Building your Recurrent Neural Network - Step by Step

Welcome to Course 5's first assignment! In this assignment, you will implement your first Recurrent Neural Network in numpy.

Recurrent Neural Networks (RNN) are very effective for Natural Language Processing and other sequence tasks because they have "memory". They can read inputs $x^{\langle t \rangle}$ (such as words) one at a time, and remember some information/context through the hidden layer activations that get passed from one time-step to the next. This allows a uni-directional RNN to take information from the past to process later inputs. A bidirection RNN can take context from both the past and the future.

Notation:

- Superscript $[l]$ denotes an object associated with the l^{th} layer.
  - Example: $a^{[4]}$ is the 4^{th} layer activation. $W^{[5]}$ and $b^{[5]}$ are the 5^{th} layer parameters.
- Superscript (i) denotes an object associated with the i^{th} example.
  - Example: $x^{(i)}$ is the i^{th} training example input.
- Superscript $\langle t \rangle$ denotes an object at the t^{th} time-step.
  - Example: $x^{\langle t \rangle}$ is the input x at the t^{th} time-step. $x^{(i)\langle t \rangle}$ is the input at the t^{th} timestep of example i.
- Lowerscript i denotes the i^{th} entry of a vector.
  - Example: $a^{[l]}_i$ denotes the i^{th} entry of the activations in layer l.

We assume that you are already familiar with `numpy` and/or have completed the previous courses of the specialization. Let's get started!

Let's first import all the packages that you will need during this assignment.

In [1]:
```
import numpy as np
from rnn_utils import *
```

1 - Forward propagation for the basic Recurrent Neural Network

Later this week, you will generate music using an RNN. The basic RNN that you will implement has the structure below. In this example, $T_x = T_y$.

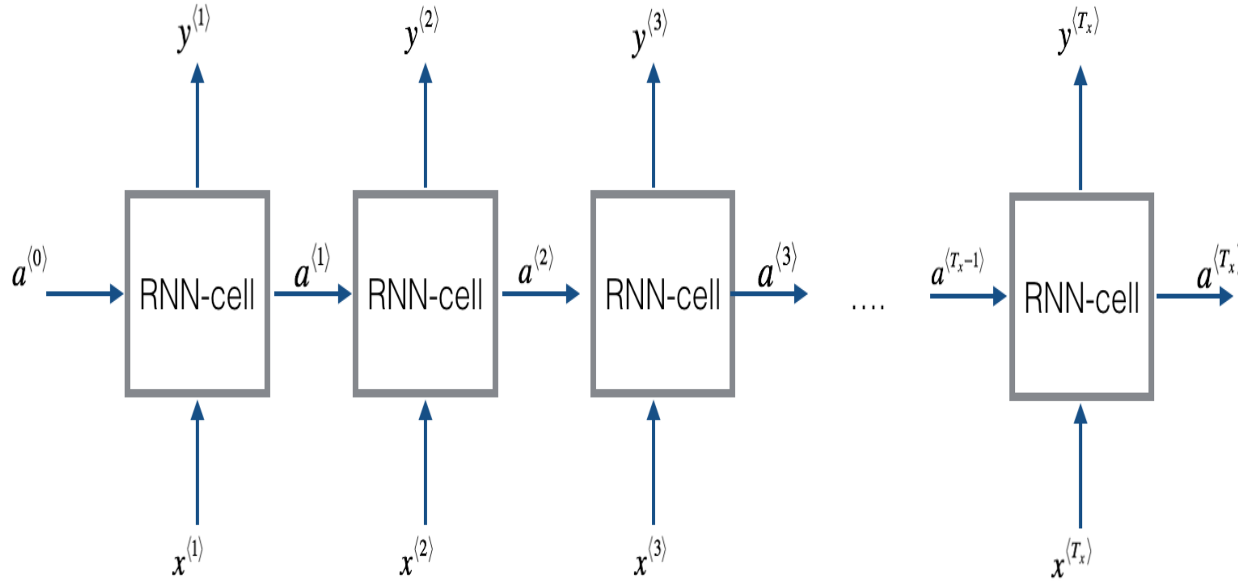


Figure 1: Basic RNN model

Here's how you can implement an RNN:

Steps:

1. Implement the calculations needed for one time-step of the RNN.
2. Implement a loop over T_x time-steps in order to process all the inputs, one at a time.

Let's go!

1.1 - RNN cell

A Recurrent neural network can be seen as the repetition of a single cell. You are first going to implement the computations for a single time-step. The following figure describes the operations for a single time-step of an RNN cell.

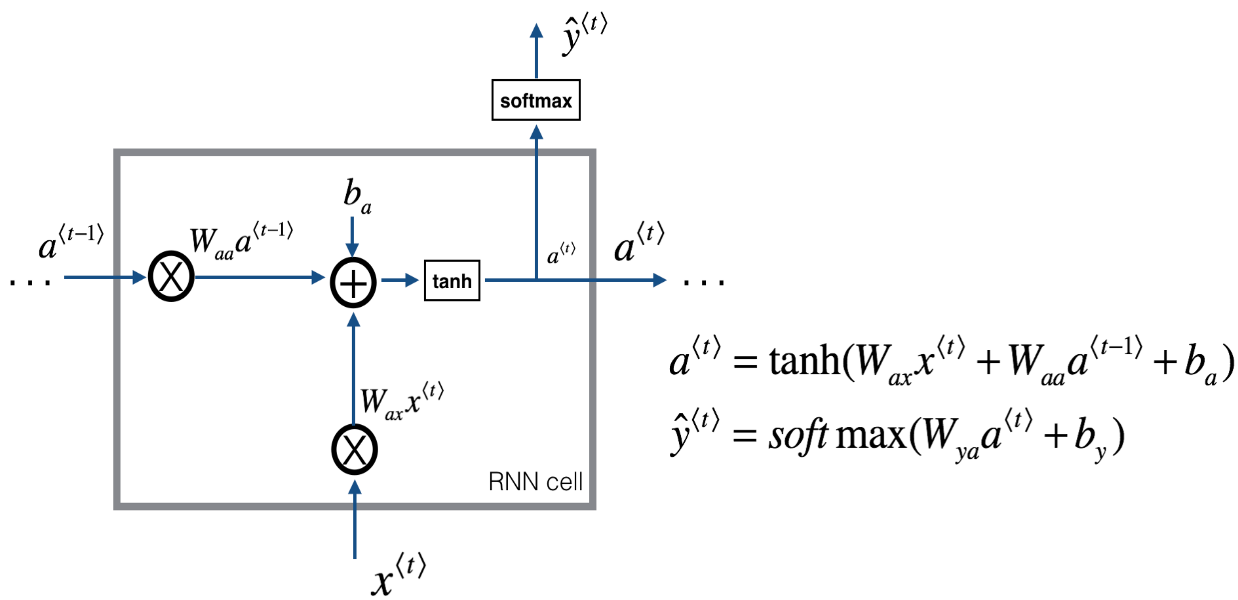


Figure 2: Basic RNN cell. Takes as input $x^{\langle t \rangle}$ (current input) and $a^{\langle t-1 \rangle}$ (previous hidden state containing information from the past), and outputs $a^{\langle t \rangle}$ which is given to the next RNN cell and also

used to predict $y^{\langle t \rangle}$

Exercise: Implement the RNN-cell described in Figure (2).

Instructions:

1. Compute the hidden state with tanh activation: $a^{\langle t \rangle} = \tanh(W_{aa} a^{\langle t-1 \rangle} + W_{ax} x^{\langle t \rangle} + b_a)$.
2. Using your new hidden state $a^{\langle t \rangle}$, compute the prediction $\hat{y}^{\langle t \rangle} = softmax(W_{ya} a^{\langle t \rangle} + b_y)$. We provided you a function: `softmax`.
3. Store $(a^{\langle t \rangle}, a^{\langle t-1 \rangle}, x^{\langle t \rangle}, parameters)$ in cache
4. Return $a^{\langle t \rangle}$, $y^{\langle t \rangle}$ and cache

We will vectorize over m examples. Thus, $x^{\langle t \rangle}$ will have dimension (n_x, m), and $a^{\langle t \rangle}$ will have dimension (n_a, m).

