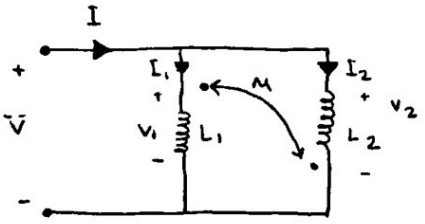
یا لطیف

رضا ادیب پور
۹۸۱۴۳۰۳
تمرینات فصل اول مدار ۲ - تراشس و تزویج

#1



$$\begin{cases} I = I_1 + I_2 \longrightarrow \dfrac{dI}{dt} = \dfrac{dI_1}{dt} + \dfrac{dI_2}{dt} \quad (I) \\ \bar{V} = \bar{V}_1 = \bar{V}_2 \end{cases}$$

$$\begin{cases} \bar{V}_1 = L_1 \dfrac{dI_1}{dt} - M \dfrac{dI_2}{dt} \\ \bar{V}_2 = L_2 \dfrac{dI_2}{dt} - M \dfrac{dI_1}{dt} \end{cases} \xrightarrow{V_1 = V_2} L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt} = L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt}$$

$$\Rightarrow \frac{dI_1}{dt}(L_1 + M) = \frac{dI_2}{dt}(L_2 + M)$$

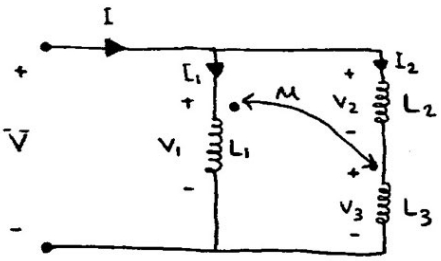
$$\Rightarrow \begin{cases} \dfrac{dI_1}{dt} = \dfrac{dI_2}{dt}\left(\dfrac{L_2+M}{L_1+M}\right) \\ \dfrac{dI_2}{dt} = \dfrac{dI_1}{dt}\left(\dfrac{L_1+M}{L_2+M}\right) \end{cases} \xrightarrow{(I)} \begin{cases} \dfrac{dI}{dt} = \dfrac{dI_2}{dt} + \dfrac{dI_2}{dt}\left(\dfrac{L_2+M}{L_1+M}\right) = \dfrac{dI_2}{dt}\left(\dfrac{L_1+L_2+2M}{L_1+M}\right) \\ \dfrac{dI}{dt} = \dfrac{dI_1}{dt} + \dfrac{dI_1}{dt}\left(\dfrac{L_1+M}{L_2+M}\right) = \dfrac{dI_1}{dt}\left(\dfrac{L_1+L_2+2M}{L_2+M}\right) \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{dI_2}{dt} = \dfrac{dI_s}{dt}\left(\dfrac{L_1+M}{L_1+L_2+2M}\right) \; * \\ \dfrac{dI_1}{dt} = \dfrac{dI}{dt}\left(\dfrac{L_2+M}{L_1+L_2+2M}\right) \; ** \end{cases} \Longrightarrow \bar{V} = L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt} \xrightarrow{*\,,\,**}$$

$$L_1 \frac{dI_s}{dt}\left(\frac{L_2+M}{L_1+L_2+2M}\right) - M \frac{dI}{dt}\left(\frac{L_1+M}{L_1+L_2+2M}\right) = \frac{dI}{dt}\left(\frac{L_1L_2 + \cancel{L_1M} - M^2 - \cancel{L_1M}}{L_1+L_2+2M}\right)$$

$$\Rightarrow \bar{V} = \frac{dI}{dt}\underbrace{\left(\frac{L_1L_2 - M^2}{L_1+L_2+2M}\right)}_{= L_{eq}}$$

#2



$$\begin{cases} V = V_1 = V_2 + V_3 & (I) \\ I = I_1 + I_2 \Rightarrow \dfrac{dI}{dt} = \dfrac{dI_1}{dt} + \dfrac{dI_2}{dt} & (II) \end{cases}$$

$$\begin{cases} V_1 = L_1 \dfrac{dI_1}{dt} + M \dfrac{dI_2}{dt} \\ V_2 = L_2 \dfrac{dI_2}{dt}, \quad V_3 = L_3 \dfrac{dI_2}{dt} + M \dfrac{dI_1}{dt} \end{cases}$$

$$\xrightarrow{(I)} L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} = L_2 \frac{dI_2}{dt} + L_3 \frac{dI_2}{dt} + M \frac{dI_1}{dt}$$

$$\Rightarrow \frac{dI_1}{dt}(L_1 - M) = \frac{dI_2}{dt}(L_2 + L_3 - M) \Rightarrow \begin{cases} \dfrac{dI_1}{dt} = \dfrac{dI_2}{dt}\left(\dfrac{L_2 + L_3 - M}{L_1 - M}\right) \\ \dfrac{dI_2}{dt} = \dfrac{dI_1}{dt}\left(\dfrac{L_1 - M}{L_2 + L_3 - M}\right) \end{cases}$$

