

#1

$$A = \begin{bmatrix} -3 & -1 \\ -2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ -2 & 2 \\ 1 & -1 \end{bmatrix}$$

$$D = 0 \quad x(0^-) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad w_1(t) = u(t), \quad w_2(t) = e^{-t} u(t)$$

$$\frac{dx(t)}{dt} - Ax(t) = Bw_1(t) \xrightarrow{t>0} \frac{dx(t)}{dt} - Ax(t) = B \rightarrow x(t) = x_p(t) + x_g(t)$$

$$\Rightarrow x(t) = \underbrace{\left(x(0) + \frac{B}{A} \right) e^{-A(t-0)}}_{x_g(t)} + \underbrace{\frac{B}{A}}_{x_p(t)} = \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}}{\begin{bmatrix} -3 & -1 \\ -2 & -1 \end{bmatrix}} \right) e^{-At} - \frac{\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}}{\begin{bmatrix} -3 & -1 \\ -2 & -1 \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} 2 & -2 \\ -1 & -2 \end{bmatrix} e^{-At} - \begin{bmatrix} 1 & -3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2e^{-3t} & -2e^{-2t} \\ -e^{-2t} & -2e^{-t} \end{bmatrix} - \begin{bmatrix} 1 & -3 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{-3t} - 1 & -2e^{-2t} + 3 \\ -e^{-2t} & -2e^{-t} + 1 \end{bmatrix} \quad \text{for } t > 0$$

$$\frac{dx(t)}{dt} - Ax(t) = Bw_2(t) \xrightarrow{t>0} \frac{dx(t)}{dt} - Ax(t) = Be^{-t} \rightarrow x(t) = x_p(t) + x_g(t)$$

$$x_g(t) = (x(0)) e^{-At}, \quad x_p(t) = Ae^{-t} \Rightarrow x(t) = x(0)e^{-At} + Ae^{-t}$$

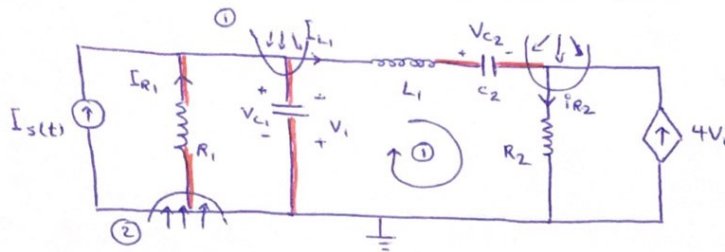
$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-At} + \begin{bmatrix} -3 & -1 \\ -2 & -1 \end{bmatrix} e^{-t} = \begin{bmatrix} e^{3t} & e^t \\ -e^{2t} & -e^t \end{bmatrix} + \begin{bmatrix} -3e^{-t} & -e^{-t} \\ -2e^{-t} & -e^{-t} \end{bmatrix} = \begin{bmatrix} e^{3t} - 3e^{-t} & e^t - e^{-t} \\ -e^{2t} - 2e^{-t} & -e^t - e^{-t} \end{bmatrix}$$

$$H(s) = \frac{Y(s)}{W(s)} \Big|_{x(0)=0} = C(SI - A)^{-1}B + D = \begin{bmatrix} 1 & 2 \\ -2 & 2 \\ 1 & -1 \end{bmatrix} \cdot \left(\begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -3 & -1 \\ -2 & -1 \end{bmatrix} \right)^{-1}$$

$$\times \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} S+7 & 2S+3 \\ -2S-2 & 2S \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 8+10 & 2S+85 \\ -2S & \end{bmatrix}$$

$$= \begin{bmatrix} 7S+27 & 4S+17 \\ -2S-6 & -2S-4 \\ S+3 & S+2 \end{bmatrix}$$

#2



3 سازه $\rightarrow \begin{bmatrix} V_{C1} \\ V_{C2} \\ i_{L1} \end{bmatrix}$ متغیرهای حالت

$$V_{C1} \rightarrow \text{درخت} \rightarrow \text{حلقه اول: } \textcircled{1}; C_1 \frac{dV_{C1}}{dt} + i_{L1} - i_{R1} - I_s = 0 \Rightarrow \frac{dV_{C1}}{dt} = \frac{I_s}{C_1} - \frac{i_{L1}}{C_1} + \frac{i_{R1}}{C_1}$$

