In The Name of God

Alireza Garmsiri

40260337

Estimation Theory

HW 7

Introduction

Simulation of a predator-prey ecosystem using a linear state-space model. This model captures the dynamics between predator and prey populations over discrete time steps, incorporating both deterministic interactions and stochastic disturbances. Such simulations are instrumental in understanding population dynamics, assessing the impact of various factors on ecosystem stability, and informing conservation strategies

State Transition Matrix (F):

Role: This matrix captures the natural interactions between predators and prey without any external influences. It determines how the populations of each species change from one time step to the next based on their current numbers.
Functionality: The elements of this matrix represent the rates at which prey and predator populations grow or decline due to their interactions. Positive values indicate growth factors, while negative values signify decline or mortality rates.

Control Input Matrix (G):

Role: This matrix introduces the possibility of external interventions or factors that can influence the predator population directly. Such controls could represent actions like conservation efforts, introduction of additional predators, or environmental changes affecting predators specifically.
Functionality: By modifying this matrix, users can simulate scenarios where external factors impact the predator population, allowing for the exploration of various management strategies.

Process Noise Covariance (Q):

Role: Real-world ecosystems are subject to unpredictable changes due to factors like weather, disease, or sudden environmental shifts. The process noise covariance matrix incorporates these random disturbances into the simulation.
Functionality: This matrix defines the extent and nature of randomness affecting each population. By adjusting its values, users can simulate environments with varying levels of unpredictability.

Simulation Function:

Inputs:

F (State Transition Matrix): Governs the natural interactions between species.
G (Control Input Matrix): Represents external influences on the predator population.
Q (Process Noise Covariance Matrix): Introduces randomness to simulate unpredictable changes.
x0 (Initial State Vector): Specifies the starting numbers of prey and predators.
u_seq (Control Sequence): An optional sequence of external interventions applied at each time step. If not provided, it defaults to no intervention.
N (Number of Steps): The total number of time steps the simulation will run. Defaults to 50 if not specified.
random_seed (Random Seed): Ensures that the random elements of the simulation can be reproduced by setting a specific starting point for the random number generator.
Process:

Initialization:

If a random seed is provided, it initializes the random number generator to ensure consistent results across runs.
Creates a data structure to store population numbers at each time step, starting with the initial populations provided.
If no control sequence is given, it assumes that there are no external interventions affecting the predator population.

Simulation Loop:

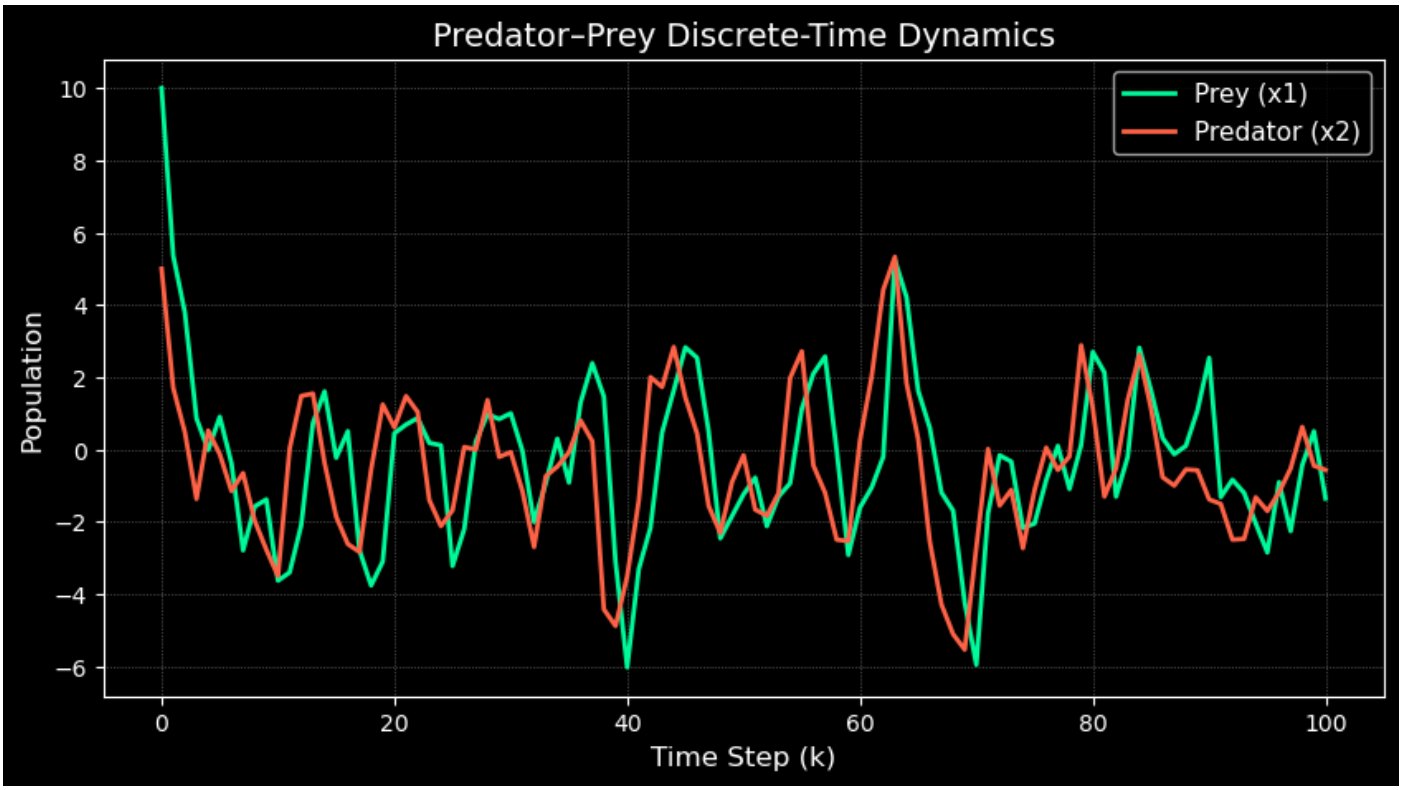
For each time step
Generates random disturbances to mimic unpredictable environmental changes affecting both prey and predator populations.
Updates the populations based on the current state, the influence of any external controls, and the random disturbances.
Records the updated populations for analysis.
Output:

