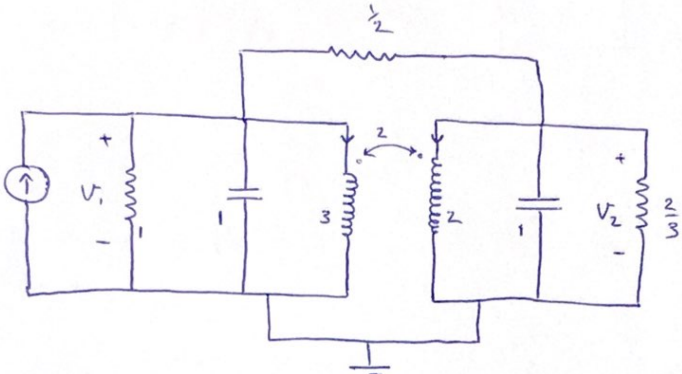
با لطیف

رضا آدینه پور
۹۸۱۴۳۰۳
تمرینات تابع شبکه مدار ۲

#1

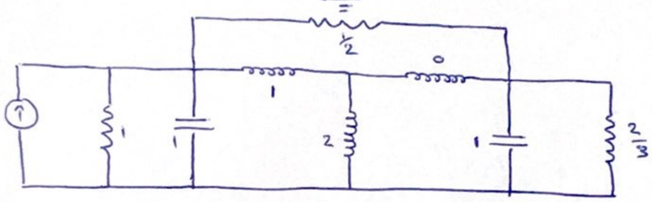


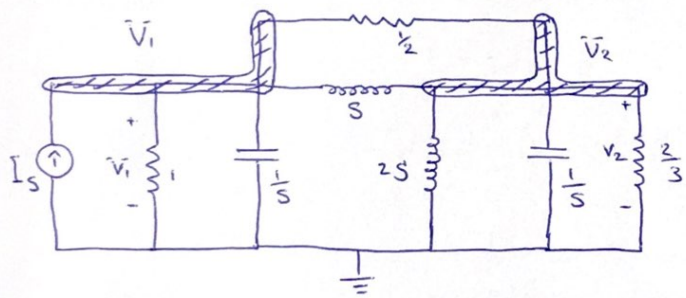
$H_1 = \frac{\bar V_1}{I_s} = ?$

$H_2 = \frac{\bar V_2}{I_s} = ?$

$H_3 = \frac{\bar V_2}{\bar V_1} = ?$

مدار معادل T :





KCL in V_1 : $-I_s + \bar V_1 + S\bar V_1 + \frac{V_1 - V_2}{S} + 2(V_1 - V_2) = 0$

$\Rightarrow V_1\left(3 + S + \frac1S\right) - V_2\left(\frac1S - 2\right) = I_s$ (II)

KCL in V_2 : $\frac32 V_2 + SV_2 + \frac{V_2 - V_1}{S} + \frac{V_2}{2S} + 2(V_2 - V_1) = 0$

$\Rightarrow V_2\left(\frac72 + S + \frac1S + \frac1{2S}\right) = V_1\left(2 + \frac1S\right)$

$\Rightarrow V_2 = \frac{2 + \frac1S}{\frac72 + S + \frac1S + \frac1{2S}} V_1 = \frac{2}{S+3} V_1$ (I)

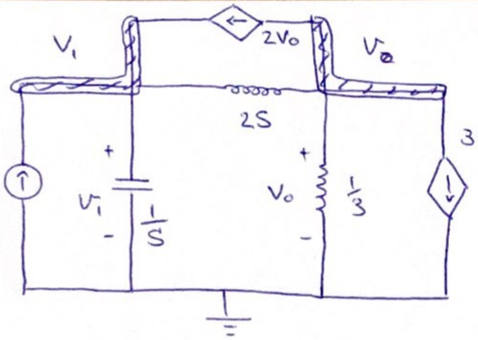
$\xrightarrow{(I) \text{ in } (II)} V_1\left(3 + S + \frac1S\right) - \frac{2}{S+3} V_1\left(\frac1S - 2\right) = I_s \Rightarrow \frac{V_1}{I_s} = \boxed{H_1 = \frac{S(S+3)}{S^3 + 6S^2 + 14S + 1}}$

$\Rightarrow \bar V_1 = \frac{\frac72 + S + \frac1S + \frac1{2S}}{2 + \frac1S} V_2 = \frac{S+3}{2} V_2$ (III) $\xrightarrow{\text{in } (II)} V_2\left(\frac{S+3}{2}\right)\left(3 + S + \frac1S\right) + V_2\left(2 - \frac1S\right)$

