Sharif University of Technology

Department of Industrial Engineering

Operations Research I Project

Supervisor:

Dr. kourosh eshghi

Advisor:

Bahare mahmoudi

By:

Alireza dehghan

Amirali omidvar

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Indexes:

i: represents the type of commodity.

j: represents number of repair center.

k: represents number of collection center.

M: the big M that represents a large number compared to other values.

Parameters:

D_{ik} = Demand for commodity i in collection center k.

TC_{jk} = Transportation cost for items between collection center k and repair center j.

TCO_j = The optimal cost of transportation of parts and replacement coming from manufacturing centers to repair center j.

E_j = Coefficient of cost of installing each unit of equipment in repair center j.

Decision Variables:

X_j = number of equipment in a repair center j.

P_{ijk} = number of item i in repair center j coming from collection center k.

y_{jk} = if repair center j is connected to collection center k.

h_{ijk} = a virtual variable used for linearizing the model.

Assumptions:

1. Transportaion cost of parts between manufacturing center and repair center is same as replacements.

Nonlinear Model Objective Function:

$$Min\ Z = \sum_{j=1}^{8} X_j E_j + 2\sum_{j=1}^{8}\sum_{k=1}^{20}\sum_{i=1}^{3} TC_{jk} y_{jk} P_{ijk} + 0.5\sum_{j=1}^{8}\sum_{k=1}^{20}\sum_{i=1}^{3} TCO_j y_{jk} P_{ijk}$$

s.t.

$$X_j \leq 130$$

$$\sum_{j=1}^{8} X_j \leq 269$$

$$\forall k : \sum_{j=1}^{8} y_{jk} = 1$$

$$\forall j : 0.8\,(8\sum_{k=1}^{20} y_{jk} P_{1jk} + 10\sum_{k=1}^{20} y_{jk} P_{2jk} + 12\sum_{k=1}^{20} y_{jk} P_{3jk}) \leq 117.5 X_j$$

$$\forall k, \forall i : \sum_{j=1}^{8} y_{jk} P_{ijk} = D_{ik}$$

$$X_j, P_{ijk} \geq 0\ ,\ y_{jk} = 0{,}1$$

Linear Model Objective Function:

$$Min\ Z = \sum_{j=1}^{8} X_j E_j + 2\sum_{j=1}^{8}\sum_{k=1}^{20}\sum_{i=1}^{3} TC_{jk} h_{ijk} + 0.5\sum_{j=1}^{8}\sum_{k=1}^{20}\sum_{i=1}^{3} TCO_j h_{ijk}$$

s.t.

1. $X_j \leq 130$

2. $\sum_{j=1}^{8} X_j \leq 269$

3. $\forall k : \sum_{j=1}^{8} y_{jk} = 1$

4. $\forall j : 0.8\,(8\sum_{k=1}^{20} h_{1jk} + 10\sum_{k=1}^{20} h_{2jk} + 12\sum_{k=1}^{20} h_{3jk}) \leq 117.5X_j$

5. $\forall k, \forall i : \sum_{j=1}^{8} h_{ijk} = D_{ik}$

6. $\forall i, \forall k, \forall j :\ h_{ijk} = y_{jk} P_{ijk}$
7. $\forall i, \forall k, \forall j :\ h_{ijk} \leq y_{jk} M$
8. $\forall i, \forall k, \forall j :\ h_{ijk} \geq -y_{jk} M$
9. $\forall i, \forall k, \forall j :\ h_{ijk} \leq P_{ijk} + (1 - y_{jk})M$
10. $\forall i, \forall k, \forall j :\ h_{ijk} \geq P_{ijk} - (1 - y_{jk})M$

$X_j, P_{ijk} \geq 0\ ,\ y_{jk} = 0{,}1$

Constraints

1. Limits number of equipment units installed in a given repair center.

2. Represents the limited number equipment units.

3. Each collection center must be connected to only one repair center.

4. Represent annual time limitation of repair center j with X_j equipment units.

5. Sum of all outgoing commodity of collection center must be equal to demand of collection center.

6. Linearization of $y_{jk}P_{ijk}$ with help of virtual variable h_{ijk}.

Notes

1. Since there are no limitations on how many parts and replacements each manufacturing center can provide, it is feasible to assume that a repair center can only use one manufacturing center for receiving parts and replacements. So it is optimal to choose the manufacturing center that has the least transportation cost to a given a repair center for that repair center. (reason for defining parameter TCO_j)

2. There is a possibility that in solving process, repair center j be active $(\forall k, \exists y_{jk} = 1)$ but the number of equipment units (X_j) be zero. Although this in practical terms is wrong, in theory does not make any problems. Since of X_j is zero the because of time constraint, P_{ijk} has to be zero for any given type of commodity.

3. Another similar problem can happen when $P_{ijk} > 0\ ,\ \forall k\ y_{jk} = 0$. since these two variables are linearized into one variable, this doesn't cause any issues.

