International Conference on Robotics and Mechatronics 2024

Dynamic Modeling of Double Segment Redundant Gough-Stewart Hybrid Manipulator based on the Principle of Virtual Work

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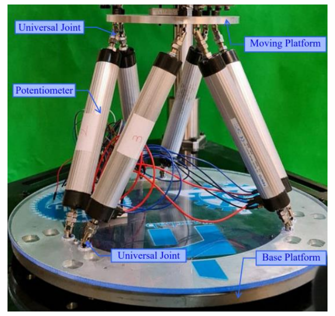
- Mehdi Tale Masouleh — Taar Lab - School of Electrical & Computer Engineering, University of Tehran
- Arash Bahrami — School of Mechanical Engineering, University of Tehran

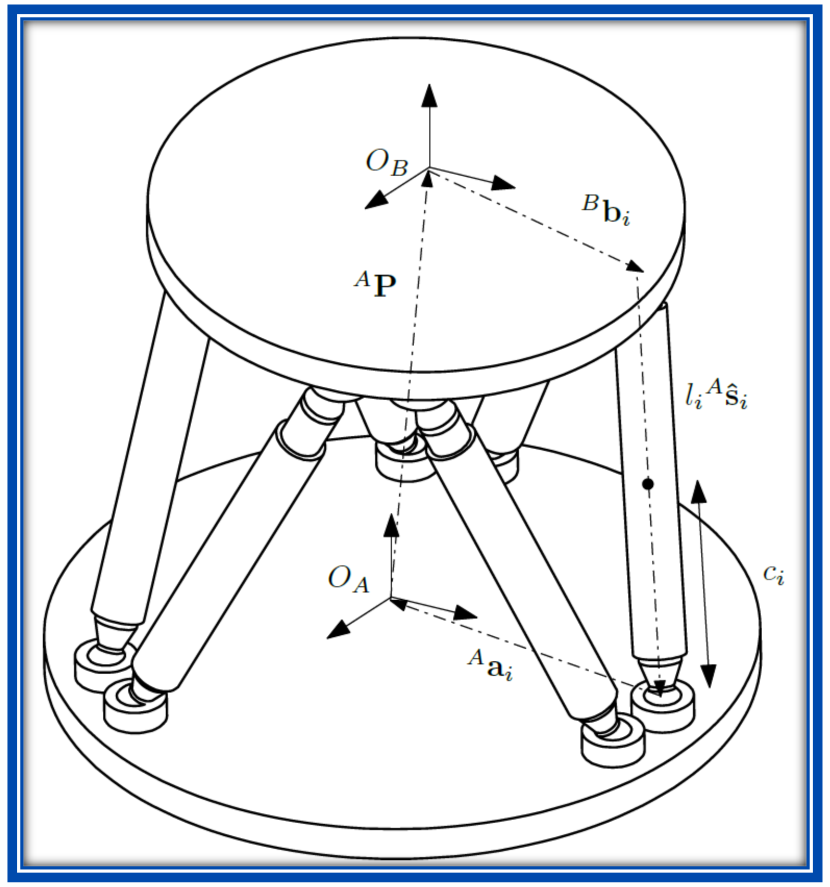
Gough-Stewart (GS) Parallel Robot

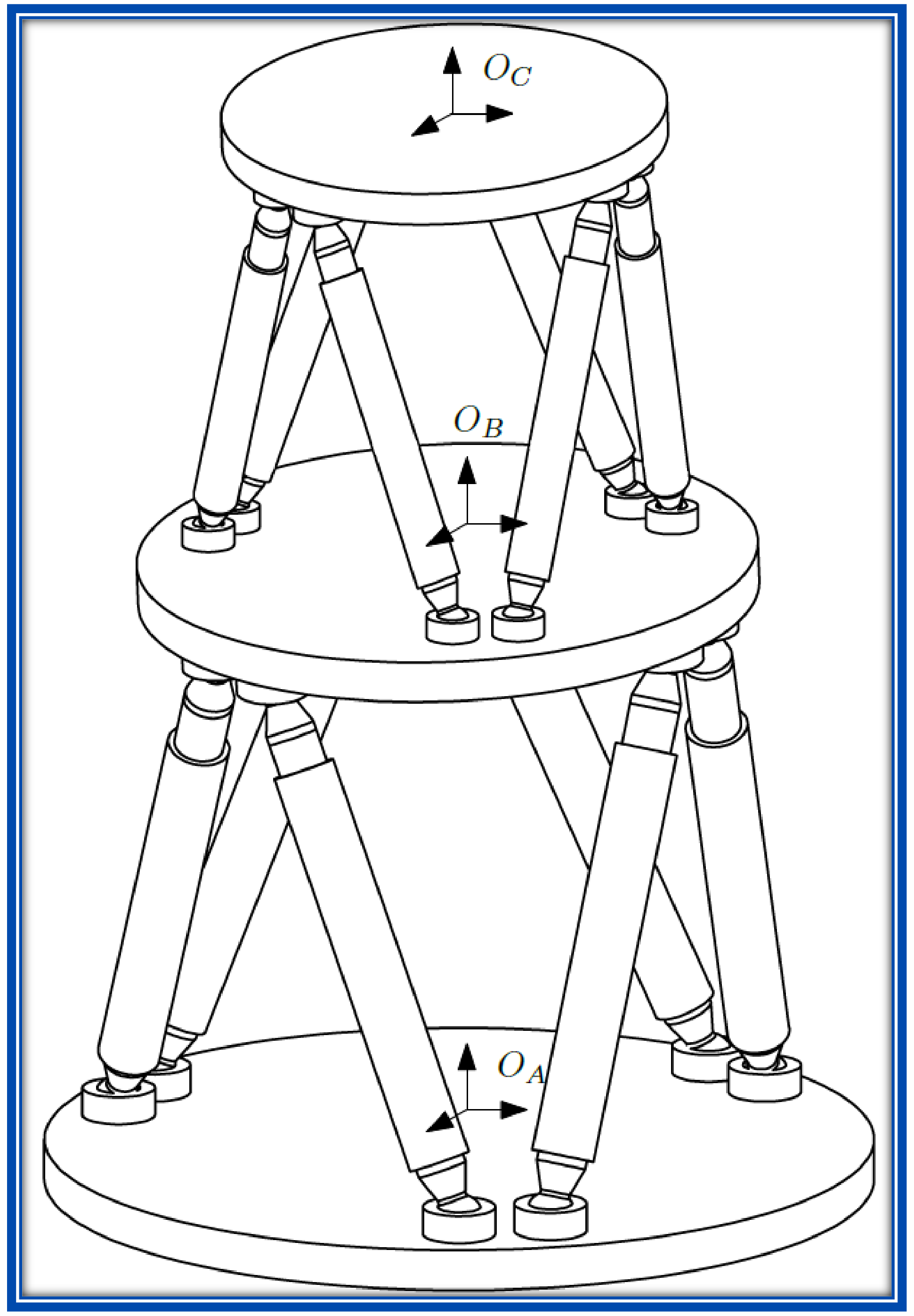


Configuration (UPS)

- Moving Platforms
- 6 Limbs (Cylinder – Pistons)
- Joints (Spherical – Prismatic – Universal)

Redundant Hybrid Serial-Parallel Manipulators





(1) M. B. M. Damnab, M. T. Masouleh and M. R. H. Yazdi, "Designing and Developing a 6-DOF Calibration Setup Based on the Gough-Stewart Platform Equipped by Potentiometer Sensors," ICROM 2023

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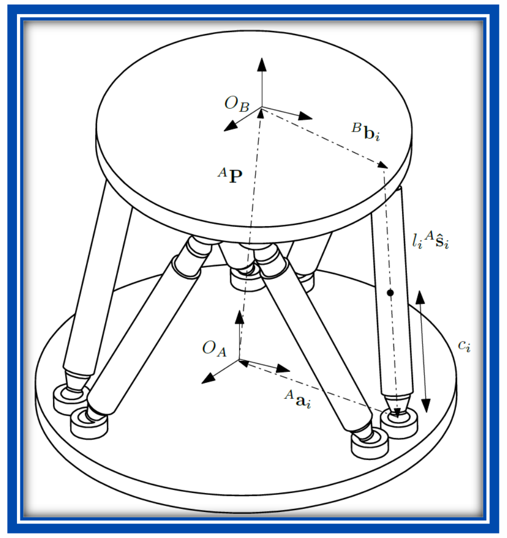

