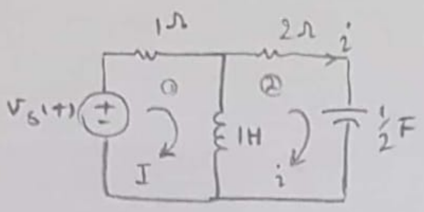
الف)



KVL@①: $-v_s + I + 1\times\dfrac{d(I-i)}{dt} = 0$ — 3

KVL@②: $+\dfrac{d(i-I)}{dt} + 2i + 2\int i\,dt = 0$

$$\downarrow$$

$$\frac{d^2 i}{dt^2} - \frac{d^2 I}{dt^2} + \frac{2di}{dt} + 2i = 0$$

$\xrightarrow[D]{\text{استفاده از عملگر}}$ $-v_s + (1+D)I - Di = 0 \Rightarrow I = \dfrac{v_s + Di}{1+D}$

$$(D^2+2D+2)i - D^2 I = 0 \Rightarrow (D^2+2D+2)i - D^2 \times \frac{v_s + Di}{1+D} = 0$$

$$(D^2+2D+2)(1+D)i - D^2 v_s - D^3 i = 0$$

$$(D^2 + \cancel{D^3} + 2D + 2D^2 + 2 + 2D)i - D^2 v_s - \cancel{D^3} i = 0$$

$$(3D^2 + 4D + 2)i = D^2 v_s \Rightarrow 3\frac{d^2 i}{dt^2} + 4\frac{di}{dt} + 2i = \frac{d^2 v_s}{dt^2}$$

$$\Rightarrow \frac{d^2 i}{dt^2} + \frac{4}{3}\frac{di}{dt} + \frac{2}{3}i = \frac{1}{3}\frac{d^2 v_s}{dt^2}$$

پاسخ پله $\left\{ \begin{array}{l} v_s = u(t) \\ \text{شرایط اولیه صفر} \end{array} \right.$

$\dfrac{3d^2 i}{dt^2} + 4\dfrac{di}{dt} + 2i = \cancel{u(t)}\ 1$ $\Leftarrow$ $\left\{ \begin{array}{l} I_L(0^-) = I_L(0^+) = 0\,A \\ v_C(0^-) = v_C(0^+) = 0\,v \end{array} \right.$

پاسخ ویژه $\Rightarrow 3s^2 + 4s + 2 = 0 \Rightarrow s_{1,2} = \dfrac{-2 \pm \sqrt{2}i}{3}$

پاسخ عمومی $\rightarrow s_g(t) = e^{-\frac{2}{3}t}\left(A\cos\frac{\sqrt{2}}{3}t + B\sin\frac{\sqrt{2}}{3}t\right)$

پاسخ خصوصی $\rightarrow s_p(t) \Rightarrow A \Rightarrow 0 + 4(0) + 2A = 1 \Rightarrow \boxed{A = \frac{1}{2}}$

می‌توان برای حل راحت مسأله سمت راست معادله را ورودی پله قرار داد و با توجه به اینکه ورودی مشتق دوم پله است پس پاسخ خروجی نیز مشتق دوم پاسخ پله خواهد بود.

$$s(t) = e^{-\frac{2}{3}t}\left(A\cos\frac{\sqrt{2}}{3}t + B\sin\frac{\sqrt{2}}{3}t\right) + \frac{1}{2}$$

$I_L(0^+) = \overset{0}{I(0^+)} - i(0^+) = 0 \Rightarrow \boxed{i(0^+) = 0}$

$v(0^+) = \frac{1}{2}\frac{di}{dt}(0^+) = 0 \Rightarrow \boxed{\frac{di}{dt}(0^+) = 0}$

$s(0^+) = 0 \Rightarrow (1)(A(1) + B(0)) + \frac{1}{2} = 0 \Rightarrow \boxed{A = -\frac{1}{2}}$

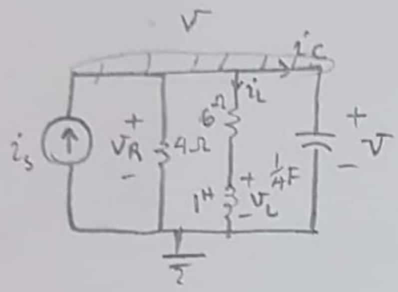
$\dfrac{ds}{dt}(0^+) = 0 \Rightarrow -\frac{2}{3}e^{-\frac{2}{3}t}\left(A\cos\frac{\sqrt{2}}{3}t + B\sin\frac{\sqrt{2}}{3}t\right) + e^{-\frac{2}{3}t}\left(-\frac{\sqrt{2}}{3}A\sin\frac{\sqrt{2}}{3}t + \frac{\sqrt{2}}{3}B\cos\frac{\sqrt{2}}{3}t\right)$

