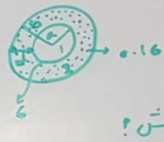


الف - ضخامت پوشش ؟

تا 50٪ کاهش مقاومت در واحد طول سیم بدون پوشش

$$R = \frac{L}{\sigma S}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{\sigma_1 S_1}{L_1} + \frac{\sigma_2 S_2}{L_2}$$

در واحد طول ⇐ $L_1 = L_2 = 1$

$$\rightarrow \frac{1}{R} = \sigma(\pi a^2) + 0.1\sigma(\pi b^2 - \pi a^2)$$

$$\rightarrow R = \frac{1}{\pi\sigma(0.9a^2 + 0.1b^2)}$$

R' ← مقاومت سیم بدون پوشش

$$R = \frac{1}{2} R'$$

$$R' = \frac{1}{\sigma \pi a^2} \rightarrow \frac{1}{2\cancel{\pi} a^2 \cancel{\sigma}} = \frac{1}{\cancel{\pi}\cancel{\sigma}(0.9a^2 + 0.1b^2)}$$

$$\rightarrow 1.1a^2 = 0.1b^2 \rightarrow b = 3.32a$$

$$d = b - a = \boxed{2.32a}$$

ب -

$$I_1 = J_1 S_1 \rightarrow I_1 = J_1(\pi a^2)$$

$$I_2 = J_2 S_2 \rightarrow I_2 = J_2(\pi b^2 - \pi a^2)$$

$$I_2 = J_2(\pi(\underbrace{3.32}_{\approx\sqrt{11}})^2 - \pi a^2)$$

$$\rightarrow \begin{cases} I_1 = J_1(\pi a^2) \\ I_2 = J_2(10\pi a^2) \end{cases}$$

$$I = I_1 + I_2 = \pi a^2(J_1 + 10J_2)$$

$$\frac{I_1}{I_2} = \frac{G_1}{G_2} = \frac{\pi a^2 \sigma}{(10\pi a^2)(0.1\sigma)} = 1 \rightarrow \boxed{I_1 = I_2}$$

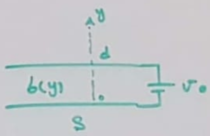
$$J_1 = 10J_2 \rightarrow \begin{cases} J_1 = \boxed{\frac{I}{2\pi a^2}} \\ J_2 = \boxed{\frac{I}{20\pi a^2}} \end{cases}$$

$$J_1 = \sigma E = \frac{I}{2\pi a^2} \rightarrow E = \boxed{\frac{I}{2\pi a^2 \sigma}}$$

$$J_2 = (0.1\sigma)E = \frac{I}{20\pi a^2} \rightarrow E = \boxed{\frac{I}{2\pi a^2 \sigma}}$$

$10-5$

الف - $R = ?$



$$R = \frac{L}{\sigma S} \Rightarrow R = \int_0^d \frac{dL}{\sigma S}, \quad dL = dy$$

$$\sigma = ay + b \rightarrow \begin{cases} \sigma(0) = \sigma_1 \rightarrow b = \sigma_1 \\ \sigma(d) = \sigma_2 \rightarrow a = \frac{\sigma_2 - \sigma_1}{d} \end{cases}$$

$$R = \int_0^d \frac{dy}{S\left(\frac{\sigma_2 - \sigma_1}{d} y + \sigma_1\right)}$$

$$R = \frac{d}{S(\sigma_2 - \sigma_1)} Ln\left(\frac{\sigma_2 - \sigma_1}{d} y + \sigma_1\right)\Big]_0^d$$

$$\rightarrow R = \frac{d}{S(\sigma_2 - \sigma_1)}\left[Ln(\sigma_2) - Ln(\sigma_1)\right]$$

$$\rightarrow R = \frac{d\, Ln(\sigma_2/\sigma_1)}{S(\sigma_2 - \sigma_1)}$$

ب -

$$\begin{cases} I = \frac{V_0}{R} \\ I = JS \end{cases}$$

$$J = \frac{V_0}{SR}$$

$$J = -\frac{V_0}{RS} a_y$$

$$E = \frac{J}{\sigma} = \frac{-V_0}{RS\sigma} a_y$$

$$\rho_s = E \cdot \varepsilon$$

$$\rho_{s_1} = \frac{-V_0 \varepsilon}{RS\sigma_1}$$

$$\rho_{s_2} = \frac{V_0 \varepsilon}{RS\sigma_2}$$

ج -

$$\rho_v = \varepsilon \cdot \nabla E$$

$$E = \frac{V_0 a_y}{SR\left(\frac{\sigma_2 - \sigma_1}{d} y + \sigma_1\right)}$$

$$\nabla \cdot E = \frac{V_0}{SR} \frac{\partial E}{\partial y} =$$

$$\frac{V_0}{RS}\left(\frac{\sigma_2 - \sigma_1}{d}\right) / \left(\left(\frac{\sigma_2 - \sigma_1}{d}\right) y + \sigma_1\right)^2$$