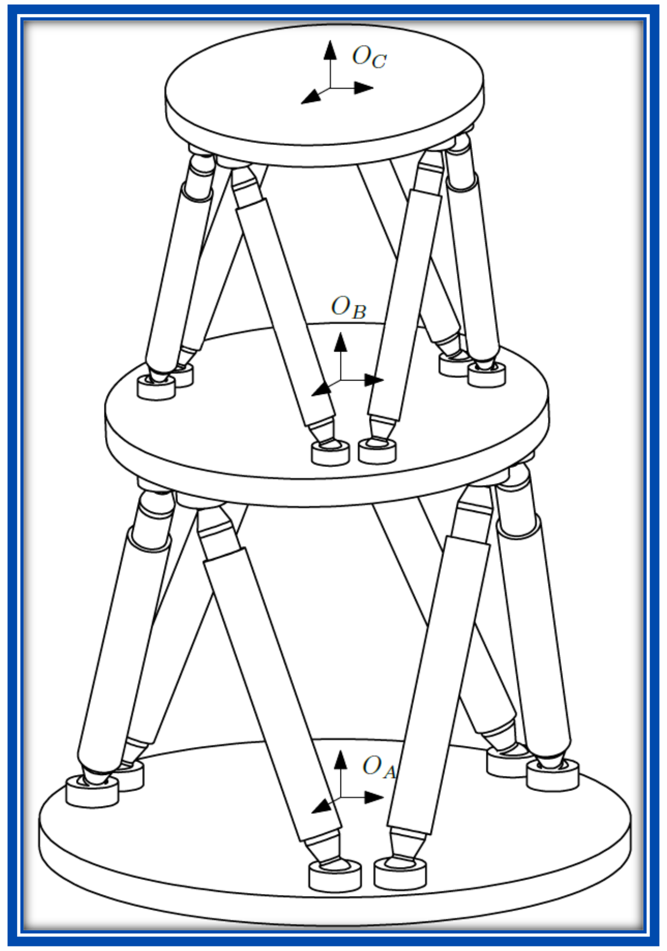

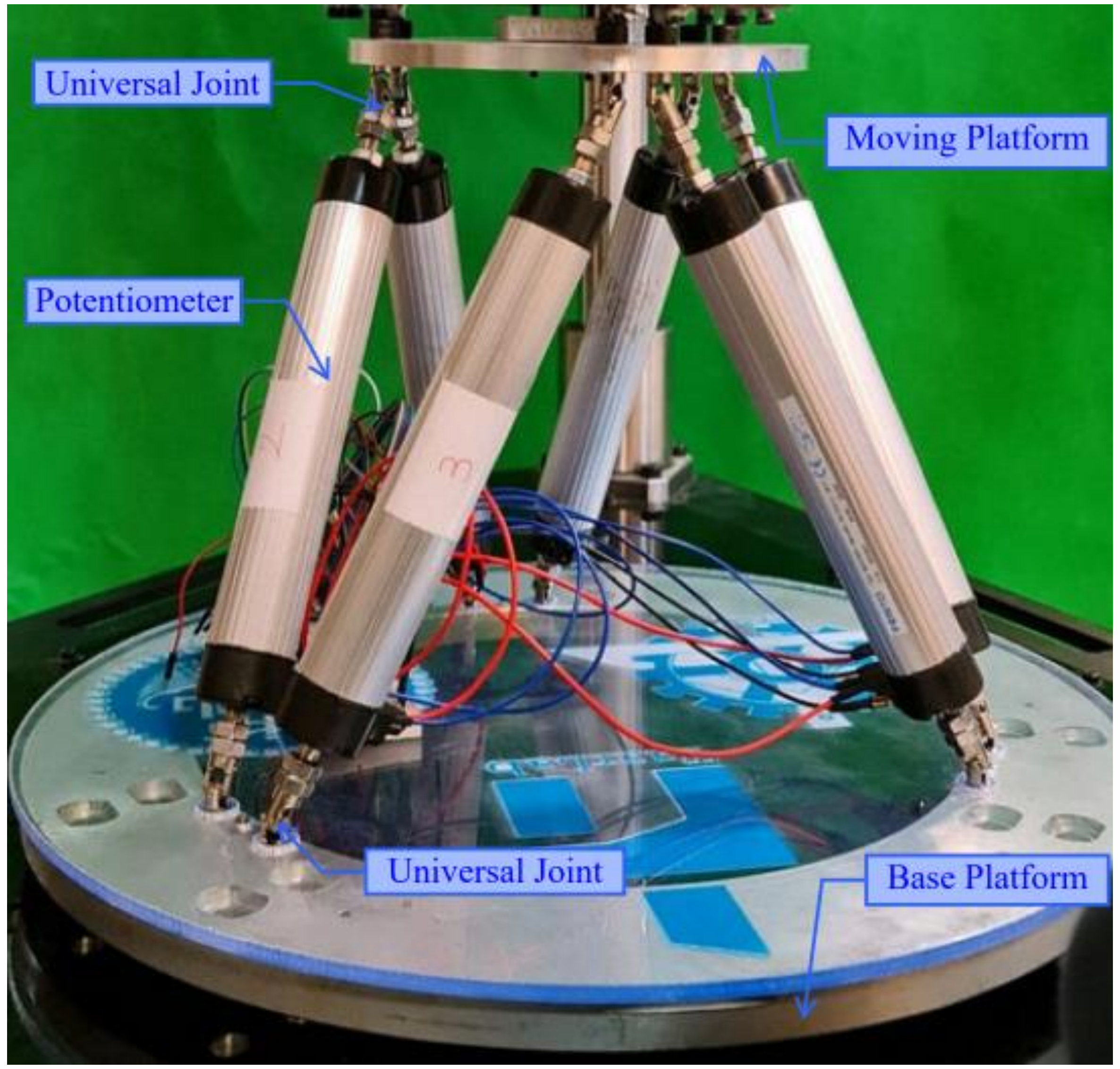

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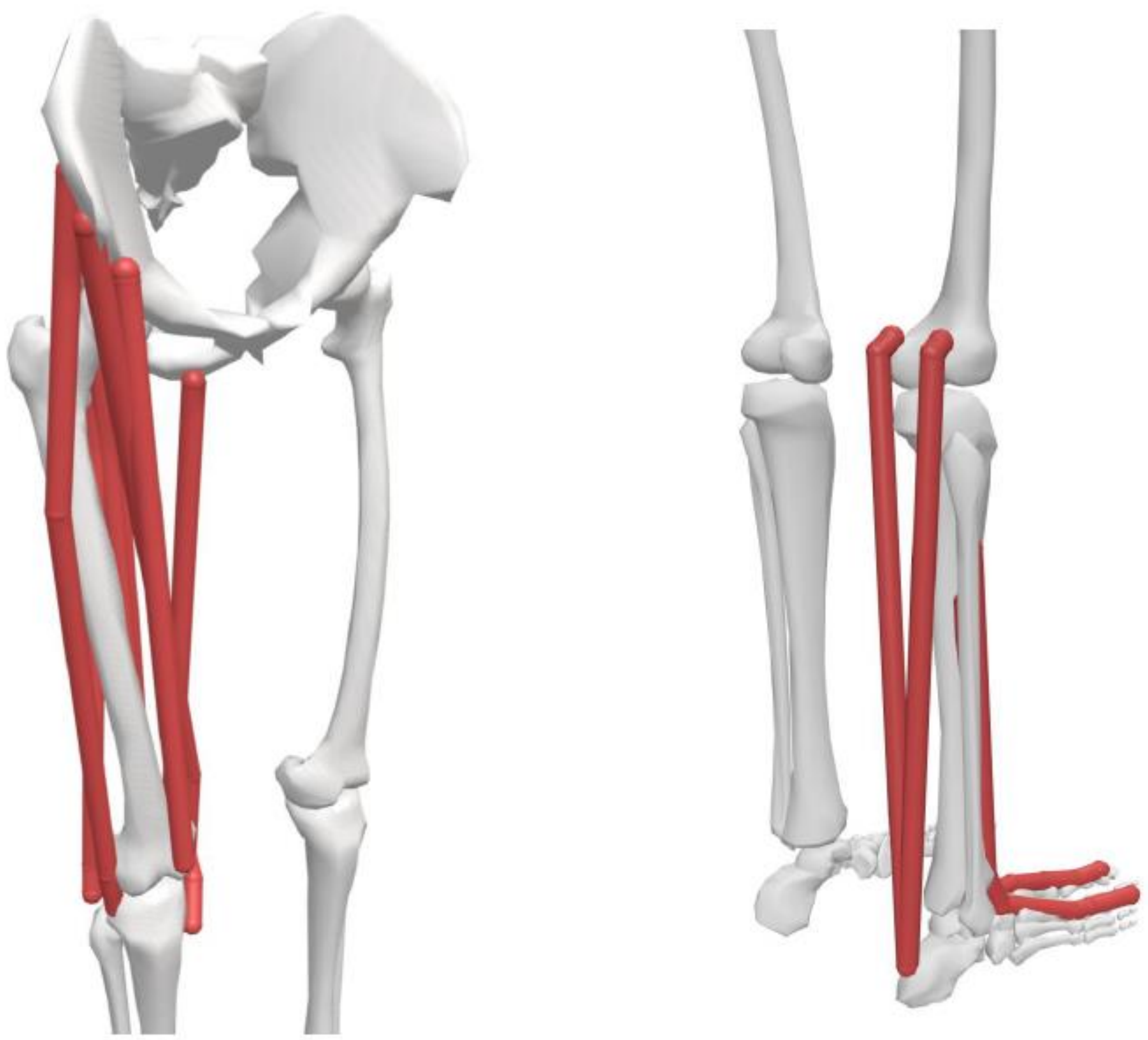
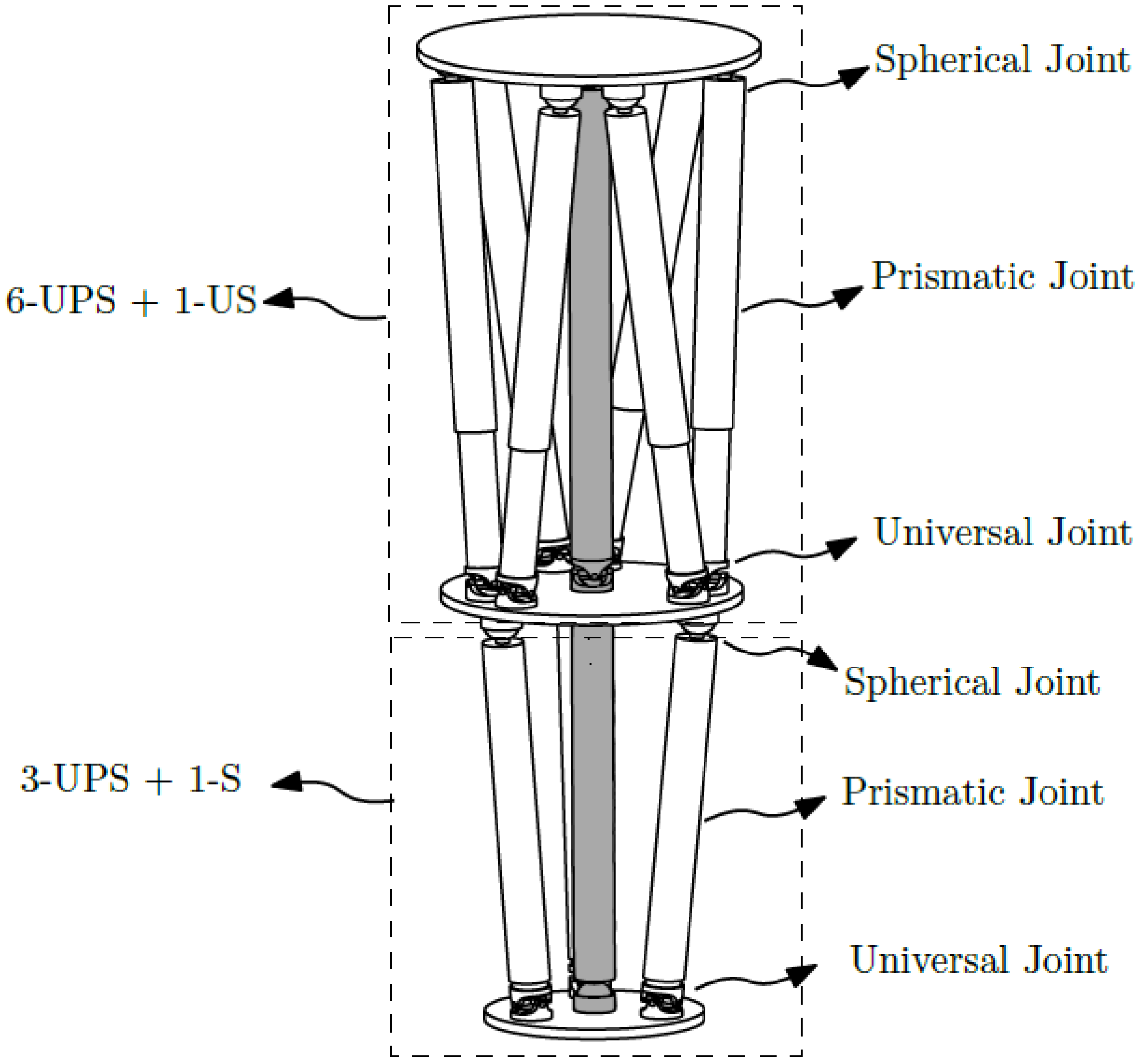
Literature Review

- Redundancy – Obstacle Avoidance - Kinematic Modeling
- Applications: Bio-Inspired – Flight Simulator

Serial-Parallel Anthropomorphic Robotic Leg





(2) P. Namazian, M. Masouleh, and M. R. Zakerzadeh, "SPAR-Leg," IEEE, 2023.

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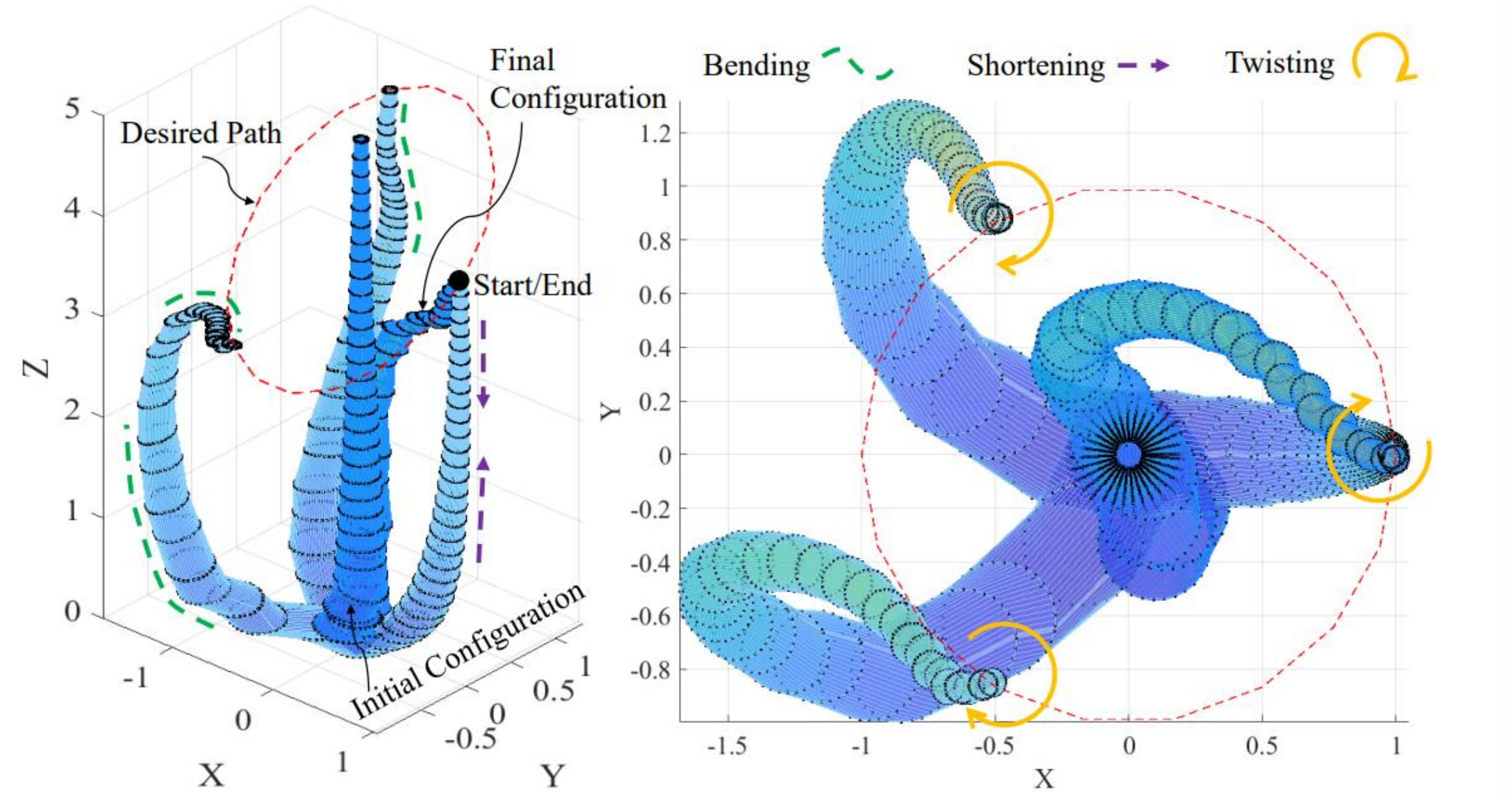
Introduction
Previews Works
Methodology
Results
Conclusion
Future Works

Literature Review

- **Redundancy – Obstacle Avoidance - Kinematic Modeling**
- **Applications: Bio-Inspired – Flight Simulator**

trajectory tracking control octopus-inspired hyper-redundant robot

Elephant’t Trunk Manipulator



(3) Hannan, Michael W. and Walker, Ian D. Kinematics and the Implementation of an Elephant's Trunk Manipulator and Other Continuum Style Robots. Journal of Robotic Systems, 2003.
(4) A. S. Lafmejani, B. Danaei, A. Kalhor, and M. T. Masouleh, "Kinematic modeling and trajectory tracking control of an octopus-inspired hyper-redundant robot,“

