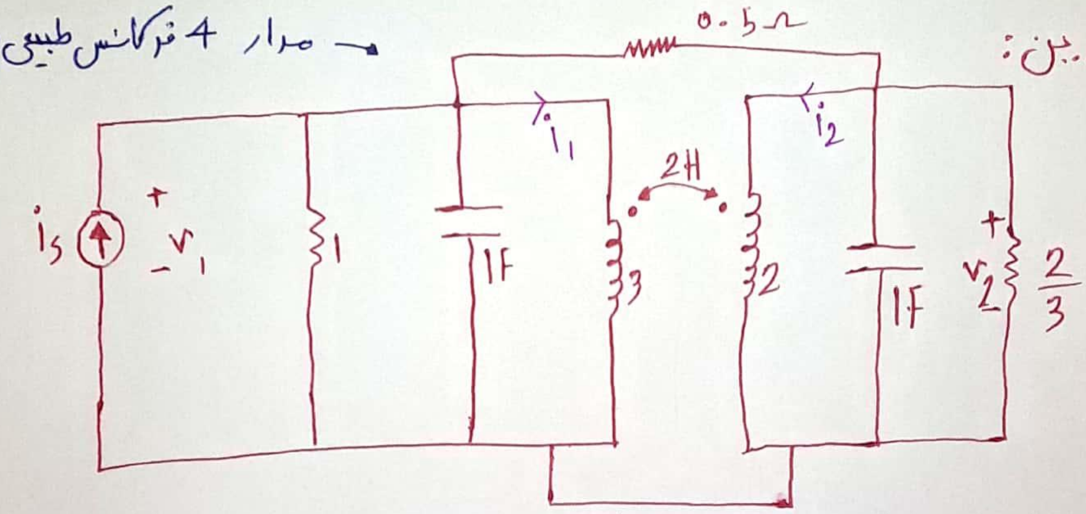
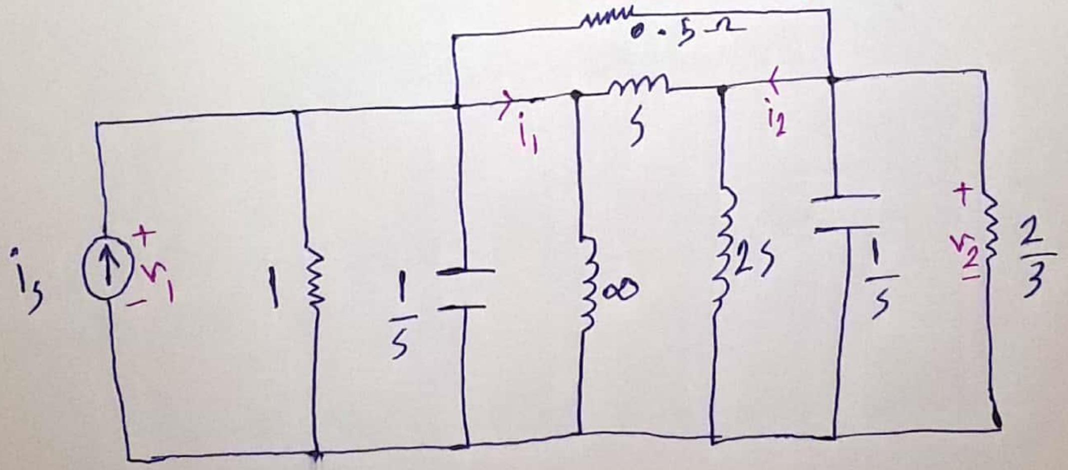
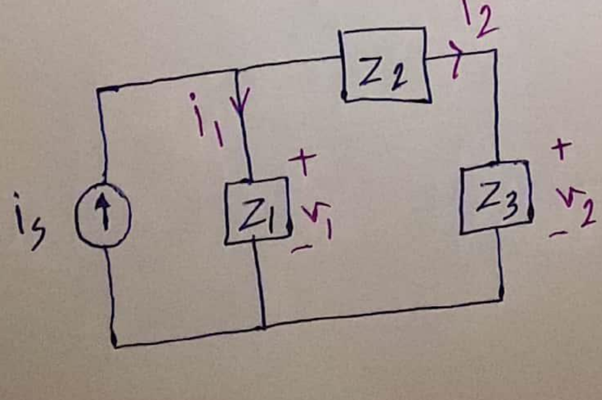
تمرین :

مدار 4 فرکانس طبیعی دارد.



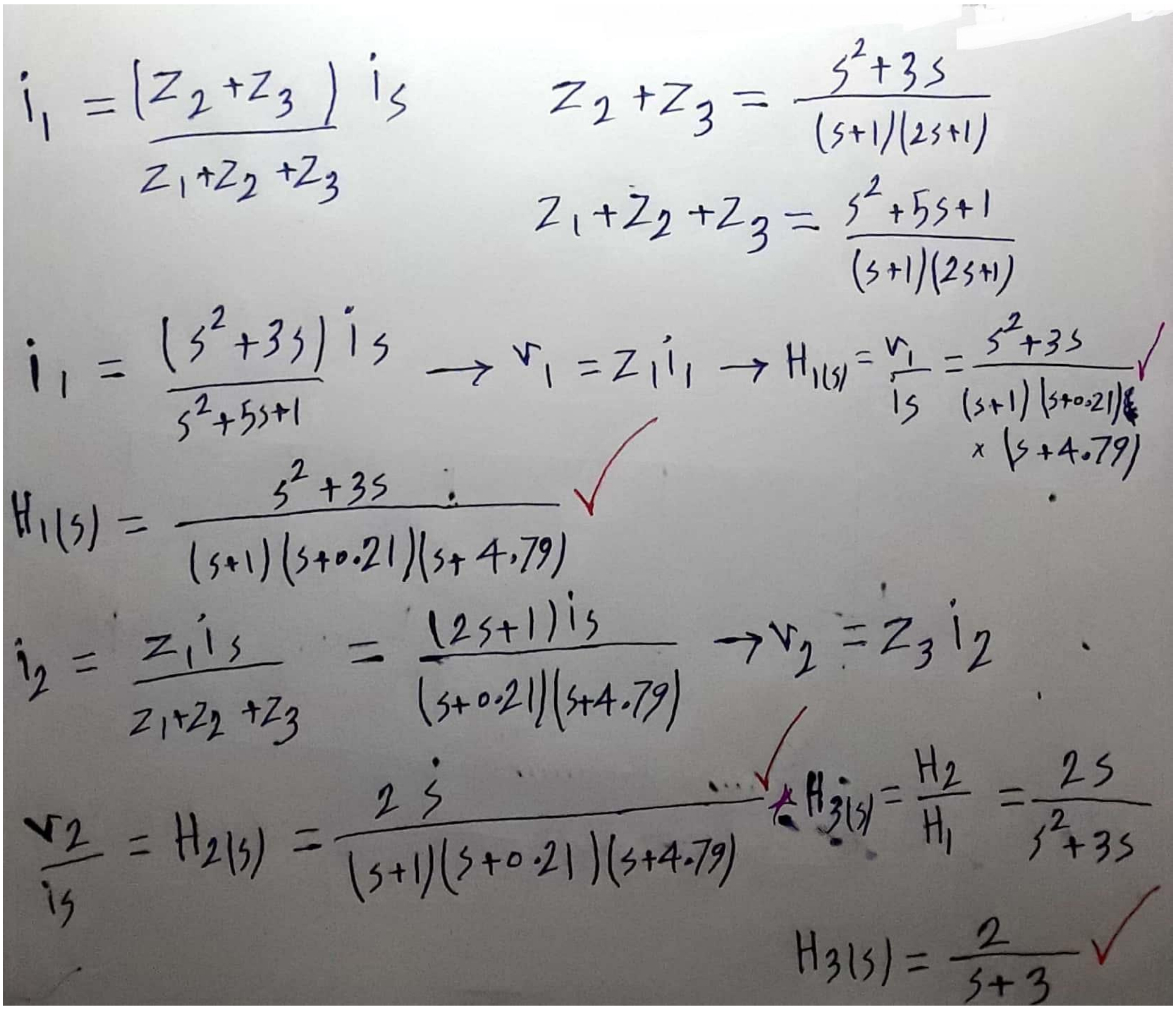




$$Z_1 = \frac{1}{s+1}$$

$$Z_2 = \frac{s}{2s+1}$$

$$Z_3 = \frac{2s}{2s^2+3s+1}$$



$$i_1 = \frac{(Z_2+Z_3)\, i_s}{Z_1+Z_2+Z_3} \qquad Z_2+Z_3 = \frac{s^2+3s}{(s+1)(2s+1)}$$

$$Z_1+Z_2+Z_3 = \frac{s^2+5s+1}{(s+1)(2s+1)}$$

$$i_1 = \frac{(s^2+3s)\, i_s}{s^2+5s+1} \rightarrow v_1 = Z_1 i_1 \rightarrow H_1(s) = \frac{v_1}{i_s} = \frac{s^2+3s}{(s+1)(s+0.21) \times (s+4.79)}$$ ✓

$$H_1(s) = \frac{s^2+3s}{(s+1)(s+0.21)(s+4.79)}$$ ✓

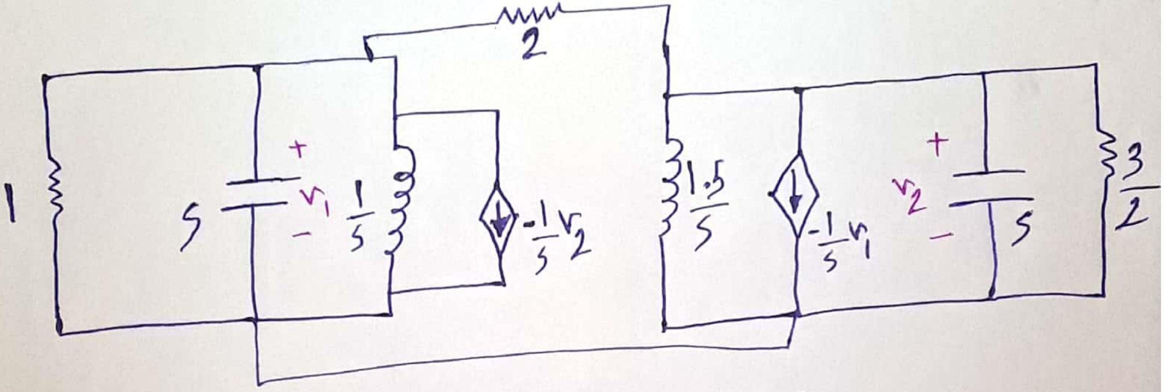
$$i_2 = \frac{Z_1 i_s}{Z_1+Z_2+Z_3} = \frac{(2s+1)\, i_s}{(s+0.21)(s+4.79)} \rightarrow v_2 = Z_3 i_2$$

$$\frac{v_2}{i_s} = H_2(s) = \frac{2s}{(s+1)(s+0.21)(s+4.79)}$$ ✓ $$H_3(s) = \frac{H_2}{H_1} = \frac{2s}{s^2+3s}$$

$$H_3(s) = \frac{2}{s+3}$$ ✓

در ادامه فرکانس های طبیعی کل مدار با روش گره پیدا می کنیم.

