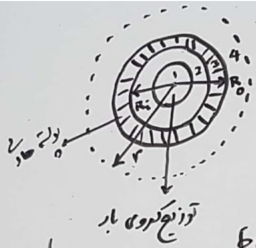


3-11

$$\rho = \rho_0\left[1-(R^2/b^2)\right]$$

$$0 \le R \le b$$

$$\oint E\cdot ds = \frac{Q}{\varepsilon_0} \rightarrow \oint E_0 \qquad r > R_o$$

$$\begin{cases} E = E_0 a_R \\ ds = R^2 \sin\theta \, d\theta \, d\phi \, a_R \end{cases}$$

$$\oint E_0 R^2 \sin\theta \, d\theta \, d\phi = \frac{Q}{\varepsilon_0}$$

$$E_0 R^2 \underbrace{\int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi}_{} = \frac{Q}{\varepsilon_0} \rightarrow E_0 = \frac{Q}{\varepsilon_0 (2\pi)(2) R^2}$$

$$-\cos\theta\Big]_0^{\pi} = -(-1-1) = 2$$

$$\phi\Big]_0^{2\pi} = 2\pi$$

$$Q = \int \rho_v \, dv = \int_0^{2\pi}\int_0^{\pi}\int_0^{b} \rho_0\left[1-(R^2/b^2)\right] R^2 \sin\theta \, dR \, d\theta \, d\phi$$

$$\rho_0 \iiint \left(R^2 - \frac{R^4}{b^2}\right) \sin\theta \, dR \, d\theta \, d\phi$$

$$\left(\frac{1}{3}R^3 - \frac{1}{5}\frac{R^5}{b^2}\right)\Big]_0^{b} = \frac{1}{3}b^3 - \frac{\cancel{b^5}\,b^3}{5\cancel{b^2}} = \boxed{\frac{2}{15}b^3}$$

$$-\cos\theta\Big]_0^{\pi} = \boxed{2} \qquad \phi\Big]_0^{2\pi} = \boxed{2\pi}$$

$$Q = \frac{8\pi b^3 \rho_0}{15}$$

$$E_0 = \frac{\cancel{8}^{\,2}\,\pi b^3 \rho_0}{15\varepsilon_0 \cancel{4\pi} R^2} = \frac{2b^3\rho_0}{15\varepsilon_0 R^2}$$

$R_i < R < R_o$ ← پوسته ی هادی بنابراین میدان درون آن صفر است.

$$\boxed{E = 0}$$

← $b < R < R_i$

$$\oint E\cdot ds = \frac{Q}{\varepsilon_0} \rightarrow \begin{cases} E = E_0 a_R \\ ds = R^2 \sin\theta \, d\theta \, d\varphi \end{cases}$$

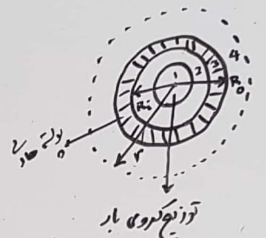
$$\int_0^{2\pi}\int_0^{\pi} E_0 R^2 \sin\theta \, d\theta \, d\varphi = \frac{Q}{\varepsilon_0} \rightarrow E_0 R^2 (2\pi)(2) = \frac{Q}{\varepsilon_0}$$

$$Q = \int \rho_v \, dv = \int_0^{2\pi}\int_0^{\pi}\int_b^{R_i} \rho_0\left[1 - \underbrace{R^2/b^2}_{\rightarrow R^2 - R^4/b^2}\right] R^2 \sin\theta \, dR \, d\theta \, d\varphi$$

$$\Rightarrow \frac{1}{3}R^3 - \frac{R^5}{5b^2}\Big]_b^{R_i}$$

$$= \frac{1}{3}R_i^3 - \frac{R_i^5}{5b^2} - \frac{1}{3}b^3 + \frac{\cancel{b^5}\,b^3}{5\cancel{b^2}} = \boxed{\frac{1}{3}R_i^3 - \frac{R_i^5}{5b^2} - \frac{2b^3}{15}}$$

$$-\cos\theta\Big]_0^{\pi} = \boxed{2} \qquad \varphi\Big]_0^{2\pi} = \boxed{2\pi}$$



