In The Name Of God

Shiraz University

Estimation Theory

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HW 1

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Introduction

The main objective of this code is to demonstrate the use of **Least Squares Estimation** (LSE) to estimate unknown parameters in various models. The code simulates and estimates parameters in three distinct scenarios: polynomial models using powers of a design variable, polynomial models with an alternative set of basis functions, and Fourier series coefficient estimation. Each scenario involves noise in the observations, and the code uses Monte Carlo simulations to evaluate the accuracy of the parameter estimates over multiple iterations.

Implementation

Description of the Problem:

Polynomial Model with Powers of a Given Variable (r): In this part, we aim to estimate the parameters of a polynomial model. The model is defined in terms of powers of a given design variable r. For this, a matrix is constructed where each row contains increasing powers of r, and the observations are calculated using these values, with some added noise to simulate real-world measurement errors. The objective is to estimate the parameters of this polynomial using the least squares method, which minimizes the difference between the observed data and the predicted values. This estimation is repeated multiple times to observe how the estimates evolve over several iterations, allowing for a better understanding of the influence of noise on the accuracy of the estimates.

Polynomial Model with an Alternative Set of Basis Functions: In this scenario, the design matrix is constructed using a different set of basis functions. These basis functions are used to represent the data, and the parameter estimation is done using a similar approach to the first scenario. Here, the polynomial terms are combined with specific basis functions, and the parameter estimation process is repeated over a large number of iterations to assess the stability and accuracy of the estimates. The results are visualized to show how the estimates converge and how the distribution of these estimates looks after many iterations.

Fourier Series Coefficient Estimation: In this scenario, we shift focus to signal processing by estimating the Fourier series coefficients. The Fourier series is used to represent periodic signals as a sum of sine and cosine waves. The goal here is to estimate the coefficients of the sine and cosine terms that make up the periodic signal. To do so, we add random noise to the signal and then use the least squares method to recover the Fourier coefficients. This estimation process is repeated many times, and the evolution of the estimated coefficients is observed. Histograms are also generated to show the distribution of these coefficients across multiple runs of the simulation, providing insight into the accuracy and variability of the estimates.

Methodology:

The core method used for all three scenarios is **Least Squares Estimation**. This method involves finding the parameters that minimize the sum of squared differences between the observed data and the predicted values. For each of the three problems:

A design matrix is constructed based on the model at hand (powers of r, basis functions, or Fourier terms).

Noisy data is simulated using the true parameters.

The least squares method is applied to estimate the parameters based on the noisy data.

The process is repeated multiple times using Monte Carlo simulations to capture the impact of random noise on the estimation process.

The Monte Carlo approach involves running the simulation several times with different random noise values to get a better understanding of the variability and accuracy of the estimated parameters. This also allows us to observe how the estimates converge to the true values over time and to analyze the distribution of the estimates.

