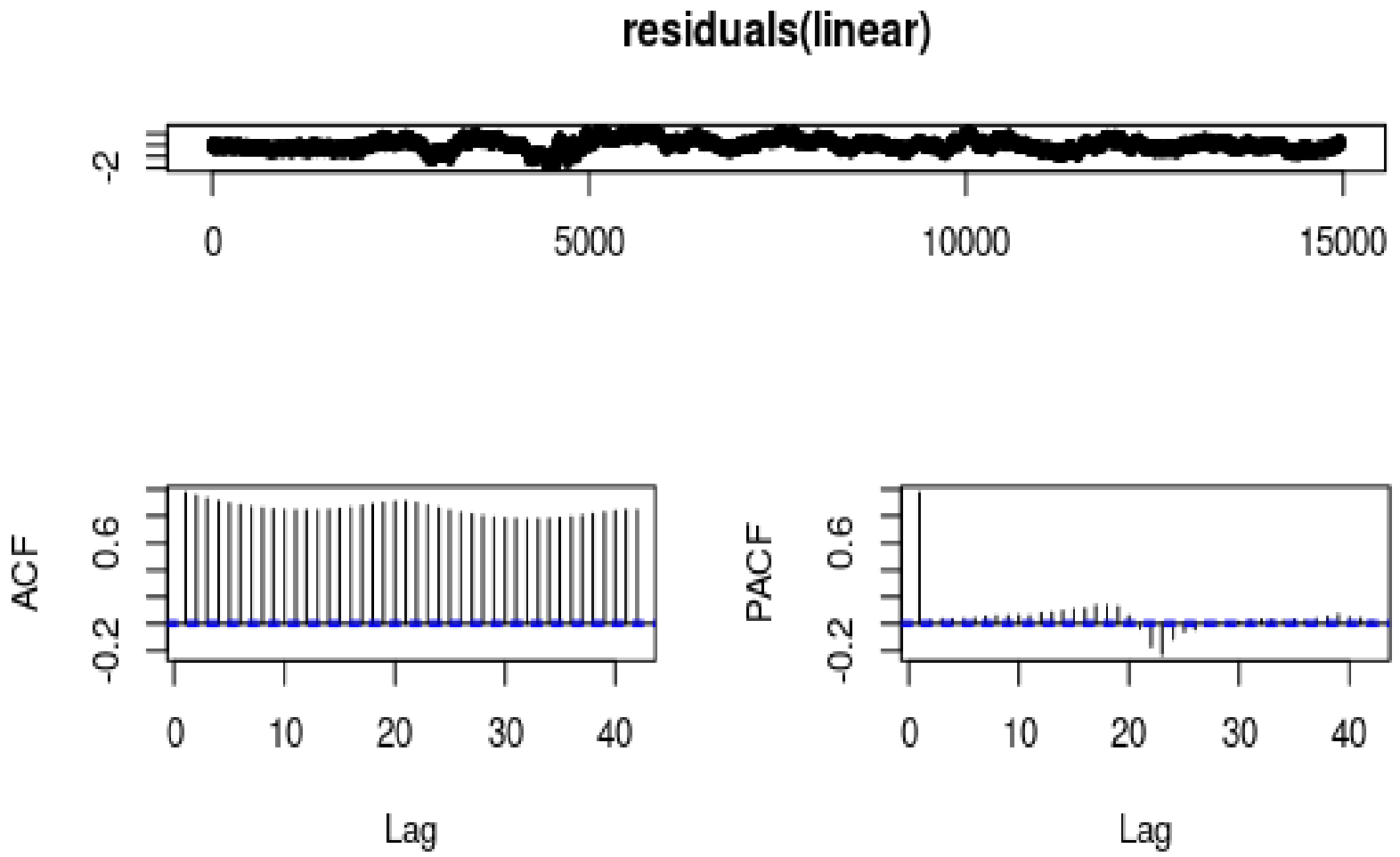

Interest Rates
Rates
0
5
10
15
1960
1970
1980
1990
2000
2010
2020
Date
1yr_rate
3yr_rate


scatter plot
3ry_rate
0
5
10
15
0
5
10
15
1yr_rate

Linear model

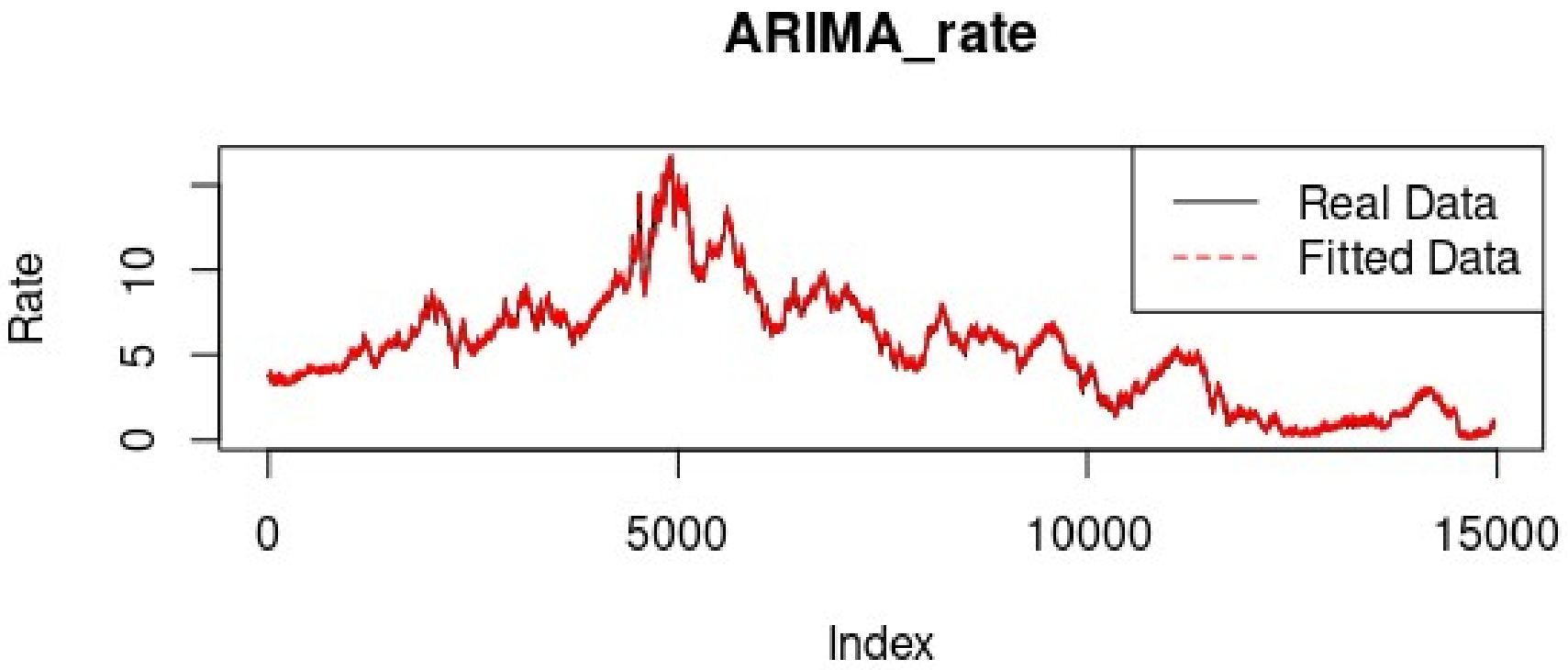
```
Call:
lm(formula = data.rate$3yr_rate ~ data.rate$series + data.rate$1yr_rate +
    data.rate$month + data.rate$year)
Residuals:
    Min      1Q  Median      3Q     Max
-2.1571 -0.3180 -0.0211  0.3175  1.5652
Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
(Intercept)         -1.058e+04  2.714e+02  -38.99   <2e-16 ***
data.rate$series    -2.161e-02  5.541e-04  -39.00   <2e-16 ***
data.rate$1yr_rate   9.266e-01  1.526e-03  607.06   <2e-16 ***
data.rate$month      4.468e-01  1.168e-02   38.25   <2e-16 ***
data.rate$year       5.393e+00  1.383e-01   38.99   <2e-16 ***
---

Residual standard error: 0.476 on 14959 degrees of freedom
Multiple R-squared:  0.9796,    Adjusted R-squared:  0.9796
F-statistic: 1.799e+05 on 4 and 14959 DF,  p-value: < 2.2e-16



AIC(lm)
[1] 20258.22
```

