In The Name Of God

Shiraz University

Estimation Theory

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HW 2

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Introduction

In time series analysis, autoregressive (AR) models are commonly used to describe systems where the current value of a series is linearly dependent on its previous values, as well as on some input variables. Estimating the parameters of such models is crucial for accurate prediction and system understanding. One standard approach for parameter estimation in the presence of noise is Least Squares Estimation (LSE), where the goal is to minimize the difference between the observed and predicted values of the system.

The objective of this report is to implement and evaluate the performance of Least Squares Estimation for an autoregressive model with exogenous inputs (ARX model), where both the parameters of the autoregressive part and the exogenous input part are estimated. The estimation is performed under different simulation settings to investigate the effect of noise variance, sample size, and iterations on the accuracy of the parameter estimates.

Implementation

Problem Description

In this simulation, we work with a simple ARX model. The model relates the output of the system to past values of the output itself and past values of the input signal, along with some noise.

AR Model:

Autoregressive (AR) coefficients a1 , a2 , a3: These coefficients determine the influence of past output values on the current output.

Input coefficients b1 , b2 , b3: These coefficients determine the influence of past input values on the current output.

Noise: The system is assumed to be corrupted by Gaussian noise, and the noise variance is an important parameter that impacts the estimation process.

Objective: The goal is to estimate the AR and input coefficients using the Least Squares Estimation method, which involves minimizing the squared differences between the observed output and the predicted output using the ARX model.

Experiments: We perform several experiments to evaluate the robustness of the Least Squares Estimation method. These experiments include:

Varying the noise variance (higher and lower values).

Changing the number of samples (larger and smaller data sets).

Evaluating the parameter estimation over multiple iterations to see how well the method converges.

Methodology

Simulating the Input and Output Signals:

The input signal is generated as white noise with a specified variance.

The output signal is generated according to the ARX model, which combines the previous output values (AR part), the previous input values (exogenous part), and Gaussian noise.

Regressor Matrix Construction:

The design matrix H is constructed using the previous values of the output signal (autoregressive part) and input signal (exogenous part). This matrix is used to express the system as a linear regression problem.

Parameter Estimation:

The Least Squares Estimation method is applied to solve for the parameters that minimize the sum of squared residuals (the difference between the observed output and the predicted output).

Monte Carlo Simulations:

A Monte Carlo approach is used, where multiple runs (iterations) are conducted with different random noise realizations. This helps simulate the effect of noise on the estimation process and provides insight into the variability and accuracy of the estimates.

Result

Parameter Estimation Over Iterations:

The plot of estimated parameters over iterations shows the convergence behavior of the estimates. As the number of iterations increases, the estimates tend to stabilize, converging towards the true parameter values. In the presence of noise, the estimates initially fluctuate but gradually reduce their variability over multiple iterations.

Effect of Noise Variance:

Higher noise variance results in more spread-out parameter estimates, with a larger degree of fluctuation. The estimation method struggles to identify the true parameters when the noise level is high, which can be observed in the wider histograms.

Lower noise variance results in more accurate and stable parameter estimates. The estimates tend to converge faster, and the histograms are narrower, indicating more precise estimations.

Effect of Sample Size:

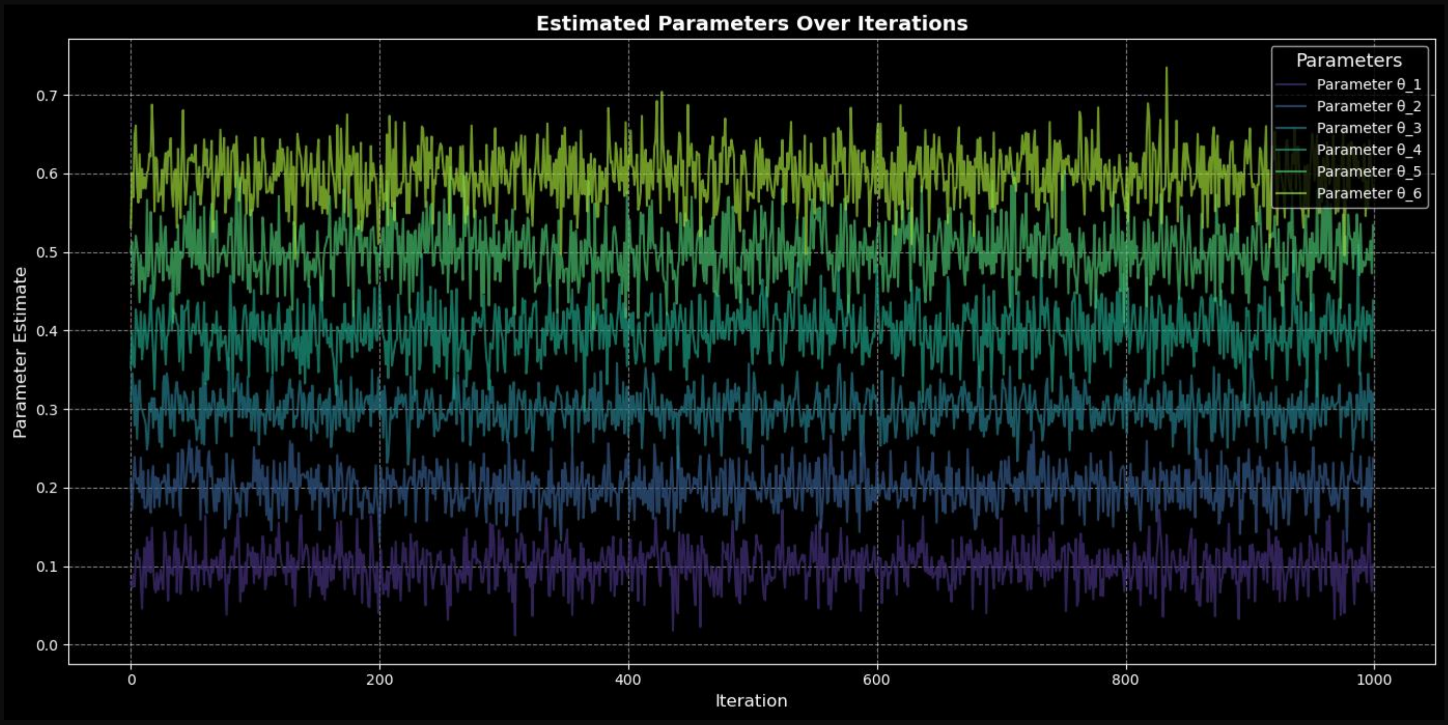
Increasing the number of samples generally leads to more accurate parameter estimates, as more data allows the model to capture the underlying relationship more effectively. With larger datasets, the estimated parameters converge to the true values faster and with less variance.

