Computational Models of Cognition: Models of Inductive Reasoning

Alireza Dehbozorgi

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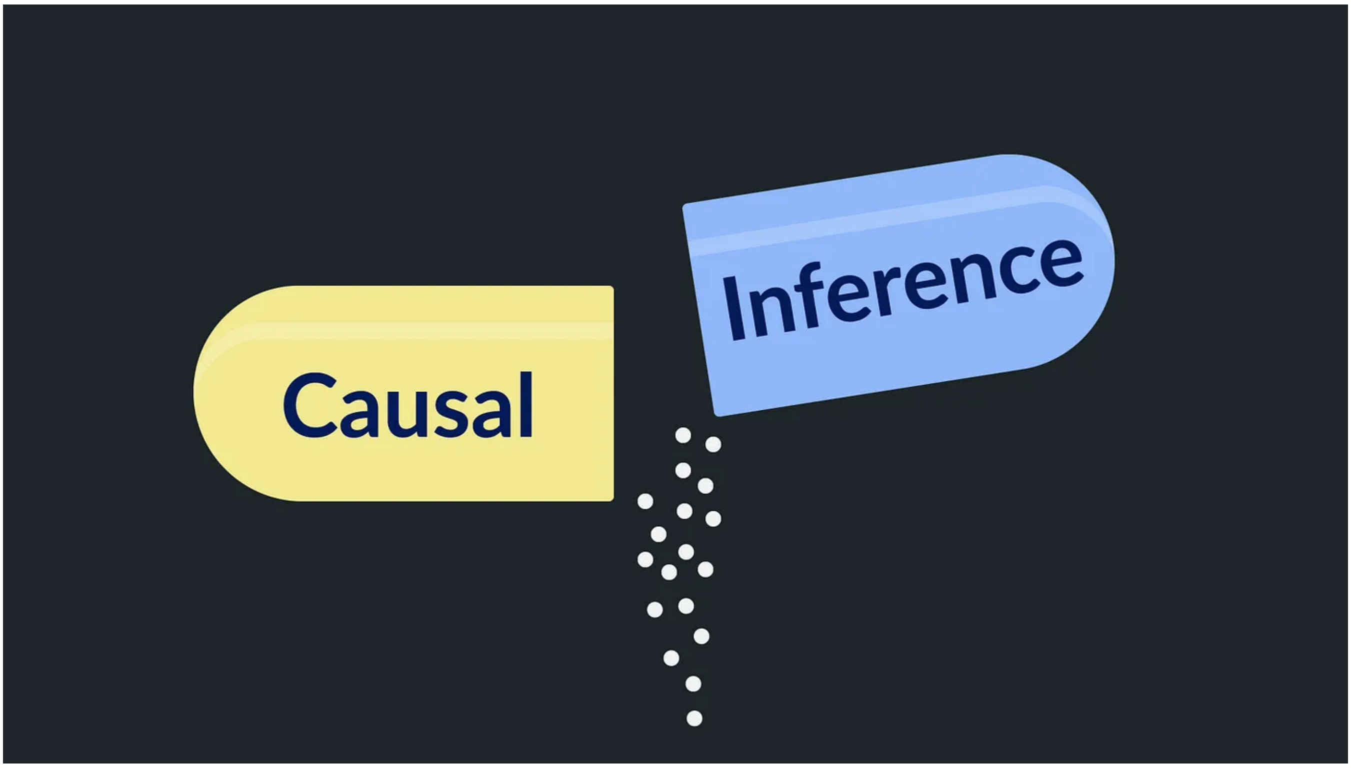

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Causal inference is the study of how actions, interventions, or treatments affect outcomes of interest.

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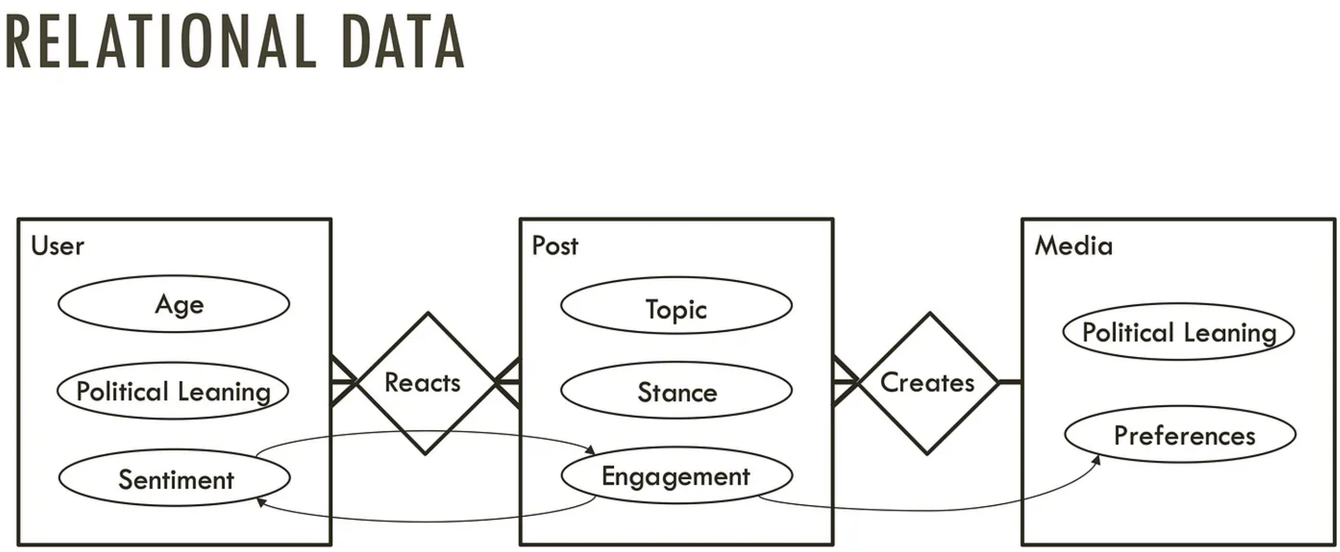
Causal Inference and Interference

Common among these questions:

1. They are concerned with causes and effects.
2. There is data from digital platforms that may help with answering them.
3. Interference: the actions of one user can affect the actions of others.

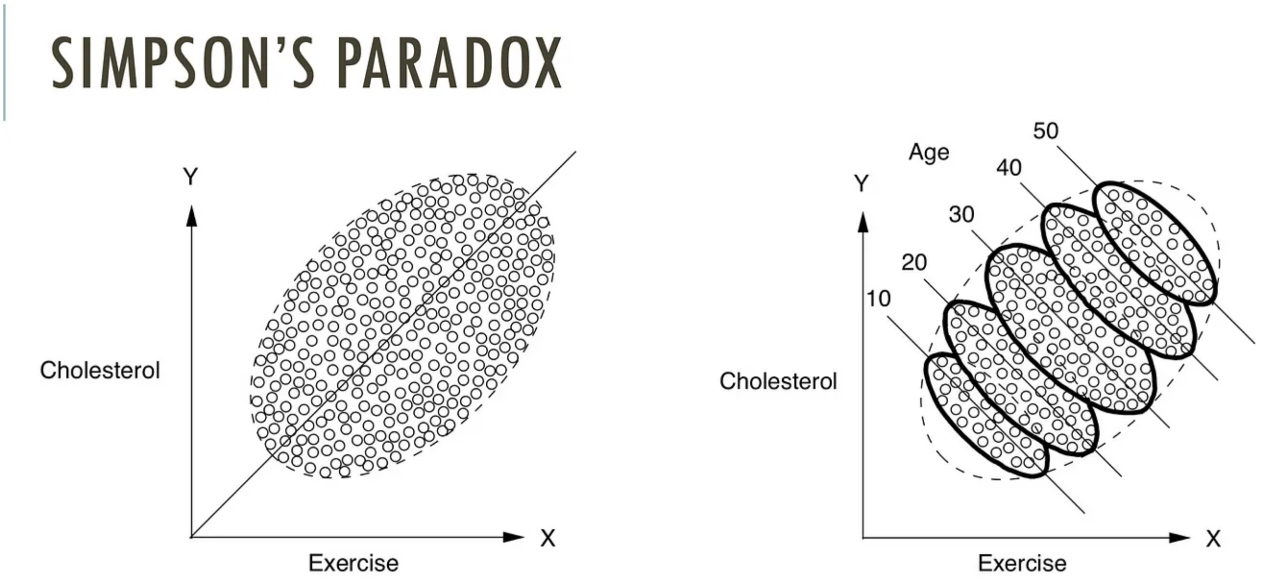
When and how can we answer causal questions of interest while accounting for interference?

Interference in the Data-Driven world

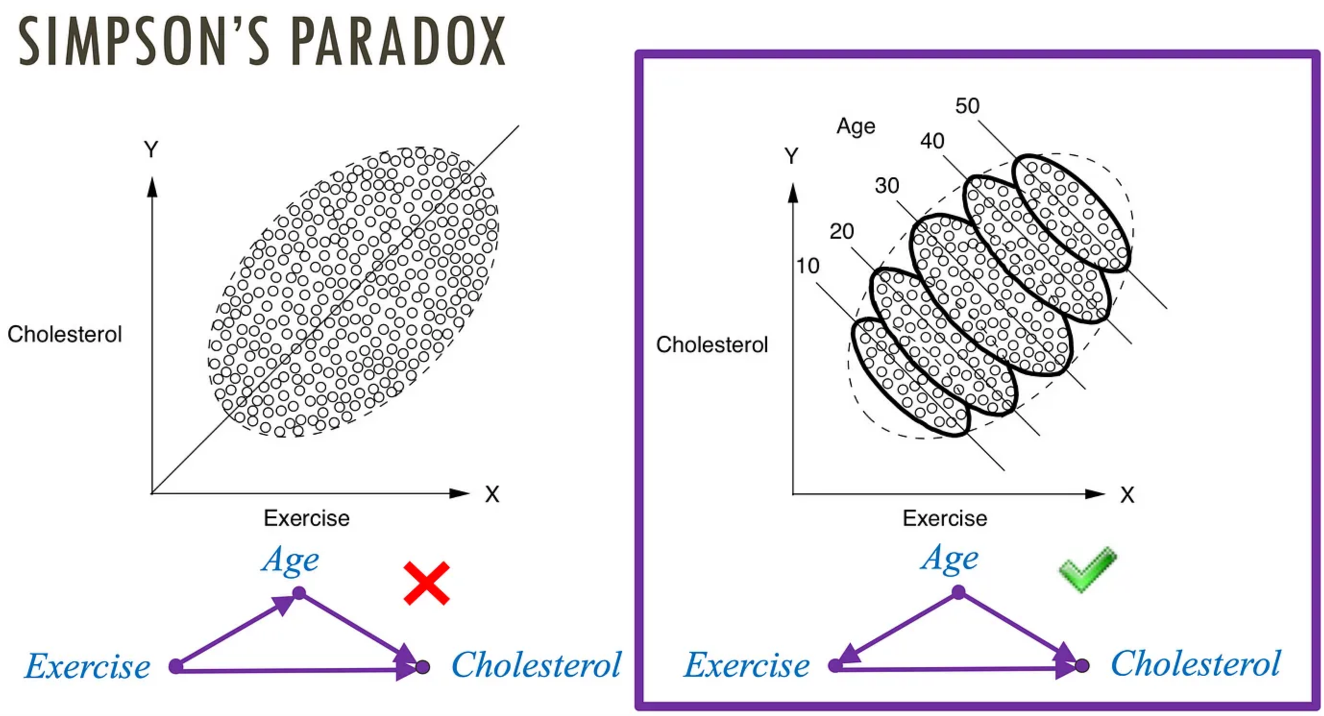


Real-world data is rarely flat!

Simpson's Paradox:



Same data can have different causal explanations!