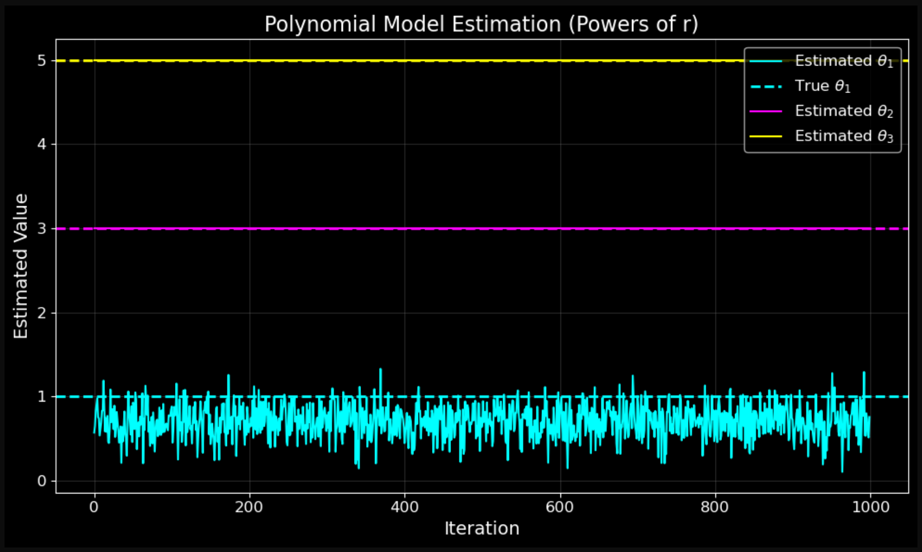
Result

Polynomial Model with Powers of a Given Variable (r): In this scenario, the parameter estimates are plotted over multiple iterations, showing how the estimates approach the true values as more data is processed. The noise added to the data causes fluctuations in the estimates, but with each iteration, the estimates begin to converge closer to the true values. This demonstrates the robustness of the least squares method in estimating parameters even in the presence of noise.

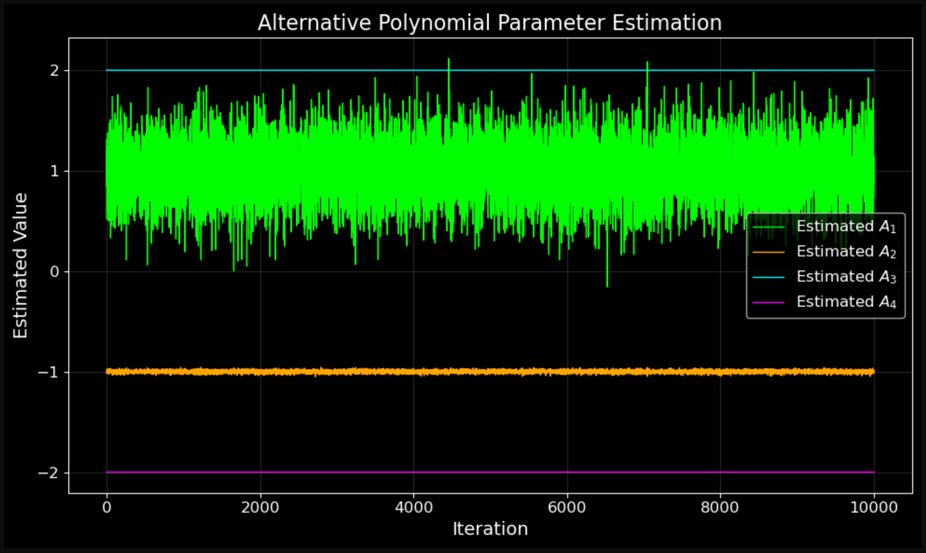
Polynomial Model with an Alternative Set of Basis Functions: Similar to the first scenario, the parameter estimates for the alternative basis functions are plotted over many iterations. The estimates initially show considerable variation due to the noise, but over time they stabilize and converge to the true values. The histograms of the estimated parameters show how the estimates are distributed across all the iterations, providing insight into the precision of the estimation process.

Fourier Series Coefficient Estimation: In the Fourier series estimation scenario, the evolution of the Fourier coefficients is plotted over multiple iterations, showing how the estimates improve with each iteration despite the noise. The histograms of the Fourier coefficients are generated to illustrate the variability of the estimates and to show how they converge to the true values over time. This part emphasizes the utility of the least squares method in signal processing tasks where periodic signals need to be modeled and estimated.