Phase II:

We inputted the mathematical model of the previous Phase, into the Lingo software using the appropriate syntax. Outcomes of this process are briefly reported as below to observe.

```
Global optimal solution found.
Objective value:                              145982.8
Objective bound:                              145982.8
Infeasibilities:                              0.000000
Extended solver steps:                               0
Total solver iterations:                           231
Elapsed runtime seconds:                          0.17

Model Class:                                      MILP

Total variables:                  1128
Nonlinear variables:                 0
Integer variables:                 160

Total constraints:                2506
Nonlinear constraints:               0

Total nonzeros:                   6920
Nonlinear nonzeros:                  0
```

Worth noticing that in Lingo, to use a linear optimization algorithm (solution method), there is no need to utilize constraint below (existing in the mathematical model), because it is unnecessary as the algorithm already considers it through the remaining constraints present in the mathematical model and takes it into account.

$$\boxed{6.\ \forall i, \forall k, \forall j:\ h_{ijk} = y_{jk} P_{ijk}}$$

- The results of the sensitivity analysis from this part are attached in the zip file.

For the next part of this section of project We have considered the following assumptions:

1. Among the coefficients of the objective function, one of them (the cost of each unit of repair equipment) is considered arbitrarily (E_1).

$$Min\ Z = \boxed{\sum_{j=1}^{8} X_j E_j} + 2\sum_{j=1}^{8}\sum_{k=1}^{20}\sum_{i=1}^{3} TC_{jk} y_{jk} P_{ijk} + 0.5\sum_{j=1}^{8}\sum_{k=1}^{20}\sum_{i=1}^{3} TCO_j y_{jk} P_{ijk}$$

In fact, if we separate the summation in the objective function and expand it or open it up, it will be as follows:

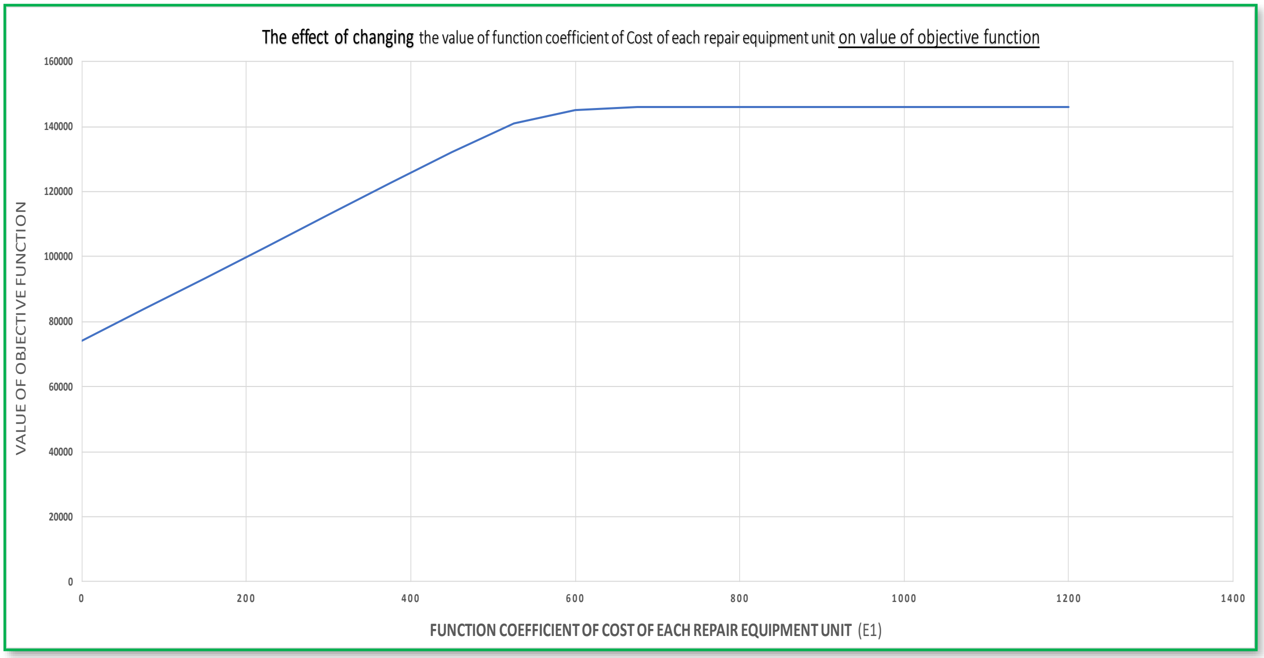
$$X_1E_1 + X_2E_2 + X_3E_3 + \cdots + X_8E_8$$