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Models of Inductive Reasoning

Inductive inference involves extrapolating from existing observations and knowledge to new observations and events. It is a fundamental cognitive capability that allows people to make predictions about the environment that can help to maximize material and social rewards and avoid harm. Much of the reasoning that people do in everyday life could be described as a form of induction. Predicting the next round of basketball results, deciding on the most suitable applicant for a job, or inferring whether your children will like a new brand of ice cream, all involve induction.

An understanding of this process is central to accounts of human reasoning, word learning, categorization, and decision-making. Inductive reasoning has also long been a central topic in philosophy (e.g., Carnap, 1968) as well as in artificial intelligence and computer science (e.g., Collins & Michalski, 1989; Sun, 1995; Sun & Zhang, 2006).

The central focus of this article, however, will be on models that have come about through the study of property induction. This paradigm, introduced by Rips (1975), typically involves learning about samples of evidence (e.g., a set of people, animals, or objects) that share some novel property, and then making an inference about whether the property generalizes to novel instances. Four decades of research using this approach has taught much about the conditions under which property generalization occurs (see Feeney, 2017; Hayes & Heit, 2018 for reviews). There remains, however, lively debate about the cognitive processes that underpin such generalization.

Induction models aim to explain key regularities in the way that people make property inferences. Many benchmark phenomena were uncovered in seminal work by Rips (1975), Osherson, Smith, Wilkie, López, and Shafir (1990), and Nisbett, Krantz, Jepson, and Kunda (1983), and have been replicated across a range of stimulus domains, tasks, and populations. Here are four particularly robust findings.

1. **Premise-conclusion similarity**. The likelihood that a novel property will be generalized increases with the similarity between premise and conclusion items. For example, a property shared by *robins* and *sparrows* is more likely to be generalized to *crows* than to *penguins*.

2. **Premise typicality**. Premise items viewed as more typical or representative are more likely to promote property generalization to general conclusion categories. For example, a property of *wolves* is more likely to be generalized to other *mammals* than a property of *dolphins*.

3. **Premise monotonicity (sample size)**. The likelihood that a novel property will be generalized to other items from the same category increases with the number of premise items known to share the property. For example, a property

shared by *chimps*, *bonobos*, *orangutans*, and *gorillas* is more likely to be generalized to other *apes* than a property shared by just *chimps* and *gorillas*.

4. **Premise diversity**. Properties shared by dissimilar members of a superordinate category are more likely to be generalized than properties shared by similar members. For example, a property of *lions* and *cows* is more likely to be generalized to other *animals* than a property of *lions* and *tigers*.

In each case, it is assumed that the learner has some knowledge about the premise and conclusion categories but knows little about the to-be-generalized property. Note also that each phenomenon involves only positive evidence (i.e., instances that have the target property).

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Similarity-Coverage Model

The similarity-coverage model of induction proposed by Osherson et al. (1990) has proven to be one of the most influential in the field. This model explains the four benchmark inductive phenomena, together with a range of other findings, using two core processes. The "similarity" component reflects the level of similarity between premise categories and the conclusion category. The "coverage" component reflects the similarity between the premise categories and members of the lowest level category that includes both premises and conclusions. Formally, the similarity component is computed as the maximal similarity between premise categories CAT(P_1) to CAT(P_n), and a conclusion category CAT(C). Coverage is computed as the mean similarity of premise categories to members of the lowest level category that includes both premises and conclusions. For any individual, "argument strength" or the likelihood that a property of the premises will be generalized to the conclusion, can be expressed as a linear weighted combination of similarity and coverage:

$$\alpha\, \mathrm{SIM}_S(\mathrm{CAT}(P_1)\ldots\mathrm{CAT}(P_n);\mathrm{CAT}(C)) \\ +(1-\alpha)\mathrm{SIM}_S(\mathrm{CAT}(P_1)\ldots\mathrm{CAT}(P_n);[\mathrm{CAT}(P_1)\ldots\mathrm{CAT}(P_n);\mathrm{CAT}\ (C)])$$

Eq. 1: SCM Equation

The parameter α is assumed to vary between 0 and 1, and represents individual differences in the weights attached to the similarity and coverage components.

Premise-conclusion similarity effects are attributed to the similarity component of the model. In the earlier example, the maximal similarity of *robins* and *sparrows* to crows will be higher than their maximal similarity to *penguins*. The other three phenomena are primarily due to the coverage component. The typicality effect arises because typical instances will be similar to more instances from a superordinate that includes premise and conclusion items, than atypical instances. Hence, a typical premise like *sparrows* will have higher levels of coverage of the category *birds* than *penguins*.

As premise diversity increases, or as the number of premise categories increases, this will also increase the mean similarity between premises and members of an inclusive superordinate category. In the earlier diversity example, *lions* and *tigers* only have high similarity to a relatively small number of *mammals*. By comparison, the diverse premises *lions* and *cows* are similar to many instances of *mammals*, increasing their overall coverage. Likewise, coverage increases as more premises are added, resulting in the premise monotonicity effect.

Note that the model predicts that premise monotonicity will only be observed when the added premises belong to the same superordinate as the conclusion. Discovering that *peacocks* have a property as well as *chimps* and *orangutans* can lead to "nonmonotonicity" with a reduction in generalization of the property to other *apes*. The additional *peacock* premise means that a much broader category needs to be considered to include all premises and the conclusion (e.g., "*animals*"), leading to lower coverage.

The similarity-coverage model has had considerable success in explaining a range of induction phenomena (Osherson et al., 1990). One concern however, is that little rationale is provided for some of the model's core assumptions. Some fundamental questions arise here:

1. Why do learners spontaneously search for the most specific category that encompasses premises and conclusions to compute coverage?

2. Is this a strategy that is learned or is it hard-wired into the cognitive architecture?

Addressing such assumptions has become an important issue in recent induction models.

A second concern is that some aspects of coverage computation are underspecified. It seems safe to assume that only a sample of the members of broad categories like mammals is considered when computing coverage, but exactly how this sample is generated is not explained.

Perhaps most seriously, even though the notion of "similarity" is the core of similarity-coverage, the model includes no formal description of how similarity between premise and conclusion items should be computed. Instead Osherson et al., (1990) derived estimates of similarity functions from empirical similarity ratings. In this respect, the model treats similarity as a fixed property derived from object or category comparisons. As detailed in later sections, this has turned out to be a major limitation in the explanatory scope of the similarity-coverage model.

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Feature-Based Induction

Sloman's (1993) feature-based induction (FBI) model offers a more principled method for computing the similarity between premises and conclusions in inductive problems. This model was implemented as a connectionist network in which premise and conclusion items are represented by vectors of features. When presented with a set of premises that share some novel property *p*, the network encodes input unit weights that correspond to features that are shared by the premises. Argument strength or the generalization of *p* then depends on the overlap between the features of the premises and conclusion items.

The details of the generalization process are captured in the equation below. This equation describes argument strength as the activation of the unit that corresponds to novel property a *p* given premises P_1 to P_n and the conclusion *C*.

$$a_p(\mathrm{C},\, P_1 \ldots\ldots P_n) = \frac{W(P_1 \ldots\ldots P_n) \cdot A(C)}{|A(C)|^2}$$

Eq. 2: FBI Induction Model

The numerator is the dot product of the vectors that represent the features that are shared by the premises W(*P1 . . . Pn*) and the vector representing the features of the conclusion *A(C)*. The dot product is a measure of the overlap between these vectors. In calculating this overlap, as premises are added, more weight is given to non-redundant features, i.e. premise features that overlap with the conclusion but were not associated with earlier premises. In the denominator, the vertical bars represent the length of the conclusion vector, referred to as the "magnitude" of the conclusion. Hence, argument strength is proportional to the overlap between the features of premises and conclusion and inversely proportional to the amount already known about the features of the conclusion.

Consideration of conclusion magnitude in determining argument strength is a particularly novel aspect of this model. This captures the intuition that when faced with two arguments with a similar level of overlap between premise and conclusion features, property generalization will be stronger when the conclusion category has fewer known distinctive or salient features. To illustrate, consider Arguments 1 and 2 below. According to Sloman (1993), collies and horses have a similar level of feature overlap to collies and Persian cats, so that the arguments have similar numerators in Equation 2. However, it is assumed that most people know more about the distinctive features of horses than Persian cats, meaning that the magnitude of the conclusion vector for Argument 1 is larger than Argument 2. This leads to the prediction, confirmed by Sloman (1993), that Argument 2 is perceived as stronger.

$$\frac{\text{Collies have property } p}{\text{Horses have property } p}$$

Argument 1

$$\frac{\text{Collies have property } p}{\text{Persian cats have property } p}$$

Argument 2

A key difference between the FBI model and the similarity-coverage model is that FBI does not require the learner to access knowledge about hierarchical category relations. FBI treats specific and general categories in exactly the same way, decomposing them into feature vectors. Nevertheless, the feature-based model can account for many of the same inductive phenomena as similarity coverage. Premise-conclusion similarity arises because of both the numerator and denominator components of FBI. For example, premise items robins and sparrows have more features in common with the conclusion category *crows* than *penguins*. Moreover, the more distinctive conclusion penguins will have a higher magnitude than *crows*. In FBI, premise diversity and premise monotonicity effects are both explained by increases in the overlap between nonredundant features of premise and conclusion items. This overlap will generally increase as premises are added or when dissimilar (diverse) premises are presented. Likewise, more typical premises like wolves will share more features in common with superordinates like mammals than will atypical premises, leading to stronger inductive generalization.

Under some circumstances, the predictions of the FBI model diverge from similarity-coverage. The FBI model, for example, predicts an effect of inclusion similarity. This can be illustrated in Arguments 3 and 4 below:

$$\frac{\text{Birds have property } p}{\text{Robins have property } p}$$

Argument 3

$$\frac{\text{Birds have property } p}{\text{Penguins have property } p}$$

Argument 4

According to similarity-coverage both arguments should be judged as perfectly strong because the premise category birds is the same as the lowest-level category that includes both premises and conclusions (i.e. perfect coverage). FBI however predicts that there will be greater feature overlap between premise and category features in 3 than 4, and that the conclusion in 4 will have higher magnitude. Hence, the model predicts that 3 should be viewed as a stronger argument, a prediction supported by empirical ratings of argument strength (Sloman, 1993, 1998).

A problematic issue for the FBI model is that it predicts that adding premises to an argument can only have a monotonic effect on inductive argument strength (i.e. strength increases or remains the same). As mentioned earlier however, Osherson et al. (1990) reported cases of nonmonotonicity where an added premise reduced property generalization. More recent work, discussed in detail later on (e.g., Medin, Coley, Storms, & Hayes, 2003; Ransom, Perfors, & Navarro, 2016), has found further evidence of nonmonotonic induction. Sloman (1993) suggests ways that FBI could be revised to account for such findings but these modifications are largely ad-hoc and have yet to be implemented in a revised model.

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Relevance, Property Knowledge, and Flexible Similarity

Although similarity-coverage and FBI account for an impressive range of phenomena, both models rely on a “static” conception of similarity; comparisons between a given set of premise and conclusion items yield fixed similarity values. Many however, have suggested that assessments of similarity are dynamic, depending on the goals of the learner and the context in which comparisons between premises and conclusions are made (e.g., Goodman, 1972; Murphy & Medin, 1985). Ample evidence with property induction tasks supports this view.

One factor that can alter the way that similarity is computed in induction is knowledge about the properties being generalized. Heit and Rubinstein (1994), for example, compared ratings of the strength of arguments like those below.

