(روابط مثلثاتی)

$$\begin{cases}\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta\\ \sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\sin\beta\cos\alpha\end{cases}$$

تمرینات سری هشتم: تاریخ تحویل: ۱/۱۰/۹۸

فصل هفتم کتاب جیبدار: ۱ـ ۲ـ ۴ـ ۷ـ ۹ـ ۱۰ـ ۱۳ـ ۱۹ـ ۳۵ـ ۴۸ـ ۶۳ـ ۷۰ـ

کتاب جیبدار:

سوال ۱:

ص ۴۱۴ کتاب ✓

الف) $2\sin(2t+18^\circ)-3\cos(2t+35^\circ)+2\frac{d^2}{dt^2}\sin(2t-25^\circ)$

$$2\frac{d^2}{dt^2}\sin(2t-25^\circ)=2\frac{d}{dt}2\cos(2t-25^\circ)=-8\sin(2t-25^\circ)$$

$$=-8\cos(2t-25^\circ-90^\circ)=-8\cos(2t-115^\circ)$$

$$\rightarrow 2\sin(2t+18^\circ)-3\cos(2t+35^\circ)-8\cos(2t-115^\circ)=2\sin2t\overset{0.95}{\cos18}$$

$$+2\sin18\cos2t-3\cos2t\cos35^\circ+3\sin2t\sin35^\circ-8\cos2t\cos115$$

$$-8\sin2t\sin115=\underline{1.9\sin2t}+\underline{0.61\cos2t}-\underline{2.45\cos2t}+\underline{1.72\sin2t}$$

$$+\underline{3.38\cos2t}-\underline{7.25\sin2t}=1.54\cos2t-3.63\sin2t=$$

$$\sqrt{(1.54)^2+(3.63)^2}\cos(2t+\tan^{-1}\underbrace{\frac{3.63}{1.54}}_{2.35})=\boxed{3.94\cos(2t+66.9)}$$

ب) $\cos2t+\cos(2t+120^\circ)+\cos(2t+240^\circ)=\cos2t+\cos2t\cos120$

$$-\sin2t\sin120+\cos2t\cos240-\sin2t\sin240=\cos2t-0.5\cos2t$$

$$\cancel{-0.86\sin2t}-0.5\cos2t+\cancel{0.86\sin2t}=\cos2t-\cos2t=0$$

س ۴:

الف) $\underset{-\omega^2}{\dfrac{d^2i}{dt^2}} + 2\dfrac{di}{dt} + i = 5\sin(2t+30^\circ) \rightarrow \omega = 2$

$(j\omega)^2 I + 2(j\omega) I + I = 5\cos(2t + \overset{-60}{\cancel{30-90}}) = 5\angle -60$

$-4I + 4jI + I = 2.5 - 4.33j \rightarrow (-3+4j)I = 2.5 - 4.33j$

$\rightarrow I = \dfrac{2.5-4.33j}{-3+4j} = \dfrac{5\angle -60}{5\angle 126.86} \rightarrow |I| = 1 \; , \angle I = -186.86$

$\rightarrow \boxed{i = \cos(2t - 186.86)}$

ب) $\dfrac{d^2i}{dt^2} + 3\dfrac{di}{dt} + 2i = 2\cos(2t-30^\circ) + 3\sin(2t+45^\circ) \rightarrow \omega = 2$

$2\cos(2t-30) + 3\sin(2t+45) = 2\cos(2t-30) + 3\cos(2t-45)$

$= (2\angle -30) + (3\angle -45) = (1.73 - j) + (2.12 - 2.12j) = 3.85 - 3.12j$

$-\omega^2 I + 3j\omega I + 2I = -4I^2 + 6jI + 2I = 3.85 - 3.12j$

$\rightarrow I = \dfrac{3.85 - 3.12j}{-2+6j} = \dfrac{4.95\angle -39}{6.32\angle 108.93} \rightarrow$

$|I| = \dfrac{4.95}{6.32} = 0.78 \; , \angle I = -147.43 \rightarrow \boxed{i = 0.78\cos(2t - 147.43)}$

ب) $\dfrac{d^2i}{dt^2} + 2\dfrac{di}{dt} + 2i = 2\cos t + 3\sin 2t$ ← جمع آثار فرکانس های متفاوت

$\dfrac{d^2i_1}{dt^2} + 2\dfrac{di_1}{dt} + 2i_1 = 2\cos t \rightarrow \omega = 1$

$-I_1 + 2jI_1 + 2I_1 = 2 \rightarrow I_1 = \dfrac{2}{1-2j} = \dfrac{2\angle 0}{2.23\angle -63.43} \rightarrow$

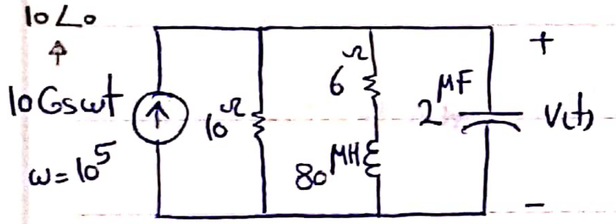
$|I_1| = 0.89 \; , \angle I_1 = 63.43 \rightarrow i_1 = 0.89\cos(t + 63.43)$

$$\frac{d^2 i_2}{dt^2} + 2\frac{di_2}{dt} + 2i_2 = 3\sin 2t = 3\cos\left(2t - \frac{\pi}{2}\right) \longrightarrow \omega = 2$$

$$-4I_2 + 4jI_2 + 2I_2 = -3j \longrightarrow I_2 = \frac{-3j}{-2+4j} = \frac{3\angle -90}{4.47\angle 116.5}$$

$$\longrightarrow |I_2| = 0.67 \ , \ \angle I_2 = 206.5 \longrightarrow i_2 = 0.67\cos(2t + 206.5)$$

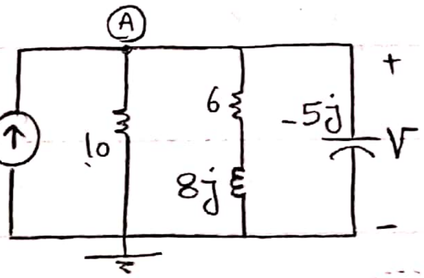
$$\Longrightarrow \boxed{i = i_1 + i_2 = 0.89\cos(t + 63.43) + 0.67\cos(2t + 206.5)}$$



$$Z_C = \frac{1}{Cj\omega} = \frac{1}{2\times10^{-6}\times10^{5}j} =$$ [illegible] $\downarrow$ $\boxed{-5j}$

$$Z_L = Lj\omega = 80\times10^{-6}\times10^{5}j = \boxed{8j}$$

مدار معادل در حوزه فازور



$$\text{kcl}Ⓐ: -10 + \frac{V}{10} + \frac{V}{6+8j} + \frac{V}{-5j} = 0 \xrightarrow{\times 10(6+8j)}$$

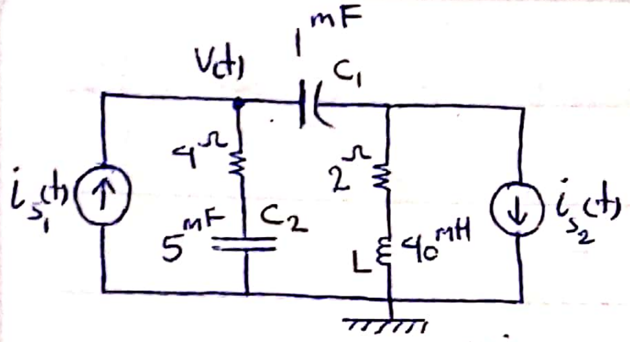
$\frac{V}{5}j$

$$(6+8j)V + 10V + 2j(6+8j)V = 100(6+8j) \longrightarrow$$

$$(\cancel{6} + 8j + \cancel{10} + 12j - \cancel{16})V = 600 + 800j \longrightarrow V = \frac{600+800j}{20j}$$

$$\longrightarrow V = \frac{1000\angle 53.13}{20\angle 90} = 50\angle -36.87 \downarrow$$

$$\boxed{V(t) = 50\cos(10^5 t - 36.87)}$$



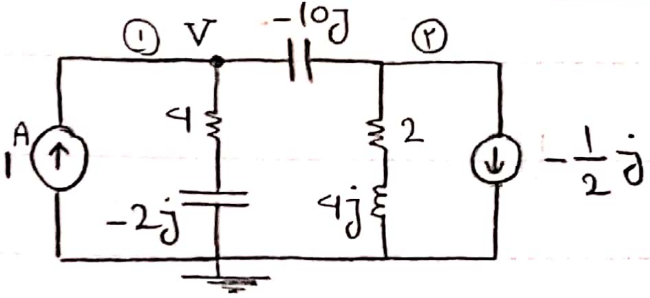
س۹:

$i_{s_1}(t) = \cos 100t$

$i_{s_2}(t) = \frac{1}{2}\sin 100t = \frac{1}{2}\cos(100t - \pi/2)$

$\boxed{\omega = 100}$ $= \frac{1}{2}\angle -\pi/2 = -\frac{1}{2}j$

مدار معادل در حوزه فازور:



$Z_L = Lj\omega = 40\times10^{-3}\times100j = \boxed{4j}$

$Z_{C_1} = \frac{1}{C_1 j\omega} = \frac{1}{1\times10^{-3}\times100j} = \boxed{-10j}$, $Z_{C_2} = \frac{1}{C_2 j\omega} = \frac{1}{5\times10^{-3}\times100j} = \boxed{-2j}$

kcl①: $-1 + \frac{V}{4-2j} + \frac{V-V_2}{-10j} = 0 \rightarrow -10(4-2j) + 10V + j(4-2j)(V-V_2) = 0$

$\rightarrow (20j-40) + (10+4j+2)V = (4j+2)V_2 \rightarrow$

$V_2 = \frac{(-40+20j)+(12+4j)V}{4j+2} = \boxed{10j + (2-2j)V}$ ⊛

kcl②: $-\frac{1}{2}j + \frac{V_2}{2+4j} + \frac{V_2 - V}{-10j} = 0 \rightarrow$

$-5(2+4j)j + 10V_2 + j(4+2j)(V_2 - V) = 0 \rightarrow$

$(-10j+20) + (10+4j-2)V_2 + (-4j+2)V = 0 \xrightarrow{⊛}$

$(20-10j) + (8+4j)(10j + (2-2j)V) + (2-4j)V = 0 \rightarrow$

$(20-10j+80j-40) + (16-16j+8j+8+2-4j)V = 0$

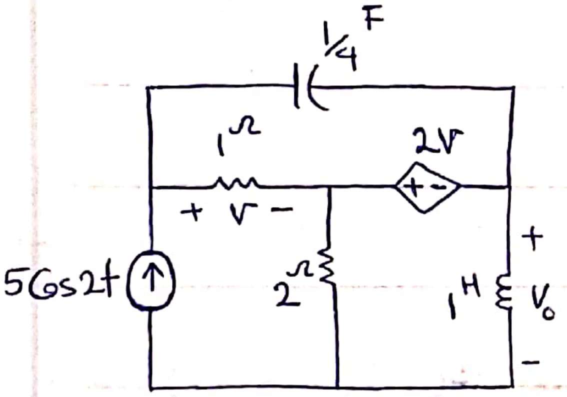
$\rightarrow (-20+70j) + (26-12j)V = 0 \rightarrow$

$$V=\frac{20-70j}{26-12j}\times\frac{26+12j}{26+12j}=\frac{520+240j-1820j+840}{676+144}$$

$$=\frac{1360-1580j}{820}=1.65-1.92j=2.53\angle-49.32$$

$$\rightarrow \boxed{v(t)=2.53\,\text{Cos}(100t-49.32)}$$

س ۱۵:

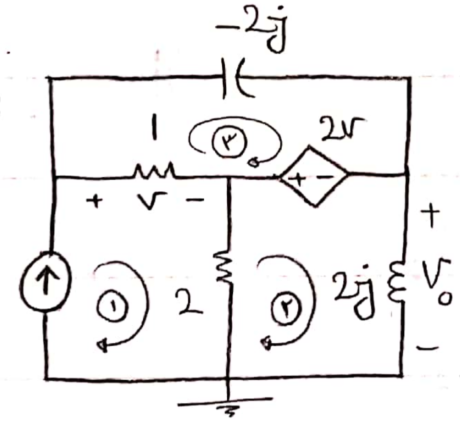


$\omega=2$

$Z_L=Lj\omega=1\times2j=2j$

$$Z_C=\frac{1}{Cj\omega}=\frac{1}{\frac{1}{4}\times2j}=-2j$$

مدار معادل در حوزه فازور:



$\boxed{I_1=5\,A}$

$V=I_1-I_3=5-I_3$

$\text{kvl}②: 2(I_2-5)+2(5-I_3)+(2j)I_2=0\rightarrow$

$2I_2-\cancel{10}+\cancel{10}-2I_3+2jI_2=0\rightarrow\boxed{(2+2j)I_2-2I_3=0}$ ⊛

$\text{kvl}③: (-2j)I_3-2(5-I_3)+(I_3-5)=0\rightarrow$

$-2jI_3-10+2I_3+I_3-5=0\rightarrow(3-2j)I_3=15$

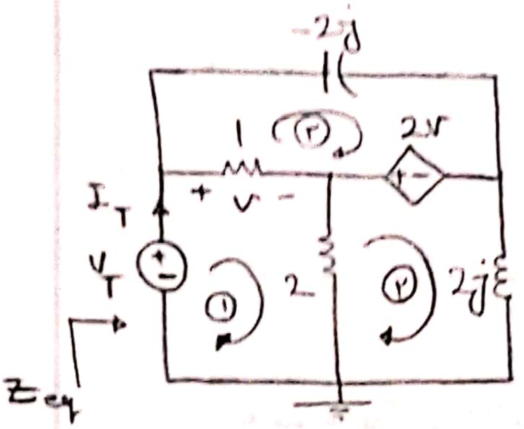
$\rightarrow\boxed{I_3=\frac{15}{3-2j}}$ ⊛⊛

⊛⊛ in ⊛ $\rightarrow (2+2j)I_2=2\left(\frac{15}{3-2j}\right)$

$$\rightarrow I_2 = \frac{30}{3-2j} \times \frac{1}{2+2j} = \frac{30}{6+6j-4j+4} = \frac{30}{10+2j}$$

$$V_o = 2j\, I_2 = 2j \times \frac{30}{10+2j} = \frac{60j}{10+2j} = \frac{60\angle 90^\circ}{10.19\angle 11.30^\circ} = 5.88\angle 78.7^\circ$$

$$\boxed{V_o(t) = 5.88 \cos(2t + 78.7^\circ)}$$



$$\text{kvL ①}: -V_T + I_1 - I_3 + 2I_1 - 2I_2 = 0 \quad \text{(I)}$$

$$\text{kvL ②}: 2I_2 - 2I_1 + 2V + 2j I_2 = 0$$

$$\rightarrow (2+2j)I_2 - 2I_1 + 2(I_1 - I_3) = 0$$

$$\rightarrow (2+2j)I_2 - \cancel{2I_1} + \cancel{2I_1} - 2I_3 = 0 \quad \text{(II)}$$

$$\text{kvL ③}: -2jI_3 - 2V + I_3 - I_1 = 0 \rightarrow -2jI_3 - 2(I_1 - I_3) + I_3 - I_1 = 0$$

$$\rightarrow (3-2j)I_3 - 3I_1 = 0 \rightarrow I_3 = \frac{3I_1}{3-2j} \quad ⊛$$

$$⊛ \text{ in (II)} \rightarrow (2+2j)I_2 - 2\left(\frac{3I_1}{3-2j}\right) = 0 \rightarrow I_2 = \frac{6I_1}{(3-2j)(2+2j)}$$

$$\rightarrow I_2 = \frac{6I_1}{10+2j} \quad ⊛⊛$$

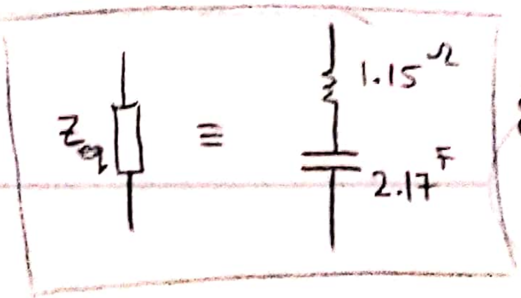
$$⊛, ⊛⊛ \text{ in ①} \rightarrow -V_T + 3I_1 - \frac{3I_1}{3-2j} - 2\left(\frac{6I_1}{10+2j}\right) = 0 \rightarrow$$

$$\left(\frac{\overset{12}{\cancel{3(10+2j) - 3(2+2j)}} - 12}{10+2j}\right) I_1 = V_T \xrightarrow{I_T = I_1}$$