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Kinematic Modeling

- Forward/Inverse Kinematic Problem (FKP/IKP)
  GS Manipulator
- Jacobian Matrix

$$\mathbf{J} = \left[\mathbf{V}_1 \, {}^{G}\mathbf{J}_1^{-1} \quad \mathbf{W}_1 \, {}^{1}\mathbf{J}_2^{-1}\right]_{6\times 12}$$

$$\mathbf{V}_1 = \begin{bmatrix} 1_{3\times 3} & -[{}^{G}\mathbf{p}_2]_\times \\ 0_{3\times 3} & 1_{3\times 3} \end{bmatrix}_{6\times 6} \qquad \mathbf{W}_1 = \begin{bmatrix} {}^{G}\mathbf{R}_1 & 0_{3\times 3} \\ 0_{3\times 3} & {}^{G}\mathbf{R}_1 \end{bmatrix}_{6\times 6}$$

$$\mathbf{J} = \begin{bmatrix} \hat{\mathbf{s}}_1^{\mathrm{T}} & (\mathbf{b}_1 \times \hat{\mathbf{s}}_1)^{\mathrm{T}} \\ \hat{\mathbf{s}}_2^{\mathrm{T}} & (\mathbf{b}_2 \times \hat{\mathbf{s}}_2)^{\mathrm{T}} \\ \vdots & \vdots \\ \hat{\mathbf{s}}_6^{\mathrm{T}} & (\mathbf{b}_6 \times \hat{\mathbf{s}}_6)^{\mathrm{T}} \end{bmatrix}_{6\times 6}$$

(5) P. Namazian, M. T. Masouleh, and M. R. Zakerzadeh, "A general formulation for kinematic analysis and redundancy resolution of hyper-redundant Gough-Stewart hybrid platforms," Technical Report, 2023..

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Dynamic Modeling

Approach Selection

Name: **Virtual Work**
Advantage: Fast Computational Algorithm

Dynamic Equation → **Static Equation + Fictitious Wrench**

$$\sum f_{\text{ext}} - m\,a_c = 0$$
$$\sum {}^c n_{\text{ext}} - ({}^c I\,\dot{\omega} + \omega \times ({}^c I\,\omega)) = 0.$$

→

$$\sum \hat{f}_{\text{ext}} = 0$$
$$\sum {}^c \hat{n}_{\text{ext}} = 0,$$

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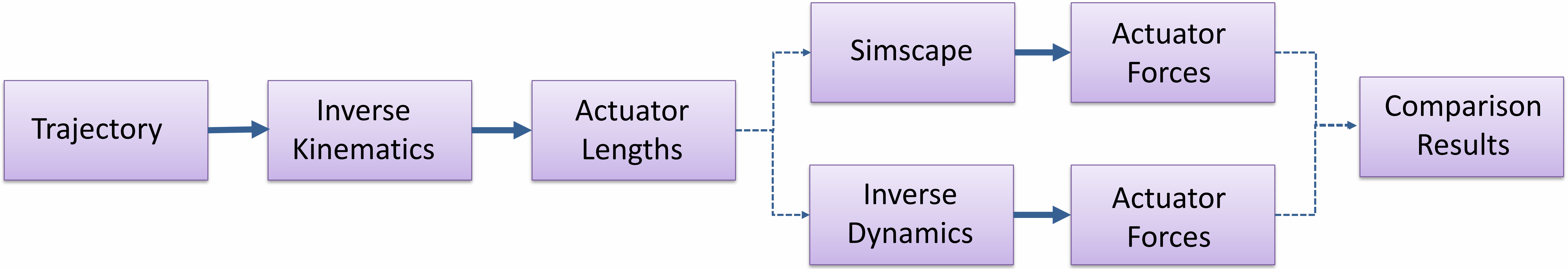
Road Map



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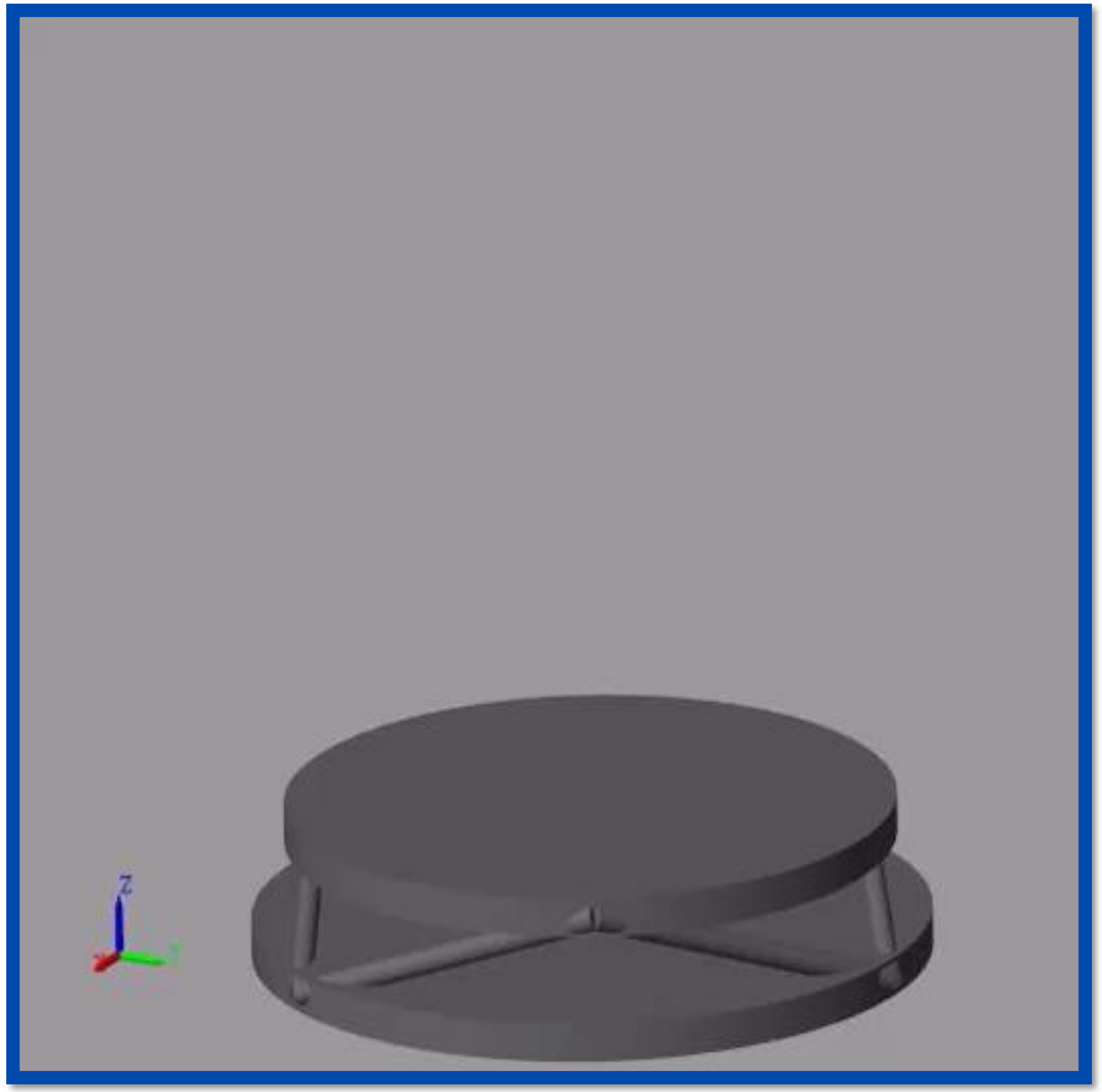
Single Gough-Stewart Simulation



Results - Helical Motion

$$\mathbf{J}^{\mathrm{T}}\hat{\boldsymbol{f}} + \mathbf{f}_p + \sum_{i=1}^{6}\left(\mathbf{J}_{\mathrm{i,cyl}}^{\mathrm{T}}\mathbf{f}_{\mathrm{cyl}} + \mathbf{J}_{\mathrm{i,pis}}^{\mathrm{T}}\mathbf{f}_{\mathrm{pis}}\right) = 0$$

