Hassan Khosravi's Booklet

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Contents

Preface

I decided to fair copy my hand written general mathematics booklet using LaTeX. This book is completely Open Source. You can contribute to this book and help to improve it. For additional information visist this repository. My main goal for writing this book is learning LaTeXand create collaborative platform to write and improve technical documents on Azad University.

Chapter 1

Limit

Chapter 2

Derivative

2.1 Definition

Derivative limit definition:

$$\frac{df}{dx} = f'(x)$$
$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Proof of the Derivative of Constant: $\frac{df}{dx}(c)$

$$\begin{aligned} f(x) &= c \\ f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{c - c}{h} \\ &= \lim_{h \to 0} 0 = 0 \end{aligned}$$

2.2 Common formulas

2.2.1 1

$$(y = c) \to y' = 0$$

Example:

$$(\sqrt{2})' = (\frac{3}{2})' = (\sin^{-1}\frac{1}{10})' = 0$$

Note: $(\sin\infty)'$ does not exists.[1]

2.2.2 2

$$(ax^n)' = a(x^n)' = anx^{n-1}$$

Note: The n coefficient have no effect on derivative.
Examples:

$$\begin{aligned}
&(3x^7)' = 21x^6\\
&(2x^{-9})' = -18x^{-10}\\
&(x^2\sqrt[7]{x^3})' = (x^2x^{\frac{3}{7}})' = (x^{\frac{17}{7}})' = \frac{17}{7}x^{\frac{10}{7}}
\end{aligned}$$

2.2.3 3

$$(u \pm v)' = u' \pm v'$$

Example:

$$(z+1)' = (z)' + (1)' = 1 + 0 = 1$$

2.2.4 4

$$(uv)' = u'v + uv'$$

Example:

$$(\underset{u}{x}\underset{v}{\sin x})' = u'v + uv' = \left\{\begin{array}{l} u' = 1 \\ v' = \cos x \end{array}\right.$$

[1]https://math.stackexchange.com/questions/635135/infinite-derivatives-of-a-trigonometric-function

2.2.5 5

$$(au^n)' = a(u^n)' = anu^{n-1}u'$$

Example:

$$(3(x^2 - x^{-2} + \sqrt{3})^{10})' = anu^{n-1}u' = 3(u^9u') \rightarrow u' = 2x + 2x^{-3} + 0$$

Example for derivation with radical:

$$\begin{aligned}(3\sqrt[7]{(x^{-3} + \frac{1}{x})^2})' &= anu^{n-1}u' \\ &= 3(\underbrace{(x^{-3} + \frac{1}{x})^2}_{u})^{\frac{2}{7}} \rightarrow \\ u' = x^{-3} \times \frac{1}{x} &= \underbrace{x^{-3}}_{w_1} \times \underbrace{x^{-1}}_{w_2} \\ &= w_1'w_2 + w_1w_2' = \begin{cases} w_1' = -3x^{-4} \\ w_2' = -x^{-2} \end{cases}\end{aligned}$$

2.2.6 6

$$(\frac{u}{v})' = \frac{u'v - uv'}{v^2} \rightarrow (\frac{1}{x})' = \frac{-1}{x^2}$$

Example:

$$(\frac{\overbrace{1 - 2x^{-7}}^{u}}{\underbrace{2x^3 - \frac{2}{\sqrt{3}}}_{v}})' = \frac{u'v - uv'}{v^2} \rightarrow \begin{cases} u' = 0 + 14x^{-8} = 14x^{-8} \\ v' = 6x^2 - 0 = 6x^2 \end{cases}$$

2.2.7 7

$$(\sqrt{u})' = \frac{u'}{2\sqrt{u}} \rightarrow (\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

Example:

$$(\sqrt{\underbrace{x^2 - x + x^{-4}}_{u}})' = \frac{u'}{2\sqrt{u}}$$

$$u' = 2z - 1 - 4x^{-5}$$

2.2.8 16

$$(\ln u)' = \frac{u'}{u} \to (\ln x)' = \frac{1}{x}$$

2.2.9 17

$$(a^u)' \xrightarrow{a>0} u'a^u \ln a \to (a^x)' = a^x \ln a$$

2.2.10 18

$$(e^u)' = u'e^u \ln e = u'e^u \to (e^{ax+b})' = ae^{ax+b}$$

2.2.11 19

$$(u^v)' \xrightarrow{u>0} u^v(v' \ln u + \frac{u'v}{u}) \to (x^x)' = (1 + \ln x)$$

2.3 Trigonometric derivatives

2.3.1 21

$(\sin u)' = u' \cos u \longrightarrow (sin(ax+b))' = a\cos(ax+b) \longrightarrow (\sin x)' = \cos x$

Example:

$$(\sin(\ln x))' = (\sin u)' = u'cosu \to u' = \frac{1}{x}$$

2.3.2 22

$(\cos u)' = -u' \sin u \longrightarrow (cos(ax+b))' = -a\sin(ax+b) \to (\cos x)' = -\sin x$
$(\cos^n x) = (\cos x)^n$

Example:

$$((\cos x)^3)' = (u^3)' = 3u^2u' \to u' = -\sin x$$

2.3.3 23

$(\tan u)' = u'(1 + \tan^2 u) = u' \sec^2 u \to (\tan x)' = 1 + \tan^2 x = \sec^2 x$