3-11

$$\rho = \rho_0 \left[1 - (R^2/b^2) \right]$$

$$0 \leq R \leq b$$

$$E_0 = \frac{\frac{2}{15} b^3 (2\pi)(2) \rho_0}{\varepsilon_0 (\cancel{2\pi})(\cancel{2}) R^2} = \boxed{\frac{2 b^3 \rho_0}{15 \varepsilon_0 R^2}}$$

پوسته‌ی هادی بنابراین میدان درون آن صفر است. ← $R_i < R < R_0$

$\boxed{E = 0}$

← $b < R < R_i$

$$\oint E \cdot ds = \frac{Q}{\varepsilon_0} \rightarrow \begin{cases} E = E_0 a_R \\ ds = R^2 \sin\theta \, d\theta \, d\varphi \end{cases}$$

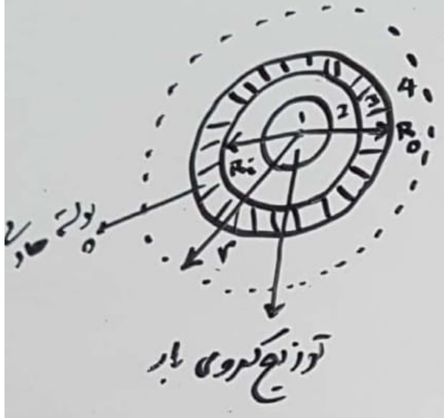
$$\int_0^{2\pi}\int_0^{\pi} E_0 R^2 \sin\theta \, d\theta \, d\varphi = \frac{Q}{\varepsilon_0} \rightarrow E_0 R^2 (2\pi)(2) = \frac{Q}{\varepsilon_0}$$

$$Q = \int \rho_v \, dv = \int_0^{2\pi}\int_0^{\pi}\int_0^{b} \rho_0 \left[1 - \frac{R^2}{b^2} \right] R^2 \sin\theta \, dR \, d\theta \, d\varphi$$

$$\left[1 - \frac{R^2}{b^2} \right] R^2 \rightarrow R^2 - \frac{R^4}{b^2}$$

$$\rightarrow \frac{1}{3} R^3 - \frac{R^5}{5 b^2} \Big]_0^b = \frac{1}{3} b^3 - \frac{b^5}{5 b^2} = \boxed{\frac{2}{15} b^3}$$

$$-\cos\theta \Big]_0^{\pi} = \boxed{2} \qquad \varphi \Big]_0^{2\pi} = \boxed{2\pi}$$



3-11

$$\rho = \rho_0 \left[1 - (R^2/b^2) \right]$$

$$0 \leq K \leq b$$

$0 \leq R \leq b$

$$\oint E \cdot ds = \frac{Q}{\varepsilon_0} \rightarrow \begin{cases} E = E_0 a_R \\ ds = R^2 \sin\theta \, d\theta \, d\varphi \end{cases}$$

$$\int_0^{2\pi}\int_0^{\pi} E_0 R^2 \sin\theta \, d\theta \, d\varphi = \frac{Q}{\varepsilon_0} \rightarrow E_0 = \frac{Q}{\varepsilon_0 (2\pi)(2) R^2}$$

$$Q = \int \rho_v \, dv = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \rho_0 \left[1 - \frac{R^2}{b^2} \right] R^2 \sin\theta \, dR \, d\theta \, d\varphi$$

$$\Rightarrow \frac{1}{3} R^3 - \frac{R^5}{5b^2} \Big]_0^R = \boxed{\frac{1}{3} R^3 - \frac{R^5}{5b^2}}$$

$$-\cos\theta \Big]_0^{\pi} = \boxed{2} \qquad \varphi \Big]_0^{2\pi} = \boxed{2\pi}$$

$$E_0 = \frac{\left(\frac{1}{3} - \frac{R^2}{5b^2} \right) R^3 \rho_0 (\cancel{2})(2\pi)}{\varepsilon_0 (4\pi) R^2} = \boxed{\frac{\rho_0 R}{\varepsilon_0} \left(\frac{1}{3} - \frac{R^2}{5b^2} \right)}$$

3-12

الف. E در تمام نقاط ؟

به دلیل تقارن موجود می‌توان از قانون گوس برای تعیین E استفاده کرد.