- Introduction
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- **Methodology**
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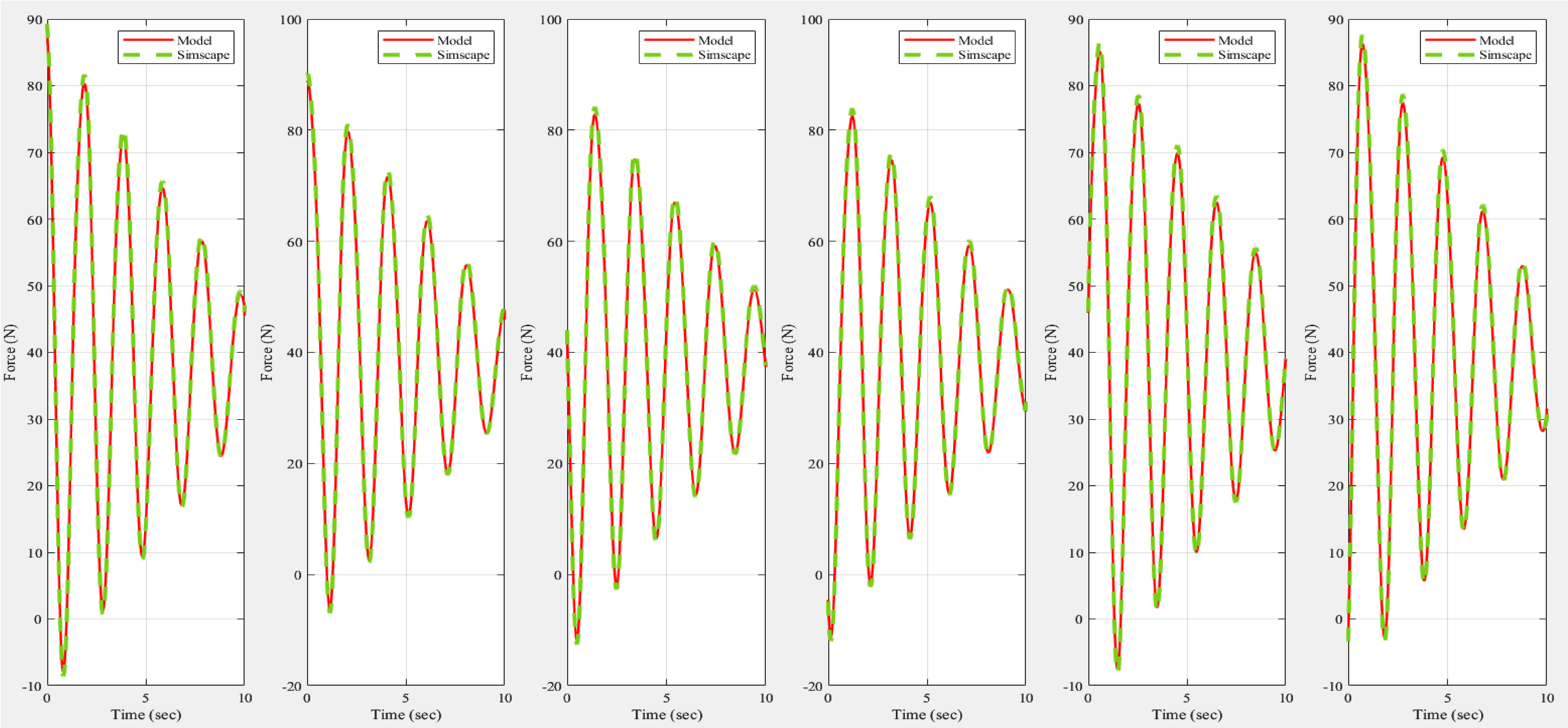

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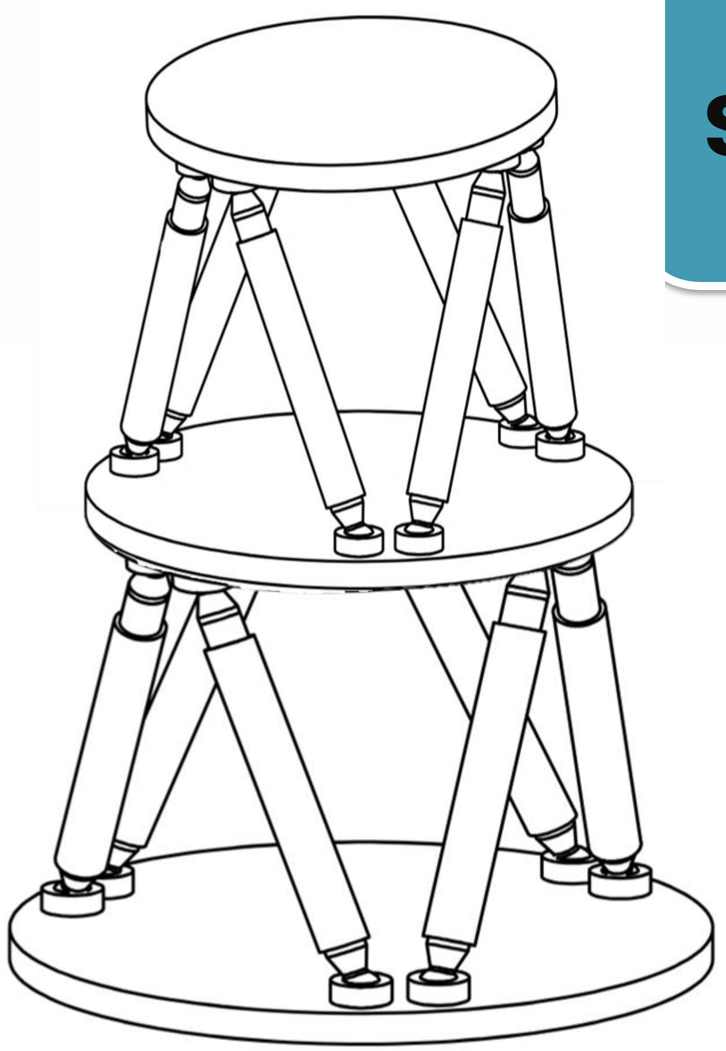
Dynamic Modeling

Dynamic Model of the Redundant Manipulator

Double Gough-Stewart



Solving IDP for the upper segment

Transferring Actuator Forces to C.M. of the lower segment

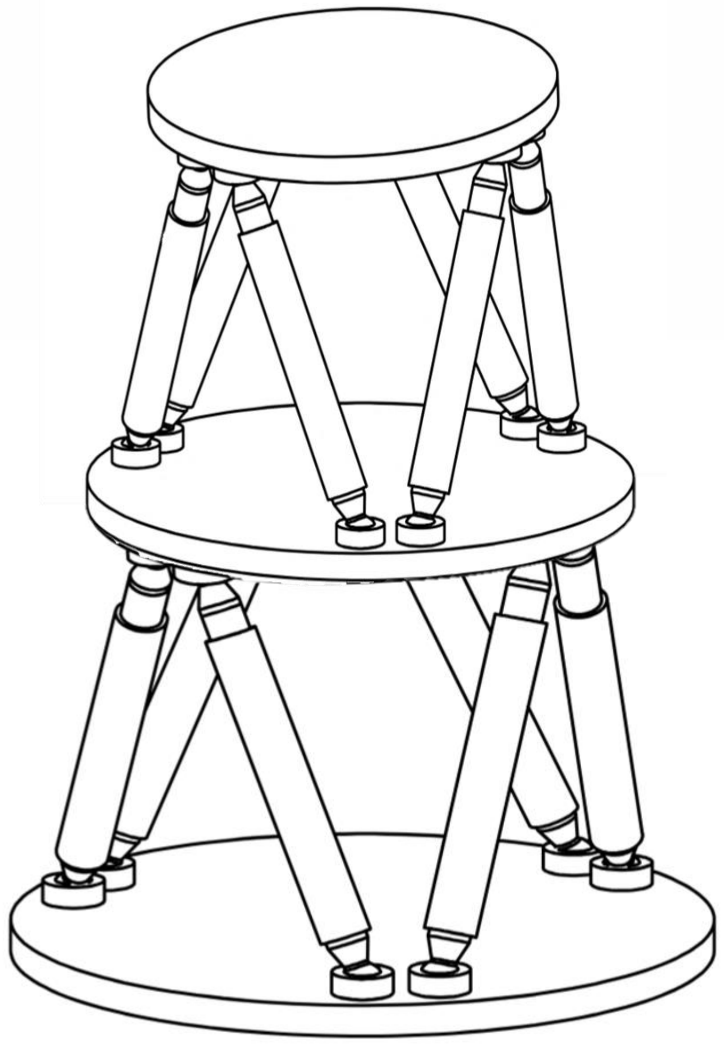
Solving IDP for the lower segment – Finding Actuator Forces

Dynamic Modeling

Generated Trajectory → IKP

↓

IDP for Upper Segment ← Length of Actuators



$$ {}^{1}\mathbf{J}_2^{\mathrm{T}}\,\hat{\boldsymbol{f}}_2 + \mathbf{f}_{b_2} + \mathbf{f}_{e_2} + \sum_{\mathrm{i}=7}^{12} \left(\mathbf{J}_{\mathrm{i,cyl}}^{\mathrm{T}}\,\mathbf{f}_{\mathrm{cyl}} + \mathbf{J}_{\mathrm{i,pis}}^{\mathrm{T}}\,\mathbf{f}_{\mathrm{pis}}\right) = 0 $$

↓

Transferring Outputs → IDP for Lower Segment

$$ \mathbf{f}_{\mathrm{Tr}} = \begin{bmatrix} \hat{\boldsymbol{f}}_2 + m_{\mathrm{cyl}}\mathbf{g} \\ -\mathbf{a}_{\mathrm{i}_\times}\hat{\boldsymbol{f}}_2 + m_{\mathrm{cyl}}(\mathbf{d}_{\mathrm{i}_\times}\mathbf{g}) \end{bmatrix} $$

$$ {}^{G}\mathbf{J}_1^{\mathrm{T}}\,\hat{\boldsymbol{f}}_1 + \mathbf{f}_{\mathrm{Tr}} + \mathbf{f}_{e_1} + \sum_{i=1}^{6} \left(\mathbf{J}_{\mathrm{i,cyl}}^{\mathrm{T}}\,\mathbf{f}_{\mathrm{cyl}} + \mathbf{J}_{\mathrm{i,pis}}^{\mathrm{T}}\,\mathbf{f}_{\mathrm{pis}}\right) = 0 $$

