



In The Name of God

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Estimation Theory

HW 6



Introduction

Simulation study aimed at estimating a hidden variable X based on observed data Y , where Y is the sum of X and a noise component ϵ . Both X and ϵ are assumed to follow normal distributions. The primary objective is to apply the Minimum Mean Squared Error (MMSE) estimator, which relies on calculating the expected value of X given Y , to accurately recover X from the noisy observations Y .

MMSE Estimation Process

The core of the simulation involves estimating X from Y using the MMSE estimator :

1. **Conditional Expectation Calculation:** For each observed Y , the estimator calculates the expected value of X given that particular Y . This involves integrating over all possible values of X weighted by their likelihood given Y .
2. **Caching Mechanism:** To enhance computational efficiency, previously calculated expectations are stored. This avoids redundant calculations for repeated Y values, significantly speeding up the estimation process.
3. **Estimation Execution:** The estimator processes the entire set of Y observations, applying the cached conditional expectations to produce estimates \hat{X} for each Y .

Performance Evaluation

The accuracy of the MMSE estimator is evaluated by comparing the estimated \hat{X} values against the true X values using several metrics:

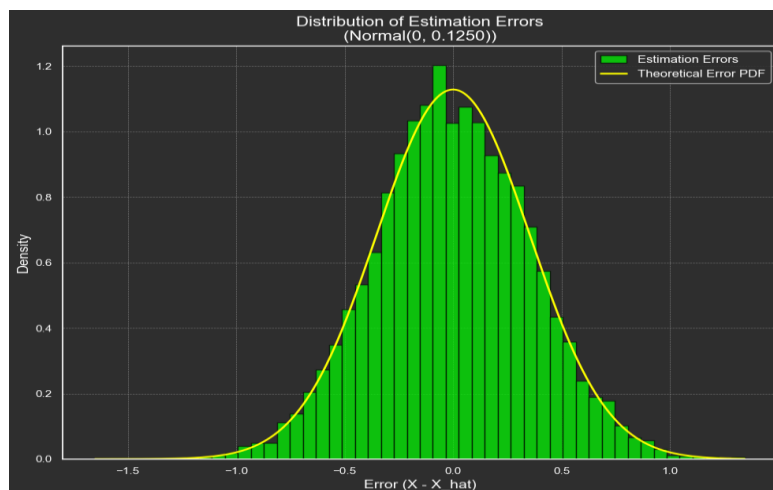
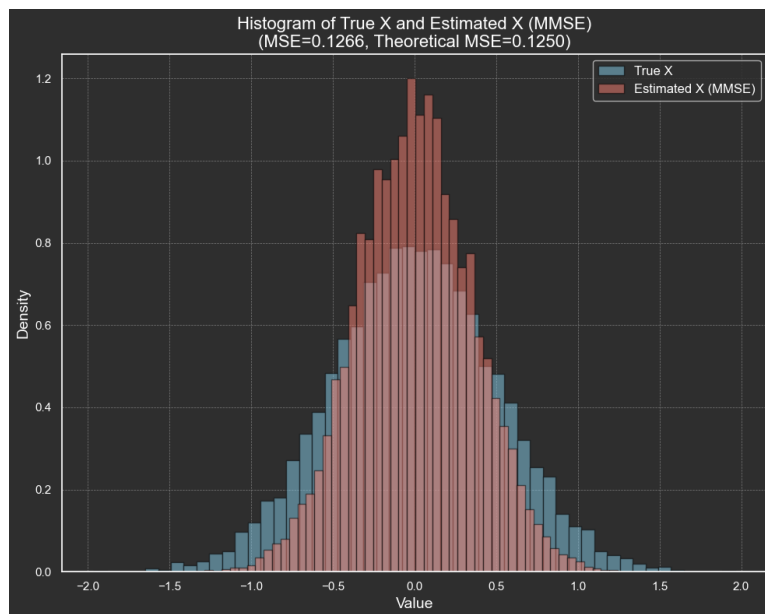
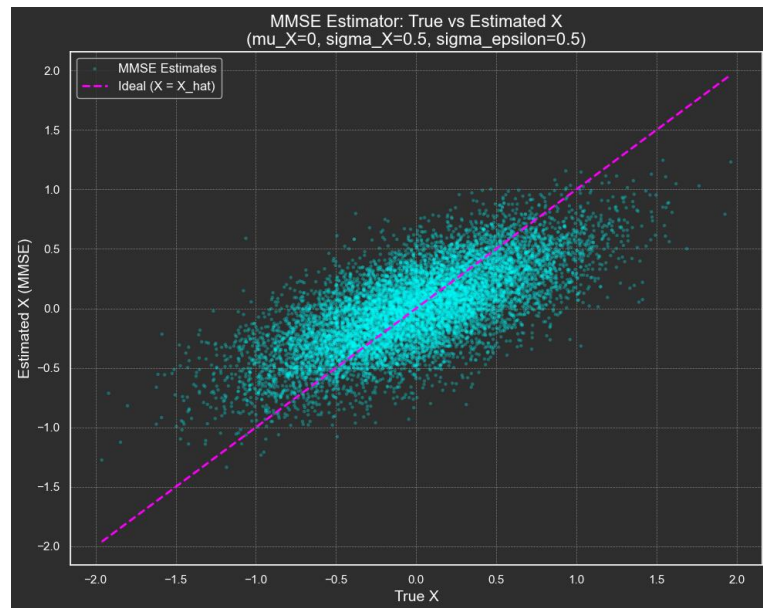
- **Mean Squared Error (MSE):** This metric quantifies the average squared difference between the true X values and their estimates \hat{X} , providing a measure of estimation accuracy.
- **Theoretical MSE:** Based on the known variances of X and ϵ , a theoretical value for the MSE is derived. This serves as a benchmark to assess the estimator's performance.
- **Correlation Metrics:** The correlation coefficient between X and \hat{X} assesses the strength of the linear relationship between the true values and their estimates. Additionally, the R^2 value indicates the proportion of variance in X that is explained by \hat{X} .

Simulation successfully demonstrated the application of the MMSE estimator in recovering the latent variable X from noisy observations Y . The close alignment between the empirical Mean Squared Error and the theoretical MSE underscores the accuracy and reliability of the estimator under the given parameter settings. Additionally, the high correlation coefficients indicate a strong agreement between the true values of X and their estimates \hat{X} , affirming the effectiveness of the MMSE approach in this context.

