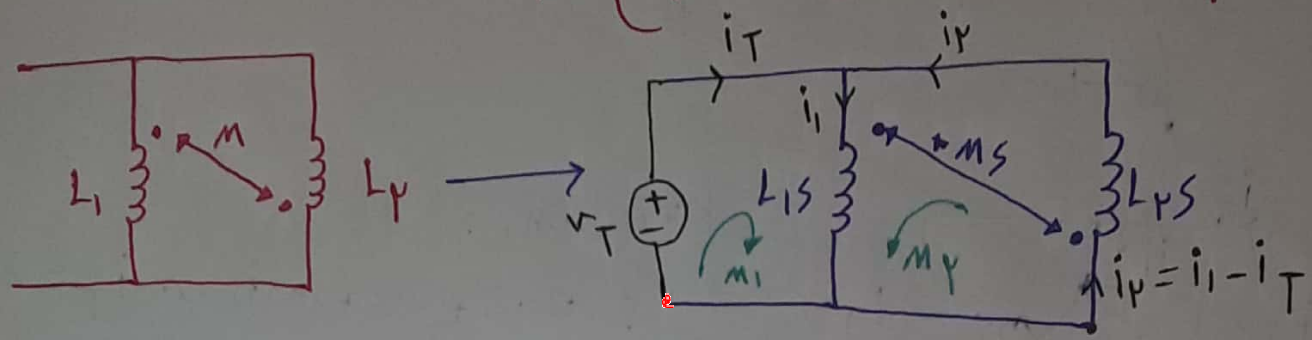
ص ۱

حل تمرین سری ۱:

* نکته * $L \xrightarrow{\text{امپدانس}} Lj\omega \xrightarrow[\text{لاپلاس}]{j\omega = s} Ls$

تمرین) مقدار سلف معادل در مدار های زیر:



KVL @ m_1: $-v_T + L_1 s i_1 + M s i_1 - M s i_T = 0$

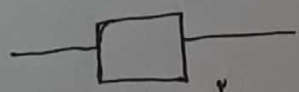
$\rightarrow v_T = (L_1 + M) s i_1 - M s i_T$ (I)

KVL @ m_2: $L_2 s i_1 - L_2 s i_T + M s i_1 + L_1 s i_1 + M s i_1 - M s i_T = 0$

$\rightarrow (L_1 + L_2 + 2M) i_1 = (M + L_2) i_T \rightarrow i_1 = \frac{(M + L_2)}{L_1 + L_2 + 2M} i_T$ (II)

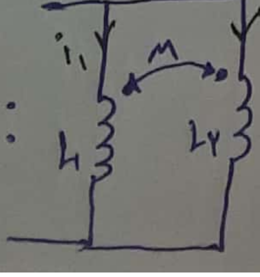
$\xrightarrow{(II),(I)} v_T = s\frac{(L_1 + M)(M + L_2)}{L_1 + L_2 + 2M} i_T - M s i_T \rightarrow$

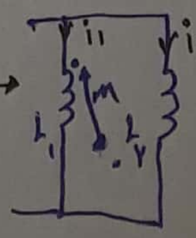
$$\frac{v_T}{i_T} = \frac{(\cancel{L_1 M} + L_1 L_2 + M^2 + \cancel{M L_2} - \cancel{L_1 M} - \cancel{L_2 M} - 2M^2)s}{L_1 + L_2 + 2M}$$

$\frac{v_T}{i_T} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} s \rightarrow$  $\rightarrow L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$

$\underbrace{\frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}}_{L_{eq}} \times s$ ← معادل سلف

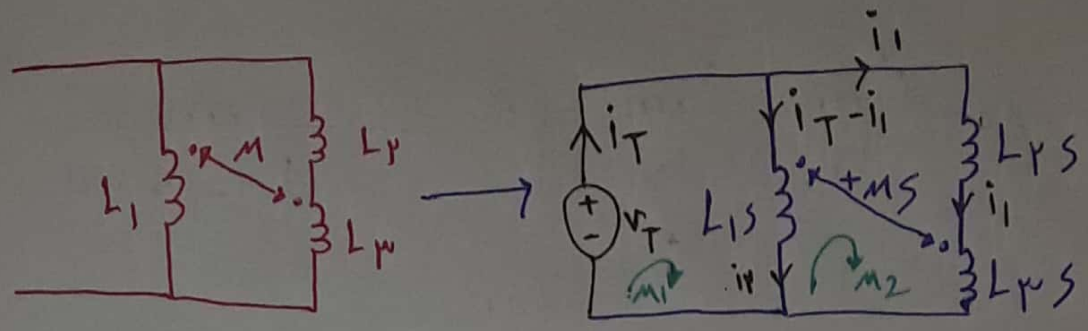
روش دوم حل:

۱- می دانیم: 

$\equiv \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \rightarrow$ 

$\overset{M \to -M}{\equiv} \frac{L_1 L_2 - (-M)^2}{L_1 + L_2 + 2M}$

ص ۲



KVL @ M_1: $-v_T + L_1 s i_T - L_1 s i_1 + M s i_1 = 0 \to$

$-v_T + L_1 s i_T + (M - L_1) s i_1 = 0$ (I)

KVL @ M_2: $-L_1 s \underbrace{i_2}_{(i_T - i_1)} - M s i_1 + L_2 s i_1 + L_3 s i_1 + M s i_T - M s i_1 = 0$

$$\to (L_1 - M + L_2 + L_3 - M) i_1 = (L_1 - M) i_T$$

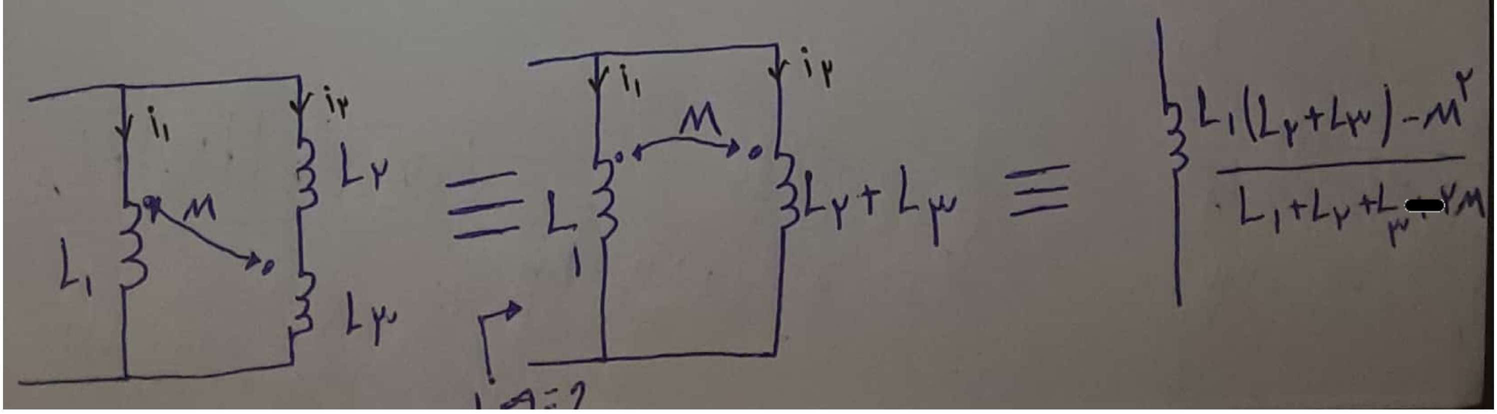
$$\to i_1 = \frac{(L_1 - M)}{(L_1 + L_2 + L_3 - 2M)} i_T \quad (\text{II})$$

$$\xrightarrow{\text{I, II}} v_T = s\left(L_1 + (M - L_1)\left(\frac{L_1 - M}{L_1 + L_2 + L_3 - 2M}\right)\right) i_T \to$$

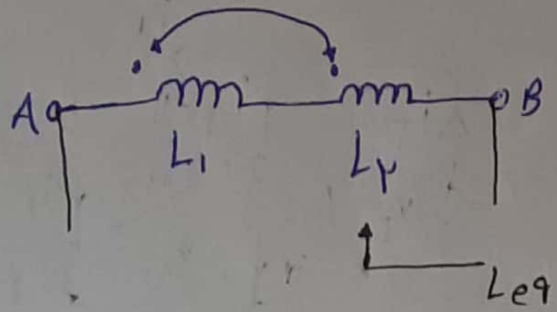
$$\frac{v_T}{i_T} = s\left(\frac{\cancel{L_1 M} - M^2 - \cancel{L_1^2} + \cancel{L_1 M} + \cancel{L_1^2} + L_1 L_2 + L_1 L_3 - \cancel{2ML_1}}{L_1 + L_2 + L_3 - 2M}\right)$$