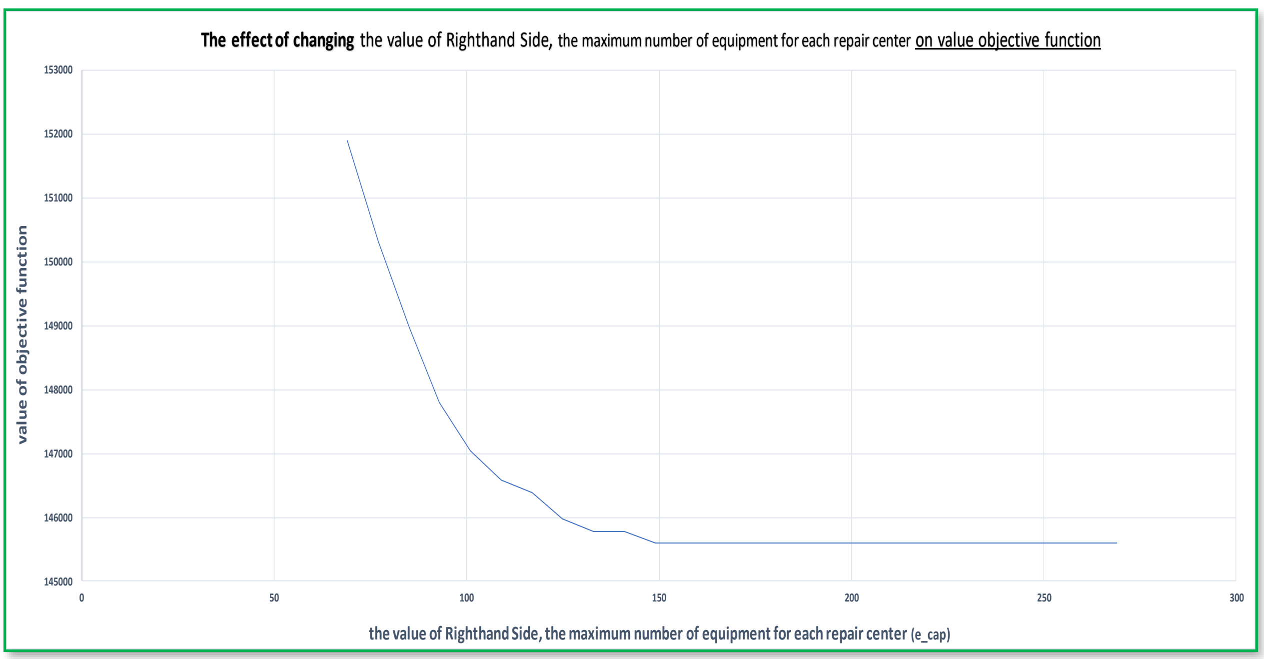
According to the question askes, it is only one coefficient of the desired objective function to consider the effects of its changes on the value of the objective function that we have arbitrarily considered it as E_1.

2. In order to contemplate the impact of altering this coefficient, all other circumstances are assumed to remain constant and unaltered.

3. To depict the plot, it is presumed that we have meticulously assigned numerical values (within a sensible range and evenly spaced) and documented the outcome in an Excel spreadsheet, which is enclosed within the sent zip file.

4. Due to the time limit and the limitation of the software's capacity to render a three-dimensional diagram, two separate and independent diagrams have been drawn. These visual aids vividly depict the impact and effect that each variable on the right-hand side (representing the maximum allowable allocation of equipment per repair center) and the coefficient of the objective function (reflecting the cost per unit of repair equipment) exerts on the overall value of said objective function.


The effect of changing the value of function coefficient of Cost of each repair equipment unit on value of objective function
VALUE OF OBJECTIVE FUNCTION
160000
140000
120000
100000
80000
60000
40000
20000
0
0
200
400
600
800
1000
1200
1400
FUNCTION COEFFICIENT OF COST OF EACH REPAIR EQUIPMENT UNIT (E1)


The effect of changing the value of Righthand Side, the maximum number of equipment for each repair center on value objective function
value of objective function
153000
152000
151000
150000
149000
148000
147000
146000
145000
0
50
100
150
200
250
300
the value of Righthand Side, the maximum number of equipment for each repair center (e_cap)

By analyzing the graphical representations, it becomes evident that as the price of each unit of repair equipment rises, the objective function's value - signifying the overall cost - also increases.

However, beyond a specific threshold (600 units), any further escalation in the cost of repair equipment units fails to yield a discernible alteration in the objective function. This occurs due to the fact that, under such circumstances, the allocation of repair equipment units to said repair center would be deemed null.

Furthermore, within the second illustration, it is clear that through augmenting the maximum capacity for equipment allocation, the objective function's value, emblematic of the overall expenditure, diminishes. This phenomenon can be attributed to a multitude of factors, encompassing the amplification in the allocation potential of devices, as well as the reduction in transportation expenses between disparate centers, consequent to the heightened service provisions.

Likewise, when the maximum capacity for assigning equipment surpasses a specific threshold of 150 units, there is an absence of noteworthy alteration in the objective function's value, as there exists no discernible distinction from the preceding state. It is crucial to bear in mind that we are addressing the maximum quantity rather than the specific count.