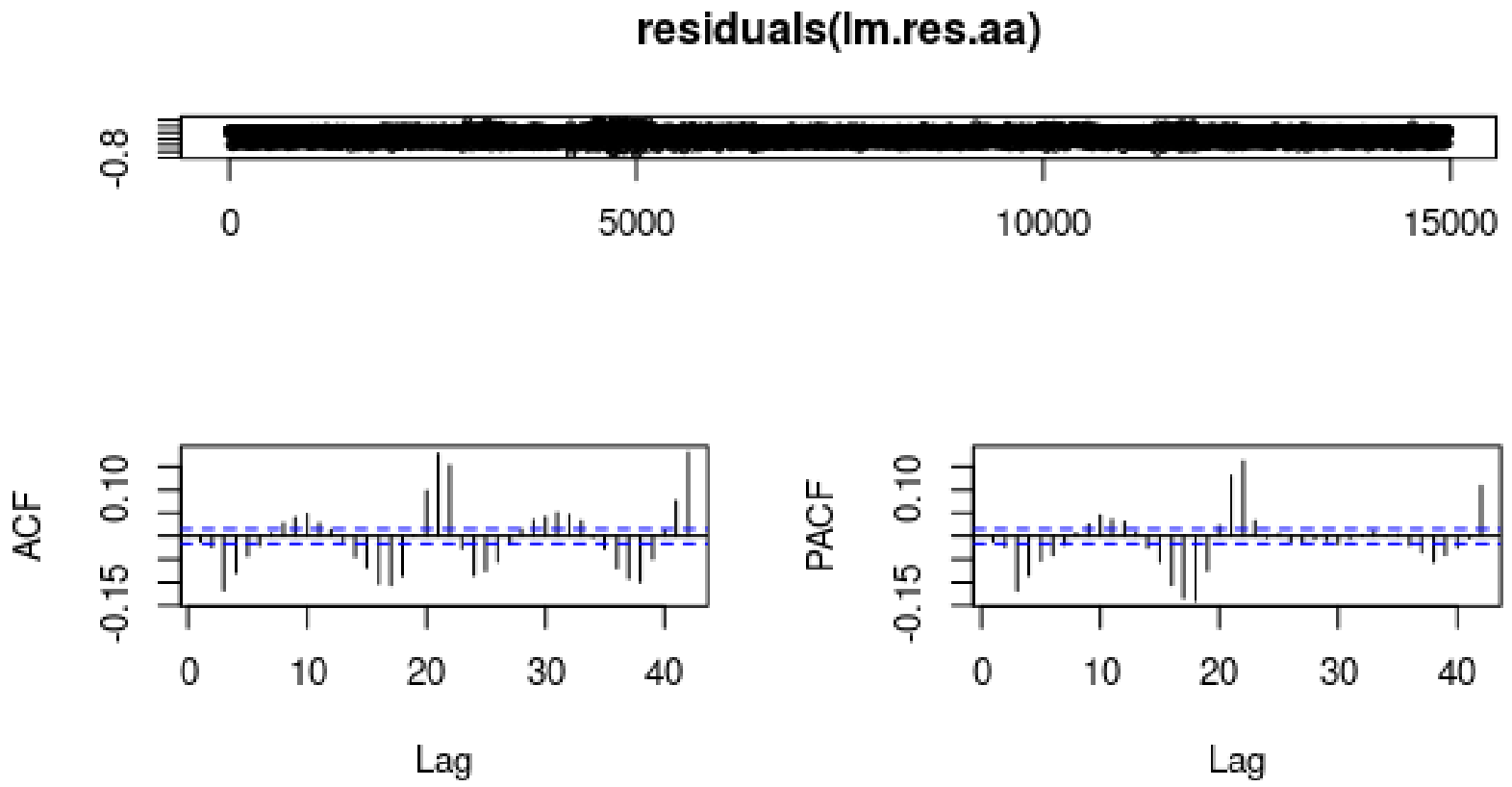

Linear model
Real Data
Fitted Data
Rate
0
5
10
15
0
5000
10000
15000
Index


residuals(linear)
-2
0
5000
10000
15000
ACF
0.6
-0.2
0
10
20
30
40
Lag
PACF
0.6
-0.2
0
10
20
30
40
Lag

ARIMA model of residuals of linear model

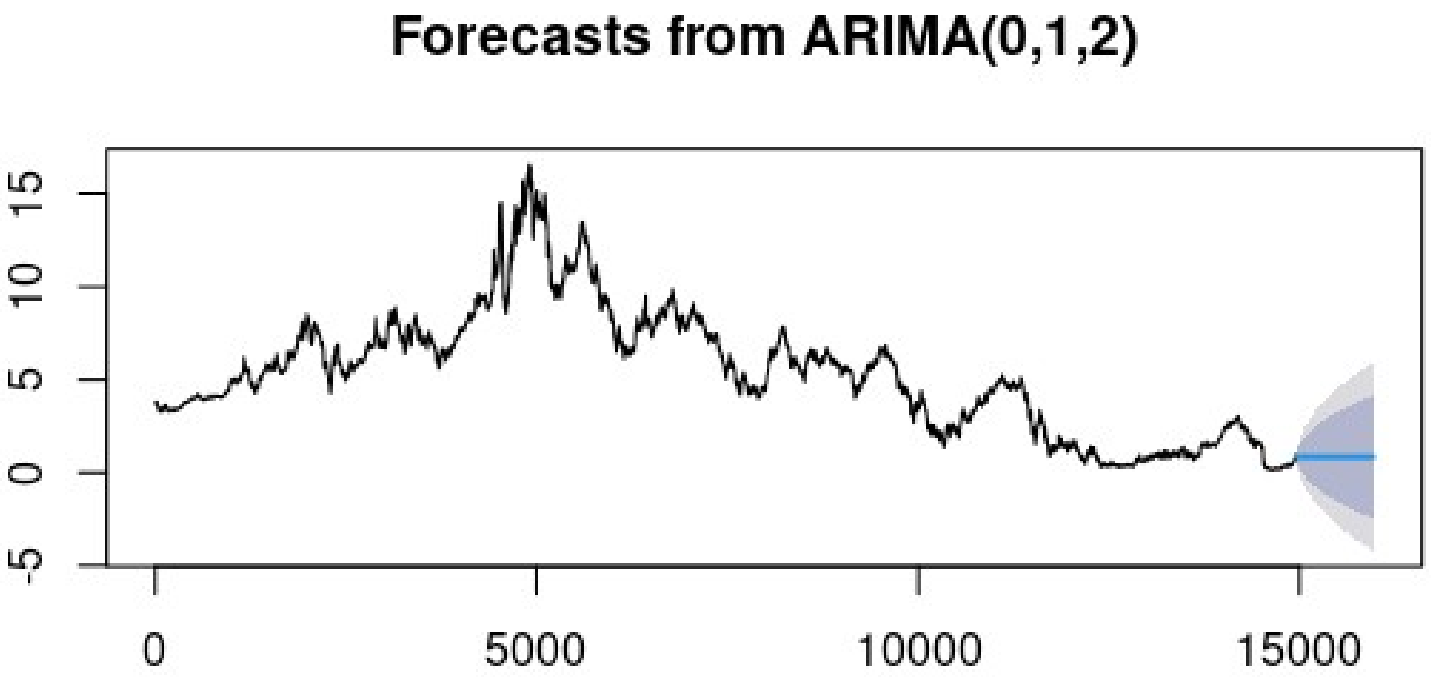
```
Series: lm.res
ARIMA(4,1,2)
Coefficients:
         ar1      ar2     ar3      ar4      ma1     ma2
      1.7519  -0.8349  0.0905  -0.1181  -1.8995  0.9652
s.e.  0.0084   0.0166  0.0165   0.0083   0.0023  0.0025
sigma^2 estimated as 0.009975:  log likelihood=13242.88
AIC=-26471.76   AICc=-26471.75   BIC=-26418.47
Training set error measures:
                      ME       RMSE        MAE       MPE     MAPE     MASE        ACF1
Training set 3.23819e-05 0.09984908 0.05811628 -1.637927 99.15122 1.081892 -0.01230773
```