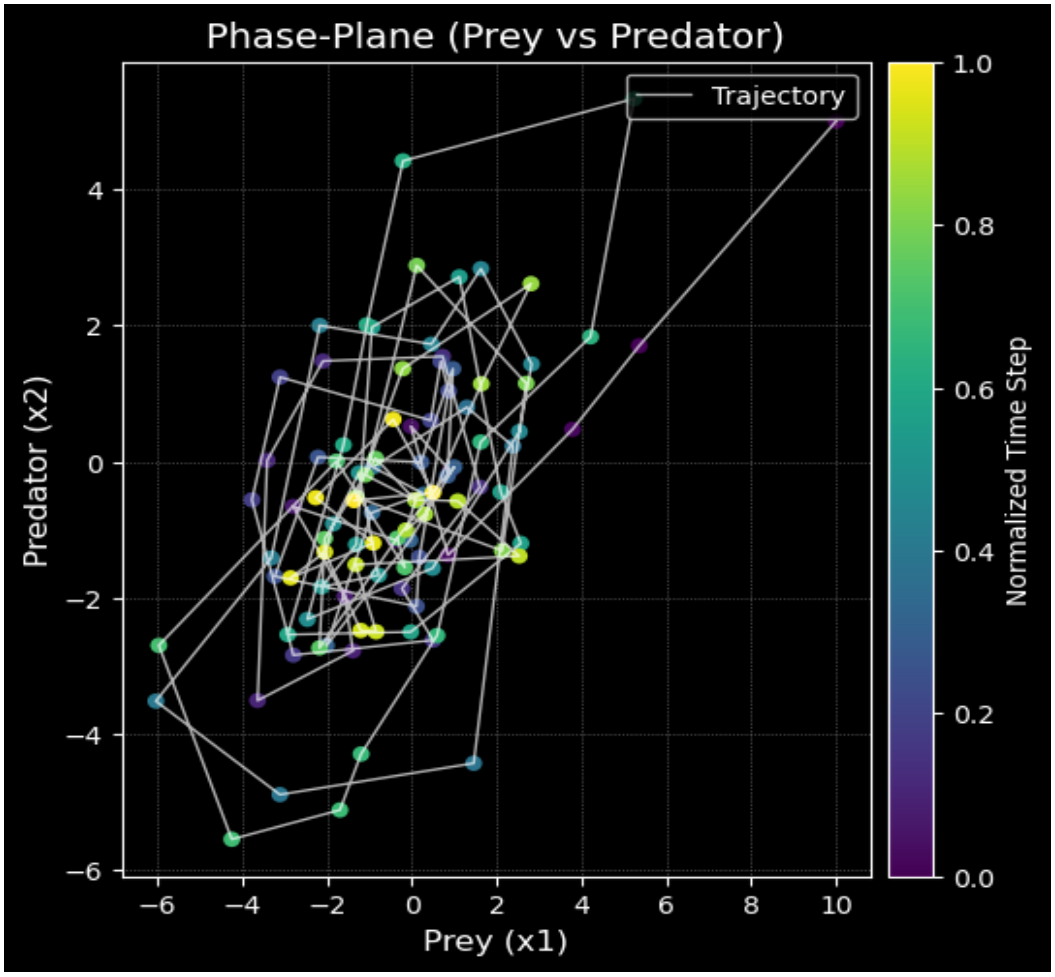
After completing all time steps, the function returns a comprehensive record of prey and predator populations throughout the simulation period