In [2]:
```
# GRADED FUNCTION: rnn_cell_forward

def rnn_cell_forward(xt, a_prev, parameters):
    """
    Implements a single forward step of the RNN-cell as described in Figure

    Arguments:
    xt -- your input data at timestep "t", numpy array of shape (n_x, m).
    a_prev -- Hidden state at timestep "t-1", numpy array of shape (n_a, m)
    parameters -- python dictionary containing:
                        Wax -- Weight matrix multiplying the input, numpy ar
                        Waa -- Weight matrix multiplying the hidden state, n
                        Wya -- Weight matrix relating the hidden-state to th
                        ba --  Bias, numpy array of shape (n_a, 1)
                        by -- Bias relating the hidden-state to the output,
    Returns:
    a_next -- next hidden state, of shape (n_a, m)
    yt_pred -- prediction at timestep "t", numpy array of shape (n_y, m)
    cache -- tuple of values needed for the backward pass, contains (a_next,
    """

    # Retrieve parameters from "parameters"
    Wax = parameters["Wax"]
    Waa = parameters["Waa"]
    Wya = parameters["Wya"]
    ba = parameters["ba"]
    by = parameters["by"]

    ### START CODE HERE ### (≈2 lines)
    # compute next activation state using the formula given above
    a_next = np.tanh(np.dot(Waa, a_prev) + np.dot(Wax, xt) + ba)
    # compute output of the current cell using the formula given above
    yt_pred = softmax(np.dot(Wya, a_next) + by)
    ### END CODE HERE ###

    # store values you need for backward propagation in cache
    cache = (a_next, a_prev, xt, parameters)

    return a_next, yt_pred, cache
```

In [3]:
```
np.random.seed(1)
xt = np.random.randn(3,10)
a_prev = np.random.randn(5,10)
Waa = np.random.randn(5,5)
Wax = np.random.randn(5,3)
Wya = np.random.randn(2,5)
ba = np.random.randn(5,1)
by = np.random.randn(2,1)
parameters = {"Waa": Waa, "Wax": Wax, "Wya": Wya, "ba": ba, "by": by}

a_next, yt_pred, cache = rnn_cell_forward(xt, a_prev, parameters)
print("a_next[4] = ", a_next[4])
print("a_next.shape = ", a_next.shape)
print("yt_pred[1] =", yt_pred[1])
print("yt_pred.shape = ", yt_pred.shape)
```

```
a_next[4] =  [ 0.59584544  0.18141802  0.61311866  0.99808218  0.85016201
  0.99980978
 -0.18887155  0.99815551  0.6531151   0.82872037]
a_next.shape =  (5, 10)
yt_pred[1] = [ 0.9888161   0.01682021  0.21140899  0.36817467  0.98988387
  0.88945212
  0.36920224  0.9966312   0.9982559   0.17746526]
yt_pred.shape =  (2, 10)
```

Expected Output:

a_next[4]:	[0.59584544 0.18141802 0.61311866 0.99808218 0.85016201 0.99980978 -0.18887155 0.99815551 0.6531151 0.82872037]
a_next.shape:	(5, 10)
yt[1]:	[0.9888161 0.01682021 0.21140899 0.36817467 0.98988387 0.88945212 0.36920224 0.9966312 0.9982559 0.17746526]
yt.shape:	(2, 10)

1.2 - RNN forward pass

You can see an RNN as the repetition of the cell you've just built. If your input sequence of data is carried over 10 time steps, then you will copy the RNN cell 10 times. Each cell takes as input the hidden state from the previous cell ($a^{\langle t-1 \rangle}$) and the current time-step's input data ($x^{\langle t \rangle}$). It outputs a hidden state ($a^{\langle t \rangle}$) and a prediction ($y^{\langle t \rangle}$) for this time-step.

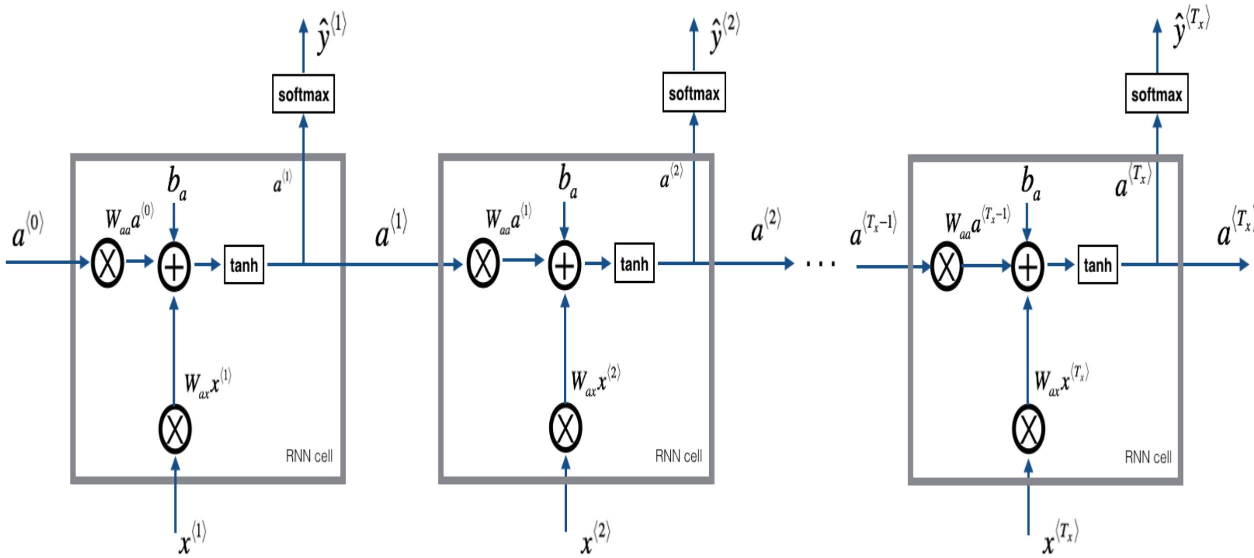


Figure 3: Basic RNN. The input sequence $x = (x^{\langle 1 \rangle}, x^{\langle 2 \rangle}, \ldots, x^{\langle T_x \rangle})$ is carried over T_x time steps. The network outputs $y = (y^{\langle 1 \rangle}, y^{\langle 2 \rangle}, \ldots, y^{\langle T_x \rangle})$.

Exercise: Code the forward propagation of the RNN described in Figure (3).

Instructions:

1. Create a vector of zeros (a) that will store all the hidden states computed by the RNN.
2. Initialize the "next" hidden state as a_0 (initial hidden state).
3. Start looping over each time step, your incremental index is t :
    - Update the "next" hidden state and the cache by running `rnn_cell_forward`
    - Store the "next" hidden state in a (t^{th} position)
    - Store the prediction in y
    - Add the cache to the list of caches
4. Return a, y and caches

In [10]:
```
# GRADED FUNCTION: rnn_forward

def rnn_forward(x, a0, parameters):
    """
    Implement the forward propagation of the recurrent neural network descri

    Arguments:
    x -- Input data for every time-step, of shape (n_x, m, T_x).
    a0 -- Initial hidden state, of shape (n_a, m)
    parameters -- python dictionary containing:
                        Waa -- Weight matrix multiplying the hidden state, r
                        Wax -- Weight matrix multiplying the input, numpy ar
                        Wya -- Weight matrix relating the hidden-state to th
                        ba --  Bias numpy array of shape (n_a, 1)
                        by -- Bias relating the hidden-state to the output,

    Returns:
    a -- Hidden states for every time-step, numpy array of shape (n_a, m, T_
    y_pred -- Predictions for every time-step, numpy array of shape (n_y, m,
    caches -- tuple of values needed for the backward pass, contains (list c
    """

    # Initialize "caches" which will contain the list of all caches
    caches = []

    # Retrieve dimensions from shapes of x and parameters["Wya"]
    n_x, m, T_x = x.shape
    n_y, n_a = parameters["Wya"].shape

    ### START CODE HERE ###

    # initialize "a" and "y" with zeros (≈2 lines)
    a = np.zeros((n_a, m, T_x))
    y_pred = np.zeros((n_y, m, T_x))

    # Initialize a_next (≈1 line)
    a_next = a0

    # loop over all time-steps
    for t in range(T_x):
        # Update next hidden state, compute the prediction, get the cache (≈
        a_next, yt_pred, cache = rnn_cell_forward(x[:,:,t], a_next, paramete
        # Save the value of the new "next" hidden state in a (≈1 line)
        a[:,:,t] = a_next
        # Save the value of the prediction in y (≈1 line)
        y_pred[:,:,t] = yt_pred
        # Append "cache" to "caches" (≈1 line)
        caches.append(cache)

    ### END CODE HERE ###

    # store values needed for backward propagation in cache
    caches = (caches, x)

    return a, y_pred, caches
```