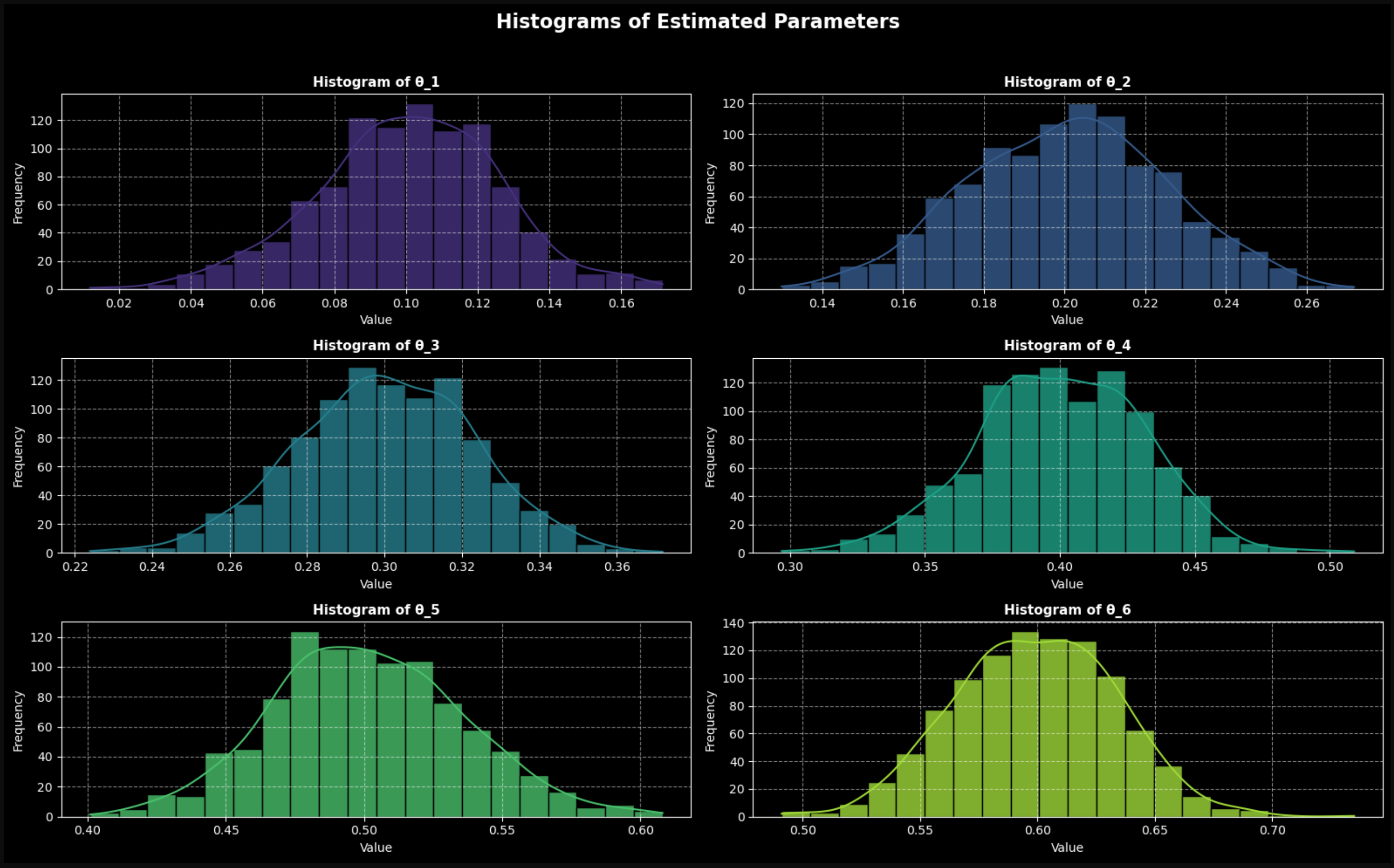
Decreasing the number of samples results in more variability in the estimates, as the model has fewer data points to make reliable predictions. In such cases, the histograms of the estimated parameters are broader, reflecting the uncertainty in the estimates.

Histograms of Estimated Parameters:

The histograms show the distribution of each estimated parameter over the multiple iterations. In the case of high noise variance, the histograms are broader, reflecting the higher uncertainty in the estimates. In contrast, with lower noise variance and larger sample sizes, the histograms become more concentrated around the true values.

Plots


Estimated Parameters Over Iterations
Parameters
Parameter θ_1
Parameter θ_2
Parameter θ_3
Parameter θ_4
Parameter θ_5
Parameter θ_6
Parameter Estimate
Iteration


