Parameters Estimation of LFM Echoes Based on Relationship of Time Delay and Doppler Shift

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Abstract—**Linear frequency modulation (LFM) signal is widely used in pulse compression radars, whereas faced with the time delay bias caused by Doppler shift of radar echoes, which must be accounted for in some applicative occasions where the linear frequency modulation ratio is very low or Doppler shift is very large. The method of time delay estimation based on cross-ambiguity function (CAF) could solve the problem but need two-dimensional search around the entire time delay and Doppler shift plane, which increases computational complexity heavily and brings great challenge to real-time processing. An effective time delay estimation method is proposed in this paper based on the relationship of time delay and Doppler shift, which can reduce two-dimensional search to one-dimensional search along LFM signal energy line, and diminish the computation complexity greatly with the same performance of parameters estimation. Simulations proved the validity of the method proposed.**

Keywords- LFM signals; time delay and Doppler shift coupling; parameters estimation

I. Introduction

Pulse Compression, resolving the conflict between detection range and range resolution, is widely used by modern radars. Due to easily produced and high Doppler tolerance, linear frequency modulation (LFM) signal [1]-[3] is one of usually used signals in pulse compression radar. There exists time delay and Doppler coupling (TDC) [1][2][4]-[6] in LFM signals: Doppler shift will interfere with time delay estimation, and lead to time delay estimation bias, which is severe in occasions when LFM rate is very low or Doppler shift is very high, such as space debris detection.

Cross ambiguity function (CAF) [7]-[10] is an effect tool for time delay estimation. However, it needs two-dimensional (2-D) search in time delay and Doppler shift plane, which leads to a huge computation load and brings great challenge to real-time processing. Against this problem, the relationship of time delay and Doppler shift is induced in this paper, and two-dimensional search is simplified to one-dimensional search along a line in the time delay and Doppler shift plane. By this method, the computational complexity of signal processing is diminished to only twice correlation processing, and time delay estimation is avoided from Doppler shift search internal in CAF method.

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In this paper, signal model and the character of time delay and Doppler shift coupling are described in Section II, and then the conventional CAF method is described in Section III. In Section IV, the new parameters estimation method based on relationship of time delay and Doppler shift is proposed, and its performance analysis is given. Section V compares the computational complexities of the two methods. Section VI gives simulation results to prove the validity of the method proposed. Finally, the conclusion is given in Section VII.

II. SINGNAL MODEL

A. Pulse Compression

The LFM echo can be expressed as

$$s_r(t) = rect\left(\frac{t-\tau^0}{T}\right)\times \times\exp\left(j\pi K_r\left(t-\tau^0\right)^2 + j2\pi f_d^0 t\right) + w(t) \tag{1}$$

where T is pulse width, $\tau^0$and f_d^0 are the time delay and Doppler shift of the echo, K_r is the LFM rate, $w(t)$ is additive complex white Gaussian noise (WGN) with the variance σ^2. $rect(\bullet)$is the rectangle function with the expression

$$rect\left(\frac{t}{T}\right) = \begin{cases} 1 & 0 \le t \le T \\ 0 & \text{others} \end{cases}$$

Reference signal for pulse compression is expressed as

$$s_{ref}(t) = rect\left(\frac{t}{T}\right)\exp\left(j\pi K_r t^2\right) \tag{2}$$

And then, pulse compression can be expressed as

$$H_0(\tau) = \int_{-\infty}^{+\infty} s_r(t)\, s_{ref}(t-\tau)^* \, dt \tag{3}$$

Time delay estimation by pulse compression result is

$$\hat{\tau}^0 = \arg\max_{\tau}\left(E\left(\left\|H_0(\tau)\right\|^2\right)\right) \tag{4}$$

where $\max(\bullet)$ denotes maximum operation, $E(\bullet)$ denotes the expectation, $\|\bullet\|^2$ denotes modulus square operation, and * denotes the complex conjugation.