Giraffes have cells with small amounts of zinc

Bats have cells with small amounts of zinc

Argument 5a

Sparrows have cells with small amounts of zinc

Bats have cells with small amounts of zinc

Argument 5b

Giraffes frequently travel for hours without stopping

Bats frequently travel for hours without stopping

Argument 5c

Sparrows frequently travel for hours without stopping

Bats frequently travel for hours without stopping

Argument 5d

Arguments like *5a* were rated as stronger than *5b*. An anatomical property of giraffes was judged more likely to generalize to bats than an anatomical property of sparrows. Arguments *5c* and *5d* contain the same premises and conclusions but involve a behavioral property. Here, strength ratings were reversed, with stronger generalization from sparrows to bats than from *giraffes* to *bats*. These and related findings (e.g., Shafto, Coley, & Baldwin, 2007) are clearly problematic for models with static notions of similarity. They suggest that different kinds of properties shift attention to different types of similarity (e.g., anatomical vs. behavioral) in induction. One might object that such findings only apply to cases where familiar properties are used. Even when abstract properties are used however, learners infer what these properties are likely to be (Coley & Vasilyeva, 2010; Feeney & Heit, 2011) and generalize accordingly.

Inductive inferences are also often driven by considerations that are not easily captured by any kind of straightforward similarity computation. Bright and Feeney

(2014), for example, found that people were more likely to generalize a disease property from *flies* to *frogs* than from *flies* to *ants*, even though the latter items are more similar taxonomically. This, together with a range of other findings (e.g., Hayes & Thompson, 2007; Rehder, 2009; Shafto, Kemp, Bonawitz, Coley, & Tenenbaum, 2008), suggests that people often prefer to generalize based on causal relations between premises and conclusions rather than overall similarity.

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Relevance Theory and Key Relevance Phenomena

Such findings have stimulated the development of approaches that move beyond static notions of similarity. One of the most influential approaches is relevance theory (Medin et al., 2003). To date, relevance theory has not been fully implemented as a formal model (although see the model developed by Blok, Medin, & Osherson, 2007 which shares some assumptions with relevance theory). Nevertheless, it deserves some consideration here because, (a) it led to the discovery of several new inductive phenomena that have subsequently become benchmarks for theory testing, and (b) it influenced the development of formal Bayesian and connectionist models.

Relevance theory suggests that when a property is associated with a premise, learners consider *why* this particular premise is relevant to the conclusion. When the property is unfamiliar, properties of premise and conclusion items that are highly distinctive (in an information-theoretic sense) are seen as candidates for guiding inductive generalization. For example, given the premise that "*skunks* have property *p*" and the conclusion "*zebras* have property *p*," the learner may infer that the property is "striped." Comparisons between premises can also suggest relevant relations for induction. Learning that *polar bears* and *penguins* share a property, suggests that it is associated with living in a cold climate. Learning that *grass* and *horses* share a property, suggests that it may be something transmitted via the food chain. These examples highlight that induction is not limited to consideration of taxonomic relations; thematic or causal relations are often more distinctive and hence more likely to guide inferences.

The relevance framework led to discovery of several novel phenomena that challenge many key assumptions of similarity-based models and set empirical benchmarks for more recent models. One notable finding was that the premise monotonicity effect can be reversed when premises share a distinctive feature that is not shared by a conclusion category. This *nonmonotonicity* effect is illustrated in Arguments *6a* and *6b*.

Buffalo have property X1

Argument 6a

Brown Bears, Polar Bears, Black Bears and Grizzly Bears have property X1

Buffalo have property X1

Argument 6b

Although argument *6b* has more premises with the property, people were less likely to generalize this property to the conclusion than in *6a*. It appears these additional premises led people to conclude that the property was something distinctively connected with bears. Likewise, Medin et al. (2003) found that the effects of premise diversity can be reversed by reinforcing distinctive relations between premises. For example, a property of penguins and polar bears was less likely to be generalized to other mammals than a property of polar bears and antelopes, even though the former premises were judged as more diverse. Another important finding was "conjunction fallacy by property reinforcement" (Feeney, Shafto, & Dunning, 2007; Medin et al., 2003), illustrated below. People are more likely to generalize a property from a single premise category to multiple conclusion categories that share a distinctive relation with the premises (*7a*) than to individual conclusion categories (*7b–c*) as shown below.

People from the Andes have Property J41

People from the Himalayas and People from the Alps have Property J41

Argument 7a

People from the Andes have Property J41

People from the Himalayas have Property J41

Argument 7b

People from the Andes have Property J41

People from the Alps have Property J41

Argument 7c

Parallel effects were found for inductive arguments involving causal relations. For example, Medin et al. (2003) found that a property of *sparrows* and *cats* (causally linked via a food chain), was judged less likely to generalize to other *animals* than a property of *cats* and *rhinos*, despite the greater diversity of the first pair.

It is possible that nonmonotonicity and nondiversity effects could be accommodated by adding selective attention mechanisms to the similarity-coverage and FBI models. Selective attention to distinctive features could lead to systematic changes in the way people compute the similarity of premises and conclusions (e.g., Heit & Feeney, 2005). It is harder to see however, how such a mechanism could explain conjunction fallacies. More significantly, similarity-coverage and FBI do not contain any core principles that would explain why learners would search for and attend to distinctive relations.

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Bayesian Induction Models

It is not an overstatement to say that recent theoretical progress in the field of inductive reasoning has been dominated by Bayesian models. One of the reasons these models are attractive is because they offer considerable flexibility in how people make property inferences from a given set of premises or sample of evidence. This section outlines a number of specific Bayesian accounts and examines how they have advanced understanding of inductive inference.

Heit (1998, 2000) proposed a Bayesian approach in which induction is conceived of as a process of learning which categories do or do not possess a property. The learner approaches the property induction task with a prior distribution of possible hypotheses *p(h`)* about how far a property p extends (e.g., *only sparrows* have property *p*, *all birds* have property *p*, all animals have property *p*). The exhaustive and mutually exclusive set of hypotheses is denoted by *H*. The learner also has some theory about the world that specifies the likelihood of observing some evidence *x* (e.g., a premise category that has the property) if hypothesis h were true. The likelihood is expressed as *p(h`)*. Observing a sample of evidence leads to revision of prior beliefs about the probabilities of competing hypotheses about property extension, *p(h`)*, increasing beliefs in some but weakening others. The process of belief updating follows Bayes' rule (Equation 3). The resulting posterior beliefs guide subjective judgments about the strength of an inductive argument.

$$p(h|x) = \frac{p(x|h)p(h)}{\sum_{h' \in H} p(x|h')p(h')}$$

Equation 3

An influential refinement of this approach was proposed by Tenenbaum and Griffiths (2001), who suggested that the form of the likelihood function is determined by the beliefs about the process by which the observed evidence x was generated (also see Sanjana & Tenenbaum, 2003). One possibility is that the observed evidence (e.g., sparrows have p) originated via random selection; i.e., the example was chosen randomly from a set containing instances that have the property as well as instances that do not. Such weak sampling is consistent with early Bayesian approaches to induction in cognitive science (e.g., Anderson, 1991; Heit, 1998) and machine learning (Mitchell, 1997).

Tenenbaum and Griffiths (2001), however, argue that in many learning contexts people are likely to assume strong sampling; x is sampled from the more restricted set of things that have the property (i.e., positive instances). In some cases (e.g., Shafto, Goodman, & Frank, 2012), even stronger assumptions are warranted. The instance may have been selected by a helpful agent or teacher to guide the learner's inferences (referred to as "pedagogical" or "helpful" sampling).

Tenenbaum and Griffiths (2001) formalize weak sampling by assuming that the likelihood simply reflects whether a specific hypothesis is consistent with the observed example:

$$p(x|h) = \begin{cases} 1 & \textit{if } x \in h \\ 0 & \text{otherwise} \end{cases}$$

Under strong sampling, the likelihood function is such that each observation or premise added to the sample provides more information about the true extension of a property than under weak sampling. If one assumes a uniform probability distribution over the members of *h*, then:

$$p(x|h) = \begin{cases} \dfrac{1}{|h|^n} & \textit{if } x_1, x_2 \ldots\ldots x_n \in h \\ 0 \text{ otherwise} \end{cases}$$

Here $|h|$ indicates the specificity or scope of a hypothesis and n is the number of observed examples or premises. An important implication of Equation 3 is that under strong sampling, as premise items with the target property are observed, "smaller" or more specific hypotheses (e.g., small birds have *p*, birds have *p*) will generally receive higher probabilities than more general hypotheses (e.g., animals have *p*). The effect of this "size principle" increases exponentially with the number of observed instances.

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Bayesian Explanations of Inductive Phenomena

It follows from the size principle that as one observes more instances of a category that share a property, belief in the hypothesis that the property is shared by all category members should increase. In other words, this aspect of the Bayesian models predicts the effects of premise monotonicity. It also follows that increasing the number of observed category members with a property should reduce belief that the property generalizes to other, more distant categories. This provides a ready explanation of the nonmonotonicity effects reported by Medin et al. (2003). Observing that many types of bears have a property increases belief that the property is shared by all bears, but reduces belief that it is shared by buffalos and other animals. This “tightening” of inductive inferences with additional positive instances has been firmly established in a range of property induction studies (Navarro, Dry, & Lee, 2012; Ransom et al., 2016; Xie, Hayes, & Navarro, 2018). It has also been found in other tasks that involve evidence-based inferences including word learning (Xu & Tenenbaum, 2007) and judgments about object similarity (Navarro & Perfors, 2010).

This Bayesian model also predicts the benchmark effect of premise diversity (Hayes, Navarro, Stephens, Ransom, & Dilevski, 2019). However, the Bayesian explanation of this effect differs from that provided by models like similaritycoverage. These previous accounts emphasized the impact of observing a diverse set of evidence on property generalization. The Bayesian account however emphasizes the role of nondiverse evidence in constraining hypotheses about how far a property generalizes (Hayes, Navarro, et al., 2019). Observing that many similar instances (i.e., a nondiverse set) share a property, increases the likelihood that the property does not generalize very far beyond those instances.

Bayesian induction models have also led to the discovery of novel empirical phenomena. Because this approach focuses on how observations are used to evaluate rival hypotheses, it makes predictions about the effects of negative evidence (instances that do not have a property) as well as positive evidence. For example, Voorspoels, Navarro, Perfors, Ransom, and Storms (2015) presented learners with positive premises (e.g., learning that Mozart’s music elicits alpha waves) and then asked them to evaluate the strength of a conclusion (e.g., Nirvana’s music elicits alpha waves). Subsequent presentation of negative evidence (e.g., waterfalls do not elicit alpha waves) led to an increase in belief in the original conclusion (cf. Lee, Lovibond, Hayes, & Navarro, 2019).

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The Role of Sampling Assumptions

A crucial prediction of the Bayesian account is that learners will make different kinds of inferences from the same set of observations depending on beliefs about how the information was sampled. This has been confirmed in studies where learners are presented with a common set of observations but given cover stories that imply either strong or weak (random) sampling (e.g., Hayes, Navarro, et al., 2019; Navarro et al., 2012; Ransom et al., 2016; Voorspoels et al., 2015). These studies reveal that benchmark phenomena such as premise monotonicity (Ransom et al., 2016) and diversity (Hayes, Navarro, et al., 2019), depend on an assumption of strong sampling. When learners believe that premise items were selected randomly, such effects are weakened or eliminated.

