1-2

$A = a_x + 2a_y - 3a_z$

$B = -4a_y + a_z$

$C = 5a_x - 2a_z$

الف

$a_A = ? \rightarrow a_A = \dfrac{\vec{A}}{|\vec{A}|} = \boxed{\dfrac{a_x + 2a_y - 3a_z}{\underbrace{\sqrt{1+4+9}}_{\sqrt{14}}}}$

ب

$|A-B| = ? \rightarrow A - B = (1-0)a_x + (2-(-4))a_y + (-3-1)a_z = a_x + 6a_y - 4a_z \rightarrow |A-B| = \boxed{\sqrt{1+36+16}}$

پ

$A \cdot B = ? \longrightarrow A \cdot B = (\overbrace{1 \times 0}^{0}) + (\overbrace{2 \times (-4)}^{-8}) + (\overbrace{-3 \times 1}^{-3}) = \boxed{-11}$

ت

$\theta_{AB} = ? \longrightarrow A \cdot B = |A||B| \cos\theta_{AB} \Rightarrow \theta_{AB} = \cos^{-1}\left(\dfrac{A \cdot B}{|A||B|}\right) \Rightarrow \theta_{AB} = \cos^{-1}\left(\dfrac{-11}{(\underbrace{\sqrt{1+4+9}}_{\sqrt{14}})(\underbrace{\sqrt{16+1}}_{\sqrt{17}})}\right) \Rightarrow \boxed{\theta_{AB} \simeq 135.5^\circ}$

ث

مولفه $A$ در $C$ $= ? \rightarrow A_C = \vec{A} \cdot a_C = \vec{A} \cdot \dfrac{\vec{C}}{|\vec{C}|} \rightarrow a_C = \dfrac{5a_x - 2a_z}{\underbrace{\sqrt{25+4}}_{\sqrt{29}}} \rightarrow A_C = \dfrac{(a_x + 2a_y - 3a_z) \cdot (5a_x - 2a_z)}{\sqrt{29}} = \dfrac{5+0+6}{\sqrt{29}} = \boxed{\dfrac{11}{\sqrt{29}}}$

ج

$A \times C = ? \rightarrow A \times C = \begin{vmatrix} a_x & a_y & a_z \\ 1 & 2 & -3 \\ 5 & 0 & -2 \end{vmatrix} = a_x(\underbrace{(2)(-2) - (0)(-3)}_{-4}) + a_y(\underbrace{(-3)(5) - (1)(-2)}_{-13}) + a_z(\underbrace{(1)(0) - (2)(5)}_{-10})$

$\longrightarrow -4a_x - 13a_y - 10a_z$

$A = a_x + 2a_y - 3a_z$

$B = -4a_y + a_z$

$C = 5a_x - 2a_z$

1–2

ز

$$(A \times B)\cdot C = ? \rightarrow A \times B = \begin{vmatrix} a_x & a_y & a_z \\ 1 & 2 & -3 \\ 0 & -4 & 1 \end{vmatrix} = a_x\big(\underbrace{(2)(1) - (-3)(-4)}_{-10}\big) + a_y\big(\underbrace{(-3)(0) - (1)(1)}_{-1}\big) + a_z\big(\underbrace{(1)(-4) - (2)(0)}_{-4}\big)$$

$$(A \times B)\cdot C = (-10a_x - a_y - 4a_z)\cdot(5a_x - 2a_z) = \underbrace{(5)(-10)}_{-50} + \underbrace{(-1)(0)}_{0} + \underbrace{(-4)(-2)}_{8} = \boxed{-42}$$

$$A\cdot(B \times C) = ? \rightarrow B \times C = \begin{vmatrix} a_x & a_y & a_z \\ 0 & -4 & 1 \\ 5 & 0 & -2 \end{vmatrix} = a_x\big(\underbrace{(-4)(-2) - (1)(0)}_{8}\big) + a_y\big(\underbrace{(1)(5) - (0)(-2)}_{5}\big) + a_z\big(\underbrace{(0)(0) - (-4)(5)}_{20}\big)$$

$$A\cdot(B \times C) = (a_x + 2a_y - 3a_z)\cdot(8a_x + 5a_y + 20a_z) = \underbrace{(1)(8)}_{8} + \underbrace{(2)(5)}_{10} + \underbrace{(-3)(+20)}_{-60} = \boxed{-42}$$