In [11]:
```
np.random.seed(1)
x = np.random.randn(3,10,4)
a0 = np.random.randn(5,10)
Waa = np.random.randn(5,5)
Wax = np.random.randn(5,3)
Wya = np.random.randn(2,5)
ba = np.random.randn(5,1)
by = np.random.randn(2,1)
parameters = {"Waa": Waa, "Wax": Wax, "Wya": Wya, "ba": ba, "by": by}

a, y_pred, caches = rnn_forward(x, a0, parameters)
print("a[4][1] = ", a[4][1])
print("a.shape = ", a.shape)
print("y_pred[1][3] =", y_pred[1][3])
print("y_pred.shape = ", y_pred.shape)
print("caches[1][1][3] =", caches[1][1][3])
print("len(caches) = ", len(caches))
```

```
a[4][1] =  [-0.99999375  0.77911235 -0.99861469 -0.99833267]
a.shape =  (5, 10, 4)
y_pred[1][3] = [ 0.79560373  0.86224861  0.11118257  0.81515947]
y_pred.shape =  (2, 10, 4)
caches[1][1][3] = [-1.1425182  -0.34934272 -0.20889423  0.58662319]
len(caches) =  2
```

Expected Output:

a[4][1]:	[-0.99999375 0.77911235 -0.99861469 -0.99833267]
a.shape:	(5, 10, 4)
y[1][3]:	[0.79560373 0.86224861 0.11118257 0.81515947]
y.shape:	(2, 10, 4)
cache[1][1][3]:	[-1.1425182 -0.34934272 -0.20889423 0.58662319]
len(cache):	2

Congratulations! You've successfully built the forward propagation of a recurrent neural network from scratch. This will work well enough for some applications, but it suffers from vanishing gradient problems. So it works best when each output $y^{\langle t \rangle}$ can be estimated using mainly "local" context (meaning information from inputs $x^{\langle t' \rangle}$ where t' is not too far from t).

In the next part, you will build a more complex LSTM model, which is better at addressing vanishing gradients. The LSTM will be better able to remember a piece of information and keep it saved for many timesteps.

2 - Long Short-Term Memory (LSTM) network

This following figure shows the operations of an LSTM-cell.

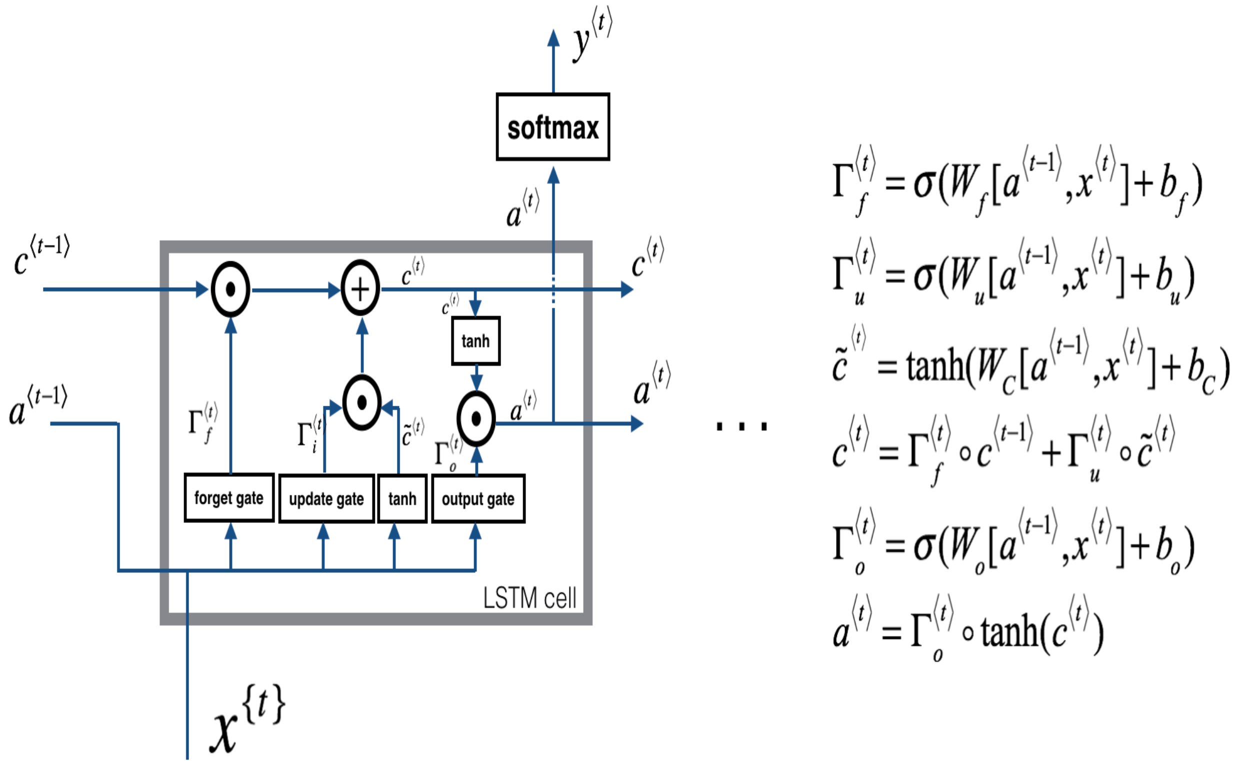


Figure 4: LSTM-cell. This tracks and updates a "cell state" or memory variable $c^{\langle t \rangle}$ at every time-step, which can be different from $a^{\langle t \rangle}$.

Similar to the RNN example above, you will start by implementing the LSTM cell for a single time-step. Then you can iteratively call it from inside a for-loop to have it process an input with T_x time-steps.

About the gates

- Forget gate

For the sake of this illustration, lets assume we are reading words in a piece of text, and want use an LSTM to keep track of grammatical structures, such as whether the subject is singular or plural. If the subject changes from a singular word to a plural word, we need to find a way to get rid of our previously stored memory value of the singular/plural state. In an LSTM, the forget gate lets us do this:

$$\Gamma_f^{\langle t \rangle} = \sigma(W_f[a^{\langle t-1 \rangle}, x^{\langle t \rangle}] + b_f) \tag{1}$$

Here, W_f are weights that govern the forget gate's behavior. We concatenate $[a^{\langle t-1 \rangle}, x^{\langle t \rangle}]$ and multiply by W_f. The equation above results in a vector $\Gamma_f^{\langle t \rangle}$ with values between 0 and 1. This forget gate vector will be multiplied element-wise by the previous cell state $c^{\langle t-1 \rangle}$. So if one of the values of $\Gamma_f^{\langle t \rangle}$ is 0 (or close to 0) then it means that the LSTM should remove that piece of information (e.g. the singular subject) in the corresponding component of $c^{\langle t-1 \rangle}$. If one of the values is 1, then it will keep the information.

- Update gate

Once we forget that the subject being discussed is singular, we need to find a way to update it to reflect that the new subject is now plural. Here is the formulat for the update gate:

$$\Gamma_u^{\langle t \rangle} = \sigma(W_u[a^{\langle t-1 \rangle}, x^{\{t\}}] + b_u) \tag{2}$$

Similar to the forget gate, here $\Gamma_u^{\langle t \rangle}$ is again a vector of values between 0 and 1. This will be multiplied element-wise with $\tilde{c}^{\langle t \rangle}$, in order to compute $c^{\langle t \rangle}$.

- Updating the cell

To update the new subject we need to create a new vector of numbers that we can add to our previous cell state. The equation we use is:

$$\tilde{c}^{\langle t \rangle} = \tanh(W_c[a^{\langle t-1 \rangle}, x^{\langle t \rangle}] + b_c) \tag{3}$$

Finally, the new cell state is:

$$c^{\langle t \rangle} = \Gamma_f^{\langle t \rangle} * c^{\langle t-1 \rangle} + \Gamma_u^{\langle t \rangle} * \tilde{c}^{\langle t \rangle} \tag{4}$$

- Output gate

To decide which outputs we will use, we will use the following two formulas:

$$\Gamma_o^{\langle t \rangle} = \sigma(W_o[a^{\langle t-1 \rangle}, x^{\langle t \rangle}] + b_o) \tag{5}$$
$$a^{\langle t \rangle} = \Gamma_o^{\langle t \rangle} * \tanh(c^{\langle t \rangle}) \tag{6}$$

Where in equation 5 you decide what to output using a sigmoid function and in equation 6 you multiply that by the $\tanh$ of the previous state.

2.1 - LSTM cell

Exercise: Implement the LSTM cell described in the Figure (3).

Instructions:

1. Concatenate $a^{\langle t-1 \rangle}$ and $x^{\langle t \rangle}$ in a single matrix: $concat = \begin{bmatrix} a^{\langle t-1 \rangle} \\ x^{\langle t \rangle} \end{bmatrix}$
2. Compute all the formulas 1-6. You can use `sigmoid()` (provided) and `np.tanh()`.
3. Compute the prediction $y^{\langle t \rangle}$. You can use `softmax()` (provided).