Of course, in practice, learners may be uncertain about the exact nature of the data generating process. They may view some observations as having been selected via strong sampling while other observations appear to have been generated randomly. Such cases can be accommodated by a mixture model (Navarro et al., 2012), illustrated in Equation 4. Here θ denotes the probability that a given observation is strongly sampled and $1 - \theta$ is the probability that the observation is weakly sampled. X is the set of all possible stimuli and $|X|$ counts its size. When $\theta = 0$ this model is equivalent to weak sampling; when $\theta = 1$ the model is equivalent to strong sampling. For intermediate values of θ, the model reflects a mixture of beliefs, with only some proportion θ of the observations believed to have been strongly sampled.

$$p(x|h, \theta) = \begin{cases} (1-\theta)\dfrac{1}{|\mathcal{X}|} + \theta\,\dfrac{1}{|h|} & \textit{if } x \in h \\ 0 & \text{otherwise} \end{cases}$$

Equation 4

The mixture model can capture variability in beliefs about sampling assumptions across different induction tasks or scenarios, and between individuals presented with the same scenario. Applications of the mixture model reveal that some form of strong sampling is the default assumption in most experimental contexts — learners rarely assume that the observations presented to them have been generated via a random process (Hayes, Navarro, et al., 2019; Ransom et al., 2016). Notably though, within a given experimental context, assumptions about strong sampling can vary across items and between individuals (e.g., Navarro et al., 2012).

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Inferences with Censored Samples

Bayesian models of induction have been extended to deal with situations where the sample of evidence available to the learner is subject to selective sampling or "censorship." In these cases, only some types of evidence can be observed while other evidence is systematically excluded. Such selective sampling could occur in situations where an agent "cherry picks" the data to influence the learner's inferences. For example, those who want to deny the existence of climate change may select sub-sets of temperature records to suggest a "pause" in warming trends. Selective samples of evidence can also arise through the strategies that learners use to search for information (Le Mens & Denrell, 2011) or simply because environmental constraints prevent one from obtaining large, representative samples (Hogarth, Lejarraga, & Soyer, 2015).

A handful of studies have examined whether learners incorporate information about selective sampling into their property inferences (e.g., Hayes, Banner, Forrester, & Navarro, 2019; Lawson & Kalish, 2009). In these studies, learners see a common training sample of instances that have a property (e.g., ten small birds with plaxium blood) and are asked to infer whether the property generalizes to test items that vary in similarity to the sample. Crucially, different groups are given alternative "sampling frames" or explanations of how instances in the training sample were selected. For example, in a category frame condition, learners are told that due to time/resource constraints, only a single type of animal (e.g., small birds) could have been observed in the sample (i.e. there was no opportunity to observe other animals). In a property frame condition, learners are told that the sample was selected because they were the first instances found to possess the target property (e.g., a screening test showed that they were "plaxium positive"). In the category frame condition, the absence of animals other than small birds is attributable to the selection mechanism, so the hypothesis that other animals share the novel property remains viable. In the property frame condition, the absence of instances outside the single category of small birds is more informative — suggesting that the property does not generalize beyond that category.

Hayes et al. (2019) formalized these predictions into a Bayesian framework where the posterior probability of a hypothesis h about property extension is a joint function of the prior probability of the hypothesis, the likelihood given the observations and a survivor function *S(x)* which determines what types of observations can be observed.

$$p(h|\ x,\ S) \propto S(x)p(x|h)p(h)$$

Hayes et al. (2019) found that property inferences were generally consistent with the key prediction of this model — learners were less likely to generalize a novel property beyond the sample category when sampling was constrained by a property frame as compared to a category frame (see Figure 1). Consistent with Bayesian model predictions, this "sampling frames" effect was moderated by a number of other factors. For example, the divergence in generalization gradients between category and property frame conditions shown in Figure 1 increased when learners observed more instances in the training sample.

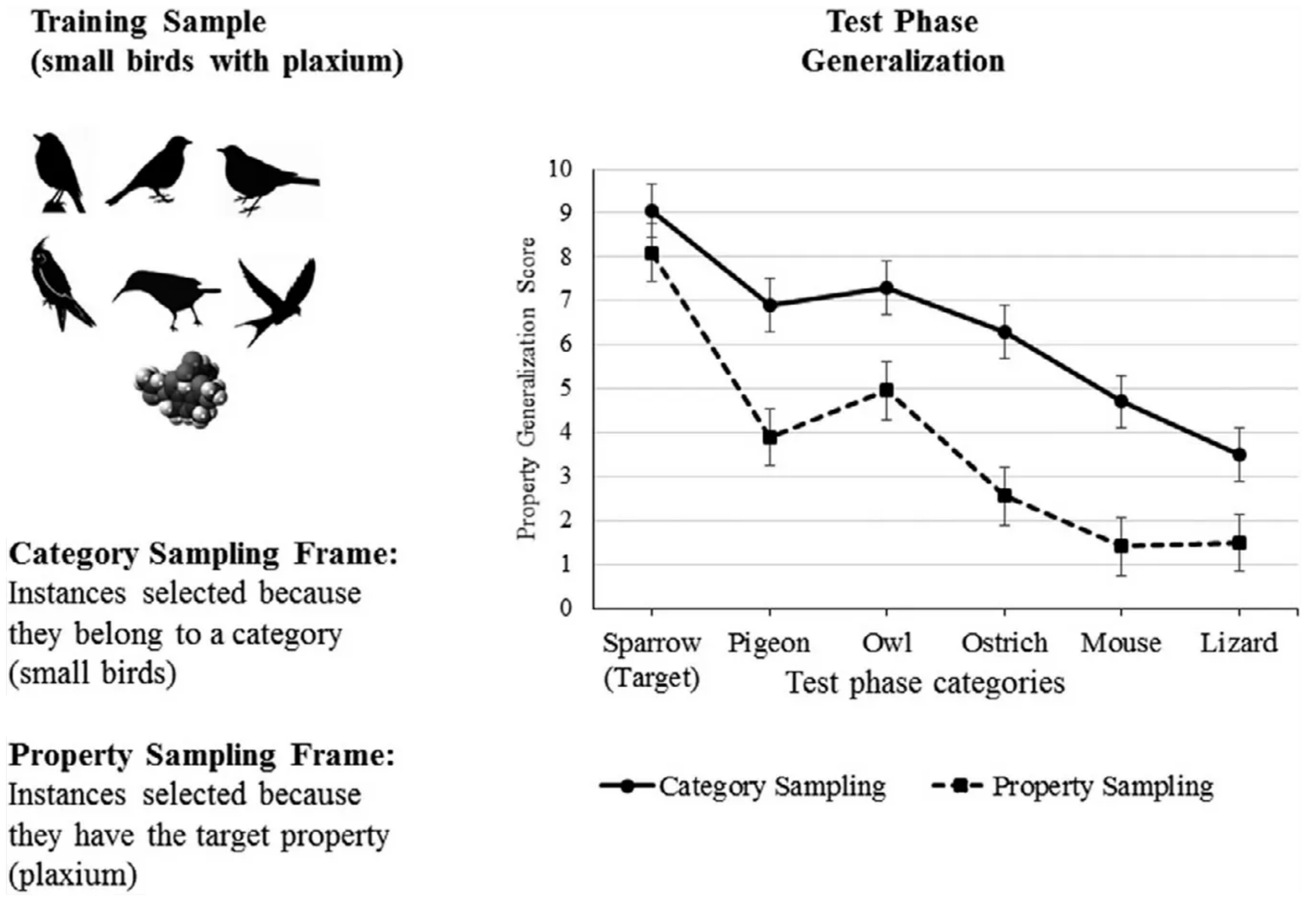

Figure 1 Illustration of the sampling frames effect (adapted from Hayes, Banner, & Navarro, 2017). All participants are presented with the same training sample (small birds that have a novel property plaxium). Category and property sampling groups are given different explanations of how the sample was selected. Those in the property sampling condition subsequently showed narrower generalization of the property to novel test items.

Given that outside the laboratory observing selectively biased or restricted samples is likely to be the norm rather than the exception, the model proposed by Hayes et al. (2019) has the potential for broad application. Future work is required however to examine how well the model accounts for inductive inference when evidence samples are subject to other types of selection mechanisms (e.g., data truncation where only quantitative properties above/below some threshold can be observed — see Feiler, Tong, and Larrick, 2012 for an example).

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Structured Bayesian Models

Adding sampling assumptions to Bayesian models has greatly increased their ability to explain the complexity and flexibility of human induction. There are some types of phenomena however, that are unlikely to be explained by variations in sampling assumptions alone. The different patterns of induction that arise when induction involves causal rather than taxonomic relations (e.g., Bright & Feeney, 2014; Hayes & Thompson, 2007; Medin et al., 2003) is one example.

Such phenomena are addressed by Bayesian models that focus on how people apply different prior beliefs about the relations that are most relevant for property induction in different learning contexts. In particular, Kemp and Tenenbaum (2009) outlined a Bayesian framework based on different types of *structured statistical models*. This class of models employs a Bayesian belief updating mechanism that has much in common with other models (e.g., Tenenbaum & Griffiths, 2001). A key innovation is that learners apply different structural representations S about the relevant relations between objects and object properties depending on the type of property being generalized.

This idea is illustrated in the top panel of Figure 2 with biological (animal) categories. When the target property is a structural biological feature (e.g., "has plaxium blood") learners represent object relations in terms of a taxonomic or hierarchical tree structure. When the property is associated with some physical property (e.g., weight), object relations are organized according to a low dimensional similarity space. When the property is causal, object relations are organized within a directed graph. Beliefs about the relevant stochastic process for transmitting properties from one object to another, T, also vary according to property type. In the taxonomic case, the process is "diffusion," where it is expected that the property will be smoothly distributed over the tree structure. Hence, for any pair of adjacent category members (e.g., *gazelles* and *giraffes* in Figure 2) it is likely that both will share the property or neither will have it. For quantitative properties, a "drift" process captures the expectation that categories towards one end of the dimension are more likely to have the property. Hence, discovering that gazelles are heavy enough to trigger a trap implies that this property generalizes to other animals that lie above it on the weight dimension. In the causal case, properties are generalized via a domain-specific causal process (e.g., predation). In the Figure 2 example, discovering that gazelles have a disease implies that the disease could be passed on to first-order predators (e.g., cheetahs) and in turn to second-order predators (e.g., *hyenas*). In each case, the prior distribution of object properties or features f is given by $p(f \mid S,T)$.

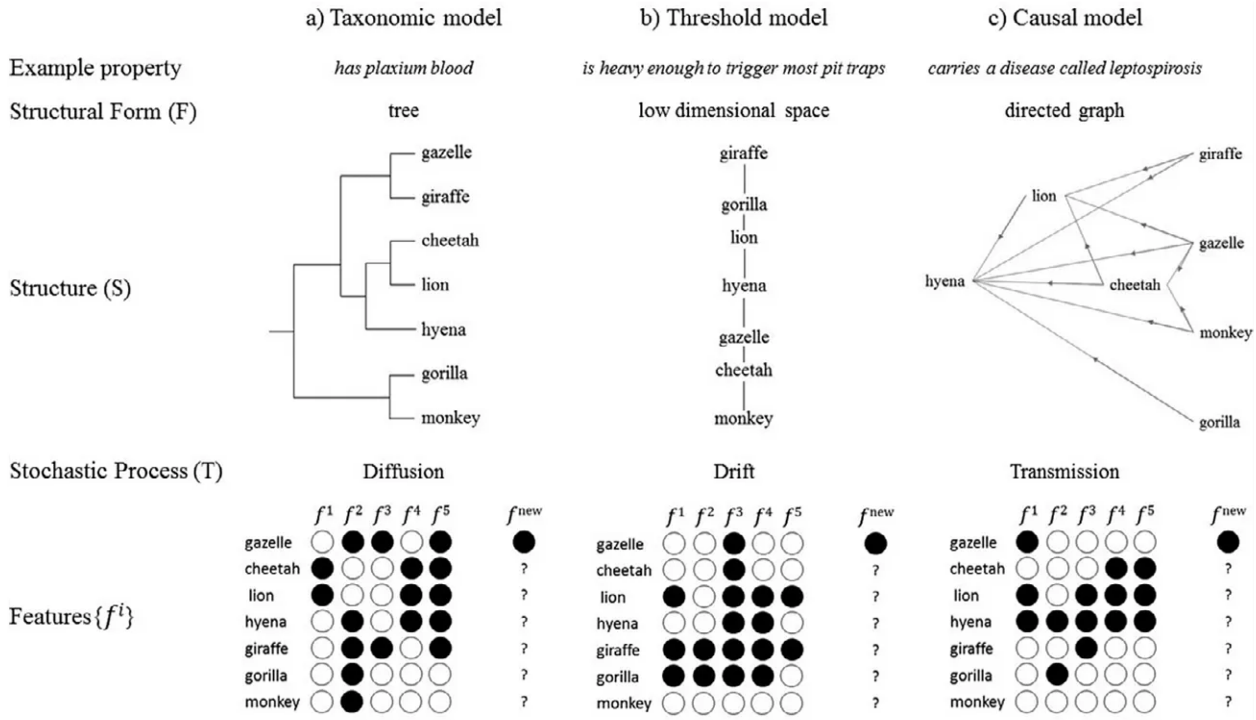

Figure 2: Examples of three structured statistical models for property induction (adapted from Kemp & Tenenbaum, 2009). Each model deals with generalization of a different type of property. Each assumes a different structure S and a stochastic process T to generate a prior distribution p(f|S, T), on properties. The bottom row shows properties (f) with high prior probability according to each model (filled circles). The inductive task is to make inferences about the extension of a novel property that has so far only been observed in a single premise category (gray circle).