$\Rightarrow -\frac{2}{3}(A) + \left(\frac{\sqrt{2}}{3}B\right) = 0 \Rightarrow \frac{\sqrt{2}}{3}B = -\frac{1}{3} \Rightarrow B = -\frac{\sqrt{2}}{2}$

$$s(t) = \left(e^{-\frac{2}{3}t}\left(-\frac{1}{2}\cos\sqrt{\tfrac{2}{3}}\,t - \frac{\sqrt{2}}{2}\sin\sqrt{\tfrac{2}{3}}\,t\right) + \frac{1}{2}\right)u(t)$$

که دوبار از این پاسخ مشتق می‌گیریم تا به جواب اصلی برسیم.

ب)



$$KCl: -i_s + \frac{v}{4} + i_L + \frac{1}{4}\frac{dv}{dt} = 0$$

$$i_L = \frac{v - v_L}{6} = \frac{v}{6} - \frac{1}{6}(1)\frac{di_L}{dt}$$

$$i_L = \int v_L\,dt = \int (v - 6i_L)\,dt \Rightarrow \frac{di_L}{dt} = v - 6i_L$$

$$i_L = i_s - \frac{v}{4} - \frac{1}{4}\frac{dv}{dt}$$

$$\Rightarrow i_L - \frac{v}{6} + \frac{1}{6}\frac{di_L}{dt} = 0 \Rightarrow i_s + \boxed{\frac{v}{4}} - \frac{1}{4}\frac{dv}{dt} - \boxed{\frac{v}{6}} + \frac{1}{6}\frac{di_s}{dt} - \frac{1}{24}\frac{dv}{dt} - \frac{1}{24}\frac{d^2v}{dt^2} = 0$$

$$\frac{1}{24}\frac{d^2v}{dt^2} + \frac{7}{24}\frac{dv}{dt} + \frac{5}{12}v = \frac{1}{6}\frac{di_s}{dt} + i_s$$

$$\Rightarrow \frac{d^2v}{dt^2} + 7\frac{dv}{dt} + 10v = 4\frac{di_s}{dt} + 24 i_s$$

$\frac{di_s}{dt} \to \delta(t)$, $i_s \to u(t)$

پاسخ عمومی $\Rightarrow s^2 + 7s + 10 = 0 \Rightarrow \begin{cases} s_1 = -5 \\ s_2 = -2 \end{cases} \Rightarrow s_g(t) = Ae^{-5t} + Be^{-2t}$

$\hookrightarrow (s+2)(s+5) = 0$

برای بدست آوردن پاسخ می‌توان از جمع آثار استفاده کرد یعنی ابتدا پاسخ به ورودی پله را به دست آوردیم و سپس با مشتق گیری از پاسخ پله، پاسخ ضربه به دست می‌آید و سپس جوابها را با هم جمع می‌کنیم.

پاسخ خصوصی $\Rightarrow K \Rightarrow (0) + 7(0) + 10K = 24 \Rightarrow \boxed{K = \frac{24}{10}}$

$$s(t) = Ae^{-5t} + Be^{-2t} + \frac{24}{10}$$

$$v_C(0^+) = v(0^+) = 0 \Rightarrow A + B = -\frac{24}{10}$$

$$i_L(0^+) = 0 \Rightarrow \underbrace{i_L(0^+)}_{0} = \cancel{i_s(0^+)} - \underbrace{\frac{v}{4}(0^+)}_{0} - \frac{1}{4}\frac{dv}{dt}(0^+) \Rightarrow \boxed{\frac{dv}{dt}(0^+) = 4}$$

$$\frac{dv}{dt} = -5Ae^{-5t} - 2Be^{-2t} \Rightarrow \begin{cases} -5A - 2B = 4 \\ 10A + 10B = -24 \end{cases} \Rightarrow \begin{cases} -10A - 4B = 8 \\ 10A + 10B = -24 \end{cases}$$

$$6B = -16$$

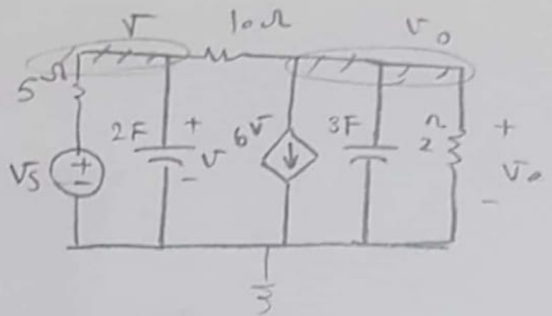
$$B = -\frac{16}{6}$$

$$A = -\frac{24}{10} + \frac{16}{6}$$

$$\Rightarrow v(t) = 4h(t) + 24s(t)$$

$$\boxed{h(t) = \frac{ds(t)}{dt}} \rightarrow$$ و پاسخ ضربه هم به دست آوردیم.

