جواب میان ترم الکترومغناطیس — آبان ۹۸

۱-

$$\bar{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_L \bar{R}' dl}{|R'|^3}$$

$\bar{R}' = -b a_r + z_p a_z$ (۰/۲۵) , $|R|^3 = (b^2+z_p^2)^{3/2}$ (۰/۲۵)

$$\Rightarrow \bar{E} = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{\cos\varphi(-b\overset{(\sin\varphi a_y + \cos\varphi a_x)}{a_r} + z_p a_z)\, b\, d\varphi}{(b^2+z_p^2)^{3/2}}$$ (۰/۵) (۰/۵)

$dl = b\,d\varphi$, $\rho_L = \cos\varphi$ (۰/۲۵)

$$\bar{E} = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{-b^2\cos^2\varphi\, d\varphi}{(b^2+z_p^2)^{3/2}} a_x + \underbrace{\frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{-b^2\cos\varphi\sin\varphi\, d\varphi}{(b^2+z_p^2)^{3/2}} a_y}_{0} + \underbrace{\frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{b z_p\cos\varphi\, d\varphi}{(b^2+z_p^2)^{3/2}} a_z}_{0}$$ (۰/۲۵) (۰/۲۵)

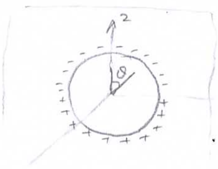
$$\bar{E} = \frac{-b^2}{4\pi\varepsilon_0 (b^2+z_p^2)^{3/2}} a_x$$ (۰/۲۵)

۲-

$\bar{P} = P_0 a_z \Rightarrow$ چگالی بارهای مقید

$\rho_v = -\nabla\cdot\bar{P} = 0$ (۰/۵)

$\rho_s = \bar{P}\cdot a_n = P_0 a_z\cdot(-a_R) = -P_0\cos\theta$ (۰/۵)



$$\bar{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho_s \bar{R}' ds}{|R'|^3} = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{\pi} +P_0\cos\theta\, a_R \sin\theta\, d\theta\, d\varphi$$ (۰/۷۵)

$a_R = \sin\theta\cos\varphi\, a_x + \sin\theta\sin\varphi\, a_y + \cos\theta\, a_z$ (۰/۲۵)

$\bar{R}' = -b a_R$ (۰/۵)

$ds = b^2\sin\theta\, d\theta\, d\varphi$ (۰/۵)

$$\bar{E} = \frac{P_0}{4\pi\varepsilon_0}\left[\underbrace{\int_0^{2\pi}\int_0^{\pi}\sin^2\theta\cos\theta\cos\varphi\, d\theta d\varphi\, a_x}_{0} + \underbrace{\int_0^{2\pi}\int_0^{\pi}\sin^2\theta\cos\theta\sin\varphi\, d\theta d\varphi\, a_y}_{0} + \underbrace{\int_0^{2\pi}\int_0^{\pi}\sin\theta\cos^2\theta\, d\theta d\varphi\, a_z}_{\frac{4\pi}{3}a_z}\right]$$ (۰/۵) (۰/۵)

$$\bar{E} = \frac{P_0}{3\varepsilon_0} a_z$$ (۰/۵)

۳-

$\oint \bar{D}\cdot d\bar{s} = Q$ (۰/۵) $\Rightarrow$ کل شار خالص خروجی از سطح مکعب برابر است با کل بار خالص موجود در داخل مکعب

$Q =$ حجم کره × چگالی بار $= \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 \times \rho_0 = \frac{\pi a^3}{6}\rho_0$ (۰/۵)