The addition of the structured priors means that different patterns of inductive generalization can result from the same set of premise and conclusion categories depending on the nature of the property being generalized. For example, as shown in Figure 13.2, learning that a gazelle has some biological property (e.g., plaxium blood) should increase the likelihood that it is shared by adjacent items in the taxonomic tree (e.g., giraffe). However, learning that a gazelle passes the threshold of being "heavy enough to trigger pit traps" should increase the likelihood that this property is shared by other items that have a higher value on the weight dimension. In the case of properties that are causally transmitted (e.g., disease), learning that a gazelle has the property should increase the likelihood that known predators have the property.

The predictions of the structured Bayesian model were tested against taxonomic induction data from Osherson et al. (1990) and Smith, López, and Osherson (1992), threshold induction data from Blok et al. (2007), and causal induction data from Shafto et al. (2008). The model's overall performance was impressive (mean correlation with the data *r = 0.91*). The complete structured Bayesian model provided a better fit to the three data sets than the similaritycoverage model and simplified versions of the model that included only a single type of structured representation.

The structured representation model therefore seems like a prime candidate for future theoretical and empirical work. One limitation is that the model currently assumes random sampling of premise items, and hence cannot explain nonmonotonicity effects. This may be relatively easy to address by adding likelihood functions that reflect strong sampling like those surveyed earlier. A more fundamental challenge is to explain how people learn different structured representations, and how they recognize which representation to apply when faced with a new induction problem. Kemp and Tenenbaum (2009) outline a hierarchical Bayesian extension of their approach that deals with learning and recognizing structured representations, but this model has not yet been fully implemented or tested.

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Bayesian Induction Models: Normative or Descriptive?

Marr's (1982) influential framework for organizing theories of information processing, suggests that theorizing can take place at three distinct levels. The computational level of analysis represents an abstract and normative solution to information processing problems. The algorithmic level specifies the cognitive processes needed to execute the solution. The implementation level specifies the neural "hardware" required to implement the algorithm. Bayesian models, like those reviewed here, have often been cast as computational solutions — providing a normative or "rational" standard against which human inference can be judged. One problem with viewing Bayesian models in this way is that they can become overly flexible — by selection of appropriate priors and likelihoods, the Bayesian framework can provide an account of virtually any pattern of observed behavior (Bowers & Davis, 2012; Cassey, Hawkins, Donkin, & Brown, 2016).

This review however suggests that the application of Bayesian models to property induction has been more nuanced. It is true that these models typically begin with a high-level "normative" description (e.g., Equation 13.7 for the sampling frames problem). When applied to specific induction tasks however, such models have often incorporated more "algorithmic" assumptions about how people process information. For example, the key role of sampling assumptions in these models implies that learners are engaged in effortful interpretation of the social and environmental mechanisms that generate observations. This has led some to argue that the sorts of Bayesian models reviewed in this article sit somewhere between Marr's computational and algorithmic levels (Griffiths, Lieder, & Goodman, 2015) or that they should be regarded as descriptive rather than normative theories (McKenzie, 2003; Tauber, Navarro, Perfors, & Steyvers, 2017).

Such an argument seems reasonable. However, there is much work to be done to flesh out the algorithmic details of Bayesian induction models. Given the potentially large number of specific hypotheses that could be considered for even the simplest induction problem, it is clear that learners rely on some form of approximation of Bayesian probabilistic calculations. The details of these approximations however are still a matter of some debate (cf. Gershman & Beck, 2018; Sanborn & Chater, 2016; Shi, Griffiths, Feldman, & Sanborn, 2010). A related challenge is incorporating human limitations in computation, attention, and memory into the processes of retrieving priors, considering sampling processes, and revising beliefs as new observations are made (e.g., Frank, Goldwater, Griffiths, & Tenenbaum, 2010; Sanborn & Chater, 2016). In other words, while Bayesian models have advanced understanding of the principles by which people can combine existing beliefs with new observations in induction, the details of this learning and inference process have yet to be specified.

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Connectionist Models of Semantic Cognition

This article has already dealt with one type of connectionist model of induction — Sloman's (1993) FBI. This section discusses a connectionist framework with considerably greater scope. Rogers and McClelland (2004, 2014) describe a connectionist approach to semantic cognition that can explain a range of induction phenomena including shifts in generalization patterns across different learning contexts (e.g., Heit & Rubenstein, 1994; Medin et al., 2003). Part of their model is illustrated in Figure 13.3. This is a feedforward network consisting of input layers, corresponding to objects and their relations, a representation layer, a hidden unit layer, and an output layer. The units in the input layers project to multiple units in the intermediate layers through weighted connections, and units in the hidden layer project to multiple output units. Note that the relation layer contains units that respond to a variety of possible object relations including structural relations ("HAS", "IS"), behavioral relations ("CAN"), and taxonomic relations ("IS A").

The network is trained by presenting correct pairings of conceptual input (e.g., "an oak HAS") and output (e.g., "bark", "roots"). In this training example, the input units, "oak" and "HAS" are activated and this activity is fed forward through hidden units to output units. Output unit activation is then compared to the correct output (i.e., activation of "bark" and "roots" should be 1 and activation of other units should be 0). Connection weights are adjusted by exposure to training exemplars to reduce the error between the correct and obtained activations (see Rogers & McClelland, 2004, for details of the leaning algorithms applied to unit weights). Error back-propagates through the network, so that changes in unit weights will spread beyond the given input to affect related conceptual representations. For example, if the network predicts incorrectly that "an oak HAS petals," the changes in activation weights due to the error will affect representation units for pines as well as oaks (this *backpropagation* process is not illustrated in Figure 3).

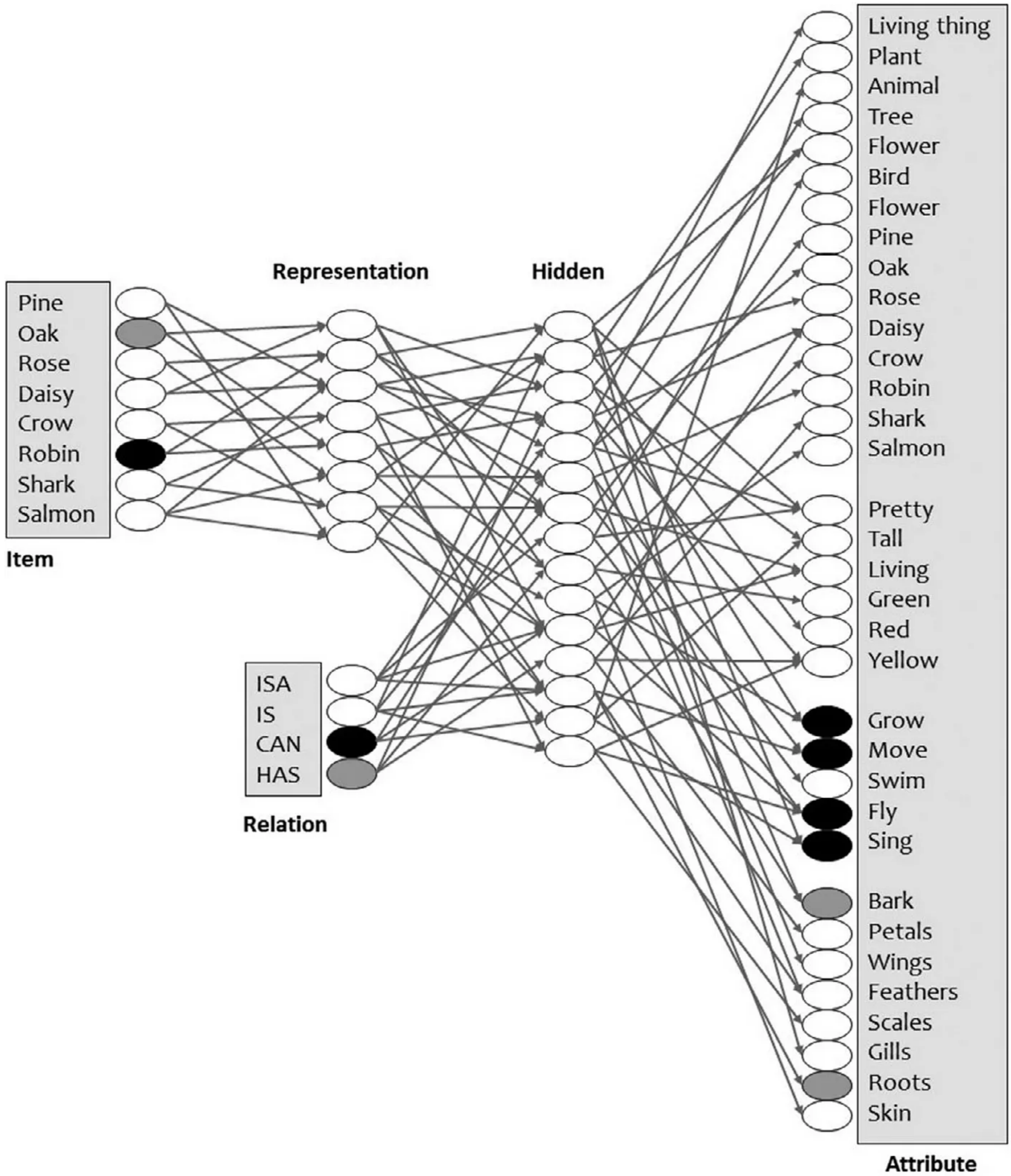

Figure 3: Illustration of a connectionist network that can learn taxonomic structures and make inductive inferences (adapted from McClelland & Rogers, 2003). Inputs (items, relations) are presented on the left and network activation propagates from left to right. Network activation is illustrated for two item-relation pairs (an oak HAS…; a robin CAN…;).

Before training, activation weights in the network are small and randomly distributed. Rogers and McClelland (2004) showed that, after extensive training, their network could learn to differentiate the properties of different animals and grouped together animals with similar properties in something approaching a taxonomic tree. Crucially, once trained, the network can make inferences about the generalization of novel properties. In many cases, these mimic the inferences of human reasoners. For example, when taught that “a robin CAN queen,” the model predicts that this novel property is likely to be shared by similar birds. The model can also simulate changes in patterns of induction due to knowledge about different types of properties (e.g., biological vs. behavioral properties in Arguments IIIa–IIId). This can arise because the network is sensitive to patterns of “coherent covariation” between objects, conceptual domains, relations and observed properties. For example, the model learns that taxonomically similar objects share many biological features, whereas behavioral features often covary with different factors such as predation or habitat.