$$\rho_v = \frac{\varepsilon V_0 \left(\frac{\sigma_2 - \sigma_1}{d}\right) d}{RS\left(\frac{\sigma_2 - \sigma_1}{d} y + \sigma_1\right)^2}$$

$$Q = \int \rho_v \cdot dv$$

$$dx\,dz = S$$

$$\rightarrow Q = \iiint \frac{d\varepsilon V_0 (\sigma_2 - \sigma_1)\, dy \overbrace{dz\, dx}^{S}}{RS((\sigma_2 - \sigma_1) y + d\sigma_1)^2}$$

$$\rightarrow Q = \int_0^d \frac{d\varepsilon V_0 (\sigma_2 - \sigma_1)}{R((\sigma_2 - \sigma_1) y + d\sigma_1)^2} dy$$

$$Q = \frac{d\varepsilon V_0 (\sigma_2 - \sigma_1)}{R} \int_0^d \frac{dy}{((\sigma_2 - \sigma_1) y + d\sigma_1)^2}$$

$$(\sigma_2 - \sigma_1) y + d\sigma_1 = A$$

$$(\sigma_2 - \sigma_1) dy = dA \rightarrow dy = \frac{dA}{\sigma_2 - \sigma_1}$$

$$\int \frac{dA}{A^2(\sigma_2 - \sigma_1)} = \frac{1}{\sigma_2 - \sigma_1}\left(-\frac{1}{A}\right)$$

$$Q = \frac{\varepsilon V_0 d}{R}\left(-\frac{1}{(\sigma_2 - \sigma_1) y + d\sigma_1}\right)\Big|_0^d$$

$$\rightarrow Q = -\frac{\varepsilon V_0}{R}\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right) = \frac{\varepsilon V_0}{R}\left(\frac{1}{\sigma_1} - \frac{1}{\sigma_2}\right)$$

1. دستگاه مختصات کروی
2. اختلاف پتانسیل V_0
3. تعیین پتانسیل هر نقطه دلخواه
4. تعیین $\bar{E}=-\nabla V$
5. تعیین $\bar{J}=\sigma\bar{E}$
6. تعیین $I=\int \bar{J}\cdot d\bar{s}$

$$\nabla^2 V = 0$$

زیرا V فقط با R تغییر می کند.

$$\frac{1}{R^2\sin\theta}\frac{\partial}{\partial R}\Big(\underbrace{R^2\sin\theta\frac{\partial V}{\partial R}}_{0}\Big)=0$$

$$R^2\frac{\partial V}{\partial R}=A \rightarrow \frac{\partial V}{\partial R}=\frac{A}{R^2}$$

$$V=-\frac{A}{R}+B \qquad \begin{cases} V(R_1)=-\frac{A}{R_1}+B=0 \\ V(R_2)=-\frac{A}{R_2}+B=V_0 \end{cases}$$

$$-A\left(\frac{1}{R_2}-\frac{1}{R_1}\right)=V_0$$

$$V=\frac{V_0R_1R_2}{(R_2-R_1)R}+\frac{V_0R_2}{R_2-R_1} \Leftarrow \boxed{A=\frac{V_0R_1R_2}{R_2-R_1}}$$

$$B=\frac{A}{R_1}=\frac{V_0R_2}{R_2-R_1}$$

$$E=-\nabla V=-\frac{\partial V}{\partial R}a_R=\frac{-V_0R_1R_2}{(R_2-R_1)R^2}a_R$$

$$\bar{J}=\sigma\bar{E}=\frac{-V_0\sigma R_1R_2}{(R_2-R_1)R^2}a_R \rightarrow I=\int\bar{J}\cdot d\bar{s} \qquad d\bar{s}=R^2\sin\theta\, d\theta\, d\varphi\,(-a_R)$$

$$I=\iint\frac{-V_0\sigma R_1R_2}{(R_2-R_1)\cancel{R^2}}\cancel{R^2}\sin\theta\, d\theta\, d\varphi=\frac{V_0\sigma R_1R_2}{(R_2-R_1)}(-\cos\theta)\Big|_0^{\pi}\varphi\Big|_0^{2\pi}$$

$$=\frac{V_0\sigma R_1R_2}{(R_2-R_1)}(2)(2\pi)=\frac{V_0\sigma R_1R_2}{(R_2-R_1)}(4\pi)$$

$$R=\frac{V_0}{I}=\frac{V_0(R_2-R_1)}{V_0\sigma R_1R_2 4\pi}=\boxed{\frac{R_2-R_1}{\sigma R_1R_2(4\pi)}}$$