In [12]:
```
# GRADED FUNCTION: lstm_cell_forward

def lstm_cell_forward(xt, a_prev, c_prev, parameters):
    """
    Implement a single forward step of the LSTM-cell as described in Figure

    Arguments:
    xt -- your input data at timestep "t", numpy array of shape (n_x, m).
    a_prev -- Hidden state at timestep "t-1", numpy array of shape (n_a, m)
    c_prev -- Memory state at timestep "t-1", numpy array of shape (n_a, m)
    parameters -- python dictionary containing:
                        Wf -- Weight matrix of the forget gate, numpy array
                        bf -- Bias of the forget gate, numpy array of shape
                        Wi -- Weight matrix of the update gate, numpy array
                        bi -- Bias of the update gate, numpy array of shape
                        Wc -- Weight matrix of the first "tanh", numpy array
                        bc --  Bias of the first "tanh", numpy array of shap
                        Wo -- Weight matrix of the output gate, numpy array
                        bo --  Bias of the output gate, numpy array of shape
                        Wy -- Weight matrix relating the hidden-state to the
                        by -- Bias relating the hidden-state to the output,

    Returns:
    a_next -- next hidden state, of shape (n_a, m)
    c_next -- next memory state, of shape (n_a, m)
    yt_pred -- prediction at timestep "t", numpy array of shape (n_y, m)
    cache -- tuple of values needed for the backward pass, contains (a_next,

    Note: ft/it/ot stand for the forget/update/output gates, cct stands for
          c stands for the memory value
    """

    # Retrieve parameters from "parameters"
    Wf = parameters["Wf"]
    bf = parameters["bf"]
    Wi = parameters["Wi"]
    bi = parameters["bi"]
    Wc = parameters["Wc"]
    bc = parameters["bc"]
    Wo = parameters["Wo"]
    bo = parameters["bo"]
    Wy = parameters["Wy"]
    by = parameters["by"]

    # Retrieve dimensions from shapes of xt and Wy
    n_x, m = xt.shape
    n_y, n_a = Wy.shape

    ### START CODE HERE ###
    # Concatenate a_prev and xt (≈3 lines)
    concat = np.zeros((n_x + n_a, m))
    concat[: n_a, :] = a_prev
    concat[n_a:, :] = xt

    # Compute values for ft, it, cct, c_next, ot, a_next using the formulas
    ft = sigmoid(np.dot(Wf,concat)+bf)
```

```
    it = sigmoid(np.dot(Wi,concat)+bi)
    cct = np.tanh(np.dot(Wc,concat)+bc)
    c_next = ft*c_prev + it*cct
    ot = sigmoid(np.dot(Wo,concat)+bo)
    a_next = ot*np.tanh(c_next)

    # Compute prediction of the LSTM cell (≈1 line)
    yt_pred = softmax(np.dot(Wy, a_next) + by)
    ### END CODE HERE ###

    # store values needed for backward propagation in cache
    cache = (a_next, c_next, a_prev, c_prev, ft, it, cct, ot, xt, parameters

    return a_next, c_next, yt_pred, cache
```

In [13]:
```
np.random.seed(1)
xt = np.random.randn(3,10)
a_prev = np.random.randn(5,10)
c_prev = np.random.randn(5,10)
Wf = np.random.randn(5, 5+3)
bf = np.random.randn(5,1)
Wi = np.random.randn(5, 5+3)
bi = np.random.randn(5,1)
Wo = np.random.randn(5, 5+3)
bo = np.random.randn(5,1)
Wc = np.random.randn(5, 5+3)
bc = np.random.randn(5,1)
Wy = np.random.randn(2,5)
by = np.random.randn(2,1)

parameters = {"Wf": Wf, "Wi": Wi, "Wo": Wo, "Wc": Wc, "Wy": Wy, "bf": bf, "b

a_next, c_next, yt, cache = lstm_cell_forward(xt, a_prev, c_prev, parameters
print("a_next[4] = ", a_next[4])
print("a_next.shape = ", c_next.shape)
print("c_next[2] = ", c_next[2])
print("c_next.shape = ", c_next.shape)
print("yt[1] =", yt[1])
print("yt.shape = ", yt.shape)
print("cache[1][3] =", cache[1][3])
print("len(cache) = ", len(cache))
```

```
a_next[4] =  [-0.66408471  0.0036921   0.02088357  0.22834167 -0.85575339
  0.00138482
  0.76566531  0.34631421 -0.00215674  0.43827275]
a_next.shape =  (5, 10)
c_next[2] =  [ 0.63267805  1.00570849  0.35504474  0.20690913 -1.64566718
  0.11832942
  0.76449811 -0.0981561  -0.74348425 -0.26810932]
c_next.shape =  (5, 10)
yt[1] = [ 0.79913913  0.15986619  0.22412122  0.15606108  0.97057211  0.3
1146381
  0.00943007  0.12666353  0.39380172  0.07828381]
yt.shape =  (2, 10)
cache[1][3] = [-0.16263996  1.03729328  0.72938082 -0.54101719  0.0275207
4 -0.30821874
  0.07651101 -1.03752894  1.41219977 -0.37647422]
len(cache) =  10
```

Expected Output:

a_next[4]:	[-0.66408471 0.0036921 0.02088357 0.22834167 -0.85575339 0.00138482 0.76566531 0.34631421 -0.00215674 0.43827275]
a_next.shape:	(5, 10)
c_next[2]:	[0.63267805 1.00570849 0.35504474 0.20690913 -1.64566718 0.11832942 0.76449811 -0.0981561 -0.74348425 -0.26810932]
c_next.shape:	(5, 10)
yt[1]:	[0.79913913 0.15986619 0.22412122 0.15606108 0.97057211 0.31146381 0.00943007 0.12666353 0.39380172 0.07828381]

yt.shape:	(2, 10)
cache[1][3]:	[-0.16263996 1.03729328 0.72938082 -0.54101719 0.02752074 -0.30821874 0.07651101 -1.03752894 1.41219977 -0.37647422]
len(cache):	10

2.2 - Forward pass for LSTM

Now that you have implemented one step of an LSTM, you can now iterate this over this using a for-loop to process a sequence of T_x inputs.

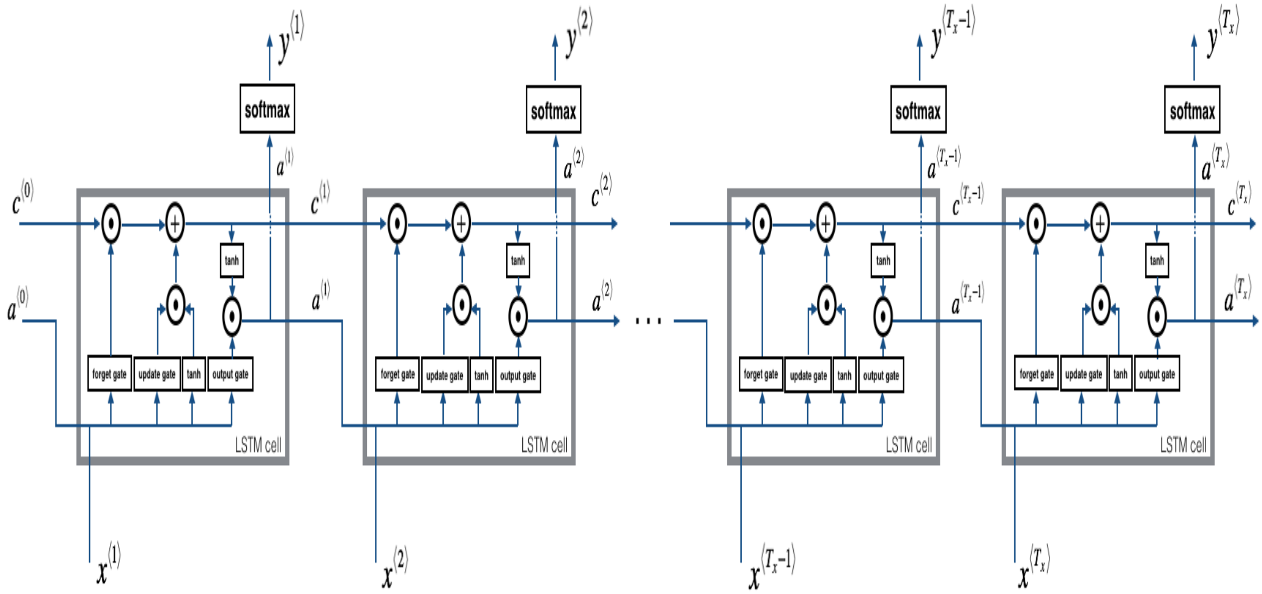


Figure 4: LSTM over multiple time-steps.