$\Rightarrow \boxed{H_2 = \frac{V_2}{I_s} = \frac{2S}{S^3 + 6S^2 + 14S + 1}}$, $H_3 = \frac{V_1}{V_2} = \frac{\frac72 + S + \frac1S + \frac1{2S}}{2 + \frac1S} = \frac{S+3}{2}$

$$\Rightarrow \underbrace{\begin{bmatrix} S + 1 + 2 + \frac1S & -2 - \frac1S \\ -2 - \frac1S & S + 2 + \frac32 + \frac1{2S} + \frac1S \end{bmatrix}}_{(I)} \Rightarrow \det\{I\} = 0 \Rightarrow \begin{cases} S = -1 \\ S = -\frac12 \\ S = -\frac52 + \sqrt{21} \\ S = -\frac52 - \sqrt{21} \end{cases}$$

#2



$$H = \frac{V_o}{I_s} = ?$$

$$\text{kcl in } V_1: -I_s + sV_1 - 2V_o + \frac{V_1 - V_o}{2s} = 0$$

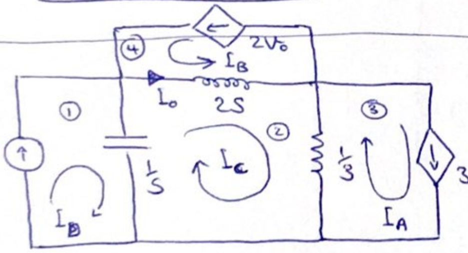
$$\Rightarrow V_1\left(s + \frac{1}{2s}\right) - V_o\left(2 + \frac{1}{2s}\right) = I_s \qquad (I)$$

$$\text{kcl in } V_o: 3V_o + 3V_1 + \frac{V_o - V_1}{2s} + 2V_o = 0 \Rightarrow V_o\left(3 + \frac{1}{2s} + 2\right) = V_1\left(-3 + \frac{1}{2s}\right)$$

$$\Rightarrow V_1 = \frac{5 + \frac{1}{2s}}{-3 + \frac{1}{2s}} V_o = \frac{10s+1}{-6s+1} V_o \quad (II) \xrightarrow{\text{in } (I)} \left(\frac{10s+1}{-6s+1}\right)\left(s + \frac{1}{2s}\right)V_o + V_o\left(-2 - \frac{1}{2s}\right) = I_s$$

$$\Rightarrow \frac{V_o}{I_s} = \boxed{H = \frac{-6s+1}{10s^2 + 13s + 6}}$$

#3



$$H = \frac{I_o}{I_s} = ?$$

$$\text{KVL in 2: } 2s(I_C + I_B) + \frac{1}{3}(I_C - I_A) + \frac{1}{s}(I_C - I_D) = 0$$

$$\frac{I_D = I_s,\ I_B = 2V_o = \frac{2}{3}(I_C - I_A)}{I_A = 3V_1 = \frac{3}{s}(I_D - I_C)} \longrightarrow 2s\left(I_C + \frac{2}{3}(I_C - I_A)\right) + \frac{1}{3}\left(I_C - \frac{3}{s}(I_D - I_C)\right) + \frac{1}{s}(I_C - I_D)$$

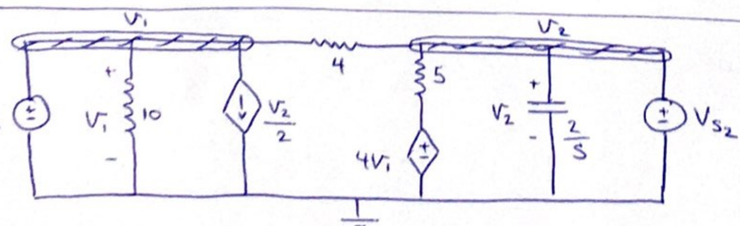
$$\Rightarrow 2sI_C + \frac{4}{3}sI_C - \frac{4}{3}sI_A + \frac{1}{3}I_C - \frac{1}{s}I_D + \frac{1}{s}I_C + \frac{1}{s}I_C - \frac{1}{s}I_D = 0$$

$$I_C\left(2s + \frac{4}{3}s + \frac{1}{3} + \frac{1}{s} + \frac{1}{s}\right) - I_s\left(\frac{1}{s} + \frac{1}{s}\right) = \frac{4}{3}s\left(\frac{3}{s}(I_D - I_C)\right)$$

$$I_C\left(2s + \frac{4}{3}s + \frac{1}{3} + \frac{1}{s} + \frac{1}{s} + 12\right) = I_s\left(\frac{1}{s} + \frac{1}{s} + 4\right) \xrightarrow{I_C = I_o}$$