B. Time Delay and Doppler Shift Coupling

According to equations (1) and (2), we obtain

$$\begin{aligned}
& s_r(t)s_{ref}(t-\tau)^* \\
&= rect\left(\frac{t-\tau^0}{T}\right)rect\left(\frac{t-\tau}{T}\right)\times \\
&\times \exp\begin{pmatrix} j\pi K_r(t-\tau^0)^2 - j\pi K_r(t-\tau)^2 \\ +j2\pi f_d^0 t \end{pmatrix} + \hat{w}(t) \\
&= rect\left(\frac{t-\max(\tau,\tau^0)}{T-|\tau-\tau^0|}\right)\times \\
&\times \exp\left(j2\pi\left[K_r(\tau-\tau^0)+f_d^0\right]t + j\hat{\varphi}\right)+\hat{w}(t)
\end{aligned} \tag{5}$$

where

$$\hat{\varphi} = \pi K_r\left(\tau^{02}-\tau^2\right)$$

is a constant phase, $\hat{w}(t)$ is also complex white Gaussian noise, whose expression is

$$\hat{w}(t) = w(t)rect\left(\frac{t-\tau}{T}\right)\exp\left(j\pi K_r(t-\tau)^2\right)$$

Substitute (5) to (3), we obtain

$$\begin{aligned}
H_0(\tau) &= \int_{-\infty}^{+\infty} s_r(t)s_{ref}(t-\tau)^* dt \\
&= \int_{\max(\tau,\tau^0)}^{\min(\tau,\tau^0)+T} \exp\left(j2\pi\left[K_r(\tau-\tau^0)+f_d^0\right]t + j\hat{\varphi}\right)dt \\
&\quad + \int_{\tau}^{\tau+T} \hat{w}(t)dt
\end{aligned} \tag{6}$$

where $\min(\bullet)$ denotes minimum operation.

From equations (4) and (6), time delay estimation $\hat{\tau}^0$ can be solved as follows

$$\begin{aligned}
\hat{\tau}^0 &= \arg\max_{\tau}\left(E\left(\|H_0(\tau)\|^2\right)\right) \\
&= \left\{\tau \mid s.t.\ K_r(\tau-\tau^0)+f_d^0 = 0\right\} \\
&= \tau^0 - \frac{f_d^0}{K_r}
\end{aligned} \tag{7}$$

Equation (7) describes the time delay and Doppler shift coupling character of LFM signal: when there is relative movement between the target and the radar, Doppler shift f_d^0 is not equal to 0, and then it will interfere with time delay estimation result by the coefficient $-1/K_r$.

In practical radar echo processing, pulse compression in (3) is usually computed by Fast Fourier Transform (FFT) for computation economy, that is

$$H_0(\tau) = IFFT\left(FFT(s_r(t))\cdot FFT(s_{ref}(t))^*\right) \tag{8}$$

where $FFT(\bullet)$ and $IFFT(\bullet)$ denote Fast Fourier Transform and its invert transform, separately.

III. Prepare Your Paper Before Styling

From equations (6) and (7), the peak of $\|H_0(\tau)\|^2$ is deduced as follows:

$$\begin{aligned}
G_0 &= \max\left(\|H_0(\tau)\|^2\right) \\
&= \left\|\int_{-\infty}^{+\infty} s_r(t)s_{ref}\left(t-\tau^0-\frac{f_d^0}{K_r}\right)^* dt\right\|^2 \\
&= \left\|\left[T-\frac{|f_d^0|}{K_r}\right]\exp(j\hat{\varphi})+\tilde{w}\right\|^2
\end{aligned} \tag{9}$$

where

$$\tilde{w} = \int_{\tau}^{\tau+T}\hat{w}(t)dt$$

denotes complex Gaussian noise with variance $T\sigma^2$.