$$L=\begin{bmatrix}3 & 2\\ 2 & \cancel{3}2\end{bmatrix}\rightarrow \Gamma=\frac{1}{6-4}\begin{bmatrix}2 & -2\\ -2 & 3\end{bmatrix}=\begin{bmatrix}1 & -1\\ -1 & 1.5\end{bmatrix}$$



$$\underbrace{\begin{bmatrix}s+1+\frac{1}{s}+2 & -2-\frac{1}{s}\\ -2-\frac{1}{s} & 2+\frac{1.5}{s}+s+1.5\end{bmatrix}}_{A}\begin{bmatrix}E_1\\ E_2\end{bmatrix}=\begin{bmatrix}\frac{1}{s}v_2\\ \frac{1}{s}v_1\end{bmatrix}$$

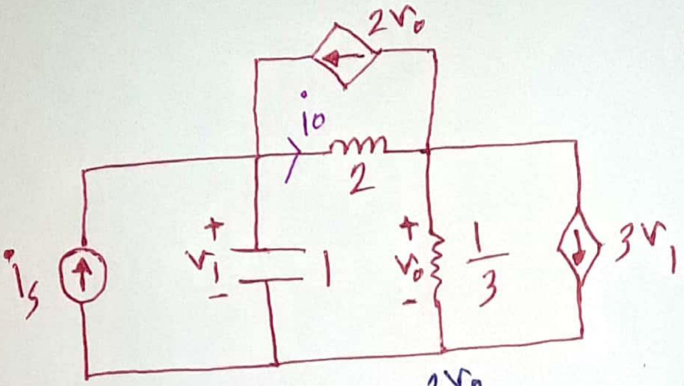
$$\det\{A\}=\frac{2s^4+13s^3+18s^2+8s+1}{2s^2}\rightarrow \det\{A\}=0 \rightarrow \begin{cases}s_1=-4.79\\ s_2=-1\\ s_3=-0.5\\ s_4=-0.21\end{cases}$$

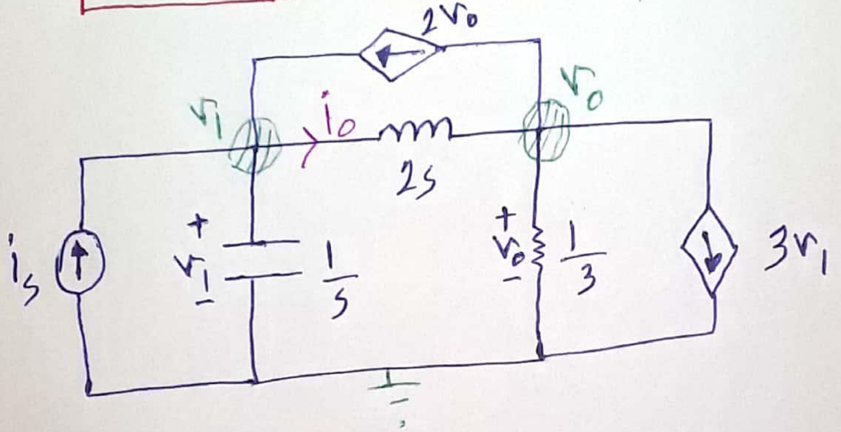
$H_1(s)$ شامل s_1 و s_2 و s_4 * $H_2(s)$ شامل s_1 و s_2 و s_4

$H_3(s)$ اصلا تابع انتقال نیست که قطب های آن شامل $s_{1,2}$ و 3 و 4 باشد.
تابع انتقال (نقطه تحریک یا متقابل) باید بر حسب خروجی/ورودی نوشته شود.

تمرین:

$H_2 = \frac{i_o}{i_s} \quad H_1 = \frac{v_o}{i_s}$





$$\text{KCL}@v_o: \frac{v_o - v_1}{2s} + 3v_o + 3v_1 + 2v_o = 0$$

$$v_1 = \frac{5 + \frac{1}{2s}}{\frac{1}{2s} - 3} v_o \quad (I)$$

$$\text{KCL}@v_1: i_s = s v_1 + \frac{v_1 - v_o}{2s} - 2v_o$$

$$\left(s + \frac{1}{2s}\right) v_1 - \left(2 + \frac{1}{2s}\right) v_o = i_s \xrightarrow{(I)}$$

$$\left[\frac{\left(s + \frac{1}{2s}\right)\left(5 + \frac{1}{2s}\right)}{\frac{1}{2s} - 3} - \left(2 + \frac{1}{2s}\right)\right] v_o = i_s$$

$$\frac{5s+\frac{2.5}{s}+\frac{1}{2}+\cancelto{0}{\frac{1}{4s^2}}-\left(\frac{1}{s}+\cancelto{0}{\frac{1}{4s^2}}-6-\frac{3}{2s}\right)}{\frac{1-6s}{2s}}\;v_0=i_s$$

$$\times$$

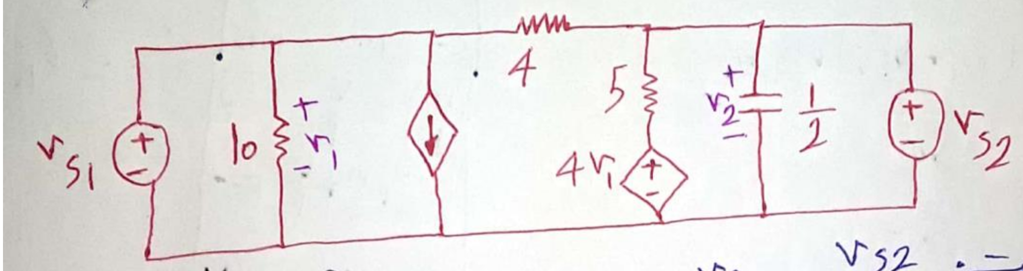
$$H_1(s)=\frac{v_o}{i_s}=\frac{1-6s}{10s^2+13s+6s}\ \checkmark\ (\mathrm{II})$$

$$H_2(s)=\frac{i_o}{i_s}=?$$

$$i_o=\frac{v_1-v_o}{2s}\xrightarrow{(\mathrm{I})}v_{1(s)}=\frac{10s+1}{1-6s}v_o\rightarrow v_1-v_o=\left(\frac{16s}{1-6s}\right)v_o$$

$$i_o=\frac{16s}{(1-6s)2s}v_o\xrightarrow{(\mathrm{II})}i_o=\frac{16\cancel{s}}{\cancel{(1-6s)}(2\cancel{s})}\times\frac{\cancel{(1-6s)}\,i_s}{10s^2+13s+6s}$$

$$\rightarrow H_2(s)=\frac{i_o}{i_s}=\frac{8}{10s^2+13s+6s}\ \checkmark$$



مقدار ثابت: $H_{(s)} = \frac{v_2}{v_1} = \frac{v_{s2}}{v_{s1}}$

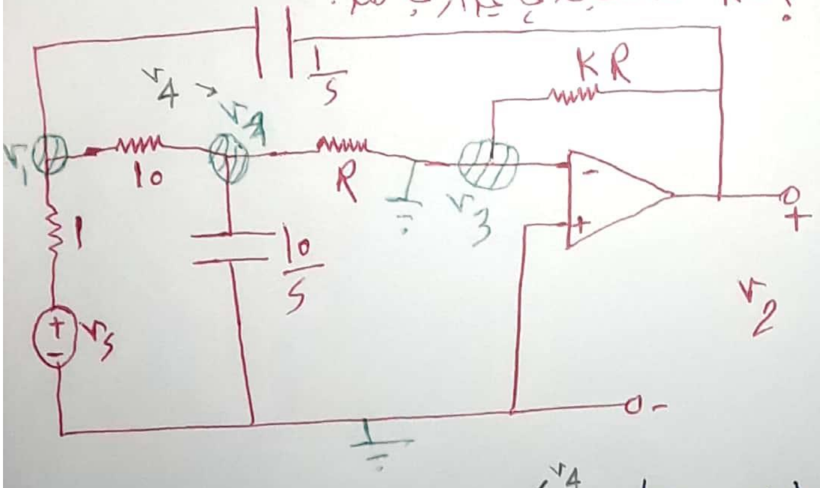
ص ۴

تمرین: $? = \frac{v_2}{v_s}$ $K = ?$ که شبکه پایدار باشد.

op → ایده آل

$A_v = \infty, R_{in} = \infty$

$R_{out} = 0$



$$KCL@v_1: v_1 - v_s + \frac{v_1 - \overset{v_4}{v_4}}{10} + (v_1 - v_2)s = 0$$

$$(s + 1.1)v_1 - 0.1v_4 - sv_2 = v_s \quad (I)$$

$$KCL@v_4: \frac{v_4 - v_1}{10} + \frac{sv_4}{10} + \frac{v_4 - \overset{0}{\cancel{v_3}}}{R} = 0$$

$$-0.1v_1 + \left(\frac{1}{R} + \frac{s}{10} + \frac{1}{10}\right)v_4 = 0 \quad (II)$$

$$KCL@v_3: \frac{\overset{0}{\cancel{v_3}} - v_4}{R} + \frac{\overset{0}{\cancel{v_3}} - v_2}{KR} = 0 \to +\frac{v_4}{R} + \frac{v_2}{KR} = 0 \quad (III)$$

$$v_2 = \frac{\begin{vmatrix} s+1.1 & -0.1 & v_s \\ -0.1 & \frac{1}{R}+\frac{s}{10}+\frac{1}{10} & 0 \\ 0 & \frac{1}{R} & 0 \end{vmatrix}}{\begin{vmatrix} s+1.1 & -0.1 & -s \\ -0.1 & \frac{1}{R}+\frac{s}{10}+\frac{1}{10} & 0 \\ 0 & \frac{1}{R} & \frac{1}{KR} \end{vmatrix}} = \frac{\frac{-v_s}{10R}}{\frac{10Rs^2 + \left(\frac{21}{10}R + \frac{100}{R} + 10KR\right)s + \frac{110}{R} + 10R}{100KR^2}}$$

$$\frac{v_2}{v_s} = -\frac{K}{s^2+\left(2.1+\frac{10}{R}+K\right)s+1+\frac{11}{R}}$$