→ $$ \hat{\boldsymbol{f}} = \begin{bmatrix} \hat{\boldsymbol{f}}_1 \\ \hat{\boldsymbol{f}}_2 \end{bmatrix} $$

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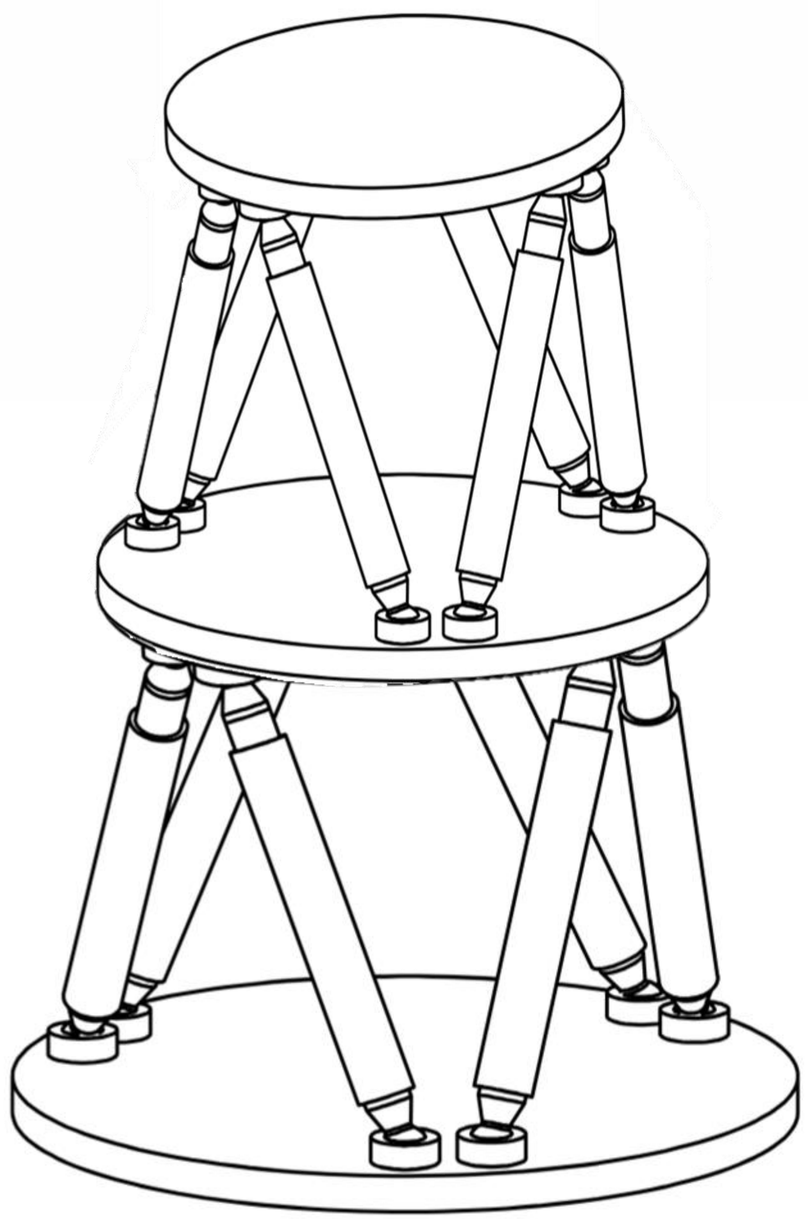
Dynamic Modeling



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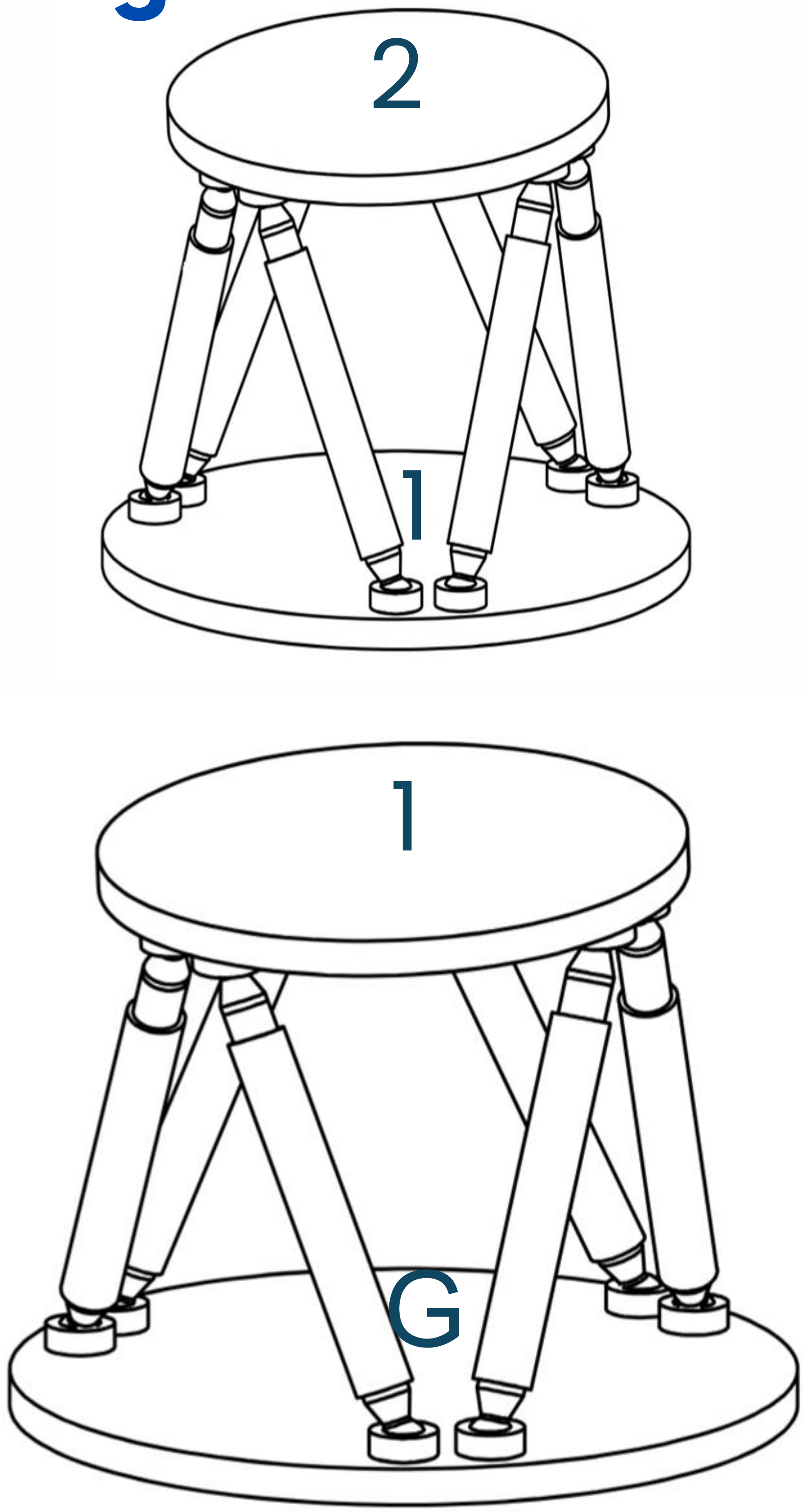
Dynamic Modeling