ح

$$A \times (B \times C) = ? \rightarrow B \times C = \begin{vmatrix} a_x & a_y & a_z \\ 0 & -4 & 1 \\ 5 & 0 & -2 \end{vmatrix} = 8a_x + 5a_y + 20a_z \rightarrow A \times (B \times C) = \begin{vmatrix} a_x & a_y & a_z \\ 1 & 2 & -3 \\ 8 & 5 & 20 \end{vmatrix}$$

$$= a_x\big(\underbrace{(2)(20) - (-3)(5)}_{55}\big) + a_y\big(\underbrace{(-3)(8) - (1)(20)}_{-44}\big) + a_z\big(\underbrace{(1)(5) - (2)(8)}_{-11}\big) = \boxed{55a_x - 44a_y - 11a_z}$$

$$(A \times B) \times C = ? \rightarrow A \times B = -10a_x - a_y - 4a_z \rightarrow (A \times B) \times C = \begin{vmatrix} a_x & a_y & a_z \\ -10 & -1 & -4 \\ 5 & 0 & -2 \end{vmatrix}$$

$$= a_x\big(\underbrace{(-1)(-2) - (-4)(0)}_{2}\big) + a_y\big(\underbrace{(-4)(5) - (-2)(-10)}_{-40}\big) + a_z\big(\underbrace{(-10)(0) - (-1)(5)}_{5}\big) = \boxed{2a_x - 40a_y + 5a_z}$$

## 2.2

$$A = a_x - 2a_y + 3a_z$$
$$B = a_x + a_y - 2a_z$$

بردار واحد C که هم بر A و هم بر B عمود است؟

بنابراین بردار C حاصل ضرب خارجی A و B می‌باشد که بر هر دوی آنها عمود است و واحد شده است.

$$A \times B = \begin{vmatrix} a_x & a_y & a_z \\ 1 & -2 & 3 \\ 1 & 1 & -2 \end{vmatrix} = a_x\big(\underbrace{(-2)(-2) - (3)(1)}_{1}\big) + a_y\big(\underbrace{(3)(1) - (1)(-2)}_{5}\big) + a_z\big(\underbrace{(1)(1) - (-2)(1)}_{3}\big)$$

$$\Rightarrow C = \frac{A \times B}{|A \times B|} = \frac{a_x + 5a_y + 3a_z}{\underbrace{\sqrt{1 + 25 + 9}}_{\sqrt{35}}}$$

---

## 3.2

$$A = A_x a_x + A_y a_y + A_z a_z$$
$$B = B_x a_x + B_y a_y + B_z a_z$$

اگر A و B موازی باشند، رابطه بین مولفه های A و B

زمانی که دو بردار A و B باهم موازی باشند، ضرب خارجی آنها برابر صفر است بنابراین از این مطلب برای یافتن رابطه بین مولفه ها استفاده می‌کنیم.

$$A \times B = 0 \rightarrow A \times B = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = 0$$

$$a_x(\underbrace{A_y B_z - A_z B_y}_{0}) + a_y(\underbrace{A_z B_x - A_x B_z}_{0}) + a_z(\underbrace{A_x B_y - A_y B_x}_{0}) = 0$$

$$\begin{cases} A_y B_z = A_z B_y \xrightarrow{/(B_z B_y)} \dfrac{A_y}{B_y} = \dfrac{A_z}{B_z} \\ A_z B_x = A_x B_z \xrightarrow{/(B_x B_z)} \dfrac{A_z}{B_z} = \dfrac{A_x}{B_x} \\ A_x B_y = A_y B_x \xrightarrow{/(B_y B_x)} \dfrac{A_x}{B_x} = \dfrac{A_y}{B_y} \end{cases} \Rightarrow \frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$$