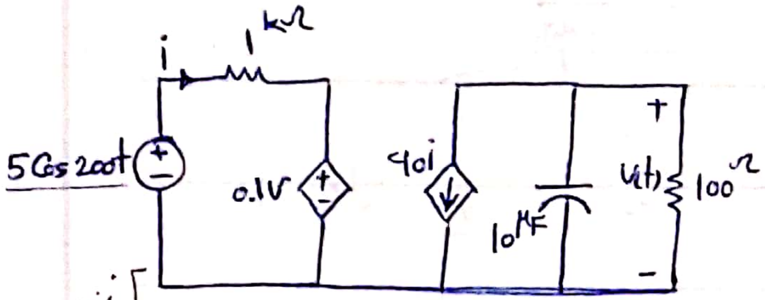
$$Z_{eq} = \frac{V_T}{I_T} = \frac{12}{10+2j} \times \frac{10-2j}{10-2j} = \frac{120-24j}{100+4} = \overbrace{1.15}^{\text{مقاومت}} \overbrace{-0.23j}^{\text{خازن}}$$

$$\frac{1}{Cj\omega} = \frac{-j}{2C} = -0.23j$$

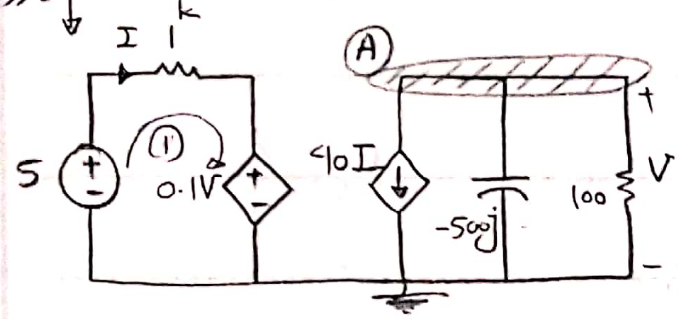
$$\rightarrow \frac{1}{2C} = \frac{23}{100} \rightarrow \boxed{C = 2.17}$$



س ۱۴ :



فازور



$\omega = 200$

$$Z_c = \frac{1}{cj\omega} = \frac{1}{10\times10^{-6}\times200j} = -500j$$

$$\text{KVL①}: -5 + 1000I + 0.1V = 0$$

$$\rightarrow I = \frac{5-0.1V}{1000} \quad ⊛$$

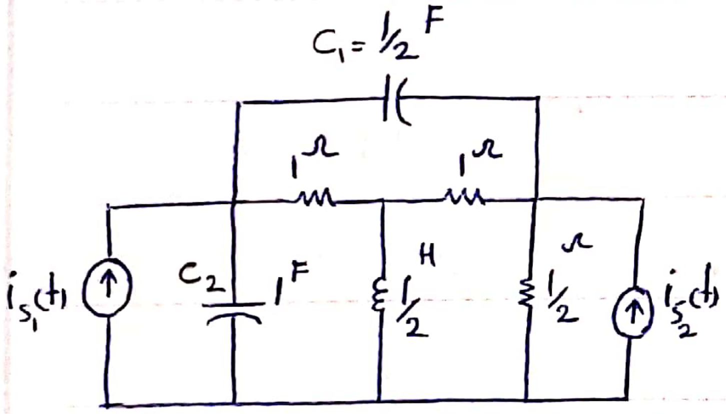
$$\text{KCl Ⓐ}: 40I + \frac{V}{-500j} + \frac{V}{100} = 0 \xrightarrow{⊛} \frac{200-4V}{1000} + \frac{V}{500}j + \frac{V}{100} = 0$$

$$\rightarrow 200 - 4V + 2Vj + 10V = 0 \rightarrow (6+2j)V = -200 \rightarrow$$

$$V = \frac{-200}{6+2j} = \frac{200\angle180}{6.32\angle18.43} = 31.64\angle161.57$$

$$\boxed{v(t) = 31.64\,\text{Cos}(200t + 161.57)}$$

س ۱۹ :



الف) $i_{s_1}(t) = 5\,\text{Cos}\,2t$

$i_{s_2}(t) = 2\,\text{Sin}\,2t = 2\,\text{Cos}(2t - 90)$

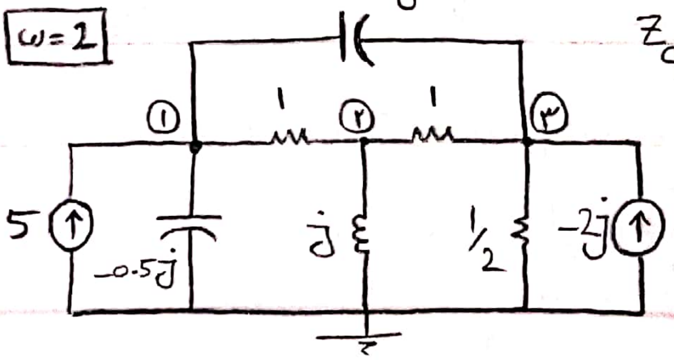
ب) $i_{s_1}(t) = 5\,\text{Cos}\,t$

$i_{s_2}(t) = 2\,\text{Sin}\,2t = 2\,\text{Cos}(2t - 90)$

مدار (الف : $\boxed{\omega = 2}$

معادل درحوزه

فازور



$$Z_{C_1} = \frac{1}{C_1 j\omega} = \frac{1}{\frac{1}{2}\times2j} = -j$$

$$Z_{C_2} = \frac{1}{1\times2j} = -0.5j$$

$$Z_L = \frac{1}{2}\times2j = j$$

kcl ①: $-5+\frac{V_1}{-0.5j}+\frac{V_1-V_2}{1}+\frac{V_1-V_3}{-j}=0 \longrightarrow$

$-5+2jV_1+V_1-V_2+jV_1-jV_3=0 \longrightarrow (1+3j)V_1-V_2-jV_3=5$ Ⅰ

kcl ②: $\frac{V_2-V_1}{1}+\frac{V_2-V_3}{1}+\frac{V_2}{j}=0 \longrightarrow (2-j)V_2-V_1-V_3=0$ Ⅱ

kcl ③: $+2j+\frac{V_3}{1/2}+\frac{V_3-V_1}{-j}=0 \longrightarrow 2j+2V_3+jV_3-jV_1=0 \longrightarrow$

$(2+j)V_3-jV_1=-2j \longrightarrow V_1=\frac{-2j-(2+j)V_3}{-j}=-2j-(2+j)V_3$ ⊛

⊛ in Ⅰ $\longrightarrow (1+3j)(-2j-(2+j)V_3)-V_2-jV_3=5 \longrightarrow$

$-2j\underbrace{-2V_3}\underline{-jV_3}+6-6\underline{jV_3}+\underbrace{3V_3}-V_2\underline{-jV_3}=5 \longrightarrow$

$V_2=(1-2j)+(1-8j)V_3$ ⊛⊛

⊛⊛, ⊛ in Ⅱ $\longrightarrow (2-j)\big((1-2j)+(1-8j)V_3\big)-(-2j-2V_3-jV_3)-V_3=0 \longrightarrow$

$\cancel{2}-4j+\underbrace{2V_3}-16jV_3-j\cancel{-2}\cancel{-jV_3}\underbrace{-8V_3}+2j+\underbrace{2V_3}+\cancel{jV_3}\underbrace{-V_3}=0 \longrightarrow$

