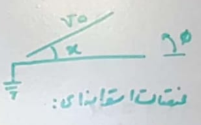
4 - 4

جواب معادله لاپلاس ؟

$$V_1 = C_1 / R$$

$$\nabla^2 V_1 = \frac{1}{R^2} \frac{\partial}{\partial R} \left(R^2 \frac{\partial V}{\partial R} \right) + \frac{1}{R^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial V}{\partial \theta} \right) + \frac{\partial^2 V}{R^2 \sin\theta \, \partial \phi 2}$$

$$\nabla^2 V_1 = \frac{1}{R^2} \frac{\partial}{\partial R} \left(-\frac{R^2 C_1}{R^2} \right) = \frac{1}{R^2} \frac{\partial}{\partial R} (-C_1) = 0 \checkmark$$



$0 < \phi < \alpha \qquad V = ?$

مختصات استوانه‌ای:

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

که فقط در راستای ϕ مولفه داریم:

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} = 0 \rightarrow \frac{\partial^2 V}{\partial \phi^2} = 0$$

$$\downarrow$$

$$\frac{\partial V}{\partial \phi} = C_1$$

$$\Rightarrow V(\phi) = C_1\phi + C_2$$

$$\begin{cases} V(0) = 0 \rightarrow C_1(0) + C_2 = 0 \rightarrow \boxed{C_2 = 0} \\ V(\alpha) = V_0 \rightarrow C_1(\alpha) + C_2 = V_0 \rightarrow \boxed{C_1 = V_0/\alpha} \end{cases}$$

$$\Rightarrow \boxed{V(\phi) = \frac{V_0}{\alpha}\phi}$$

$V(\phi) = C_1\phi + C_2 \qquad \alpha < \phi < 2\pi \qquad V = ?$

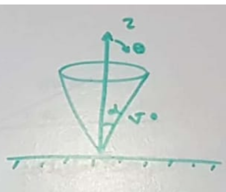
$$\begin{cases} V(\alpha) = V_0 \rightarrow C_1(\alpha) + C_2 = V_0 \\ V(2\pi) = 0 \rightarrow C_1(2\pi) + C_2 = 0 \end{cases}$$

$$C_1(\alpha - 2\pi) = V_0 \rightarrow C_1 = \frac{V_0}{\alpha - 2\pi}$$

$$C_2 = -2\pi C_1 = \frac{-2\pi V_0}{\alpha - 2\pi}$$

$$\Rightarrow V(\phi) = \frac{V_0}{\alpha - 2\pi}\phi - \frac{2\pi V_0}{\alpha - 2\pi}$$

Scanned by CamScanner



الف - $\alpha < \theta < \frac{\pi}{2}$ ، $V(\theta) = ?$

دستگاه کروی :

که فقط مولفه‌ی θ داریم :

$$\frac{1}{R^2 \sin\theta} \frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial V}{\partial\theta}\right) = 0$$

$$\Rightarrow \sin\theta \frac{\partial V}{\partial\theta} = C_1 \rightarrow \frac{\partial V}{\partial\theta} = \frac{C_1}{\sin\theta}$$

$$V = \int \frac{C_1}{\sin\theta} d\theta$$

نکته : $\boxed{\int \frac{dx}{\sin ax} = \frac{1}{a} Ln\left(\tan \frac{ax}{2}\right) + C}$

$$V = C_1 Ln\left(\tan \theta/2\right) + C_2$$

$$\begin{cases} V(\alpha) = V_0 \rightarrow C_1 Ln(\tan \alpha/2) + C_2 = V_0 \\ V(\pi/2) = 0 \rightarrow \underbrace{C_1 Ln(\underbrace{\tan \pi/4}_{1})}_{0} + C_2 = 0 \rightarrow C_2 = 0 \end{cases}$$

$$\Rightarrow C_1 = \frac{V_0}{Ln(\tan \alpha/2)}$$

$$V(\theta) = \frac{V_0}{Ln(\tan \alpha/2)} Ln\left(\tan \theta/2\right)$$

ب - $\alpha < \theta < \pi/2$ ، $E = ?$

$$E = -\nabla V = -\frac{1}{R}\frac{\partial V}{\partial\theta} = \frac{-V_0}{R\, Ln(\tan \alpha/2)} \times \frac{\frac{1}{2}(1+\tan^2 \theta/2)}{\tan \theta/2}\, a_\theta$$