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$\boxed{1}$ مولفه ها را جداگانه بررسی میکنیم.

$$S_xS_y - S_yS_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \hbar\left(\frac{\hbar}{4}\right)\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \hbar\left(\frac{\hbar}{4}\right)\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$$

$$= i\hbar\left(\frac{\hbar}{2}\right)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = i\hbar S_z = (\vec{S}\times\vec{S})_z$$

$$(\vec{S}\times\vec{S})_y = S_zS_x - S_xS_z = \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$-\frac{\hbar^2}{4}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{\hbar^2}{4}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - \frac{\hbar^2}{4}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$= i\hbar\left(\frac{\hbar}{2}\right)\begin{pmatrix} 0 & 1/i \\ -1/i & 0 \end{pmatrix} = i\hbar\left(\frac{\hbar}{2}\right)\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = i\hbar S_y$$

$$(i^2 = -1 \Rightarrow 1/i = -i)$$

$$(\vec{S}\times\vec{S})_x = S_yS_z - S_zS_y = \frac{\hbar^2}{4}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\cdot\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \frac{\hbar^2}{4}\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} - \frac{\hbar^2}{4}\begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} = i\hbar\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= i\hbar S_x$$

I

$$\Rightarrow \quad \vec{S} \times \vec{S} = i\hbar \vec{S}$$

در واقع هر مولفه‌ی این بردار جابه‌جاگر دو ماتریس را نشان می‌دهد.

وضمنا می‌بینیم که مثلا $S_x S_y = -S_y S_x$

$\boxed{2}$

$$|\psi\rangle = A\begin{pmatrix} 1 \\ 1+3i \end{pmatrix} \qquad \langle\psi|\psi\rangle = AA^* \begin{pmatrix} 1 & 1-3i \end{pmatrix} \begin{pmatrix} 1 \\ 1+3i \end{pmatrix}$$

$$= AA^* \left(1 + (1+9)\right) = 11\, AA^* = 1$$

$$|A| = \frac{1}{\sqrt{11}} \longrightarrow A = \frac{e^{i\theta}}{\sqrt{11}}$$

با توجه به بی ارزش بودن فاز در $|\psi\rangle$ می‌توان A را همان $\frac{1}{\sqrt{11}}$ در نظر گرفت.

$$\longrightarrow |\psi\rangle = \frac{1}{\sqrt{11}} \begin{pmatrix} 1 \\ 1+3i \end{pmatrix}$$

$$\langle\psi|S_x|\psi\rangle = \frac{\hbar}{2} \cdot \frac{1}{11} \begin{pmatrix} 1 & 1-3i \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1+3i \end{pmatrix}$$

$$= \frac{\hbar}{22} \begin{pmatrix} 1 & 1-3i \end{pmatrix} \begin{pmatrix} 1+3i \\ 1 \end{pmatrix} = \frac{\hbar}{22}(1+3i+1-3i) = \frac{\hbar}{11} \Big]_x$$

$$\langle\psi|S_y|\psi\rangle = \frac{\hbar}{2} \cdot \frac{1}{11} \begin{pmatrix} 1 & 1-3i \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1+3i \end{pmatrix} = \frac{\hbar}{22} \begin{pmatrix} 1 & 1-3i \end{pmatrix} \begin{pmatrix} -i+3 \\ i \end{pmatrix}$$

$$= \frac{\hbar}{22}(-i+3+i+3) = \frac{6\hbar}{22} \Big]_y$$

$$\langle\psi|S_z|\psi\rangle = \frac{\hbar}{2} \cdot \frac{1}{11} \begin{pmatrix} 1 & 1-3i \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1+3i \end{pmatrix} = \frac{\hbar}{22} \begin{pmatrix} 1 & 1-3i \end{pmatrix} \begin{pmatrix} 1 \\ -1-3i \end{pmatrix}$$

$$= \frac{\hbar}{22}(1-1-9) = \frac{-9\hbar}{22} \Big]_z$$

$$S_x^2 = \frac{\hbar^2}{4}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}^2 = \frac{\hbar^2}{4}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$S_y^2 = \frac{\hbar^2}{4}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}^2 = \frac{\hbar^2}{4}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$S_z^2 = \frac{\hbar^2}{4}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = \frac{\hbar^2}{4}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\Rightarrow \quad S_x^2 = S_y^2 = S_z^2 = \frac{\hbar^2}{4} I$$

$$\sigma_{S_a}^2 = \langle \psi | S_a^2 | \psi \rangle - \langle S_a \rangle^2 = \langle \psi | \frac{\hbar^2}{4} I | \psi \rangle - \langle S_a \rangle^2$$

$$= \frac{\hbar^2}{4} \underbrace{\langle \psi | \psi \rangle}_{1} - \langle S_a \rangle^2 = \frac{\hbar^2}{4} - \langle S_a \rangle^2$$

$$\sigma_{S_x}^2 = \frac{\hbar^2}{4} - \left(\frac{\hbar}{11}\right)^2 = \frac{117\hbar^2}{484} \Longrightarrow \sigma_{S_x} = \frac{3\sqrt{13}}{22}\hbar$$

$$\sigma_{S_y}^2 = \frac{\hbar^2}{4} - \left(\frac{6\hbar}{22}\right)^2 = \frac{85\hbar^2}{484} \Longrightarrow \sigma_{S_y} = \frac{\sqrt{85}}{22}\hbar$$

$$\sigma_{S_z}^2 = \frac{\hbar^2}{4} - \left(\frac{-9\hbar}{22}\right)^2 = \frac{10\hbar^2}{121} \Longrightarrow \sigma_{S_z} = \frac{\sqrt{10}}{11}\hbar$$

$$\sigma_{S_x}\sigma_{S_y} = \frac{3\sqrt{13}}{22}\cdot\frac{\sqrt{85}}{22}\hbar^2$$

$$\frac{\hbar}{2}\langle S_z \rangle = \frac{\hbar}{2}\left(\frac{-9\hbar}{22}\right) = \frac{-9}{44}\hbar^2$$

$$\overbrace{\sigma_{S_x}\sigma_{S_y}}^{\simeq 0.206\hbar^2} \geqslant \overbrace{\frac{\hbar}{2}\langle S_z \rangle}^{\simeq -0.205\hbar^2} \quad \checkmark$$

$$, \quad \sigma_{S_x}\sigma_{S_y} \geqslant -\frac{\hbar}{2}\langle S_z \rangle$$

$$\sigma_{S_y}\sigma_{S_z} = \frac{\sqrt{85}}{22}\cdot\frac{\sqrt{10}}{11}\hbar^2$$

$$\frac{\hbar}{2}\langle S_x \rangle = \frac{\hbar}{2}\left(\frac{\hbar}{11}\right) = \frac{\hbar^2}{22}$$

$$\overbrace{\sigma_{S_y}\sigma_{S_z}}^{\simeq 0.12\hbar^2} \geqslant \overbrace{\frac{\hbar}{2}\langle S_x \rangle}^{\simeq 0.05\hbar^2} \quad \checkmark$$

$$\sigma_{S_z}\sigma_{S_x} = \frac{\sqrt{10}}{11}\cdot\frac{3\sqrt{13}}{22}\hbar^2$$

III $$\frac{\hbar}{2}\langle S_y \rangle = \frac{\hbar}{2}\left(\frac{6\hbar}{22}\right) = \frac{3\hbar^2}{22}$$

$$\overbrace{\sigma_{S_z}\sigma_{S_x}}^{\simeq 0.141\hbar^2} \geqslant \overbrace{\frac{\hbar}{2}\langle S_y \rangle}^{0.136\hbar^2} \quad \checkmark$$