$-3j+(-5-16j)V_3=0 \longrightarrow \boxed{V_3=\frac{3j}{-5-16j}}$

$\xrightarrow{⊛} V_1=-2j-(2+j)\frac{3j}{-5-16j}=\frac{-32+10j-6j-3}{-5-16j}=\boxed{\frac{-35+4j}{-5-16j}}$

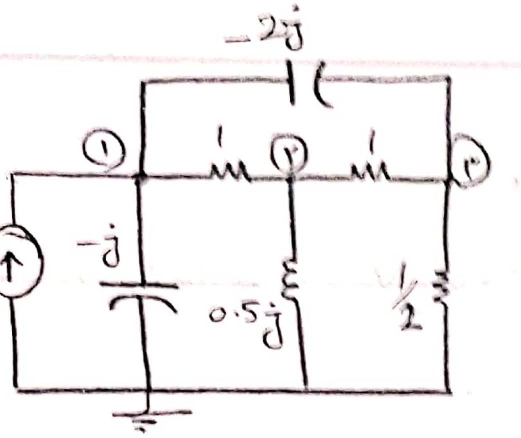
$V_{C_1}=V_1-V_3=\frac{-35+4j-3j}{-5-16j}=\boxed{\frac{-35+j}{-5-16j}}$

ب) با توجه به اینکه فرکانس منابع موجود در مدار با هم متفاوت است از جمع آثار استفاده می‌کنیم.

حالت اول : $i_{s_1}=5\cos t$
$i_{s_2}=0$

حالت دوم : $i_{s_1}=0$
$i_{s_2}=2\sin 2t$

حالت اول
$\omega = 1$



$$Z_{C_1} = \frac{1}{\frac{1}{2} \times j \times 1} = -2j$$

$$Z_{C_2} = \frac{1}{1 \times j \times 1} = -j$$

$$Z_L = \frac{1}{2} \times j \times 1 = 0.5j$$

kcl①:

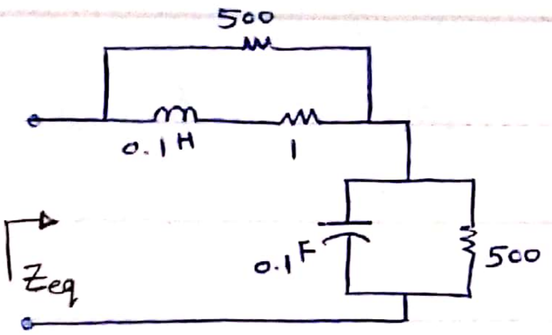
$$\frac{V_1}{-j} + \frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{-2j} - 5 = 0 \longrightarrow 2jV_1 + 2V_1 - 2V_2 + jV_1 - jV_3 - 10 = 0$$

kcl②:

$$\frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{1} + \frac{V_2}{0.5j} = 0 \longrightarrow 2V_2 - V_1 - V_3 - 2jV_2 = 0$$

kcl③: $$\frac{V_3}{1/2} + \frac{V_3 - V_1}{-2j} + \frac{V_3 - V_2}{1} = 0 \longrightarrow 4V_3 + jV_3 - jV_1 + 2V_3 - 2V_2 = 0$$

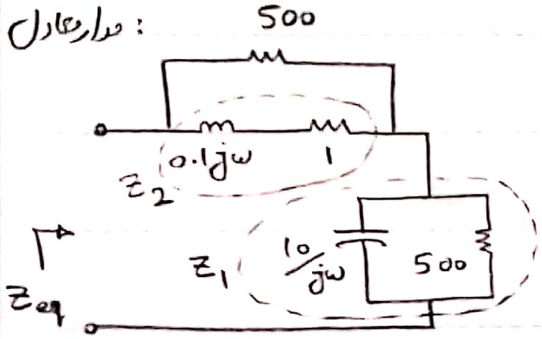
س ۳۵:



$Z_L = 0.1j\omega$

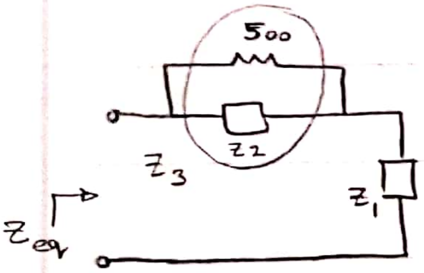
$Z_C = \frac{1}{0.1j\omega} = \frac{10}{j\omega}$

مدار معادل:



$Z_1 = \frac{10}{j\omega} \parallel 500 = \frac{\frac{10}{j\omega} \times 500}{\frac{10}{j\omega} + 500} = \frac{5\times10^3/j\omega}{\frac{10+500j\omega}{j\omega}} = \frac{5\times10^3}{10+500j\omega} = \frac{500}{1+50j\omega}$

$Z_2 = 0.1j\omega + 1$



$Z_3 = Z_2 \parallel 500 = \frac{(0.1j\omega+1)500}{0.1j\omega+1+500} = \frac{50j\omega+500}{0.1j\omega+501}$

$$Z_{eq} = Z_3 + Z_1 = \frac{50j\omega+500}{0.1j\omega+501} + \frac{500}{1+50j\omega} = \left(\frac{50j\omega+500}{0.1j\omega+501} \times \frac{501-0.1j\omega}{501-0.1j\omega}\right)$$

$$+\left(\frac{500}{1+50j\omega} \times \frac{1-50j\omega}{1-50j\omega}\right) = \frac{501(50j\omega)+5\omega^2+500(501)-50j\omega}{(501)^2+(0.1\omega)^2} +$$

$$\frac{500-500(50j\omega)}{1+(50\omega)^2} = \overbrace{\left[\frac{5\omega^2+500(501)}{(501)^2+(0.1\omega)^2} + \frac{500}{1+(50\omega)^2}\right]}^{Re} + \overbrace{\left[(50\omega j) \times \left(\frac{500}{(501)^2+(0.1\omega)^2} - \frac{500}{1+(50\omega)^2}\right)\right]}^{Im}$$

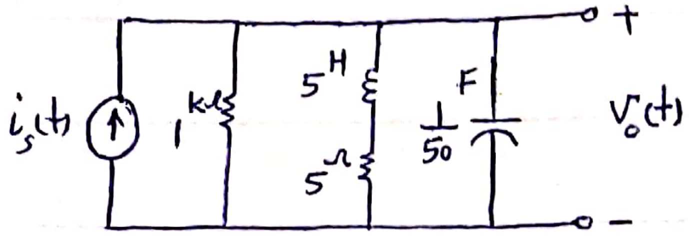
; $Im\{Z_{eq}\} = 0 \longrightarrow \omega =$ فرکانس تشدید

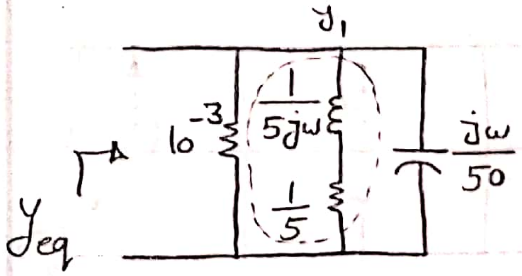
$$\frac{500}{(501)^2+(0.1\omega)^2} - \frac{500}{1+(50\omega)^2} = 0 \longrightarrow 500\left[(1+(50\omega)^2) - (501^2 + \underset{0}{\cancel{(0.1\omega)^2}})\right] = 0$$

$$\longrightarrow 2500\omega^2 - 251000 = 0 \longrightarrow \omega^2 = 100.4 \longrightarrow \boxed{\omega = 10.02}$$

فرکانس تشدید

س ۴۸ : $i_s(t) = 0.42 \cos \omega t$





$$y_1 = \frac{1}{5j\omega} \parallel \frac{1}{5} = \frac{\frac{1}{5j\omega} \times \frac{1}{5}}{\frac{1}{5j\omega} + \frac{1}{5}} = \frac{\frac{1}{25j\omega}}{\frac{1+j\omega}{5j\omega}}$$

$$= \frac{\cancel{5j\omega}}{\cancel{25j\omega}(1+j\omega)} = \frac{1}{5(1+j\omega)}$$

$$y_{eq} = \frac{1}{1000} + \frac{1}{5(1+j\omega)} + \frac{j\omega}{50} = \frac{1+20j\omega}{1000} + \frac{1}{5+5j\omega} \times \frac{5-5j\omega}{5-5j\omega}$$