$\frac{v_T}{i_T} = \left(\frac{L_1(L_2 + L_3) - M^2}{L_1 + L_2 + L_3}\right.$ [redacted] $\left.2M\right) s \to L_{eq} = \frac{L_1(L_2 + L_3) - M^2}{L_1 + L_2 + L_3}$ [redacted] $2M$

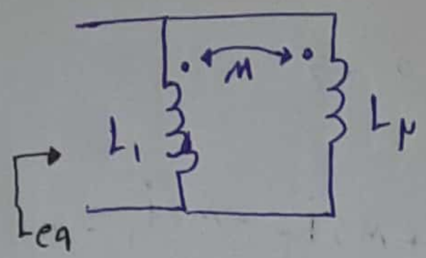
(روش دوم حل)



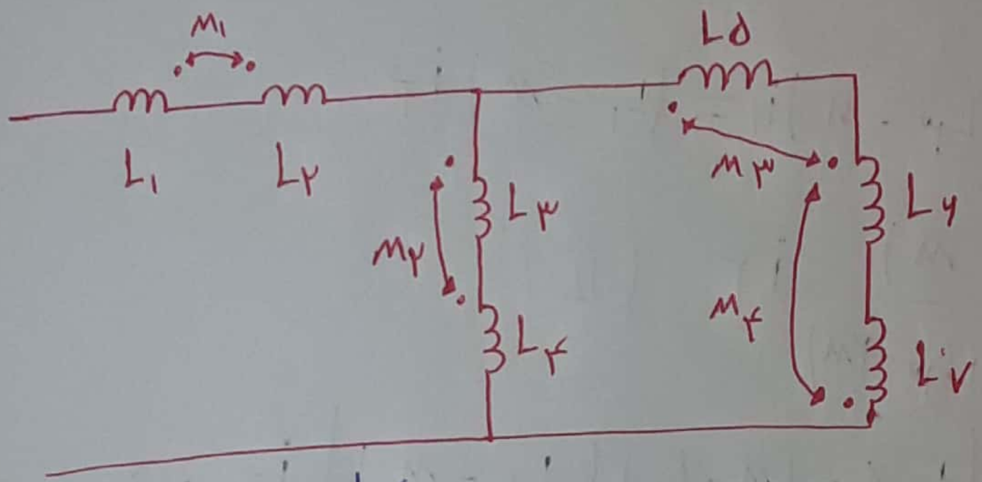
* نکته *:

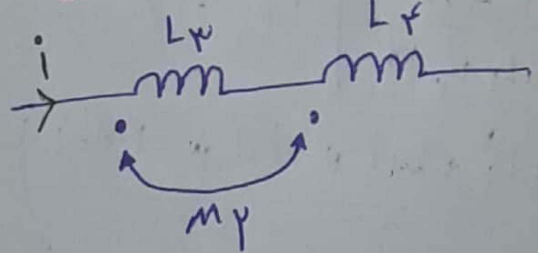


$\equiv$ L_1+L_2+2M

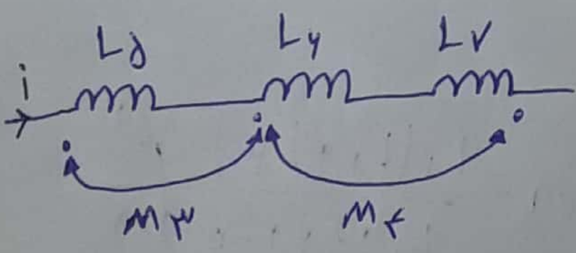


$\equiv \dfrac{L_1L_2-M^2}{L_1+L_2-2M}$

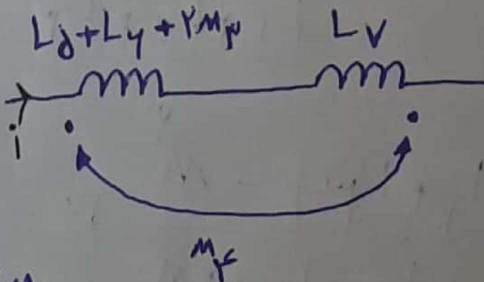




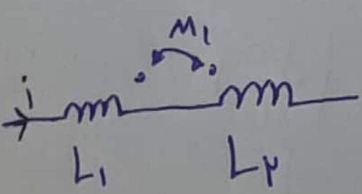
$\equiv L_3+L_4+2M_2$



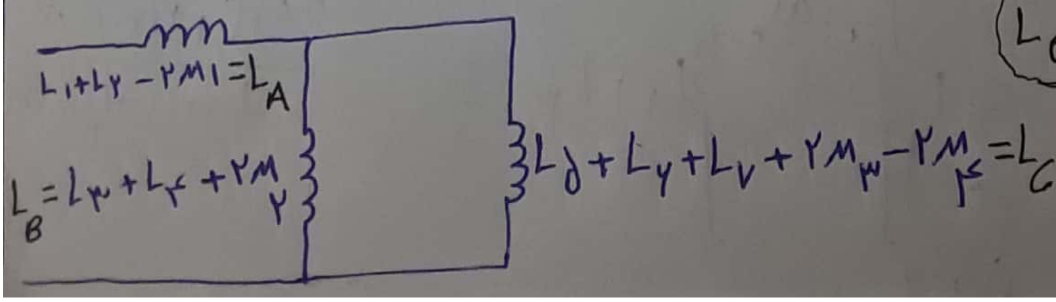
$\equiv$



$\equiv L_5+L_6+2M_3+L_7-2M_4$



$\equiv L_1+L_2-2M_1$



$$L_{eq}=(L_B \| L_C)+L_A$$

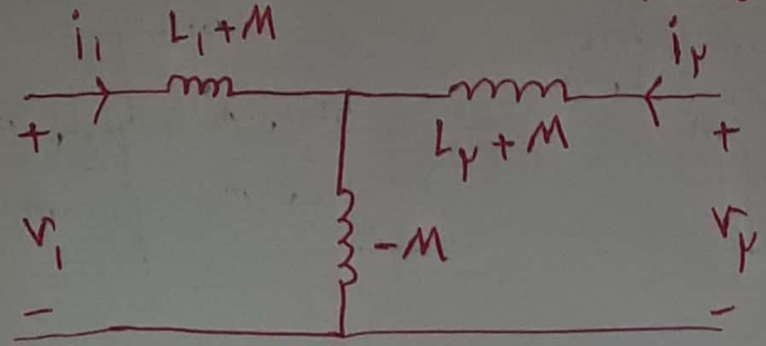
ادامه حل در ص ۴

ص 4

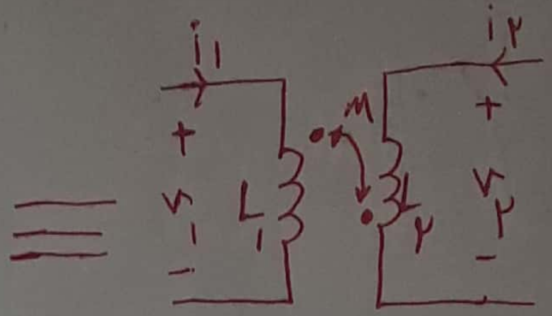
$$L_{eq} = L_A + L_B \parallel L_C = L_A + \frac{L_B \cdot L_C}{L_B + L_C}$$

$$L_{eq} = L_۱ + L_۲ - ۲M_۱ + \frac{(L_۳ + L_۴ + ۲M_۲)(L_۵ + L_۶ + L_۷ - ۲M_۳ - ۲M_۴)}{L_۳ + L_۴ + L_۵ + L_۶ + L_۷ + ۲(M_۲ - M_۳ - M_۴)}$$

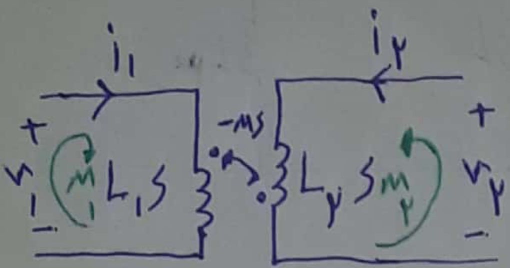
تمرین: ثابت کنید 2 مدار معادل هستند.