$$H = \frac{I_o}{I_s} = \frac{3(2+4s)}{6s^2 + 6 + 49s}$$

#4



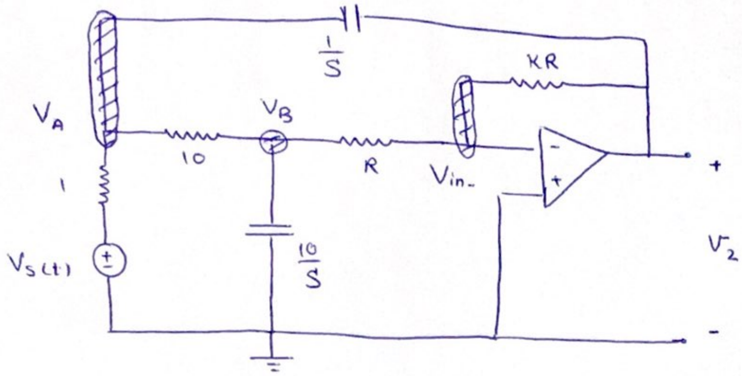
$$H = \frac{V_2}{V_1} = ?$$

$$V_1 = V_{s1}$$

$$V_2 = V_{s2}$$

$$V_{s1} = \frac{V_1}{10}, \quad V_{s2} = \frac{V_2}{\frac{2}{s}} = \frac{s}{2}V_2 \Rightarrow H = \frac{V_2}{V_1} = \frac{\frac{V_1}{10}}{\frac{s}{2}V_2} = \boxed{\frac{2}{10s}} = \frac{1}{5s}$$

#5



$H = \frac{V_2}{V_S}$

$V_{in-} = V_{in+} = 0$

KCL in V_A: $\frac{V_A - V_2}{\frac{1}{S}} + \frac{V_A - V_B}{10} + \frac{V_A - V_S}{1} = 0 \Rightarrow V_A\left(S + \frac{11}{10}\right) - SV_2 - \frac{V_B}{10} = V_S$ (I)

KCL in V_B: $\frac{V_B - V_A}{10} + \frac{V_B - \cancel{V_{in-}}^{\,0}}{R} + \frac{S}{10}V_B = 0 \Rightarrow V_A = V_B\left(1 + S - \frac{10}{R}\right)$ (II)

KCL in V_{in-}: $\frac{0 - V_B}{R} + \frac{0 - V_2}{KR} = 0 \Rightarrow \frac{V_B}{\cancel{R}} = \frac{V_2}{K\cancel{R}} \Rightarrow V_B = \frac{V_2}{K}$ (III)

$\xrightarrow{\text{(II) in (III)}} V_A = \frac{V_2}{K}\left(1 + S - \frac{10}{R}\right)$ (IV) $\xrightarrow{\text{in (I)}}$

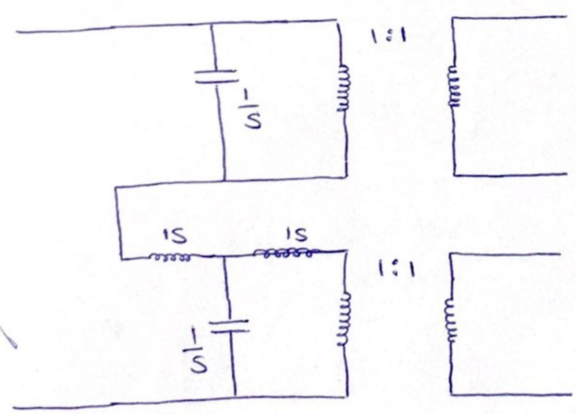
$\left(1 + S - \frac{10}{R}\right)\left(S + \frac{11}{10}\right)\frac{V_2}{K} - SV_2 - \frac{1}{10K}V_2 = V_S \Rightarrow H = \frac{V_2}{V_S} = \frac{10RK}{10S^2R + 21SR - 10SRK + 10R - 100S - 110}$

$\xrightarrow{R=1}$ $10S^2 + 21S - 10SK + 10 - 100S - 110 = 0$

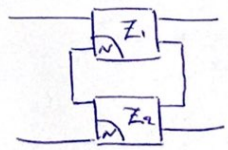
$\Rightarrow K = \frac{10S^2 - 79S - 100}{10S}$

تمرینات دو قطبی :

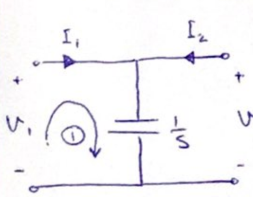
#1

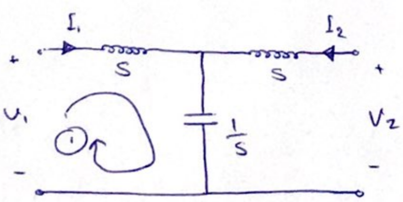


$\Rightarrow$ ترانس‌ها با هم سری هستند



$Z = Z_1 + Z_2$





KVL in 1: $-V_1 + \frac{1}{S}(I_1 + I_2) = 0$

$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = \frac{1}{S}$ $\Rightarrow Z_1 = \begin{bmatrix} \frac{1}{S} & \frac{1}{S} \\ \frac{1}{S} & \frac{1}{S} \end{bmatrix}$

KVL in I_1: $-V_1 + SI_1 + \frac{1}{S}(I_1 + I_2)$

$V_1 = SI_1 + \frac{1}{S}(I_1 + I_2)$

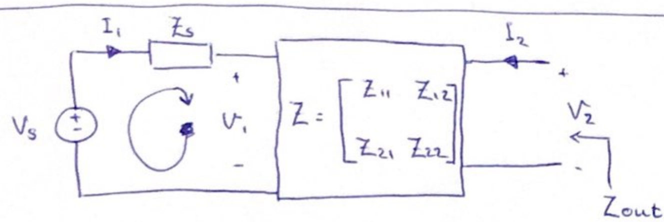
KVL in 2: $+V_2 = SI_2 + \frac{1}{S}(I_1 + I_2)$