Histograms of Estimated Parameters
Histogram of θ_1
Histogram of θ_2
Histogram of θ_3
Histogram of θ_4
Histogram of θ_5
Histogram of θ_6
Frequency
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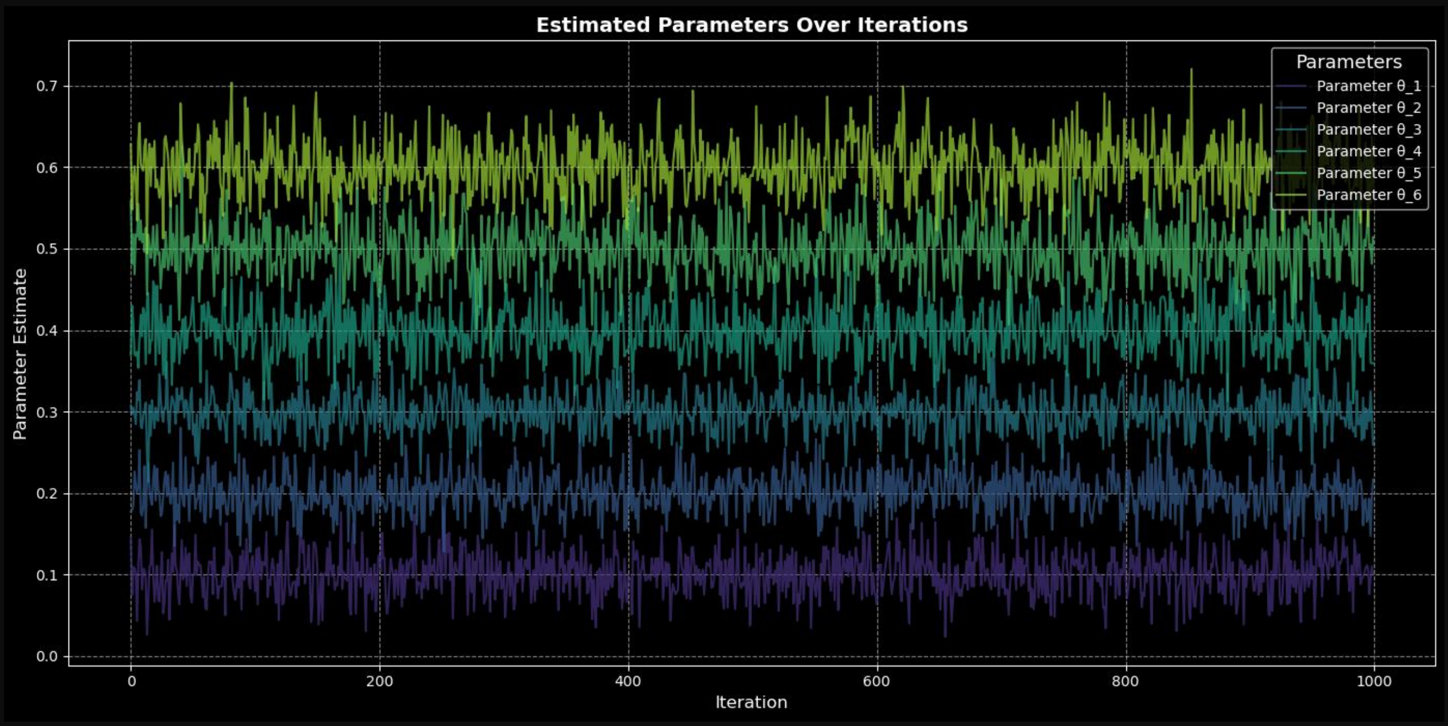

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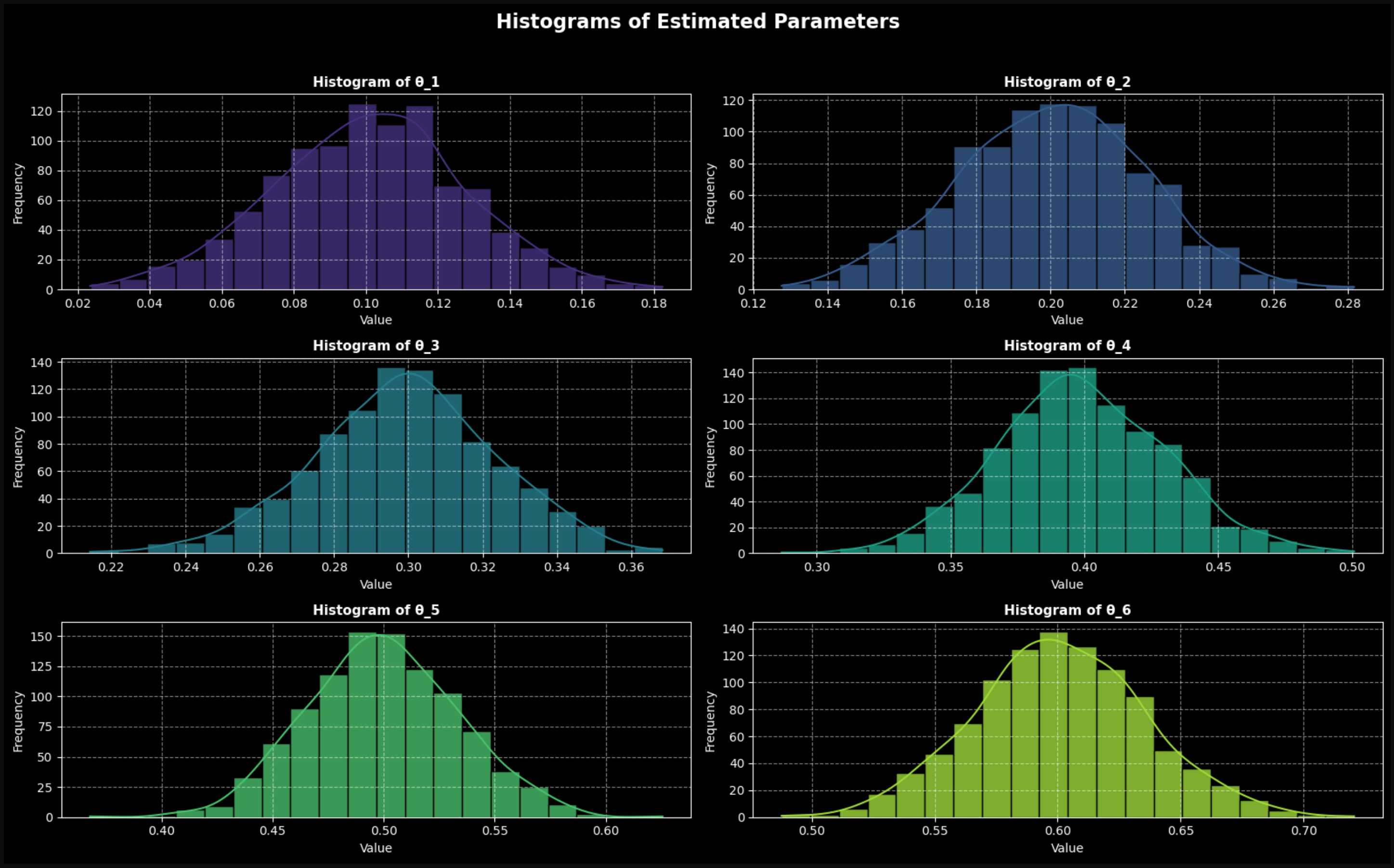