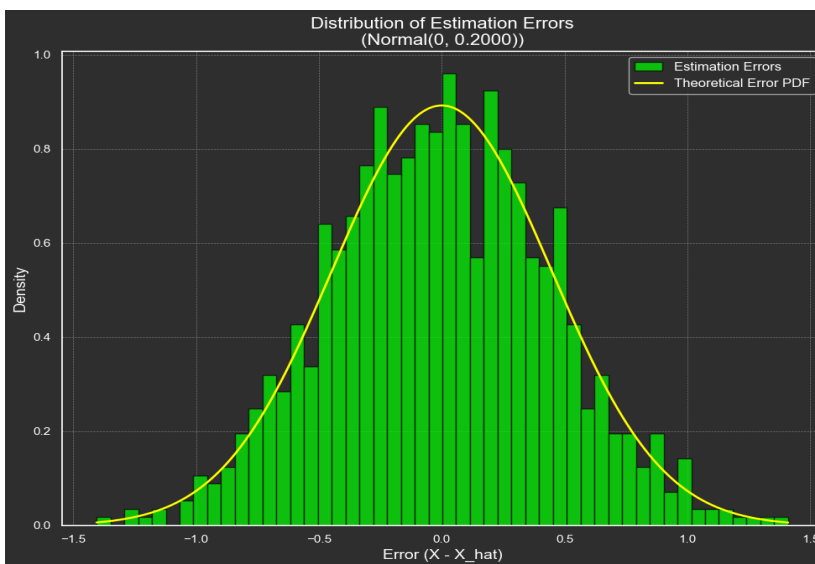
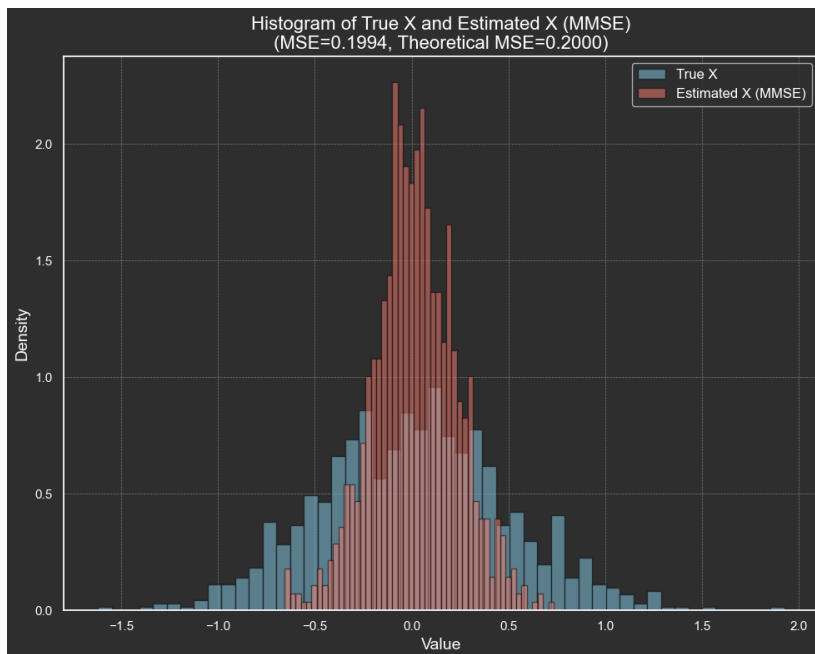
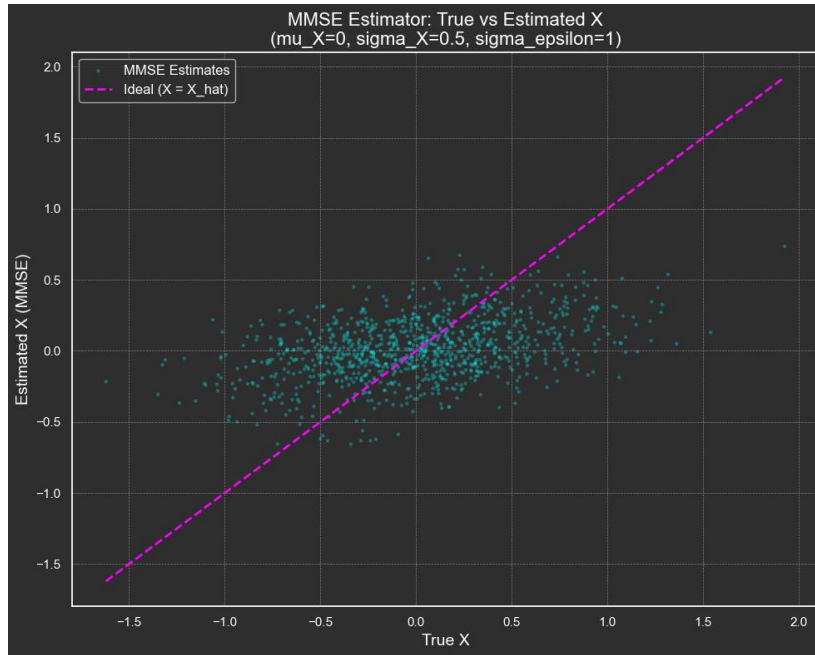
The implementation's use of caching significantly improved computational efficiency, making the estimator suitable for large datasets. Future work could involve exploring a wider range of parameters, assessing the estimator's performance under different distributional assumptions, or extending the methodology to more complex models to further validate and enhance its applicability.