$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = \frac{1}{S}$

$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = S + \frac{1}{S}$

$$\Rightarrow Z_2 = \begin{bmatrix} s+\frac{1}{s} & \frac{1}{s} \\ \frac{1}{s} & s+\frac{1}{s} \end{bmatrix} \Rightarrow Z = \begin{bmatrix} s+\frac{1}{s} & \frac{1}{s} \\ \frac{1}{s} & s+\frac{1}{s} \end{bmatrix} + \begin{bmatrix} \frac{1}{s} & \frac{1}{s} \\ \frac{1}{s} & \frac{1}{s} \end{bmatrix} = \begin{bmatrix} \frac{s^2+2}{s} & \frac{2}{s} \\ \frac{2}{s} & \frac{s^2+2}{s} \end{bmatrix}$$

#2



$$Z_{out} = Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad (I)$

$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad (II)$

منبع Vs را اتصال کوتاه می‌کنیم

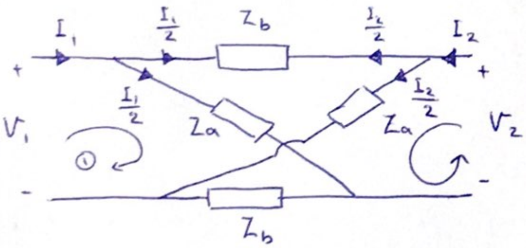
$\longrightarrow Z_s I_1 + V_1 = 0 \quad (III)$

$$\xrightarrow{(I),\,(III)} -Z_s I_1 = Z_{11} I_1 + Z_{12} I_2 \Rightarrow -I_1 (Z_s + Z_{11}) = Z_{12} I_2$$

$$\Rightarrow I_1 = \frac{-Z_{12} I_2}{Z_s + Z_{11}} \xrightarrow{\text{in } (II)} V_2 = -Z_{21}\left(\frac{Z_{12} I_2}{Z_s + Z_{11}}\right) + Z_{22} I_2$$

$$\Rightarrow V_2 = I_2\left(Z_{22} - \frac{Z_{12} Z_{21}}{Z_s + Z_{11}}\right) \Rightarrow Z_{out} = \frac{V_2}{I_2} = \boxed{Z_{22} - \frac{Z_{12} Z_{21}}{Z_s + Z_{11}}}$$

#3



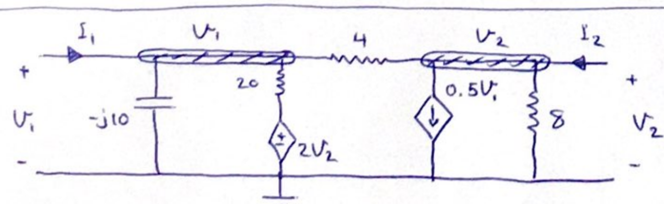
KVL in 1 : $-V_1 + Z_a\left(\frac{I_1}{2}\right) + Z_b\left(\frac{I_1}{2}\right) = 0$

$$\Rightarrow Z_{11} = Z_{22} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{Z_a + Z_b}{2}$$

KVL in 2 : $-V_2 + Z_a\left(\frac{I_2}{2}\right) + Z_b\left(\frac{I_2}{2}\right) = 0$

$$\Rightarrow Z_{12} = Z_{21} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{Z_a + Z_b}{2} \Rightarrow Z = \begin{bmatrix} \frac{Z_a+Z_b}{2} & \frac{Z_a+Z_b}{2} \\ \frac{Z_a+Z_b}{2} & \frac{Z_a+Z_b}{2} \end{bmatrix}$$

#4



kcl in V_1 : $-I_1 + \frac{V_1}{-10j} + \frac{V_1 - 2V_2}{20} + \frac{V_1 - V_2}{4} = 0$

$$\underset{(I)}{\Rightarrow} V_1\left(\frac{-1}{10j} + \frac{1}{20} + \frac{1}{4}\right) - V_2\left(\frac{1}{10} + \frac{1}{4}\right) = I_1$$

kcl in V_2 : $\frac{V_2 - V_1}{4} + 0.5V_1 + \frac{V_2}{8} - I_2 = 0 \Rightarrow V_2\left(\frac{1}{4} + \frac{1}{8}\right) = I_2 + V_1\left(\frac{-1}{2} + \frac{1}{8}\right)$

