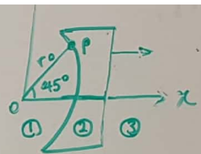


<u>3-28</u>

$P(r_0, 45°, z)$

E درناحیه‌ی 1 ← $E_1 = 5a_r - 3a_\phi$

ضریب دی‌الکتریک ؟ تا E_3 بموازات محور x باشه.

$$\begin{cases} E_{1t} = E_{2t} \\ \varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = \underbrace{\rho_s}_{0} \end{cases}$$

$E_{1t} = E_{2t} \rightarrow E_{2t} = -3 \quad , \varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = 0$

$$\downarrow$$

$$E_{2n} = \frac{\varepsilon_1}{\varepsilon_2} E_{1n} = \frac{5\varepsilon_1}{\varepsilon_2}$$

$$\rightarrow E_2 = \frac{5\varepsilon_1}{\varepsilon_2} a_r - 3a_\varphi$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{5\varepsilon_1}{\varepsilon_2} \\ -3 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} A_x = \frac{5\varepsilon_1}{\varepsilon_2}\cos\varphi + 3\sin\varphi \xrightarrow{\varphi = 45°} A_x = \frac{5\sqrt{2}\,\varepsilon_1}{2\varepsilon_2} + \frac{3\sqrt{2}}{2} \\ A_y = \frac{5\varepsilon_1}{\varepsilon_2}\sin\varphi - 3\cos\varphi \xrightarrow{\varphi = 45°} A_y = \frac{5\sqrt{2}\,\varepsilon_1}{2\varepsilon_2} - \frac{3\sqrt{2}}{2} \\ A_z = 0 \end{cases}$$

$$E_{2t} = E_{3t} = \frac{5\sqrt{2}\,\varepsilon_1}{2\varepsilon_2} - \frac{3\sqrt{2}}{2}$$

$$E_{2t} = 0 \rightarrow \frac{5\sqrt{2}\,\varepsilon_1}{\cancel{2}\varepsilon_2} = \frac{3\sqrt{2}}{\cancel{2}} \rightarrow \frac{\varepsilon_1}{\varepsilon_2} = \frac{3}{5}$$

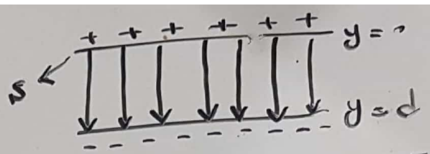
$$\downarrow$$

$$\frac{\varepsilon_0}{\varepsilon_0 \varepsilon_r} = \frac{3}{5} \rightarrow \boxed{\varepsilon_r = \frac{5}{3}}$$

$$\downarrow$$

ضریب دی‌الکتریک

3-30



گذردهی به صورت خطی از ε_1 در $y=0$ تا ε_2 در $y=d$

$$\varepsilon - \varepsilon_1 = \frac{\varepsilon_2-\varepsilon_1}{d-0}(y-0)$$

$$\boxed{\varepsilon = \frac{\varepsilon_2-\varepsilon_1}{d} y + \varepsilon_1}$$

1- در دستگاه دکارتی

2- در نظر گرفتن بار $+Q$ و $-Q$ بر روی صفحات

3- محاسبه ی E بین صفحات

4- محاسبه ی V از طریق E

5- $C = \frac{Q}{V}$

$$\int D \cdot ds = Q \qquad \begin{cases} D = D_0 a_y \\ ds = S a_y \end{cases}$$

$$\downarrow$$

$$\int D_0 S = Q$$

$$\downarrow$$

$$D_0 = \frac{Q}{S} = \rho_s \xrightarrow{D=\varepsilon E} E = \frac{\rho_s}{\varepsilon}$$

میدان ناشی از یک صفحه

$$V = -\int E \cdot dl = -\int \frac{\rho_s}{\varepsilon} dy = -\int \frac{\rho_s}{\left(\frac{\varepsilon_2-\varepsilon_1}{d}\right) y + \varepsilon_1} dy$$

$$\frac{-\rho_s d}{\varepsilon_2-\varepsilon_1} Ln\left(\frac{\varepsilon_2-\varepsilon_1}{d} y + \varepsilon_1\right)\Bigg|_0^d = \frac{-\rho_s}{\varepsilon_2-\varepsilon_1} Ln\left[\frac{\left(\frac{\varepsilon_2-\varepsilon_1}{d}\right)d + \varepsilon_1}{\varepsilon_1}\right]$$

$$V = -\frac{\rho_s d}{\varepsilon_2-\varepsilon_1} Ln \frac{\varepsilon_2}{\varepsilon_1} \Rightarrow C = \frac{\cancel{Q}}{\frac{\cancel{Q} d}{S(\varepsilon_2-\varepsilon_1)} Ln \frac{\varepsilon_1}{\varepsilon_2}} = \frac{S(\varepsilon_2-\varepsilon_1)}{d \, Ln \frac{\varepsilon_1}{\varepsilon_2}}$$

اگر نصف استوانه را در نظر بگیریم یعنی دو خازن داریم که به صورت سری به هم متصل شده‌اند
پس ظرفیت نیم استوانه بر ۲ تقسیم می‌شود.

$$C_1 = \frac{\pi \varepsilon_1 L}{\ln\left(\frac{r_o}{r_i}\right)}$$

$$C_2 = \frac{\pi \varepsilon_2 L}{\ln\left(\frac{r_o}{r_i}\right)}$$

حال ظرفیت کل از سری قرار دادن C_1 و C_2 به دست می‌آید:

$$C_T = C_1 + C_2 = \frac{\pi \varepsilon_0 \varepsilon_{r_1} L}{\ln\left(\frac{r_o}{r_i}\right)} + \frac{\pi \varepsilon_0 \varepsilon_{r_2} L}{\ln\left(\frac{r_o}{r_i}\right)}$$

$$\Rightarrow C_T = \frac{\pi \varepsilon_0 L (\varepsilon_{r_1} + \varepsilon_{r_2})}{\ln\left(\frac{r_o}{r_i}\right)}$$