6.2

سه رأس مثلث → $P_1 = (0, 1, -2)$

$P_2 = (4, 1, -3)$

$P_3 = (6, 2, 5)$

الف – آیا مثلث قائم الزاویه است؟ ← این سه نقطه، دو به دو با هم سه بردار را می‌سازند، بنابراین وقتی این مثلث قائم الزاویه است که دو بردار از این سه بردار بر هم عمود باشند و شرط عمود بودن هم صفر شدن ضرب داخلی دو بردار است:

$$\overrightarrow{P_1P_2} = (4-0)a_x + (1-1)a_y + (-3-(-2))a_z = 4a_x - a_z$$

$$\overrightarrow{P_1P_3} = (6-0)a_x + (2-1)a_y + (5-(-2))a_z = 6a_x + a_y + 7a_z$$

$$\overrightarrow{P_2P_3} = (6-4)a_x + (2-1)a_y + (5-(-3))a_z = 2a_x + a_y + 8a_z$$

$\overrightarrow{P_1P_2} \perp \overrightarrow{P_1P_3} = ? \rightarrow (4)(6) + (0)(2) + (-1)(7) \neq 0$

$\overrightarrow{P_1P_2} \perp \overrightarrow{P_2P_3} = ? \rightarrow (4)(2) + (0)(1) + (-1)(8) = 0$ ✓ → بنابراین این مثلث قائم الزاویه می‌باشد.

$\overrightarrow{P_1P_3} \perp \overrightarrow{P_2P_3} = ? \rightarrow (6)(2) + (1)(1) + (7)(8) \neq 0$

ب – سطح مثلث = ؟

$$|\overrightarrow{P_1P_2}| = \sqrt{16+1} = \sqrt{17} \longrightarrow S = \frac{(\sqrt{17})(\sqrt{69})}{2}$$

$$|\overrightarrow{P_2P_3}| = \sqrt{4+1+64} = \sqrt{69}$$

12.2

$$A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$$

$$B \times C = \begin{vmatrix} a_x & a_y & a_z \\ x_B & y_B & z_B \\ x_C & y_C & z_C \end{vmatrix} = a_x(y_B z_C - z_B y_C) + a_y(z_B x_C - x_B z_C) + a_z(x_B y_C - y_B x_C)$$

طرف اول تساوی

$$A \times (B \times C) = \begin{vmatrix} a_x & a_y & a_z \\ x_A & y_A & z_A \\ y_B z_C - z_B y_C & z_B x_C - x_B z_C & x_B y_C - y_B x_C \end{vmatrix} = a_x(x_B y_C y_A - y_B x_C y_A - z_B x_C z_A + x_B z_C z_A)$$

$$+ a_y(y_B z_C z_A - z_B y_C z_A - x_B y_C x_A + y_B x_C x_A) + a_z(z_B x_C x_A - x_B z_C x_A - y_B z_C y_A + z_B y_C y_A)$$

---

طرف دوم تساوی

$$A \cdot C = x_A x_C + y_A y_C + z_A z_C \quad , \quad A \cdot B = x_A x_B + y_A y_B + z_A z_B$$

$$B(A \cdot C) = (x_A x_C x_B + y_A y_C x_B + z_A z_C x_B)a_x + (x_A x_C y_B + y_A y_C y_B + z_A z_C y_B)a_y + (x_A x_C z_B + y_A y_C z_B + z_A z_C z_B)a_z$$

$$C(A \cdot B) = (x_A x_B x_C + y_A y_B x_C + z_A z_B x_C)a_x + (x_A x_B y_C + y_A y_B y_C + z_A z_B y_C)a_y + (x_A x_B z_C + y_A y_B z_C + z_A z_B z_C)a_z$$