Rogers and McClelland (2004) suggest a similar explanation for why causal features have high salience in property induction — this is the result of the strong covariation between observed surface or structural features and underlying causes. For example, features such as wings, feathers, and hollow bones frequently co-occur because they all reflect part of the evolved ability to fly. Hence, the priority given to causal relations in induction simply reflects prior experience that such relations are highly predictive of many other features.

Connectionist models are interesting because they explain many aspects of induction that appear to rely on high-level conceptual knowledge without assuming explicit representation of such knowledge. The absence of such representations, however, means the networks can only revise their “knowledge” about conceptual relations via extensive experience and feedback with individual instances. Hence, they have difficulty in explaining why patterns of inductive inference can shift dramatically when different explanations are given for the origins of a set of training instances (e.g., selected randomly vs. selected by a helpful agent), or when different structured relations are invoked for a given set of premises and conclusions (cf. Figure 2). Having explicit representations of relations between objects has other benefits over the connectionist approach, in that such representations support knowledge transfer. For example, if you are told that panthers are located in the same part of the taxonomic tree as cheetahs and lions, you can readily make inferences about the property of this instance without further learning.

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Challenges and New Frontiers for Induction Models

- **Individual and Developmental Differences**

A key theme in this review is the flexibility of the inductive process. Patterns of property generalization can change depending on the knowledge domain, the nature of the property being generalized, and one's beliefs about how the inductive premises were generated. Given this flexibility, it is surprising that little attention has been paid to individual differences in inductive inference. There is some evidence that such differences do exist. Feeney (2007), for example, observed that sensitivity to premise diversity and monotonicity was correlated with general cognitive ability. Navarro et al. (2012) reported considerable individual variation in the θ parameter that reflects belief in strong sampling. The origins of these differences and their stability over time and across tasks however, remain unknown.

A related issue is developmental change in inductive processes. Starting with the seminal work of Carey (1985) and Gelman and Markman (1986), there has been extensive study of how property induction develops over early- and midchildhood (see Fisher, 2015 for a review). There have been some attempts to apply models such as similarity-coverage (e.g., López, Gelman, Gutheil, & Smith, 1992) and Bayesian approaches (e.g., Bonawitz & Shafto, 2016) to children's induction. However, more work is needed to specify how key processing parameters in these models change with development.

- **Extending Models of Inductive Reasoning to Other Cognitive Domains**

An exciting possibility is that models of induction could be extended to explain other forms of inference and decision-making. Kemp and Jern (2013) analyzed the structure of a variety of inference problems, highlighting the commonalities between property induction and other tasks such as categorization (e.g., Hendrickson, Perfors, Navarro, & Ransom, 2019) and category construction (e.g., Medin, Wattenmaker, & Hampson, 1987). Kemp and Jern's taxonomy suggests that the models reviewed in this article can provide insight into the cognitive mechanisms that underlie these tasks.

An extension of some computational models of induction to other task domains has already begun. One recent advance is the development of more general reasoning models that encompass induction as well as other forms of reasoning. Traditionally, a hard distinction has been drawn between inductive reasoning and deductive reasoning. The goal of deduction is to infer whether an inference is deductively valid or necessarily follows from given premises. For example, knowing that mammals have enzyme X, and that horses are mammals, it necessarily follows that horses must also have this enzyme. Responses to such deductive problems are often thought to be due to a slow analytic processing system that differs qualitatively from the processes involved in probabilistic reasoning and inductive inference (Evans & Stanovich, 2013; Handley & Trippas, 2015). This distinction has often been maintained in formal models and computer simulations that incorporate separate modules for reasoning via logical rules and for inductive reasoning (e.g., Sun, 1995; Sun & Zhang, 2006).

A number of lines of work however have begun to challenge these approaches. In an extensive program of theory and research, Oaksford and Chater (2007, 2013) proposed that both deduction and induction are driven by a Bayesian process of assessing the conditional probability of a conclusion given the argument premises. Others have used a signal-detection framework to examine whether both induction and deduction can be explained using a single dimension for evaluating argument strength (e.g., Heit & Rotello, 2010; Stephens, Dunn, & Hayes; 2018). Stephens et al. (2018), for example, developed a model that assumes people use a single process for assessing the strength of arguments in inductive and deductive reasoning tasks, but that the decision criteria for responding can differ across tasks. This model can account for much of the data that has previously been seen as supporting the notion of separate processing systems (Hayes, Stephens, Ngo, & Dunn, 2018; Hayes, Wei, Dunn, & Stephens, 2019; Stephens, Matzke, & Hayes, 2019).

The potential reach of induction models is further highlighted by the finding that inductive principles operate when people generalize learned fear responses. Dunsmoor and Murphy (2014), for example, showed that fear generalization following pairing of stimuli belonging to natural categories (e.g., birds) with electric shock, depends on the typicality of those stimuli. Lee, et al. (2019) go further, showing that a Bayesian model incorporating strong sampling assumptions, can explain patterns of human fear generalization.

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Conclusion

This review highlights the progress that has been made in computational modeling of the processes that drive inductive reasoning, over the past three decades. There have been important advances in both the formal complexity and the explanatory scope of such models. One caveat is that much of this work has focused on demonstrating that a given model provides a good account of the induction data rather than carrying out systematic comparisons between a candidate model and its rivals (but see Kemp & Tenenbaum, 2009 for a notable exception). As induction models proliferate, there will be greater need for explicit model comparisons that take account of differences in computational complexity and flexibility. The most important principle for deciding on the best model of induction, however, will be not whether it accounts for known phenomena but whether it can generate and explain novel (and preferably counterintuitive) patterns of inductive inference.

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Selected References (In Non-Alphabetical Order):

Carnap, R. (1968). *Inductive logic and inductive intuition*. In I. Lakatos (Ed.), Studies in Logic and the Foundations of Mathematics (vol. 51, pp. 258–314). Amsterdam:
Elsevier.

Collins, A. & Michalski, R. (1989). *The logic of plausible reasoning: a core theory*. Cognitive Science, 13(1), 1–49.

Sun, R. (1995). *Robust reasoning: integrating rule-based and similarity-based reasoning*. Artificial Intelligence, 75, 241–295.

Sun, R., & Zhang, X. (2006). *Accounting for a variety of reasoning data within a cognitive architecture*. Journal of Experimental and Theoretical Artificial Intelligence, 18(2), 169–191.

Rips, L. J. (1975). *Inductive judgements about natural categories*. Journal of Verbal Learning & Verbal Behavior, 14, 665–681.

Feeney, A. (2017). *Forty years of progress on category-based inductive reasoning*. In L. J. Ball & V. A. Thompson (Eds.), **International Handbook of Thinking and Reasoning** (pp. 167–185). London: Routledge.

Hayes, B. K., & Heit, E. (2018). *Inductive reasoning 2.0*. Wiley Interdisciplinary Reviews Cognitive Science, 9(3), 1–13, e1459.

Osherson, D. N., Smith, E. E., Wilkie, O., & Lopez, A. (1990). *Category-based induction*. Psychological Review, 97, 185–200.

Nisbett, R. E., Krantz, D. H., Jepson, C., & Kunda, Z. (1983). *The use of statistical heuristics in everyday inductive reasoning*. Psychological Review, 90, 339–363.

Sloman, S. A. (1993). *Feature-based induction*. Cognitive Psychology, 25, 231–280.

Sloman, S. A. (1998). *Categorical inference is not a tree: the myth of inheritance hierarchies*. Cognitive Psychology, 35, 1–33.

Ransom, K. J., Perfors, A., & Navarro, D. J. (2016). *Leaping to conclusions: why premise relevance affects argument strength*. Cognitive Science, 40, 1775–1796.

Medin, D. L., Coley, J. D., Storms, G., & Hayes, B. K. (2003). *A relevance theory of induction*. Psychonomic Bulletin & Review, 10, 517–532.

Goodman, N. (1972). *Seven strictures on similarity*. In N. Goodman, **Problems and Projects** (pp. 437–447). Indianapolis, IN: Bobbs-Merrill.

Murphy, G. L., & Medin, D. L. (1985). *The role of theories in conceptual coherence.* Psychological Review, 92(3), 289–316.

Shafto, P., Coley, J. D., & Baldwin, D. (2007). *Effects of time pressure on context-sensitive property induction*. Psychonomic Bulletin & Review, 14, 890–894.

Coley, J. D., & Vasilyeva, N. Y. (2010). *Generating inductive inferences: premise relations and property effects*. Psychology of Learning and Motivation: Advances in Research and Theory, 53, 183–226.

Feeney, A., & Heit, E. (2011). *Properties of the diversity effect in category-based inductive reasoning*. Thinking & Reasoning, 17, 156–181.

Bright, A. K., & Feeney, A. (2014). *The engine of thought is a hybrid: roles of associative and structured knowledge in reasoning*. Journal of Experimental Psychology: General, 143(6), 2082–2102.

Hayes, B. K., & Thompson, S. P. (2007). *Causal relation and feature similarity in children's inductive reasoning*. Journal of Experimental Psychology: General, 136, 470–484.

Rehder, B. (2009). *Causal-based property generalization*. Cognitive Science, 33, 301–343.

Shafto, P., Kemp, C., Bonawitz, E. B., Coley, J. D., & Tenenbaum, J. B. (2008). *Inductive reasoning about causally transmitted properties*. Cognition, 109, 175–192.

Blok, S. V., Medin, D. L., & Osherson, D. N. (2007). *Induction as conditional probability judgment*. Memory & Cognition, 36(6), 1353–1364.

Heit, E., & Feeney, A. (2005). *Relations between premise similarity and inductive strength*. Psychonomic Bulletin & Review, 12(2), 340–344.

Heit, E. (1998). *A Bayesian analysis of some forms of inductive reasoning*. In M. Oaksford & N. Chater (Eds.), **Rational Models of Cognition** (pp. 248–274). Oxford: Oxford University Press.

Anderson, J. R. (1991). *The adaptive nature of human categorization*, Psychological Review, 98, 409–429.

Griffiths, T. L., Lieder, F., & Goodman, N. D. (2015). *Rational use of cognitive resources: level of analysis between computational and the algorithmic*. Topics in Cognitive Science, 7, 217–229.

Sanjana, N. E., & Tenenbaum, J. B. (2003). *Bayesian models of inductive generalization*. In M. I. Jordan, Y. LeCun, & S. A. Solla (Eds.), Advances in Neural Information Processing Systems (pp. 59–66). Cambridge, MA: MIT Press.

Mitchell, T. (1997). **Machine Learning**. London: McGraw-Hill.

Tenenbaum, J. B., & Griffiths, T. L. (2001). *Generalization, similarity, and Bayesian inference*. Behavioral and Brain Sciences, 24, 629–640.

Shafto, P., Goodman, N. D., & Frank, M. C. (2012). *Learning from others: the consequences of psychological reasoning for human learning*. Perspectives on Psychological Science, 7(4), 341–351.

Kemp, C., & Tenenbaum, J. B. (2009). *Structured statistical models of inductive reasoning*. Psychological Review, 116, 20–58.

Hayes, B. K., Banner, S., & Navarro, D. J. (2017). *Sampling frames, Bayesian inference and inductive reasoning*. In G. Gunzelmann, A. Howes, T. Tenbrink, & E. Davelaar (Eds.), Proceedings of the 39th Annual Meeting of the Cognitive Science Society (pp. 488–493). Austin, TX: Cognitive Science Society.

Hayes, B. K., Banner, S., Forrester, S., & Navarro, D. J. (2019). *Selective sampling and inductive inference: drawing inferences based on observed and missing evidence*. Cognitive Psychology, 113, 101221.

Marr, D. (1982). **Vision**. San Francisco, CA: W. H. Freeman.