Exercise: Implement `lstm_forward()` to run an LSTM over T_x time-steps.

Note: $c^{\langle 0 \rangle}$ is initialized with zeros.

In [16]:
```
# GRADED FUNCTION: lstm_forward

def lstm_forward(x, a0, parameters):
    """
    Implement the forward propagation of the recurrent neural network using

    Arguments:
    x -- Input data for every time-step, of shape (n_x, m, T_x).
    a0 -- Initial hidden state, of shape (n_a, m)
    parameters -- python dictionary containing:
                        Wf -- Weight matrix of the forget gate, numpy array
                        bf -- Bias of the forget gate, numpy array of shape
                        Wi -- Weight matrix of the update gate, numpy array
                        bi -- Bias of the update gate, numpy array of shape
                        Wc -- Weight matrix of the first "tanh", numpy array
                        bc -- Bias of the first "tanh", numpy array of shape
                        Wo -- Weight matrix of the output gate, numpy array
                        bo -- Bias of the output gate, numpy array of shape
                        Wy -- Weight matrix relating the hidden-state to the
                        by -- Bias relating the hidden-state to the output,

    Returns:
    a -- Hidden states for every time-step, numpy array of shape (n_a, m, T_
    y -- Predictions for every time-step, numpy array of shape (n_y, m, T_x)
    caches -- tuple of values needed for the backward pass, contains (list c
    """

    # Initialize "caches", which will track the list of all the caches
    caches = []

    ### START CODE HERE ###
    # Retrieve dimensions from shapes of x and parameters['Wy'] (≈2 lines)
    n_x, m, T_x = x.shape
    n_y, n_a = parameters['Wy'].shape

    # initialize "a", "c" and "y" with zeros (≈3 lines)
    a = np.zeros((n_a, m, T_x))
    c = np.zeros((n_a, m, T_x))
    y = np.zeros((n_y, m, T_x))

    # Initialize a_next and c_next (≈2 lines)
    a_next = a0
    c_next = np.zeros((n_a, m))

    # loop over all time-steps
    for t in range(T_x):
        # Update next hidden state, next memory state, compute the predicti
        a_next, c_next, yt, cache = lstm_cell_forward(x[:, :, t], a_next, c_
        # Save the value of the new "next" hidden state in a (≈1 line)
        a[:,:,t] = a_next
        # Save the value of the prediction in y (≈1 line)
        y[:,:,t] = yt
        # Save the value of the next cell state (≈1 line)
        c[:,:,t]  = c_next
        # Append the cache into caches (≈1 line)
        caches.append(cache)
```

```
    ### END CODE HERE ###

    # store values needed for backward propagation in cache
    caches = (caches, x)

    return a, y, c, caches
```

In [17]:
```
np.random.seed(1)
x = np.random.randn(3,10,7)
a0 = np.random.randn(5,10)
Wf = np.random.randn(5, 5+3)
bf = np.random.randn(5,1)
Wi = np.random.randn(5, 5+3)
bi = np.random.randn(5,1)
Wo = np.random.randn(5, 5+3)
bo = np.random.randn(5,1)
Wc = np.random.randn(5, 5+3)
bc = np.random.randn(5,1)
Wy = np.random.randn(2,5)
by = np.random.randn(2,1)

parameters = {"Wf": Wf, "Wi": Wi, "Wo": Wo, "Wc": Wc, "Wy": Wy, "bf": bf, "b

a, y, c, caches = lstm_forward(x, a0, parameters)
print("a[4][3][6] = ", a[4][3][6])
print("a.shape = ", a.shape)
print("y[1][4][3] =", y[1][4][3])
print("y.shape = ", y.shape)
print("caches[1][1[1]] =", caches[1][1][1])
print("c[1][2][1]", c[1][2][1])
print("len(caches) = ", len(caches))
```

```
a[4][3][6] =  0.172117767533
a.shape =  (5, 10, 7)
y[1][4][3] = 0.95087346185
y.shape =  (2, 10, 7)
caches[1][1[1]] = [ 0.82797464  0.23009474  0.76201118 -0.22232814 -0.200
75807  0.18656139
  0.41005165]
c[1][2][1] -0.855544916718
len(caches) =  2
```

Expected Output:

a[4][3][6] =	0.172117767533
a.shape =	(5, 10, 7)
y[1][4][3] =	0.95087346185
y.shape =	(2, 10, 7)
caches[1][1][1] =	[0.82797464 0.23009474 0.76201118 -0.22232814 -0.20075807 0.18656139 0.41005165]
c[1][2][1] =	-0.855544916718

len(caches) = 2

Congratulations! You have now implemented the forward passes for the basic RNN and the LSTM. When using a deep learning framework, implementing the forward pass is sufficient to build systems that achieve great performance.

The rest of this notebook is optional, and will not be graded.

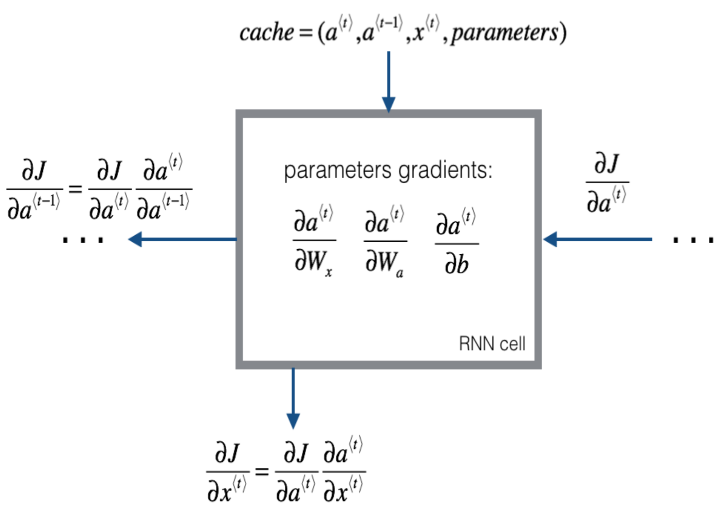
3 - Backpropagation in recurrent neural networks (OPTIONAL / UNGRADED)

In modern deep learning frameworks, you only have to implement the forward pass, and the framework takes care of the backward pass, so most deep learning engineers do not need to bother with the details of the backward pass. If however you are an expert in calculus and want to see the details of backprop in RNNs, you can work through this optional portion of the notebook.

When in an earlier course you implemented a simple (fully connected) neural network, you used backpropagation to compute the derivatives with respect to the cost to update the parameters. Similarly, in recurrent neural networks you can to calculate the derivatives with respect to the cost in order to update the parameters. The backprop equations are quite complicated and we did not derive them in lecture. However, we will briefly present them below.

3.1 - Basic RNN backward pass

We will start by computing the backward pass for the basic RNN-cell.



$$a^{\langle t \rangle} = \tanh(W_{ax} x^{\langle t \rangle} + W_{aa} a^{\langle t-1 \rangle} + b)$$

$$\frac{\partial \tanh(x)}{\partial x} = 1 - \tanh(x)^2$$

$$\frac{\partial a^{\langle t \rangle}}{\partial W_{ax}} = (1 - \tanh(W_{ax} x^{\langle t \rangle} + W_{aa} a^{\langle t-1 \rangle} + b)^2) x^{\langle t \rangle T}$$

$$\frac{\partial a^{\langle t \rangle}}{\partial W_{aa}} = (1 - \tanh(W_{ax} x^{\langle t \rangle} + W_{aa} a^{\langle t-1 \rangle} + b)^2) a^{\langle t-1 \rangle T}$$

$$\frac{\partial a^{\langle t \rangle}}{\partial b} = \sum_{batch} (1 - \tanh(W_{ax} x^{\langle t \rangle} + W_{aa} a^{\langle t-1 \rangle} + b)^2)$$

$$\frac{\partial a^{\langle t \rangle}}{\partial x^{\langle t \rangle}} = W_{ax}{}^T . (1 - \tanh(W_{ax} x^{\langle t \rangle} + W_{aa} a^{\langle t-1 \rangle} + b)^2)$$

$$\frac{\partial a^{\langle t \rangle}}{\partial a^{\langle t-1 \rangle}} = W_{aa}{}^T . (1 - \tanh(W_{ax} x^{\langle t-1 \rangle} + W_{aa} a^{\langle t-1 \rangle} + b)^2)$$

Figure 5: RNN-cell's backward pass. Just like in a fully-connected neural network, the derivative of the cost function J backpropagates through the RNN by following the chain-rule from calculus. The chain-rule is also used to calculate $(\frac{\partial J}{\partial W_{ax}}, \frac{\partial J}{\partial W_{aa}}, \frac{\partial J}{\partial b})$ to update the parameters (W_{ax}, W_{aa}, b_a).