$$B(A \cdot C) - C(A \cdot B) = (y_A y_C x_B + z_A z_C x_B - y_A y_B x_C - z_A z_B x_C)a_x + (x_A x_C y_B + z_A z_C y_B - x_A x_B y_C - z_A z_B y_C)a_y$$

$$+ (x_A x_C z_B + y_A y_C z_B - x_A x_B z_C - y_A y_B z_C)a_z$$

لذا تساوی طرف اول و دوم درستی قاعده ی back-cab در مورد ضرب سه‌گانه برداری سه بردار را اثبات می‌شود.

Scanned by CamScanner

مولفه بردار $A = -za_y + ya_z$ را در نقطه $P_1 = (0, -2, 3)$ که به سمت نقطه $P_2 = (\sqrt{3}, -60^\circ, 1)$ جهت گرفته است را بیابید.

$P_2$ در مختصات استوانه ای داده شده است، مختصات آنرا در دستگاه کارتزین می یابیم:

$$\begin{cases} x = r\cos\theta \to x = \sqrt{3}\cos(-60) = \frac{\sqrt{3}}{2} \\ y = r\sin\theta \to y = \sqrt{3}\sin(-60) = -\frac{3}{2} \end{cases} \to P_2 = \left(\frac{\sqrt{3}}{2}, -\frac{3}{2}, 1\right)$$

$$\overrightarrow{P_1P_2} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}, -2\right)$$

مولفه ی $A$ در جهت $\overrightarrow{P_1P_2}$ $= A \cdot \frac{\overrightarrow{P_1P_2}}{|\overrightarrow{P_1P_2}|}$

$$= \frac{(-za_y + ya_z) \cdot \left(\frac{\sqrt{3}}{2}a_x + \frac{1}{2}a_y - 2a_z\right)}{\underbrace{\sqrt{\frac{3}{4} + \frac{1}{4} + 4}}_{\sqrt{5}}} = \frac{-\frac{z}{2} - 2y}{\sqrt{5}} = \frac{\overbrace{-(3 + 4(-2))}^{5}}{2\sqrt{5}}$$

$$= \frac{5\sqrt{5}}{2(\cancel{5})} = \frac{\sqrt{5}}{2} \simeq \boxed{1.12}$$

17-2

$$E = \frac{25}{R^2} a_R$$

الف - $|E| = ?\ ,\ E_x = ?$

$P = (-3, 4, 5)$ درنقطه

$$|E| = \frac{25}{R^2} = \frac{25}{x^2+y^2+z^2} = \frac{25}{9+16+25} = \boxed{\frac{1}{2}}$$

$$E_x = E \cdot a_x = \frac{25}{R^2}(a_R \cdot a_x) = \frac{25}{R^2}(\sin\theta\cos\varphi) = \frac{25}{R^2}\left(\frac{r}{R} \times \frac{x}{r}\right) = \frac{25}{R^2}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right) = \frac{25}{50}\left(\frac{-3}{\sqrt{50}}\right) \simeq \boxed{-0/212}$$

$a_R$: $\cos\theta a_z + \underbrace{\sin\theta a_r}_{\sin\theta\cos\varphi a_x + \sin\theta\sin\varphi a_y}$

ب - زاویه E با بردار $B = 2a_x - 2a_y + a_z$ $= ?$

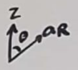
درنقطه P

$$\begin{cases} E_x \simeq -0/212 \\ E_y = E \cdot a_y = \frac{25}{R^2}(a_R \cdot a_y) = \frac{25}{R^2}(\sin\theta \underset{\frac{r}{R}\times\frac{y}{r}}{\sin\varphi}) = \frac{25}{R^2}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right) \simeq 0/282 \\ E_z = E \cdot a_z = \frac{25}{R^2}(a_R \cdot a_z) = \frac{25}{R^2}(\cos\theta) = \frac{25}{R^2}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) \simeq -0/353 \end{cases}$$

$$\theta = \cos^{-1}\left(\frac{B \cdot E}{|B||E|}\right) = \cos^{-1}\left(\frac{(2)(-0/212) + (-2)(0/282) + (1)(-0/353)}{\left(\sqrt{4+4+1}\right)\left(\sqrt{(-0/212)^2 + (0/282)^2 + (-0/353)^2}\right)}\right)$$