From equation (9), we know G_0 obeys χ_2 distribution as follows:

$$G_0 \sim \chi_2\left(T\sigma^2, \left(T-\frac{|f_d^0|}{K_r}\right)^2\right) \tag{10}$$

Therefore, the expectation of G_0 is

$$\mathrm{E}(G_0) = \left(T-\frac{|f_d^0|}{K_r}\right)^2 \tag{11}$$

According to the equation above, $\mathrm{E}(G_0)$descends with $|f_d^0|$ increases, so the existence of Doppler shift leads to the loss of pulse compression peak. Cross Ambiguous Function (CAF) method can diminish the loss and time delay estimation bias by compensating Doppler shift, the expression is written as follows:

$$\left(\hat{\tau}^1, \hat{f}_d^1\right) = \arg\max_{(\tau, f_d)}\left(\|H_1(\tau,f_d)\|^2\right) \tag{12}$$

where

$$H_1(\tau,f_d) = \int_{-\infty}^{+\infty} s_r(t)s_{ref}(t-\tau)^*\exp(-j2\pi f_d t)dt$$

Compare equations (3) and (12), we know CAF method can be viewed as two-dimension (2-D) correlation in time delay and Doppler shift. $\left\|H_1(\tau, f_d)\right\|^2$ reaches to the peak value when

$$\hat{\tau}^1 = \tau^0 \text{ and } \hat{f}_d^1 = f_d^0$$

with the peak value

$$G_1 = \max\left(\left\|H_1(\tau, f_d)\right\|^2\right) = \left\|T\exp(j\hat{\varphi}) + \tilde{w}\right\|^2 \qquad (13)$$

Similar to G_0, G_1 obeys χ_2 distribution as follows:

$$G_1 \sim \chi_2\left(T\sigma^2, T\right) \qquad (14)$$

The expectation of G_1 is T^2, which reaches to the peak in equation (11).

In practical radar echo processing, equation (14) is realized by multi-channel Doppler Bank, Doppler shift search internal between channels is chosen according to required parameter estimation precision.

IV. TIME DELAY AND DOPPLER SHIFT COUPLING METHED

A. Computational Algorithm

As discussed above, the CAF method can be viewed as a two-dimension (2-D) search by time delay and Doppler shift, and the search trace fills the entire time delay and Doppler shift plane, as shown in Figure 1 (dispersed scatters). Therefore, numbers of searches lead to heavy computation load, which brings great problem to real-time processing.

Note that the energy of the LFM echo focuses on an oblique line with the slope K_r. Thus, it is an equivalent way to search the correlation peak along the LFM signal energy line, which avoid complex two-dimensional search.

The expression of the relationship of time delay and Doppler shift of the LFM echo energy line is

$$f_d = K_r\left(\tau - \hat{\tau}^0\right) \qquad (15)$$

Substitute (15) to (12), we obtain

$$\hat{\tau}^1 = \arg\max_{\tau}\left(\left\|H_2(\tau)\right\|^2\right) \qquad (16)$$

where

$$H_2(\tau) = \int_{-\infty}^{+\infty} s_r(t)\, s_{ref}(t-\tau)^* \exp\left(-j2\pi K_r\left(\tau - \hat{\tau}^0\right)t\right)dt$$

According to (16), we can reduce a two-dimensional (2-D) search around the entire time delay and Doppler shift plane to a one-dimensional (1-D) search along the LFM signal energy line, as shown in Figure 1 (oblique line). By this method, computation complexity can be diminished greatly.

Equation (16) can be further deduced as follows:

$$s_r(t)\, s_{ref}(t-\tau)^* \exp\left(-j2\pi K_r\left(\tau - \hat{\tau}^0\right)t\right)$$