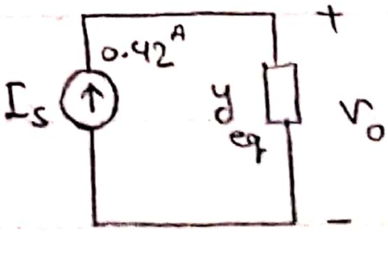
$$= \frac{1+20j\omega}{1000} + \frac{5-5j\omega}{25+25\omega^2} = \frac{(1+20j\omega)(25+25\omega^2)+1000(5-5j\omega)}{1000(25+25\omega^2)}$$

$$= \frac{25+25\omega^2+500j\omega+500j\omega^3+5000-5000j\omega}{1000(25+25\omega^2)}$$

فرکانس تشدید

$$\text{Im}\{y_{eq}\} = \frac{500j\omega(1+\omega^2-10)}{1000(25+25\omega^2)} = 0 \to \omega^2-9=0 \to \omega^2=9 \to \boxed{\omega=3}$$

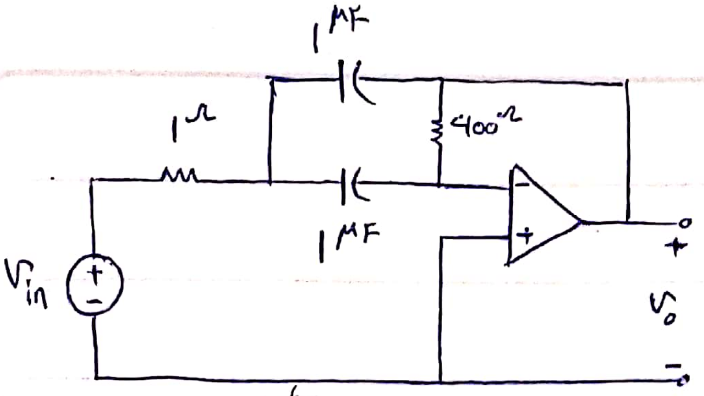
$$y_{eq}\Big|_{\omega=3} = \frac{5250}{25\times10^4} = \boxed{0.021}$$



$$V_o = \frac{1}{y_{eq}} \times I_s = \frac{0.42}{0.021} = \boxed{20}$$

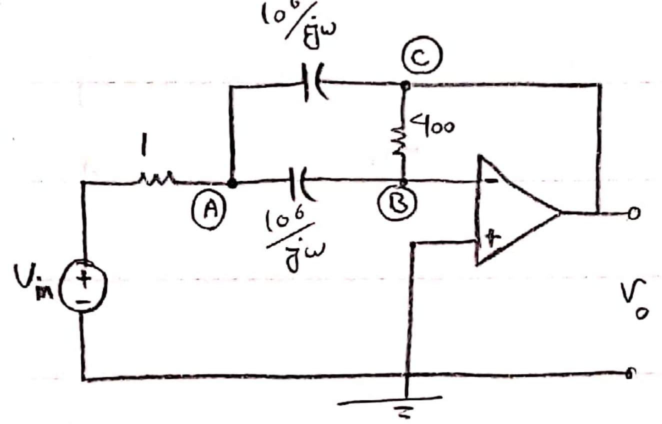
دامنه ولتاژ خروجی

- برای محاسبه Q مدار با در حوزه زمان تحلیل می‌کنیم (مشابه فصل ۵، معادلات ورشه ۲) و با توجه به مقادیر بدست آمده برای α و ω_0 جواب را بدست می‌آوریم.



$H(j\omega) = \frac{V_o}{V_{in}} = ?$: ۹۴ س

$Z_c = \frac{1}{10^{-6} j\omega} = \frac{10^6}{j\omega}$



$$\begin{cases} V_+ = V_- \\ I_- = I_+ = 0 \end{cases}$$

$\downarrow$

$V_+ = 0 \;,\; V_- = V_B$

$\hookrightarrow \boxed{V_B = 0}$

$\boxed{V_C = V_o}$

kcl Ⓐ:

$$\frac{V_A - V_{in}}{1} + \frac{V_A - V_C}{10^6/j\omega} + \frac{V_A}{10^6/j\omega} = 0 \longrightarrow$$

Ⓘ

$$\frac{10^6}{j\omega}(V_A - V_{in}) + V_A - V_o + V_A = 0 \longrightarrow \left(2 - \frac{10^6}{\omega} j\right) V_A - V_o + \frac{10^6}{\omega} j V_{in} = 0$$

kcl Ⓑ: $\frac{-V_o}{400} + \frac{-V_A}{10^6/j\omega} = 0 \longrightarrow \left(\frac{10^6}{\omega} j\right) V_o - 400 V_A = 0 \longrightarrow$

$$V_A = \left(\frac{10^6}{400\omega} j\right) V_o = \left(\frac{10^4}{4\omega} j\right) V_o \quad ⊛$$

⊛ in Ⓘ $\longrightarrow \left(2 - \frac{10^6}{\omega} j\right)\left(\frac{10^4}{4\omega} j\right) V_o - V_o + \left(\frac{10^6}{\omega} j\right) V_{in} = 0 \longrightarrow$

$$\left(\frac{10^6}{\omega} j\right) V_{in} = \left(1 - \frac{10^{10}}{4\omega^2} - \frac{10^4}{2\omega} j\right) V_o$$

$$H(j\omega) = \frac{V_o}{V_{in}} = \frac{(10^6/\omega) j}{\left(\frac{4\omega^2 - 10^{10}}{4\omega^2}\right) - \left(\frac{10^4}{2\omega}\right) j} = \frac{(4 \times 10^6 \omega) j}{(4\omega^2 - 10^{10}) - (2 \times 10^{10} \omega) j}$$

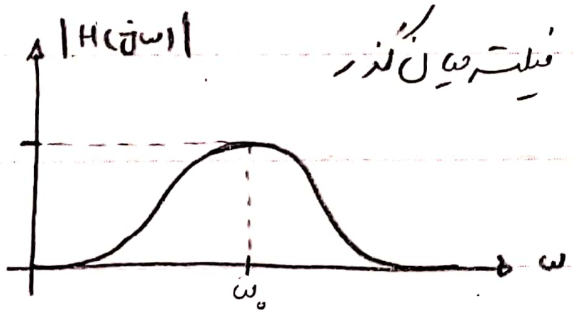
$$|H(j\omega)| = \frac{4\times10^{6}\,\omega}{\sqrt{(4\omega^{2}-10^{10})^{2}+(2\times10^{10}\,\omega)^{2}}}$$

$$\measuredangle H(j\omega) = \frac{\pi}{2} - \tan^{-1}\left(\frac{2\times10^{10}\,\omega}{4\omega^{2}-10^{10}}\right)$$

$$\begin{cases} \text{if } \omega=0 \longrightarrow |H(j\omega)|=0 \\ \text{if } \omega=\infty \longrightarrow |H(j\omega)|=0 \end{cases}$$

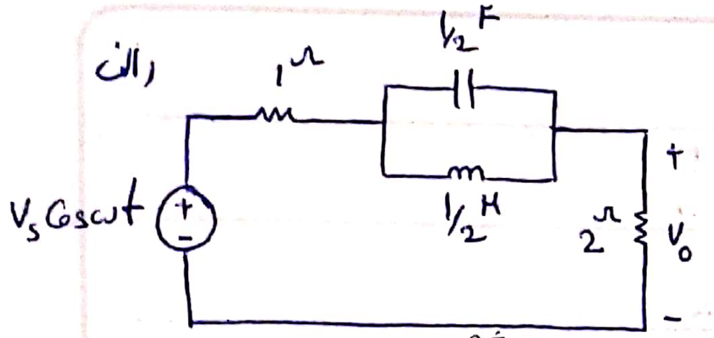
; if $\frac{d|H(j\omega)|}{d\omega}=0 \longrightarrow |H(j\omega)| = \text{Max}$