ARIMA_rate
Rate
0
5
10
0
5000
10000
15000
Index
Real Data
Fitted Data


residuals(lm.res.aa)
-0.8
0
5000
10000
15000
ACF
0.10
-0.15
0
10
20
30
40
Lag
PACF
0.10
-0.15
0
10
20
30
40
Lag

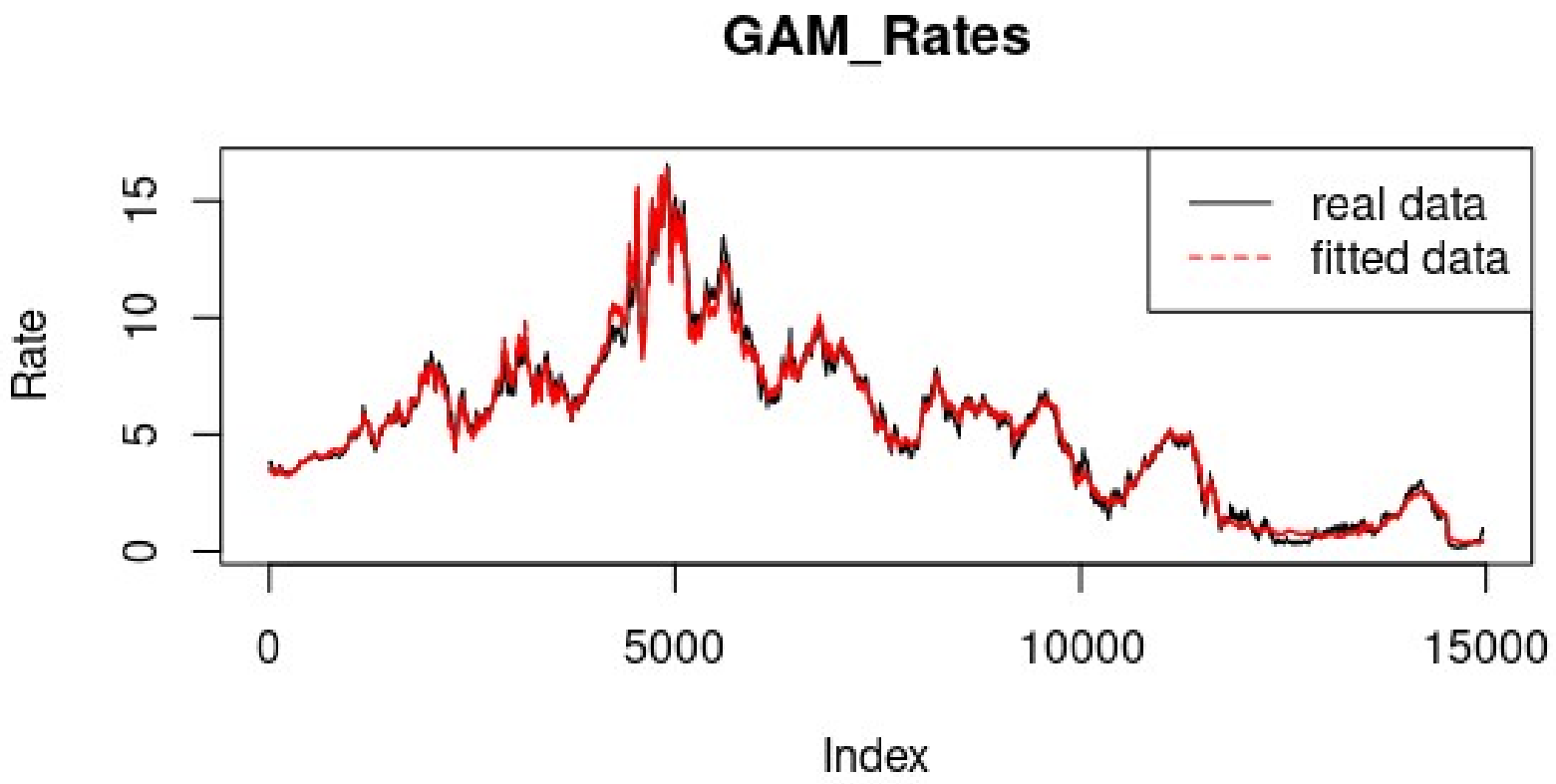
ARIMA model of 3-year maturity rate

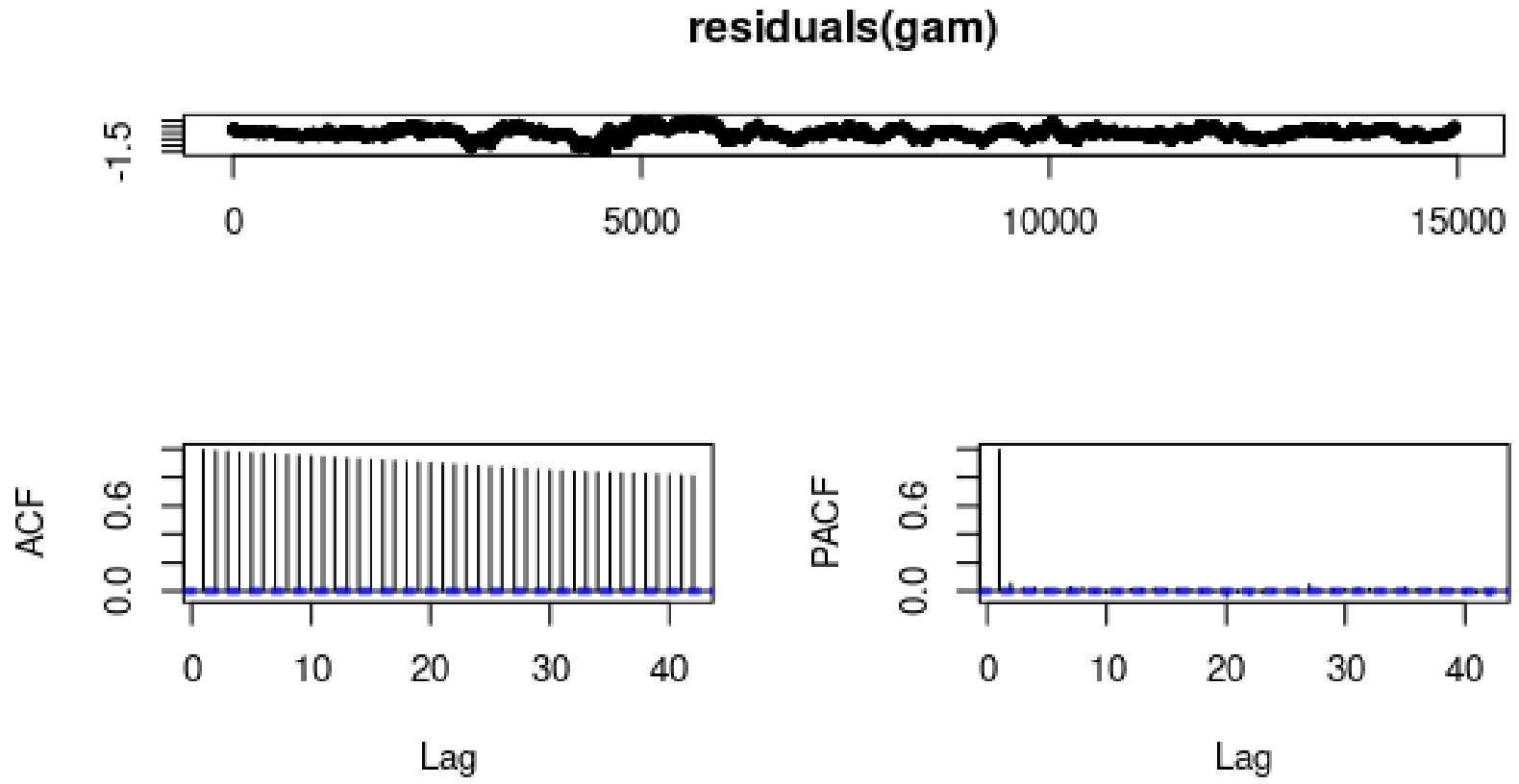
```
Series: data.rate$3yr_rate
ARIMA(0,1,2)
Coefficients:
         ma1     ma2
      0.0960  0.0201
s.e.  0.0082  0.0082
sigma^2 estimated as 0.005358:  log likelihood=17890.74
AIC=-35775.48   AICc=-35775.48   BIC=-35752.64
Training set error measures:
                       ME       RMSE        MAE         MPE     MAPE    MASE         ACF1
Training set -0.0001706578 0.07319432 0.04587743 -0.04117275 1.426316 1.00183 0.0001008334
```