قطب $P_{(s)} \xrightarrow{R=1} s^2+(12.1+K)s+12=0$

(I) 2
میرای کُند

$\Delta<0 \rightarrow s_{1,2} = \frac{-(12.1+K) \pm \sqrt{(12.1+K)^2-48}}{2}$

$\Delta<0 \rightarrow (12.1+K)^2-48<0 \rightarrow -6.9<12.1+K<6.9$

$-19<K<-5.2$ (I)

$\alpha<0$

$\alpha = -\frac{(12.1+K)}{2}<0 \rightarrow K>-12.1$ (II)

$\xrightarrow{(I)\cap(II)} -12.1<K<-5.2$ ⊛

(II) 2
میرای شدید

$\Delta>0 \rightarrow (12.1+K)^2-48>0 \rightarrow 12.1+K>6.9$
یا
$12.1+K<-6.9$

$K>-5.2$ یا $K<-19$ (III)

$-(12.1+K) \pm \sqrt{(12.1+K)^2-48}<0$ ✓ $\rightarrow K>-12.1$ (IV)

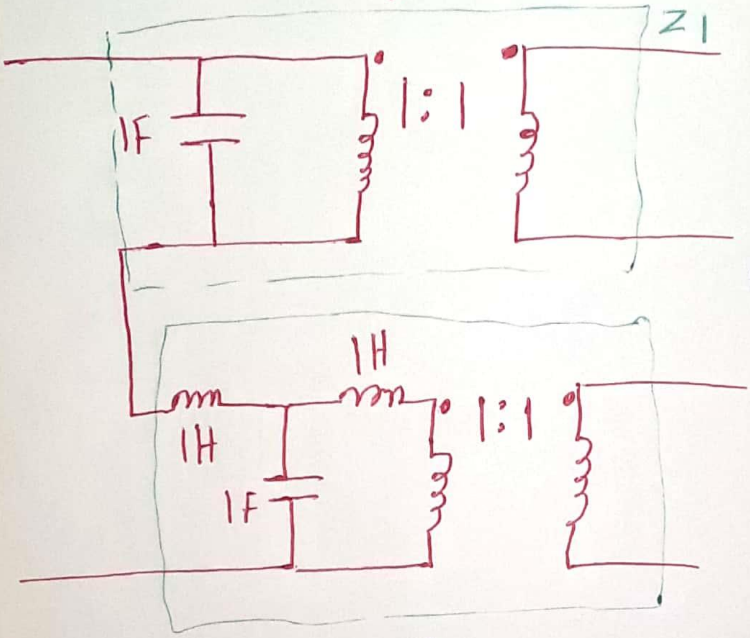
$\xrightarrow{(III)\cap(IV)} K>-5.2$ ⊛⊛ $\quad$ ⊛ $\underset{\text{اجتماع}}{\cup}$ ⊛⊛ $\longrightarrow \boxed{K>-12.1}$

به ازای $K>-12.1$ پایدار اکید

به ازای $K=-12.1$ ← $\alpha=0$ ← نوسانی ⟵ $K\geq-12.1$ پایدار مجانبی

دوقطبی :

تمرین: ماتریس امپدانس شکل مقابل:

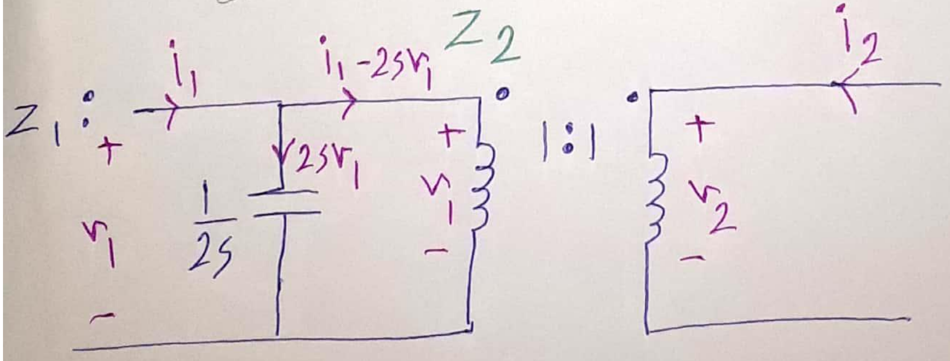


Z_1 سری با Z_2

$Z = Z_1 + Z_2$

$v_1 = Z_{11} i_1 + Z_{12} i_2$

$v_2 = Z_{21} i_1 + Z_{22} i_2$



o طبق روابط
o ترانس

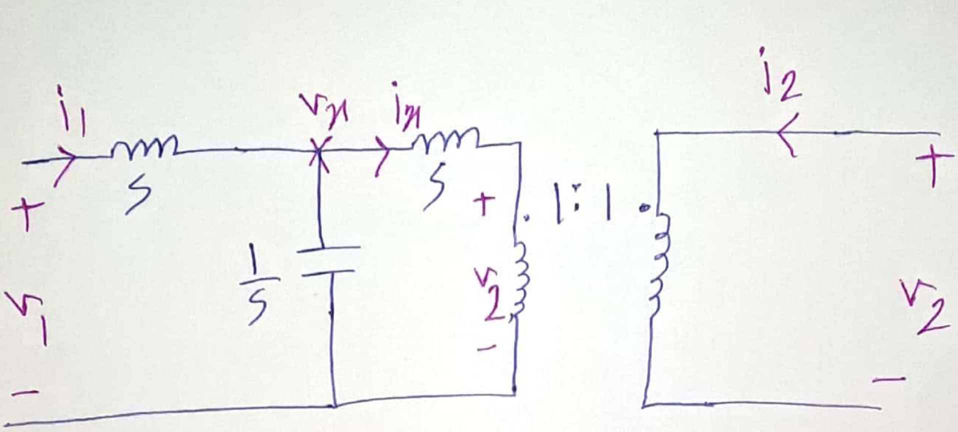
$\frac{v_1}{v_2} = 1 \qquad \frac{i_1 - 2s v_1}{i_2} = -1 \rightarrow v_1 = v_2 \rightarrow$ ماتریس Z_1 متقابل ومتقارن

$i_1 - 2s v_1 = -i_2 \rightarrow v_1 = v_2 = \frac{1}{2s} i_1 + \frac{1}{2s} i_2$

$$Z_1 = \begin{bmatrix} \frac{1}{2s} & \frac{1}{2s} \\ \frac{1}{2s} & \frac{1}{2s} \end{bmatrix}$$

ادامه حل
صفحه بعد

Z_2 :



$$\text{KCl@}v_x : -i_1 + s v_x + \frac{v_x - v_2}{s} = 0$$

$$\longrightarrow v_x = \frac{s i_1 + v_2}{s^2 + 1}$$

$$i_x = \frac{v_x - v_2}{s} = \frac{\frac{s i_1 + v_2}{s^2+1} - v_2}{s} = \frac{\frac{s i_1 + v_2 - s^2 v_2 - v_2}{s^2+1}}{\frac{s}{1}}$$

$$\frac{i_x}{i_2} = -1 \longrightarrow s i_1 + v_2 - s^2 v_2 - v_2 = -s i_2 (s^2+1)$$

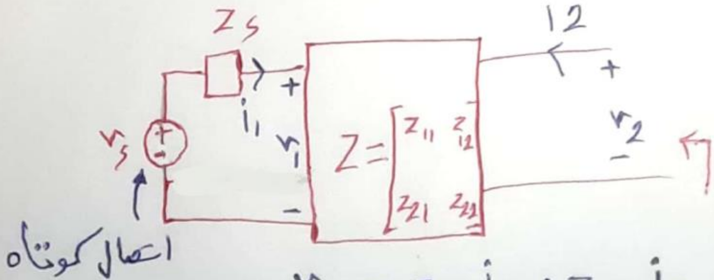
$$v_2 = \frac{1}{s} i_1 + \left(\frac{1}{s} + s\right) i_2$$

$$v_1 = s i_1 + \frac{s i_1 + v_2}{s^2 + 1} \longrightarrow v_1 = s i_1 + \frac{1}{s} i_1 + \frac{1}{s} i_2$$

$$Z_2 = \begin{bmatrix} s + \frac{1}{s} & \frac{1}{s} \\ \frac{1}{s} & \frac{1}{s} + s \end{bmatrix}$$

$$\boxed{Z = Z_1 + Z_2 = \begin{bmatrix} s + \frac{1}{s} + \frac{1}{2s} & \frac{1}{s} + \frac{1}{2s} \\ \frac{1}{s} + \frac{1}{2s} & s + \frac{1}{s} + \frac{1}{2s} \end{bmatrix}}$$

2قطبی: $Z_{out} = ?$



$$Z_{out} = ? = \frac{v_2}{i_2} = ?$$

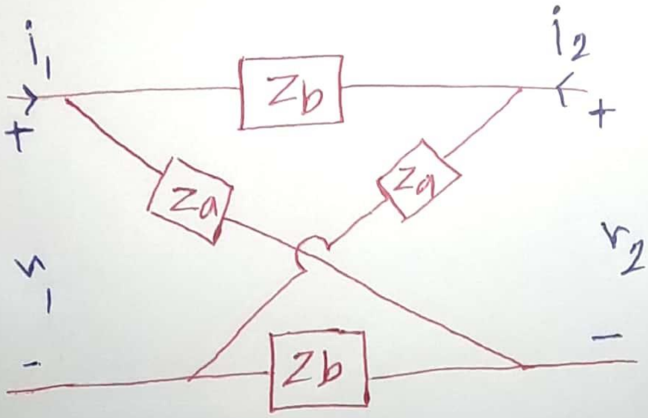
$$v_1 = Z_{11} i_1 + Z_{12} i_2$$

$$v_2 = Z_{21} i_1 + Z_{22} i_2$$

$$v_1 = -Z_s i_1 \rightarrow -Z_s i_1 = Z_{11} i_1 + Z_{12} i_2 \rightarrow i_1 = \frac{Z_{12}}{Z_s + Z_{11}} (-i_2)$$

$$v_2 = \frac{Z_{21} \times Z_{12}}{Z_s + Z_{11}} (-i_2) + Z_{22} i_2 \rightarrow Z_{out} = Z_{22} - \frac{Z_{12} \cdot Z_{21}}{Z_s + Z_{11}}$$

