باللطیف

رضا آدینه پور

۹۸۱۴۳۰۳

تمرین سری ۲ مدار ۲

#1 a) $L\left\{ \sin t\, u(t-1) \right\}$

$$= L\left\{ \sin(t-1+1)\, u(t-1) \right\} = L\left\{ \sin(t-1)\cos(1)\cdot u(t-1) + \sin(1)\cdot\cos(t-1)\cdot u(t-1) \right\}$$

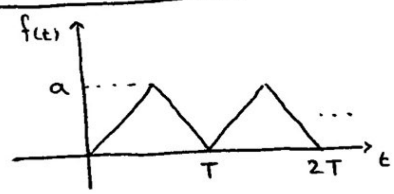
$$= \cos(1)\left(\frac{e^{-t}}{s^2+1} \right) + \sin(1)\left(e^{-t}\cdot \frac{s}{s^2+1} \right)$$

b) $L\left\{ \sin(t-1)\, u(t) \right\} = L\left\{ \sin t \cdot \cos(1)\, u(t) - \cos t\, \sin(1)\, u(t) \right\}$

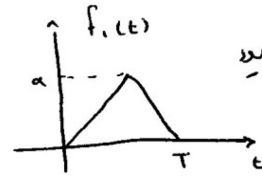
$$= \frac{\cos(1)}{s^2+1} - \sin(1) \times \frac{s}{s^2+1}$$

c) $L\left\{ \sin(t-1)\, u(t-1) \right\} = \dfrac{e^{-t}}{s^2+1}$

#2 a)



$\Rightarrow$



سیگنال در یک پریود

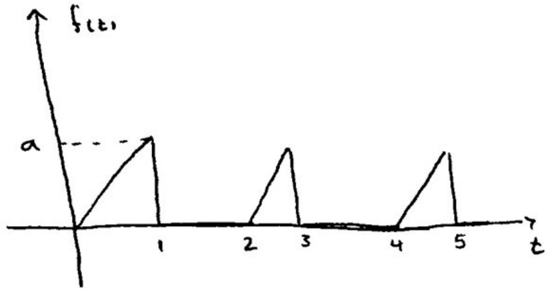
$\cancel{f_1(t) = a \;|\; t = \frac{T}{2}}$

$$f_1(t) = \frac{2a}{T} r(t) - \frac{4a}{T} r\left(t - \frac{T}{2}\right) + \frac{2a}{T} r(t-T)$$

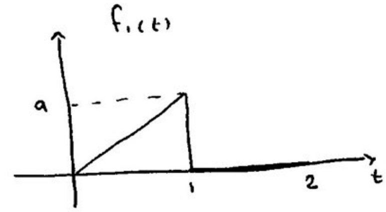
$$= \frac{2a}{T} t\,u(t) - \frac{4a}{T}\left(t - \frac{T}{2}\right) u\left(t - \frac{T}{2}\right) + \frac{2a}{T}(t-T)\,u(t-T)$$

$$F_1(s) = \left(\frac{2a}{T}\cdot\frac{1}{s^2}\right) - \left(\frac{4a}{T}\cdot\frac{e^{-\frac{T}{2}}}{s^2}\right) + \left(\frac{2a}{T}\cdot\frac{e^{-T}}{s^2}\right) = \frac{2a}{Ts^2}\left(1 - 2e^{-\frac{T}{2}} + e^{-T}\right)$$

$$\Rightarrow F(s) = \frac{\frac{2a}{T\cdot S}\left(1 - 2e^{-\frac{T}{2}} + e^{-T}\right)}{1 - e^{-ST}}$$



$\Longrightarrow$

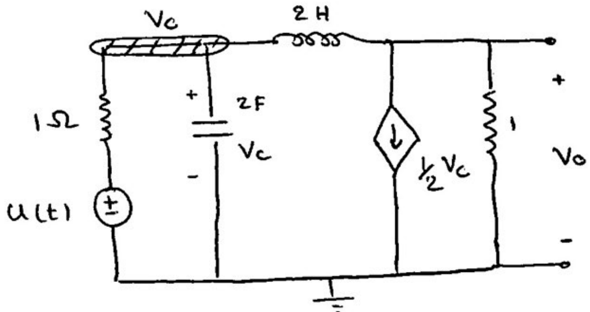


$f_1(t) = a\,r(t) - a\,u(t-1) - a\,r(t-1) = a\,t\,u(t) - a\,u(t-1) - a(t-1)u(t-1)$