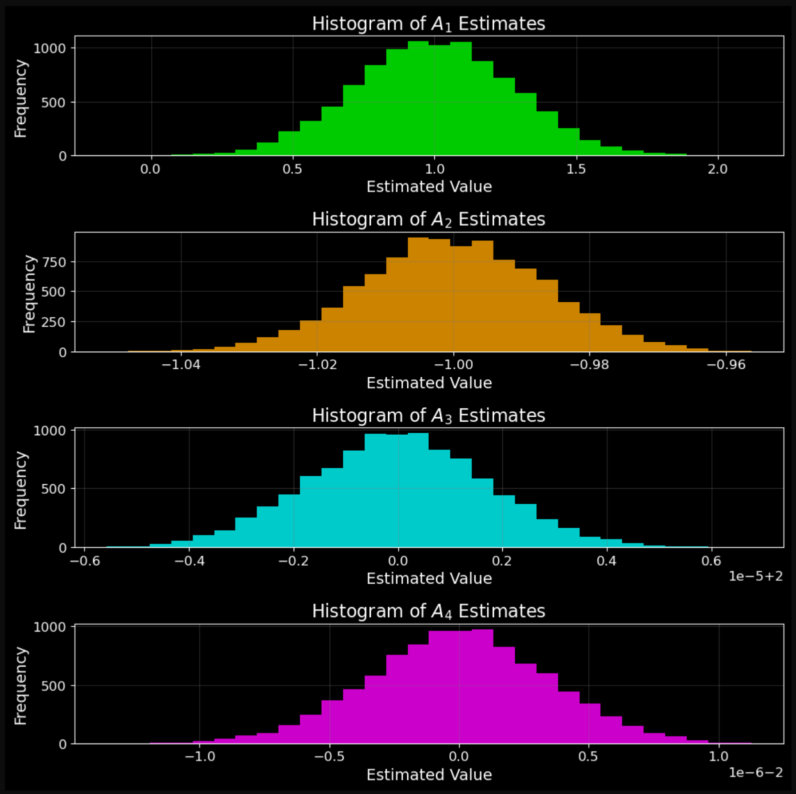
Plots



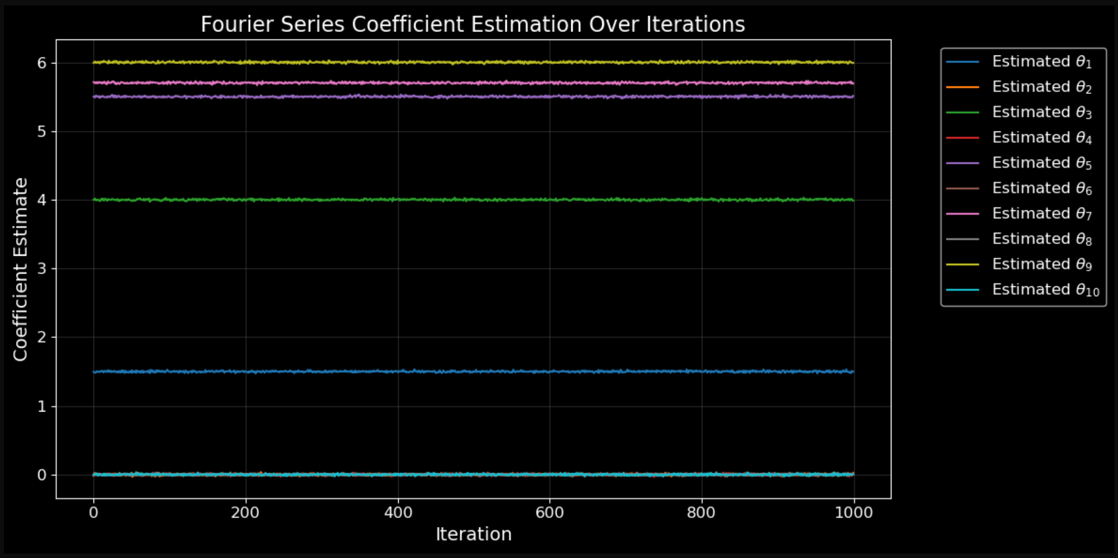
Polynomial Model Estimation (Powers of r)
Estimated θ_1
True θ_1
Estimated θ_2
Estimated θ_3
Estimated Value
Iteration
0
1
2
3
4
5
0
200
400
600
800
1000