Dynamic Modeling

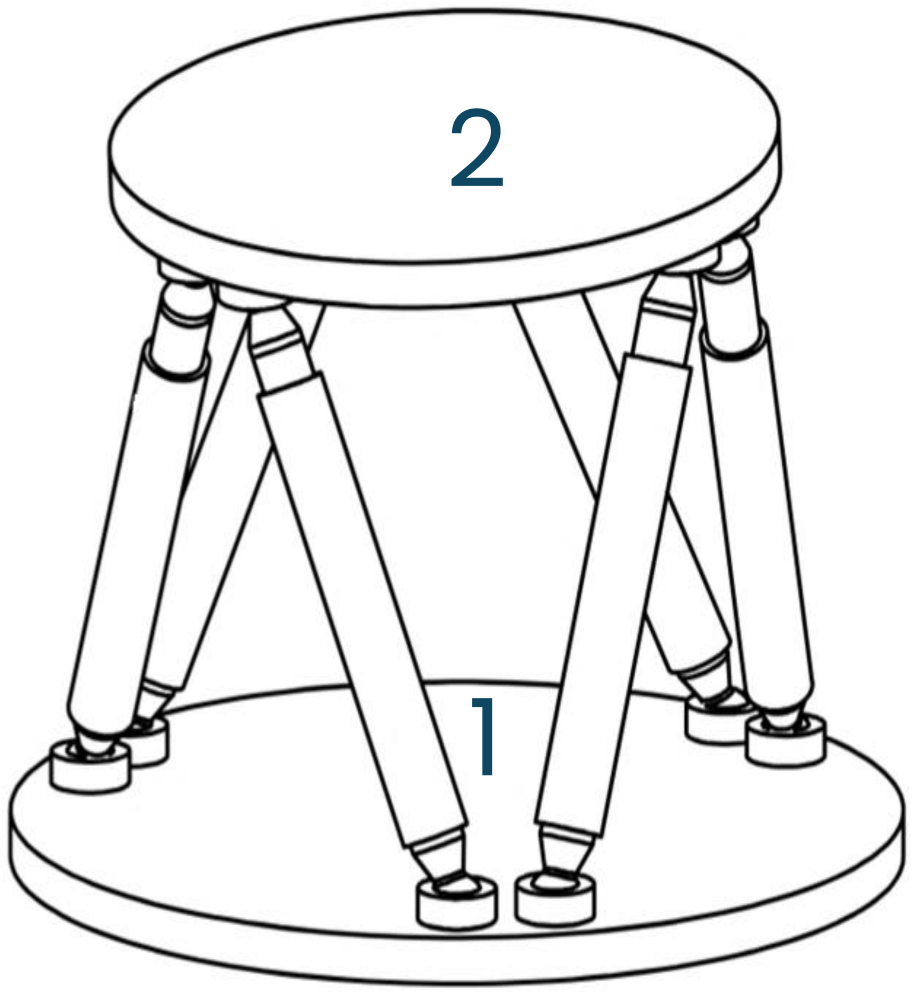
Solving IDP for Upper Segment (N=2)

$$\rightarrow \quad \hat{\boldsymbol{f}}_2$$

$$^{1}\mathbf{J}_2^{\mathrm{T}}\,\hat{\boldsymbol{f}}_2 + \mathbf{f}_{b_2} + \mathbf{f}_{e_2} + \sum_{i=7}^{12}\left(\mathbf{J}_{\mathrm{i,cyl}}^{\mathrm{T}}\,\mathbf{f}_{\mathrm{cyl}} + \mathbf{J}_{\mathrm{i,pis}}^{\mathrm{T}}\,\mathbf{f}_{\mathrm{pis}}\right) = 0$$

$$\mathbf{f}_p = \begin{bmatrix} \mathbf{f} \\ \mathbf{n} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{\mathrm{d}} + M(\mathbf{g} - \mathbf{a}_{\mathrm{p}}) \\ \mathbf{n}_{\mathrm{d}} - \mathbf{I}_{\mathrm{p}}^{\mathrm{A}}\dot{\boldsymbol{\omega}}_{\mathrm{p}} - \boldsymbol{\omega}_{\mathrm{p}} \times \mathbf{I}_{\mathrm{p}}^{\mathrm{A}}\boldsymbol{\omega}_{\mathrm{p}} \end{bmatrix}$$

$$^{1}\mathbf{J}_2 = \begin{bmatrix} \hat{\mathbf{s}}_1^{\mathrm{T}} & (\mathbf{b}_1 \times \hat{\mathbf{s}}_1)^{\mathrm{T}} \\ \hat{\mathbf{s}}_2^{\mathrm{T}} & (\mathbf{b}_2 \times \hat{\mathbf{s}}_2)^{\mathrm{T}} \\ \vdots & \vdots \\ \hat{\mathbf{s}}_6^{\mathrm{T}} & (\mathbf{b}_6 \times \hat{\mathbf{s}}_6)^{\mathrm{T}} \end{bmatrix}_{6\times 6}$$



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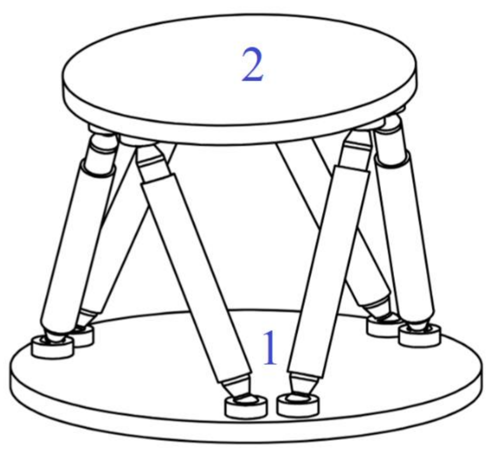
Dynamic Modeling

Solving IDP for Upper Segment (N=2)



Jacobian → $^{1}\mathbf{J}_2$

Fictitious Wrenches → $\mathbf{F}_{b_2}$, $\mathbf{F}_{e_2}$

Jacobian for Limbs →

$$\mathbf{J}_{\mathrm{i},cyl} = \frac{1}{l_{\mathrm{i}}}\begin{bmatrix} -c_{i,cyl}\mathbf{s}_{\mathrm{i}\times}^2 & c_{i,cyl}\mathbf{s}_{\mathrm{i}\times}^2\mathbf{b}_{\mathrm{i}\times} \\ \mathbf{s}_{\mathrm{i}\times} & -\mathbf{s}_{\mathrm{i}\times}\mathbf{b}_{\mathrm{i}\times} \end{bmatrix}$$

$$\mathbf{J}_{\mathrm{i},pis} = \frac{1}{l_{\mathrm{i}}}\begin{bmatrix} -c_{i,pis}\mathbf{s}_{\mathrm{i}\times}^2 + l_{\mathrm{i}}\mathbf{s}_{\mathrm{i}}\mathbf{s}_{\mathrm{i}}^T & c_{i,pis}\mathbf{s}_{\mathrm{i}\times}^2\mathbf{b}_{\mathrm{i}_\times} - l_{\mathrm{i}}\mathbf{s}_{\mathrm{i}}\mathbf{s}_{\mathrm{i}}^T\mathbf{b}_{\mathrm{i}_\times} \\ \mathbf{s}_{\mathrm{i}\times} & -\mathbf{s}_{\mathrm{i}\times}\mathbf{b}_{\mathrm{i}_\times} \end{bmatrix}$$

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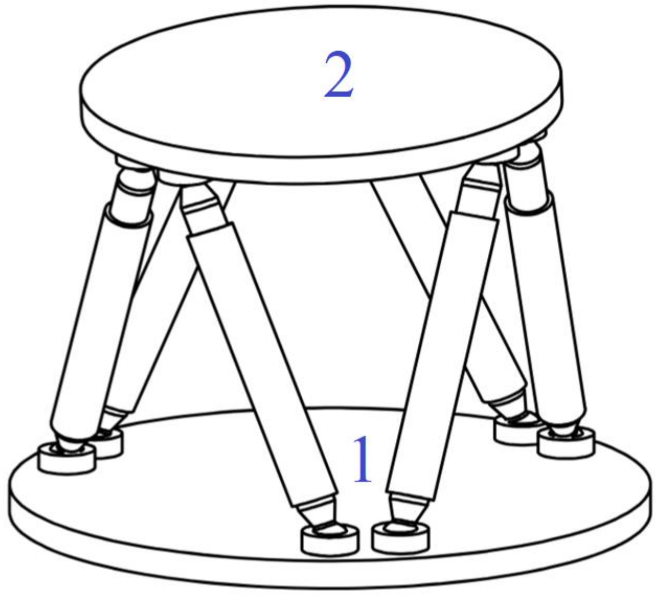
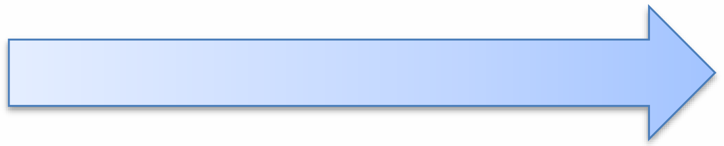
Dynamic Modeling