$\Rightarrow F_1(t) = \frac{a}{s^2} - \frac{a}{s}e^{-1} - \frac{a}{s^2}e^{-1} = \frac{a}{s}\left(\frac{1}{s} - e^{-1} - \frac{e^{-1}}{s}\right)$

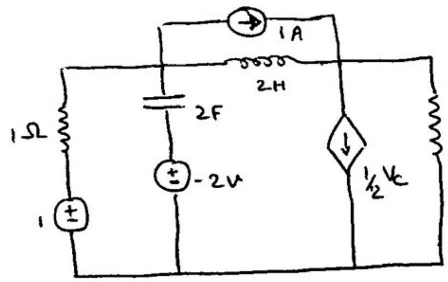
$\Rightarrow F(t) = \dfrac{\frac{a}{s}\left(\frac{1}{s} - e^{-1} - \frac{e^{-1}}{s}\right)}{1 - e^{-2s}}$

#3

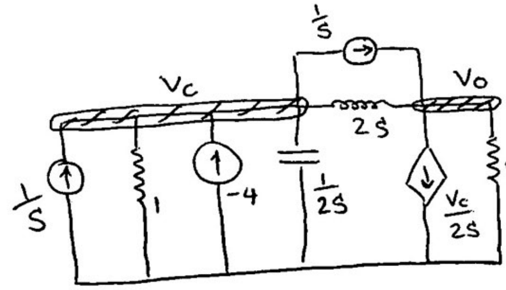


$$\begin{cases} V_c(0^-) = V_c(0^+) = -2\,V \\ I_L(0^-) = I_L(0^+) = 1A \end{cases}$$

$t > 0$:



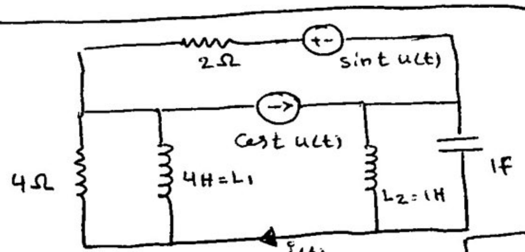
$\overset{L}{\equiv}$



$$KCL \;@\; V_o : \; -\frac{1}{S} + \cancel{\frac{V_c}{2S}} + V_o + \frac{V_o - \cancel{V_c}}{2S} = 0 \;\Rightarrow\; V_o\left(\frac{2S+1}{2S}\right) = \frac{1}{S}$$

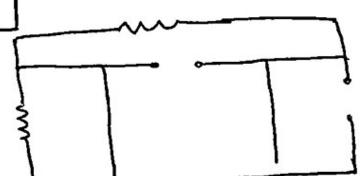
$$\Rightarrow V_o = \frac{2}{2S+1} = \frac{2 \times \frac{1}{2}}{\frac{1}{2}(S+2)} = 2 \times \frac{1}{S+2} \;\overset{L^{-1}}{=\!=}\; \boxed{2e^{-2t}\,u(t)}$$

#4



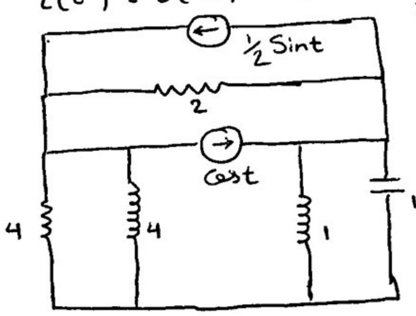
$i_{(t)} = ?$

$t < 0$: $\; t(0^-) = t(\infty) \Rightarrow$

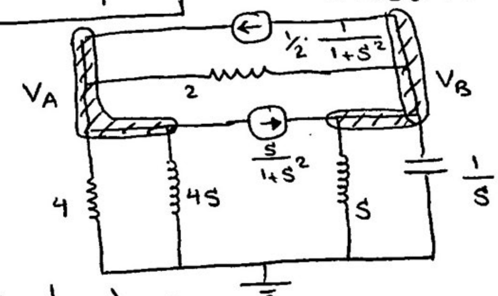


$$\Rightarrow \begin{cases} V_c(0^-) = V_c(0^+) = 0 \\ i_{L1}(0^-) = i_{L1}(0^+) = 0 \\ i_{L2}(0^-) = i_{L2}(0^+) = 0 \end{cases}$$

