Electric and Magnetic field equations

$$\overline{\mathrm{E}}(\bar{\mathrm{r}}) = \int_{c'} \frac{\rho_l(\bar{\mathrm{r}}')\mathrm{dl}'}{4\pi\varepsilon R^2}\hat{\mathrm{a}}_R \qquad \overline{\mathrm{E}}(\bar{\mathrm{r}}) = \int_{s'} \frac{\rho_s(\bar{\mathrm{r}}')\mathrm{ds}'}{4\pi\varepsilon R^2}\hat{\mathrm{a}}_R \qquad \overline{\mathrm{E}}(\bar{\mathrm{r}}) = \int_{v'} \frac{\rho_v(\bar{\mathrm{r}}')\mathrm{dv}'}{4\pi\varepsilon R^2}\hat{\mathrm{a}}_R$$

$$\mathrm{V}(\bar{\mathrm{r}}) = \int_{c'} \frac{\rho_l(\bar{\mathrm{r}}')\mathrm{dl}'}{4\pi\varepsilon R} \qquad \mathrm{V}(\bar{\mathrm{r}}) = \int_{s'} \frac{\rho_s(\bar{\mathrm{r}}')\mathrm{ds}'}{4\pi\varepsilon R} \qquad \mathrm{V}(\bar{\mathrm{r}}) = \int_{v'} \frac{\rho_v(\bar{\mathrm{r}}')\mathrm{dv}'}{4\pi\varepsilon R}$$

$$\oint_s \overline{\mathrm{E}}.\overline{\mathrm{ds}} = \frac{1}{\varepsilon_0}\int_v \rho_v\,\mathrm{dv} \qquad \nabla.\overline{\mathrm{E}} = {}^{\rho}/_{\varepsilon_0} \qquad \nabla\times\overline{\mathrm{E}} = 0 \qquad \nabla^2\mathrm{V} = -{}^{\rho_v}/_{\varepsilon_0} \qquad \nabla.\overline{\mathrm{D}} = \rho_v \qquad \overline{\mathrm{D}} = \varepsilon\overline{\mathrm{E}}$$

$$\oint_s \overline{\mathrm{D}}.\overline{\mathrm{ds}} = \mathrm{Q} \qquad V = \int_L \overline{\mathrm{E}}.\overline{\mathrm{dl}} \qquad C = \frac{Q}{V} \qquad \rho_{ps} = \overline{\mathrm{P}}.\hat{\mathrm{a}}_n \qquad \rho_{pv} = -\nabla.\overline{\mathrm{P}} \qquad \overline{\mathrm{D}} = \varepsilon_0\overline{\mathrm{E}} + \overline{\mathrm{P}} \qquad \overline{\mathrm{P}} = \varepsilon_0\chi_e\overline{\mathrm{E}}$$

$$\overline{\mathrm{B}}(\bar{\mathrm{r}}) = \int_{c'} \frac{\mu_0 I\overline{\mathrm{dl}'}\times\bar{R}}{4\pi R^3} \qquad \overline{\mathrm{B}}(\bar{\mathrm{r}}) = \int_{s'} \frac{\mu_0\vec{J}_s(\bar{\mathrm{r}}')\times\bar{R}\mathrm{ds}'}{4\pi R^3} \qquad \overline{\mathrm{B}}(\bar{\mathrm{r}}) = \int_{v'} \frac{\mu_0\vec{J}_v(\bar{\mathrm{r}}')\times\bar{R}\mathrm{dv}'}{4\pi R^3}$$

$$\overline{\mathrm{A}}(\bar{\mathrm{r}}) = \int_{c'} \frac{\mu_0 I\overline{\mathrm{dl}'}}{4\pi R} \qquad \overline{\mathrm{A}}(\bar{\mathrm{r}}) = \int_{s'} \frac{\mu_0\vec{J}_s(\bar{\mathrm{r}}')\mathrm{ds}'}{4\pi R} \qquad \mathrm{A}(\bar{\mathrm{r}}) = \int_{v'} \frac{\mu_0\vec{J}_v(\bar{\mathrm{r}}')\mathrm{dv}'}{4\pi R}$$

$$\oint_s \overline{\mathrm{B}}.\overline{\mathrm{ds}} = 0 \qquad \nabla.\overline{\mathrm{B}} = 0 \qquad \nabla\times\overline{\mathrm{B}} = \mu_0\vec{J}_v \qquad \nabla^2\overline{\mathrm{A}} = -\mu_0\vec{J}_v \qquad \nabla\times\overline{\mathrm{H}} = \vec{J}_v \qquad \overline{\mathrm{B}} = \mu_0\overline{\mathrm{H}}$$

$$\oint_L \overline{\mathrm{H}}.\overline{\mathrm{dL}} = \mathrm{I} \qquad L = \frac{\Psi}{I} \qquad \vec{J}_{ms} = \overline{\mathrm{M}}\times\hat{\mathrm{a}}_n \qquad \vec{J}_{mv} = \nabla\times\overline{\mathrm{M}} \qquad \overline{\mathrm{H}} = \frac{1}{\mu_0}\overline{\mathrm{B}} - \overline{\mathrm{M}} \qquad \overline{\mathrm{M}} = \frac{\chi_m}{\mu_0}\overline{\mathrm{B}}$$