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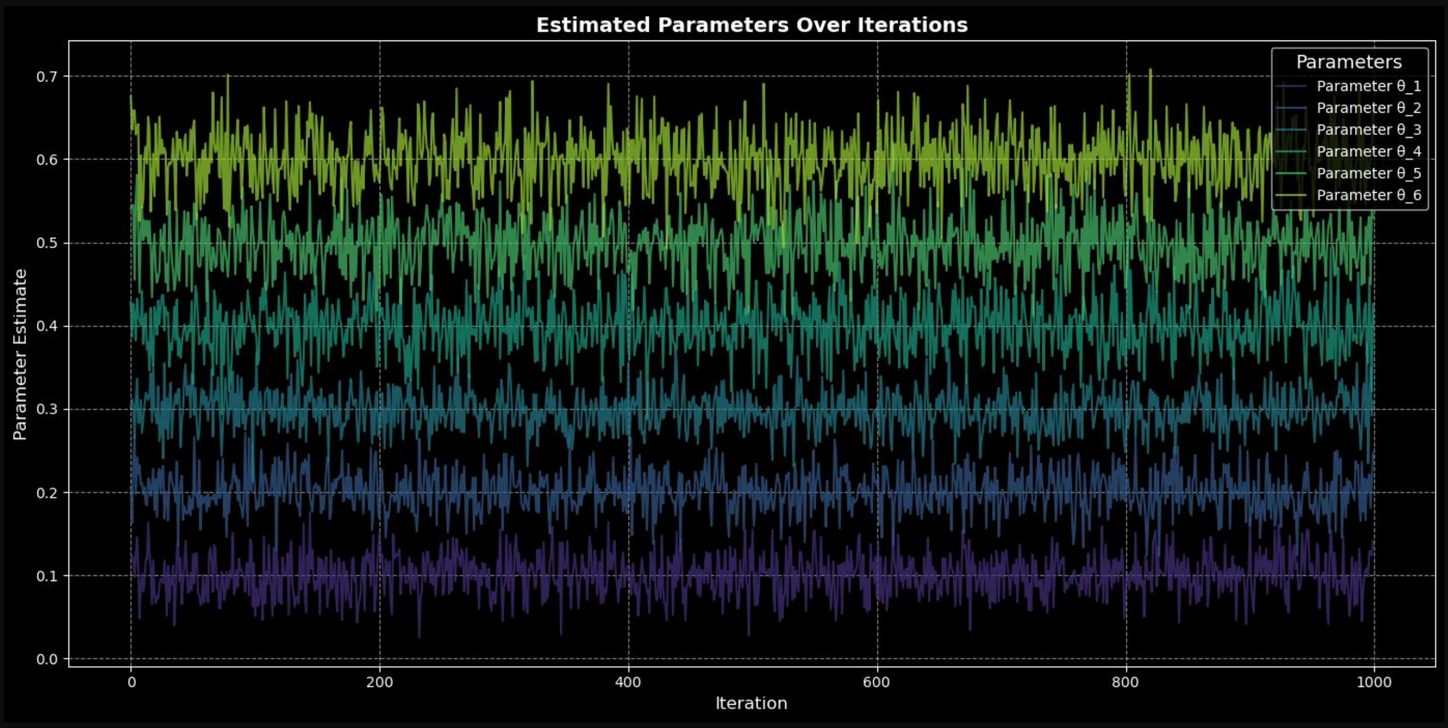
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