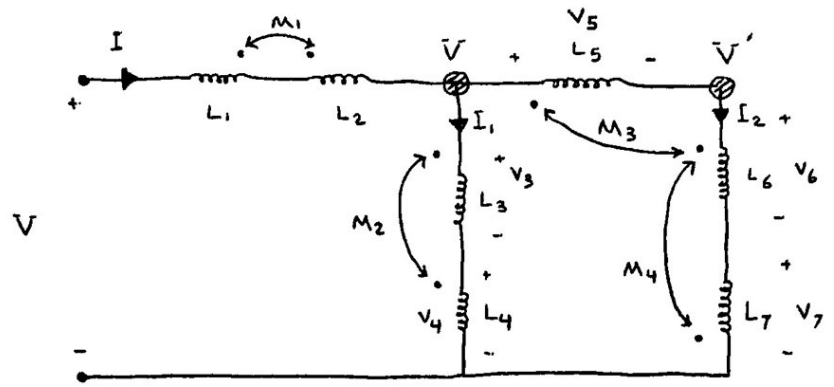
$$\xrightarrow{(II)} \begin{cases} \dfrac{dI}{dt} = \dfrac{dI_2}{dt} + \dfrac{dI_2}{dt}\left(\dfrac{L_2 + L_3 - M}{L_1 - M}\right) = \dfrac{dI_2}{dt}\left(\dfrac{L_1 + L_2 + L_3 - 2M}{L_1 - M}\right) \\ \dfrac{dI}{dt} = \dfrac{dI_1}{dt} + \dfrac{dI_1}{dt}\left(\dfrac{L_1 - M}{L_2 + L_3 - M}\right) = \dfrac{dI_1}{dt}\left(\dfrac{L_1 + L_2 + L_3 - 2M}{L_2 + L_3 - M}\right) \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{dI_1}{dt} = \dfrac{dI}{dt}\left(\dfrac{L_2 + L_3 - M}{L_1 + L_2 + L_3 - 2M}\right) * \\ \dfrac{dI_2}{dt} = \dfrac{dI}{dt}\left(\dfrac{L_1 - M}{L_1 + L_2 + L_3 - 2M}\right) ** \end{cases} \Rightarrow V = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt} \xrightarrow{*,\ **}$$

$$V = L_1 \frac{dI}{dt}\left(\frac{L_2 + L_3 - M}{L_1 + L_2 + L_3 - 2M}\right) + M \frac{dI}{dt}\left(\frac{L_1 - M}{L_1 + L_2 + L_3 - 2M}\right)$$

$$= \frac{dI_1}{dt}\left(\frac{L_1L_2 + L_1L_3 - \cancel{L_1M} + \cancel{ML_1} - M^2}{L_1 + L_2 + L_3 - 2M}\right) \Rightarrow V = \underbrace{\frac{dI}{dt}\left(\frac{L_1L_2 + L_1L_3 - M^2}{L_1 + L_2 + L_3 - 2M}\right)}_{= L_{eq}}$$

#3



$$V_3 = L_3 \frac{dI_1}{dt} + M_2 \frac{dI_1}{dt} \quad , \quad V_4 = L_4 \frac{dI_1}{dt} + M_2 \frac{dI_1}{dt}$$

$$V_6 = L_6 \frac{dI_2}{dt} + M_3 \frac{dI_2}{dt} \quad , \quad V_7 = L_7 \frac{dI_2}{dt} - M_4 \frac{dI_2}{dt}$$

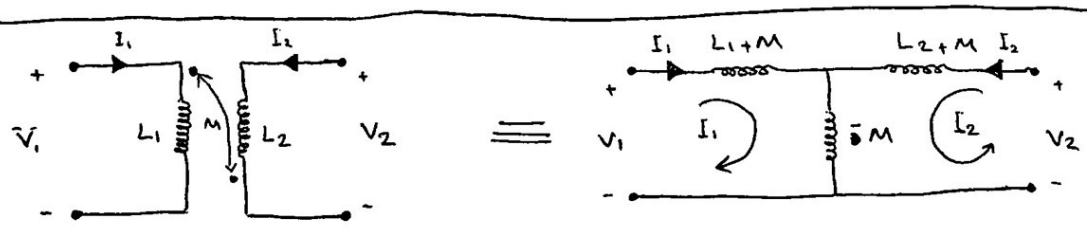
$$\Rightarrow V = V_3 + V_4 = \frac{dI_1}{dt}\left(L_3 + M_2 + L_4 + M_2\right) = \frac{dI_1}{dt}\left(L_3 + L_4 + 2M_2\right)$$

$$V' = V_6 + V_7 = \frac{dI_2}{dt}\left(L_6 + L_7 + M_3 - M_4\right) \longrightarrow I = I_1 + I_2 \xrightarrow{I=0} I_1 = -I_2$$

$$\Rightarrow V_5 = V - V' = \frac{dI_1}{dt}\left(L_3 + L_4 + 2M_2\right) - \frac{dI_1}{dt}\left(M_4 - M_3 - L_6 - L_7\right) =$$

$$= \frac{dI_1}{dt}\underbrace{\left(L_3 + L_4 + 2M_2 - M_4 + M_3 + L_6 + L_7\right)}_{L_{eq}}$$