? = Z



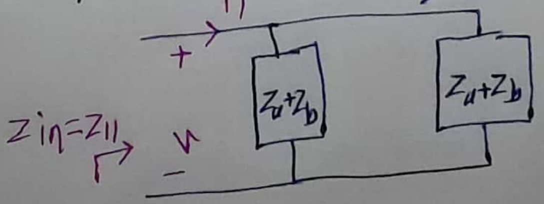
$v_1 = Z_{11} i_1 + Z_{12} i_2$

$v_2 = Z_{21} i_1 + Z_{22} i_2$

$$Z_{11} = \frac{v_1}{i_1}\Big|_{i_2=0}$$

$$\frac{v_1}{i_1} = Z_{11} = Z_{in} = (Z_a+Z_b) \| (Z_a+Z_b)$$

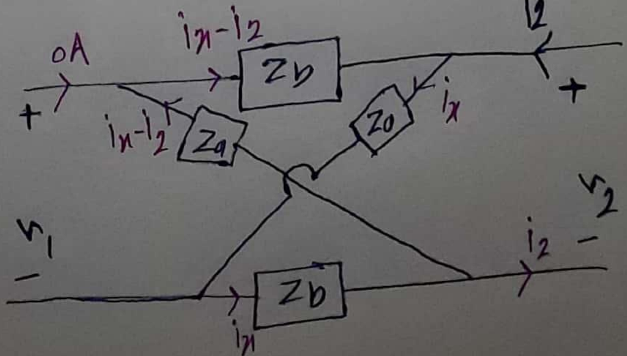
$$Z_{11} = (Z_a+Z_b) \| (Z_a+Z_b) = \frac{1}{2}(Z_a+Z_b)$$

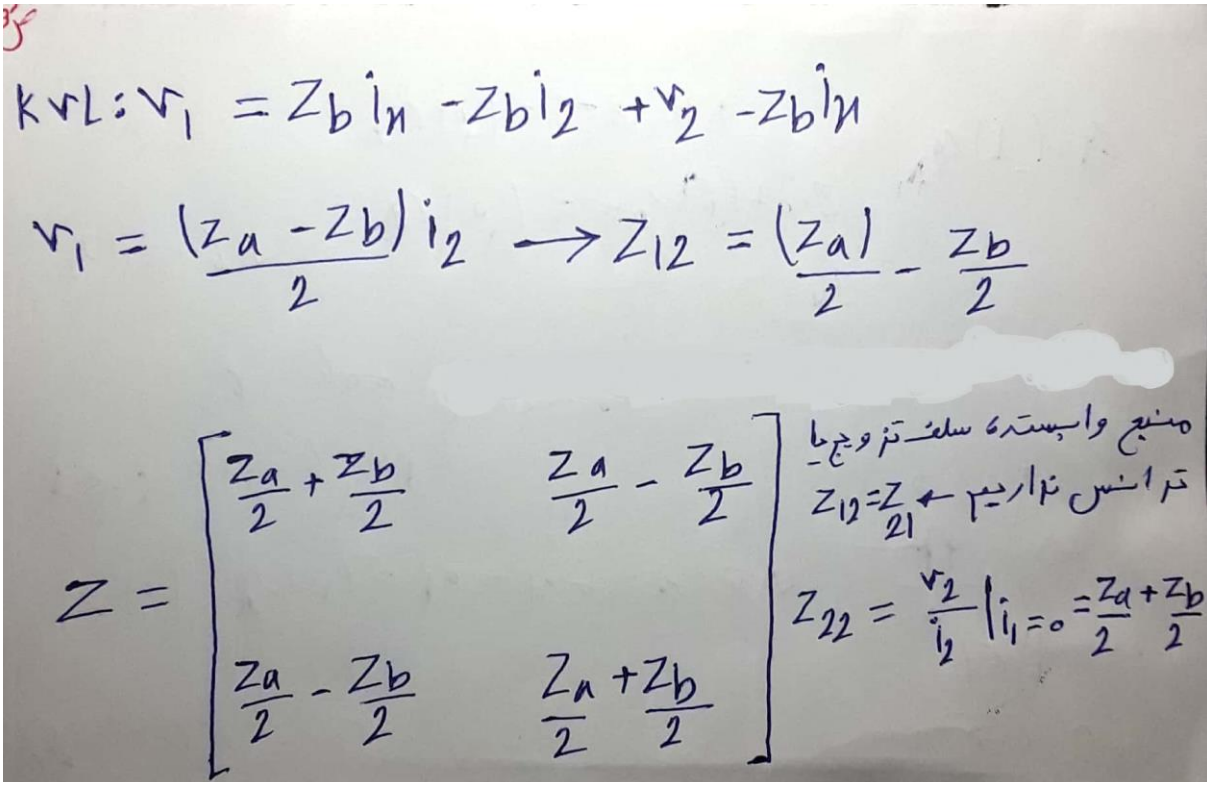


$$Z_{12} = \frac{v_1}{i_2}\Big|_{i_1=0}$$

$$i_x = \frac{v_2}{Z_a+Z_b}$$

$$v_2 = \frac{(Z_a+Z_b)\, i_2}{2}$$





KVL: $v_1 = Z_b i_x - Z_b i_2 + v_2 - Z_b i_x$

$$v_1 = \frac{(Z_a - Z_b)}{2} i_2 \longrightarrow Z_{12} = \frac{(Z_a)}{2} - \frac{Z_b}{2}$$

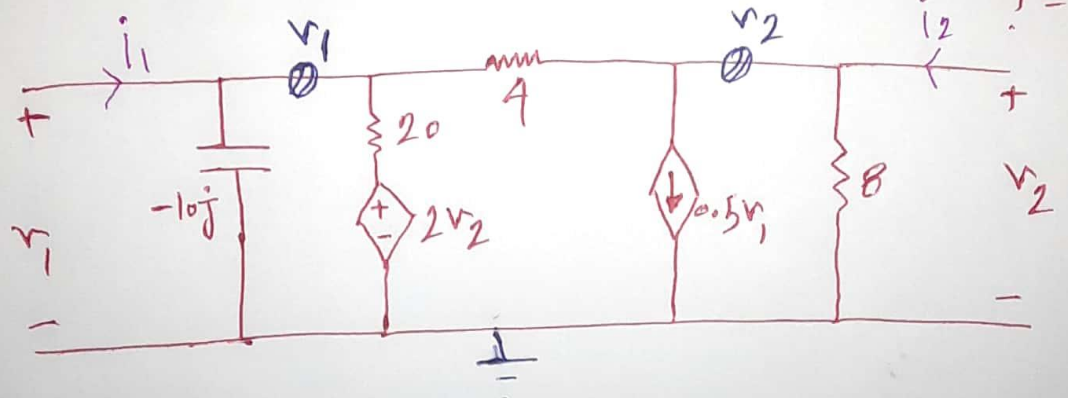
$$Z = \begin{bmatrix} \frac{Z_a}{2} + \frac{Z_b}{2} & \frac{Z_a}{2} - \frac{Z_b}{2} \\ \frac{Z_a}{2} - \frac{Z_b}{2} & \frac{Z_a}{2} + \frac{Z_b}{2} \end{bmatrix}$$

منبع وابسته‌ی سلف تزوج یا ترانس نداریم $\rightarrow Z_{12} = Z_{21}$

$$Z_{22} = \frac{v_2}{i_2}\Big|_{i_1=0} = \frac{Z_a}{2} + \frac{Z_b}{2}$$

ج4

J = Z



$$kcl@v_1 : i_1 = \frac{v_1 j}{10} + \frac{v_1 - 2v_2}{20} + \frac{v_1 - v_2}{4} \cancel{=0}$$

$$(0.3 + 0.1j)v_1 - 0.35v_2 = i_1 \quad (\text{I})$$

$$kcl@v_2 : i_2 = \frac{v_2}{8} + 0.5v_1 + \frac{v_2 - v_1}{4}$$

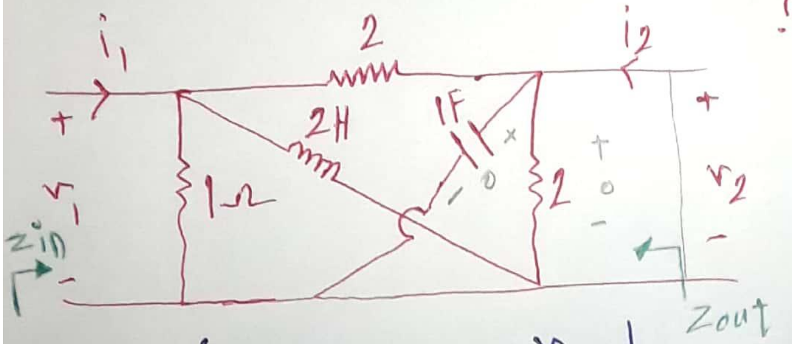
$$0.25v_1 + 0.375v_2 = i_2 \quad (\text{II})$$

$$v_1 = \frac{\begin{vmatrix} i_1 & -0.35 \\ i_2 & +0.375 \end{vmatrix}}{\begin{vmatrix} 0.3+0.1j & -0.35 \\ 0.25 & 0.375 \end{vmatrix}} = \underbrace{(1.8 - 0.34j)}_{Z_{11}} i_1 + \underbrace{(1.7 - 0.3j)}_{Z_{12}} i_2$$

$$Z = \begin{bmatrix} 1.8 - 0.34j & 1.7 - 0.3j \\ 1.2 + 0.22j & 1.54 + 0.2j \end{bmatrix}$$

$$v_2 = \frac{\begin{vmatrix} 0.3+0.1j & i_1 \\ 0.25 & i_2 \end{vmatrix}}{//} = \underbrace{(-1.2 + 0.22j)}_{Z_{21}} i_1 + \underbrace{(1.54 + 0.2j)}_{Z_{22}} i_2$$

تمرین: Z و Y = ?



$$i_1 = Y_{11} v_1 + Y_{12} v_2$$

$$i_2 = Y_{21} v_1 + Y_{22} v_2$$

$$Y_{11} = \frac{i_1}{v_1}\Big|_{v_2=0} \qquad \frac{v_1}{i_1}\Big|_{v_2=0} \rightarrow$$ مقاومت 2Ω و خازن 1F حذف می شود.

