

Hassan Khosravi's Booklet

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Preface

I decided to fair copy my hand written general mathematics booklet using \LaTeX . This book is completely Open Source. You can contribute to this book and help to improve it. For additional information visist this repository. My main goal for writing this book is learning \LaTeX and create collaborative platform to write and improve technical documents on Azad University.

Chapter 1

Limit

Chapter 2

Derivative

2.1 Definition

Derivative limit definition:

$$\begin{aligned}\frac{df}{dx} &= f'(x) \\ f'(x_0) &= \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} \\ f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}\end{aligned}$$

Proof of the Derivative of Constant: $\frac{df}{dx}(c)$

$$\begin{aligned}f(x) &= c \\ f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{c - c}{h} \\ &= \lim_{h \rightarrow 0} 0 = 0\end{aligned}$$

2.2 Common formulas

2.2.1 1

$$(y = c) \rightarrow y' = 0$$

Example:

$$(\sqrt{2})' = \left(\frac{3}{2}\right)' = \left(\sin^{-1} \frac{1}{10}\right)' = 0$$

Note: $(\sin \infty)'$ does not exist.¹

2.2.2 2

$$(ax^n)' = a(x^n)' = anx^{n-1}$$

Note: The n coefficient have no effect on derivative.

Examples:

$$(3x^7)' = 21x^6$$

$$(2x^{-9})' = -18x^{-10}$$

$$(x^2\sqrt[7]{x^3})' = (x^2x^{\frac{3}{7}})' = (x^{\frac{17}{7}})' = \frac{17}{7}x^{\frac{10}{7}}$$

2.2.3 3

$$(u \pm v)' = u' \pm v'$$

Example:

$$(z + 1)' = (z)' + (1)' = 1 + 0 = 1$$

2.2.4 4

$$(uv)' = u'v + uv'$$

Example:

$$\underset{u}{(x \sin x)}' = \underset{v}{u'v + uv'} = \begin{cases} u' = 1 \\ v' = \cos x \end{cases}$$

¹<https://math.stackexchange.com/questions/635135/infinite-derivatives-of-a-trigonometric-function>

2.2.5 5

$$(au^n)' = a(u^n)' = anu^{n-1}u'$$

Example:

$$(3(x^2 - x^{-2} + \sqrt{3})^{10})' = anu^{n-1}u' = 3(u^9u') \rightarrow u' = 2x + 2x^{-3} + 0$$

Example for derivation with radical:

$$\begin{aligned} (3\sqrt[7]{(x^{-3} + \frac{1}{x})^2})' &= anu^{n-1}u' \\ &= 3((x^{-3} + \frac{1}{x})^2)^{\frac{2}{7}} \rightarrow \\ &\quad \underbrace{\hspace{1.5cm}}^u \\ u' &= x^{-3} \times \frac{1}{x} = \underbrace{x^{-3}}_{w_1} \times \underbrace{x^{-1}}_{w_2} \\ &= w_1'w_2 + w_1w_2' = \begin{cases} w_1' = -3x^{-4} \\ w_2' = -x^{-2} \end{cases} \end{aligned}$$

2.2.6 6

$$(\frac{u}{v})' = \frac{u'v - uv'}{v^2} \rightarrow (\frac{1}{x})' = \frac{-1}{x^2}$$

Example:

$$\underbrace{(\frac{1 - 2x^{-7}}{2x^3 - \frac{2}{\sqrt{3}}})'}_v = \frac{u'v - uv'}{v^2} \rightarrow \begin{cases} u' = 0 + 14x^{-8} = 14x^{-8} \\ v' = 6x^2 - 0 = 6x^2 \end{cases}$$

2.2.7 7

$$(\sqrt{u})' = \frac{u'}{2\sqrt{u}} \rightarrow (\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

Example:

$$\begin{aligned} (\sqrt{\underbrace{x^2 - x + x^{-4}}_u})' &= \frac{u'}{2\sqrt{u}} \\ u' &= 2x - 1 - 4x^{-5} \end{aligned}$$

2.3 Derivative Cheat Sheet

$$(y = c) \rightarrow y' = 0$$

$$(ax^n)' = a(x^n)' = anx^{n-1}$$

$$(u \pm v)' = u' \pm v'$$

$$(uv)' = u'v + uv'$$

$$(au^n)' = a(u^n)' = anu^{n-1}u'$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \rightarrow \left(\frac{1}{x}\right)' = \frac{-1}{x^2}$$

$$(\sqrt{u})' = \frac{u'}{2\sqrt{u}} \rightarrow (\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

Chapter 3

Integral