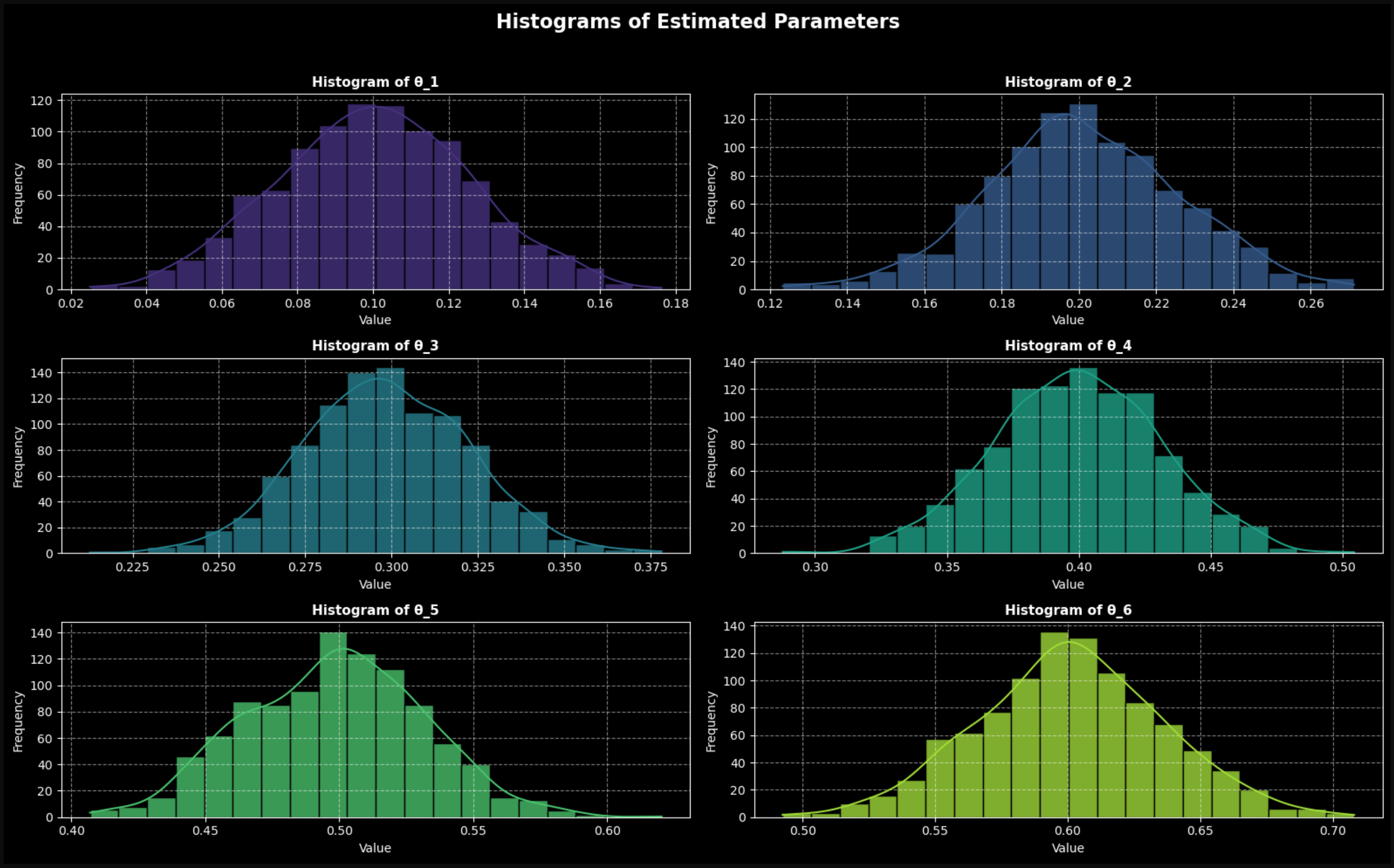
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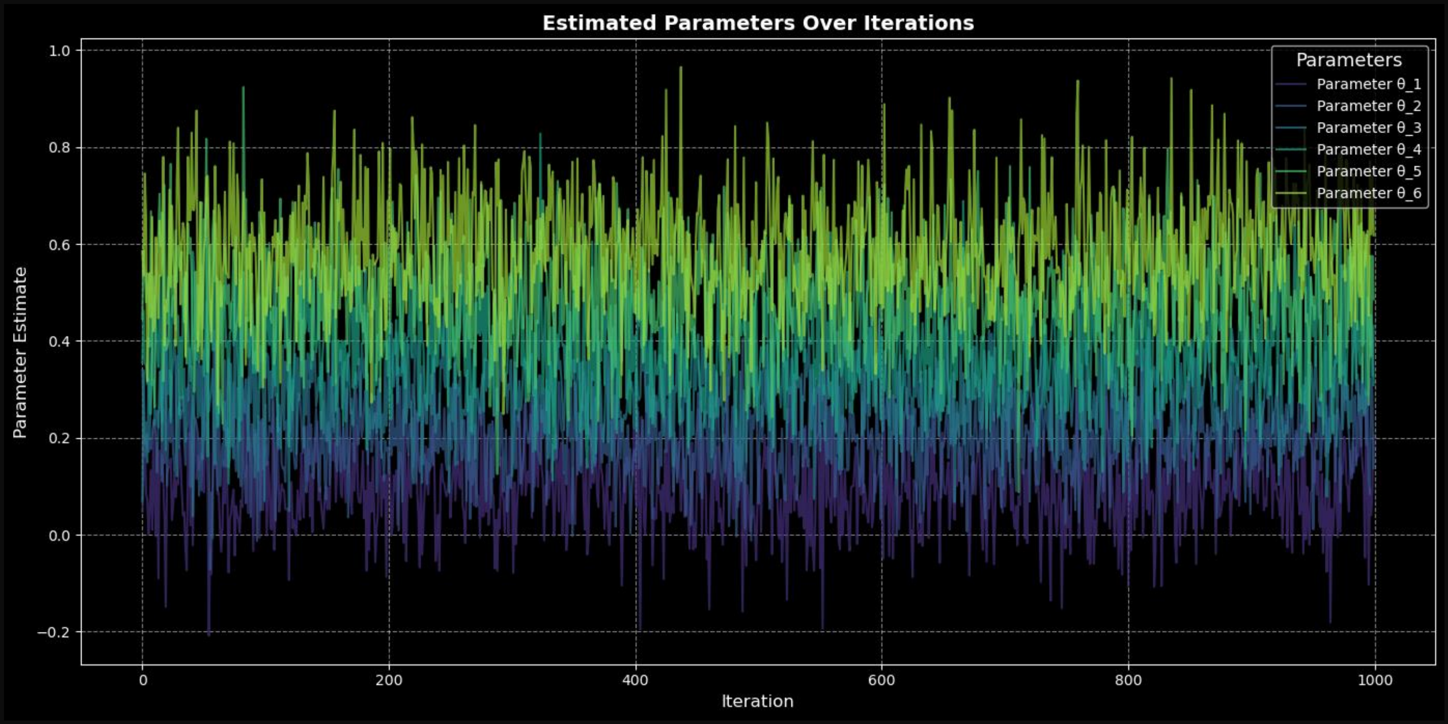