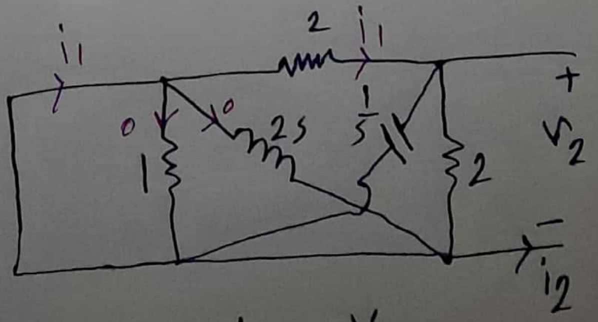
$$\frac{v_1}{i_1} = Z_{in} = 2 \,\|\, 2s \,\|\, 1 = \frac{2s}{1+3s}$$

$$\frac{i_1}{v_1} = \frac{1+3s}{2s} = Y_{11}$$

$$Y_{22} = \frac{i_2}{v_2}\Big|_{v_1=0} = ? \longrightarrow$$ مقاومت 1Ω و سلف 2H حذف می شود. (اتصال کوتاه)

$$Z_{out} = 2 \,\|\, 2 \,\|\, \frac{1}{s} = \frac{1}{s+1} \rightarrow Y_{22} = s+1$$

$$Y_{12} = \frac{i_1}{v_2}\Big|_{v_1=0}$$



$$KVL: 2i_1 + v_2 = 0 \rightarrow Y_{12} = -\frac{1}{2} = Y_{21}$$

$$Y=\begin{bmatrix}\frac{3s+1}{2s} & \frac{-1}{2}\\ \frac{-1}{2} & s+1\end{bmatrix}\rightarrow Z=\begin{bmatrix}s+1 & \frac{1}{2}\\ \frac{1}{2} & \frac{3s+1}{2s}\end{bmatrix}\times\frac{1}{\frac{(s+1)(3s+1)}{2s}-\frac{1}{4}}$$

| روش دوم حل |

$$\text{KCl}@v_1:\ i_1=\left(1+\frac{1}{2s}\right)v_1+\frac{v_1}{2}-\frac{v_2}{2}$$

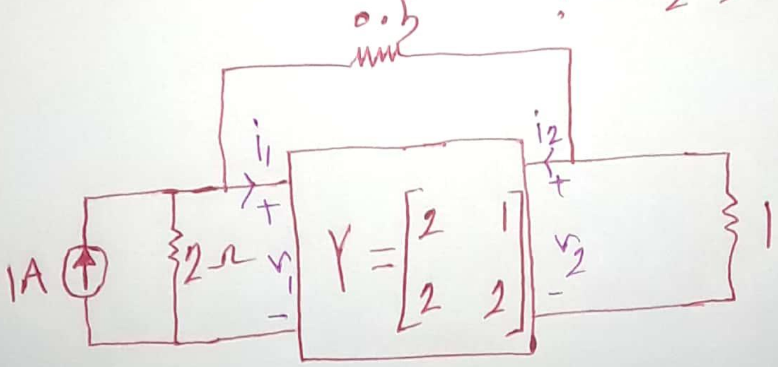
$$\frac{3}{2}v_1+\frac{1}{2s}v_1-\frac{v_2}{2}$$

$$\text{KCl}@v_2:\ i_2=\frac{v_2}{2}+sv_2+\frac{v_2-v_1}{2}\rightarrow$$

$$i_2=-\frac{v_1}{2}+(s+1)v_2$$

$$Y=\begin{bmatrix}\frac{3s+1}{2s} & \frac{-1}{2}\\ \frac{-1}{2} & s+1\end{bmatrix}$$

تمرین: v_1 و v_2 = ?



$i_1 = 2v_1 + v_2 \qquad i_2 = 2v_1 + 2v_2$

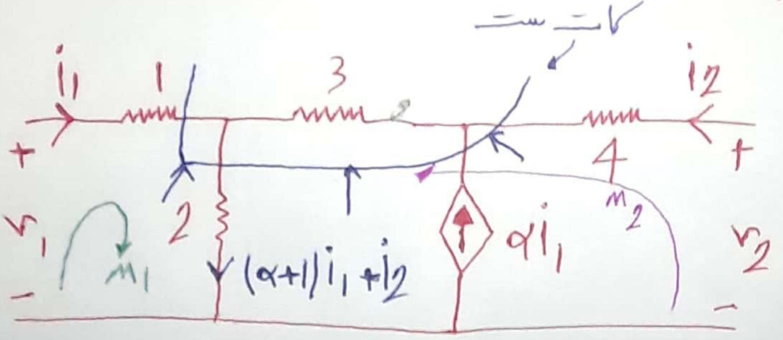
$\text{KCL @ } v_1 : 1 = \frac{v_1 - 0}{2} + 2(v_1 - v_2) + i_1$

$(4.5)v_1 - v_2 = 1$

$\text{KCL @ } v_2 : (v_2 - v_1)2 + i_2 + v_2 = 0 \longrightarrow v_2 = 0$ ✓

$v_1 = \frac{1}{4.5}\ \text{v}$ ✓

تمرین: $\alpha = ?$ تا 2 قطبی ماتریس امپدانس و ادمیتانس نداشته باشد:



$$\text{KVL @}M_1:\ v_1 = i_1 + 2(\alpha+1)i_1 + 2i_2 \rightarrow v_1 = (2\alpha+\overset{\overset{3}{\downarrow}}{3})i_1 + 2i_2 \ \star$$

$$\text{KVL @}M_2:\ v_2 = 4i_2 + 3(\alpha i_1 + i_2) + 2(\alpha+1)i_1 + 2i_2$$

$$v_2 = (5\alpha+2)i_1 + 9i_2 \ \star$$

$$Z = \begin{bmatrix} 2\alpha+3 & 2 \\ 5\alpha+2 & 9 \end{bmatrix} \quad \det|Z| = 0 \longrightarrow$$ مقادیر از α که Y وجود نداشته باشد.

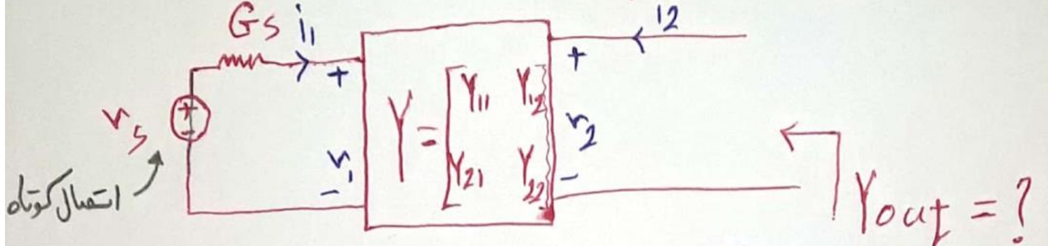
$$\det|Z| = (2a+3)9 - 2(5a+\overset{2}{\ }) = 8a + 23 = 0 \rightarrow a = -\frac{23}{8}$$

به ازا $a = \frac{-23}{8}$ ماتریس ادمیتانس ندارد.

$$\text{inv}(Z) = \begin{bmatrix} \frac{9}{8a+23} & \frac{-2}{8a+23} \\ \frac{-(5a+2)}{8a+23} & \frac{2a+3}{8a+23} \end{bmatrix} = Y \rightarrow \det|Y| = \frac{1}{8a+23}$$

$$\frac{1}{8\alpha+23} = 0 \longrightarrow$$ به ازا هیچ مقدار α $|Y|$ صفر نمی شود پس به ازا همه مقادیر α ماتریس Z وجود دارد.

تمرین: ادمیتانس خروجی = ?



$i_1 = Y_{11} v_1 + Y_{12} v_2 \ (\text{I})$ $\quad i_2 = Y_{21} v_1 + Y_{22} v_2 \ (\text{II})$

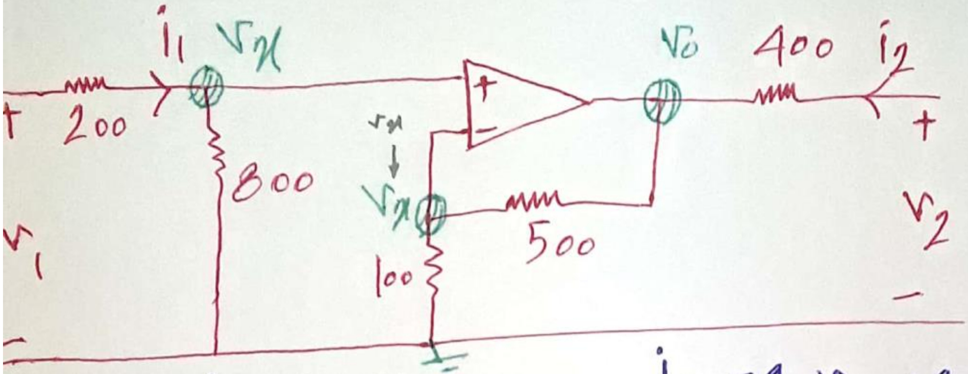
$$Y_{out} = \frac{i_2}{v_2} = ?$$

$$i_1 = -G_s v_1 \ (\text{III}) \xrightarrow{(\text{I})} (-G_s - Y_{11}) v_1 = Y_{12} v_2$$

$$v_1 = \frac{Y_{12}}{-G_s - Y_{11}} v_2 \xrightarrow{(\text{II})} \frac{i_2}{v_2} = Y_{22} - \frac{Y_{12} Y_{21}}{G_s + Y_{11}} = Y_{out}$$

$$Y_{22} \neq Y_{out}$$

ماتریس G و H ، 2 قطبی زیر به صورت جداگانه



$$\left.\begin{aligned} v_1 &= h_{11} i_1 + h_{12} v_2 \\ i_2 &= h_{21} i_1 + h_{22} v_2 \end{aligned}\right\} H \qquad \left.\begin{aligned} i_1 &= g_{11} v_1 + g_{12} i_2 \\ v_2 &= g_{21} v_1 + g_{22} i_2 \end{aligned}\right| G$$

$$v_x = \frac{800}{200+800} v_1 \rightarrow v_x = \frac{4}{5} v_1 \ (\mathrm{I})$$

$$v_x = \frac{100\, v_o}{100+500} \rightarrow v_x = \frac{v_o}{6} \ (\mathrm{II}) \xrightarrow{(\mathrm{I},\mathrm{II})} v_o = \frac{24}{5} v_1$$

$$\frac{v_2 - \frac{24}{5} v_1}{400} = i_2 \rightarrow v_2 = 4.8 v_1 + 400 i_2 \ (\mathrm{III})$$

$$i_1 = \frac{v_1 - \frac{4}{5} v_1}{200} \rightarrow i_1 = \frac{1}{1000} v_1 \ (\mathrm{IV})$$

$$\xrightarrow{(\mathrm{III},\mathrm{IV})} G = \begin{bmatrix} \frac{1}{1000} & 0 \\ 4.8 & 400 \end{bmatrix}$$