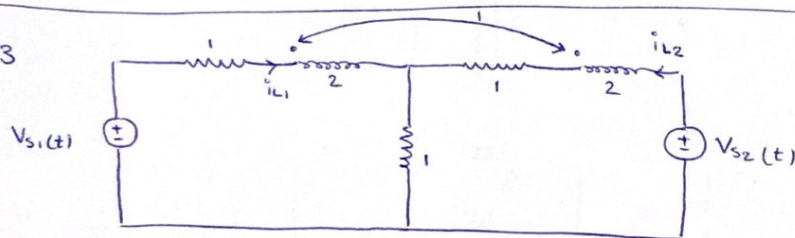
$$\textcircled{2} \rightarrow +I_s + i_{R1} = 0 \rightarrow I_s = -i_{R1} \longrightarrow \frac{dV_{C1}}{dt} = \frac{I_s}{C_1} - \frac{i_{L1}}{C_1} - \frac{I_s}{C_1} \Rightarrow \frac{dV_{C1}}{dt} = -\frac{i_{L1}}{C_1}$$

$$V_{C2} \rightarrow \text{درخت} \rightarrow \text{حلقه دوم: } -C_2 \frac{dV_{C2}}{dt} + i_{R2} + 4V_{C1} = 0 \Rightarrow \frac{dV_{C2}}{dt} = \frac{i_{R2}}{C_2} + 4 \frac{V_{C1}}{C_2}$$

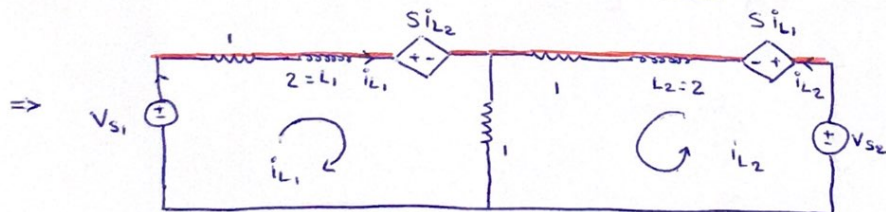
$$\text{حلقه اول: } R_2 i_{R2} - V_{C1} + L_1 \frac{di_{L1}}{dt} + V_{C2} = 0 \Rightarrow i_{R2} = \frac{1}{R_2} (V_{C1} - V_{C2} - L_1 \frac{di_{L1}}{dt})$$

$$\Rightarrow \frac{dV_{C2}}{dt} = \frac{1}{R_2 C_2} (V_{C1} - V_{C2} - L_1 \frac{di_{L1}}{dt}) + 4 \frac{V_{C1}}{C_2}$$

#3



2 سازه $\rightarrow \begin{bmatrix} i_{L1} \\ i_{L2} \end{bmatrix}$

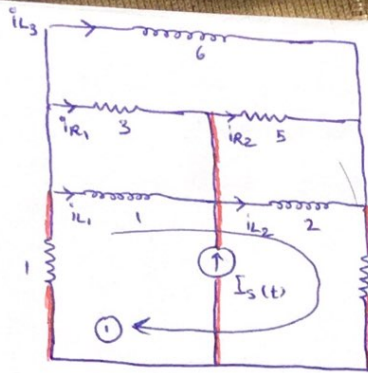


$$\text{KVL in } i_{L1}: -V_{S1} + i_{L1} + 2 \frac{di_{L1}}{dt} + S i_{L2} + (i_{L1} + i_{L2}) = 0 \Rightarrow 2 \frac{di_{L1}}{dt} = V_{S1} - S i_{L2} - 2 i_{L1} - i_{L2}$$

$$\Rightarrow \frac{di_{L1}}{dt} = \frac{1}{2} (V_{S1} - i_{L2}(S+1) - 2 i_{L1})$$

$$\text{KVL in } i_{L2}: S i_{L1} + 2 \frac{di_{L2}}{dt} + i_{L2} + (i_{L2} + i_{L1}) - V_{S2} = 0 \Rightarrow \frac{di_{L2}}{dt} = \frac{1}{2} (V_{S2} - 2 i_{L2} - i_{L1}(S+1))$$

#4



5 4 2 1
 متغيرات → 3 → متغيرات: 3 → $\begin{bmatrix} i_{L1} \\ i_{L2} \\ i_{L3} \end{bmatrix}$

