۴) تبدیل فوریه :

الف) $f(t) = e^{-t}\cos\omega_0 t \, u(t)$ $\qquad \cos\omega_0 t = \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t}$

$$\Longrightarrow f(t) = \frac{1}{2}e^{j\omega_0 t}e^{-t}u(t) + \frac{1}{2}e^{-j\omega_0 t}e^{-t}u(t)$$

(خاصیت انتقال در حوزه فرکانس)

$$e^{-\alpha t}u(t) \xrightarrow{F} \frac{1}{\alpha+j\omega} \Longrightarrow e^{-t}u(t) = \frac{1}{1+j\omega}$$

$$\Longrightarrow F(\omega) = \frac{1}{2}\frac{1}{1+j(\omega-\omega_0)} + \frac{1}{2}\frac{1}{1+j(\omega+\omega_0)}$$ ✓

ب) $f(t) = t^2 e^{-t}u(t) \Longrightarrow F(\omega) = \frac{۲}{(1+j\omega)^3}$ ✓

$$e^{-\alpha t}u(t) \xrightarrow{F} \frac{1}{\alpha+j\omega}$$

$$t e^{-\alpha t}u(t) \xrightarrow{F} \frac{1}{(\alpha+j\omega)^2}$$

$$f(t) = t \times \underbrace{t e^{-t}u(t)}_{f_1(t)}$$

(مشتق گیری در حوزه فرکانس)

$$F(\omega) = \frac{-1}{j}\frac{dF_1(\omega)}{d\omega} = \frac{-1}{j} \times \frac{-۲j}{(\alpha+j\omega)^3} = \frac{۲}{(\alpha+j\omega)^3}$$

ا) $f(t) = \ln t$

خاصیت مشتق گیری در حوزه زمان.

$$\mathrm{Sign}(t) \xrightarrow{F} \frac{2}{j\omega}$$

$$\frac{2}{jt} \xrightarrow{F} 2\pi\,\mathrm{Sign}(-\omega)$$ خاصیت دوگانی

$$\frac{j}{2} \times \left(\frac{2}{jt} \xrightarrow{F} -2\pi\,\mathrm{Sign}(\omega) \right)$$

$$\frac{1}{t} \xrightarrow{F} -j\pi\,\mathrm{Sign}(\omega)$$

$$f'(t) = \frac{1}{t} \xrightarrow{F} -j\pi\,\mathrm{sig}(\omega)$$

ز) $f(t) = e^{-\alpha|t|}(1+\alpha|t|) \rightarrow f(t) = \underbrace{e^{-\alpha|t|}}_{f_1(t)} + \underbrace{e^{-\alpha|t|}\,\alpha|t|}_{f_2(t)}$

$$F(\omega) = F_1(\omega) + F_2(\omega) = \frac{2\alpha}{\alpha^2+\omega^2} + F_2(\omega)$$

$$f_2(t) = \alpha|t|\,e^{-\alpha|t|} = \begin{cases} \alpha t e^{-\alpha t} & t>0 \\ -\alpha t e^{\alpha t} & t<0 \end{cases} \xrightarrow{F} F_2(\omega) = \begin{cases} \alpha \dfrac{1}{(\alpha+j\omega)^2} & \omega>0 \\ \dfrac{-j}{(\alpha-j\omega)^2} & \omega<0 \end{cases}$$

۵- تبدیل فوریه معکوس:

الف) $F(\omega)=\dfrac{k^2}{k^2+\omega^2}$ $\qquad * \; e^{-k|t|} \xrightarrow{F} \dfrac{2k}{k^2+\omega^2}$

$$F(\omega)=\frac{k\times k}{k^2+\omega^2} \xrightarrow{F^{-1}} \frac{k}{2}e^{-k|t|} \quad \checkmark$$

ب) $$F(\omega)=\frac{\sin\omega T\cos\omega T}{\omega} \Longrightarrow F(\omega)=\frac{\sin 2\omega T}{2\omega} \xrightarrow{\times T/T} \frac{T\sin 2\omega T}{2\omega T}=T\,\mathrm{sinc}\,2\omega T$$

$$\xrightarrow{F^{-1}} f(t)=\Pi\left(\frac{t}{T}\right)$$

ج) $F(\omega)=\dfrac{1}{(j\omega+k)^3}$ $\qquad * \; t^2e^{-kt}u(t) \xrightarrow{F} \dfrac{2}{(k+j\omega)^3}$

$$\xrightarrow{F^{-1}} f(t)=\frac{t^2}{2}e^{-kt}u(t)$$

د) $$F(\omega)=\frac{2\pi}{1+2j}(j\omega)\left[\delta(\omega+1)+\delta(\omega-1)\right]$$

$\cos\omega_0 t=\pi\left[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\right]$

$$\xrightarrow{F^{-1}} f(t)=\frac{2}{1+2j}\,\frac{d\cos t}{dt}=\frac{-2\sin t}{1+2j}$$

$\dfrac{dx(t)}{dt} \xrightarrow{F} j\omega F(\omega)$

۶-

$$\frac{1}{\alpha^2-1-2j\alpha}=\int_0^{\infty} f(t)e^{-j\alpha t}\,dt \longrightarrow f(t)=?$$

$* \; te^{-\alpha t}u(t) \xrightarrow{F} \dfrac{1}{(\alpha+j\omega)^2}$

$$F(\omega)=\frac{1}{\alpha^2-1-2j\alpha} \xrightarrow{F^{-1}} f(t)=te^{-\alpha t}u(t)$$

$\alpha^2+2\alpha j\omega-\omega^2$

$\omega=-1 \rightarrow \alpha^2-2j\alpha-1$