Forecasts from ARIMA(0,1,2)
-5
0
5
10
15
0
5000
10000
15000

Generalized Additive Model

```
Call: gam(formula = data.rate$3yr_rate ~ s(data.rate$1yr_rate) +
    s(data.rate$series) + s(data.rate$month) + s(data.rate$year))
Deviance Residuals:
       Min         1Q     Median         3Q        Max
-1.6976411 -0.2052273  0.0004767  0.2256401  1.3661756
(Dispersion Parameter for gaussian family taken to be 0.1315)
    Null Deviance: 166440.5 on 14963 degrees of freedom
Residual Deviance: 1965.138 on 14947 degrees of freedom
AIC: 12123.8
Number of Local Scoring Iterations: NA
Anova for Parametric Effects
                         Df Sum Sq Mean Sq    F value    Pr(>F)
s(data.rate$1yr_rate)     1 135197  135197 1.0283e+06 < 2.2e-16 ***
s(data.rate$series)       1   1047    1047 7.9638e+03 < 2.2e-16 ***
s(data.rate$month)        1      5       5 3.7664e+01 8.616e-10 ***
s(data.rate$year)         1      2       2 1.5523e+01 8.187e-05 ***
Residuals             14947   1965       0
---

Anova for Nonparametric Effects
                      Npar Df  Npar F     Pr(F)
(Intercept)
s(data.rate$X1yr_rate)       3  215.72 < 2.2e-16 ***
s(data.rate$series)          3 1179.25 < 2.2e-16 ***
s(data.rate$month)           3   39.88 < 2.2e-16 ***
s(data.rate$year)            3 1146.69 < 2.2e-16 ***
---
```


GAM_Rates
real data
fitted data
Rate
0
5
10
15
0
5000
10000
15000
Index

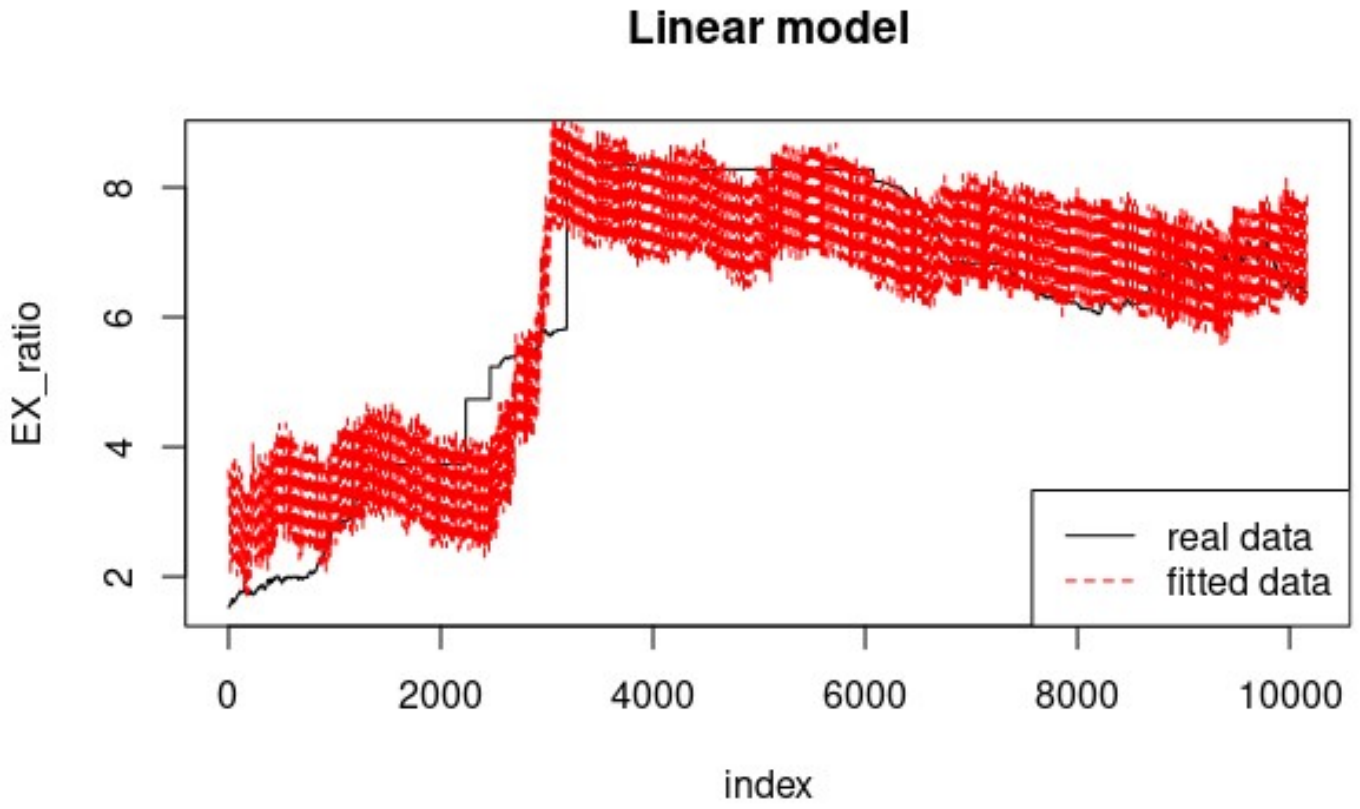

residuals(gam)
-1.5
0
5000
10000
15000
ACF
0.0
0.6
0
10
20
30
40
Lag
PACF
0.0
0.6
0
10
20
30
40
Lag