مدار ۲



مدار ۱



$$KVL@m_1: -v_1 + L_1 s i_1 - M s i_2 = 0 \rightarrow$$

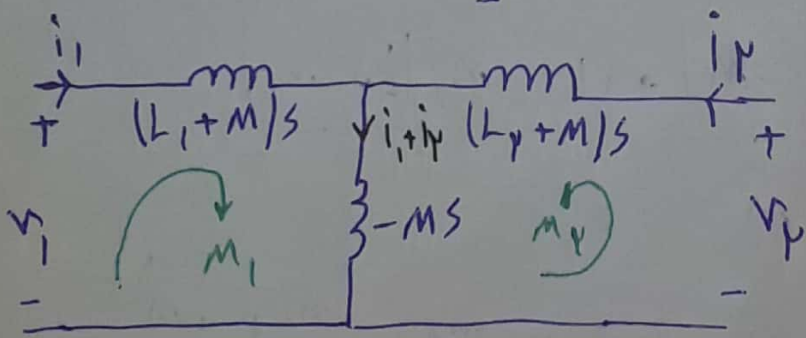
$$v_1 = L_1 s i_1 - M s i_2 \quad (\mathrm{I})$$

$$KVL@m_2: v_2 = L_2 s i_2 - M s i_1 \quad (\mathrm{II})$$

درصورتی که همین روابط برای مدار ۲ صادق باشد ← مدار ۲ معادل مدار ۱

به عبارت دیگر باید ثابت کنیم در مدار دوم

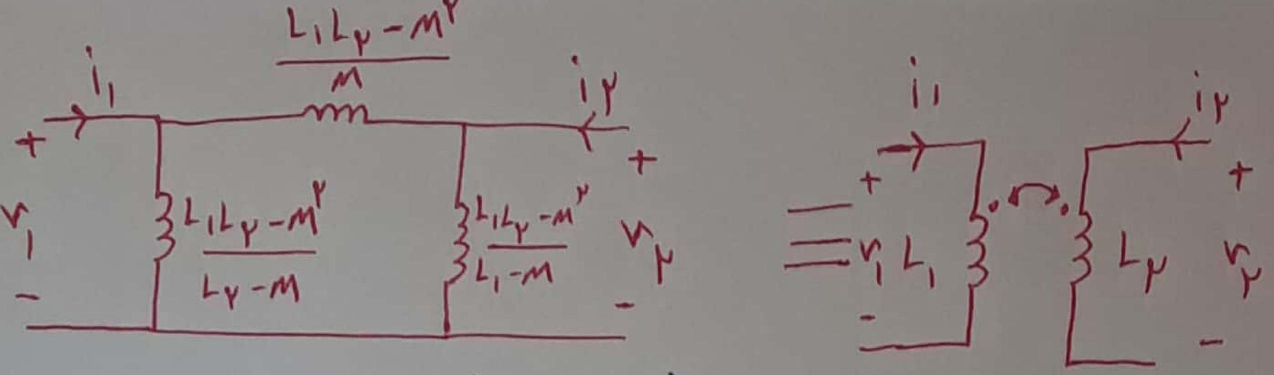
$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_1 s & -Ms \\ -Ms \ \cancel{L_2 s} & +L_2 s \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$



$$KVL@m_1: v_1 = (L_1+M)s i_1 - Ms(i_1+i_2) \rightarrow v_1 = L_1 s i_1 - M s i_2 \quad (\mathrm{I})$$

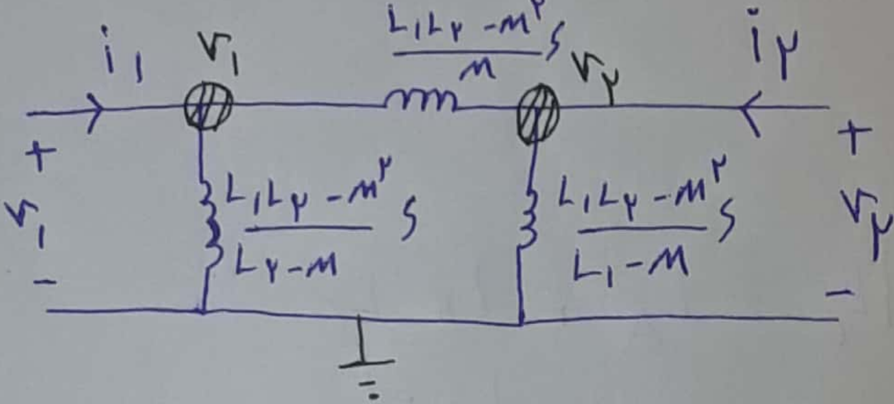
$$KVL@m_2: v_2 = (L_2+M)s i_2 - Ms(i_1+i_2) \rightarrow v_2 = L_2 s i_2 - M s i_1 \quad (\mathrm{II})$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_1 s & -Ms \\ -Ms \ \cancel{L_2 s} & L_2 s \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$