Boundary conditions

$$\hat{\mathrm{a}}_{n21}\times(\overline{\mathrm{E}}_1 - \overline{\mathrm{E}}_2) = 0 \qquad \hat{\mathrm{a}}_{n21}.(\overline{\mathrm{D}}_1 - \overline{\mathrm{D}}_2) = \rho_s$$

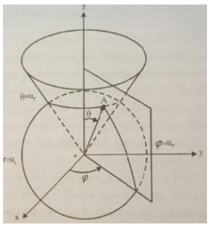
$$\hat{\mathrm{a}}_{n21}\times(\overline{\mathrm{H}}_1 - \overline{\mathrm{H}}_2) = \vec{J}_s \qquad \hat{\mathrm{a}}_{n21}.(\overline{\mathrm{B}}_1 - \overline{\mathrm{B}}_2) = 0$$

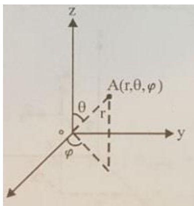
Some useful integrals

$$\int \frac{\mathrm{dx}}{(\mathrm{x}^2+\mathrm{a}^2)^{3/2}} = \frac{\mathrm{x}}{\mathrm{a}^2(\mathrm{x}^2+\mathrm{a}^2)^{1/2}} \qquad \int \frac{\mathrm{xdx}}{(\mathrm{x}^2+\mathrm{a}^2)^{3/2}} = \frac{-1}{(\mathrm{x}^2+\mathrm{a}^2)^{1/2}}$$

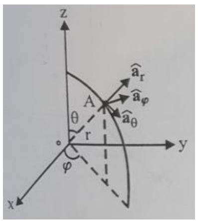
Spherical to Cartesian coordinate transforms

Inner product (.)	$\hat{a}_x$	$\hat{a}_y$	$\hat{a}_z$
$\hat{a}_r$	sinθ cosφ	sin θ sinφ	cosθ
$\hat{a}_\theta$	cosθ cosφ	cosθ sinφ	- sinθ
$\hat{a}_\varphi$	- sinφ	cosφ	0



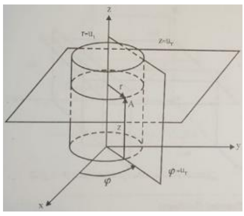


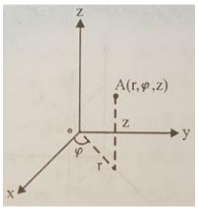
$$x = r\sin\theta\cos\varphi$$
$$y = r\sin\theta\sin\varphi$$
$$z = r\cos\theta$$
$$r = (x^2+y^2+z^2)^{\frac{1}{2}}$$
$$\varphi = \tan^{-1}\frac{y}{x}$$
$$\theta = \cos^{-1}\frac{z}{r}$$



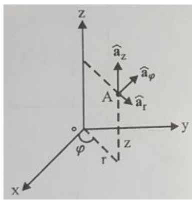
Cylindrical to Cartesian coordinate transforms

Inner product (.)	$\hat{a}_x$	$\hat{a}_y$	$\hat{a}_z$
$\hat{a}_r$	cosφ	sinφ	0
$\hat{a}_\varphi$	-sinφ	cosφ	0
$\hat{a}_z$	0	0	1





$$x = r\cos\varphi$$
$$y = r\sin\varphi$$
$$r = (x^2+y^2)^{\frac{1}{2}}$$
$$\varphi = \tan^{-1}\frac{y}{x}$$



$$\nabla.\overline{\mathrm{A}} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(h_2h_3A_1) + \frac{\partial}{\partial u_2}(h_1h_3A_2) + \frac{\partial}{\partial u_3}(h_1h_2A_3)\right]$$

$$\nabla\times\overline{\mathrm{A}} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{a}_{u_1} & h_2\hat{a}_{u_2} & h_3\hat{a}_{u_3} \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1A_1 & h_2A_2 & h_3A_3 \end{vmatrix}$$

$$\nabla f = \frac{1}{h_1}\frac{\partial f}{\partial u_1}\hat{a}_{u_1} + \frac{1}{h_2}\frac{\partial f}{\partial u_2}\hat{a}_{u_2} + \frac{1}{h_3}\frac{\partial f}{\partial u_3}\hat{a}_{u_3}$$

$$\nabla^2 f = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2h_3}{h_1}\frac{\partial f}{\partial u_1}\right) + \frac{\partial}{\partial u_{21}}\left(\frac{h_1h_3}{h_2}\frac{\partial f}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1h_2}{h_3}\frac{\partial f}{\partial u_3}\right)\right]$$