-10



KVL @ v_0 : $\frac{v_0}{2} + 3\frac{dv_0}{dt} + 6v + \frac{v_0 - v}{10} = 0$

KVL @ v : $\frac{v - v_S}{5} + 2\frac{dv}{dt} + \frac{v - v_0}{10} = 0$

$\Rightarrow 5v_0 + 30\frac{dv_0}{dt} + 60v + v_0 - v = 0 \Rightarrow v = \frac{1}{59}\left(-6v_0 - 30\frac{dv_0}{dt}\right)$

$\Rightarrow 2v - 2v_S + 20\frac{dv}{dt} + v - v_0 = 0 \Rightarrow 20\frac{dv}{dt} + 3v - v_0 = 2v_S$

$\Rightarrow \frac{20}{59}\left(-\underbrace{6\frac{dv_0}{dt}}_{}^{-\frac{120}{59}} - 30\frac{d^2v_0}{dt^2}\right) + \frac{3}{59}\left(-6v_0 - 30\frac{dv_0}{dt}\right) - v_0 = 2v_S$ $\quad -\frac{90}{59}$ $\quad -\frac{18}{59}$

$-\frac{600}{59}\frac{d^2v_0}{dt^2} - \frac{210}{59}\frac{dv_0}{dt} - \frac{77}{59}v_0 = 2v_S$

$600\frac{d^2v_0}{dt^2} + 210\frac{dv_0}{dt} + 77v_0 = -118v_S \to u(t)$

شرایط اولیه $\begin{cases} v_S = u(t) \\ v_0(0^+) = v(0^+) = 0 \end{cases}$

$600s^2 + 210s + 77 = 0$

$\hookrightarrow s_{1,2} = -0.175 \pm 0.312j$

$s_g(t) = e^{-0.175t}(A\cos 0.312t + B\sin 0.312t)$

$s_p(t) = K \Rightarrow 600(0) + 210(0) + 77K = -118 \Rightarrow K = -\frac{118}{77}$

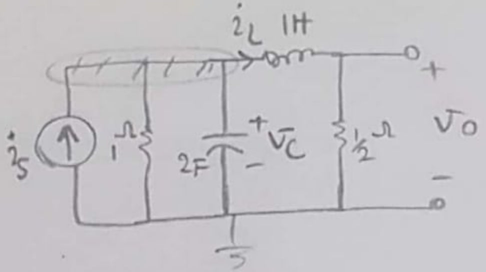
$s(t) = e^{-0.175t}(A\cos 0.312t + B\sin 0.312t) - \frac{118}{77}$

$v_0(0^+) = 0 \Rightarrow \boxed{A = \frac{118}{77}}$ $\quad v(0^+) = \frac{1}{59}\left(-6v_0(0^+) - 30\frac{dv_0}{dt}(0^+)\right)$

$\Rightarrow \frac{dv_0}{dt}(0^+) = 0$

$\Rightarrow \frac{dv_0}{dt}(0^+) = -0.175A + 0.312B = 0 \Rightarrow B = \frac{(0.175)(\frac{118}{77})}{0.312} \simeq 0.86$

پاسخ ضربه هم مشتق تابع پله است چون مدار LTI است.



$$\frac{i_L(0)=1}{v_C(0)=1}$$

$$\text{kcl } v_C: \; -i_S + \frac{v_C}{1} + 2\frac{dv_C}{dt} + i_L = 0$$

$$\text{kcl } v_o: \; 2v_o - i_L = 0 \Rightarrow \boxed{i_L = 2v_o}$$

$$\frac{di_L}{dt} = v_C - v_o \Rightarrow v_C = \frac{di_L}{dt} + v_o = \boxed{2\frac{dv_o}{dt} + v_o}$$

$$2\frac{dv_o}{dt} + v_o + 4\frac{d^2v_o}{dt^2} + 2\frac{dv_o}{dt} + 2v_o = i_S$$

$$4\frac{d^2v_o}{dt^2} + 4\frac{dv_o}{dt} + 3v_o = i_S \xrightarrow{} u(t)$$

$$4s^2 + 4s + 3 = 0 \rightarrow s_{1,2} = -\frac{1}{2} \pm \frac{\sqrt{2}}{2}j$$

$$v_{og}(t) = e^{-\frac{1}{2}t}\left(A\cos\frac{\sqrt{2}}{2}t + B\sin\frac{\sqrt{2}}{2}t\right)$$