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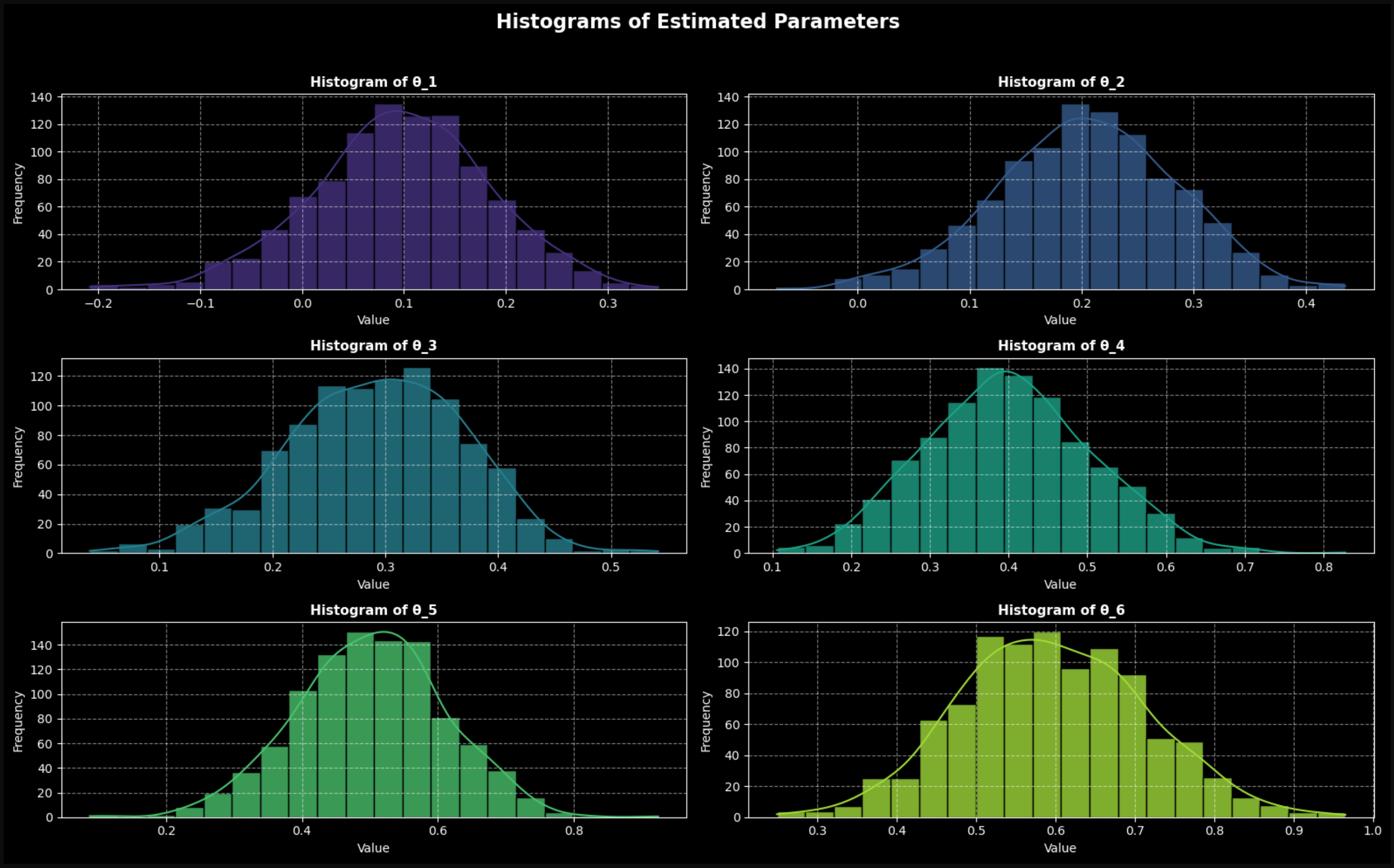
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Discussion