& $\omega_0 =$ فرکانس تشدید



فیلتر میان گذر

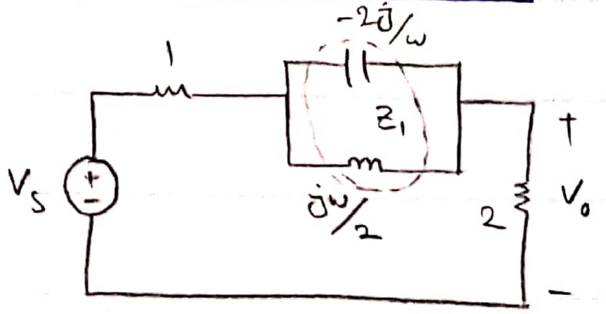
الف)



س : $\underline{V_o}$

$$Z_L = \frac{1}{2} j\omega = \frac{j\omega}{2}$$

$$Z_C = \frac{1}{\frac{1}{2} j\omega} = \frac{-2j}{\omega}$$



$$Z_1 = \frac{-2j}{\omega} \| \frac{j\omega}{2} = \frac{\frac{-2j}{\omega} \times \frac{j\omega}{2}}{\frac{-2j}{\omega} + \frac{j\omega}{2}} = \frac{2j\omega}{4-\omega^2}$$

تقسیم امپدانس‌ها ← مقسم ولتاژ :

$$V_o = \frac{2}{1+2+\frac{2j\omega}{4-\omega^2}} \times V_s \longrightarrow \frac{V_o}{V_s} = \frac{2}{3+\frac{2j\omega}{4-\omega^2}} = \frac{2(4-\omega^2)}{12-3\omega^2+2j\omega}$$

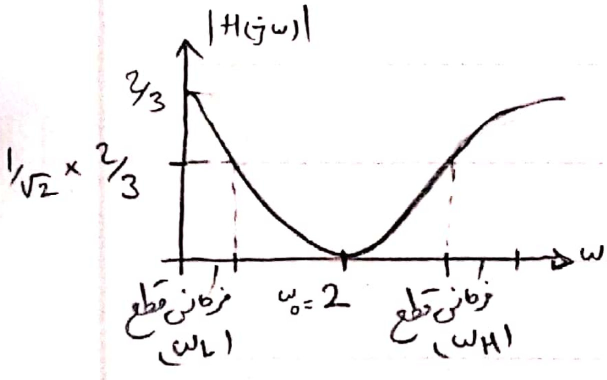
$$\longrightarrow H(j\omega) = \frac{8-2\omega^2}{12-3\omega^2+2j\omega} \longrightarrow |H(j\omega)| = \frac{8-2\omega^2}{\sqrt{(12-3\omega^2)^2 + \underset{4\omega^2}{\cancel{(2j\omega)^2}}}}$$

$$\begin{cases} \text{if } \omega = 0 \longrightarrow |H(j\omega)| = 2/3 \\ \text{if } \omega = \infty \longrightarrow |H(j\omega)| = 2/3 \end{cases}$$

; if $|H(j\omega)| = 0 \longrightarrow \omega_0 =$ فرکانس تشدید

$$\hookrightarrow 8-2\omega^2 = 0 \longrightarrow \omega^2 = 4 \longrightarrow \boxed{\omega_0 = 2}$$

فیلتر میان نگذر

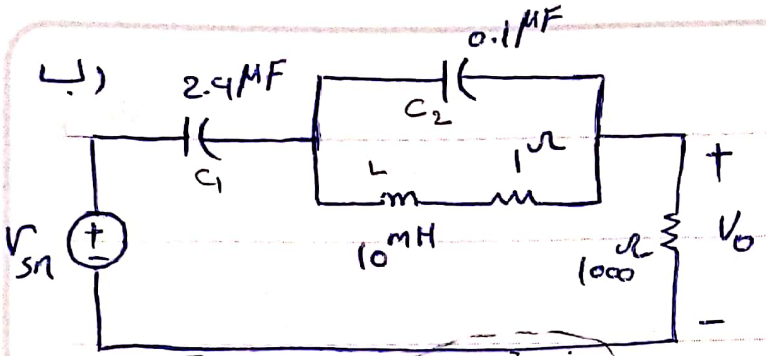


فرکانس قطع :

$$|H(j\omega)| = \frac{\sqrt{2}}{2} A_{max} = \frac{\sqrt{2}}{2} \times \frac{2}{3} = \frac{\sqrt{2}}{3}$$

$$\longrightarrow \frac{8-\omega^2}{\sqrt{(12-3\omega^2)^2 + 4\omega^2}} = \frac{\sqrt{2}}{3} \longrightarrow \begin{cases} \omega_L = 1.69 \\ \omega_H = 2.36 \end{cases}$$

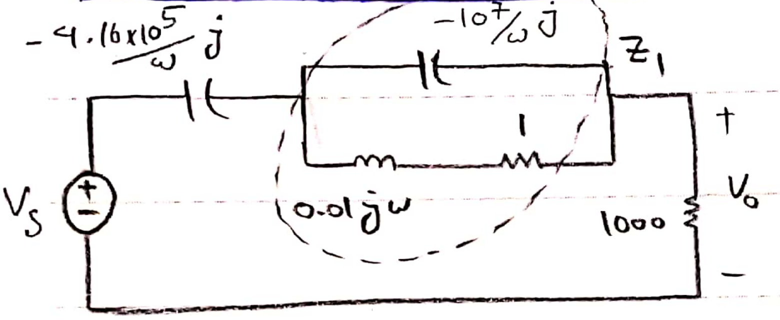
ب)



$$Z_{C_1} = \frac{1}{2.4\times10^{-6} j\omega} = \frac{-4.16\times10^{5}}{\omega} j$$

$$Z_{C_2} = \frac{1}{0.1\times10^{-6} j\omega} = \frac{-10^{7}}{\omega} j$$

$$Z_L = 10\times10^{-3} j\omega = 0.01 j\omega$$



$$Z_1 = (1+0.01j\omega) \parallel \left(\frac{-10^{7}}{\omega} j\right) = \frac{(1+0.01j\omega)\left(\frac{-10^{7}}{\omega} j\right)}{1+0.01j\omega + \frac{10^{-7}}{\omega} j} = \frac{-10^{7} j + 10^{5}\omega}{-10^{7} j + \omega + 10^{-2} j\omega^{2}}$$

(تقسیم امپدانسی) مقسم ولتاژ

$$\frac{V_o}{V_s} = \frac{1000}{1000 + \frac{-4.16\times10^{5}}{\omega} j + \frac{-10^{7} j + 10^{5}\omega}{-10^{7} j + \omega + 10^{-2} j\omega^{2}}} =$$

«تمرینات سری ششم» تاریخ تحویل: ۹۸/۱۰/۸

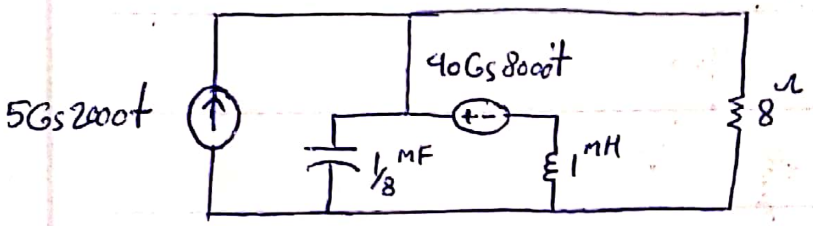
فصل هفتم کتاب چهاردار: ۵۰ - ۵۳ - ۵۶ - ۵۷ - ۸۸ - ۱۱۴

فصل هشتم کتاب چهاردار: ۱۱ - ۱۲ - ۲۱ - ۲۲.

