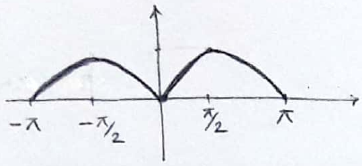
بنام خدا

# پاسخ تمرین سری 4 (ریاضی مهندسی)

سری فوریه کسینوسی متناظر با تابع $0<x<\pi$ ، $f(x)=\sin x$ . آنگاه سری فوریه تابع $\cos x$ ؟




$$\begin{cases} \sin x & 0<x<\pi \\ -\sin x & -\pi<x<0 \end{cases} \Rightarrow \underline{\underline{T=2\pi}}$$

$a_n = \checkmark$
$a_0 = \checkmark$
$b_n = 0$

$$a_0 = \frac{1}{T}\int_{T} f(x)\,dx = \frac{1\times 2}{2\pi}\int_0^{\pi} \sin x\,dx = -\frac{1}{\pi}\cos x\Big]_0^{\pi} = \frac{1}{\pi}+\frac{1}{\pi} = \boxed{\frac{2}{\pi}}$$

$$a_n = \frac{2}{T}\int_{T} f(x)\cos\frac{2n\pi}{T}x\,dx = \frac{2\times 2}{2\pi}\int_0^{\pi}\sin x\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi}\frac{1}{2}\left[\sin(1+n)x+\sin(1-n)x\right]dx$$

$$= \frac{1}{\pi}\left(-\frac{1}{1+n}\cos(1+n)x - \frac{1}{1-n}\cos(1-n)x\right)\Big|_0^{\pi} = \frac{1}{\pi}\left(\frac{-\cos(1+n)\pi}{1+n} - \frac{\cos(1-n)\pi}{1-n} + \frac{1}{1+n} + \frac{1}{1-n}\right)$$

$$= \frac{1}{\pi}\left(\frac{1-\overset{-(-1)^n}{\cos(1+n)\pi}}{1+n} + \frac{1-\overset{-(-1)^n}{\cos(1-n)\pi}}{1-n}\right) = \frac{1}{\pi}\left(\frac{2+2(-1)^n}{1-n^2}\right) = \frac{2\left(1+(-1)^n\right)}{\pi(1-n^2)} \qquad \underline{\underline{n\neq 1}}$$

$$a_1 = \frac{2}{T}\int f(x)\cos\frac{2n\pi}{T}x\,dx = \frac{2}{\pi}\int_0^{\pi}\sin x\cos x\,dx = \frac{2}{\pi}\int_0^{\pi}\frac{1}{2}\left[\sin 2x + \cancel{\sin 0}\right]dx$$

$$= \frac{1}{\pi}\left(-\frac{1}{2}\cos 2x\right)\Big|_0^{\pi} = \frac{1}{\pi}\left(\frac{-1}{2}+\frac{1}{2}\right) = \boxed{0} \qquad \underline{n=1}$$

$$\longrightarrow a_n = \begin{cases} n=2m & \dfrac{4}{\pi(1-4m^2)} \\ n=2m-1 & 0 \end{cases} \qquad \Longrightarrow \sin x = \frac{2}{\pi} + \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{1}{(1-4m^2)}\cos 2mx$$

$$\xrightarrow[\text{مشتق میگیریم}]{\text{از } \sin x} \cos x = 0 + \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{1}{(1-4m^2)}\times -2m\times\sin 2mx$$

$$\longrightarrow \boxed{\cos x = \frac{-8}{\pi}\sum_{m=1}^{\infty}\frac{m\sin 2mx}{(1-4m^2)}} \checkmark$$