$$v_{op}(t) = K \Rightarrow 4(0) + 4(0) + 3K = 1 \Rightarrow \boxed{K = \frac{1}{3}}$$

$$v_o(t) = e^{-\frac{1}{2}t}\left(A\cos\frac{\sqrt{2}}{2}t + B\sin\frac{\sqrt{2}}{2}t\right) + \frac{1}{3}$$

$$2v_o(0) = i_L(0) = 1 \longrightarrow \boxed{v_o(0) = \frac{1}{2}}$$

$$v_C(0) = 2\frac{dv_o}{dt}(0) + \overset{\frac{1}{2}}{v_o(0)} = 1 \Rightarrow \boxed{\frac{dv_o(0)}{dt} = \frac{1}{4}}$$

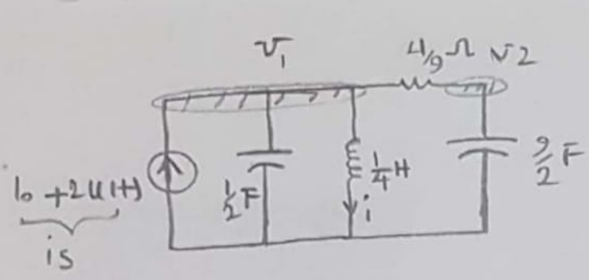
$$v_o(0) = A + \frac{1}{3} = \frac{1}{2} \Rightarrow \boxed{A = \frac{1}{6}}$$

$$\frac{dv_o}{dt}(0) = -\frac{1}{2}A + \frac{\sqrt{2}}{2}B = \frac{1}{4} \Rightarrow B = \frac{2}{\sqrt{2}}\Big(\overbrace{\frac{1}{4} + \frac{1}{12}}^{\frac{4}{12} \to \frac{1}{3}}\Big) = \boxed{\frac{2}{3\sqrt{2}}} = \frac{\sqrt{2}}{3}$$

⇐ و پاسخ به ورودی ضربه، هم از مشتق‌گیری از پاسخ به ورودی پله بدست می‌آید. ⇐

توجه: پاسخ به ورودی پله و ورودی ضربه با $\underbrace{\text{پاسخ پله و ضربه}}$ متفاوت است.

↲ شرایط اولیه صفر است ⇐ تفاوت



$$KVL: \; -i_s + \frac{1}{2}\frac{dv_1}{dt} + i + \frac{v_1 - v_2}{4/9} = 0$$

$$KVL\,v_2: \; \frac{v_2 - v_1}{4/9} + \frac{9}{2}\frac{dv_2}{dt} = 0$$

$$\boxed{v_1 = \frac{1}{4}\frac{di}{dt}}$$

$$v_2 = -\frac{4}{9}i_s + \frac{2}{9}\frac{dv_1}{dt} + \frac{4}{9}i + v_1$$

$$v_2 - v_1 + 2\frac{dv_2}{dt} = 0 \Rightarrow -\frac{4}{9}i_s + \frac{1}{18}\frac{d^2i}{dt^2} + \frac{4}{9}i + \cancel{\frac{1}{4}\frac{di}{dt}} - \cancel{\frac{1}{4}\frac{di}{dt}}$$

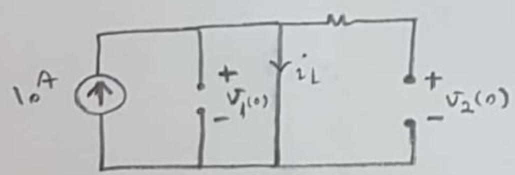
$$-\frac{8}{9}\frac{di_s}{dt} + \frac{4}{9}\left(\frac{1}{4}\right)\frac{d^3i}{dt^3} + \frac{8}{9}\frac{di}{dt} + \frac{1}{2}\frac{d^2i}{dt^2} = 0$$

$$\frac{1}{9}\frac{d^3i}{dt^3} + \frac{10}{18}\frac{d^2i}{dt^2} + \frac{8}{9}\frac{di}{dt} + \frac{4}{9}i = \frac{4}{9}i_s + \frac{8}{9}\frac{di_s}{dt}$$

$$\Rightarrow \frac{d^3i}{dt^3} + 5\frac{d^2i}{dt^2} + 8\frac{di}{dt} + 4i = \underbrace{4i_s + 8\frac{di_s}{dt}}_{40 + 8u(t) + 16\delta(t)}$$