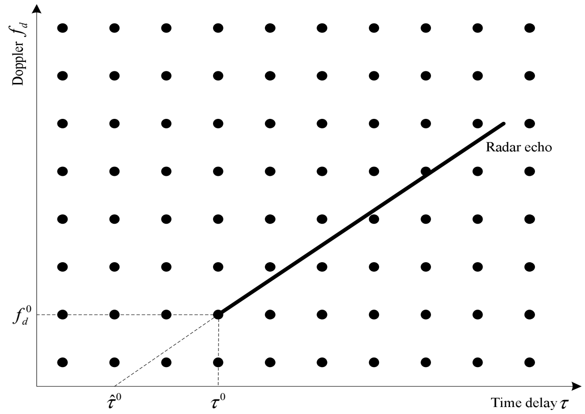


Figure 1. Time delay and Doppler shift search traces

$$= s_r(t)\, rect\left(\frac{t-\tau}{T}\right)\exp\left(-j\pi K_r\left(t^2 - 2\hat{\tau}^0 t + \tau^2\right)\right)$$

$$= s_r(t)\exp\left(-j\pi K_r\left(t - \hat{\tau}^0\right)^2\right) rect\left(\frac{t-\tau}{T}\right)\exp\left(j\tilde{\varphi}\right) \qquad (17)$$

where $\tilde{\varphi} = -\pi K_r\left(\tau^2 - \hat{\tau}^{02}\right)$ is a constant phase.

Substitute (17) to (16), we can obtain

$$\hat{\tau}^1 = \arg\max_{\tau}\left(\left\|H_2(\tau)\right\|^2\right)$$

$$= \arg\max_{\tau}\left(\left\|\int_{-\infty}^{+\infty} R(t)\, rect\left(\frac{t-\tau}{T}\right)\exp\left(j\tilde{\varphi}\right)dt\right\|^2\right)$$

$$= \arg\max_{\tau}\left(\left\|\int_{-\infty}^{+\infty} R(t)\, rect\left(\frac{t-\tau}{T}\right)dt\right\|^2\right) \qquad (18)$$

where

$$R(t) = s_r(t)\exp\left(-j\pi K_r\left(t - \hat{\tau}^0\right)^2\right)$$

Compare equations (18) and (3), we obtain that equation (18) can be realized by FFT, that is

$$H_2(\tau) = \int_{-\infty}^{+\infty} R(t)\, rect\left(\frac{t-\tau}{T}\right)dt$$

$$= IFFT\left(FFT\left(R(t)\right)\cdot \sin c(Tf)\exp\left(j\pi Tf\right)\right) \qquad (19)$$

where

$$\sin c(t) = \frac{\sin(\pi t)}{\pi t}$$

The time delay and Doppler shift coupling (TDC) method is summarized as follows:

Step 1: Estimate time delay initial value $\hat{\tau}^0$ by equations (4) and (8);

Step 2: Construct function $R(t)$ by $\hat{\tau}^0$ and equation (18);

Step 3: Estimate time delay improved value $\hat{\tau}^1$ by equations (16) and (19);

Step 4: Estimate Doppler shift by equation (15).

B. *Performance Analysis*

It is proved that the CAF method is a maximum likelihood (ML) estimator [11], whose performance can be described by the Cramer-Rao lower bound (CRLB), which is given by [12] as follows:

$$\mathrm{var}(\hat{\tau}) = \frac{1}{D_0}\frac{\delta^2}{\beta^2\delta^2 - \alpha^2} \tag{20}$$

$$\mathrm{var}(\hat{f}_d) = \frac{1}{D_0}\frac{\beta^2}{\beta^2\delta^2 - \alpha^2} \tag{21}$$

where

$$D_0 = E/2N_0$$

$$\alpha = \overline{\omega t} - \overline{\omega}\,\overline{t}$$

$$\beta = \overline{\omega^2} - \overline{\omega}^2$$

$$\delta^2 = \overline{t^2} - \overline{t}^2$$

$$E = \int_{-\infty}^{\infty} |s(t)|^2 dt$$

$$\overline{t} = \frac{1}{E}\int_{-\infty}^{\infty} t|s(t)|^2 dt$$

$$\overline{t^2} = \frac{1}{E}\int_{-\infty}^{\infty} t^2 |s(t)|^2 dt$$

$$\overline{\omega} = \frac{1}{E}\mathrm{Im}\int_{-\infty}^{\infty} s^*(t)\dot{s}(t)\,dt$$

$$\overline{\omega^2} = \frac{1}{E}\int_{-\infty}^{\infty} |\dot{s}(t)|^2 dt$$

$$\overline{\omega t} = \frac{1}{E}\mathrm{Im}\int_{-\infty}^{\infty} ts^*(t)\dot{s}(t)\,dt$$