Sanborn, A. N., & Chater, N. (2016). *Bayesian brains without probabilities*. Trends in Cognitive Sciences, 20(12), 883–893.

McClelland, J. L., & Rogers, T. T. (2003). *The parallel distributed processing approach to semantic cognition*. Nature Reviews Neuroscience, 4(4), 310–322.

Rogers, T. T., & McClelland, J. L. (2004). **Semantic Cognition: A Parallel Distributed Processing Approach**. Cambridge, MA: MIT Press.

Rogers, T. T., & McClelland, J. L. (2014). *Parallel distributed processing at 25: further explorations in the microstructure of cognition*. Cognitive Science, 38, 1024–1077.

Bonawitz, E., & Shafto, P. (2016). *Computational models of development, social influences*. Current Opinion in Behavioral Sciences, 7, 95–100.

Fisher, A. V. (2015). *Development of inductive generalization*. Child Development Perspectives, 9(3), 172–177.

Hendrickson, A. T., Perfors, A., Navarro, D. J., & Ransom, K. (2019). *Sample size, number of categories and sampling assumptions: exploring some differences between categorization and generalization*. Cognitive Psychology, 111, 80–102.

Kemp, C., & Jern, A. (2013). *A taxonomy of inductive problems*. Psychological Bulletin and Review, 21, 23–46.

Medin, D. L., Wattenmaker, W. D., & Hampson, S. E. (1987). *Family resemblance, conceptual cohesiveness, and category construction*. Cognitive Psychology, 19, 242–279.

Stephens, R. G., Dunn, J. C., & Hayes, B. K. (2018). *Are there two processes in reasoning? The dimensionality of inductive and deductive inferences*. Psychological Review, 125(2), 218–244.

Sun, R., & Zhang, X. (2006). *Accounting for a variety of reasoning data within a cognitive architecture*. Journal of Experimental and Theoretical Arti”cial Intelligence, 18(2), 169–191.

Stephens, R. G., Matzke, D., & Hayes, B. K. (2019). *Disappearing dissociations in experimental psychology: using state-trace analysis to test for multiple processes*. Journal of Mathematical Psychology, 90, 3–22.

Lee, J. C., Lovibond, P. F., Hayes, B. K., & Navarro, D. (2019). *Negative evidence and inductive reasoning in generalization of associative learning*. Journal of Experimental Psychology: General, 148, 289–303.

Dunsmoor, J. E., & Murphy, G. L. (2014). *Stimulus typicality determines how broadly fear is generalized*. Psychological Science, 25, 1816–1821.

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Alireza Dehbozorgi

https://www.linkedin.com/in/alireza-dehbozorgi-8055702a/

Email: ***alirezadehbozorgi83@yahoo.com***

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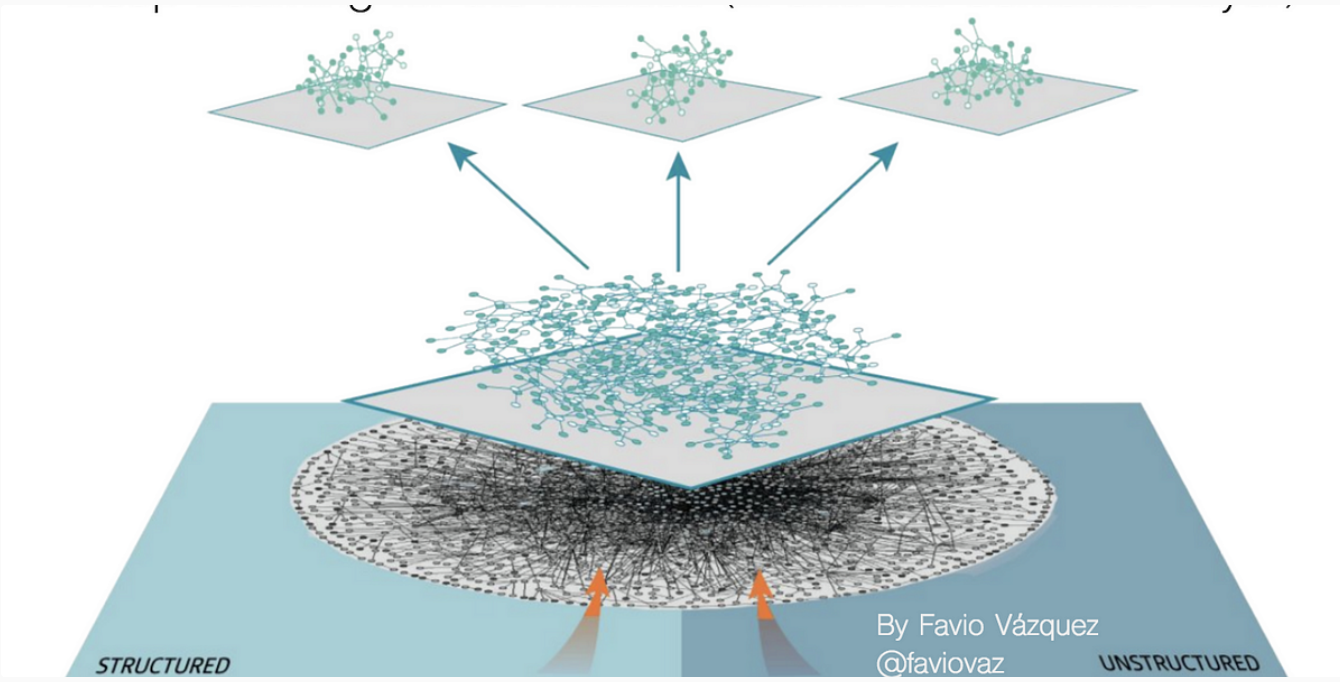

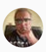

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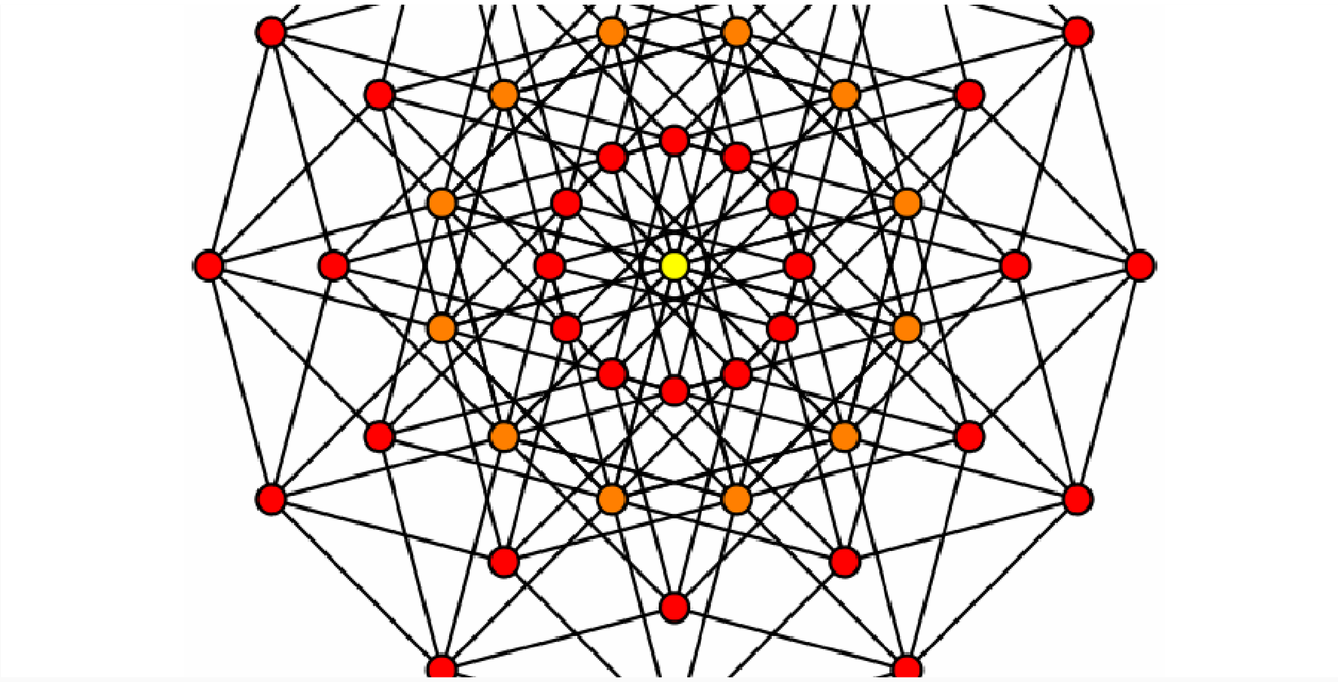

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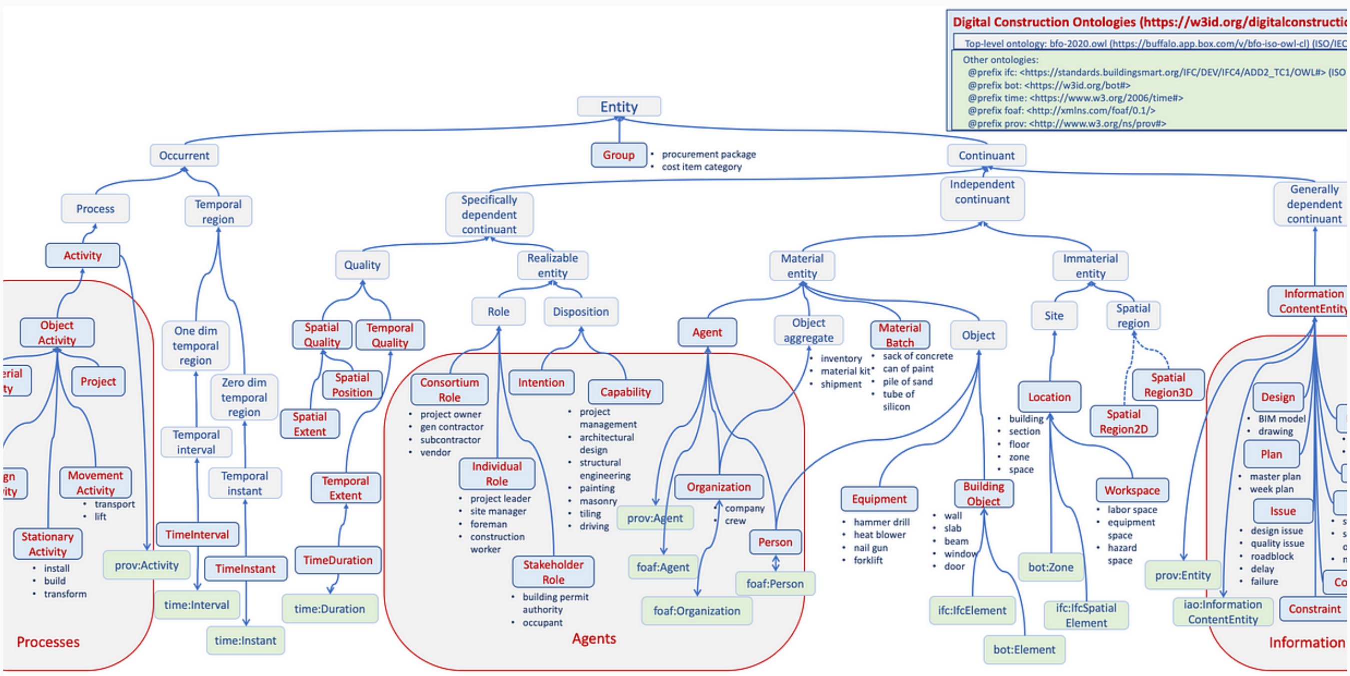