$r>b$

$$\oint E\cdot ds = \frac{Q}{\varepsilon_0} \rightarrow \begin{cases} E = E_0 a_r \\ ds = r\,d\varphi\,dz \end{cases}$$

$$\int_0^L\int_0^{2\pi} E_0\, r\,d\varphi\,dz = \frac{Q}{\varepsilon_0} \rightarrow E_0 = \frac{Q}{2\pi r L \varepsilon_0}$$

$$Q = \int \rho_s\,ds = \int \rho_{sa}\,ds + \int \rho_{sb}\,ds$$

$$= \int_0^L\int_0^{2\pi} \rho_{sa}\, r\,d\varphi\,dz + \int_0^L\int_0^{2\pi} \rho_{sb}\, r\,d\varphi\,dz$$

$$= \rho_{sa}\, r(2\pi)(L) + \rho_{sb}\, r(2\pi)(L)$$

$$\Rightarrow Q = \boxed{2\pi L a \rho_{sa} + 2\pi L b \rho_{sb}}$$

$$E_0 = \frac{2\pi L a\rho_{sa} + 2\pi L b\rho_{sb}}{2\pi r L \varepsilon_0}$$

$$\Rightarrow E = \boxed{\frac{a\rho_{sa} + b\rho_{sb}}{r\varepsilon_0}\, a_r}$$

$a<r<b$

$$\oint E\cdot ds = \frac{Q}{\varepsilon_0} \rightarrow \begin{cases} E = E_0 a_r \\ ds = r\,d\varphi\,dz \end{cases}$$

$$\int_0^L\int_0^{2\pi} E_0\, r\,d\varphi\,dz = \frac{Q}{\varepsilon_0} \rightarrow E_0 = \frac{Q}{2\pi r L \varepsilon_0}$$

$$Q = \rho_{sa}(2\pi r L) = 2\pi a L \rho_{sa}$$

$$\Rightarrow E_0 = \frac{\cancel{2\pi} a \cancel{L} \rho_{sa}}{\cancel{2\pi} r \cancel{L} \varepsilon_0} \rightarrow E = \boxed{\frac{\rho_{sa}\, a}{r\varepsilon_0}\, a_r}$$

که زیرا این ناحیه فقط چگالی بار سطحی ρ_{sa} را شامل می‌شود.

$r<a \rightarrow E=0$ که زیرا این ناحیه باری را شامل نمی‌شود، بنابراین میدان در این ناحیه صفر است.

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3-12

ب - رابطه‌ی بین a و b ؟

که در $r>b$ ← $E=0$

$$E = \frac{a\rho_{sa} + b\rho_{sb}}{\varepsilon_0 r} a_r \Leftarrow r>b$$

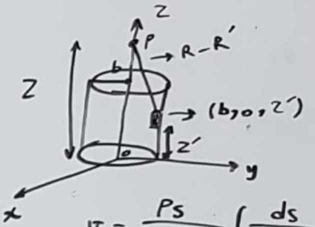
$$\downarrow$$

$$E=0 \rightarrow a\rho_{sa} + b\rho_{sb} = 0$$

$$a\rho_{sa} = -b\rho_{sb}$$

$$\boxed{\frac{a}{b} = -\frac{\rho_{sb}}{\rho_{sa}}}$$

3-19



$$V=\frac{\rho_s}{4\pi\varepsilon_0}\int\frac{ds}{|R-R'|}$$

$$R-R'=(z-z')a_z-b\,a_r$$

$$|R-R'|=\sqrt{(z-z')^2+b^2}$$

$$ds=r\,d\varphi\,dz$$

$$V=\frac{\rho_s}{4\pi\varepsilon_0}\int_0^h\int_0^{2\pi}\frac{r\,d\varphi\,dz'}{\sqrt{(z-z')^2+b^2}}$$

$$\Rightarrow V=\frac{\rho_s r}{4\pi\varepsilon_0}(2\pi)\int\frac{dz'}{\sqrt{(z-z')^2+b^2}}$$

$$\boxed{(\sinh^{-1}x)'=\frac{1}{\sqrt{1+x^2}}}$$

$$\int\frac{dz'}{b\sqrt{\left(\frac{z-z'}{b}\right)^2+1}}=\frac{1}{b}\times\sinh^{-1}\left(\frac{z-z'}{b}\right)\times(-b)$$