As discussed above, it can be seen that the principle of the TDC method is the same as that of the CAF method, but restricts the search zone around the energy line. The primary possible values of the parameters can be also searched by the TDC method, that is, the two methods have different search efficiencies but the same accuracies. So the accuracy of parameter estimation by the TDC method can also achieve the CRLB in equations (20) and (21).

It is stressed that the step distance of search is also an important factor to influence the performances of the two methods. The CRLB can only describe the estimation accuracies regardless of computational complexities of the two methods. In the case of the same computational complexities, the TDC method has a little better performance than the CAF method, statistically, for its nearer search steps, which is proved in section VI.

V. COMPUTATIONAL COMPLEXITIES COMPARISON

In this section, we discuss the computational complexities of the CAF method and the TDC methods described above.

We use the number of the complex multiplications (NCM) as a figure of computation complexity merit. To reduce computation complexity, the frequency spectrum of reference signal in (2) can be pre-computed and stored. Thus, the NCM needed in the CAF method is expressed:

$$L_1 = 2MN\left[\log_2(N) + 1\right] \tag{22}$$

where M denotes the number of Doppler shift searches, N denotes the number of points in signal processing.

As described in Section IV, the TDC method contains four steps, whose NCM needed in each step is given in Table I, relatively, and the total NCM needed in the TDC method is

$$L_2 = N\left[4\log_2(N) + 3\right] \tag{23}$$

Compare the computation complexities of two methods:

$$\frac{L_1}{L_2} = \frac{2MN\left[\log_2(N)+1\right]}{N\left[4\log_2(N)+3\right]} \approx \frac{M}{2} \tag{24}$$

M depends on the need of time delay estimation precision. As shown in equation (7), time delay estimation bias coursed by Doppler shift bias is written as

$$\Delta\tau = \frac{\Delta f_d}{K_r} \tag{25}$$

where Δf_d denotes Doppler shift remains after Doppler shift compensating in CAF method, and it associates with Doppler shift search internal.

If the need of time delay estimation precision is $\Delta\tau_{precision}$, and then the maximum Doppler shift search internal is $K_r\Delta\tau_{precision}$, so the minimum number of Doppler shift search become

$$M_{\min} = \frac{f_{d\max} - f_{d\min}}{K_r\Delta\tau_{precision}} \tag{26}$$

where $f_{d\max}$ and $f_{d\min}$ denote the upper and lower bounds of Doppler shift search range, respectively.

Assume that Doppler shift range is -20KHz ~ +20KHz, the need of time delay estimation precision $\Delta\tau_{precision}$=100 ns, radar pulse width is 500 us, bandwidth is 5 MHz, then the minimum number of Doppler shift search $M_{\min}$=100. That is, the computational complexity of the TDC method is only 1/50 of that of CAF method.

TABLE I. NCM IN EACH STEP OF THE TDC METHOD

Step	NCM
1	$2N\log_2(N)+N$
2	N
3	$2N\log_2(N)+N$
4	--

Therefore, the TDC method has much lower computational complexity than the CAF method.

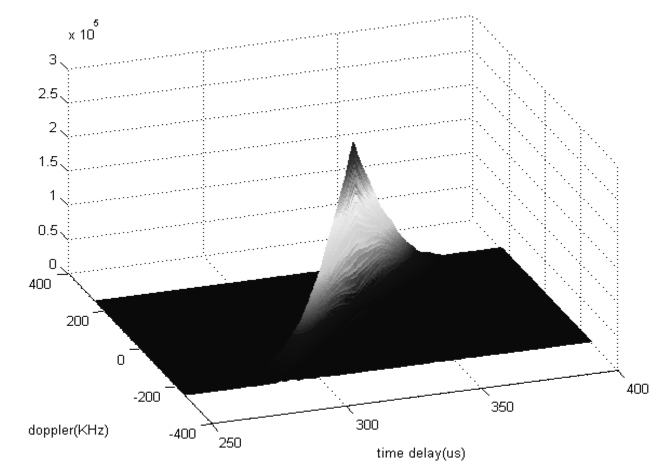
VI. SIMULATION RESULTS

In this section, simulation results are presented to demonstrate the performance of the proposed TDC method. Simulation parameters are listed in Table II.