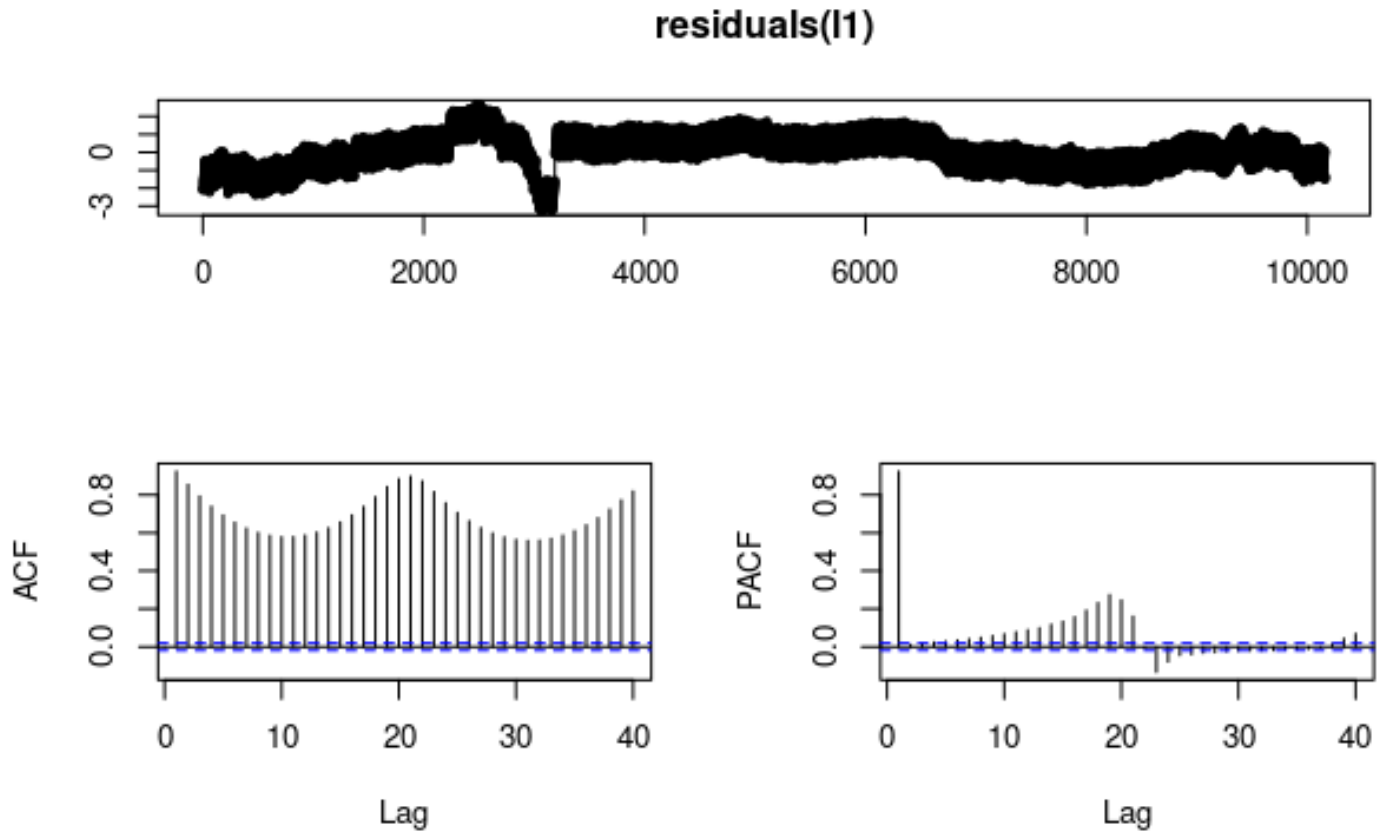
Yuan Exchange rate


scatter plot
Yuan_ratio
2
4
6
8
0
5
10
15
1yr_rate

Linear model of Yuan Exchange rate

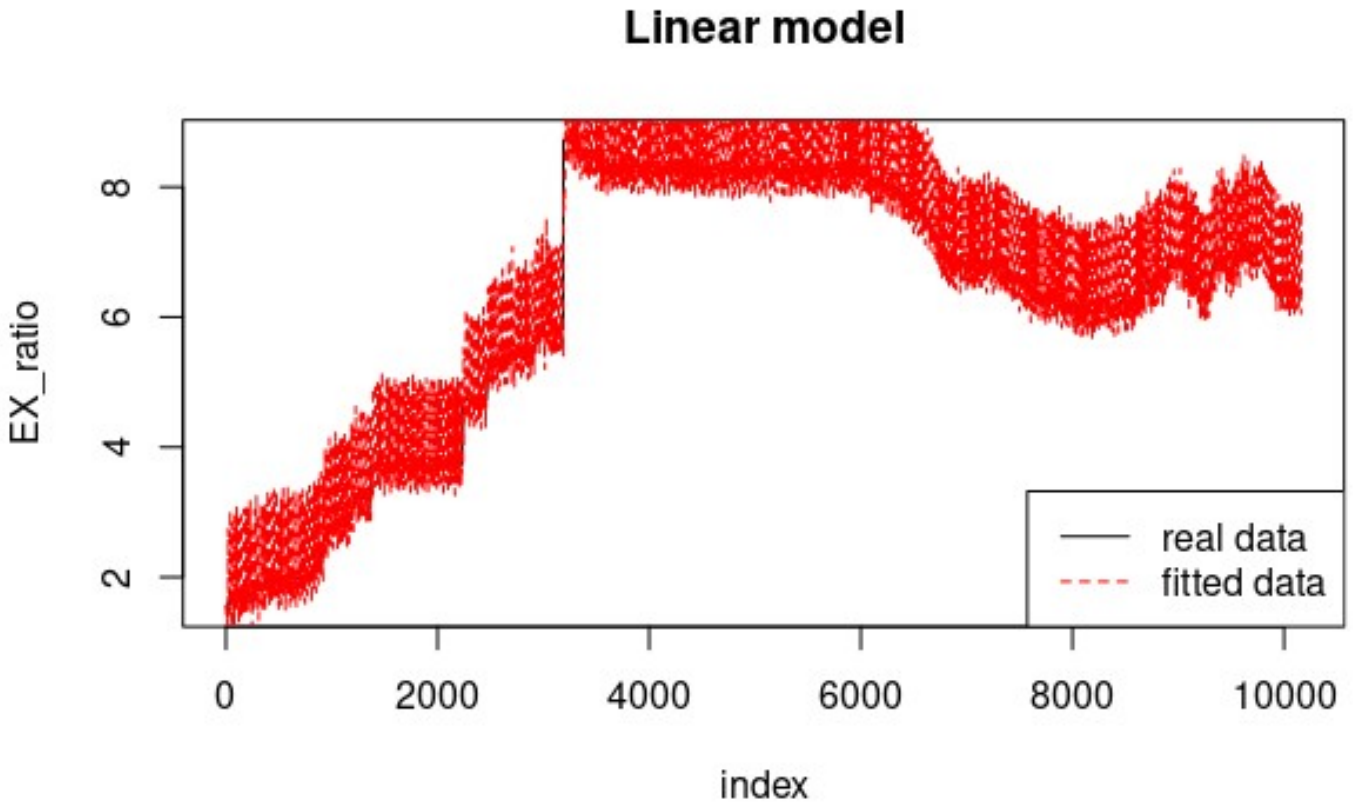
```
Call:
lm(formula = china$ex_ch ~ china$new_series + china$`1yr_rate` +
    china$month + china$year)
Residuals:
    Min      1Q  Median      3Q     Max
-3.2748 -0.6159  0.0348  0.6697  2.6719
Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)      -3.953e+04  2.748e+02 -143.85   <2e-16 ***
china$new_series -8.011e-02  5.574e-04 -143.71   <2e-16 ***
china$1yr_rate`  -1.081e-01  5.234e-03  -20.65   <2e-16 ***
china$month       1.674e+00  1.197e-02  139.84   <2e-16 ***
china$year        1.995e+01  1.387e-01  143.87   <2e-16 ***
---

Residual standard error: 0.9145 on 10157 degrees of freedom
Multiple R-squared:  0.8039,    Adjusted R-squared:  0.8038
F-statistic: 1.041e+04 on 4 and 10157 DF,  p-value: < 2.2e-16


AIC(lm_ch2)
[1] 38187.18
```