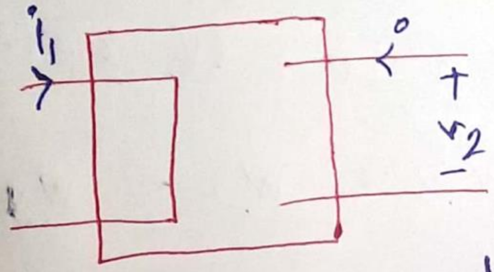
$\xrightarrow{(IV)} v_1 = 1000\, i_1 \quad (VI)$

$\xrightarrow[(VI)]{(III)} v_2 = 4.8\left(1000\, i_1\right) + 400\, i_2 \rightarrow i_2 = -12\, i_1 + \frac{1}{400} v_2$

$$H = \begin{bmatrix} 1000 & 0 \\ -12 & \frac{1}{400} \end{bmatrix}$$

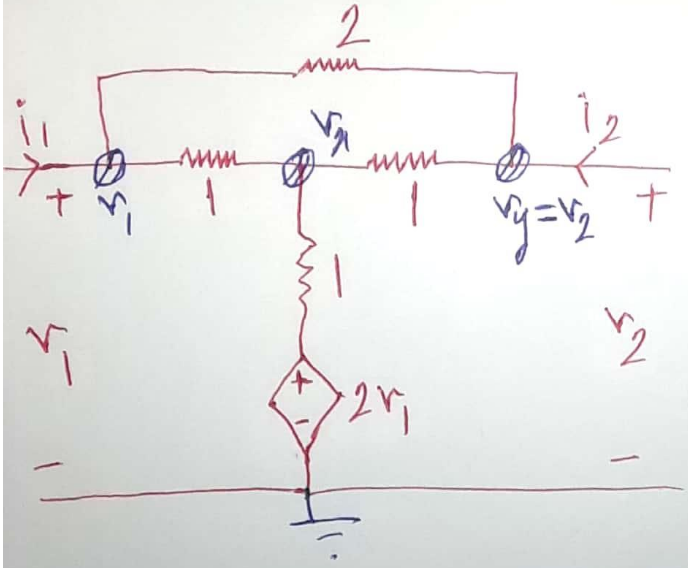
انتظار داریم $H = G^{-1}$ ←

$$G^{-1} = \frac{1000}{400} \begin{bmatrix} 400 & 0 \\ -4.8 & \frac{1}{1000} \end{bmatrix} = H \ ✓$$



$$\left.\begin{array}{l} v_1 = 0 \\ i_2 = 0 \end{array}\right\} \rightarrow H = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\det\{H\} = 0 \rightarrow$ G وجود ندارد



H
$$\begin{cases} v_1 = h_{11} i_1 + h_{12} v_2 \\ i_2 = h_{21} i_1 + h_{22} v_2 \end{cases}$$

G
$$\begin{cases} i_1 = g_{11} v_1 + g_{12} i_2 \\ v_2 = g_{21} v_1 + g_{22} i_2 \end{cases}$$

$KCL @ v_1 : v_1 - v_x + \frac{v_1 - v_2}{2} = i_1 \quad (I)$

$KCL @ v_x : v_x - v_1 + v_x - 2v_1 + v_x - v_2 = 0 \rightarrow v_x = v_1 + \frac{v_2}{3} \leftarrow 3$

$\xrightarrow{(I)} \left(\frac{3}{2}\right) v_1 - \frac{v_2}{2} - \left(v_1 + \frac{v_2}{3}\right) = i_1 \rightarrow i_1 = \frac{1}{2} v_1 - \frac{5}{6} v_2 \quad (II)$

$KCL @ v_2 : i_2 = \frac{v_2 - \left(v_1 + \frac{v_2}{3}\right)}{1} + \frac{v_2 - v_1}{2}$

$i_2 = -\frac{3}{2} v_1 + \frac{7}{6} v_2 \quad (III)$

$\xrightarrow{(II)} v_1 = 2 i_1 + \frac{5}{3} v_2 \quad (IV) \xrightarrow{(III, IV)} i_2 = -\frac{3}{2}\left(2 i_1 + \frac{5}{3} v_2\right) + \frac{7}{6} v_2$

$i_2 = -3 i_1 - \frac{4}{3} v_2 \quad (V) \xrightarrow{(IV, V)} H = \begin{bmatrix} 2 & \frac{5}{3} \\ -3 & -\frac{4}{3} \end{bmatrix}$

$$\xrightarrow{(\text{V})} v_2 = -\frac{9}{4} i_1 - \frac{3}{4} i_2 \quad (\text{VI})$$

$$\xrightarrow{(\text{II})} i_1 = \frac{1}{2} v_1 - \frac{5}{6}\left(-\frac{9}{4} i_1 - \frac{3}{4} i_2\right)$$

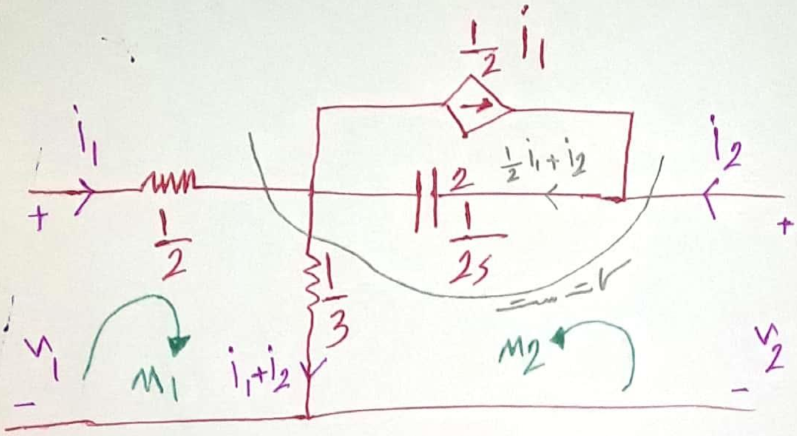
$$\longrightarrow i_1 = -\frac{8}{7}\left(\frac{1}{2} v_1 + \frac{5}{8} i_2\right) \rightarrow i_1 = -\frac{4}{7} v_1 - \frac{5}{7} i_2 \quad (\text{VII})$$

$$\xrightarrow{(\text{VI}, \text{VII})} v_2 = -\frac{9}{4}\left(-\frac{4}{7} v_1 - \frac{5}{7} i_2\right) - \frac{3}{4} i_2$$

$$v_2 = \frac{9}{7} v_1 + \frac{6}{7} i_2 \quad (\text{VIII})$$

$$\xrightarrow{(\text{VII}, \text{VIII})} G = \begin{bmatrix} -\frac{4}{7} & -\frac{5}{7} \\ \frac{9}{7} & \frac{6}{7} \end{bmatrix}$$

برای چک کردن پاسخ می توان از رابطه $H = G^{-1}$ استفاده کرد.



$$\left.\begin{aligned} v_1 &= h_{11} i_1 + h_{12} v_2 \\ i_2 &= h_{21} i_1 + h_{22} v_2 \end{aligned}\right\} H \qquad \left.\begin{aligned} i_1 &= g_{11} v_1 + g_{12} i_2 \\ v_2 &= g_{21} v_1 + g_{22} i_2 \end{aligned}\right\} G$$

$$\text{KVL @ } m_1 : v_1 = \frac{1}{2} i_1 + \frac{1}{3}(i_1 + i_2) \rightarrow v_1 = \frac{5}{6} i_1 + \frac{1}{3} i_2 \quad (\text{I})$$

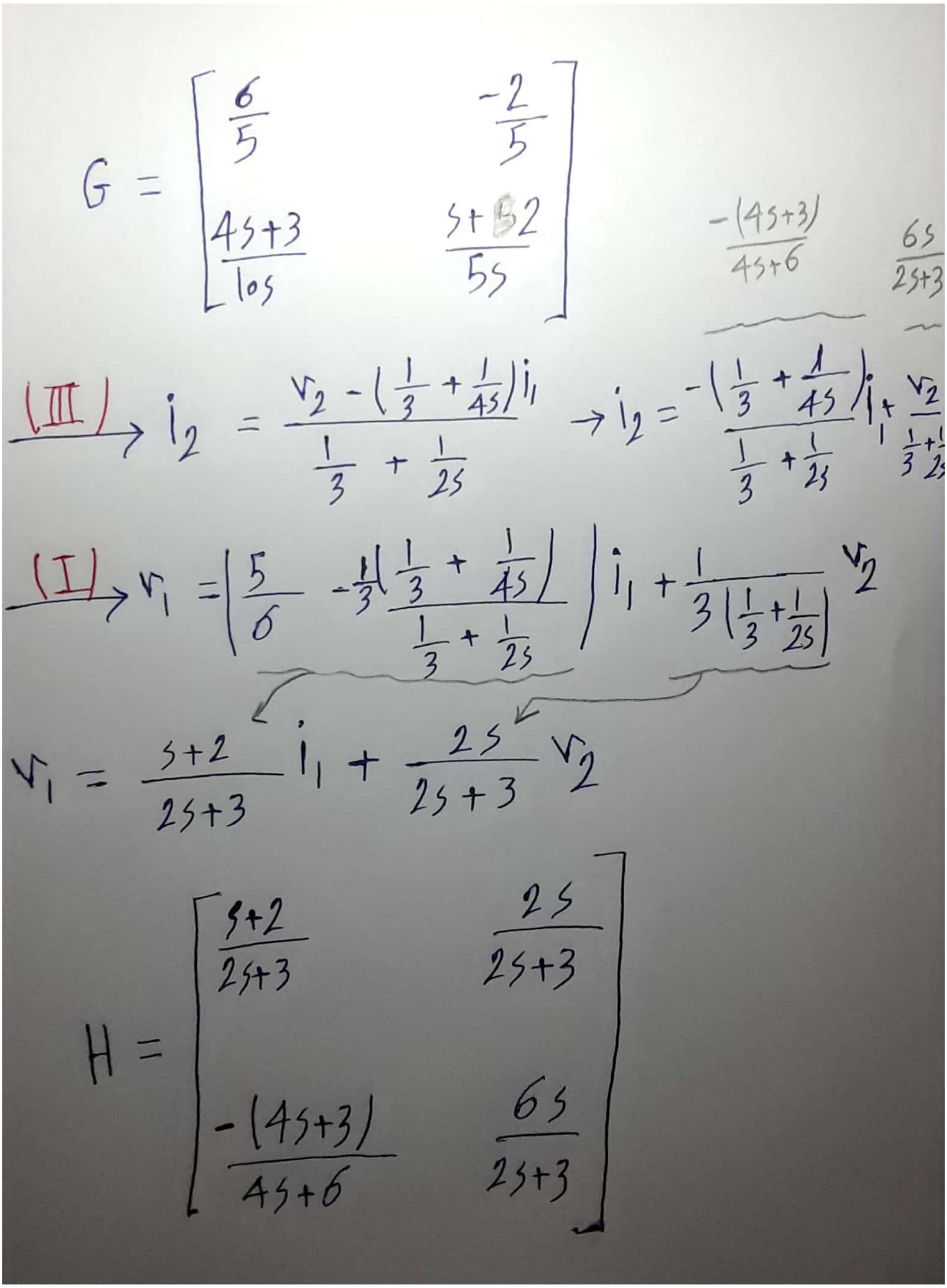
$$\text{KVL @ } m_2 : v_2 = \frac{1}{2s}\left(\frac{1}{2} i_1 + i_2\right) + \frac{1}{3}(i_1 + i_2)$$