KVL in ①: $i_{L1} + \frac{d(i_{L1} - i_{R1})}{dt} + 2 \frac{d(i_{L2} - i_{R2})}{dt} + i_{L2} = 0$

$$\Rightarrow \frac{di_{L1}}{dt} - \frac{di_{R1}}{dt} + 2 \frac{di_{L2}}{dt} - 2 \frac{di_{R2}}{dt} + i_{R2} = 0$$

$$\boxed{i_{L2} - i_{L1} = I_s(t)}$$

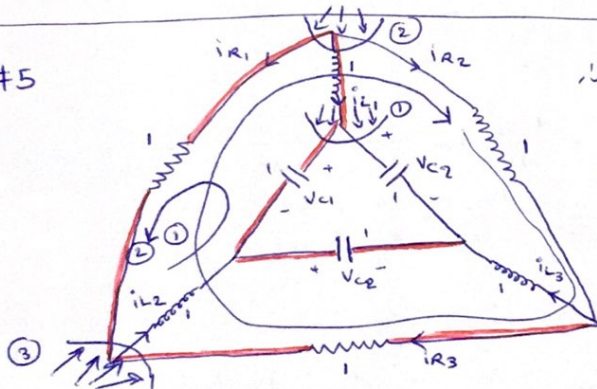
KVL in i_{R1} : $3(i_{R1} - i_{L3}) + \frac{d(i_{R1} - i_{L1})}{dt} = 0 \Rightarrow \frac{di_{R1}}{dt} + 3i_{R1} = 3i_{L3} + \frac{di_{L1}}{dt}$

KVL in i_{R2} : $5(i_{R2} - i_{L3}) + 2 \frac{d(i_{R2} - i_{L2})}{dt} = 0 \Rightarrow 2 \frac{di_{R2}}{dt} + 5i_{R2} = 5i_{L3} + 2 \frac{di_{L2}}{dt}$

KVL in i_{L3} : $6 \frac{di_{L3}}{dt} + 5(i_{L3} - i_{R2}) + 3(i_{L3} - i_{R1}) = 0 \Rightarrow 6 \frac{di_{L3}}{dt} = -5i_{L3} + 5i_{R2} - 3i_{L3} + 3i_{R1}$

$$\Rightarrow \frac{di_{L3}}{dt} = -\frac{4}{3}i_{L3} + 5i_{R2} + 3i_{R1}$$

#5



متغيرات → 4 → متغيرات: 4 → $\begin{bmatrix} V_{C1} \\ V_{C2} \\ i_{L1} \\ i_{L2} \end{bmatrix}$

KVL in ①: $\frac{dV_{C1}}{dt} + \frac{dV_{C2}}{dt} + i_{L1} = 0$

$$\Rightarrow \frac{dV_{C1}}{dt} = -\frac{dV_{C2}}{dt} - i_{L1}$$

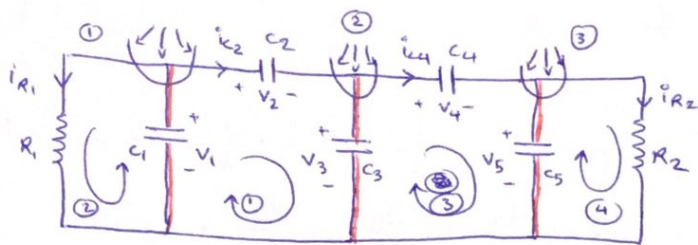
②: $i_{R1} + i_{R2} + i_{L1} = 0 \Rightarrow i_{L1} = -(i_{R1} + i_{R2})$

KVL in ①: $i_{R2} + i_{R3} - i_{R1} = 0$

③: $-i_{R1} + i_{L2} - i_{R3} = 0 \Rightarrow$

KVL in ②: $\frac{di_{L2}}{dt} - V_{C1} - \frac{di_{L1}}{dt} + i_{R1} = 0 \Rightarrow \frac{di_{L2}}{dt} = V_{C1} + \frac{di_{L1}}{dt} - i_{R1}$