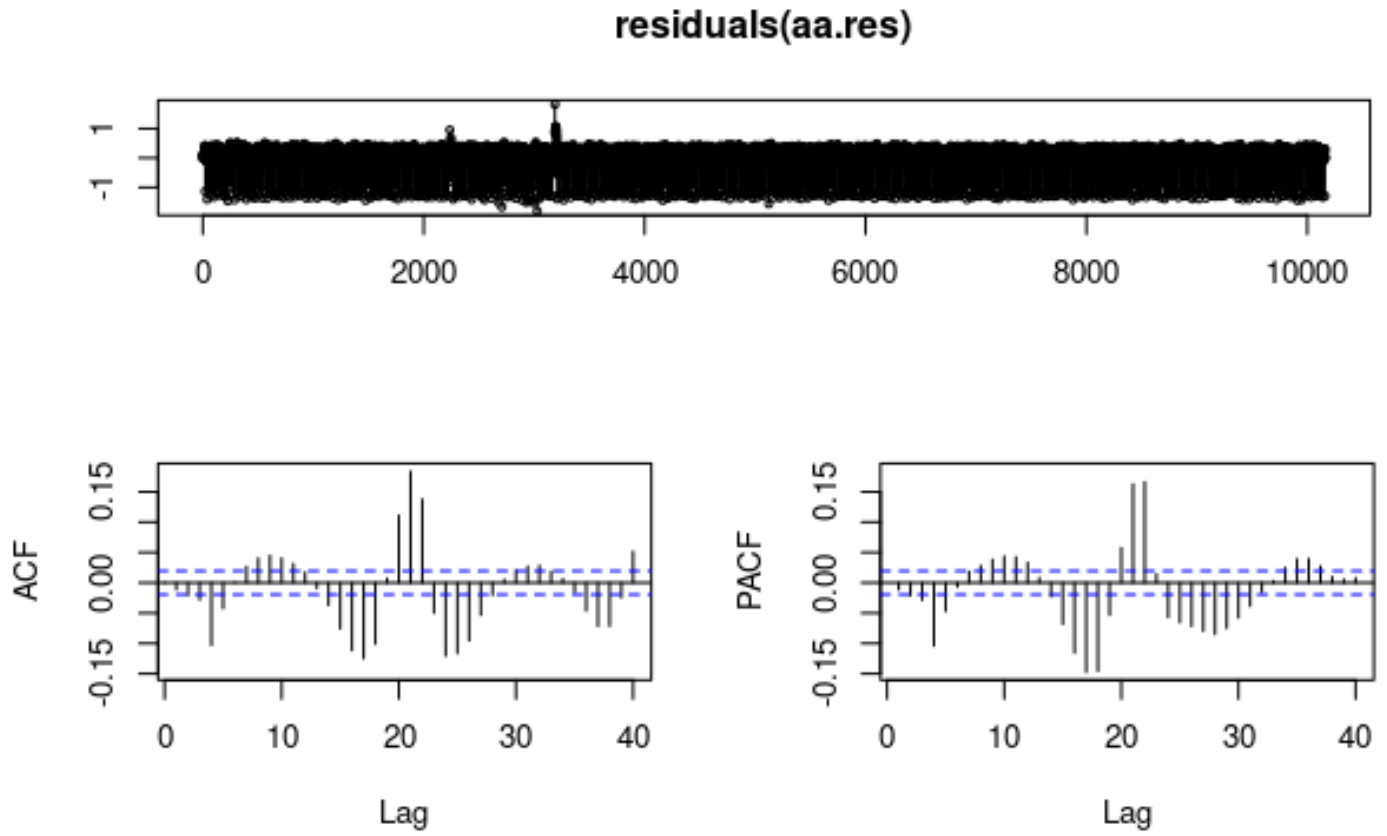

Linear model
EX_ratio
2
4
6
8
0
2000
4000
6000
8000
10000
index
real data
fitted data


residuals(l1)
-3
0
0
2000
4000
6000
8000
10000
ACF
0.0
0.4
0.8
0
10
20
30
40
Lag
PACF
0.0
0.4
0.8
0
10
20
30
40
Lag

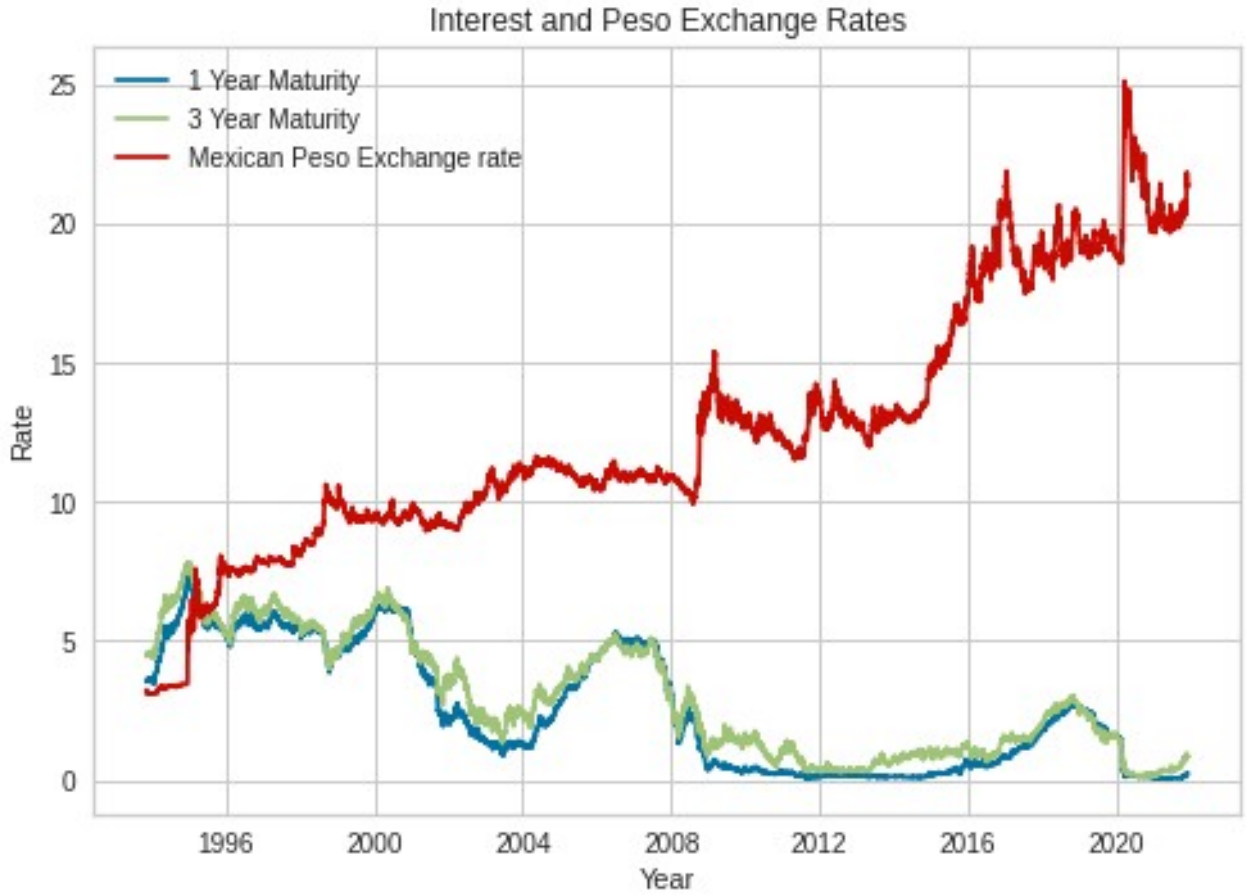
ARIMA model of residuals of linear model

```
Series: res
ARIMA(5,1,2)
Coefficients:
         ar1      ar2      ar3     ar4      ar5      ma1     ma2
      1.6309  -0.7102  -0.0083  0.0666  -0.1114  -1.8903  0.9191
s.e.  0.0107   0.0192   0.0202  0.0191   0.0104   0.0043  0.0045
sigma^2 estimated as 0.1022:  log likelihood=-2827.63
AIC=5671.26   AICc=5671.28   BIC=5729.07
Training set error measures:
                      ME      RMSE      MAE      MPE     MAPE     MASE        ACF1
Training set 0.0004817025 0.3195456 0.168004 -36.4293 206.9595 1.100659 -0.01118128
```


Linear model
EX_ratio
2
4
6
8
0
2000
4000
6000
8000
10000
index
real data
fitted data


residuals(aa.res)
-1
1
0
2000
4000
6000
8000
10000
ACF
-0.15
0.00
0.15
0
10
20
30
40
Lag
PACF
-0.15
0.00
0.15
0
10
20
30
40
Lag