$$v_2 = \left(\frac{1}{3} + \frac{1}{4s}\right) i_1 + \left(\frac{1}{3} + \frac{1}{2s}\right) i_2 \quad (\text{III})$$

$$\xrightarrow{(\text{I})} i_1 = \left(v_1 - \frac{1}{3} i_2\right)\frac{6}{5} = \frac{6}{5} v_1 - \frac{2}{5} i_2$$

$$v_2 = \left(\frac{1}{3} + \frac{1}{4s}\right)\left(\frac{6}{5} v_1 - \frac{2}{5} i_2\right) + \left(\frac{1}{3} + \frac{1}{2s}\right) i_2$$

$$v_2 = \frac{6}{5}\left(\frac{1}{3} + \frac{1}{4s}\right) v_1 + \left[\underbrace{\left(\frac{1}{3} + \frac{1}{4s}\right)\left(-\frac{2}{5}\right) + \frac{1}{3} + \frac{1}{2s}}_{\frac{1}{5} + \frac{2}{5s}}\right] i_2 \quad (\text{II})$$



$$G = \begin{bmatrix} \frac{6}{5} & \frac{-2}{5} \\ \frac{4s+3}{10s} & \frac{s+2}{5s} \end{bmatrix}$$

$$\underbrace{\frac{-(4s+3)}{4s+6}}_{} \qquad \underbrace{\frac{6s}{2s+3}}_{}$$

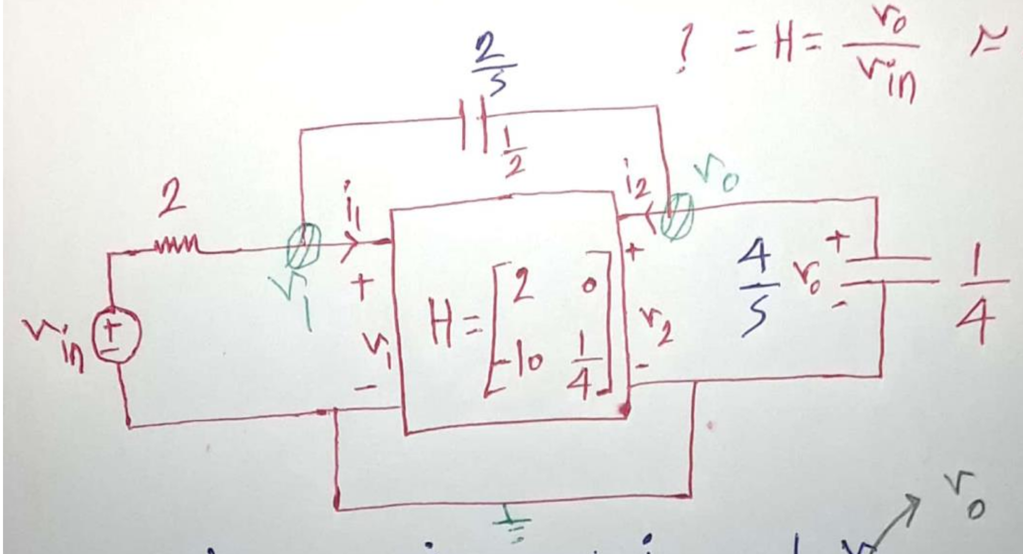
$$\xrightarrow{(\mathrm{III})} i_2 = \frac{v_2 - \left(\frac{1}{3} + \frac{1}{4s}\right) i_1}{\frac{1}{3} + \frac{1}{2s}} \rightarrow i_2 = \frac{-\left(\frac{1}{3} + \frac{1}{4s}\right)}{\frac{1}{3} + \frac{1}{2s}} i_1 + \frac{v_2}{\frac{1}{3} + \frac{1}{2s}}$$

$$\xrightarrow{(\mathrm{I})} v_1 = \left(\underbrace{\frac{5}{6} - \frac{1}{3}\frac{\left(\frac{1}{3} + \frac{1}{4s}\right)}{\frac{1}{3} + \frac{1}{2s}}}_{}\right) i_1 + \underbrace{\frac{1}{3\left(\frac{1}{3} + \frac{1}{2s}\right)}}_{} v_2$$

$$v_1 = \frac{s+2}{2s+3} i_1 + \frac{2s}{2s+3} v_2$$

$$H = \begin{bmatrix} \frac{s+2}{2s+3} & \frac{2s}{2s+3} \\ \frac{-(4s+3)}{4s+6} & \frac{6s}{2s+3} \end{bmatrix}$$

در مدار زیر $H = \frac{v_o}{v_{in}} =$ ؟



$v_1 = 2 i_1$ $\qquad i_2 = -10 i_1 + \frac{1}{4} \overset{v_0}{\cancel{v_2}}$

$i_1 = \frac{v_1}{2}$

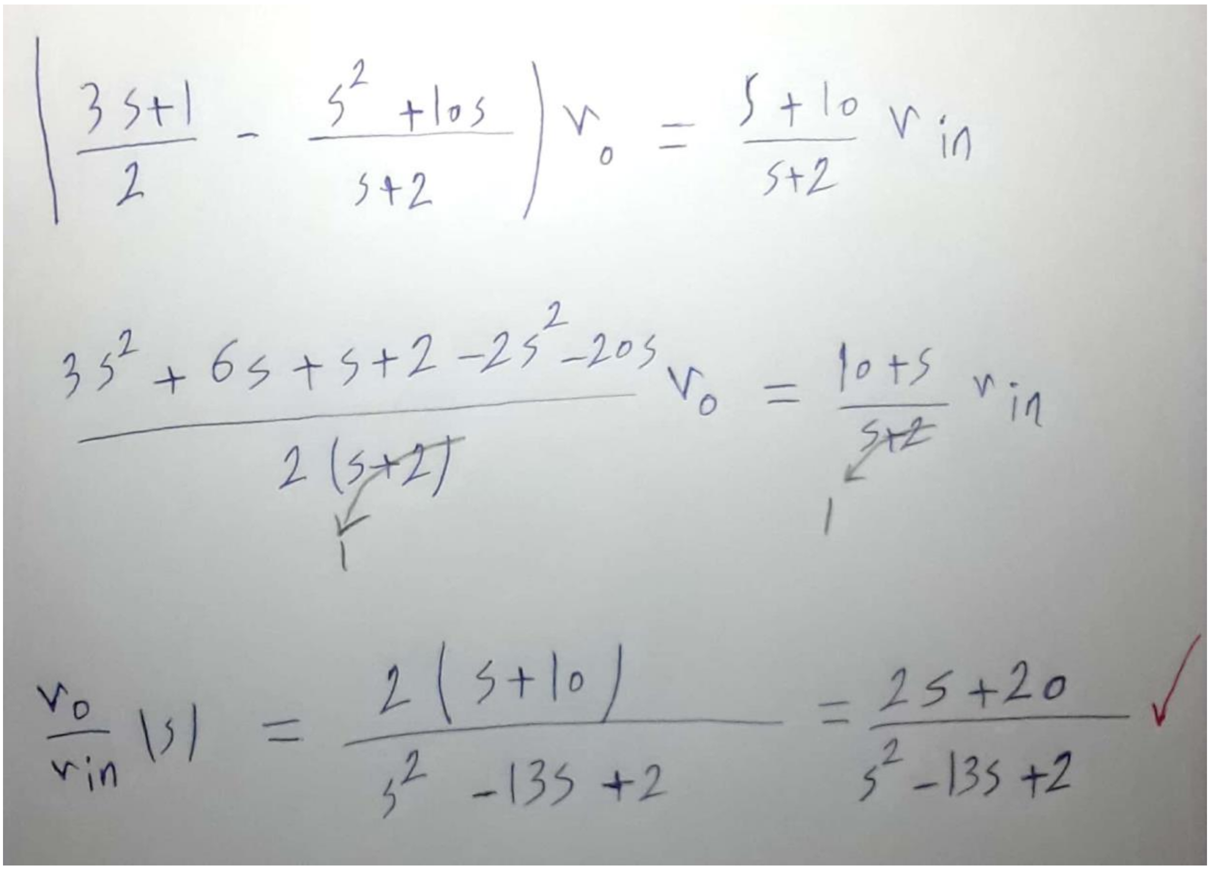
KCl @v_1: $\frac{v_1 - v_{in}}{2} + \frac{v_1}{2} + \frac{(v_1 - v_0)s}{2} = 0$

