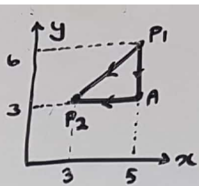

$$F = xy\,a_x + (3x - y^2)\,a_y$$

$\int F \cdot dL = ? \rightarrow$ از $P_1 = (5,6)$ تا $P_2(3,3)$

الف) در امتداد مسیر مستقیم P_1P_2

$$\int F \cdot dL = \int (xy\,a_x + (3x - y^2)\,a_y) \cdot (dx\,a_x + dy\,a_y + dz\,a_z)$$

$$= \int_5^3 xy\,dx + \int_6^3 (3x - y^2)\,dy$$

معادله مسیر $P_1P_2 \leftarrow$

$m = \frac{6-3}{5-3} = \frac{3}{2}$

$y - 3 = \frac{3}{2}(x-3) \Rightarrow y = \frac{3}{2}x - \frac{9}{2} + 3$

$\Rightarrow y = \frac{3}{2}x - \frac{3}{2}$

$\downarrow$

$x = \frac{2}{3}(y + \frac{3}{2}) = \frac{2}{3}y + 1$

$$= \int_5^3 x(\tfrac{3}{2}x - \tfrac{3}{2})\,dx + \int_6^3 (3(\tfrac{2}{3}y + 1) - y^2)\,dy$$

$$\Rightarrow \int_5^3 (\tfrac{3}{2}x^2 - \tfrac{3}{2}x)\,dx = \tfrac{3}{6}x^3 - \tfrac{3}{4}x^2 \Big]_5^3$$

$$= \tfrac{3}{6}(27) - \tfrac{3}{4}(9) - \tfrac{3}{6}(125) + \tfrac{3}{4}(25)$$

$$= -37$$

$$\Rightarrow \int_6^3 (2y + 3 - y^2)\,dy = y^2 + 3y - \tfrac{1}{3}y^3 \Big]_6^3$$

$$= 9 + 9 - \tfrac{1}{3}(27) - 36 - 18 + \tfrac{1}{3}(216) = 27$$

$$\Rightarrow \int F \cdot dL = \boxed{-10}$$

20
21
24

20-2

ب) در امتداد مسیر P_1AP_2

$$\int F \cdot dL = \int_{P_1A} F \cdot dL + \int_{AP_2} F \cdot dL$$

$P_1A \downarrow$ $dL = dy\,a_y$ $\quad$ $AP_2 \downarrow$ $dL = dx\,a_x$

$$\int_{P_1A} F \cdot dL = \int (3\underset{\underline{x=5}}{x} - y^2)\,dy = -\tfrac{1}{3}y^3 + 15y \Big]_6^3$$

$$= -\tfrac{1}{3}(27) + 15(3) + \tfrac{216}{3} - 15(6) = +18$$

$$\int_{AP_2} F \cdot dL = \int (x\underset{y=3}{y})\,dx = \tfrac{3}{2}x^2 \Big]_5^3$$

$$= +\tfrac{3}{2}(9) - \tfrac{3}{2}(25) = -24$$

$$\Rightarrow \int F \cdot dL = +18 - 24 = \boxed{-6}$$

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$E = y a_x + x a_y$

$\int E \cdot dl \rightarrow$ از نقطه $P_1 = (2, 1, -1)$ تا $P_2 = (8, 2, -1)$

الف

در امتداد سهمی

$x = 2y^2 \rightarrow y = \sqrt{\frac{x}{2}}$

$$\int E \cdot dl = \int (y a_x + x a_y) \cdot (dx\, a_x + dy\, a_y) = \int_2^8 y\, dx + \int_1^2 x\, dy$$

$$= \int_2^8 \frac{\sqrt{x}}{\sqrt{2}} dx + \int_1^2 2y^2 dy = \frac{2}{3} x^{3/2} \times \frac{1}{\sqrt{2}} \Big]_2^8 + \frac{2}{3} y^3 \Big]_1^2$$

$\sqrt{x} \rightarrow x^{\frac{1}{2}}$

$$= \frac{2}{3\sqrt{2}} \Big[\underbrace{\sqrt{8^3}}_{8\sqrt{8}} - \underbrace{\sqrt{2^3}}_{2\sqrt{2}} \Big] + \frac{2}{3} [\underbrace{8 - 1}_{7}] = \frac{28}{3} + \frac{14}{3} = \frac{42}{3} = \boxed{14}$$

$$\frac{\overset{32}{\cancel{16 \times 2}} - 4}{3} = \frac{28}{3}$$

ب

در امتداد خط راست رابط دو نقطه

$$y - 1 = \frac{2-1}{8-2}(x - 2) \Rightarrow y - 1 = \frac{1}{6}(x-2) \Rightarrow y = \frac{1}{6}x - \frac{1}{3} + 1 = \frac{1}{6}x + \frac{2}{3}$$

$$x = 6\left(y - \frac{2}{3}\right)$$

$$\begin{cases} y = \frac{1}{6}x + \frac{2}{3} \\ x = 6y - 4 \\ z = -1 \end{cases}$$

$$\int E \cdot dl = \int (y a_x + x a_y) \cdot (dx\, a_x + dy\, a_y + dz\, a_z)$$

$$= \int y\, dx + \int x\, dy = \int_2^8 \left(\frac{1}{6}x + \frac{2}{3}\right) dx + \int_1^2 (6y - 4)\, dy$$

$$= \frac{1}{12}x^2 + \frac{2}{3}x \Big]_2^8 + \frac{6y^2}{2} - 4y \Big]_1^2 = \frac{1}{12}(64) + \frac{2}{3}(8) - \frac{1}{12}(4) - \frac{2}{3}(2) + 3(4) - 4(2) - 3(1) + 4(1) = \boxed{14}$$

21-2

$\frac{20}{21}/28$

اگر E یک میدان دفع شونده باشد باید $\nabla \times E$ برابر صفر باشد.

زیرا کرل گرادیان هر کمیت عددی برابر صفر است

$\nabla \times (\nabla V) = 0$

$\nabla V = E$

$$\nabla \times E = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) a_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) a_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) a_z$$

$$= (0 - 0) a_x + (0 - 0) a_y + (1 - 1) a_z = 0$$

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$$\oint_S (3\sin\theta\, a_R)\cdot ds \rightarrow$$ محاسبه انتگرال روی سطح کره ای با شعاع 5 و مرکز مبدأ مختصات

$$\oint (3\sin\theta\, a_R)\cdot(R^2\sin\theta\, d\theta\, d\varphi\, a_R) = \int_0^{2\pi}\int_0^{\pi} 3\sin^2\theta\, R^2\, d\theta\, d\varphi$$

$$\Rightarrow \int \sin^2\theta\, d\theta = \int \frac{1-\cos 2\theta}{2}\, d\theta = \frac{1}{2}\theta - \frac{1}{2}\times\frac{1}{2}\sin 2\theta \Big]_0^{\pi}$$

$$= \frac{1}{2}(\pi) - \frac{1}{4}(0) = \boxed{\frac{\pi}{2}}$$

$$\int d\varphi = \varphi\Big]_0^{2\pi} = \boxed{2\pi}$$

$$\Rightarrow (2\pi)\left(\frac{\pi}{2}\right)(3)(5)^2 = \boxed{75\pi^2}$$