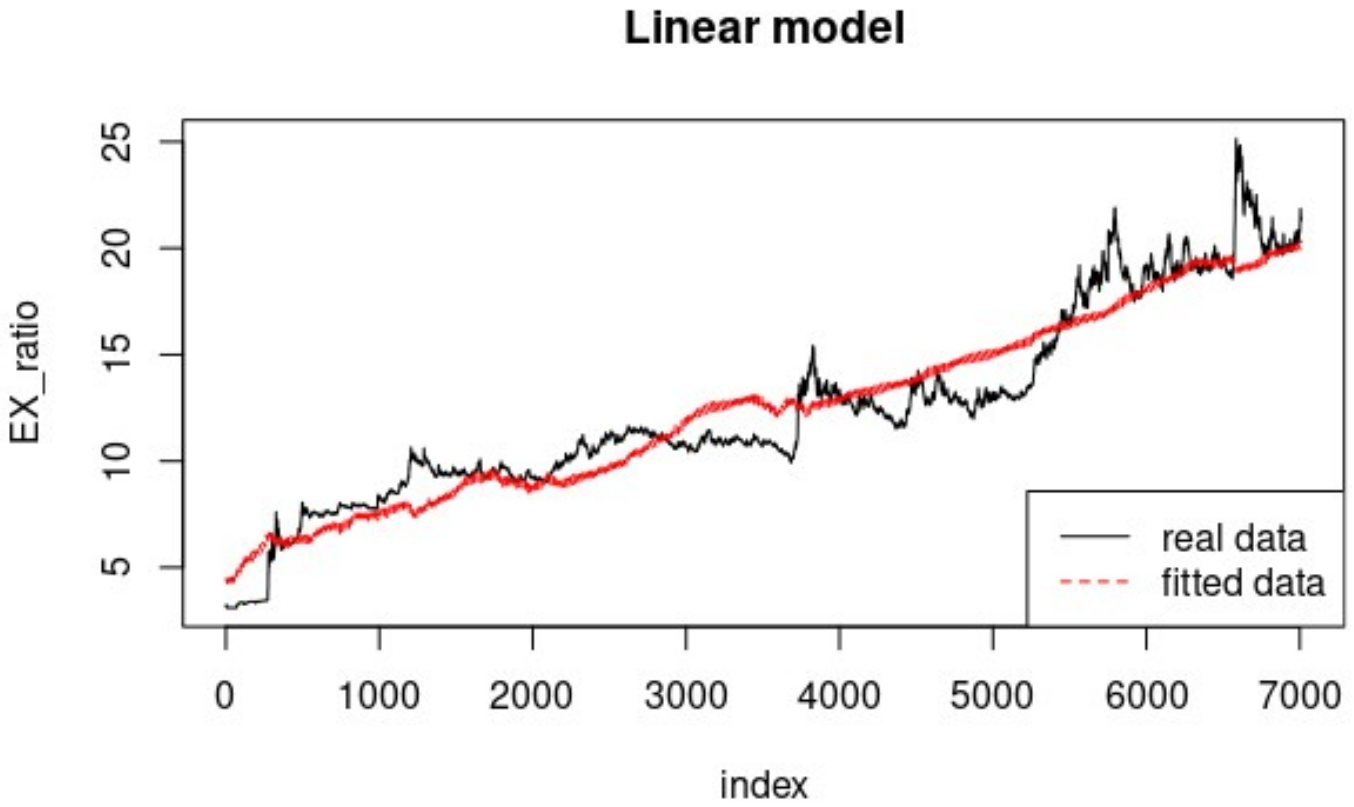
Exchange rate of Mexico

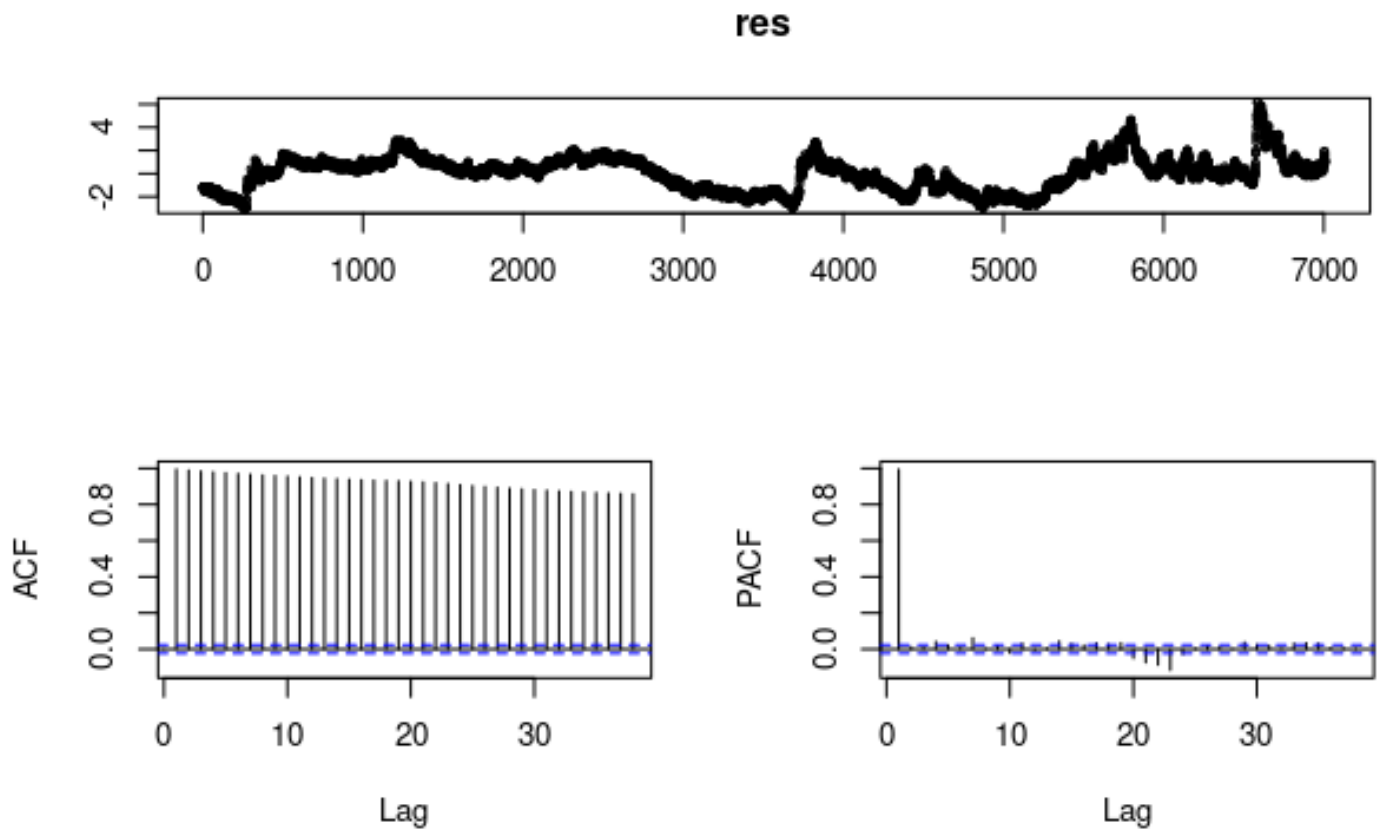



scatter plot
Peso_ratio
5
10
20
0
2
4
6
1yr_rate

Linear model of Peso

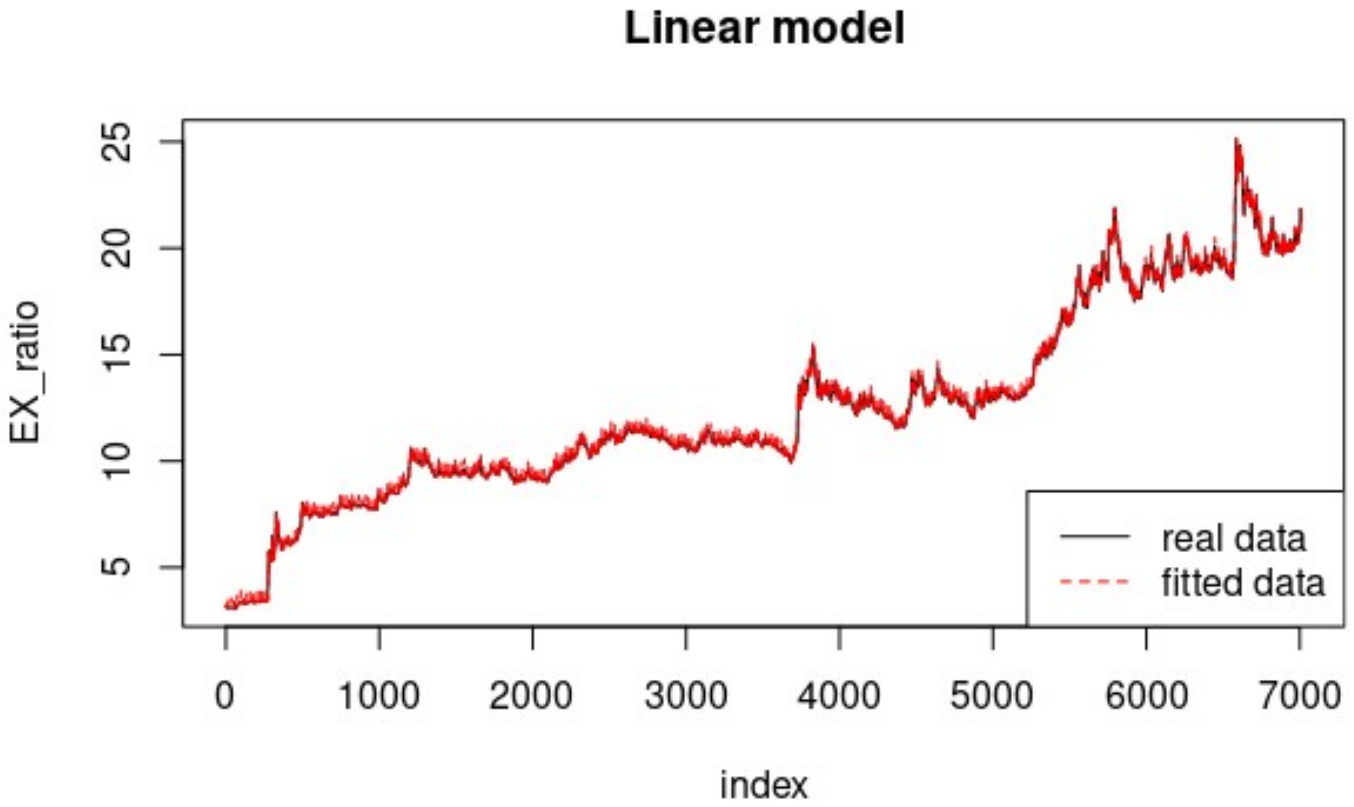
```
Call:
lm(formula = mexico$ex_mx ~ mexico$`1yr_rate` + mexico$new_series +
    mexico$month + mexico$year)
Residuals:
    Min      1Q  Median      3Q     Max
-3.0695 -1.2159  0.2157  0.9313  6.1658
Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)       -1.097e+04  1.341e+03  -8.185 3.20e-16 ***
mexico$`1yr_rate`  3.439e-01  1.260e-02  27.286  < 2e-16 ***
mexico$new_series -1.964e-02  2.691e-03  -7.298 3.24e-13 ***
mexico$month       4.589e-01  5.678e-02   8.082 7.47e-16 ***
mexico$year        5.505e+00  6.724e-01   8.188 3.14e-16 ***
---

Residual standard error: 1.431 on 7004 degrees of freedom
Multiple R-squared:  0.8987,    Adjusted R-squared:  0.8986
F-statistic: 1.553e+04 on 4 and 7004 DF,  p-value: < 2.2e-16


AIC(lm_mx2)
[1] 24975.46
```