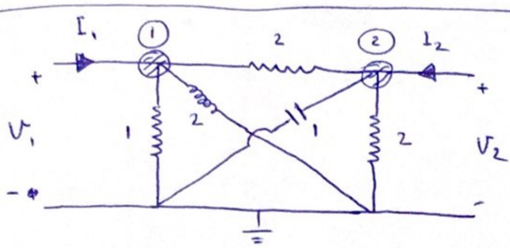
$$\Rightarrow V_2 = \frac{8}{3} I_2 - V_1 \quad (II) \xrightarrow{\text{in}(I)} V_1\left(\frac{3}{10} + \frac{1}{10}j\right) - \frac{7}{20}\left(\frac{8}{3} I_2 - V_1\right) = I_1$$

$$\Rightarrow V_1\left(\frac{13}{20} - \frac{1}{10}j\right) = \frac{14}{15} I_2 + I_1 \Rightarrow V_1 = \underbrace{\left(\frac{728}{519} + j\frac{112}{519}\right)}_{Z_{12}} I_2 + \underbrace{\left(\frac{260}{173} + j\frac{40}{173}\right)}_{Z_{11}} I_1$$

$$V_2\left(\frac{41}{20} - j\frac{7}{20}\right) = \frac{8}{3} I_2 - (3-j) I_1 \Rightarrow V_2 = \left(\frac{656}{519} + j\frac{112}{519}\right) I_2 + \left(\frac{-260}{173} + j\frac{40}{173}\right) I_1$$

$$\Rightarrow Z = \begin{bmatrix} \frac{260}{173} + j\frac{40}{173} , & \frac{728}{519} + j\frac{112}{519} \\ \frac{-260}{173} + j\frac{40}{173} , & \frac{656}{519} + j\frac{112}{519} \end{bmatrix}$$

#5



KCL in ①: $-I_1 + \frac{V_1}{1} + \frac{V_1 - V_2}{2} + \frac{V_1 - 0}{2S} = 0$

$V_1\left(\frac{3}{2} + \frac{1}{2S}\right) - \frac{V_2}{2} = I_1 \quad (I)$

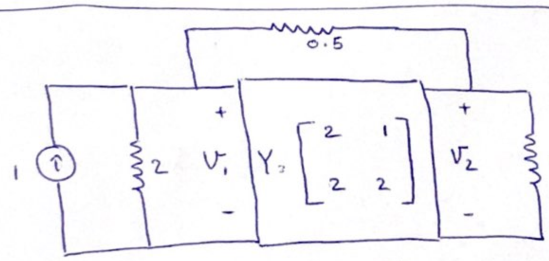
KCL in ②: $-I_2 + \frac{V_2}{2} + \frac{V_2 - V_1}{2} + SV_2 = 0$

$\Rightarrow V_2(1+S) - \frac{V_1}{2} = I_2 \quad (II)$

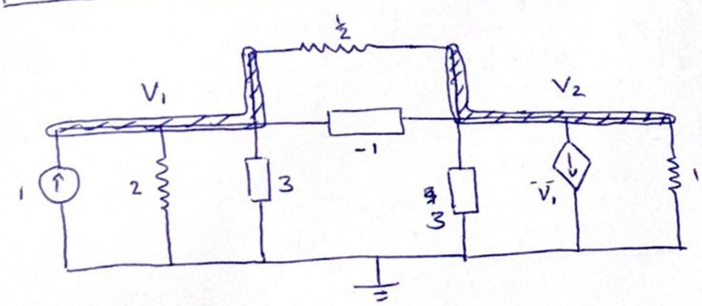
$$\Rightarrow Y = \begin{bmatrix} \frac{3}{2} + \frac{1}{2S} & \frac{-1}{2} \\ \frac{-1}{2} & 1+S \end{bmatrix} \longrightarrow Z = Y^{-1} = \frac{1}{\left(\frac{3S+1}{2S}\right)(1+S) - \frac{1}{4}} \begin{bmatrix} 1+S & \frac{1}{2} \\ \frac{1}{2} & \frac{3S+1}{2S} \end{bmatrix}$$

$$\Rightarrow Z = \begin{bmatrix} \frac{(1+S)(6S^2+7S+2)}{4S} & \frac{6S^2+7S+2}{8S} \\ \frac{6S^2+7S+2}{8S} & \frac{(3S+1)(6S^2+7S+2)}{8S^2} \end{bmatrix}$$

#6



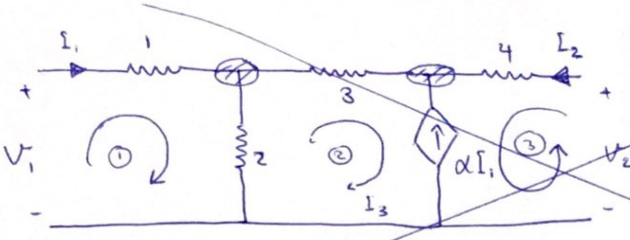
$\Rightarrow$



$$\begin{bmatrix} \frac{1}{2} + 2 + 3 - 1 & -2+1 \\ -2+1+1 & 2-1+3+1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -V_1 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{9}{2} & -1 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{cases} \frac{9}{2}V_1 - V_2 = 1 \longrightarrow \frac{9}{2}V_1 = 1 \Rightarrow \underline{V_1 = \frac{2}{9}} \\ 5V_2 = 0 \longrightarrow \underline{V_2 = 0} \end{cases}$$