$$\sigma = \sigma_0 (1 + K/R)$$

$$R = \frac{L}{\sigma S} \rightarrow dR = \frac{dL}{\sigma S}$$

$$R = \int \frac{dL}{\sigma S} \xrightarrow[S = 4\pi R^2]{dL = dR}$$

$$R = \int \frac{dR}{\sigma_0 (1 + K/R)(4\pi R^2)}$$

$$R = \frac{1}{4\pi\sigma_0} \int \frac{dR}{R^2 + KR}$$

$$R(R+K)$$

تفکیک $\rightarrow \left(\frac{1}{R} - \frac{1}{R+K} \right) \times 1/K$

$$R = \frac{1}{4\pi\sigma_0 K} \int_{R_1}^{R_2} \left(\frac{1}{R} - \frac{1}{R+K} \right) dR$$

$$= \frac{1}{4\pi\sigma_0 K} \left(\ln(R) - \ln(R+K) \right) \Big|_{R_1}^{R_2}$$

$$= \frac{1}{4\pi\sigma_0 K} \left[\ln\left(\frac{R_2}{R_1}\right) - \ln\frac{(R_2+K)}{(R_1+K)} \right]$$

$$R = \frac{1}{4\pi\sigma_0 K} \ln\left(\frac{R_2(R_1+K)}{R_1(R_2+K)} \right) \Leftarrow$$

6-5

$\varepsilon = 1.2\varepsilon_0$

$\sigma = 10\,(S/m) \qquad r = 0.1\,(m)$

بار کل $= Q = 1\,(mC)$

الف-

$$\frac{\partial \rho}{\partial t} + \frac{\sigma \rho}{\varepsilon} = 0$$

$\varepsilon = 1.2\varepsilon_0$

$$\frac{\partial \rho}{\partial t} + \frac{\sigma \rho}{1.2\varepsilon_0} = 0$$

$$A + \frac{\sigma}{1.2\varepsilon_0} = 0 \rightarrow A = -\frac{\sigma}{1.2\varepsilon_0}$$

$$\Rightarrow \rho = k e^{At}$$

$$\rho = k e^{-\frac{\sigma}{1.2\varepsilon_0}}$$

$$\rho_0 \left(\tfrac{4}{3}\pi R^3\right) = Q$$

$$\rho_0 \left(\tfrac{4}{3}\pi (0.1)^3\right) = 10^{-3} \rightarrow \rho_0 = \frac{3}{4\pi}$$

$$\rho = k e^{-\frac{\sigma}{1.2\varepsilon_0}t}$$
$$\rho_0 = \frac{3}{4\pi} \rightarrow \rho = \frac{3}{4\pi} e^{-\frac{\sigma}{1.2\varepsilon_0}t}$$

$$\nabla \cdot E = \frac{\rho}{\varepsilon} \rightarrow \nabla \cdot E = \frac{\rho}{1.2\varepsilon_0}$$

$$\frac{1}{R^2}\left(\frac{\partial}{\partial R}\left(R^2 E_R\right)\right) = \frac{\rho}{1.2\varepsilon_0}$$

$$\frac{\partial R^2 E_R}{\partial R} = \frac{R^2 \rho}{1.2\varepsilon_0}$$

$$\hookrightarrow R^2 E_R = \frac{R^3 \rho}{3.6\varepsilon_0} + C$$

$\underline{E_R = 0}$ $\leftarrow$ $\underline{\rho = 0}$ $\Leftarrow t \to \infty$

$\rho = 0$
$E_R = 0 \Rightarrow C = 0$

$$\downarrow$$

$$E_R = \frac{R\rho}{3.6\varepsilon_0}$$

برای درون کره :

$$E = \frac{R}{3.6\varepsilon_0} \times \frac{3}{4\pi} e^{-\frac{\sigma t}{1.2\varepsilon_0}}$$

$$\hookrightarrow E = \frac{R}{4.8\varepsilon_0 \pi} e^{-\frac{\sigma}{1.2\varepsilon_0}t}\, a_R$$

ب- بیرون کره :

$$\nabla \cdot E = \frac{\rho}{\varepsilon}, \quad \rho = 0 \Rightarrow \nabla \cdot E = 0$$

$$\rightarrow \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 E_R\right) = 0$$

$$\downarrow$$

$$E_R R^2 = C \rightarrow E_R = C/R^2$$

$$E = C/R^2\, a_R$$

$$E_R = C/R^2 \rightarrow \varepsilon_0 E_R = \frac{C\varepsilon_0}{R^2}$$
$$\rho_s = \varepsilon_0 E_R$$

$$\rho_s = \frac{\varepsilon_0 C}{R^2} = \frac{\varepsilon_0 C}{(0.1)^2}$$

$$\rho_s = \frac{Q}{4\pi R^2} = \frac{10^{-3}}{4\pi (0.1)^2} = \frac{1}{40\pi}$$

$$\frac{1}{40\pi} = \frac{C\varepsilon_0}{(0.1)^2} \rightarrow C = \frac{1}{4000\pi\varepsilon_0}$$

$$\boxed{E = \frac{1}{4000\pi\varepsilon_0 R^2}\, a_R}$$