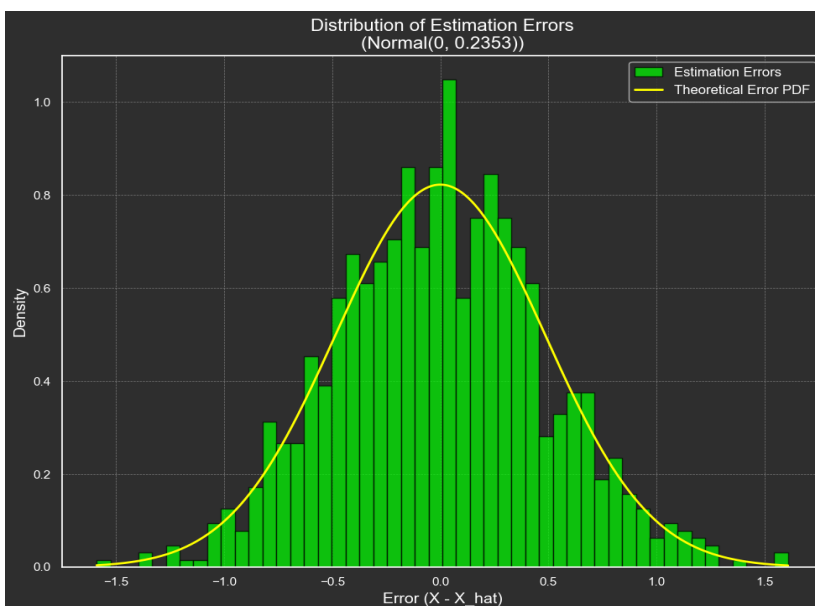
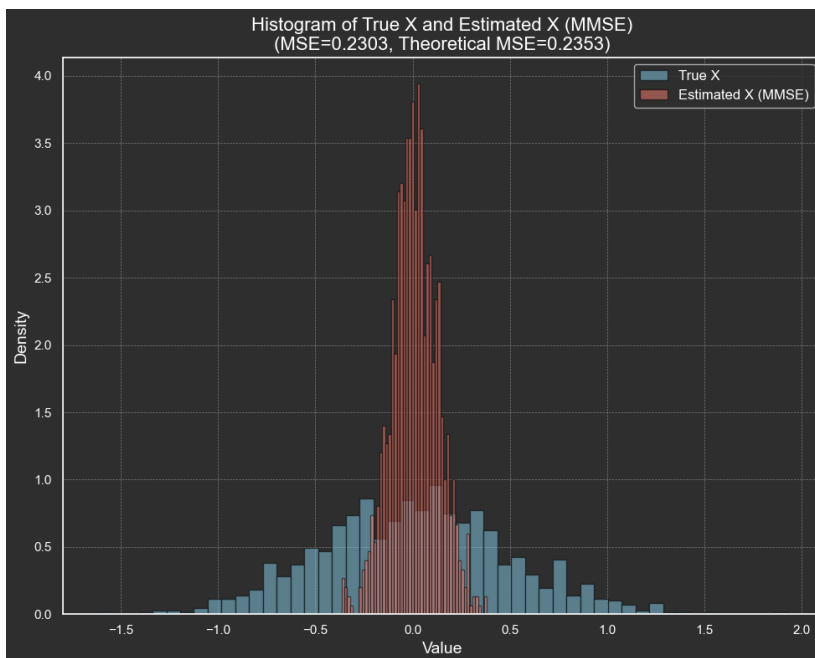
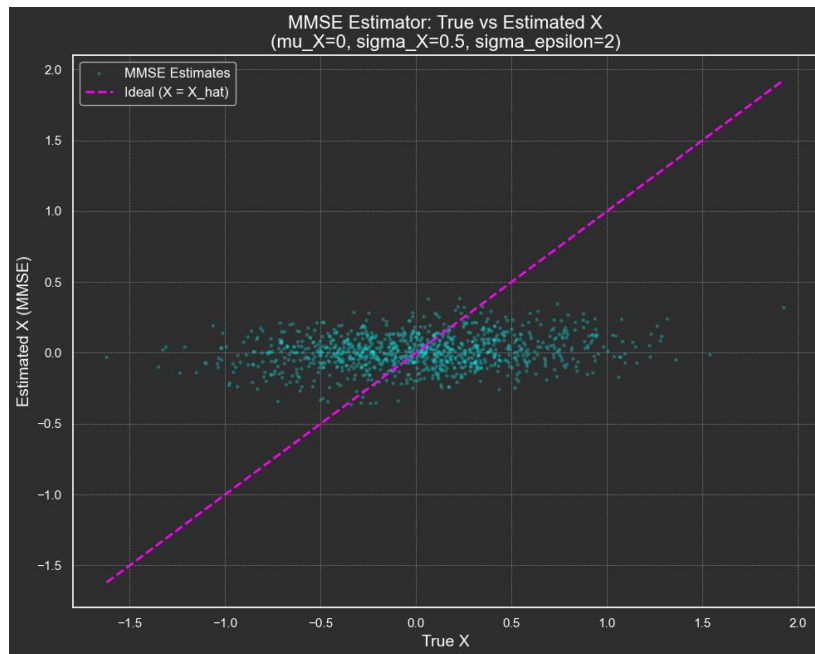
Result

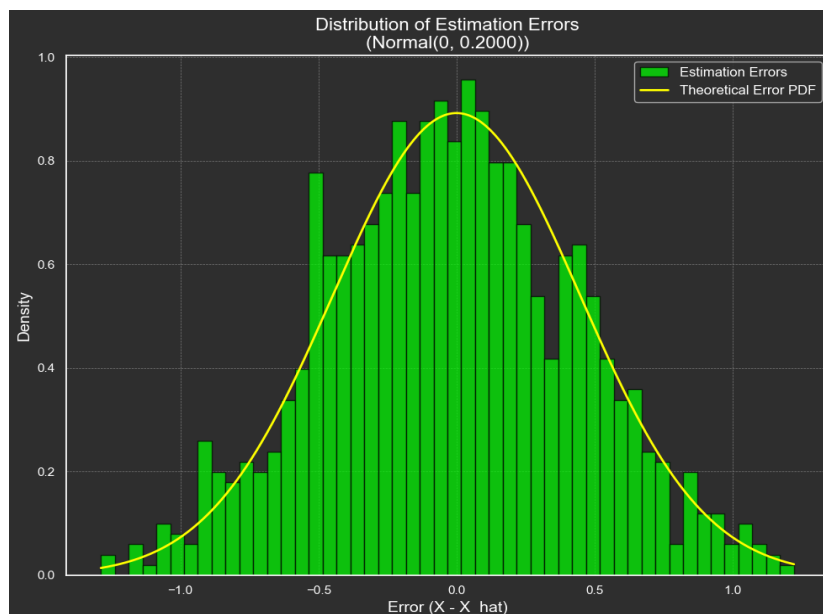
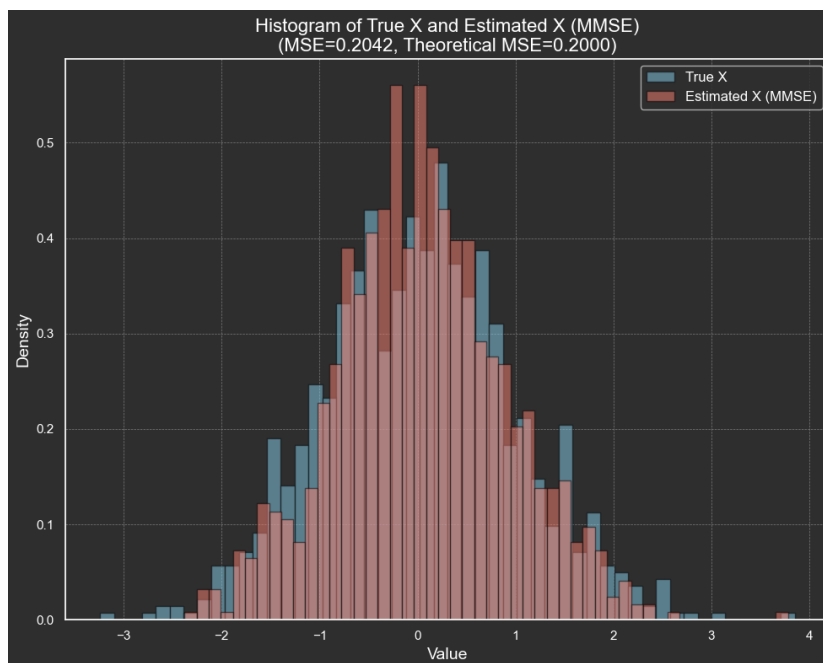
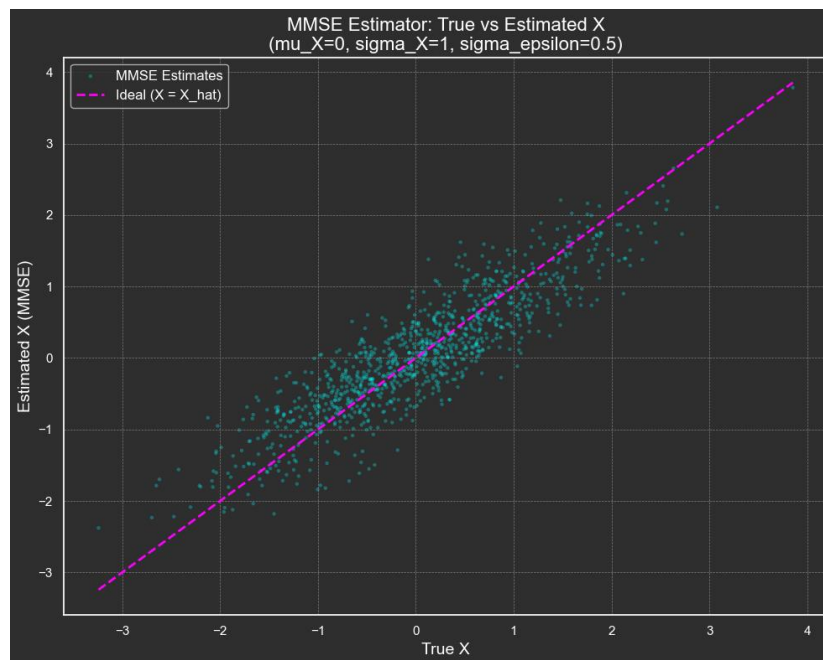
Main