Deriving the one step backward functions:

To compute the `rnn_cell_backward` you need to compute the following equations. It is a good exercise to derive them by hand.

The derivative of $\tanh$ is $1 - \tanh(x)^2$. You can find the complete proof here (https://www.wyzant.com/resources/lessons/math/calculus/derivative_proofs/tanx). Note that: $\text{sech}(x)^2 = 1 - \tanh(x)^2$

Similarly for $\frac{\partial a^{\langle t \rangle}}{\partial W_{ax}}, \frac{\partial a^{\langle t \rangle}}{\partial W_{aa}}, \frac{\partial a^{\langle t \rangle}}{\partial b}$, the derivative of $\tanh(u)$ is $(1 - \tanh(u)^2)du$.

The final two equations also follow same rule and are derived using the $\tanh$ derivative. Note that the arrangement is done in a way to get the same dimensions to match.

In [ ]:
```
def rnn_cell_backward(da_next, cache):
    """
    Implements the backward pass for the RNN-cell (single time-step).

    Arguments:
    da_next -- Gradient of loss with respect to next hidden state
    cache -- python dictionary containing useful values (output of rnn_cell_

    Returns:
    gradients -- python dictionary containing:
                        dx -- Gradients of input data, of shape (n_x, m)
                        da_prev -- Gradients of previous hidden state, of sh
                        dWax -- Gradients of input-to-hidden weights, of sha
                        dWaa -- Gradients of hidden-to-hidden weights, of sh
                        dba -- Gradients of bias vector, of shape (n_a, 1)
    """

    # Retrieve values from cache
    (a_next, a_prev, xt, parameters) = cache

    # Retrieve values from parameters
    Wax = parameters["Wax"]
    Waa = parameters["Waa"]
    Wya = parameters["Wya"]
    ba = parameters["ba"]
    by = parameters["by"]

    ### START CODE HERE ###
    # compute the gradient of tanh with respect to a_next (≈1 line)
    dtanh = None

    # compute the gradient of the loss with respect to Wax (≈2 lines)
    dxt = None
    dWax = None

    # compute the gradient with respect to Waa (≈2 lines)
    da_prev = None
    dWaa = None

    # compute the gradient with respect to b (≈1 line)
    dba = None

    ### END CODE HERE ###

    # Store the gradients in a python dictionary
    gradients = {"dxt": dxt, "da_prev": da_prev, "dWax": dWax, "dWaa": dWaa,

    return gradients
```

In [ ]:
```
np.random.seed(1)
xt = np.random.randn(3,10)
a_prev = np.random.randn(5,10)
Wax = np.random.randn(5,3)
Waa = np.random.randn(5,5)
Wya = np.random.randn(2,5)
b = np.random.randn(5,1)
by = np.random.randn(2,1)
parameters = {"Wax": Wax, "Waa": Waa, "Wya": Wya, "ba": ba, "by": by}

a_next, yt, cache = rnn_cell_forward(xt, a_prev, parameters)

da_next = np.random.randn(5,10)
gradients = rnn_cell_backward(da_next, cache)
print("gradients[\"dxt\"][1][2] =", gradients["dxt"][1][2])
print("gradients[\"dxt\"].shape =", gradients["dxt"].shape)
print("gradients[\"da_prev\"][2][3] =", gradients["da_prev"][2][3])
print("gradients[\"da_prev\"].shape =", gradients["da_prev"].shape)
print("gradients[\"dWax\"][3][1] =", gradients["dWax"][3][1])
print("gradients[\"dWax\"].shape =", gradients["dWax"].shape)
print("gradients[\"dWaa\"][1][2] =", gradients["dWaa"][1][2])
print("gradients[\"dWaa\"].shape =", gradients["dWaa"].shape)
print("gradients[\"dba\"][4] =", gradients["dba"][4])
print("gradients[\"dba\"].shape =", gradients["dba"].shape)
```

Expected Output:

gradients["dxt"][1][2] =	-0.460564103059
gradients["dxt"].shape =	(3, 10)
gradients["da_prev"][2][3] =	0.0842968653807
gradients["da_prev"].shape =	(5, 10)
gradients["dWax"][3][1] =	0.393081873922
gradients["dWax"].shape =	(5, 3)
gradients["dWaa"][1][2] =	-0.28483955787
gradients["dWaa"].shape =	(5, 5)
gradients["dba"][4] =	[0.80517166]
gradients["dba"].shape =	(5, 1)

Backward pass through the RNN

Computing the gradients of the cost with respect to $a^{\langle t \rangle}$ at every time-step t is useful because it is what helps the gradient backpropagate to the previous RNN-cell. To do so, you need to iterate through all the time steps starting at the end, and at each step, you increment the overall db_a, dW_{aa}, dW_{ax} and you store dx.

Instructions:

Implement the `rnn_backward` function. Initialize the return variables with zeros first and then loop through all the time steps while calling the `rnn_cell_backward` at each time timestep, update the other variables accordingly.

In [ ]:
```
def rnn_backward(da, caches):
    """
    Implement the backward pass for a RNN over an entire sequence of input d

    Arguments:
    da -- Upstream gradients of all hidden states, of shape (n_a, m, T_x)
    caches -- tuple containing information from the forward pass (rnn_forwar

    Returns:
    gradients -- python dictionary containing:
                        dx -- Gradient w.r.t. the input data, numpy-array of
                        da0 -- Gradient w.r.t the initial hidden state, nump
                        dWax -- Gradient w.r.t the input's weight matrix, nu
                        dWaa -- Gradient w.r.t the hidden state's weight mat
                        dba -- Gradient w.r.t the bias, of shape (n_a, 1)
    """

    ### START CODE HERE ###

    # Retrieve values from the first cache (t=1) of caches (≈2 lines)
    (caches, x) = None
    (a1, a0, x1, parameters) = None

    # Retrieve dimensions from da's and x1's shapes (≈2 lines)
    n_a, m, T_x = None
    n_x, m = None

    # initialize the gradients with the right sizes (≈6 lines)
    dx = None
    dWax = None
    dWaa = None
    dba = None
    da0 = None
    da_prevt = None

    # Loop through all the time steps
    for t in reversed(range(None)):
        # Compute gradients at time step t. Choose wisely the "da_next" and
        gradients = None
        # Retrieve derivatives from gradients (≈ 1 line)
        dxt, da_prevt, dWaxt, dWaat, dbat = gradients["dxt"], gradients["da_
        # Increment global derivatives w.r.t parameters by adding their der
        dx[:, :, t] = None
        dWax += None
        dWaa += None
        dba += None

    # Set da0 to the gradient of a which has been backpropagated through all
    da0 = None
    ### END CODE HERE ###

    # Store the gradients in a python dictionary
    gradients = {"dx": dx, "da0": da0, "dWax": dWax, "dWaa": dWaa,"dba": dba

    return gradients
```

In [ ]:
```
np.random.seed(1)
x = np.random.randn(3,10,4)
a0 = np.random.randn(5,10)
Wax = np.random.randn(5,3)
Waa = np.random.randn(5,5)
Wya = np.random.randn(2,5)
ba = np.random.randn(5,1)
by = np.random.randn(2,1)
parameters = {"Wax": Wax, "Waa": Waa, "Wya": Wya, "ba": ba, "by": by}
a, y, caches = rnn_forward(x, a0, parameters)
da = np.random.randn(5, 10, 4)
gradients = rnn_backward(da, caches)

print("gradients[\"dx\"][1][2] =", gradients["dx"][1][2])
print("gradients[\"dx\"].shape =", gradients["dx"].shape)
print("gradients[\"da0\"][2][3] =", gradients["da0"][2][3])
print("gradients[\"da0\"].shape =", gradients["da0"].shape)
print("gradients[\"dWax\"][3][1] =", gradients["dWax"][3][1])
print("gradients[\"dWax\"].shape =", gradients["dWax"].shape)
print("gradients[\"dWaa\"][1][2] =", gradients["dWaa"][1][2])
print("gradients[\"dWaa\"].shape =", gradients["dWaa"].shape)
print("gradients[\"dba\"][4] =", gradients["dba"][4])
print("gradients[\"dba\"].shape =", gradients["dba"].shape)
```

Expected Output:

gradients["dx"][1][2] =	[-2.07101689 -0.59255627 0.02466855 0.01483317]
gradients["dx"].shape =	(3, 10, 4)
gradients["da0"][2][3] =	-0.314942375127
gradients["da0"].shape =	(5, 10)
gradients["dWax"][3][1] =	11.2641044965
gradients["dWax"].shape =	(5, 3)
gradients["dWaa"][1][2] =	2.30333312658
gradients["dWaa"].shape =	(5, 5)
gradients["dba"][4] =	[-0.74747722]
gradients["dba"].shape =	(5, 1)

3.2 - LSTM backward pass

3.2.1 One Step backward

The LSTM backward pass is slighltly more complicated than the forward one. We have provided you with all the equations for the LSTM backward pass below. (If you enjoy calculus exercises feel free to try deriving these from scratch yourself.)