19_2

ت $a_R \cdot a_r = ?$




$(\cos\theta\, a_z + \sin\theta\, a_r) \cdot a_r = \boxed{\sin\theta}$

ث $a_y \cdot a_R = ?$

$a_y \cdot (\cos\theta\, a_z + \underbrace{\sin\theta\, a_r}_{\sin\theta\cos\varphi\, a_x + \sin\theta\sin\varphi\, a_y})$

$\rightarrow a_y \cdot a_R = \boxed{\sin\theta\sin\varphi}$

ج. $a_R \cdot a_z = ?$

$(\cos\theta\, a_z + \sin\theta\, a_r) \cdot a_z = \boxed{\cos\theta}$

چ $a_R \times a_z = ?$

$(\cos\theta\, a_z + \sin\theta\, a_r) \times a_z =$

$\cos\theta(\underbrace{a_z \times a_z}_{0}) + \sin\theta\,(a_r \times a_z) =$

$a_r = \cos\varphi\, a_x + \sin\varphi\, a_y$

$\sin\theta\Big(\cos\varphi(\underbrace{a_x \times a_z}_{-a_y}) + \sin\varphi\,(\underbrace{a_y \times a_z}_{a_x})\Big) = \boxed{-\sin\theta\, a_\varphi}$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\varphi & \cos\theta\cos\varphi & -\sin\varphi \\ \sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{bmatrix}$$

$\Rightarrow a_\varphi = -\sin\varphi\, a_x + \cos\varphi\, a_y$

الف $a_x \cdot a_\varphi = ?$

$a_x \cdot (-\sin\varphi\, a_x + \cos\varphi\, a_y) = \boxed{-\sin\varphi}$

ب $a_\theta \cdot a_y = ?$


$(\underbrace{\cos(90+\theta)}_{-\sin\theta} a_z + \underbrace{\underbrace{\sin(90+\theta)}_{\cos\theta} a_r}_{\cos\theta\cos\varphi\, a_x + \cos\theta\sin\varphi\, a_y}) \cdot a_y = \boxed{\cos\theta\sin\varphi}$

پ $a_r \times a_x = ?$

$|a_r||a_x|\sin\varphi = \sin\varphi$ اندازه‌ی ضرب خارجی

جهت از طریق قاعده‌ی دست راست تعیین می‌شود $\Rightarrow a_r \times a_x = \boxed{-\sin\varphi\, a_z}$

ح $a_\theta \cdot a_z = ?$

$(\overset{-\sin\theta}{\cos(90+\theta)}\, a_z + \underbrace{\overset{\cos\theta}{\sin(90+\theta)}\, a_r}_{\cos\theta\cos\varphi\, a_x + \cos\theta\sin\varphi\, a_y}) \cdot a_z = -\sin\theta$

$a_x \perp a_z \qquad a_z \perp a_y$

خ $a_z \times a_\theta = a_z \times (-\sin\theta\, a_z + \cos\theta\cos\varphi\, a_x + \cos\theta\sin\varphi\, a_y)$

$= -\sin\theta(\underbrace{a_z \times a_z}_{0}) + \cos\theta\cos\varphi(\underbrace{a_z \times a_x}_{a_y}) + \cos\theta\sin\varphi(\underbrace{a_z \times a_y}_{-a_x})$

$\cos\theta(\underbrace{-\sin\varphi\, a_x + \cos\varphi\, a_y}_{a_\varphi}) = \cos\theta\, a_\varphi$