$$=-\sinh^{-1}\left(\frac{z-z'}{b}\right)$$

$$\sinh^{-1}x=\ln\left(x+\sqrt{1+x^2}\right)$$

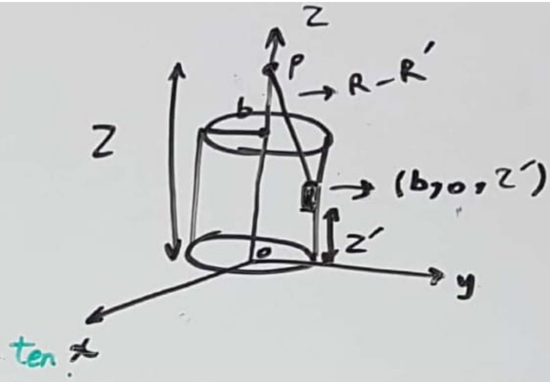
$$-\sinh^{-1}\left(\frac{z-z'}{b}\right)=-\ln\left(\frac{z-z'}{b}+\sqrt{1+\left(\frac{z-z'}{b}\right)^2}\right)$$

$$\Rightarrow\ =\frac{-\rho_s b}{2\varepsilon_0}\ln\left(\frac{(z-z')+\sqrt{b^2+(z-z')^2}}{b}\right)\Bigg]_0^h$$

$$=\frac{-\rho_s b}{2\varepsilon_0}\left[\ln\left(\frac{(z-h)+\sqrt{b^2+(z-h)^2}}{b}\right)-\ln\left(\frac{z+\sqrt{b^2+z^2}}{b}\right)\right]$$

$$=\frac{-\rho_s b}{2\varepsilon_0}\ln\left(\frac{(z-h)+\sqrt{b^2+(z-h)^2}}{z+\sqrt{b^2+z^2}}\right)$$

$$=\frac{\rho_s b}{2\varepsilon_0}\ln\left(\frac{z+\sqrt{b^2+z^2}}{(z-h)+\sqrt{b^2+(z-h)^2}}\right)$$



3-19

$$V = \frac{\rho_s b}{2\varepsilon_0}\left[\ln\left(z + \sqrt{b^2+z^2}\right) - \ln\left(z-h+\sqrt{b^2+(z-h)^2}\right)\right]$$

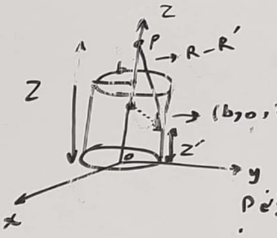
$$E = -\frac{\partial V}{\partial z} = \frac{-\rho_s b}{2}$$

$$\frac{1+\frac{\cancel{2}z}{\cancel{2}\sqrt{b^2+z^2}}}{z+\sqrt{b^2+z^2}} - \frac{1+\frac{\cancel{2}(z-h)}{\cancel{2}\sqrt{b^2+(z-h)^2}}}{z-h+\sqrt{b^2+(z-h)^2}}$$

$$\frac{\cancel{\sqrt{b^2+z^2}+z}}{\sqrt{b^2+z^2}\,\cancel{\left(z+\sqrt{b^2+z^2}\right)}} - \frac{\cancel{\sqrt{b^2+(z-h)^2}+z-h}}{\sqrt{b^2+(z-h)^2}\,\cancel{\left(z-h+\sqrt{b^2+(z-h)^2}\right)}}$$

$$E = \frac{-\rho_s b}{2}\left[\frac{1}{\sqrt{b^2+z^2}} - \frac{1}{\sqrt{b^2+(z-h)^2}}\right]$$