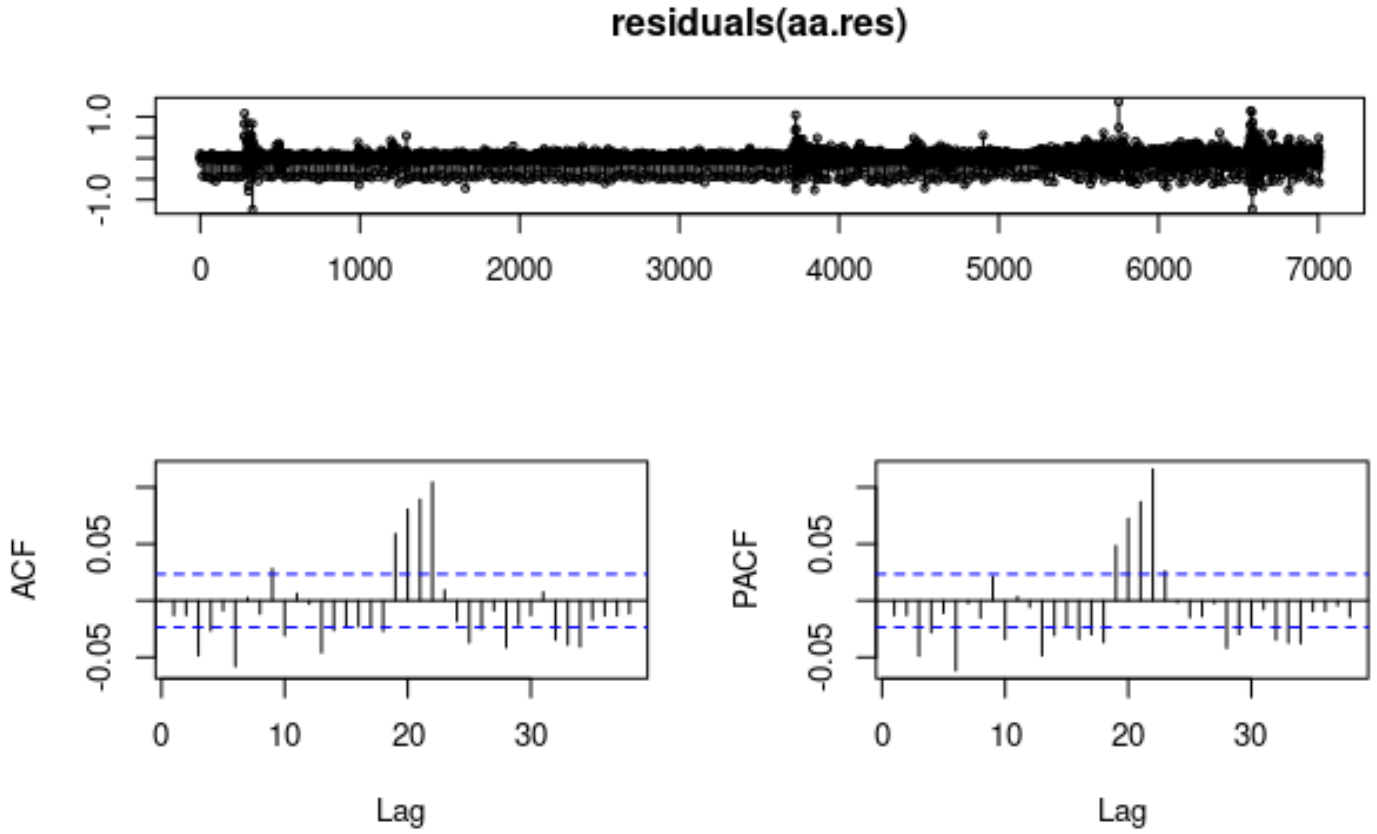

Linear model
EX_ratio
5
10
15
20
25
0
1000
2000
3000
4000
5000
6000
7000
index
real data
fitted data


res
-2
4
0
1000
2000
3000
4000
5000
6000
7000
ACF
0.0
0.4
0.8
0
10
20
30
Lag
PACF
0.0
0.4
0.8
0
10
20
30
Lag

ARIMA model of the residuals of linear model

```
Series: res
ARIMA(0,1,0)
sigma^2 estimated as 0.02231:  log likelihood=3380.83
AIC=-6759.66   AICc=-6759.66   BIC=-6752.8
Training set error measures:
                      ME      RMSE        MAE      MPE     MAPE      MASE        ACF1
Training set 0.0003479613 0.1493543 0.08739674 -45.3447 107.7707 0.9998595 -0.01287183
```


Linear model
EX_ratio
5
10
15
20
25
0
1000
2000
3000
4000
5000
6000
7000
index
real data
fitted data


residuals(aa.res)
1.0
-1.0
0
1000
2000
3000
4000
5000
6000
7000
ACF
0.05
-0.05
0
10
20
30
Lag
PACF
0.05
-0.05
0
10
20
30
Lag

Gradient Boosting Model

- https://colab.research.google.com/drive/1Mv6D8YDvoj13D-uT_XO8TdB5fSOfnvzp#scrollTo=N631jbcywGTH