سری اول:

۱۴- $f(t+2)=f(t) \underset{\downarrow}{} \; T=2$ ، $f(t)=\begin{cases} t & 0<t<1 \\ 2-t & 1<t<2 \end{cases}$ ، سری فوریه $f(t)$ ؟

$$f(t)=a_0+\sum_{n=1}^{\infty} a_n\cos\frac{2n\pi}{T}t+b_n\sin\frac{2n\pi}{T}t$$

$$a_0=\frac{1}{T}\int f(t)\,dt=\frac{1}{2}\left[\int_0^1 t\,dt+\int_1^2(2-t)\,dt\right]=\frac{1}{2}\left[\frac{t^2}{2}\Big]_0^1+2t\Big]_1^2-\frac{t^2}{2}\Big]_1^2\right]$$

$$=\frac{1}{2}\left[\frac{1}{2}+\cancel{4}-\cancel{2}-\cancel{2}+\frac{1}{2}\right]=\boxed{\frac{1}{2}}$$

$$a_n=\frac{2}{T}\int f(t)\cos\frac{2n\pi}{T}t\,dt=\frac{2}{2}\left[\int_0^1 t\cos n\pi t\,dt+\int_1^2(2-t)\cos n\pi t\,dt\right]$$

$$=\left[\cancel{\frac{t}{n\pi}\sin n\pi t}+\frac{1}{n^2\pi^2}\cos n\pi t\right]_0^1+\cancel{2\times\frac{1}{n\pi}\sin n\pi t}\Big]_1^2-\cancel{\frac{t}{n\pi}\sin n\pi t}-\frac{1}{n^2\pi^2}\cos n\pi t\Big]_1^2$$

مشتق	انتگرال
t (+)	$\cos n\pi t$
1 (−)	$\frac{1}{n\pi}\sin n\pi t$
0	$\frac{-1}{n^2\pi^2}\cos n\pi t$

$$=\left[\frac{\cos n\pi}{n^2\pi^2}-\frac{1}{n^2\pi^2}-\frac{\overset{1}{\cos 2n\pi}}{n^2\pi^2}+\frac{\cos n\pi}{n^2\pi^2}\right]$$

$$\cos n\pi=(-1)^n$$

$$=\boxed{\frac{2}{n^2\pi^2}\left((-1)^n-1\right)}\longrightarrow\begin{cases}0 & n=2k \quad \text{زوج}\\ \dfrac{-4}{n^2\pi^2} & n=2k-1 \quad \text{فرد}\end{cases}$$

$$b_n=\frac{2}{T}\int f(t)\sin\frac{2n\pi}{T}t\,dt=\frac{2}{2}\left[\int_0^1 t\sin n\pi t\,dt+\int_1^2(2-t)\sin n\pi t\,dt\right]$$

$$=\left[\frac{-t}{n\pi}\cos n\pi t+\cancel{\frac{1}{n^2\pi^2}\sin n\pi t}\right]_0^1-\frac{2}{n\pi}\cos n\pi t\Big]_1^2+\frac{t}{n\pi}\cos n\pi t-\cancel{\frac{1}{n^2\pi^2}\sin n\pi t}\Big]_1^2$$

مشتق	انتگرال
t (+)	$\sin n\pi t$
1 (−)	$\frac{-1}{n\pi}\cos n\pi t$
0	$\frac{-1}{n^2\pi^2}\sin n\pi t$

$$=\left[\frac{-\cos n\pi}{n\pi}-\frac{2\cos 2n\pi}{n\pi}+\frac{2\cos n\pi}{n\pi}+\frac{2\cos 2n\pi}{n\pi}-\frac{\cos n\pi}{n\pi}\right]=\boxed{0}$$

$$\Longrightarrow f(t)=\frac{1}{2}+\sum_{n=1}^{\infty}\frac{2}{n^2\pi^2}\left((-1)^n-1\right)\cos n\pi t$$

$$\Longrightarrow f(t)=\frac{1}{2}-\sum_{k=1}^{\infty}\frac{4}{(2k-1)^2\pi^2}\cos(2k-1)\pi t$$

نرگس ب

$T=2\pi \;,\; f(x)=x^3$ (۷

$b_n \checkmark \;,\; a_n=0 \;,\; a_0=0 \Leftarrow \underbrace{x}_{\text{فرد}} \times \underbrace{x^2}_{\text{زوج}} =$ فرد

$$f(x)=\cancel{a_0}+\sum_{n=1}^{\infty} \cancel{a_n \cos\frac{2n\pi}{T}x} + b_n \sin\frac{2n\pi}{T}x$$

$$b_n=\frac{2\times2}{T}\int_0^{\pi} f(x)\sin\frac{2n\pi}{T}x\,dx=\frac{4}{2\pi}\int_0^{\pi} x^3 \sin nx\,dx$$

مشتق	انتگرال
x^3	$\sin nx$
$3x^2$ (+)	$\frac{-1}{n}\cos nx$
$6x$ (−)	$\frac{-1}{n^2}\sin nx$
6 (+)	$\frac{+1}{n^3}\cos nx$
0 (−)	$\frac{1}{n^4}\sin nx$

$$=\frac{2}{\pi}\left[\frac{-x^3}{n}\cos nx+\cancel{\frac{3x^2}{n^2}\sin nx}+\frac{6x}{n^3}\cos nx-\cancel{\frac{6}{n^4}\sin nx}\right]_0^{\pi}$$

$$=\frac{2}{\pi}\left[\frac{-\pi^3}{n}\cos n\pi+\frac{6\pi}{n^3}\cos n\pi\right]=\frac{2(-1)^n}{n}\left[-\pi^2+\frac{6}{n^2}\right]$$

$$\begin{cases} \text{زوج } n: & \frac{2}{n}\left[-\pi^2+\frac{6}{n^2}\right] \\ \text{فرد } n, & \frac{-2}{n}\left[-\pi^2+\frac{6}{n^2}\right] \end{cases}$$

$$\longrightarrow f(x)=\sum_{n=1}^{\infty}\frac{2(-1)^n}{n}\left[\frac{6}{n^2}-\pi^2\right]\sin nx$$

$$x=\frac{\pi}{2}\longrightarrow f(\pi/2)=\sum_{n=1}^{\infty}\frac{2(-1)^n}{n}\left[\frac{6}{n^2}-\pi^2\right]\sin\frac{n\pi}{2}$$

$n=2k-1 \rightarrow \sin\frac{n\pi}{2}=-(-1)^n$

$n=2k+1 \rightarrow \sin\frac{n\pi}{2}=(-1)^n$