3-19



ب ـ در نقطه‌ای داخل استوانه P

$$V = \frac{\rho_s}{4\pi\varepsilon_0}\int \frac{ds}{|R-R'|} \quad , \quad ds = r\,d\varphi\,dz\,a_r$$

$$V = \frac{\rho_s}{4\pi\varepsilon_0}\left[\int_0^{2\pi}\int_0^{z'} \frac{b\,d\varphi\,dz'}{\sqrt{b^2+(z-z')^2}} + \int_0^{2\pi}\int_z^{h} \frac{b\,d\varphi\,dz'}{\sqrt{b^2+(z'-z)^2}}\right]$$

$$b\sqrt{1+\left(\frac{z-z'}{b}\right)^2}$$

$$= \frac{b\rho_s(2\pi)}{4\pi\varepsilon_0}\left[-\sinh^{-1}\left(\frac{z-z'}{b}\right)\Big|_0^z + \sinh^{-1}\left(\frac{z'-z}{b}\right)\Big|_z^h\right]$$

$$-\ln\left(\sqrt{1+\left(\frac{z-z'}{b}\right)^2}+\frac{z-z'}{b}\right)\Big|_0^z \quad \ln\left(\frac{z'-z}{b}+\sqrt{1+\left(\frac{z'-z}{b}\right)^2}\right)\Big|_z^h$$

$$= \frac{\rho_s b}{2\varepsilon_0}\left[\ln\left(\frac{\sqrt{b^2+z^2}+z}{b}\right)+\ln\left(\frac{h-z+\sqrt{b^2+(h-z)^2}}{b}\right)\right]$$

$$= \frac{\rho_s b}{2\varepsilon_0}\ln\left(\frac{1}{b^2}\times\left(\sqrt{b^2+z^2}+z\right)\left(h-z+\sqrt{b^2+(h-z)^2}\right)\right)$$

$$E = -\nabla V = -\frac{\partial V}{\partial z}a_z$$

$$\left(\frac{\sqrt{b^2+z^2}+z}{\sqrt{b^2+z^2}}\right)$$

$$\left(1+\frac{2z}{2\sqrt{b^2+z^2}}\right)\left((h-z)+\sqrt{b^2+(h-z)^2}\right)$$

$$+\left(z+\sqrt{b^2+z^2}\right)\left(-1-\frac{2(h-z)}{2\sqrt{b^2+(h-z)^2}}\right) \to -\left(\frac{\sqrt{b^2+(h-z)^2}+(h-z)}{\sqrt{b^2+(h-z)^2}}\right)$$

$$E = \frac{-b\rho_s}{2\varepsilon_0}\times\frac{1}{b^2}\times\frac{\cdots}{\frac{1}{b^2}\left(z+\sqrt{b^2+z^2}\right)\left((h-z)+\sqrt{b^2+(h-z)^2}\right)}$$

$$\to E = -\frac{b\rho_s}{2\varepsilon_0}\times\frac{\frac{1}{\sqrt{b^2+z^2}}-\frac{1}{\sqrt{b^2+(h-z)^2}}}{1}$$

$$\boxed{E = \frac{b\rho_s}{2\varepsilon_0}\,\frac{\sqrt{b^2+z^2}-\sqrt{b^2+(h-z)^2}}{\left(\sqrt{b^2+z^2}\right)\left(\sqrt{b^2+(h-z)^2}\right)}\,a_z}$$

3-16

الف - محاسبه V در صفحه منصف بار خطی P

$$V = \frac{\rho_L}{4\pi\varepsilon_0}\int \frac{dL}{|R-R'|} \qquad \begin{cases} R_1 = y a_y + z a_z \\ R_2 = +x a_x \end{cases}$$

$$R - R' = R_1 - R_2 = y a_y + z a_z - x a_x$$

$$|R-R'| = \sqrt{x^2+y^2+z^2}$$

$$V = \frac{\rho_L}{4\pi\varepsilon_0}\int_{-L/2}^{L/2} \frac{dx}{\sqrt{x^2+y^2+z^2}}$$

$$\downarrow$$

$$\frac{dx}{\sqrt{y^2+z^2}\sqrt{\left(\frac{x}{\sqrt{y^2+z^2}}\right)^2+1}}$$

$$V = \frac{\rho_L}{4\pi\varepsilon_0}\sinh^{-1}\left(\frac{x}{\sqrt{y^2+z^2}}\right)\Bigg|_{-L/2}^{L/2} = \frac{\rho_L}{4\pi\varepsilon_0}\, Ln\left(\frac{x}{\sqrt{y^2+z^2}} + \sqrt{1+\frac{x^2}{y^2+z^2}}\right)$$

$$= \frac{\rho_L}{4\pi\varepsilon_0}\, Ln\left(\frac{L/2 + \sqrt{y^2+z^2+\frac{L^2}{4}}}{-L/2 + \sqrt{y^2+z^2+\frac{L^2}{4}}}\right)$$

$$V = \frac{\rho_L}{4\pi\varepsilon_0}\, Ln\left(\frac{L+\sqrt{4y^2+4z^2+L^2}}{-L+\sqrt{4y^2+4z^2+L^2}}\right)$$