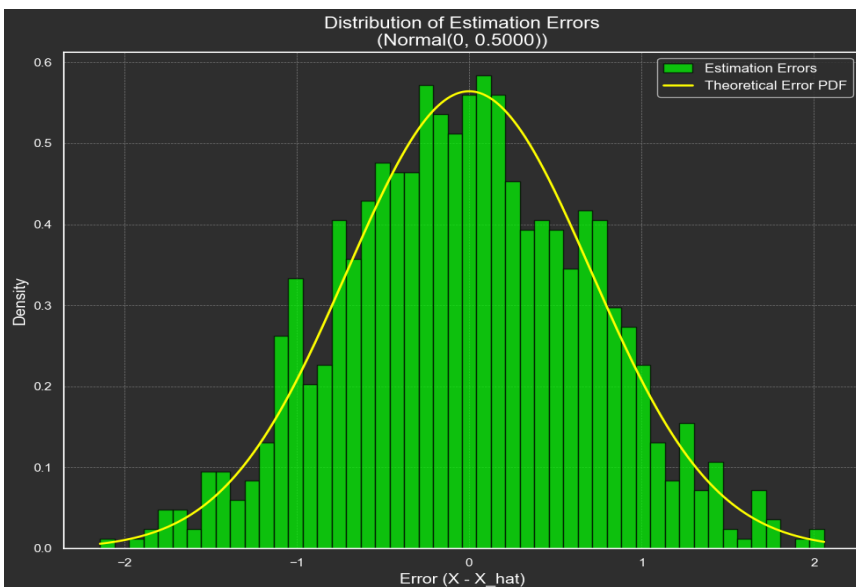
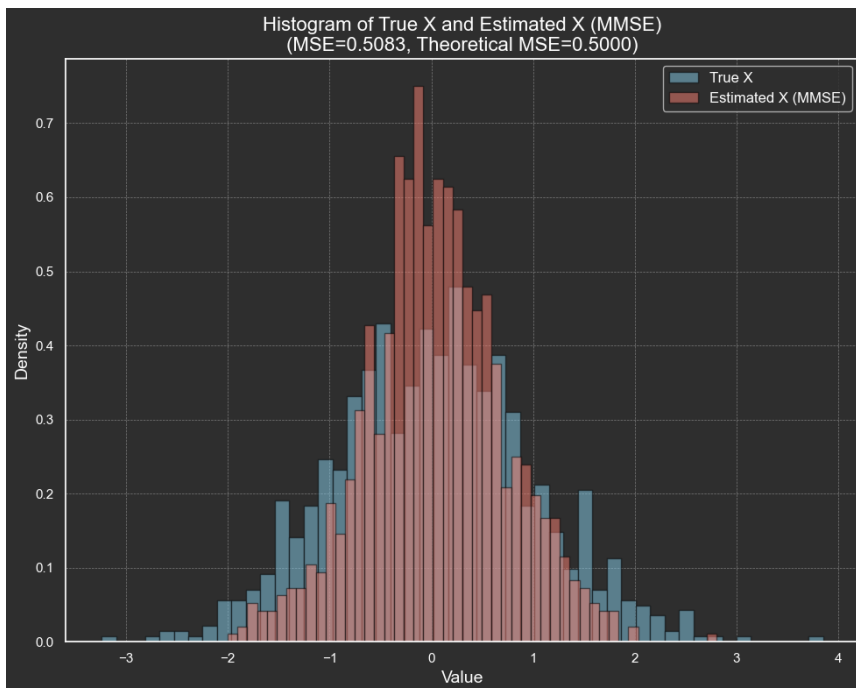
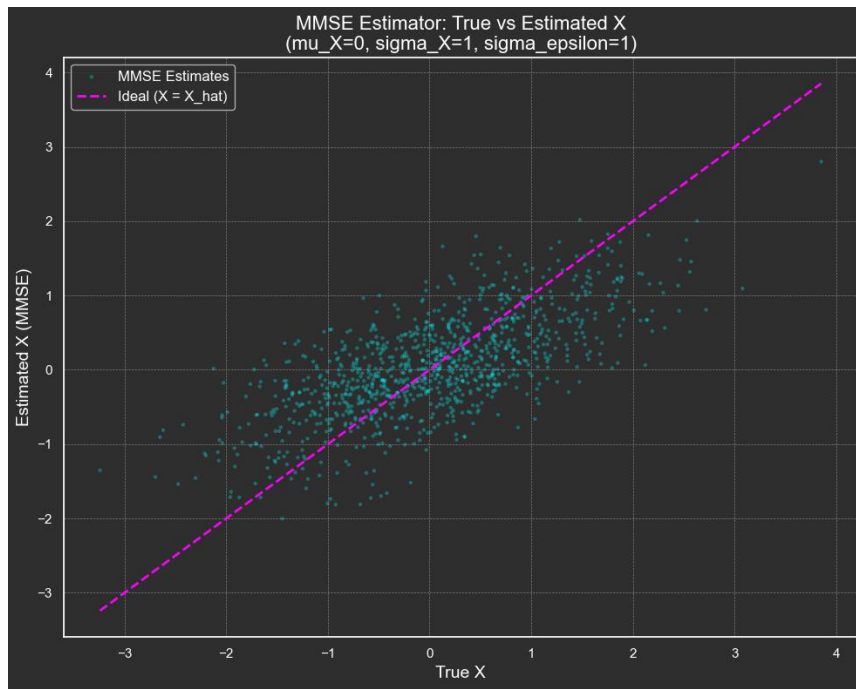