2.3.4 24

$(\cot u)' = -u'(1+\cot^2 u) = -u'\csc^2 u \longrightarrow (\cot x)' = -(1+\cot^2 x) = -\csc^2 x$

2.3.5 25

$$(\sec u)' = u'.\sec u.\tan u \longrightarrow (secx)' = x'\sec x.\tan x$$

Example:

$$(\sec(\csc x))' = (\sec u)' = u'\sec u\tan u \longrightarrow u' = -\cot x\csc x$$

2.3.6 26

$$(cscu)' = -u'.\csc u.\cot u \longrightarrow (\csc x)' = -\cot x.\csc x$$

2.4 Inverse trigonometric derivative

2.4.1 31

$$(\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}} \longrightarrow (\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$$

2.4.2 32

$$(\cos^{-1} u)' = \frac{-u'}{\sqrt{1-u^2}} \longrightarrow (\cos^{-1} x)' = \frac{-1}{\sqrt{1-x^2}}$$

2.4.3 33

$$(\tan^{-1} u)' = \frac{u'}{1+u^2} \longrightarrow (\tan^{-1} x)' = \frac{1}{1+x^2}$$

2.4.4 34

$$(\cot^{-1} u)' = \frac{-u'}{1+u^2} \longrightarrow (\cot^{-1} x)' = \frac{-1}{1+x^2}$$

2.4.5 35

$$(\sec^{-1} u)' = \frac{u'}{u\sqrt{u^2-1}} \longrightarrow (\sec^{-1} x)' = \frac{1}{x\sqrt{x^2-1}}$$

2.4.6 36

$$(\csc^{-1} u)' = \frac{-u'}{u\sqrt{u^2-1}} \to (\csc^{-1} x)' = \frac{-1}{x\sqrt{x^2-1}}$$

2.5 Hyperbolic derivative

2.5.1 45

$(\sinh u)' = u' \cosh u \to (\sinh(ax+b))' = a\cosh(ax+b) \to (\sinh x)' = \cosh x$

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$(\cosh u)' = u' \sinh u \to (\cosh(ax+b))' = a\sinh(ax+b) \to (\cosh x)' = \sinh x$

2.6 Derivative Cheat Sheet

$(y = c) \longrightarrow y' = 0$
$(ax^n)' = a(x^n)' = anx^{n-1}$
$(u \pm v)' = u' \pm v'$
$(uv)' = u'v + uv'$
$(au^n)' = a(u^n)' = anu^{n-1}u'$
$(\frac{u}{v})' = \frac{u'v - uv'}{v^2} \longrightarrow (\frac{1}{x})' = \frac{-1}{x^2}$
$(\sqrt{u})' = \frac{u'}{2\sqrt{u}} \longrightarrow (\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$(\ln u)' = \frac{u'}{u} \longrightarrow (\ln x)' = \frac{1}{x}$
$(a^u)' \overset{a>0}{\longrightarrow} u'a^u \ln a \longrightarrow (a^x)' = a^x \ln a$
$(e^u)' = u'e^u \ln e = u'e^u \longrightarrow (e^{ax+b})' = ae^{ax+b}$
$(u^v)' \overset{u>0}{\longrightarrow} u^v(v' \ln u + \frac{u'v}{u}) \longrightarrow (x^x)' = (1 + \ln x)$

$(\sin u)' = u' \cos u \longrightarrow (sin(ax+b))' = a\cos(ax+b) \longrightarrow (\sin x)' = \cos x$
$(\tan u)' = u'(1 + \tan^2 u) = u' \sec^2 u \longrightarrow (\tan x)' = 1 + \tan^2 x = \sec^2 x$
$(\cot u)' = -u'(1 + \cot^2 u) = -u'csc^2u \longrightarrow (\cot x)' = -(1 + \cot^2 x) = -csc^2x$
$(\sec u)' = u' \sec u \tan u \longrightarrow (secx)' = x' \sec x \tan x$
$(cscu)' = -u'. \csc u. \cot u \longrightarrow (\csc x)' = -\cot x. \csc x$

$(\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}} \longrightarrow (\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$
$(\cos^{-1} u)' = \frac{-u'}{\sqrt{1-u^2}} \longrightarrow (\cos^{-1} x)' = \frac{-1}{\sqrt{1-x^2}}$
$(\tan^{-1} u)' = \frac{u'}{1+u^2} \longrightarrow (\tan^{-1} x)' = \frac{1}{1+x^2}$
$(\cot^{-1} u)' = \frac{-u'}{1+u^2} \longrightarrow (\cot^{-1} x)' = \frac{-1}{1+x^2}$
$(\sec^{-1} u)' = \frac{u'}{u\sqrt{u^2-1}} \longrightarrow (\sec^{-1} x)' = \frac{1}{x\sqrt{x^2-1}}$
$(\csc^{-1} u)' = \frac{-u'}{u\sqrt{u^2-1}} \longrightarrow (\csc^{-1} x)' = \frac{-1}{x\sqrt{x^2-1}}$

$(\sinh u)' = u' \cosh u \longrightarrow (\sinh(ax+b))' = a\cosh(ax+b) \longrightarrow (\sinh x)' = \cosh x$
$(\cosh u)' = u' \sinh u \longrightarrow (\cosh(ax+b))' = a\sinh(ax+b) \longrightarrow (\cosh x)' = \sinh x$

Chapter 3

Integral