#6

مستوی مدار $\rightarrow 3$ تعداد حالات $\rightarrow 3 \rightarrow$

$$\begin{bmatrix} V_{C1} \\ V_{C3} \\ V_{C5} \end{bmatrix}$$

$$\textcircled{1}: i_{R1} + C_1 \frac{dV_{C1}}{dt} + i_{C2} = 0 \Rightarrow \frac{dV_{C1}}{dt} = -\frac{1}{C_1} i_{R1} - \frac{i_{C2}}{C_1}$$

$$\text{KVL in } \textcircled{1}: +V_{C2} + V_{C3} - V_{C1} = 0 \Rightarrow \frac{dV_{C2}}{dt} + \frac{dV_{C3}}{dt} - \frac{dV_{C1}}{dt} = 0 \Rightarrow \frac{i_{C2}}{C_2} = \frac{dV_{C1}}{dt} - \frac{dV_{C3}}{dt}$$

$$\Rightarrow i_{C2} = C_2 \frac{dV_{C1}}{dt} - C_2 \frac{dV_{C3}}{dt}$$

$$\text{KVL in } \textcircled{2}: R_1 i_{R1} - V_{C1} = 0 \Rightarrow i_{R1} = \frac{V_{C1}}{R_1} \Rightarrow \frac{dV_{C1}}{dt} = \frac{1}{C_1} \left(\frac{V_{C1}}{R_1} \right) - \frac{1}{C_1} \left(C_2 \frac{dV_{C1}}{dt} - C_2 \frac{dV_{C3}}{dt} \right)$$

$$\Rightarrow \frac{dV_{C1}}{dt} = \frac{-V_{C1}}{C_1 R_1} - \frac{C_2}{C_1} \left(\frac{dV_{C1}}{dt} + \frac{dV_{C3}}{dt} \right)$$

$$\text{KCL in } \textcircled{2}: -i_{C2} + C_3 \frac{dV_{C3}}{dt} + i_{C4} = 0 \Rightarrow \frac{dV_{C3}}{dt} = \frac{i_{C2}}{C_3} - \frac{i_{C4}}{C_3}$$

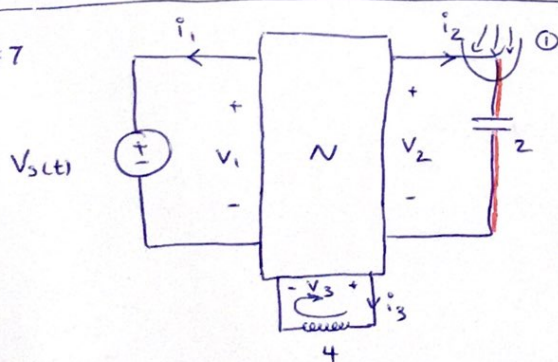
$$\text{KVL in } \textcircled{3}: +V_{C4} + V_{C5} - V_{C3} = 0 \Rightarrow \frac{dV_{C4}}{dt} + \frac{dV_{C5}}{dt} = \frac{dV_{C3}}{dt} \Rightarrow \frac{i_{C4}}{C_4} + \frac{dV_{C5}}{dt} = \frac{dV_{C3}}{dt}$$

$$\Rightarrow i_{C4} = C_4 \frac{dV_{C3}}{dt} - C_4 \frac{dV_{C5}}{dt} \Rightarrow \frac{dV_{C3}}{dt} = \frac{C_2}{C_3} \left(\frac{dV_{C1}}{dt} - \frac{dV_{C3}}{dt} \right) - \frac{C_4}{C_3} \left(\frac{dV_{C3}}{dt} + \frac{dV_{C5}}{dt} \right)$$

$$\text{KCL in } \textcircled{3}: -i_{C4} + C_5 \frac{dV_{C5}}{dt} + i_{R2} = 0 \Rightarrow \frac{dV_{C5}}{dt} = \frac{i_{C4}}{C_5} - \frac{i_{R2}}{C_5}$$

$$\text{KVL in } \textcircled{4}: R_2 i_{R2} - V_{C5} = 0 \Rightarrow i_{R2} = \frac{V_{C5}}{R_2} \Rightarrow \frac{dV_{C5}}{dt} = \frac{C_4}{C_5} \left(\frac{dV_{C3}}{dt} - \frac{dV_{C5}}{dt} \right) - \frac{V_{C5}}{C_5 R_2}$$

#7



$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

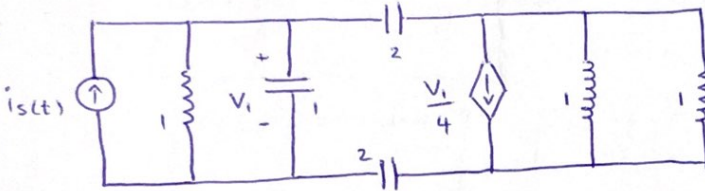
$$\text{مستوی مدار: } \begin{bmatrix} V_2 \\ i_3 \end{bmatrix}$$

$$\text{KCL in } \textcircled{1}: -i_2 + 2 \frac{dV_2}{dt} = 0 \Rightarrow \frac{dV_2}{dt} = \frac{i_2}{2} = \frac{1}{2} (V_3 - V_1) \Rightarrow \frac{dV_2}{dt} = 2 \frac{di_3}{dt} - \frac{V_1}{2}$$