<u>3-33</u>

۱- استوانه‌ها
2- در نظر گرفتن Q و $-Q$
3- محاسبه‌ی E
4- محاسبه‌ی V
5- $C = \frac{Q}{V}$

$$\boxed{D = D_0 a_r}$$

$$\int D \cdot ds = Q$$

$$\boxed{ds = r\, d\varphi\, dz\, a_r}$$

$$\int_0^L \int_0^{2\pi} D_0 r\, d\varphi\, dz = Q \longrightarrow D_0 = \frac{Q}{2\pi L r}$$

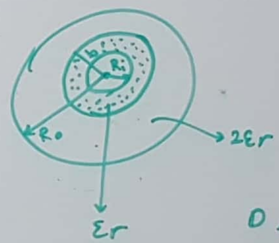
$$E_0 = \frac{Q}{\varepsilon 2\pi r L} \qquad \begin{cases} E = \frac{Q}{2\pi \varepsilon r L} a_r \\ dl = dr\, a_r \end{cases}$$

$$V = -\int E \cdot dl = -\int \frac{Q}{2\pi \varepsilon r L} dr$$

$$= \frac{-Q}{2\pi \varepsilon L} \ln r \Big]_{r_o}^{r_i} = \frac{Q}{2\pi \varepsilon L} \ln\left(\frac{r_o}{r_i}\right)$$

$$C = \frac{Q}{V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{2\pi \varepsilon L} \ln\left(\frac{r_o}{r_i}\right)} = \frac{2\pi \varepsilon L}{\ln\left(\frac{r_o}{r_i}\right)}$$

3-37



الف) گاس با E و D

در تمام نقاط فضا برحسب ولتاژ اعمال شده V ؟

$$\oint D\cdot ds = Q \qquad \begin{cases} D = D_0\, a_R \\ ds = R^2 \sin\theta\, d\theta\, d\varphi\, a_R \end{cases}$$

$\left.\begin{matrix} R < R_i \\ R > R_0 \end{matrix}\right\}$ ← برای این ۲ ناحیه Q برابر صفر است بنابراین E و D صفر هستند.

$$\int_0^{2\pi}\int_0^{\pi} D_0 R^2 \sin\theta\, d\theta\, d\varphi = Q$$

$$D_0 R^2 (-\cos\theta)\Big|_0^{\pi}\ \varphi\Big|_0^{2\pi} = Q$$

$$D_0 = \frac{Q}{R^2(2\pi)(2)} = \frac{\rho_s(4\pi R_i^2)}{4\pi R^2} = \frac{\rho_s R_i^2}{R^2}$$

$$\underline{D_{1n} = D_{2n} = \frac{\rho_s R_i^2}{R^2} a_R}$$ ← شرایط مرزی

$$V = -\int E\cdot dL = -\left[\int E_1\cdot dL + \int E_2\cdot dL\right]$$

$$= -\left[\int_{R_0}^{b} \frac{Q}{4\pi R^2 \varepsilon_0 (2\varepsilon_r)} dR + \int_b^{R_i} \frac{Q}{4\pi R^2 \varepsilon_0 (\varepsilon_r)} dR\right]$$

$$= -\frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left[\underbrace{-\frac{1}{2R}\Big|_{R_0}^{b} - \frac{1}{R}\Big|_b^{R_i}}\right]$$

$$-\left[\underbrace{\frac{1}{2b} - \frac{1}{2R_0} + \frac{1}{R_i} - \frac{1}{b}}\right]$$

$$-\frac{1}{2b} + \frac{1}{R_i} - \frac{1}{2R_0}$$

$$\Rightarrow V = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left[\frac{1}{R_i} - \frac{1}{2b} - \frac{1}{2R_0}\right]$$

$$Q = \frac{4\pi\varepsilon_0\varepsilon_r V}{\left[\frac{1}{R_i} - \frac{1}{2b} - \frac{1}{2R_0}\right]} \Rightarrow D_{1n} = D_{2n} = D_1 = D_2 = \frac{4\pi\varepsilon_0\varepsilon_r V a_R}{4\pi R^2\left[\frac{1}{R_i} - \frac{1}{2b} - \frac{1}{2R_0}\right]}$$

$R_i < R < b$ ←

$$E_1 = \frac{D_1}{\varepsilon_1} = \frac{D_1}{\varepsilon_0\varepsilon_r} = \frac{V a_R}{R^2\left[\frac{1}{R_i} - \frac{1}{2b} - \frac{1}{2R_0}\right]}$$

$\rightarrow b < R < R_0$

$$E_2 = \frac{D_2}{\varepsilon_2} = \frac{D_2}{\varepsilon_0(2\varepsilon_r)} = \frac{V a_R}{2R^2\left[\frac{1}{R_i} - \frac{1}{2b} - \frac{1}{2R_0}\right]}$$

ب - C ؟

$$C = \frac{Q}{V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\varepsilon_0\varepsilon_r}\left[\frac{1}{R_i} - \frac{1}{2b} - \frac{1}{2R_0}\right]} = \frac{4\pi\varepsilon_0\varepsilon_r}{\left[\frac{1}{R_i} - \frac{1}{2b} - \frac{1}{2R_0}\right]}$$