فصل یازدهم کتاب هیت: ۲۴ - ۴۷ - ۵۱ فصل سیزدهم کتاب هیت: ۲۹ - ۳۹ - ۴۲

کتاب چهاردار:

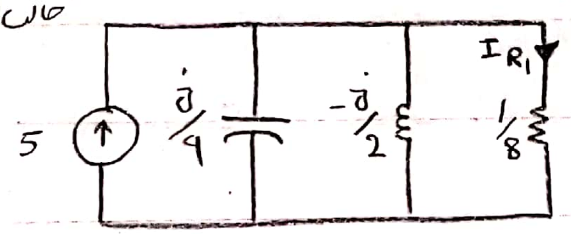
س ۵۰:



$$P_R = \frac{VI}{2} = \frac{RI^2}{2}$$

جمع آثار

حالت ①

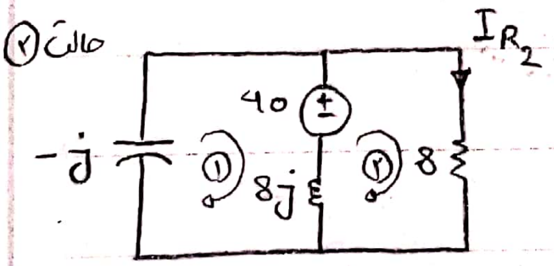


$$y_C = \frac{1}{8} \times 10^{-3} \times 2 \times 10^3 j = \frac{1}{4} j$$

$$y_L = \frac{1}{1 \times 10^{-3} \times 2 \times 10^3 j} = -0.5j$$

تقسیم جریان: $I_{R_1} = \frac{1/8}{1/8 + j/4 - j/2} \times 5 = \frac{1}{1+2j-4j} \times 5 = \frac{5}{1-2j} = \frac{5\angle 0}{2.23\angle -63.43}$

حالت ②



kvl ①: $-jI_1 + 40 + 8jI_1 - 8jI_2 = 0$

$\longrightarrow 40 + 7jI_1 - 8jI_2 = 0 \longrightarrow$

$I_1 = \frac{-40 + 8jI_2}{7j}$ ⊛

kvl ②: $8jI_2 - 8jI_1 - 40 + 8I_2 = 0 \longrightarrow (8+8j)I_2 - 8jI_1 = 40$

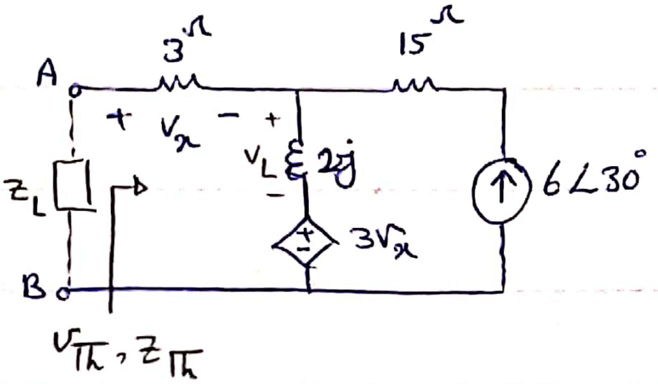
$\xrightarrow{⊛} (8+8j)I_2 + \frac{320}{7} - \frac{64jI_2}{7} = 40 \longrightarrow (8-1.14j)I_2 = -5.71$

$\longrightarrow I_2 = I_{R_2} = \frac{-5.71}{8-1.14j} = \frac{5.71\angle 180}{8.08\angle 8.11}$

$I_R = I_{R_1} + I_{R_2} \longrightarrow |I_R| = |I_{R_1}| + |I_{R_2}| = 2.24 + 0.64 = \boxed{2.88}$

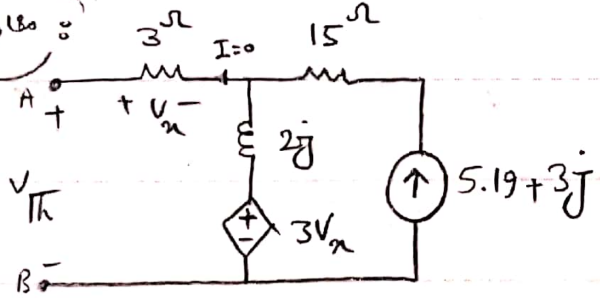
$P_{R_{avg}} = \frac{R|I|^2}{2} = \frac{8(2.88)^2}{2} = 4 \times 8.29 = \boxed{33.16}$ w

س ۳ه



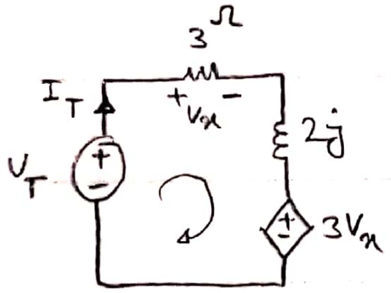
$$\begin{cases} V_{Th} = V_L + \cancel{3V_x} = V_L \\ V_L = Z_L(I) \end{cases}$$

مدار تونن :



$\rightarrow V_{Th} = 2j(5.19+3j)$

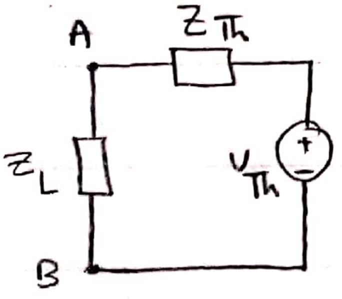
$= -6 + 10.39j = 12\angle 120$



kvl: $-V_T + 3I_T + 2j\,I_T + 3V_x = 0$

$(3+2j)I_T + 3(3I_T) = V_T \rightarrow$

$(12+2j)I_T = V_T \rightarrow Z_{Th} = \frac{V_T}{I_T} = 12+2j$



$\boxed{\bar{Z}_L = Z_{Th}} \rightarrow Z_L = \boxed{12-2j}$

مقسم ولتاژ

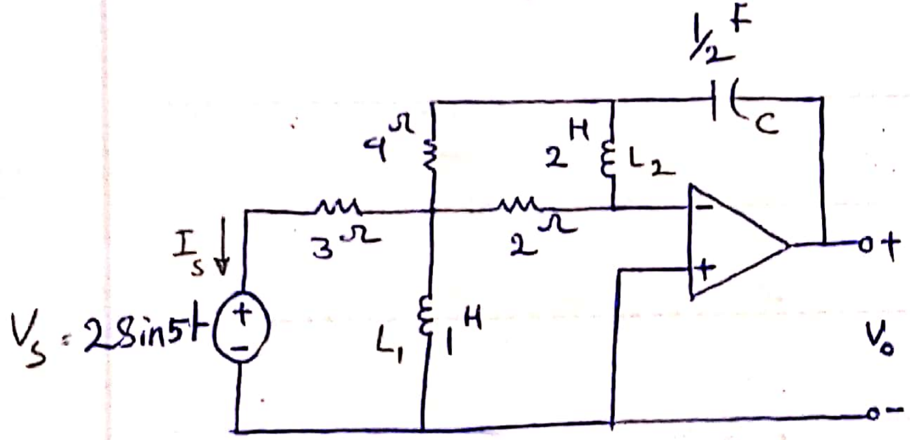
$V_{AB} = \frac{Z_L}{Z_L+Z_{Th}} \times V_{Th} = \frac{12-2j}{24} \times (-6+10.39j) = \frac{-72+124.68j+12j+20.78}{24}$

$= \frac{-51.22+136.68j}{24} = -2.13+5.7j = 6\angle 110.5$

$$I_{AB} = \frac{V_{AB}}{Z_L} = \frac{-2.13+5.7j}{12-2j} = \frac{6\angle 110.5}{12.16\angle -9.5} = 0.5\angle 120$$

$$P_{AB_{max}} = \frac{|V_{AB}||I_{AB}|}{2} \cos(\varphi_{V_{AB}} - \varphi_{I_{AB}}) = \frac{6(0.5)}{2} \cos(110.5-120)$$

$$= 1.5\cos(-9.5) = 1.5\times 0.98 = \boxed{1.47}^{w}$$

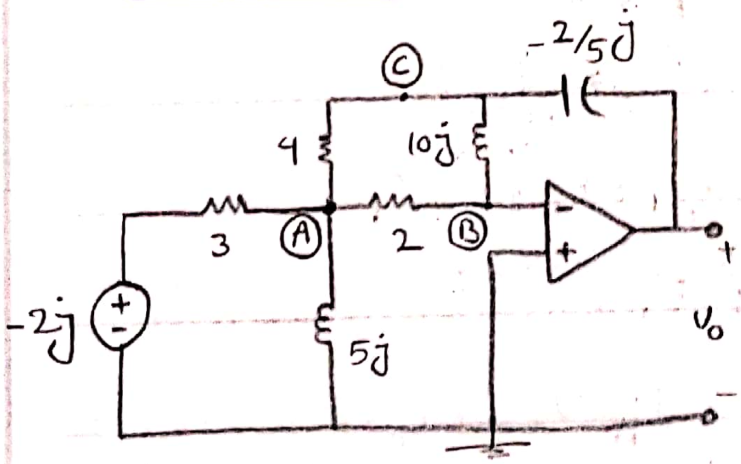


$\omega = 5$ حل :