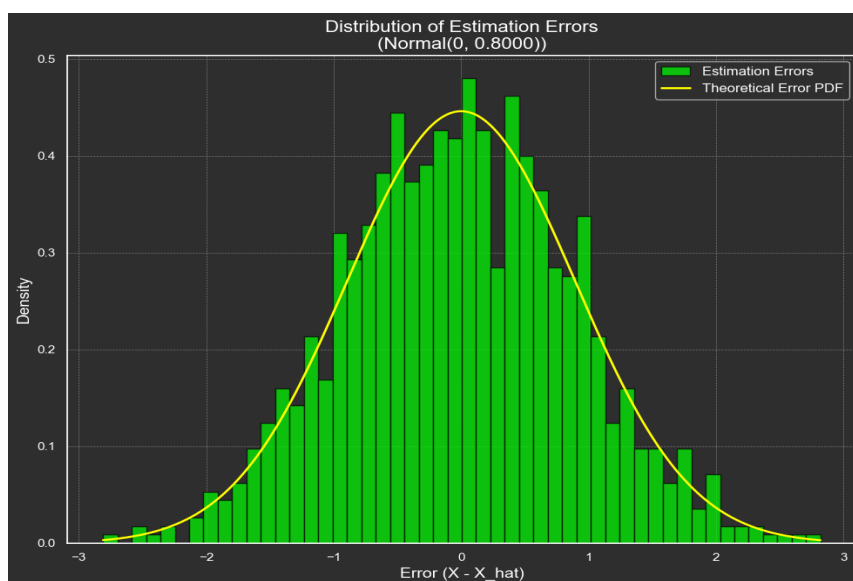
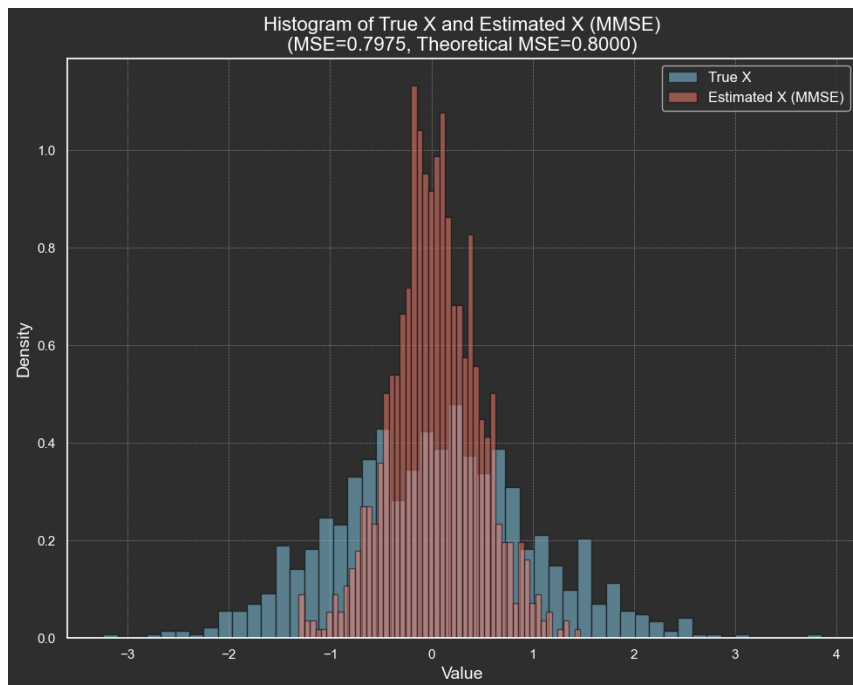
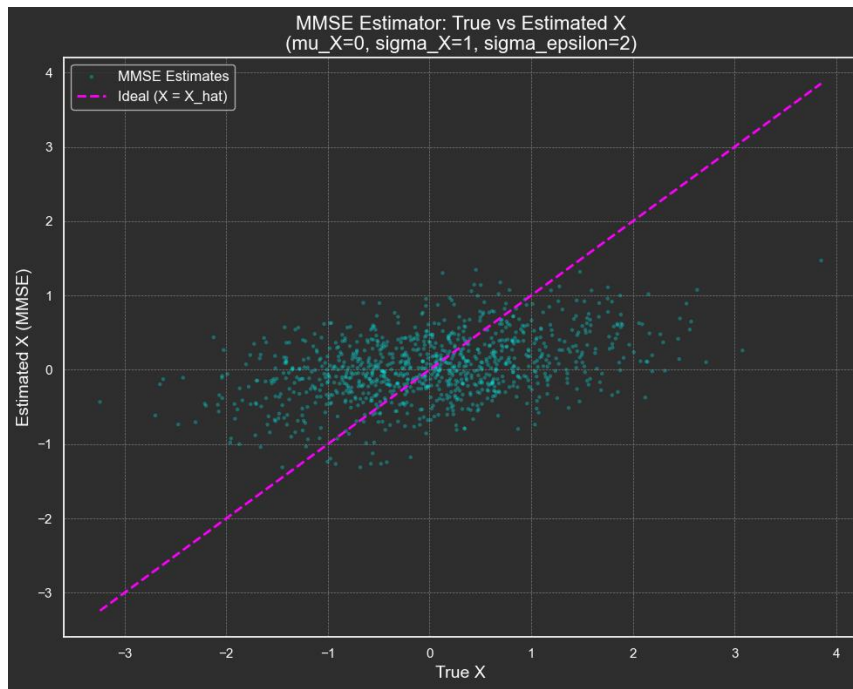
Experiment : Different Variance