3.2.2 gate derivatives

$$d\Gamma_o^{\langle t \rangle} = da_{next} * \tanh(c_{next}) * \Gamma_o^{\langle t \rangle} * (1 - \Gamma_o^{\langle t \rangle}) \tag{7}$$

$$d\tilde{c}^{\langle t \rangle} = dc_{next} * \Gamma_u^{\langle t \rangle} + \Gamma_o^{\langle t \rangle}(1 - \tanh(c_{next})^2) * i_t * da_{next} * \tilde{c}^{\langle t \rangle} * (1 - \tanh(\tilde{c})^2) \tag{8}$$

$$d\Gamma_u^{\langle t \rangle} = dc_{next} * \tilde{c}^{\langle t \rangle} + \Gamma_o^{\langle t \rangle}(1 - \tanh(c_{next})^2) * \tilde{c}^{\langle t \rangle} * da_{next} * \Gamma_u^{\langle t \rangle} * (1 - \Gamma_u^{\langle t \rangle}) \tag{9}$$

$$d\Gamma_f^{\langle t \rangle} = dc_{next} * \tilde{c}_{prev} + \Gamma_o^{\langle t \rangle}(1 - \tanh(c_{next})^2) * c_{prev} * da_{next} * \Gamma_f^{\langle t \rangle} * (1 - \Gamma_f^{\langle t \rangle}) \tag{10}$$

3.2.3 parameter derivatives

$$dW_f = d\Gamma_f^{\langle t \rangle} * \begin{pmatrix} a_{prev} \\ x_t \end{pmatrix}^T \tag{11}$$

$$dW_u = d\Gamma_u^{\langle t \rangle} * \begin{pmatrix} a_{prev} \\ x_t \end{pmatrix}^T \tag{12}$$

$$dW_c = d\tilde{c}^{\langle t \rangle} * \begin{pmatrix} a_{prev} \\ x_t \end{pmatrix}^T \tag{13}$$

$$dW_o = d\Gamma_o^{\langle t \rangle} * \begin{pmatrix} a_{prev} \\ x_t \end{pmatrix}^T \tag{14}$$

To calculate db_f, db_u, db_c, db_o you just need to sum across the horizontal (axis= 1) axis on $d\Gamma_f^{\langle t \rangle}, d\Gamma_u^{\langle t \rangle}, d\tilde{c}^{\langle t \rangle}, d\Gamma_o^{\langle t \rangle}$ respectively. Note that you should have the `keep_dims = True` option.

Finally, you will compute the derivative with respect to the previous hidden state, previous memory state, and input.

$$da_{prev} = W_f^T * d\Gamma_f^{\langle t \rangle} + W_u^T * d\Gamma_u^{\langle t \rangle} + W_c^T * d\tilde{c}^{\langle t \rangle} + W_o^T * d\Gamma_o^{\langle t \rangle} \tag{15}$$

Here, the weights for equations 13 are the first n_a, (i.e. $W_f = W_f[: n_a, :]$ etc...)

$$dc_{prev} = dc_{next}\Gamma_f^{\langle t \rangle} + \Gamma_o^{\langle t \rangle} * (1 - \tanh(c_{next})^2) * \Gamma_f^{\langle t \rangle} * da_{next} \tag{16}$$

$$dx^{\langle t \rangle} = W_f^T * d\Gamma_f^{\langle t \rangle} + W_u^T * d\Gamma_u^{\langle t \rangle} + W_c^T * d\tilde{c}_t + W_o^T * d\Gamma_o^{\langle t \rangle} \tag{17}$$

where the weights for equation 15 are from n_a to the end, (i.e. $W_f = W_f[n_a :, :]$ etc...)

Exercise: Implement `lstm_cell_backward` by implementing equations $7 - 17$ below. Good luck! :)

In [ ]:
```
def lstm_cell_backward(da_next, dc_next, cache):
    """
    Implement the backward pass for the LSTM-cell (single time-step).

    Arguments:
    da_next -- Gradients of next hidden state, of shape (n_a, m)
    dc_next -- Gradients of next cell state, of shape (n_a, m)
    cache -- cache storing information from the forward pass

    Returns:
    gradients -- python dictionary containing:
                        dxt -- Gradient of input data at time-step t, of sha
                        da_prev -- Gradient w.r.t. the previous hidden state
                        dc_prev -- Gradient w.r.t. the previous memory state
                        dWf -- Gradient w.r.t. the weight matrix of the forg
                        dWi -- Gradient w.r.t. the weight matrix of the upda
                        dWc -- Gradient w.r.t. the weight matrix of the memo
                        dWo -- Gradient w.r.t. the weight matrix of the outp
                        dbf -- Gradient w.r.t. biases of the forget gate, of
                        dbi -- Gradient w.r.t. biases of the update gate, of
                        dbc -- Gradient w.r.t. biases of the memory gate, of
                        dbo -- Gradient w.r.t. biases of the output gate, of
    """

    # Retrieve information from "cache"
    (a_next, c_next, a_prev, c_prev, ft, it, cct, ot, xt, parameters) = cach

    ### START CODE HERE ###
    # Retrieve dimensions from xt's and a_next's shape (≈2 lines)
    n_x, m = None
    n_a, m = None

    # Compute gates related derivatives, you can find their values can be fo
    dot = None
    dcct = None
    dit = None
    dft = None

    # Code equations (7) to (10) (≈4 lines)
    dit = None
    dft = None
    dot = None
    dcct = None

    # Compute parameters related derivatives. Use equations (11)-(14) (≈8 li
    dWf = None
    dWi = None
    dWc = None
    dWo = None
    dbf = None
    dbi = None
    dbc = None
    dbo = None

    # Compute derivatives w.r.t previous hidden state, previous memory state
    da_prev = None
```

```
    dc_prev = None
    dxt = None
    ### END CODE HERE ###

    # Save gradients in dictionary
    gradients = {"dxt": dxt, "da_prev": da_prev, "dc_prev": dc_prev, "dWf":
                "dWc": dWc,"dbc": dbc, "dWo": dWo,"dbo": dbo}

    return gradients
```

In [ ]:

```
np.random.seed(1)
xt = np.random.randn(3,10)
a_prev = np.random.randn(5,10)
c_prev = np.random.randn(5,10)
Wf = np.random.randn(5, 5+3)
bf = np.random.randn(5,1)
Wi = np.random.randn(5, 5+3)
bi = np.random.randn(5,1)
Wo = np.random.randn(5, 5+3)
bo = np.random.randn(5,1)
Wc = np.random.randn(5, 5+3)
bc = np.random.randn(5,1)
Wy = np.random.randn(2,5)
by = np.random.randn(2,1)

parameters = {"Wf": Wf, "Wi": Wi, "Wo": Wo, "Wc": Wc, "Wy": Wy, "bf": bf, "b

a_next, c_next, yt, cache = lstm_cell_forward(xt, a_prev, c_prev, parameters

da_next = np.random.randn(5,10)
dc_next = np.random.randn(5,10)
gradients = lstm_cell_backward(da_next, dc_next, cache)
print("gradients[\"dxt\"][1][2] =", gradients["dxt"][1][2])
print("gradients[\"dxt\"].shape =", gradients["dxt"].shape)
print("gradients[\"da_prev\"][2][3] =", gradients["da_prev"][2][3])
print("gradients[\"da_prev\"].shape =", gradients["da_prev"].shape)
print("gradients[\"dc_prev\"][2][3] =", gradients["dc_prev"][2][3])
print("gradients[\"dc_prev\"].shape =", gradients["dc_prev"].shape)
print("gradients[\"dWf\"][3][1] =", gradients["dWf"][3][1])
print("gradients[\"dWf\"].shape =", gradients["dWf"].shape)
print("gradients[\"dWi\"][1][2] =", gradients["dWi"][1][2])
print("gradients[\"dWi\"].shape =", gradients["dWi"].shape)
print("gradients[\"dWc\"][3][1] =", gradients["dWc"][3][1])
print("gradients[\"dWc\"].shape =", gradients["dWc"].shape)
print("gradients[\"dWo\"][1][2] =", gradients["dWo"][1][2])
print("gradients[\"dWo\"].shape =", gradients["dWo"].shape)
print("gradients[\"dbf\"][4] =", gradients["dbf"][4])
print("gradients[\"dbf\"].shape =", gradients["dbf"].shape)
print("gradients[\"dbi\"][4] =", gradients["dbi"][4])
print("gradients[\"dbi\"].shape =", gradients["dbi"].shape)
print("gradients[\"dbc\"][4] =", gradients["dbc"][4])
print("gradients[\"dbc\"].shape =", gradients["dbc"].shape)
print("gradients[\"dbo\"][4] =", gradients["dbo"][4])
print("gradients[\"dbo\"].shape =", gradients["dbo"].shape)
```