Convergence Behavior:

The convergence of the estimated parameters over iterations is a key result. The method appears to be stable and converges to the true values as the number of iterations increases, particularly in the absence of excessive noise. However, the noise variance significantly impacts the speed and stability of convergence. High noise variance causes the estimates to fluctuate more and converge more slowly.

Impact of Noise:

The simulation clearly shows that noise has a detrimental effect on the accuracy of parameter estimation. In practical applications, careful consideration must be given to the noise characteristics of the system. Techniques such as regularization, filtering, or using robust estimation methods may be needed when working with high-noise data.

Sample Size and Estimation Quality:

The results demonstrate that having a larger sample size improves the accuracy of the estimates. This is expected, as more data provides more information for the estimation process. In real-world scenarios, collecting more data is often a key strategy for improving model accuracy.

Robustness of Least Squares Estimation:

The Least Squares Estimation method is effective in estimating the parameters of the ARX model, but its performance is sensitive to noise levels and sample sizes. While it performs well under low-noise conditions with sufficient data, it can become less reliable when noise levels are high or when data is sparse.

Conclusion

The implementation of **Least Squares Estimation** for parameter estimation in an ARX model demonstrates the fundamental principles of regression-based estimation in time series analysis. The simulation results provide valuable insights into how the accuracy of parameter estimates is affected by noise, sample size, and the number of iterations.

Higher noise variance leads to less accurate parameter estimates.

Larger sample sizes help improve the reliability and accuracy of the estimates.

The method is robust and effective under normal conditions but may require modifications for handling noisy or limited data.