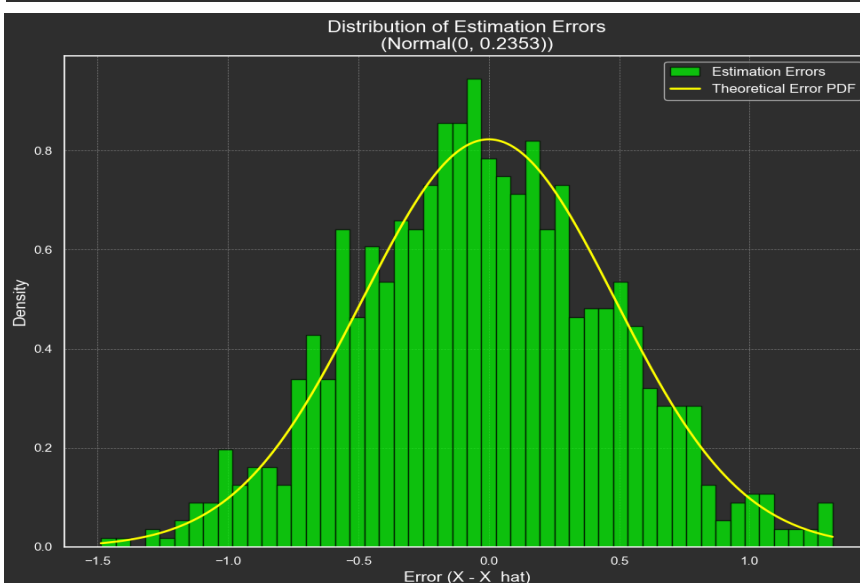
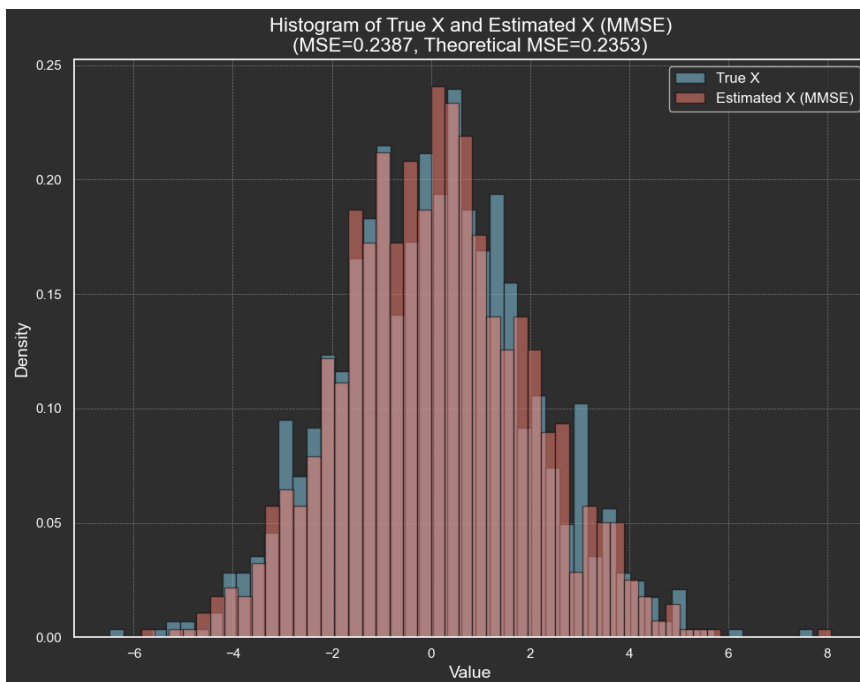
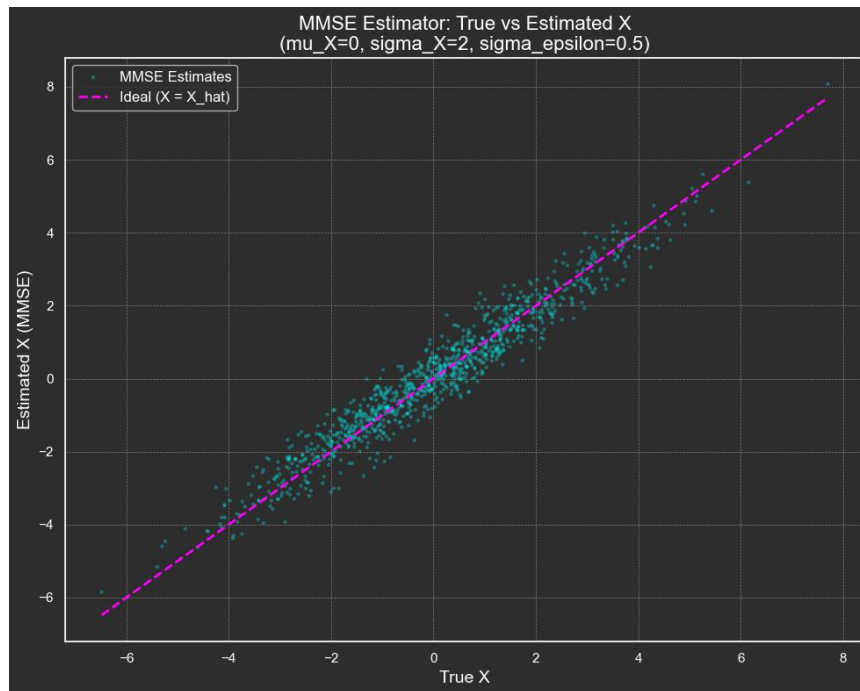


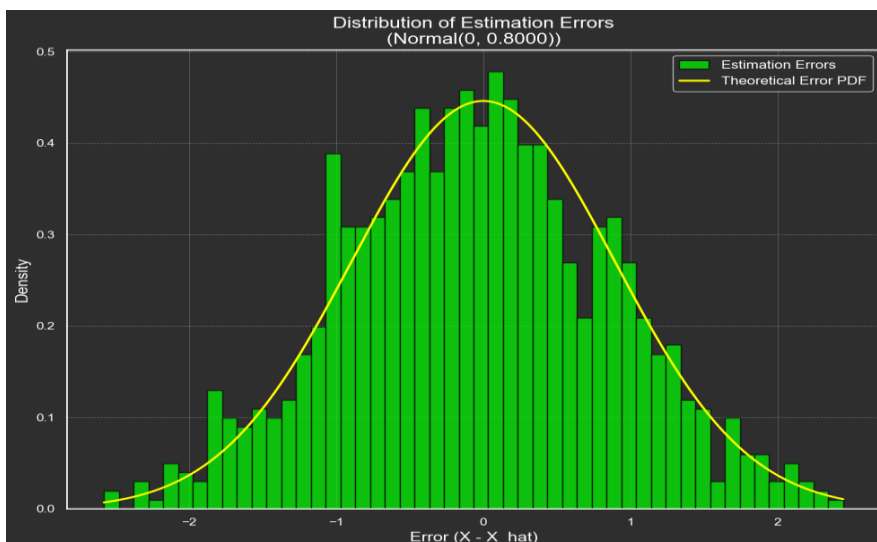
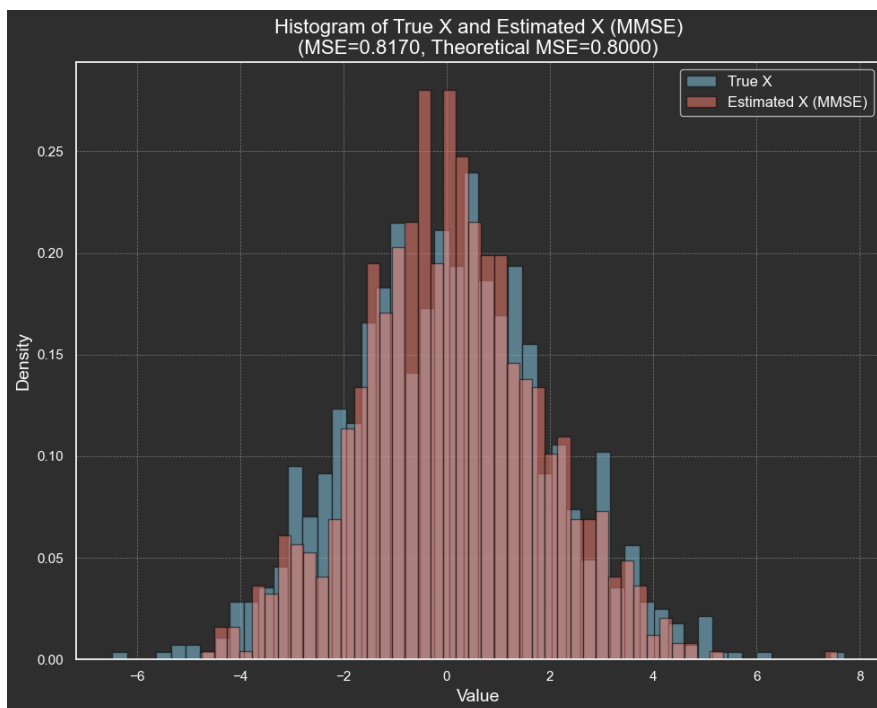
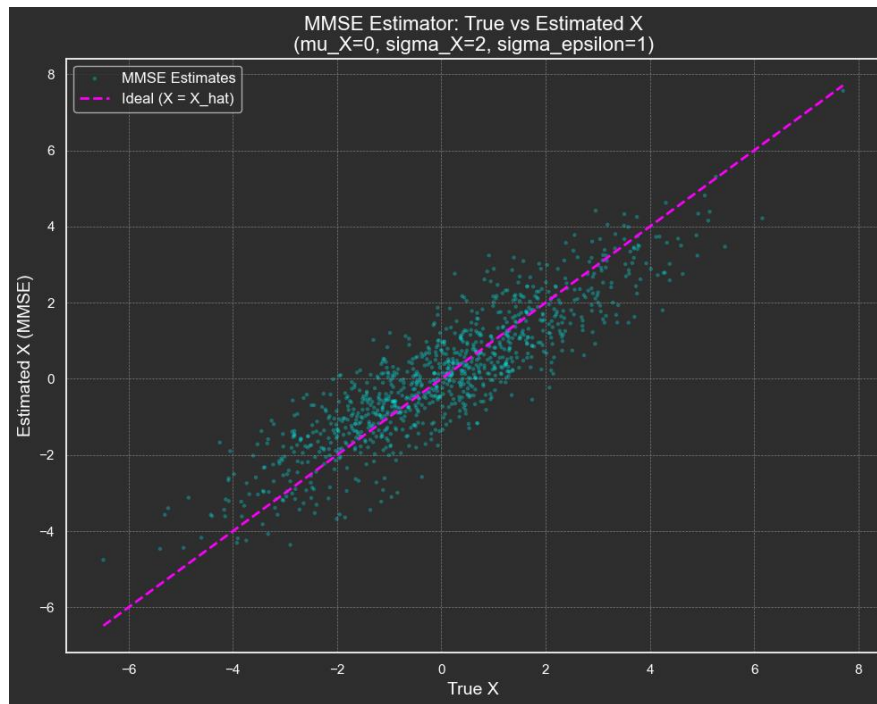