باید ثابت کنیم:

$$v_1 = L_1 s\, i_1 + M s\, i_2$$
$$v_2 = L_2 s\, i_2 + M s\, i_1$$



$$KCL\ @v_1:\ i_1 = \frac{(v_1-0)(L_2-M)}{(L_1L_2-M^2)s} + \frac{(v_1-v_2)M}{(L_1L_2-M^2)s} \xrightarrow{\times(L_1L_2-M^2)s}$$

$$i_1 s(L_1L_2-M^2) = v_1L_2 - v_1M + v_1M - v_2M \rightarrow L_2v_1 - Mv_2 = (L_1L_2-M^2)i_1 s \quad (I)$$

$$KCL\ @v_2:\ \frac{(v_2-0)(L_1-M)}{(L_1L_2-M^2)s} + \frac{(v_2-v_1)M}{(L_1L_2-M^2)s} = i_2 \rightarrow -Mv_1 + L_1v_2 = i_2 s(L_1L_2-M^2) \quad (II)$$

$$\left.\begin{aligned} L_2v_1 - Mv_2 &= (L_1L_2-M^2)i_1 s \\ -Mv_1 + L_1v_2 &= (L_1L_2-M^2)i_2 s \end{aligned}\right| \rightarrow \text{حل به روش کرامر}$$

$$\begin{bmatrix} L_2 & -M \\ -M & L_1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} (L_1L_2-M^2)i_1 s \\ (L_1L_2-M^2)i_2 s \end{bmatrix}$$

ادامه حل در ص 7

ص

$$\begin{vmatrix} L_2 & -M \\ -M & L_1 \end{vmatrix} = A \quad , \quad \begin{vmatrix} (L_1L_2 - M^2) i_1 s & -M \\ (L_1L_2 - M^2) i_2 s & L_1 \end{vmatrix} = B$$

$$\begin{vmatrix} L_2 & (L_1L_2 - M^2) i_1 s \\ -M & (L_1L_2 - M^2) i_2 s \end{vmatrix} = C \qquad \begin{cases} v_1 = \dfrac{B}{A} \\ v_2 = \dfrac{C}{A} \end{cases}$$

$$A = L_1L_2 - M^2 \quad , \quad B = L_1 i_1 s (L_1L_2 - M^2) + M s i_2 (L_1L_2 - M^2) \Rightarrow$$

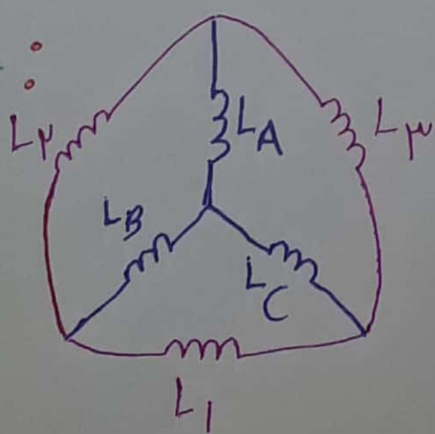
$$B = (L_1L_2 - M^2)(L_1 i_1 + M i_2) s$$

$$C = (L_1L_2 - M^2)(L_2 s i_2 + M s i_1)$$

$$v_1 = \frac{B}{A} = L_1 s i_1 + M s i_2 \quad , \quad v_2 = \frac{C}{A} = L_2 s i_2 + M s i_1$$

روش دوم حل :

* نکته * :