ب - محاسبه‌ی E با استفاده از قانون کولمب P

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_L R\, dL}{|R|^3} = \frac{\rho_L}{4\pi\varepsilon_0}\int \frac{-x a_x + y a_y + z a_z}{(x^2+y^2+z^2)^{3/2}}\, dx$$

$$R = -x a_x + y a_y + z a_z$$

به دلیل تقارن موجود E مولفه‌ی a_x ندارد.

$$E = \frac{\rho_L}{4\pi\varepsilon_0}\left[\int_{-L/2}^{L/2} \frac{y a_y + z a_z}{(x^2+y^2+z^2)^{3/2}}\, dx\right]$$

$$\downarrow$$

$$\frac{1}{\left(\sqrt{y^2+z^2}\right)^3\left(\left(\frac{x}{\sqrt{y^2+z^2}}\right)^2+1\right)^{3/2}} \Rightarrow \frac{x}{\sqrt{y^2+z^2}} = \tan\theta$$

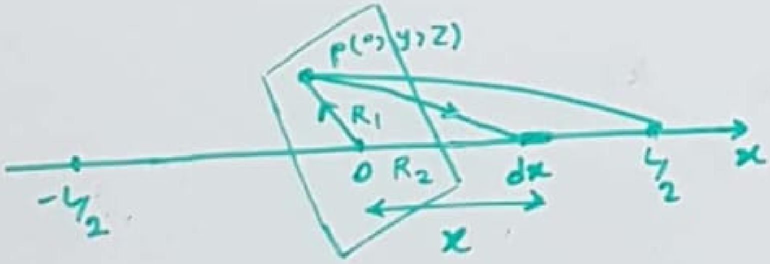
$$\downarrow$$

$$dx = \sqrt{y^2+z^2}\,(1+\tan^2\theta)\, d\theta$$

$$\int \frac{\sqrt{y^2+z^2}\,\overbrace{(1+\tan^2\theta)}^{\frac{1}{\cos^2\theta}}\, d\theta}{\underbrace{(1+\tan^2\theta)^{3/2}}_{\frac{1}{\cos^3\theta}}\left(\sqrt{y^2+z^2}\right)^3} = \frac{1}{y^2+z^2}\int \underbrace{\cos\theta\, d\theta}_{\sin\theta}$$

$$E = \frac{\rho_L\,(y a_y + z a_z)}{4\pi\varepsilon_0\,(y^2+z^2)} \times \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} =$$

$$\hookrightarrow \frac{x}{\sqrt{y^2+z^2}}\Bigg|_{-L/2}^{L/2} = \frac{L}{\sqrt{y^2+z^2}}$$



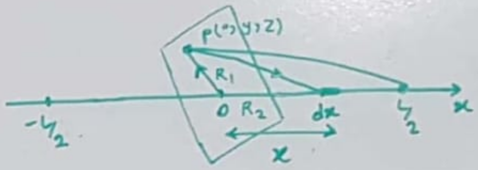
3-16

$$\frac{\frac{x}{\sqrt{y^2+z^2}}}{\sqrt{1+\frac{x^2}{y^2+z^2}}} = \frac{x}{\sqrt{y^2+z^2+x^2}}\Bigg|_{-L/2}^{L/2}$$

$$= \frac{L}{\sqrt{\frac{L^2}{4}+y^2+z^2}} = \frac{2L}{\sqrt{L^2+4y^2+4z^2}}$$

$$\Rightarrow E = \frac{\rho_L (y a_y + z a_z)}{4\pi\varepsilon_0 (y^2+z^2)} \times \frac{2L}{\sqrt{L^2+4y^2+4z^2}}$$

$$= \frac{\rho_L L (y a_y + z a_z)}{2\pi \varepsilon_0 (y^2+z^2)\sqrt{L^2+4y^2+4z^2}}$$