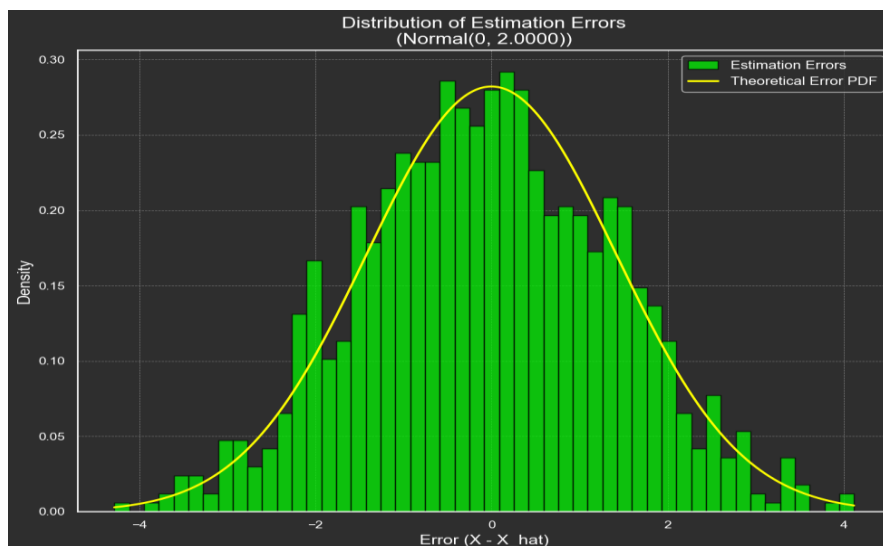
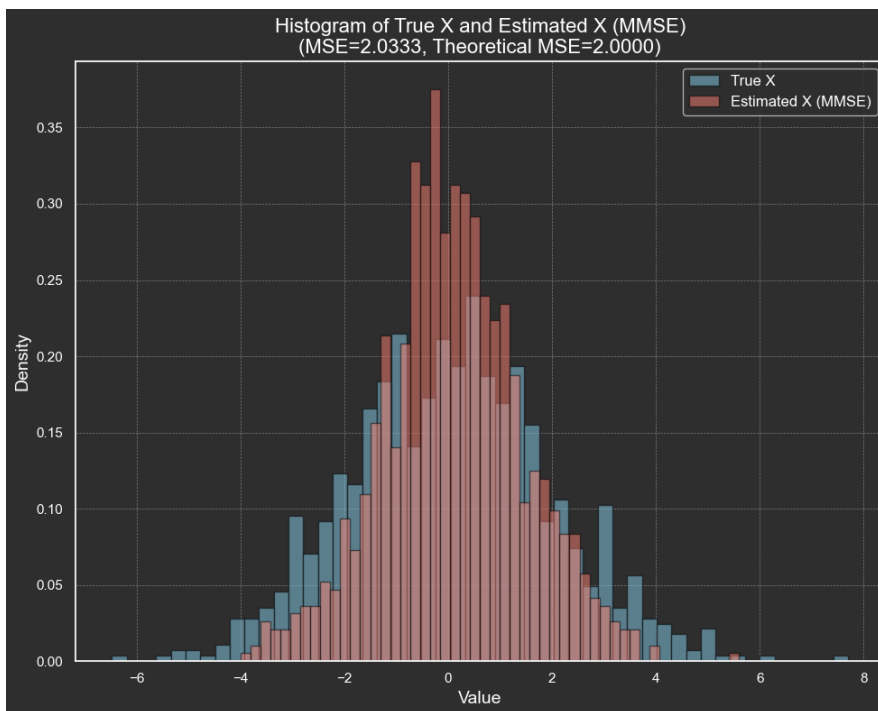
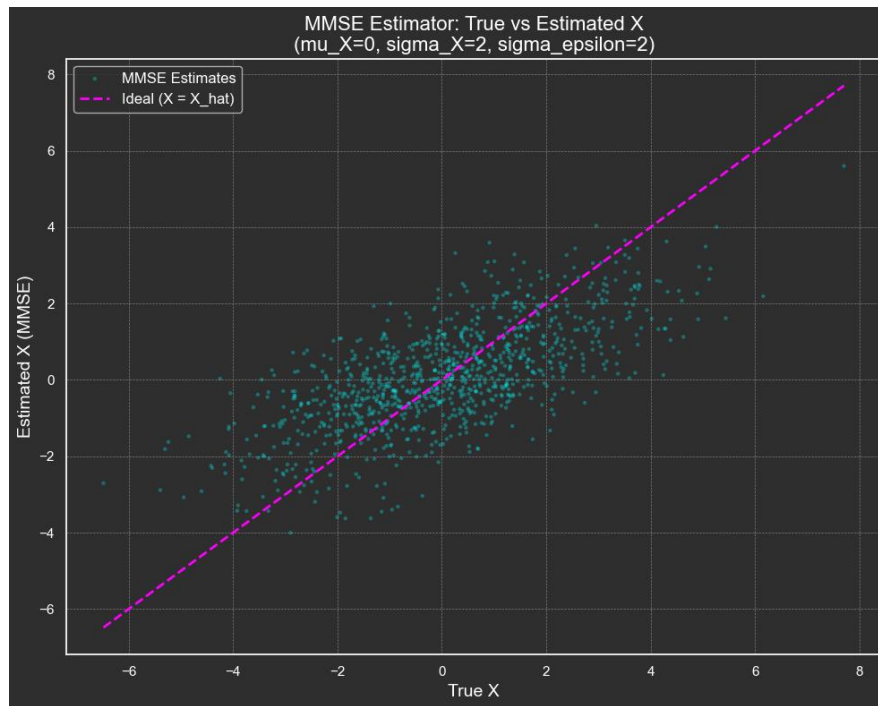