$t > 0$:



$\xrightarrow{L}$



$$KCL \;@\; V_A : \; \frac{V_A}{4} + \frac{V_A}{4S} + \frac{V_A - V_B}{2} + \frac{S}{1+S^2} - \frac{1}{2}\left(\frac{1}{1+S^2}\right) = 0 \qquad (I)$$

$$\Rightarrow V_A\left(\frac{3}{4} + \frac{1}{4S}\right) + \frac{S}{1+S^2} - \frac{1}{2(1+S^2)} = \frac{V_B}{2} \Rightarrow \boxed{V_B = V_A\left(\frac{3}{2} + \frac{1}{2S}\right) + \frac{2S}{1+S^2} - \frac{1}{1+S^2}}$$

$$KCL \;@\; V_B : \; SV_B + \frac{V_B}{S} - \frac{S}{1+S^2} + \frac{V_B - V_A}{2} + \frac{1}{2(1+S^2)} = 0 \qquad \frac{1}{S^2}$$

$$\Rightarrow \boxed{V_A = V_B\left(2S + \frac{2}{S} + 1\right) + \frac{1-2S}{1+S^2}} \quad (II)$$

$\xrightarrow{(II) \text{ in } (I)}$

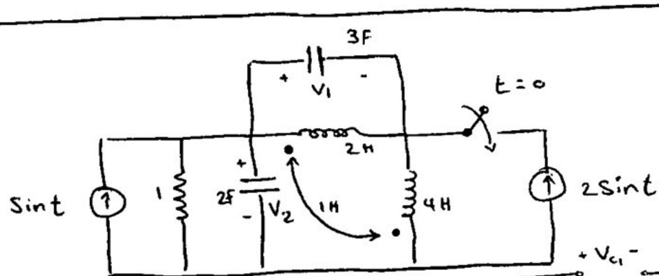
$$\Rightarrow V_B = \left(V_B\left(2S + \frac{2}{S} + 1\right) + \frac{1-2S}{1+S^2}\right)\cdot\left(\frac{3}{2} + \frac{1}{2S}\right) + \frac{2S-1}{1+S^2}$$

$$\Rightarrow V_B = -\frac{-2S^3 - S^2 + S}{12S^4 + 12S^3 + 17S^2 + 11S + 2} = \frac{2S^3 + S^2 + S}{(S+\frac{1}{2})(S+\frac{1}{3})} = \frac{A}{(S+\frac{1}{2})} + \frac{B}{(S+\frac{1}{3})}$$

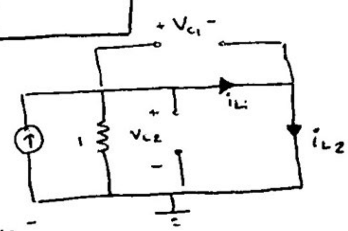
$$\Rightarrow A = (S+\tfrac{1}{2})\,V_B\Big|_{S=-\frac{1}{2}} = 3 \quad , \quad B = (S+\tfrac{1}{3})\,V_B\Big|_{S=\frac{-1}{3}} = -\frac{16}{9}$$

$$\Rightarrow V_B = 3\times\frac{1}{(S+\frac{1}{2})} - \frac{16}{9}\times\frac{1}{(S+\frac{1}{3})} = \left(3e^{-\frac{t}{2}} - \frac{16}{9}e^{-\frac{t}{3}}\right)u(t)$$

5

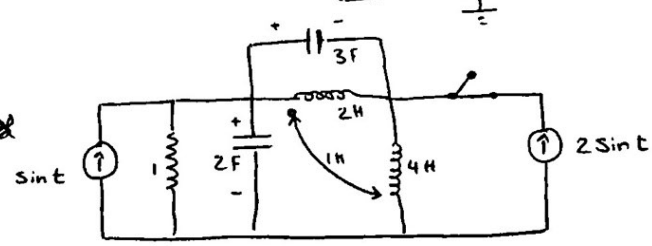


$t<0 \longrightarrow t(0^-) = t(\infty)$:

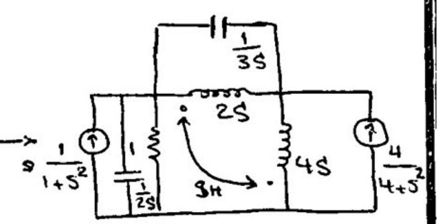


$$\Rightarrow \begin{cases} V_{C1}(0^-) = V_{C1}(0^+) = V_{C2}(0^-) = V_{C2}(0^+) = 0 \\ i_{L1}(0^-) = i_{L2}(0^+) = i_{L1}(0^+) = i_{L2}(0^-) = 0 \end{cases}$$

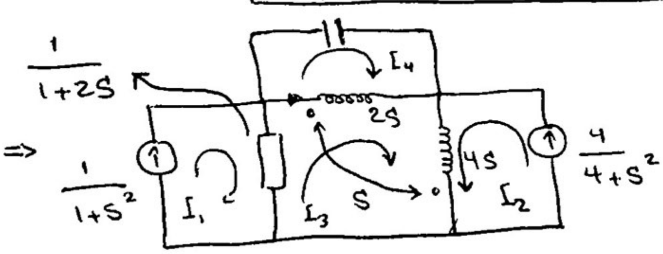
$t>0 \longrightarrow$



$\xrightarrow{L}$



$\Rightarrow$



$$I_1 = \frac{1}{1+S^2}$$

$$I_2 = \frac{4}{4+S^2}$$

$$\text{KVL @ } I_3: \frac{1}{1+2S}\left(I_3 - I_1\right) + 2S\left(I_3 - I_4\right) - S\left(I_2 + I_3\right) + 4S\left(I_2 + I_3\right) - S\left(I_3 - I_4\right) = 0$$

$$\Rightarrow \frac{1}{1+2S}\left(I_3 - \frac{1}{1+S^2}\right) + 2S\left(I_3 - I_4\right) - S\left(\frac{4}{4+S^2} + I_3\right) + 4S\left(\frac{4}{4+S^2} + I_3\right) - S\left(I_3 - I_4\right) = 0$$

$$= \frac{1}{1+2S}I_3 - \frac{1}{(1+2S)(1+S^2)} + \cancel{2S I_3} - \underline{\underline{2S I_4}} - \frac{4S}{4+S^2} - \cancel{S I_3} + \frac{16S}{4+S^2} + \cancel{4S I_3} - \cancel{S I_3} + \underline{\underline{S I_4}} = 0$$

$$\Rightarrow I_3\left(4S + \frac{1}{1+2S}\right) + \frac{16S}{4+S^2} - \frac{4S}{4+S^2} - \frac{1}{(1+2S)(1+S^2)} = S I_4$$

5 ادامه :

$$I_4 = I_3\left(S + \frac{1}{S(1+2S)}\right) + \frac{16}{4+S^2} - \frac{4}{4+S^2} - \frac{1}{S(1+2S)(1+S^2)} \quad \text{(I)}$$

$$\text{KVL @ } I_4 : \frac{1}{\cancel{3}S}(\cancel{I_4}) + 2S(\cancel{I_4} - I_3) - S\left(I_3 + \frac{4}{4+S^2}\right) = 0$$

$$I_4\left(\frac{1}{3S} + 2S\right) - \frac{4S}{4+S^2} = 3S\,I_3 \Rightarrow \boxed{I_3 = I_4\left(\frac{1}{9S^2} + \frac{2}{3}\right) - \frac{4}{3(4+S^2)}} \quad \text{(II)}$$

$$\xrightarrow{\text{(II) in (I)}} I_4 = \left(I_4\left(\frac{1}{9S^2} + \frac{2}{3}\right) - \frac{4}{3(4+S^2)}\right)\cdot\left(S + \frac{1}{S(1+2S)}\right) + \frac{16}{4+S^2} - \frac{4}{4+S^2} - \frac{1}{S(1+2S)(1+S^2)}$$

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