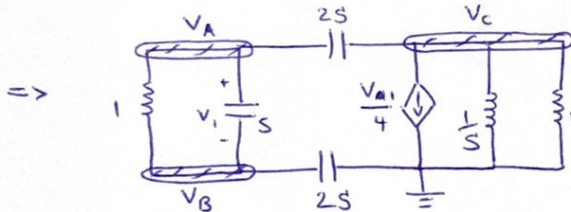
$$\text{KVL: } 4 \frac{di_3}{dt} - V_3 = 0 \Rightarrow 4 \frac{di_3}{dt} = V_3 \xrightarrow{V_3 = i_2 + V_1} \frac{di_3}{dt} = V_1 + i_2 \xrightarrow{i_2 = 2 \frac{dV_1}{dt}}$$

$$\frac{di_3}{dt} = V_1 + 2 \frac{dV_1}{dt}$$

#1



V_i : نمادهاش هر



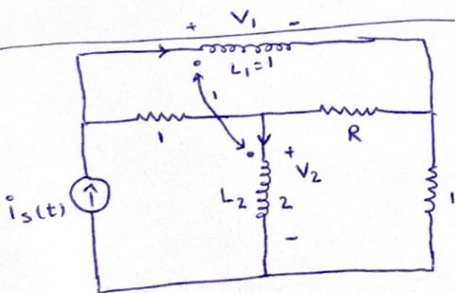
$$\Rightarrow \begin{bmatrix} 3S+1 & -1-S & -2S \\ -1-S & 3S+1 & 0 \\ -2S+\frac{1}{4} & 0-\frac{1}{4} & 2S+\frac{1}{S}+1 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ -\frac{V_i}{4} \end{bmatrix} \xrightarrow{V_i = (V_A - V_B)}$$

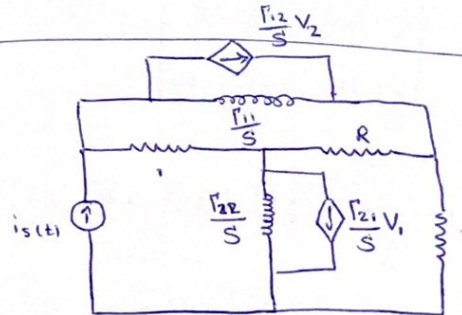
$$\det(Y_n) = 0 \rightarrow$$

$$\begin{cases} S_1 = -2 \\ S_{2,3} = -\frac{5}{8} \pm j\sqrt{\frac{7}{8}} \end{cases}$$

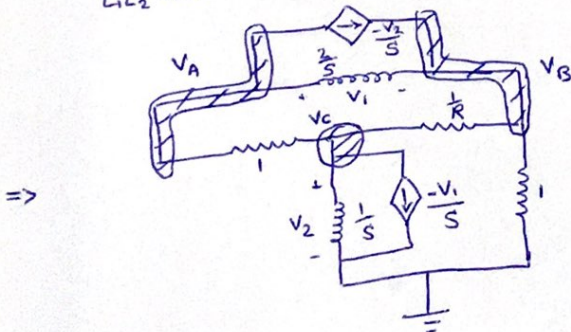
#2



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$$L^{-1} = \frac{1}{L_1 L_2 - M^2} \begin{bmatrix} L_2 & -M \\ -M & L_1 \end{bmatrix} = \frac{1}{2-1} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