#7



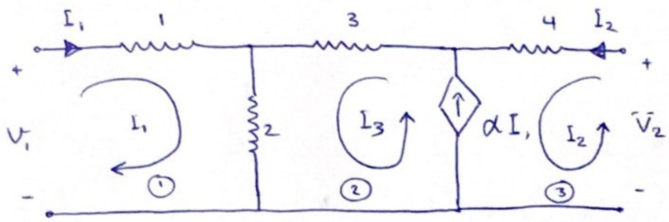
KVL in ① : $-V_1 + \cancel{I_1} + 2(\cancel{I_1} - I_3) = 0$

$I_3 = \frac{-V_1}{2}$

KVL in ②,③ : $3I_3 + 4(\cancel{I_2}) + V_2 + 2(I_3 - \cancel{I_1}) = 0 \Rightarrow 5I_3 = -V_2$

$\Rightarrow I_3 = \frac{-V_2}{5}$, $\alpha I_1 = -I_3$

#7



KVL in I_1 : $-V_1 + I_1 + 2(I_1 + I_3) = 0 \Rightarrow I_1(1+2) - V_1 = -2I_3 \Rightarrow I_1 = \frac{V_1}{3} - \frac{2}{3}I_3$ (I)

KVL in I_3, I_2 : $3I_3 + 2(I_1 + I_3) - V_2 + 4(I_2) = 0 \Rightarrow 5I_3 + 2I_1 + 4I_2 = V_2$ (II)

از طرفی: $I_3 - I_2 = \alpha I_1 \Rightarrow I_3 = \alpha I_1 + I_2$ (III)

$\xrightarrow{\text{(III) in I, II}}$ $\begin{cases} I_1 = \frac{V_1}{3} - \frac{2}{3}(\alpha I_1 + I_2) \\ 5(\alpha I_1 + I_2) + 2I_1 + 4I_2 = V_2 \end{cases} \Rightarrow \begin{cases} I_1 = \frac{1}{3(1+\frac{2}{3}\alpha)}V_1 - \frac{2}{3(1+\frac{2}{3}\alpha)}I_2 \\ 9I_2 + I_1(2+5\alpha) = V_2 \end{cases}$

$$\Rightarrow 9I_2 + (2+5\alpha)\left(\frac{1}{3(1+\frac{2}{3}\alpha)}V_1 - \frac{2}{3(1+\frac{2}{3}\alpha)}I_2\right) = V_2$$

$$\Rightarrow I_2\left(9 - \frac{2(2+5\alpha)}{3(1+\frac{2}{3}\alpha)}\right) = -\frac{(2+5\alpha)}{3(1+\frac{2}{3}\alpha)}V_1 + V_2 \longrightarrow \boxed{I_2 = -\frac{2+5\alpha}{8\alpha+23}V_1 + \frac{3+2\alpha}{8\alpha+23}V_2}$$

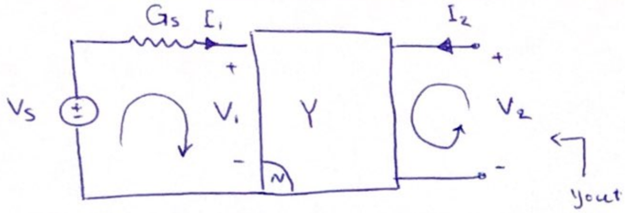
$$\longrightarrow I_2 = \frac{V_2}{9} - \frac{I_1}{9}(2+5\alpha) \longrightarrow I_1 = \frac{1}{3(1+\frac{2}{3}\alpha)}V_1 - \frac{2}{3(1+\frac{2}{3}\alpha)}\left(\frac{V_2}{9} - \frac{I_1}{9}(2+5\alpha)\right)$$

$$\Rightarrow I_1\left(1 - \frac{1}{3(1+\frac{2}{3}\alpha)}\left(\frac{2}{9}(2+5\alpha)\right)\right) = \frac{V_1}{3(1+\frac{2}{3}\alpha)} - \frac{2V_2}{27(1+\frac{2}{3}\alpha)}$$

$$\Rightarrow \boxed{I_1 = \frac{9}{8\alpha+23}V_1 - \frac{2}{8\alpha+23}V_2} \Rightarrow Y = \begin{bmatrix} \frac{9}{8\alpha+23} & \frac{-2}{8\alpha+23} \\ \frac{-2-5\alpha}{8\alpha+23} & \frac{3+2\alpha}{8\alpha+23} \end{bmatrix}$$

$\det(Y) = 0 \Rightarrow \frac{1}{8\alpha+23} = 0$???

