رضا ادینه پور
۹۸۱۴۳۰۳
تمرین سری دوم ریاضی مهندسی

#1 می دانیم:
$$\begin{cases} c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} f(x)\, e^{-jn\omega_0 x} \\ e^{jx} = \cos x + j\sin x \end{cases} \Rightarrow \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(x)\,(\cos n\omega_0 x - j\sin n\omega_0 x)\, dx$$

$$= \underbrace{\frac{1}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} f(x)\cos n\omega_0 x\, dx}_{\frac{a_n}{2}} - j\underbrace{\frac{1}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} f(x)\sin n\omega_0 x\, dx}_{\frac{b_n}{2}} = \frac{a_n}{2} - j\frac{b_n}{2} = c_n \quad **$$

و از طرفی می دانیم یک تابع زوج تنها هارمونیک های کسینوسی دارد و یک تابع فرد فقط هارمونیک های سینوسی دارد.

#2
$$\text{if} \quad f(x) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 x} \overset{?}{\Longrightarrow} \frac{1}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} f(x)\cdot f^*(x) = \sum_{-\infty}^{+\infty} |c_n|^2$$

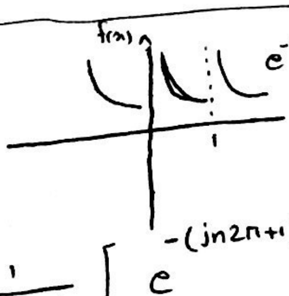
$$\Rightarrow \frac{1}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} c_n e^{jn\omega_0 x}\cdot c_n e^{-jn\omega_0 x} = \frac{1}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} c_n^2 e^{j(n\omega_0 x - n\omega_0 x)} = \frac{1}{T}\int_{-\frac{T}{2}}^{+\frac{T}{2}} c_n^2 \times \underbrace{e^0}_{=1}$$

$$= \sum_{n=-\infty}^{+\infty} |c_n|^2$$

#3
$$\text{if} \begin{cases} f(x) \xrightarrow{\text{ضرایب}} c_n \\ g(x) \longrightarrow d_n \end{cases} \Rightarrow \begin{cases} f(x) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 x} \\ g(x) = \sum_{n=-\infty}^{+\infty} d_n e^{jn\omega_0 x} \end{cases}$$

$$\Rightarrow f(x)\cdot g(x) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 x} \times \sum_{n=-\infty}^{+\infty} d_n e^{jn\omega_0 x} = \sum_{n=-\infty}^{+\infty} c_n d_n e^{2jn\omega_0 x} = h(x)$$

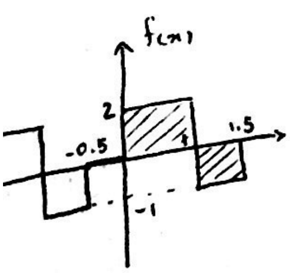
#4



$$\begin{cases} T = 1 \\ \omega_0 = \frac{2\pi}{T} = 2\pi \end{cases} \qquad c_0 = \int_0^1 e^{-x}\, dx = -e^{-x}\Big|_0^1 = 1 - \frac{1}{e}$$

$$c_n = \int_0^1 e^{-x} e^{-jn2\pi x}\, dx = \int_0^1 e^{(jn2\pi+1)-x}\, dx = \frac{-1}{jn2\pi+1} e^{-x(jn2\pi+1)}\Big|_0^1$$

$$= \frac{1}{1+2\pi jn}\left[e^{-(jn2\pi+1)} - 1\right] = \frac{1-jn2\pi}{1+4n^2\pi^2}\left(e^{-(jn2\pi+1)} - 1\right) =$$



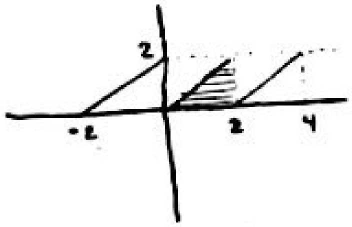
$$\begin{cases} T = 1.5 \\ \omega_0 = \frac{2\pi}{1.5} = \frac{4\pi}{3} \end{cases} \qquad \begin{aligned} c_0 &= (2\times 1) - (0.5\times 1) = 1.5 \\ c_n &= \int_0^1 2e^{-jn\frac{4\pi}{3}x}\, dx + \int_1^{1.5} -e^{-jn\frac{4\pi}{3}x}\, dx \end{aligned}$$

$$= \frac{-2}{jn\frac{4\pi}{3}} e^{-jn\frac{4\pi}{3}x}\Big|_0^1 + \frac{1}{jn\frac{4\pi}{3}} e^{-jn\frac{4\pi}{3}x}\Big|_1^{1.5} = \frac{-6}{jn4\pi} e^{-jn\frac{4\pi}{3}x}\Big|_0^1 + \frac{3}{jn4\pi} e^{-jn\frac{4\pi}{3}x}\Big|_1^{1.5}$$

#4 حلها

$$= \frac{1}{jn4\pi}\left[-6e^{-jn\frac{4\pi}{3}} + 6 + 3e^{-jn2\pi} - 3e^{-jn\frac{4\pi}{3}}\right] = \frac{1}{jn4\pi}\left[-6e^{-jn\frac{4\pi}{3}} + 3e^{-jn2\pi} + 6\right]$$

$$= \frac{-j}{4n\pi}\left[-6e^{-jn\frac{4\pi}{3}} + 3e^{-jn2\pi} + 6\right] = C_n$$



$$\begin{cases} T = 2 \\ \omega_0 = \frac{2\pi}{2} = \pi \end{cases}$$

$$C_0 = \frac{1}{2} \times 2 \times 2 = 2$$

$$C_n = \frac{1}{2}\int_0^2 x e^{jn\pi x}\, dx =$$

مشتق	انتگرال
x (+)	$e^{jn\pi x}$
1 (−)	$\frac{1}{jn\pi} e^{jn\pi x}$
0	$\frac{-1}{n^2\pi^2} e^{jn\pi x}$

$$= \frac{1}{2}\left[\frac{x}{jn\pi} e^{jn\pi x} + \frac{1}{n^2\pi^2} e^{jn\pi x}\right]_0^2 = \frac{1}{2}\left[\left(\frac{2}{jn\pi} e^{jn\pi 2} + \frac{1}{n^2\pi^2} e^{jn\pi 2}\right)\right.$$

$$\left. - \left(0 + \frac{1}{n^2\pi^2}\right)\right] = \frac{1}{2}\left(\frac{2}{jn\pi} e^{jn\pi 2} + \frac{1}{n^2\pi^2} e^{jn\pi 2} - \frac{1}{n^2\pi^2}\right)$$

$$= \frac{1}{2}\left(e^{jn\pi 2}\left(\frac{2}{jn\pi} + \frac{1}{n^2\pi^2}\right) - \frac{1}{n^2\pi^2}\right)$$