$t = \infty \approx t = 0^-$

با وجود منبع: خازن: اتصال باز ، سلف: اتصال کوتاه



$$\begin{cases} v_1(0^-) = v_1(0^+) = 0V \\ v_2(0^-) = v_2(0^+) = 0V \\ i_L(0^-) = i_L(0^+) = 10^A \end{cases}$$

[illegible] $\Rightarrow \; s^3 + 5s^2 + 8s + 4 = 0 \Rightarrow s_1 = -1, \; s_{2,3} = -2$

$$s_g(t) = k_1e^{-t} + k_2e^{-2t} + k_3te^{-2t}$$

سمت راست را ورودی پله در نظر می‌گیریم: $u(t) = 1$

$$s_p(t) = k \Rightarrow 0 + 5(0) + 8(0) + 4k = 1 \Rightarrow k = \frac{1}{4}$$

$$i(t) \to s(t) = k_1e^{-t} + k_2e^{-2t} + k_3te^{-2t} + \frac{1}{4}$$

$$s(0) = 10 \Rightarrow k_1 + k_2 + k_3(0) + \frac{1}{4} = 10 \Rightarrow \boxed{k_1 + k_2 = \frac{39}{4}}$$

$$v_1(0) = \frac{1}{4}\frac{di}{dt}(0) = 0 \Rightarrow \boxed{\frac{di}{dt}(0) = 0}$$

$$\frac{1}{4}\frac{d^2i}{dt^2}(0) = 0 \to \boxed{\frac{d^2i(0)}{dt^2} = 0}$$

$$\frac{di}{dt}(0) = -k_1 - 2k_2 + k_3 = 0$$

$$v_2(0) = -\frac{4}{9}(10) + \frac{2}{9}\frac{dv_1}{dt}(0) + \frac{4}{9}i(0) + v_1(0) \Rightarrow \frac{dv_1}{dt}(0) = 0$$

$$\frac{d^2 i}{dt^2}(0) = k_1 + 4k_2 - 2k_3 - 2k_3 = 0$$

$$\Rightarrow \begin{cases} k_1 = 39 \\ k_2 = -29.25 \\ k_3 = -19.5 \end{cases}$$

$$i'(t) = 39e^{-t} - 29.25e^{-2t} - 19.5te^{-2t} + \frac{1}{4}$$

چون مدار LTI است پس پاسخ کامل مدار است با :

$$\boxed{i(t) = 48\, i'(t) + 16 \frac{d i'(t)}{dt}}$$

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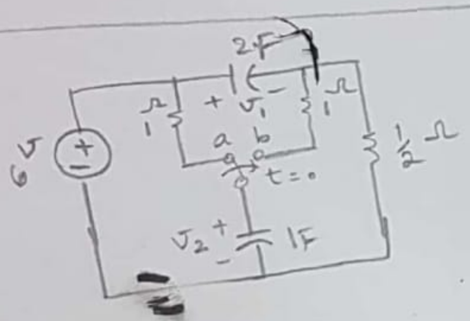
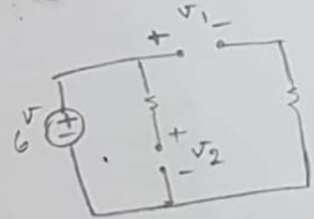
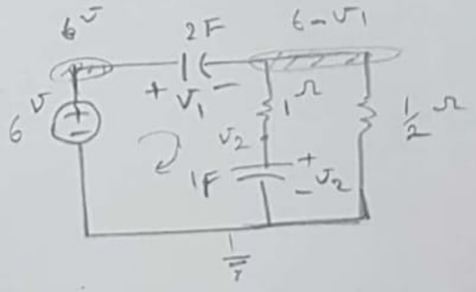


$t = -\infty \leftarrow t = 0^-$

خازن اتصال باز



$$\begin{cases} v_2^{(0^-)} = 6v \\ v_1^{(0^-)} = 6v \end{cases}$$



$\Leftarrow t > 0$

$$\text{kcl}: \frac{6-v_1}{\frac{1}{2}\Omega} + \frac{6-v_1-v_2}{1\Omega} - 2\frac{dv_1}{dt} = 0$$

$$\text{KVL}: -6 + v_1 + \frac{dv_2}{dt} + v_2 = 0$$

$$\hookrightarrow v_1 = 6 - v_2 - \frac{dv_2}{dt}$$

$$\Rightarrow 12 - 2v_1 + 6 - v_1 - v_2 - 2\frac{dv_1}{dt} = 0 \Rightarrow 18 - 3v_1 - v_2 - 2\frac{dv_1}{dt} = 0$$