Trigonometric Identities

$$\sin^2\alpha = \frac{1-\cos(2\alpha)}{2}, \qquad \cos^2\alpha = \frac{1+\cos(2\alpha)}{2}$$

$$\sin^3\alpha = \frac{1}{4}(-\sin(3\alpha) + 3\sin\alpha)$$

$$\cos^3\alpha = \frac{1}{4}(\cos(3\alpha) + 3\cos\alpha)$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

$$\sin 2\alpha = \frac{1}{2}\sin\alpha\cos\alpha\ \alpha$$

Cartesian coordinate

$\overline{\mathrm{dl}}_x = \mathrm{dx}\hat{a}_x,\quad \overline{\mathrm{dl}}_y = \mathrm{dy}\hat{a}_y,\quad \overline{\mathrm{dl}}_z = \mathrm{dz}\hat{a}_z$

$\overline{\mathrm{dl}} = \mathrm{dx}\hat{a}_x + \mathrm{dy}\hat{a}_y + \mathrm{dz}\hat{a}_z$

$\overline{\mathrm{ds}}_x = \mathrm{dydz}\hat{a}_x,\ \overline{\mathrm{ds}}_y = \mathrm{dxdz}\hat{a}_y,\ \overline{\mathrm{ds}}_z = \mathrm{dxdy}\hat{a}_z$

$\mathrm{dv} = \mathrm{dxdydz}$

$\bar{\mathrm{r}} = \mathrm{x}\hat{a}_x + \mathrm{y}\hat{a}_y + \mathrm{z}\hat{a}_z$

Cylindrical coordinate

$\overline{\mathrm{dl}}_r = \mathrm{dr}\hat{a}_r,\quad \overline{\mathrm{dl}}_\varphi = \mathrm{rd}\varphi\hat{a}_\varphi,\quad \overline{\mathrm{dl}}_z = \mathrm{dz}\hat{a}_z$

$\overline{\mathrm{dl}} = \mathrm{dr}\hat{a}_r + \mathrm{rd}\varphi\hat{a}_\varphi + \mathrm{dz}\hat{a}_z$

$\overline{\mathrm{ds}}_r = \mathrm{rd}\varphi\mathrm{dz}\hat{a}_r,\ \overline{\mathrm{ds}}_\varphi = \mathrm{drdz}\hat{a}_\varphi,\ \overline{\mathrm{ds}}_z = \mathrm{rdrd}\varphi\hat{a}_z$

$\mathrm{dv} = \mathrm{rdrd}\varphi\mathrm{dz}$

$\bar{\mathrm{r}} = \mathrm{r}\hat{a}_r + \mathrm{z}\hat{a}_z$

Spherical coordinate

$\overline{\mathrm{dl}}_r = \mathrm{dr}\hat{a}_r,\quad \overline{\mathrm{dl}}_\theta = \mathrm{rd}\theta\hat{a}_\theta,\quad \overline{\mathrm{dl}}_\varphi = \mathrm{r}\sin\theta\mathrm{d}\varphi\hat{a}_\varphi$

$\overline{\mathrm{dl}} = \mathrm{dr}\hat{a}_r + \mathrm{rd}\theta\hat{a}_\theta + \mathrm{r}\sin\theta\mathrm{d}\varphi\hat{a}_\varphi$

$\overline{\mathrm{ds}}_r = \mathrm{r}^2\sin\theta\mathrm{d}\theta\mathrm{d}\varphi\hat{a}_r,\ \overline{\mathrm{ds}}_\theta = \mathrm{r}\sin\theta\mathrm{drd}\varphi\hat{a}_\theta,\ \overline{\mathrm{ds}}_\varphi = \mathrm{rdrd}\theta\hat{a}_\varphi$

$\mathrm{dv} = \mathrm{r}^2\sin\theta\mathrm{drd}\theta\mathrm{d}\varphi$

$\bar{\mathrm{r}} = \mathrm{r}\hat{a}_r$