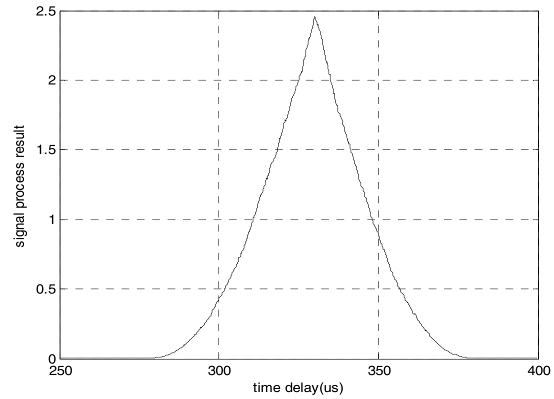
Simulation results of correlation by the CAF method and the TDC method are illustrated in Figure2 (a) and (b), respectively. It can be seen that $\left\|H_2(\tau)\right\|^2$ by the TDC method is the "knife-edge" of the correlation result $\left\|H_1(\tau, f_d)\right\|^2$ by the CAF method, which adequately embodies the idea of this paper: concentrate the two-dimensional (2-D) search around the entire time delay and Doppler shift plane into one-dimensional (1-D) search along the LFM signal energy line (the "knife-edge" in Figure 2 (a)).

TABLE II. SIMULATION PARAMETERS

Item	Value
RF Frequency	2.1GHz
Pulse width	500us
Bandwidth	5.0MHz
Signal to noise ratio	5.0dB



(a)



(b)

Figure 2. Correlation results (a) by CAF method; (b) by the proposed TDC method.

In order to compare the performances of the two methods, root mean square errors (RMSE) of time delay estimation under the same computational complexities, are simulated versus signal to noise ratio (SNR) in Figure 3. It can be seen that the RMSEs of the two methods descend with the SNRs increasing, and the TDC method has a little better performance than CAF method, for it has nearer search steps.

VII. CONLUSION

In this paper, a LFM echo parameter estimation method is proposed based on the relationship of time delay and Doppler shift. By this method, complex two-dimensional (2-D) search in CAF method is restricted to one-dimensional (1-D) search along LFM signal energy line. Because the processing is reduced to only twice correlation operations, the proposed TDC method has far lower computational complexity than that of the CAF method, without influencing the accuracies of parameters estimation. On the other hand, under the same computational complexity, the TDC method has a litter better estimation performance for its smaller step distance, which is proved by simulation results.

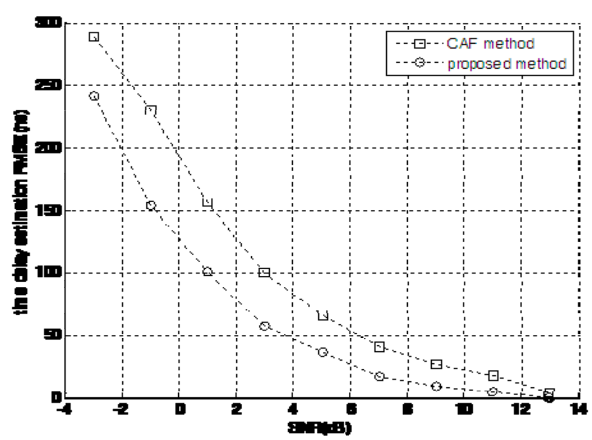


Figure 3. Time delay estimation MSE vesus SNR

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