$$\cancel{18} - \cancel{18} + 3v_2 + 3\frac{dv_2}{dt} - v_2 + 2\frac{dv_2}{dt} + 2\frac{d^2v_2}{dt^2} = 0$$

$$2\frac{d^2v_2}{dt^2} + 5\frac{dv_2}{dt} + 2v_2 = 0$$

$$2s^2 + 5s + 2 = 0 \rightarrow \begin{cases} s_1 = -\frac{1}{2} \\ s_2 = -2 \end{cases} \Rightarrow v_2(t) = k_1 e^{-\frac{1}{2}t} + k_2 e^{-2t}$$

$$v_2(0) = 6 \Rightarrow \boxed{k_1 + k_2 = 6}$$

$$\overset{6}{v_1(0)} = 6 - \overset{6}{v_2(0)} - \frac{dv_2}{dt}(0)$$

$$\frac{dv_2}{dt}(0) = -\frac{1}{2}k_1 - 2k_2 = -6$$

$$\frac{dv_2}{dt}(0) = -6$$

$$\begin{cases} k_1 = 4 \\ k_2 = 2 \end{cases} \Rightarrow v_2(t) = 4e^{-\frac{1}{2}t} + 2e^{-2t}$$

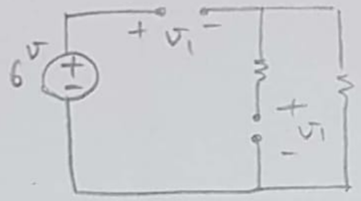
$$v_1 = 6 - 4e^{-\frac{1}{2}t} - 2e^{-2t} + 2e^{-\frac{1}{2}t} + 4e^{-2t}$$

$$\Rightarrow v_1 = -2e^{-\frac{1}{2}t} + 2e^{-2t} + 6$$

$$\lim_{t\to\infty} v_1(t) = 6 \quad , \quad \lim_{t\to\infty} v_2(t) = \infty$$

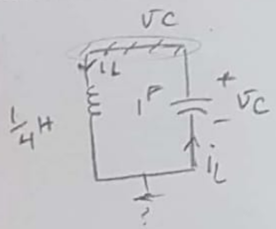
$t \to \infty$
خازن اتصال باز

$\Rightarrow$



$$\Rightarrow \begin{cases} v_1(\infty) = 6^V \\ v_1(\infty) = 0^V \end{cases}$$

-24



$$v_C(0) = 1^V$$
$$i_L(0) = 2^A$$

$$i_L + \frac{dv_C}{dt} = 0$$

$$v_C = \frac{1}{4}\frac{di_L}{dt} \quad \Rightarrow \quad i_L + \frac{1}{4}\frac{d^2 i_L}{dt^2} = 0$$

$$i_L = -\frac{dv_C}{dt} \quad \Rightarrow \quad v_C = -\frac{1}{4}\frac{d^2 v_C}{dt^2} \Rightarrow \frac{d^2 v_C}{dt^2} + 4v_C = 0$$

$$s^2 + 4 = 0 \Rightarrow s_{1,2} = \pm 2i$$

$$v_C(t) = A\cos 2t + B\sin 2t$$

$$v_C(0) = \boxed{A = 1} \quad , \quad i_L(0) = -\frac{dv_C}{dt}(0) = 2$$

$$\Rightarrow \frac{dv_C}{dt}(0) = 2B = -2 \Rightarrow \boxed{B = -1}$$

$$\boxed{v_C(t) = \cos 2t - \sin 2t}$$

$$i_L = -\frac{dv_C}{dt} = 2\sin 2t + 2\cos 2t$$

$$P_C = v_C(t) \cdot i_C(t) = v_C(t) \cdot (-i_L)$$

$$P_C = (\cos 2t - \sin 2t)(-2\sin 2t - 2\cos 2t)$$

$$= \cancel{-2\sin 2t\cos 2t} - 2\cos^2 2t + 2\sin^2 2t + \cancel{2\sin 2t\cos 2t}$$

$$w_C = \int_0^t P_C(t)\,dt = 2\int (\underbrace{\sin^2 2t - \cos^2 2t}_{-\cos 4t})\,dt = -\frac{2}{4}\sin 4t \Big|_0^t = \boxed{-\frac{2}{4}\sin 4t}$$

$$P_L = v_L \cdot i_L = (-v_C)(i_L) = (-\cos 2t + \sin 2t)(2\sin 2t + 2\cos 2t)$$