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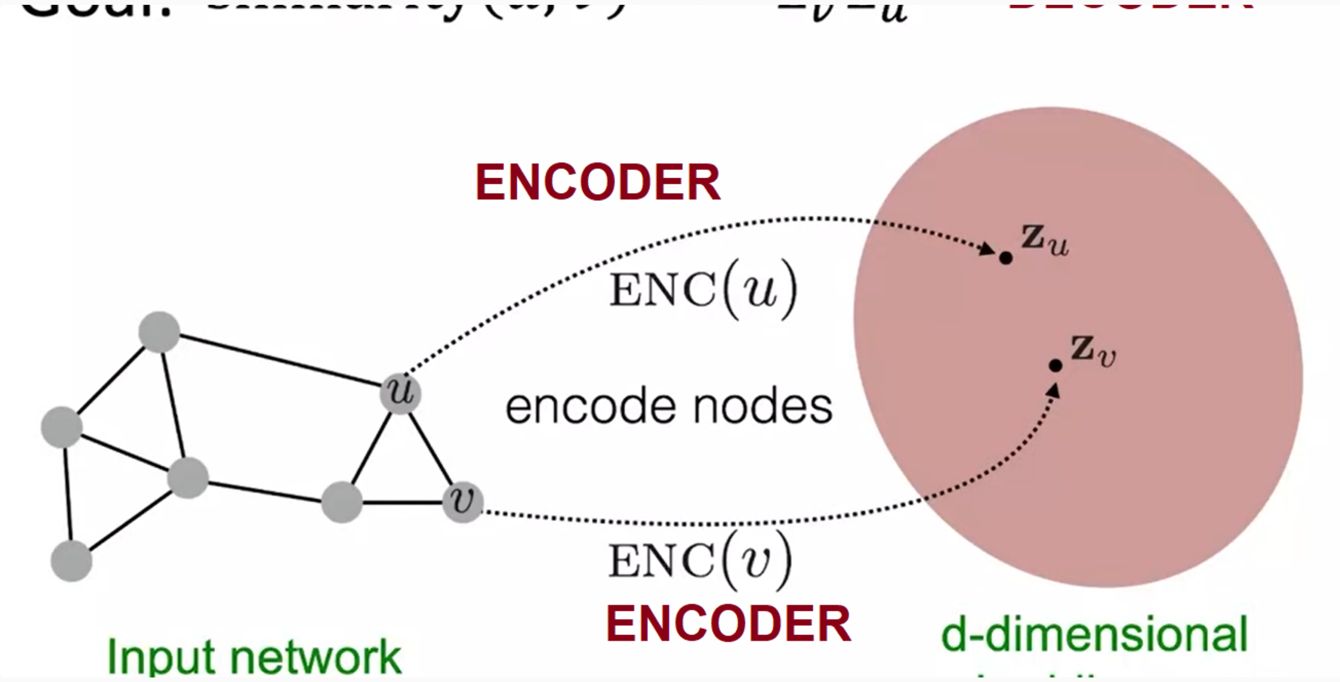

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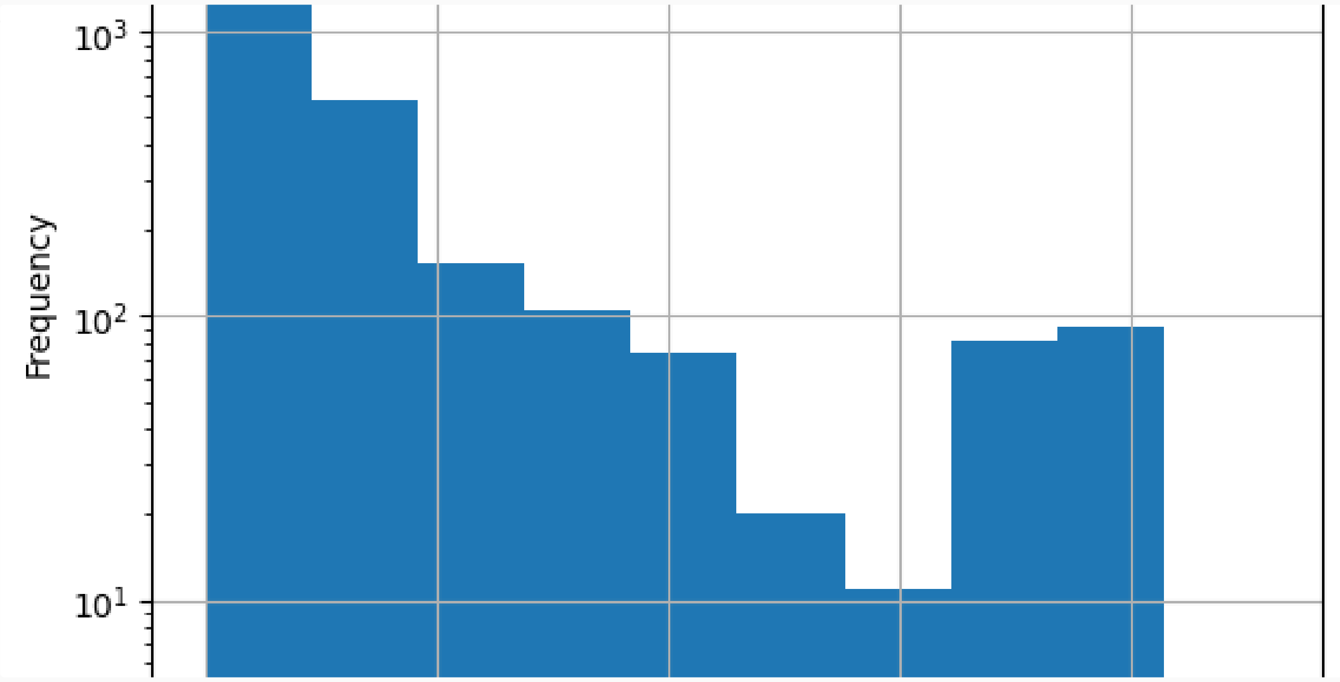

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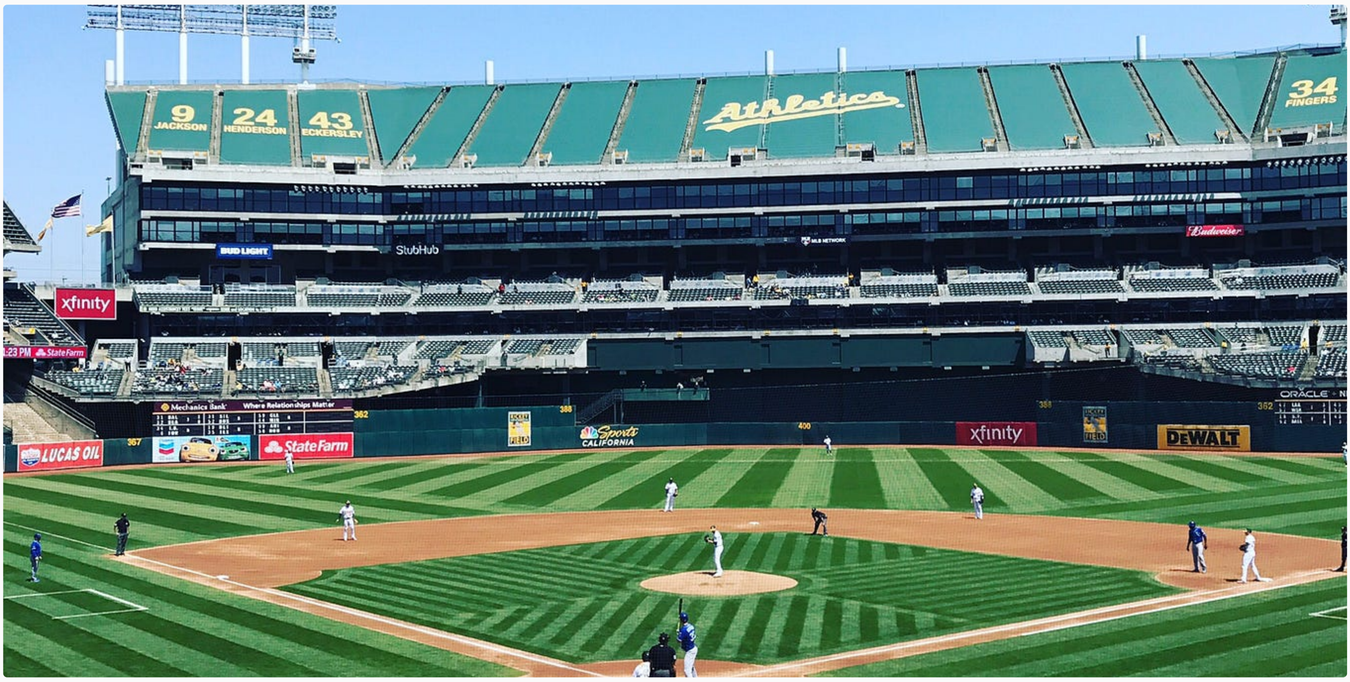

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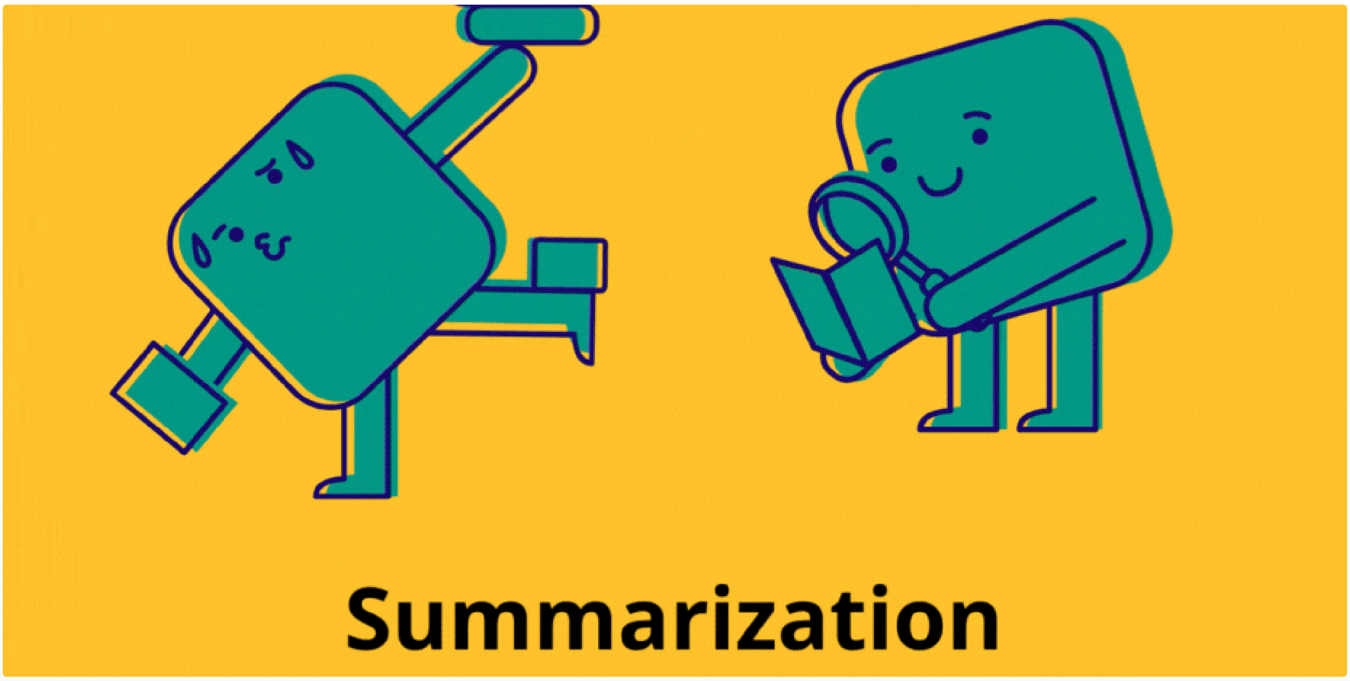

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