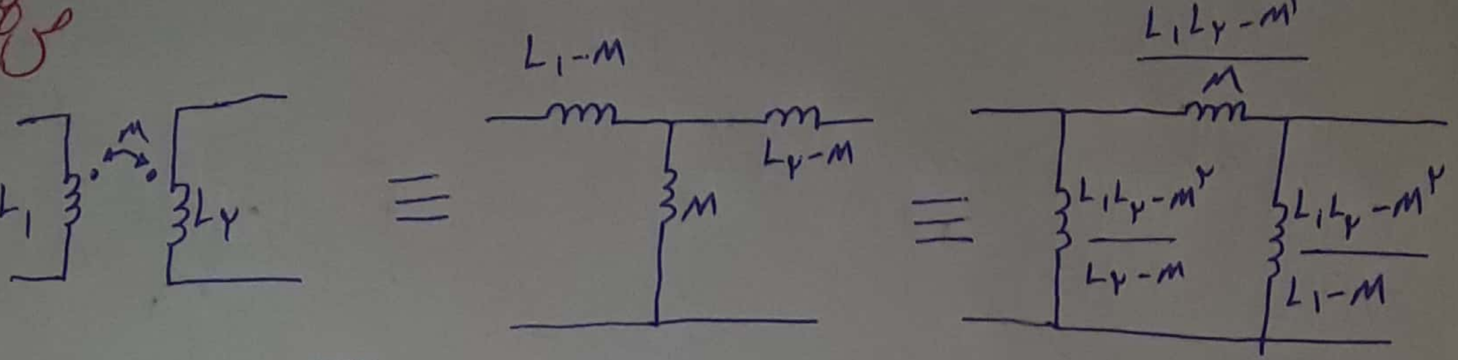
تبدیل ستاره به مثلث

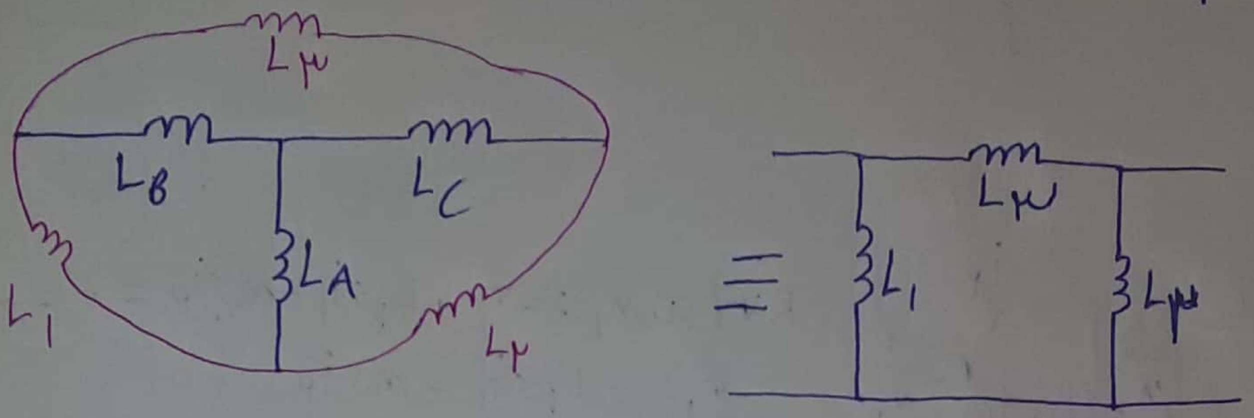
$$\begin{cases} L_1 = \dfrac{L_AL_B + L_AL_C + L_CL_B}{L_A} \\ L_2 = \dfrac{L_AL_B + L_AL_C + L_CL_B}{L_C} \\ L_3 = \dfrac{L_AL_B + L_AL_C + L_CL_B}{L_B} \end{cases}$$

تبدیل مثلث به ستاره

$$\begin{cases} L_A = \dfrac{L_2L_3}{L_1 + L_2 + L_3} \\ L_B = \dfrac{L_1L_2}{L_1 + L_2 + L_3} \\ L_C = \dfrac{L_1L_3}{L_1 + L_2 + L_3} \end{cases}$$

ادامه حل در ص ۶





$$L_1 = \frac{L_A L_B + L_A L_C + L_B L_C}{L_C}, \quad L_2 = \frac{//}{L_B}, \quad L_3 = \frac{//}{L_A}$$

$$L_A = M, \quad L_B = L_1 - M, \quad L_C = L_2 - M$$

$$L_1 = \frac{M(L_1 - M) + M(L_2 - M) + (L_1 - M)(L_2 - M)}{L_2 - M}$$

$$L_1 = \frac{ML_1 - M^2 + ML_2 - M^2 + L_1 L_2 - ML_1 - ML_2 + M^2}{L_2 - M}$$

$$L_1 = \frac{L_1 L_2 - M^2}{L_2 - M}$$

به طریق مشابه: $L_3 = \frac{L_1 L_2 - M^2}{M}$,

$$L_2 = \frac{L_1 L_2 - M^2}{L_1 - M}$$