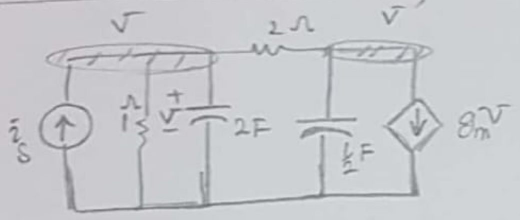
$w_L = \int_0^t p_L \, dt = \boxed{\frac{1}{2}\sin 4t}$

$w_{C_0} = \frac{1}{2} C v_C^2 = \frac{1}{2}(1)(1)^2 = \frac{1}{2}$, $w_{L_0} = -\frac{1}{2} L i_L^2 = \frac{1}{2}(\frac{1}{4})(2)^2 = -\frac{1}{2}$

$w_0 = w_{C_0} + w_{L_0} = 0$

$w = w_L + w_C = \frac{1}{2}\sin 4t - \frac{1}{2}\sin 4t = 0$

—3



$g_m = ? \rightarrow$ میرای شدید

KCL @ v: $-i_s + v + 2\frac{dv}{dt} + \frac{v - v'}{2} = 0 \rightarrow v' = -2i_s + 2v + 4\frac{dv}{dt} + v$

KCL @ v': $\frac{v' - v}{2} + \frac{1}{2}\frac{dv'}{dt} + g_m v = 0$

$\hookrightarrow v' - v + \frac{dv'}{dt} + 2g_m v = 0$

$\Rightarrow -2i_s + 2v + 4\frac{dv}{dt} + \cancel{v} - \cancel{v} - 2\frac{di_s}{dt} + 2\frac{dv}{dt} + 4\frac{d^2v}{dt^2} + \frac{dv}{dt} + 2g_m v = 0$

$2\frac{d^2v}{dt^2} + \frac{7}{2}\frac{dv}{dt} + (1 + g_m)v = i_s + \frac{di_s}{dt}$

$2s^2 + \frac{7}{2}s + (1 + g_m) = 0$

$\Delta > 0 \Leftarrow$ شرط میرای شدید

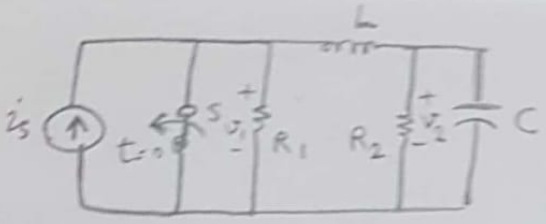
$\Delta = (\frac{7}{2})^2 - 4(2)(1 + g_m) > 0 \Rightarrow 1 + g_m < \frac{49}{32} \Rightarrow \boxed{g_m < \frac{17}{32}}$

$\frac{49}{4}$

—ب $Q = 1 \Rightarrow g_m = ?$

$Q = \frac{\omega_0}{2\alpha} = \frac{\sqrt{1 + g_m}/\sqrt{2}}{\frac{7}{2\times 2}} = 1$

$\frac{1 + g_m}{2} = \frac{49}{16} \Rightarrow g_m = \frac{49}{8} - 1 = \boxed{\frac{41}{8}}$



$v_1(0^+) = ?$

$v_2(0^+) = ?$

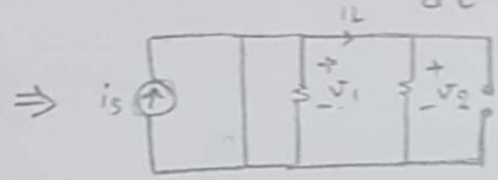
$\frac{dv_1}{dt}(0^+) = ?$

$\frac{d^2 v_2}{dt^2}(0^+) = ?$

$t<0 \Rightarrow t = -\infty$

خازن : اتصال باز
سلف : اتصال کوتاه

$\Rightarrow$

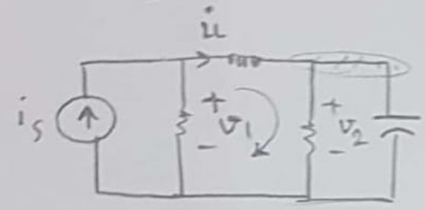


$v_1(0^-) = 0V$

$v_2(0^+) = v_2(0^-) = 0V$

$i_L(0^+) = i_L(0^-) = 0A$

$v_1(0^+) = R_1 i_s$



$$(*)\quad \underbrace{\frac{v_2(0^+)}{R}}_{0} - \underbrace{i_L(0^+)}_{0} + C\frac{dv_2}{dt}(0^+) = 0 \Rightarrow \boxed{\frac{dv_2}{dt}(0) = 0}$$