Expected Output:

gradients["dxt"][1][2] =	3.23055911511
gradients["dxt"].shape =	(3, 10)
gradients["da_prev"][2][3] =	-0.0639621419711
gradients["da_prev"].shape =	(5, 10)
gradients["dc_prev"][2][3] =	0.797522038797
gradients["dc_prev"].shape =	(5, 10)
gradients["dWf"][3][1] =	-0.147954838164
gradients["dWf"].shape =	(5, 8)
gradients["dWi"][1][2] =	1.05749805523
gradients["dWi"].shape =	(5, 8)
gradients["dWc"][3][1] =	2.30456216369
gradients["dWc"].shape =	(5, 8)
gradients["dWo"][1][2] =	0.331311595289
gradients["dWo"].shape =	(5, 8)
gradients["dbf"][4] =	[0.18864637]
gradients["dbf"].shape =	(5, 1)
gradients["dbi"][4] =	[-0.40142491]
gradients["dbi"].shape =	(5, 1)
gradients["dbc"][4] =	[0.25587763]
gradients["dbc"].shape =	(5, 1)
gradients["dbo"][4] =	[0.13893342]
gradients["dbo"].shape =	(5, 1)

3.3 Backward pass through the LSTM RNN

This part is very similar to the `rnn_backward` function you implemented above. You will first create variables of the same dimension as your return variables. You will then iterate over all the time steps starting from the end and call the one step function you implemented for LSTM at each iteration. You will then update the parameters by summing them individually. Finally return a dictionary with the new gradients.

Instructions: Implement the `lstm_backward` function. Create a for loop starting from T_x and going backward. For each step call `lstm_cell_backward` and update the your old gradients by adding the new gradients to them. Note that `dxt` is not updated but is stored.

In [ ]:
```
def lstm_backward(da, caches):

    """
    Implement the backward pass for the RNN with LSTM-cell (over a whole sequ

    Arguments:
    da -- Gradients w.r.t the hidden states, numpy-array of shape (n_a, m, T_
    dc -- Gradients w.r.t the memory states, numpy-array of shape (n_a, m, T_
    caches -- cache storing information from the forward pass (lstm_forward)

    Returns:
    gradients -- python dictionary containing:
                        dx -- Gradient of inputs, of shape (n_x, m, T_x)
                        da0 -- Gradient w.r.t. the previous hidden state, num
                        dWf -- Gradient w.r.t. the weight matrix of the forge
                        dWi -- Gradient w.r.t. the weight matrix of the updat
                        dWc -- Gradient w.r.t. the weight matrix of the memor
                        dWo -- Gradient w.r.t. the weight matrix of the save
                        dbf -- Gradient w.r.t. biases of the forget gate, of
                        dbi -- Gradient w.r.t. biases of the update gate, of
                        dbc -- Gradient w.r.t. biases of the memory gate, of
                        dbo -- Gradient w.r.t. biases of the save gate, of sh
    """

    # Retrieve values from the first cache (t=1) of caches.
    (caches, x) = caches
    (a1, c1, a0, c0, f1, i1, cc1, o1, x1, parameters) = caches[0]

    ### START CODE HERE ###
    # Retrieve dimensions from da's and x1's shapes (≈2 lines)
    n_a, m, T_x = None
    n_x, m = None

    # initialize the gradients with the right sizes (≈12 lines)
    dx = None
    da0 = None
    da_prevt = None
    dc_prevt = None
    dWf = None
    dWi = None
    dWc = None
    dWo = None
    dbf = None
    dbi = None
    dbc = None
    dbo = None

    # loop back over the whole sequence
    for t in reversed(range(None)):
        # Compute all gradients using lstm_cell_backward
        gradients = None
        # Store or add the gradient to the parameters' previous step's gradie
        dx[:,:,t] = None
        dWf = None
        dWi = None
        dWc = None
```

```
        dWo = None
        dbf = None
        dbi = None
        dbc = None
        dbo = None
    # Set the first activation's gradient to the backpropagated gradient da_p
    da0 = None

    ### END CODE HERE ###

    # Store the gradients in a python dictionary
    gradients = {"dx": dx, "da0": da0, "dWf": dWf,"dbf": dbf, "dWi": dWi,"dbi
                "dWc": dWc,"dbc": dbc, "dWo": dWo,"dbo": dbo}

    return gradients
```

In [ ]:

```
np.random.seed(1)
x = np.random.randn(3,10,7)
a0 = np.random.randn(5,10)
Wf = np.random.randn(5, 5+3)
bf = np.random.randn(5,1)
Wi = np.random.randn(5, 5+3)
bi = np.random.randn(5,1)
Wo = np.random.randn(5, 5+3)
bo = np.random.randn(5,1)
Wc = np.random.randn(5, 5+3)
bc = np.random.randn(5,1)

parameters = {"Wf": Wf, "Wi": Wi, "Wo": Wo, "Wc": Wc, "Wy": Wy, "bf": bf, "b

a, y, c, caches = lstm_forward(x, a0, parameters)

da = np.random.randn(5, 10, 4)
gradients = lstm_backward(da, caches)

print("gradients[\"dx\"][1][2] =", gradients["dx"][1][2])
print("gradients[\"dx\"].shape =", gradients["dx"].shape)
print("gradients[\"da0\"][2][3] =", gradients["da0"][2][3])
print("gradients[\"da0\"].shape =", gradients["da0"].shape)
print("gradients[\"dWf\"][3][1] =", gradients["dWf"][3][1])
print("gradients[\"dWf\"].shape =", gradients["dWf"].shape)
print("gradients[\"dWi\"][1][2] =", gradients["dWi"][1][2])
print("gradients[\"dWi\"].shape =", gradients["dWi"].shape)
print("gradients[\"dWc\"][3][1] =", gradients["dWc"][3][1])
print("gradients[\"dWc\"].shape =", gradients["dWc"].shape)
print("gradients[\"dWo\"][1][2] =", gradients["dWo"][1][2])
print("gradients[\"dWo\"].shape =", gradients["dWo"].shape)
print("gradients[\"dbf\"][4] =", gradients["dbf"][4])
print("gradients[\"dbf\"].shape =", gradients["dbf"].shape)
print("gradients[\"dbi\"][4] =", gradients["dbi"][4])
print("gradients[\"dbi\"].shape =", gradients["dbi"].shape)
print("gradients[\"dbc\"][4] =", gradients["dbc"][4])
print("gradients[\"dbc\"].shape =", gradients["dbc"].shape)
print("gradients[\"dbo\"][4] =", gradients["dbo"][4])
print("gradients[\"dbo\"].shape =", gradients["dbo"].shape)
```

Expected Output:

gradients["dx"][1][2] =	[-0.00173313 0.08287442 -0.30545663 -0.43281115]
gradients["dx"].shape =	(3, 10, 4)
gradients["da0"][2][3] =	-0.095911501954
gradients["da0"].shape =	(5, 10)
gradients["dWf"][3][1] =	-0.0698198561274
gradients["dWf"].shape =	(5, 8)
gradients["dWi"][1][2] =	0.102371820249
gradients["dWi"].shape =	(5, 8)
gradients["dWc"][3][1] =	-0.0624983794927
gradients["dWc"].shape =	(5, 8)
gradients["dWo"][1][2] =	0.0484389131444
gradients["dWo"].shape =	(5, 8)
gradients["dbf"][4] =	[-0.0565788]
gradients["dbf"].shape =	(5, 1)
gradients["dbi"][4] =	[-0.06997391]
gradients["dbi"].shape =	(5, 1)
gradients["dbc"][4] =	[-0.27441821]
gradients["dbc"].shape =	(5, 1)
gradients["dbo"][4] =	[0.16532821]
gradients["dbo"].shape =	(5, 1)

Congratulations !

Congratulations on completing this assignment. You now understand how recurrent neural networks work!

Lets go on to the next exercise, where you'll use an RNN to build a character-level language model.