$Z_{L_1} = 5j \;,\; Z_{L_2} = 10j$

$$Z_C = \frac{1}{\frac{1}{2}\times 5j} = -\frac{2}{5}j$$

$$V_s = 2\sin 5t = 2\cos(5t - \pi/2) = \boxed{-2j}$$



آپ امپ : $V_- = V_+$

$\hookrightarrow V_B = 0^V$

$I_- = I_+ = 0^A$

$$\text{kcl}Ⓐ: \frac{V_A+2j}{3} + \frac{V_A}{5j} + \frac{V_A - V_C}{4} + \frac{V_A}{2} = 0 \rightarrow$$

$$20V_A + 40j - 12jV_A + 15V_A - 15V_C + 30V_A = 0 \rightarrow$$

$$(65-12j)V_A + 40j = 15V_C \quad (**)$$

$$\text{kcl}Ⓑ: \frac{-V_A}{2} + \frac{-V_C}{10j} = 0 \rightarrow -5V_A + V_C = 0 \rightarrow V_C = 5V_A \quad (*)$$

$$\text{kcl}\,Ⓒ:\ \frac{V_C-V_A}{4}+\frac{V_C}{10j}+\frac{V_C-V_o}{-2/5j}=0 \rightarrow 5V_C-5V_A-2jV_C+50jV_C-50jV_o=0$$

$$\rightarrow (5+48j)V_C-5V_A=50jV_o \xrightarrow{\circledast} (25+240j-5)V_A=50jV_o$$

$$(20+240j)V_A=50jV_o \rightarrow V_o=\frac{2+24j}{5}V_A \quad ①$$

$$\circledast \text{ in } \circledast\circledast\ (65-12j)V_A+40j=75V_A \rightarrow (10+12j)V_A=40j \rightarrow$$

$$V_A=\frac{40j}{10+12j} \quad ②$$

$$② \text{ in } ① \rightarrow V_o=\frac{2+24j}{\cancel{5}}\times\frac{\overset{8}{\cancel{40}}j}{10+12j}=\frac{(1+12j)(8j)}{5+6j}=\frac{-96+8j}{5+6j}$$

$$V_o=\frac{96.3\angle 175.2}{7.8\angle 50.2}=12.34\angle 125 \rightarrow V_o=-7+10.1j$$

$$V_o(t)=12.34\cos(5t+125)$$

$$V_S=-2j=2\angle -90$$

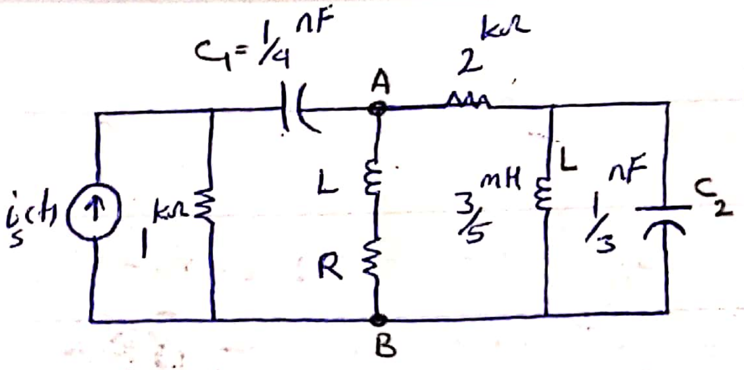
$$I_S=\frac{V_A-V_S}{3}=\frac{\frac{40j}{10+12j}+2j}{3}=\frac{1.9+1.6j+2j}{3}=\frac{1.9+3.6j}{3}\rightarrow$$

$$I_S=0.63+1.2j=1.3\angle 62.3$$

توان متوسط $P_S=\frac{V_S I_S}{2}\cos(\Phi_{V_S}-\Phi_{I_S})=\frac{2\times 1.3}{2}\overset{-0.88}{\cancel{\cos(-90-62.3)}}=\boxed{-1.14}\ ^{w}$

توان راکتیو $Q_S=\frac{V_S I_S}{2}\sin(\Phi_{V_S}-\Phi_{I_S})=\frac{2\times 1.3}{2}\overset{-0.46}{\cancel{\sin(-90-62.3)}}=\boxed{-0.59}\ \text{VAR}$

توان مختلط $S_S=P_S+jQ_S=\boxed{-1.14-0.59j}\ \text{VA}$



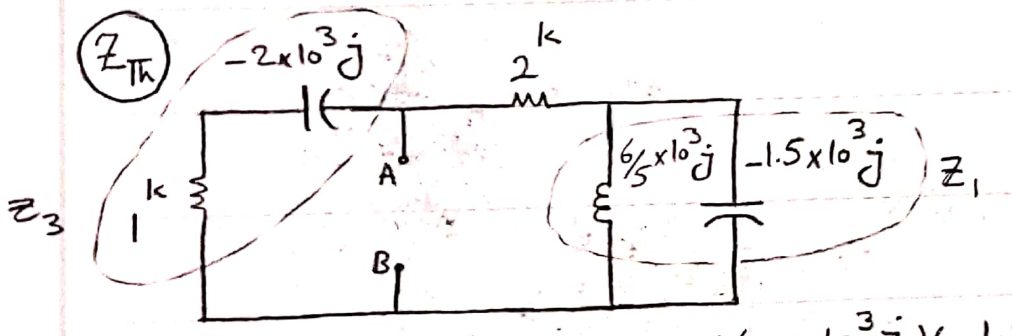
$i_s(t) = 5\cos 2\times10^6 t$ (mA) : $\underline{04}$ س

$\omega = 2\times10^6$

$Z_L = \frac{3}{5}\times10^{-3}\times2\times10^6 j = \frac{6}{5}\times10^3 j$

$Z_{C_1} = \frac{1}{\frac{1}{4}\times10^{-9}\times2\times10^6 j} = -2\times10^3 j$, $Z_{C_2} = \frac{1}{\frac{1}{3}\times10^{-9}\times2\times10^6 j} = -1.5\times10^3 j$

Z_{Th}

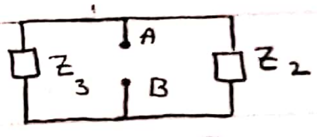


$Z_1 = (\frac{6}{5}\times10^3 j) \| (-1.5\times10^3 j) = \frac{(6/5\times10^3 j)(-1.5\times10^3 j)}{(6/5\times10^3 j)+(-1.5\times10^3 j)} = \frac{1.8\times10^6}{-0.3\times10^3 j} = \boxed{6\times10^3}$

$Z_2 = Z_1 + 2^k = 6\times10^3 + 2\times10^3 = 8\times10^3$, $Z_3 = 10^3 - 2\times10^3 j$

$Z_{Th} = Z_2 \| Z_3 = (8\times10^3) \| (10^3 - 2\times10^3 j) = \frac{8\times10^3(10^3 - 2\times10^3 j)}{8\times10^3 + 10^3 - 2\times10^3 j}$

$= \frac{8\times\cancel{10^6}(1-2j)}{\cancel{10^3}(9-2j)} = \frac{8\times10^3(1-2j)}{9-2j}\times\frac{9+2j}{9+2j}$



$= \frac{8\times10^3(9+2j-18j+4)}{81+4} = \frac{(13-16j)8\times10^3}{85} = (1.22 - 1.5j)\times10^3$

اصلاح شود!

$Z_L = Z_{AB} = \overline{Z_{Th}} = 1.22 + 1.5j = Z_R + Z_L \rightarrow \boxed{R = 1.22}\,\Omega$

$Lj\omega = 1.5j \rightarrow 2\times10^6 L = 1.5 \rightarrow L = 0.75\times10^{-6} = \boxed{0.75}\,\mu H$