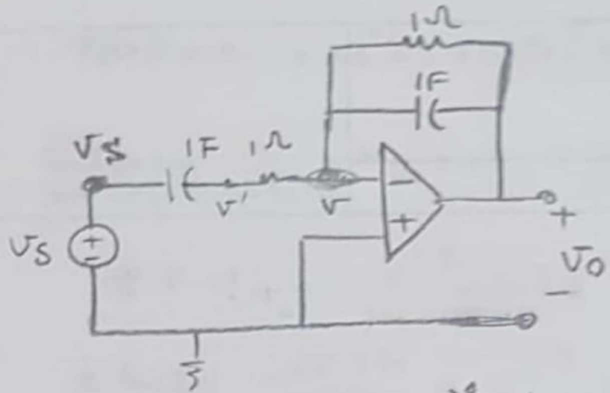
$$i_L = i_s - v_1/R_1 \Rightarrow v_1 = R_1(i_s - i_L) \Rightarrow \frac{dv_1}{dt} = R_1\Big(\underbrace{\frac{di_s}{dt}}_{0} - \underbrace{\frac{di_L}{dt}}_{\frac{v_L}{L}}\Big)$$

$$v_L(0^+) = v_1(0^+) - \underbrace{v_2(0^+)}_{0} = R_1 i_s$$

$$\Rightarrow \frac{dv_1}{dt}(0^+) = R_1\left(-\frac{R_1 i_s}{L}\right) = \boxed{-\frac{R_1^2}{L} i_s}$$

مشتق گیری از (*) $\longrightarrow$

$$\frac{1}{R}\underbrace{\frac{dv_2}{dt}(0^+)}_{0} - \underbrace{\frac{di_L}{dt}(0^+)}_{\frac{v_L}{L} = \frac{R_1 i_s}{L}} + C\frac{d^2v_2}{dt^2}(0^+) = 0 \Rightarrow \boxed{\frac{d^2v_2}{dt^2}(0^+) = \frac{R_1}{LC} i_s}$$



KCL @ v: $\frac{\cancel{v}-v'}{1\Omega} + \cancel{I^{-}} + \frac{d(\cancel{v}-v_o)}{dt} + \frac{\cancel{v}-v_o}{1\Omega} = 0$

$-v' - \frac{dv_o}{dt} - v_o = 0$

$\downarrow$

$v' = -\frac{dv_o}{dt} - v_o$

KCL @ v': $\frac{v'-\cancel{v}}{1\Omega} + \frac{d(v'-v_s)}{dt} = 0$

$v' + \frac{dv'}{dt} - \frac{dv_s}{dt} = 0$

$$\Rightarrow -\frac{dv_o}{dt} - v_o - \frac{d^2v_o}{dt^2} - \frac{dv_o}{dt} = \frac{dv_s}{dt}$$

$$\Rightarrow \frac{d^2v_o}{dt^2} + 2\frac{dv_o}{dt} + v_o = -\frac{dv_s}{dt}$$

$\delta(t)$?

$s^2 + 2s + 1 = 0$

$(s+1)^2 = 0 \Rightarrow s_{1,2} = -1$

$v_{og}(t) = k_1 e^{-t} + k_2 t e^{-t}$

$v_{op}(t) = k \Rightarrow (0) + 2(0) + k = +1 \Rightarrow \boxed{k = 1}$

$v_o(t) = k_1 e^{-t} + k_2 t e^{-t} + 1$

$v_{C_2}(0) = 0 \Rightarrow v - v_o = 0 - v_o = 0 \Rightarrow \boxed{v_o(0) = 0}$

$v_{C_1}(0) = \underset{1}{v_s(0)} - v'(0) = 0 \Rightarrow v'(0) = 1 \Rightarrow$

$-\underset{1}{v'(0)} - \frac{dv_o}{dt}(0) - \cancel{v_o(0)}^{0} = 0$

$\Rightarrow \boxed{\frac{dv_o}{dt}(0) = -1}$

$$v_o(0) = k_1 + k_2^{(0)} + 1 = 0 \Rightarrow \boxed{k_1 = -1}$$

$$\frac{dv_o}{dt}(0) = -\underset{-1}{k_1} + k_2 = -1 \Rightarrow \boxed{k_2 = -2}$$

$$\Rightarrow v_o(t) = -e^{-t} - 2te^{-t} + 1$$

$$\Rightarrow s(t) = -\frac{dv_o(t)}{dt} = e^{-t} - 2e^{-t} + 2te^{-t}$$

مدار LTI

پاسخ ضربه هم از مشتق گیری از پاسخ پله به دست می‌آید.

$$h(t) = \frac{ds(t)}{dt}$$