$v_1 = \frac{s v_0 + v_{in}}{s + 2}$

KCl@v_0: $-10 \overset{\frac{v_1}{2}}{\cancel{i_1}} + \frac{1}{4} v_0 + \frac{v_0 s}{4} + \frac{(v_0 - v_1)s}{2} = 0$

$(\frac{1}{4} + \frac{s}{4} + \frac{s}{2}) v_0 = (5 + \frac{s}{2}) v_1$

$\frac{3s + 1}{\cancel{4} \to 2} v_0 = \frac{s + 10}{\cancel{2} \to 1} \times \frac{s v_0 + v_{in}}{s + 2}$

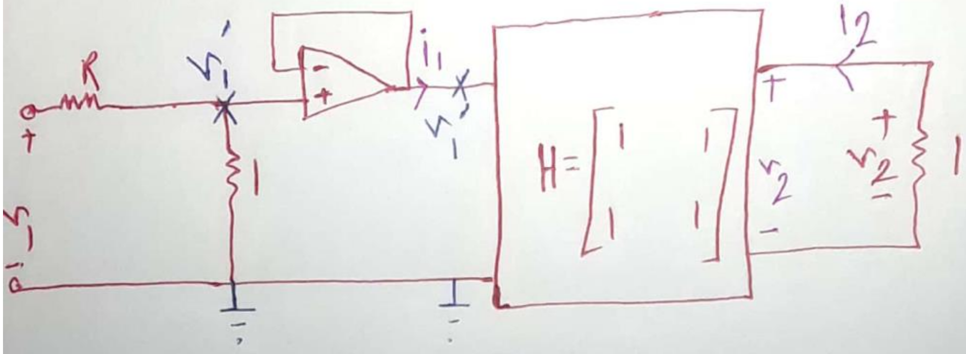


$$\left(\frac{3s+1}{2} - \frac{s^2+10s}{s+2}\right) v_o = \frac{s+10}{s+2} v_{in}$$

$$\frac{3s^2+6s+s+2-2s^2-20s}{2\cancelto{1}{(s+2)}} v_o = \frac{10+s}{\cancelto{1}{s+2}} v_{in}$$

$$\frac{v_o}{v_{in}}(s) = \frac{2(s+10)}{s^2-13s+2} = \frac{2s+20}{s^2-13s+2}$$ ✓

تمرین: در مدار زیر $\frac{v_2}{v_1}=-\frac{1}{2}$ ← $R=?$



$$v_1' = i_1 + v_2 \ (\mathrm{I}), \quad i_2 = i_1 + v_2 \ (\mathrm{II})$$

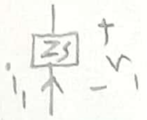
$$v_1' = \frac{v_1}{R+1} \ (\mathrm{III}), \quad i_2 = -v_2 \ (\mathrm{IV})$$

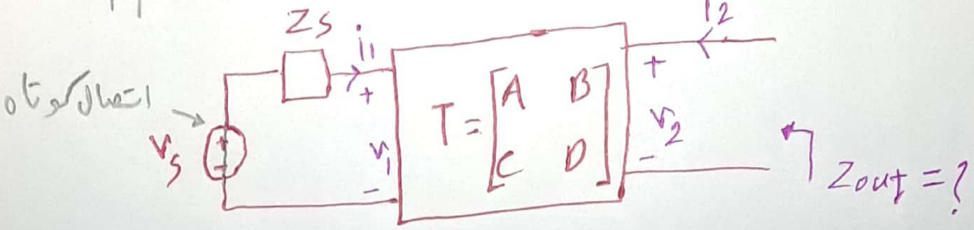
$$\xrightarrow{(\mathrm{I},\ \mathrm{III})} \frac{v_1}{R+1} = i_1 + v_2 \rightarrow i_1 = \frac{v_1}{R+1} - v_2$$

$$\xrightarrow[(\mathrm{IV})]{(\mathrm{II})} -v_2 = \frac{v_1}{R+1} - v_2 + v_2 \rightarrow \frac{v_2}{v_1} = -\frac{1}{R+1}$$

$$-\frac{1}{R+1} = -\frac{1}{2} \rightarrow R+1 = 2 \rightarrow R = 1\,\Omega \ \checkmark$$

تمرین: در مدار زیر $Z_{out} = ?$





$v_1 = A v_2 - B i_2$ $\qquad$ $v_1 = -Z_s i_1$

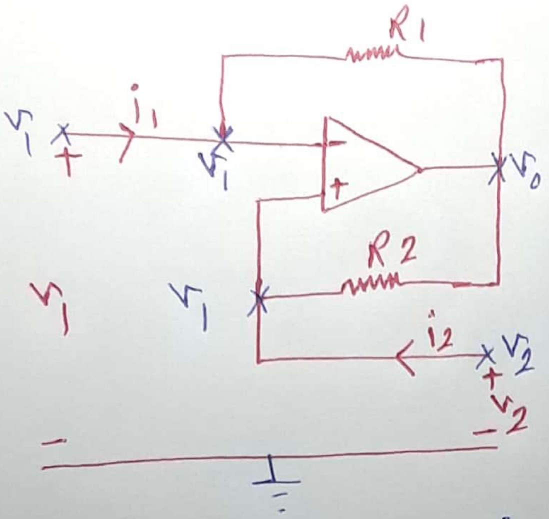
$i_1 = C v_2 - D i_2$ $\qquad$ $\frac{v_2}{i_2} = ?$

$$-Z_s i_1 = A v_2 - B i_2 \rightarrow i_1 = \frac{B i_2 - A v_2}{Z_s}$$

$$\frac{B i_2 - A v_2}{Z_s} = C v_2 - D i_2 \rightarrow \left(\frac{B}{Z_s} + D\right) i_2 = \left(\frac{A}{Z_s} + C\right) v_2$$

$$Z_{out} = \frac{v_2}{i_2} = \frac{D + \frac{B}{Z_s}}{\frac{A}{Z_s} + C} = \frac{B + D Z_s}{A + C Z_s}$$

تمرین ۱ ماتریس T و T' به صورت جداگانه



$$\left.\begin{aligned} v_1 &= A v_2 - B i_2 \\ i_1 &= C v_2 - D i_2 \end{aligned}\right\} T$$

$$\left.\begin{aligned} v_2 &= A' v_1 - B' i_1 \\ i_2 &= C' v_1 - D' i_1 \end{aligned}\right\} T'$$

$v_1 = v_2$ $\quad$ $KCl @ v_{1-} : i_1 = \frac{v_1 - v_0}{R_1} \rightarrow v_0 = v_1 - R_1 i_1$ (I)

opamp - سر

$KCl @ v_{1+} : i_2 = \frac{v_1 - v_0}{R_2} \rightarrow v_1 - v_1 + R_1 i_1 = R_2 i_2 \rightarrow i_1 = \frac{R_2}{R_1} i_2$

opamp + سر

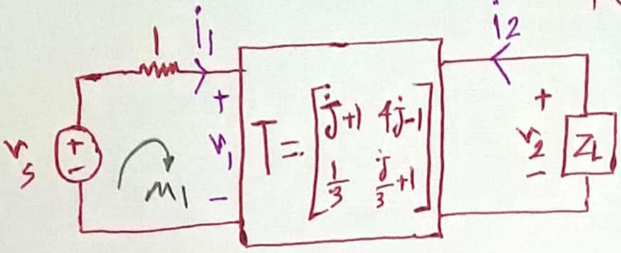
$$\left.\begin{aligned} v_1 &= v_2 - 0 i_2 \\ i_1 &= 0 v_2 - \left(-\frac{R_2}{R_1}\right) i_2 \end{aligned}\right\} \longrightarrow T = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{R_2}{R_1} \end{bmatrix}$$

$$\left.\begin{aligned} v_2 &= 1 v_1 - 0 i_1 \\ i_2 &= 0 v_1 - \left(-\frac{R_1}{R_2}\right) i_1 \end{aligned}\right\} \longrightarrow T' = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{R_1}{R_2} \end{bmatrix}$$

همان طور که مشخص است $T = (T')^{-1}$

تمرین | در مدار زیر تحت چه شرایطی حداکثر توان به Z_L منتقل می‌شود؟

$V_s = 15\angle 6$ ← $V_2 = ?$ (فازور) |



$A = j + 1$

$B = 4j - 1$

$C = \frac{1}{3}$

$D = 1 + \frac{j}{3}$

برای آنکه حداکثر توان به Z_L برسد باید $Z_L = Z_{out}^*$

Z_{out} در تمرین ص 30 محاسبه شده است.

$$Z_{out} = \frac{B + DZ_s}{A + CZ_s} \overset{Z_s=1}{=} \frac{4j - 1 + j/3 + 1}{j + \underbrace{1 + \frac{1}{3}}_{4/3}}$$

$$Z_{out} = \frac{\frac{13j}{3}}{\frac{3j+4}{3}} \rightarrow Z_{out} = \frac{13j}{3j+4} \times \frac{(-3j+4)}{(-3j+4)}$$

$$Z_{out} = \frac{39 + 52j}{25} = 1.56 + 2.08j = Z_{out}$$

$$\boxed{Z_L = 1.56 - 2.08j} \leftarrow Z_L = Z_{out}^*$$

$$v_1 = (j+1)v_2 - (4j-1)i_2 \ (\text{I}) \qquad v_s = 15.6$$

$$i_1 = \frac{1}{3}v_2 - \left(1+\frac{j}{3}\right)i_2 \ (\text{II})$$

$$\text{KVL @ } m_1: v_s = i_1 + v_1 = 15.6 \ (\text{III})$$

$$v_2 = -Z_L i_2 \longrightarrow i_2 = \frac{v_2}{-1.56+2.08j} \rightarrow i_2 = (-0.2308-0.3077j)v_2$$

$$i_2 = (-0.2308-0.3077j)v_2$$

$$\xrightarrow{(\text{I})} v_1 = (j+1)v_2 - (4j-1)(-0.2308-0.3077j)v_2$$

$$v_1 = (-0.4616+1.6155j)v_2$$

$$\xrightarrow{(\text{II}),(\text{III})} i_1 + v_1 = 15.6$$

$$\frac{1}{3}v_2 - \left(1+\frac{j}{3}\right)(-0.2308-0.3077j)v_2 + (-0.4616+1.6155j)v_2 = 15.6$$

$$\rightarrow 2j\,v_2 = 15.6 \rightarrow v_2 = -7.8j = 7.8\angle -90^\circ = v_2$$