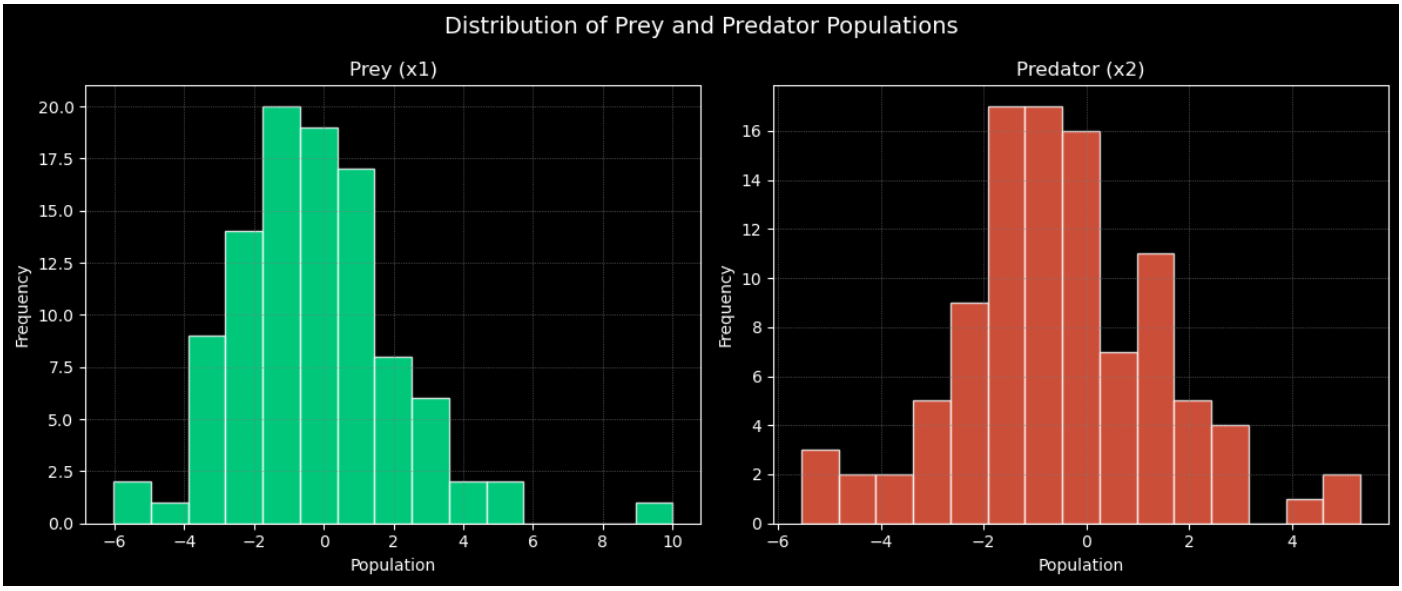
Conclusion

The predator-prey simulation offers a structured approach to modeling ecological interactions between two species. By integrating mathematical matrices that represent natural interactions, external influences, and random environmental factors, the simulation provides a dynamic and realistic portrayal of how predator and prey populations can fluctuate over time. Users can adjust various parameters, such as initial populations, interaction rates, and the degree of randomness, to explore different ecological scenarios. This tool serves as a valuable resource for studying population dynamics, testing conservation strategies, and enhancing our understanding of ecosystem stability and resilience.

Result


Predator–Prey Discrete-Time Dynamics
Prey (x1)
Predator (x2)
Population
Time Step (k)
10
8
6
4
2
0
−2
−4
−6
0
20
40
60
80
100


Phase-Plane (Prey vs Predator)
Trajectory
Predator (x2)
Prey (x1)
Normalized Time Step
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
8
10
1.0
0.8
0.6
0.4
0.2
0.0



Time-Series Plot

This shows the dynamics of the prey (green) and predator (red) populations over 100 time steps.
Both populations exhibit oscillatory behavior with fluctuations influenced by random noise.
The populations stabilize but remain dynamic, showcasing predator-prey interactions.

Phase-Plane Plot

The prey population (x-axis) is plotted against the predator population (y-axis) to form a trajectory over time.
The color gradient indicates time progression, moving from purple (early) to yellow (late).
The trajectory forms a complex pattern, reflecting the interplay and feedback between prey and predators.

Histograms

The left plot shows the distribution of prey populations, which has a central tendency around a specific range.
The right plot represents predator populations, showing a slightly wider spread.
Both populations display variability due to stochastic factors, with prey tending to be slightly more positive overall.