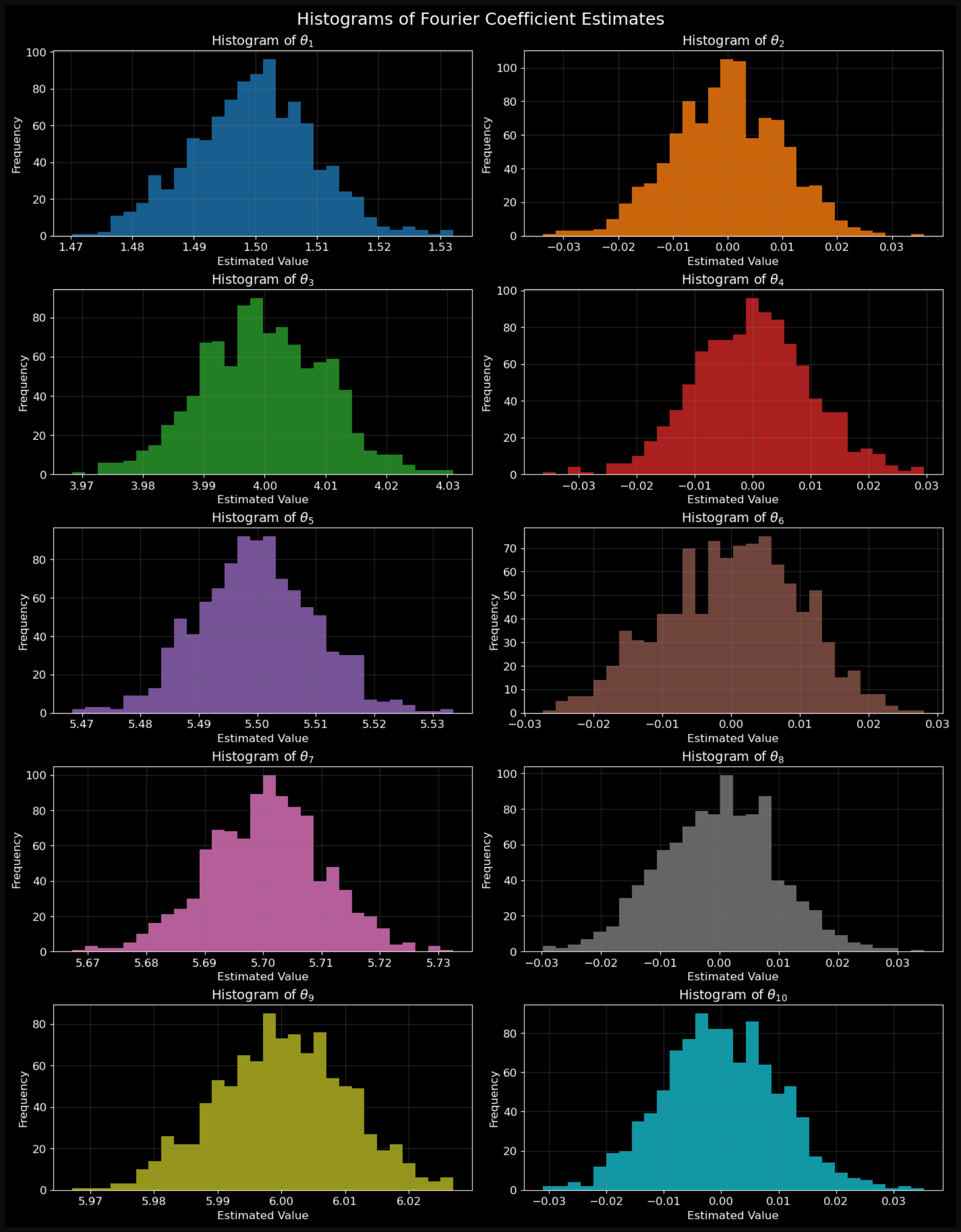
Alternative Polynomial Parameter Estimation
Estimated A_1
Estimated A_2
Estimated A_3
Estimated A_4
Estimated Value
Iteration
−2
−1
0
1
2
0
2000
4000
6000
8000
10000



Histogram of A_1 Estimates
Frequency
Estimated Value
Histogram of A_2 Estimates
Frequency
Estimated Value
Histogram of A_3 Estimates
Frequency
Estimated Value
1e−5+2
Histogram of A_4 Estimates
Frequency
Estimated Value
1e−6−2



Fourier Series Coefficient Estimation Over Iterations
Coefficient Estimate
Iteration
Estimated θ_1
Estimated θ_2
Estimated θ_3
Estimated θ_4
Estimated θ_5
Estimated θ_6
Estimated θ_7
Estimated θ_8
Estimated θ_9
Estimated θ_{10}


Histograms of Fourier Coefficient Estimates
Histogram of θ_1
Histogram of θ_2
Histogram of θ_3
Histogram of θ_4
Histogram of θ_5
Histogram of θ_6
Histogram of θ_7
Histogram of θ_8
Histogram of θ_9
Histogram of θ_{10}
Frequency
Estimated Value