#8



$$\begin{cases} I_1 = Y_{11} V_1 + Y_{12} V_2 & (\mathrm{I}) \\ I_2 = Y_{21} V_1 + Y_{22} V_2 & (\mathrm{II}) \end{cases}$$

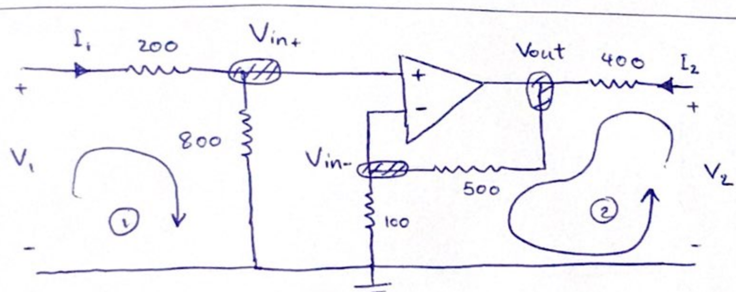
$$Y_{out} = Y_{22} = \frac{I_2}{V_2}\Big|_{V_1 = 0}$$

$$⑧ \Rightarrow \frac{I_1}{G} + V_1 = 0 \Rightarrow V_1 = \frac{-I_1}{G} \xrightarrow{\text{in}(\mathrm{I})} I_1 = Y_{11}\left(\frac{-I_1}{G}\right) + Y_{12} V_2$$

$$I_1\left(1 + \frac{Y_{11}}{G}\right) = Y_{12} V_2 \Rightarrow I_1 = \frac{Y_{12} V_2}{1 + \frac{Y_{11}}{G}} \xrightarrow{\text{in}(\mathrm{II})} I_2 = Y_{21}\left(\frac{-I_1}{G}\right) + Y_{22} V_2$$

$$\Rightarrow I_2 = \frac{-Y_{21}}{G}\left(\frac{Y_{12} V_2}{1 + \frac{Y_{11}}{G}}\right) + Y_{22} V_2 \Rightarrow I_2 = V_2 \underbrace{\left(\frac{-Y_{12} Y_{21}}{G}\left(\frac{1}{1 + \frac{Y_{11}}{G}}\right)\right)}_{Y_{out}}$$

#9



$$V_{in+} = V_{in-}$$

$$\text{KCL in } V_{in+}: \frac{V_{in+}}{800} + \frac{V_{in+} - V_1}{200} = 0 \longrightarrow V_{in+} = \frac{4}{5} V_1 \quad (\mathrm{I})$$

$$\text{KCL in } V_{in-}: \frac{V_{in+} - V_{out}}{500} + \frac{V_{in+}}{100} = 0 \longrightarrow \frac{3}{250}\left(\frac{4}{5} V_1\right) = \frac{V_{out}}{500} \Rightarrow V_{out} = \frac{24}{5} V_1 \quad (\mathrm{II})$$

$$\text{KCL in } V_{out}: \frac{V_{out} - V_2}{400} + \frac{V_{out} - V_{in+}}{500} = 0 \xrightarrow{(\mathrm{II}),\,(\mathrm{I})} \frac{1}{400}\left(\frac{24}{5} V_1\right) + \frac{1}{500}\left(\frac{24}{5} V_1\right) - \frac{1}{500}\left(\frac{4}{5} V_1\right) = \frac{V_2}{400}$$

$$\Rightarrow \frac{27}{1250} V_1 - \frac{1}{625} V_1 = \frac{V_2}{400} \Rightarrow V_2 = 8 V_1 \quad (\mathrm{III})$$

$$\text{KVL in } ①: -V_1 + 200 I_1 + \cancel{800}\left(\frac{V_{in+}}{\cancel{800}}\right) = 0 \xrightarrow{(\mathrm{III})} \boxed{V_1 = 200 I_1 + \frac{1}{10} V_2} \quad (\mathrm{IV})$$

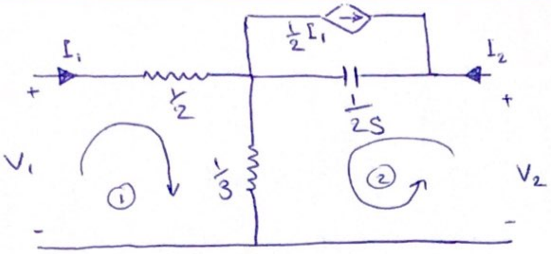
$$\text{KVL in } ②: -V_2 + 400 I_2 + \cancel{500}\left(\frac{V_{out} - V_{in+}}{\cancel{500}}\right) + \cancel{100}\left(\frac{V_{in+}}{\cancel{100}}\right) = 0$$

$$\Rightarrow -V_2 + 400 I_2 + \frac{16}{5} V_1 \xrightarrow{(\mathrm{IV})} -V_2 + 400 I_2 + \frac{16}{5}\left(200 I_1 + \frac{1}{10} V_2\right) = 0$$

$$\Rightarrow 400 I_2 = \frac{17}{25} V_2 - 640 I_1 \Rightarrow \boxed{I_2 = \frac{17}{10000} V_2 - \frac{8}{5} I_1}$$

$$\Rightarrow H = \begin{bmatrix} 200 & \frac{1}{10} \\ \frac{-8}{5} & \frac{17}{10000} \end{bmatrix} \Rightarrow G = H^{-1} = \frac{1}{\frac{17 \times 200}{10000} + \frac{8}{50}} \begin{bmatrix} \frac{17}{10000} & \frac{-1}{10} \\ \frac{8}{5} & 200 \end{bmatrix} = \begin{bmatrix} \frac{17}{5000} & \frac{-1}{5} \\ \frac{16}{5} & 400 \end{bmatrix}$$