#4



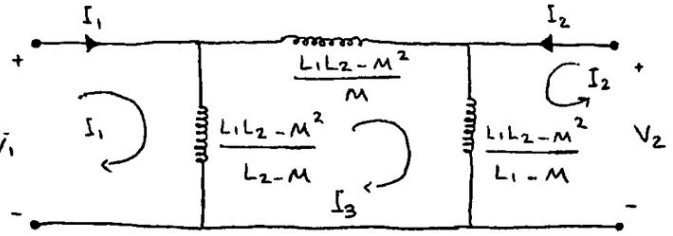
KVL in I_1 : $-V_1 + (L_1 + M)\frac{dI_1}{dt} + (-M)\frac{d}{dt}(I_1 + I_2) = 0$

$$\Rightarrow V_1 = L_1 \frac{dI_1}{dt} + M \cancel{\frac{dI_1}{dt}} - M \cancel{\frac{dI_1}{dt}} - M \frac{dI_2}{dt} \quad \Rightarrow \boxed{V_1 = L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt}}$$

KVL in I_2 : $-V_2 + (L_2 + M)\frac{dI_2}{dt} + (-M)\frac{d}{dt}(I_2 + I_1) = 0$

$$\Rightarrow V_2 = L_2 \frac{dI_2}{dt} + M \cancel{\frac{dI_2}{dt}} - M \cancel{\frac{dI_2}{dt}} - M \frac{dI_1}{dt} \quad \Rightarrow \boxed{V_2 = L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt}}$$

4



KVL in I_1 : $-V_1 + \left(\frac{L_1L_2-M^2}{L_2-M}\right)\frac{d}{dt}(I_1-I_3) = 0$ ⟹ (I)

KVL in I_2 : $-V_2 + \left(\frac{L_1L_2-M^2}{L_1-M}\right)\frac{d}{dt}(I_2+I_3) = 0$ (II)

KVL in I_3: $\frac{L_1L_2-M^2}{M}\cdot\frac{dI_3}{dt} + \frac{L_1L_2-M^2}{L_1-M}\left(\frac{d}{dt}(I_3+I_2)\right) + \frac{L_1L_2-M^2}{L_2-M}\left(\frac{d}{dt}(I_3-I_1)\right) = 0$

$$\Rightarrow \frac{dI_3}{dt}\left(\frac{L_1L_2-M^2}{M} + \frac{L_1L_2-M^2}{L_1-M} + \frac{L_1L_2-M^2}{L_2-M}\right) = \frac{L_1L_2-M^2}{L_2-M}\cdot\frac{dI_1}{dt} - \frac{L_1L_2-M^2}{L_1-M}\cdot\frac{dI_2}{dt}$$

$$\Rightarrow \frac{dI_3}{dt} = \frac{\frac{L_1L_2-M^2}{L_2-M}\cdot\frac{dI_1}{dt} - \frac{L_1L_2-M^2}{L_1-M}\cdot\frac{dI_2}{dt}}{\frac{L_1^2L_2^2 - 2M^2L_1L_2 + M^4}{M(L_1-M)(L_2-M)}} = \frac{M(L_1-M)}{L_1L_2-M^2}\cdot\frac{dI_1}{dt} - \frac{M(L_2-M)}{L_1L_2-M^2}\cdot\frac{dI_2}{dt}$$

$$\Rightarrow \frac{dI_3}{dt} = \frac{1}{L_1L_2-M^2}\left(M(L_1-M)\frac{dI_1}{dt} - M(L_2-M)\frac{dI_2}{dt}\right) \quad \text{(III)}$$

(III) in (I) ⟶ $V_1 = \frac{L_1L_2-M^2}{L_2-M}\cdot\frac{dI_1}{dt} - \frac{dI_3}{dt}\left(\frac{L_1L_2-M^2}{L_2-M}\right)$

$$\Rightarrow V_1 = \frac{L_1L_2-M^2}{L_2-M}\cdot\frac{dI_1}{dt} + \frac{-1}{L_1L_2-M^2}\left(M(L_1-M)\frac{dI_1}{dt} - M(L_2-M)\frac{dI_2}{dt}\right)\cdot\frac{L_1L_2-M^2}{L_2-M}$$

$$\Rightarrow V_1 = \frac{dI_1}{dt}\underbrace{\left(\frac{L_1L_2-M^2}{L_2-M} - \frac{M(L_1-M)}{L_2-M}\right)}_{=L_1} + \frac{dI_2}{dt}\underbrace{\left(\frac{M(L_2-M)}{\cancel{L_1L_2-M^2}}\cdot\frac{\cancel{L_1L_2-M^2}}{L_2-M}\right)}_{=M}$$

$$\Rightarrow \boxed{V_1 = L_1\frac{dI_1}{dt} + M\frac{dI_2}{dt}}$$

برای V_2 هم دقیقاً به همین صورت، (III) را در (I) جایگذاری می‌کنیم و V_2 به صورت مقابل به دست می‌آید:

$$V_2 = L_2\frac{dI_2}{dt} + M\frac{dI_1}{dt}$$