3-16

ب - محاسبه E با $P - \nabla V$

$$V = \frac{PL}{4\pi\varepsilon_0}\left[Ln\left(L + \sqrt{L^2 + 4y^2 + 4z^2}\right) - Ln\left(-L + \sqrt{L^2 + 4y^2 + 4z^2}\right)\right]$$

$$E = -\nabla V = -\frac{\partial V}{\partial y} a_y - \frac{\partial V}{\partial z} a_z$$

$$\frac{\partial V}{\partial y} = \frac{\dfrac{8y}{2\sqrt{L^2+4y^2+4z^2}}}{L + \sqrt{L^2+4y^2+4z^2}} - \frac{\dfrac{8y}{2\sqrt{L^2+4y^2+4z^2}}}{-L + \sqrt{L^2+4y^2+4z^2}}$$

$$= \frac{\dfrac{8y\left(-L + \sqrt{L^2+4y^2+4z^2}\right)}{2\sqrt{L^2+4y^2+4z^2}}}{\cancel{L^2} + 4y^2 + 4z^2 - \cancel{L^2}} - \frac{\dfrac{8y\left(L + \sqrt{L^2+4y^2+4z^2}\right)}{2\sqrt{L^2+4y^2+4z^2}}}{4y^2 + 4z^2}$$

$$\Rightarrow \frac{\partial V}{\partial y} = \frac{4y\,(-2L)}{(4y^2+4z^2)\sqrt{L^2+4y^2+4z^2}}$$

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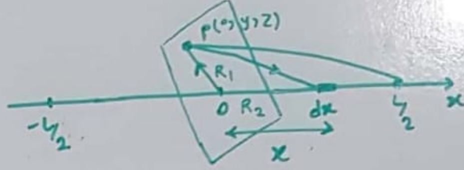


3-16

P - ∇V ، E [illegible] - ب

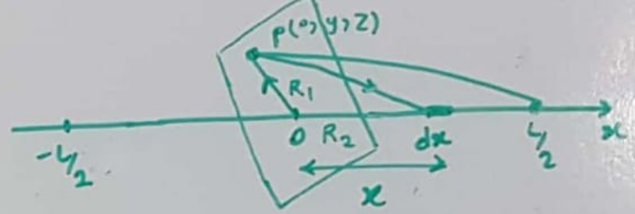
$$V = \frac{\rho_L}{4\pi\varepsilon_0}\left[\ln\left(L+\sqrt{L^2+4y^2+4z^2}\right)-\ln\left(-L+\sqrt{L^2+4y^2+4z^2}\right)\right]$$

$$E = -\nabla V = -\frac{\partial V}{\partial y}a_y - \frac{\partial V}{\partial z}a_z$$

$$\frac{\partial V}{\partial z} = \frac{\dfrac{8z}{2\sqrt{L^2+4y^2+4z^2}}}{L+\sqrt{L^2+4y^2+4z^2}} - \frac{\dfrac{8z}{2\sqrt{L^2+4y^2+4z^2}}}{-L+\sqrt{L^2+4y^2+4z^2}}$$

$$= \frac{\dfrac{8z\left(-L+\sqrt{L^2+4y^2+4z^2}\right)}{2\sqrt{L^2+4y^2+4z^2}}}{\cancel{L^2}+4y^2+4z^2-\cancel{L^2}} - \frac{\dfrac{8z\left(L+\sqrt{L^2+4y^2+4z^2}\right)}{2\sqrt{L^2+4y^2+4z^2}}}{4y^2+4z^2}$$

$$\Rightarrow \frac{\partial V}{\partial z} = \frac{4z(-2L)}{(4y^2+4z^2)\sqrt{L^2+4y^2+4z^2}}$$



ب - احسب E بـ $-\nabla V$ P

$$V = \frac{\rho_L}{4\pi\varepsilon_0}\left[\ln\left(L + \sqrt{L^2 + 4y^2 + 4z^2}\right) - \ln\left(-L + \sqrt{L^2 + 4y^2 + 4z^2}\right)\right]$$

$$E = -\nabla V = -\frac{\partial V}{\partial y} a_y - \frac{\partial V}{\partial z} a_z$$

$$E = \frac{-L\rho_L\,(y a_y + z a_z)}{2\pi\varepsilon_0 (y^2 + z^2)\sqrt{L^2 + 4y^2 + 4z^2}}$$