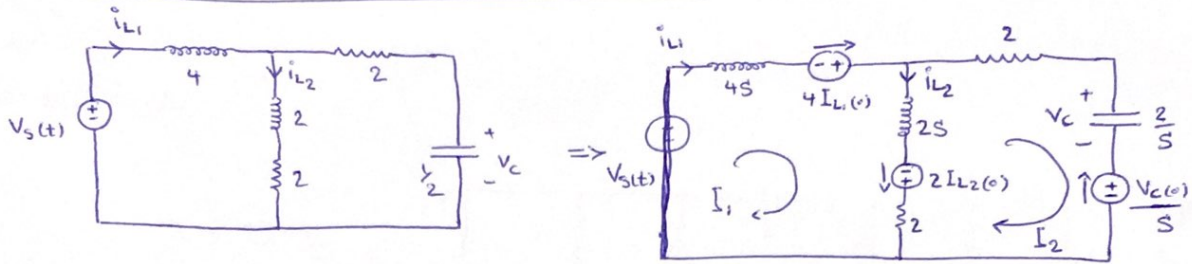


$$\Rightarrow \begin{bmatrix} 1+\frac{2}{S} & -\frac{2}{S} & -\frac{1}{S}-1 \\ -\frac{2}{S} & 1+\frac{1}{R}+\frac{2}{S} & \frac{1}{S}-\frac{1}{R} \\ -1-\frac{1}{S} & -\frac{1}{R}+\frac{1}{S} & 1+\frac{1}{R}+\frac{1}{S} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} \frac{V_2}{S} \\ -\frac{V_2}{S} \\ \frac{V_1}{S} \end{bmatrix}$$

$$V_2 = V_C, \quad V_i = V_A - V_B$$

$$\det(Y_n) = \frac{s^2 + 3s + sR + 2R + 1}{s^2 R} \xrightarrow{s=-3} R=1$$

#3



$$\begin{bmatrix} 4s + 2s + 2 & -2 - 2s \\ -2 - 2s & 2 + \frac{2}{s} + 2 + 2s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 4\dot{I}_{L1}(s) + 2\dot{I}_{L2}(s) \\ -\frac{V_c(s)}{s} - 2\dot{I}_{L2}(s) \end{bmatrix}$$

$$\det(Z_m) = 0 : \begin{cases} s_1 = -1 \rightarrow \text{کوتاه شدن} \\ s_2 = -\frac{1}{2} \rightarrow \text{باز شدن} \end{cases}$$

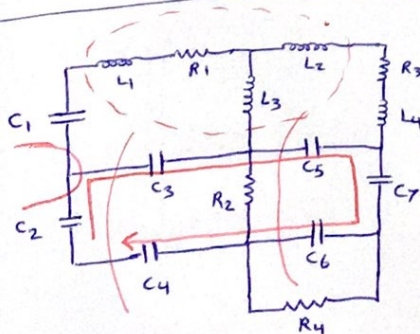
$$\Rightarrow I_1 = \frac{\begin{vmatrix} 4\dot{I}_{L1}(s) + 2\dot{I}_{L2}(s) & -2 - 2s \\ -\frac{V_c(s)}{s} - 2\dot{I}_{L2}(s) & 4 + \frac{2}{s} + 2s \end{vmatrix}}{\begin{vmatrix} 6s + 2 & -2 - 2s \\ -2 - 2s & 4 + \frac{2}{s} + 2s \end{vmatrix}} = \frac{(4 + \frac{2}{s} + 2s)(4\dot{I}_{L1}(s) + 2\dot{I}_{L2}(s)) - ((-2 - 2s)(-\frac{V_c(s)}{s} - 2\dot{I}_{L2}(s)))}{(s+1)(s+\frac{1}{2})}$$

$$s = -\frac{1}{2} \rightarrow (4 + \frac{2}{-\frac{1}{2}} + 2(-\frac{1}{2})) \cdot (4\dot{I}_{L1}(s) + 2\dot{I}_{L2}(s)) - ((-2(-\frac{1}{2}) - 2) \times (\frac{V_c(s)}{-\frac{1}{2}} - 2\dot{I}_{L2}(s)))$$

$$\Rightarrow -4\dot{I}_{L1}(s) - 2\dot{I}_{L2}(s) - 2V_c(s) - 2\dot{I}_{L2}(s) = 0 \Rightarrow -4\dot{I}_{L1}(s) - 4\dot{I}_{L2}(s) = 2V_c(s)$$

$$\Rightarrow V_c(s) = -(2\dot{I}_{L1}(s) + 2\dot{I}_{L2}(s))$$

#4



$$\text{تعداد فرکانس ها} : 11 - 1 - 1 = 9 = \text{مقادیر طبیعی}$$

$$\text{تعداد فرکانس ها} : 3 \text{ تا با 3 بیت} \rightarrow \text{مقادیر طبیعی} \rightarrow \text{مقادیر طبیعی}$$

$$\text{تعداد فرکانس ها} : 9 - 3 = 6$$