پاسخ تمرین سری 5 ریاضی مهندسی (سری فوریه)

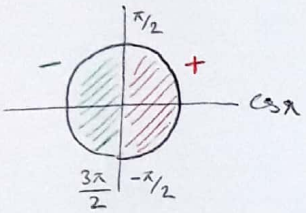
سری فوریه کسینوسی تابع $f(x)=H(\cos x)$ ، $H(x)=\begin{cases}1 & x>0\\ 0 & x<0\end{cases}$ ؟

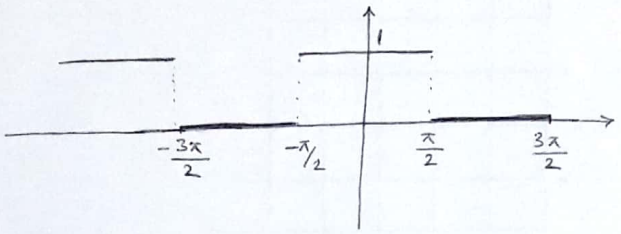
$a_0=\checkmark$

$a_n=\checkmark$

$b_n=0$

$$H(y)=\begin{cases}1 & *\ \cos x>0\\ 0 & *\ \cos x<0\end{cases}\Longrightarrow f(x)=\begin{cases}1 & -\pi/2<x<\pi/2\\ 0 & \pi/2<x<3\pi/2\end{cases}$$





تابع زوج

$T=2\pi$

$$a_0=\frac{1\times 2}{T}\int_0^{\pi/2} f(x)\,dx=\frac{2}{2\pi}\int_0^{\pi/2}1\,dx=\frac{1}{\pi}x\Big]_0^{\pi/2}=\frac{1}{\pi}\times\frac{\pi}{2}=\frac{1}{2}$$

$$a_n=\frac{2\times 2}{T}\int_0^{\pi/2} f(x)\cos\frac{2n\pi}{T}x\,dx=\frac{4}{2\pi}\int_0^{\pi/2}1\times\cos\frac{2n\pi}{2\pi}x\,dx=\frac{2}{\pi}\times\frac{1}{n}\sin nx\Big]_0^{\pi/2}=\frac{2}{n\pi}\sin\frac{n\pi}{2}$$

$$a_n=\begin{cases}n=2m, & 0\\ n=2m-1, & \dfrac{2}{(2m-1)\pi}\sin\dfrac{(2m-1)\pi}{2}\end{cases}$$

$n=2m+1\Rightarrow \sin\frac{n\pi}{2}=(-1)^n$

$n=2m-1\Rightarrow \sin\frac{n\pi}{2}=-(-1)^n$

$$\Longrightarrow f(x)=\frac{1}{2}+\sum_{m=1}^{\infty}\frac{2}{(2m-1)\pi}\sin\frac{(2m-1)\pi}{2}\times\cos(2m-1)x$$

$$\Longrightarrow f(x)=\frac{1}{2}+\frac{2}{\pi}\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{(2m-1)}\cos(2m-1)x\ \checkmark$$