#10



KVL in ① : $-V_1 + \frac{1}{2}I_1 + \frac{1}{3}(I_1 + I_2) = 0$

$V_1 = \frac{5}{6}I_1 + \frac{1}{3}I_2$ (I)

KVL in ② : $-V_2 + \frac{1}{2S}\left(I_2 + \frac{1}{2}I_1\right) + \frac{1}{3}(I_1 + I_2) = 0$

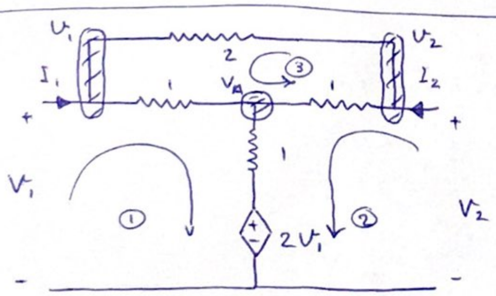
$\Rightarrow I_2\left(\frac{1}{2S} + \frac{1}{3}\right) = V_2 - I_1\left(\frac{1}{3} + \frac{1}{4S}\right) \Rightarrow \boxed{I_2 = \frac{6S}{2S+3}V_2 - \frac{4S+3}{2(3+2S)}I_1}$ (II)

$\xrightarrow{\text{(II) in (I)}} V_1 = \frac{5}{6}I_1 + \frac{1}{3}\left(\frac{6S}{2S+3}V_2 - \frac{4S+3}{6+4S}I_1\right) \Rightarrow V_1 = \left(\frac{5}{6} - \frac{4S+3}{3(4S+6)}\right)I_1 + \frac{2S}{2S+3}V_2$

$$\Rightarrow H = \begin{bmatrix} \frac{52S+93}{30(2S+3)} & \frac{2S}{2S+3} \\ \frac{-4S-3}{2(3+2S)} & \frac{6S}{2S+3} \end{bmatrix} \longrightarrow G = H^{-1} = \frac{1}{\frac{36S}{5(2S+3)}} \begin{bmatrix} \frac{6S}{2S+3} & \frac{-2S}{2S+3} \\ \frac{4S+3}{2(3+2S)} & \frac{52S+93}{30(2S+3)} \end{bmatrix}$$

$$\Rightarrow G = \begin{bmatrix} \frac{5}{6} & \frac{-5}{18} \\ \frac{5(4S+3)}{72S} & \frac{52S+93}{216S} \end{bmatrix}$$

#11



$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

kcl in V_1 : $-I_1 + \frac{V_1 - V_A}{1} + \frac{V_1 - V_2}{2} = 0 \longrightarrow V_1 = \frac{2}{3}V_A + \frac{1}{3}V_2 + I_1$ (I)

kcl in V_A: $V_A - V_1 + \frac{V_A - 2V_1}{1} + \frac{V_A - V_2}{1} = 0 \Rightarrow V_A = V_1 + \frac{1}{3}V_2$ (II)

kcl in V_2: $\frac{V_2 - V_1}{2} + \frac{V_2 - V_A}{1} - I_2 = 0 \Rightarrow I_2 = \frac{3}{2}V_2 - \frac{1}{2}V_1 - V_A$ (III)

$\xrightarrow{\text{(II) in (I)}} V_1 = \frac{2}{3}\left(V_1 + \frac{1}{3}V_2\right) + \frac{1}{3}V_2 + I_1 \Rightarrow V_1\left(1 - \frac{2}{3}\right) = \frac{5}{9}V_2 + I_1$

$\Rightarrow \boxed{V_1 = \frac{5}{3}V_2 + 3I_1}$, $I_2 = \frac{3}{2}V_2 - \frac{1}{2}\left(\frac{5}{3}V_2 + 3I_1\right) - \frac{1}{3}V_2$

$\Rightarrow \boxed{I_2 = \frac{1}{3}V_2 - \frac{3}{2}I_1}$

$$H = \begin{bmatrix} 3 & \frac{5}{3} \\ \frac{-3}{2} & \frac{1}{3} \end{bmatrix} \Rightarrow G = \frac{1}{\frac{7}{2}} \begin{bmatrix} \frac{1}{3} & \frac{-5}{3} \\ \frac{3}{2} & 3 \end{bmatrix}$$

$$\Rightarrow G = \begin{bmatrix} \frac{2}{21} & \frac{-10}{21} \\ \frac{3}{7} & \frac{6}{7} \end{bmatrix}$$