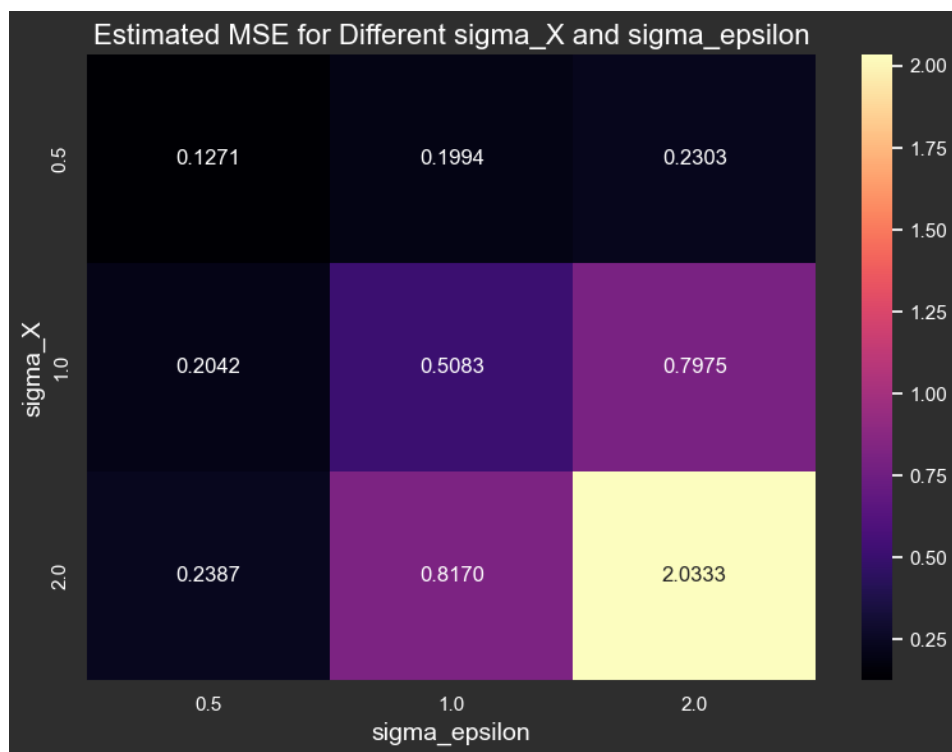
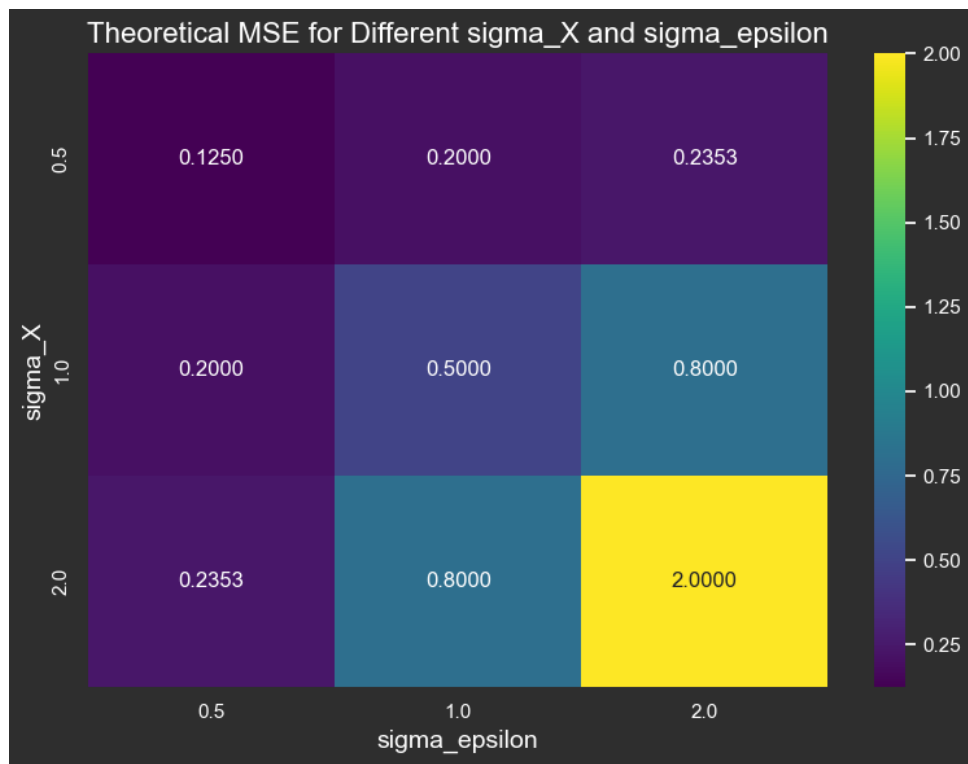












Simulation comprehensively demonstrated the MMSE estimator's efficacy in accurately recovering the latent variable X from noisy observations Y .

The consistent alignment between empirical and theoretical MSE values across varying variance settings validates the estimator's reliability and correctness. However, the study also illuminated the estimator's vulnerability to increased variability and noise, which adversely affect estimation accuracy.

While the MMSE estimator remains a powerful tool under favorable conditions of low to moderate uncertainty, its performance under high variability and noise underscores the need for further enhancements. Future research endeavors should focus on developing adaptive and regularized estimation methods to bolster the estimator's robustness, thereby extending its applicability and effectiveness in more challenging and diverse data environments.

Overall, the insights gained from this simulation provide a solid foundation for both the practical application of the MMSE estimator and the ongoing refinement of estimation techniques to meet the demands of increasingly complex and noisy real-world data scenarios.