Transferring Actuator Forces to C.M. of the lower segment

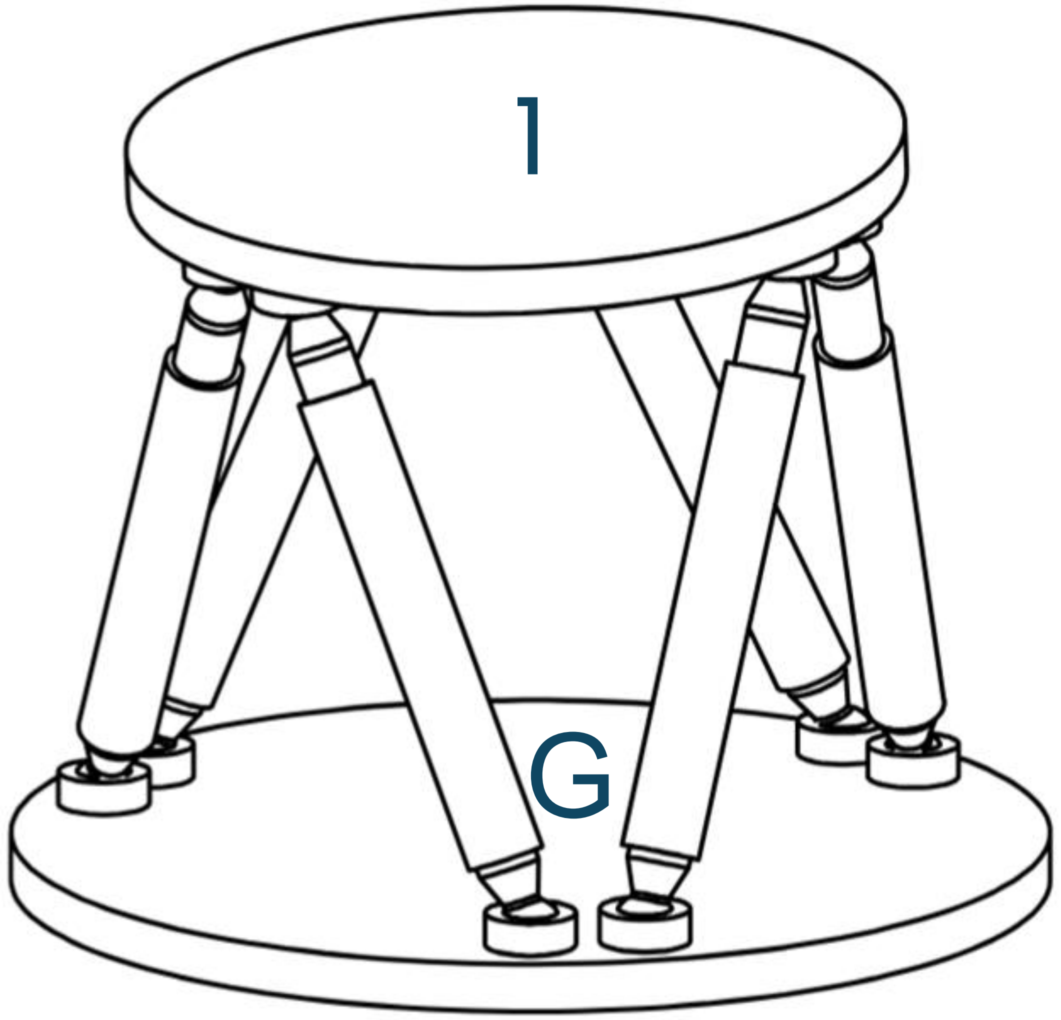
Upper Segment



$\hat{f}_2$



Lower Segment



$$\mathbf{f}_{\mathrm{Tr}} = \begin{bmatrix} \hat{f}_2 + m_{\mathrm{cyl}}\mathbf{g} \\ -\mathbf{a}_{\mathrm{i}_\times}\hat{f}_2 + m_{\mathrm{cyl}}(\mathbf{d}_{\mathrm{i}_\times}\mathbf{g}) \end{bmatrix}$$

$$\mathbf{d_i} = -\mathbf{a_i} + c_{\mathrm{i,cyl}}\mathbf{s_i}$$

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Dynamic Modeling

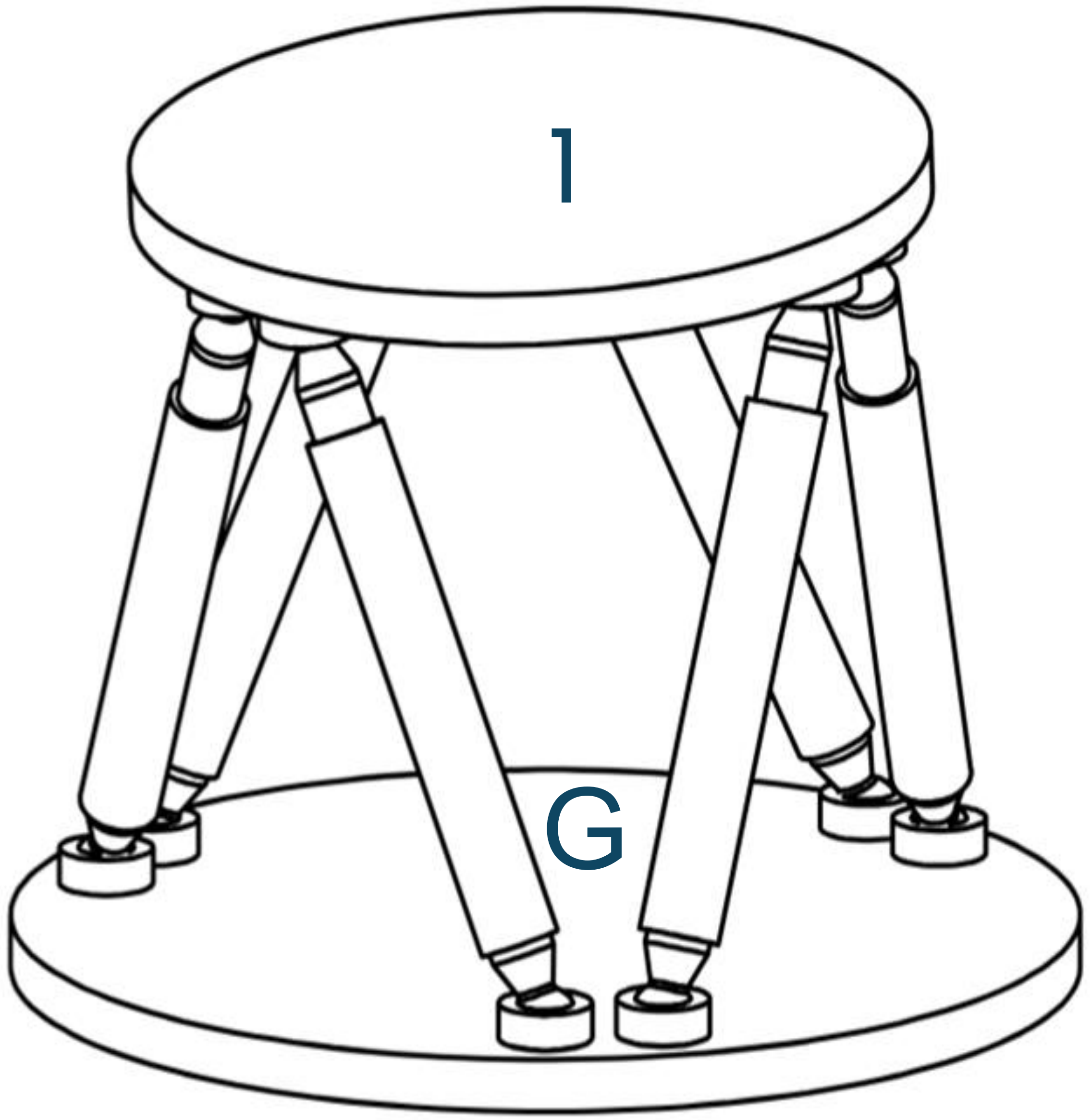
Solving IDP for Lower Segment (N=1)

$$ {}^{G}\mathbf{J}_1^{\mathrm{T}} \hat{\boldsymbol{f}}_1 + \mathbf{f}_{\mathrm{Tr}} + \mathbf{f}_{e_1} + \sum_{i=1}^{6} \left(\mathbf{J}_{\mathrm{i,cyl}}^{\mathrm{T}} \mathbf{f}_{\mathrm{cyl}} + \mathbf{J}_{\mathrm{i,pis}}^{\mathrm{T}} \mathbf{f}_{\mathrm{pis}} \right) = 0 \quad \Rightarrow \quad \hat{\boldsymbol{f}}_1 $$

$$ \mathbf{f}_p = \begin{bmatrix} \mathbf{f} \\ \mathbf{n} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{\mathrm{d}} + M(\mathbf{g} - \mathbf{a}_{\mathrm{p}}) \\ \mathbf{n}_{\mathrm{d}} - \mathbf{I}_{\mathrm{p}}^{\mathrm{A}} \dot{\boldsymbol{\omega}}_{\mathrm{p}} - \boldsymbol{\omega}_{\mathrm{p}} \times \mathbf{I}_{\mathrm{p}}^{\mathrm{A}} \boldsymbol{\omega}_{\mathrm{p}} \end{bmatrix} $$

$$ {}^{G}\mathbf{J}_1 = \begin{bmatrix} \hat{\mathbf{s}}_1^{\mathrm{T}} & (\mathbf{b}_1 \times \hat{\mathbf{s}}_1)^{\mathrm{T}} \\ \hat{\mathbf{s}}_2^{\mathrm{T}} & (\mathbf{b}_2 \times \hat{\mathbf{s}}_2)^{\mathrm{T}} \\ \vdots & \vdots \\ \hat{\mathbf{s}}_6^{\mathrm{T}} & (\mathbf{b}_6 \times \hat{\mathbf{s}}_6)^{\mathrm{T}} \end{bmatrix}_{6\times 6} $$

$$ \mathbf{f}_{\mathrm{Tr}} = \begin{bmatrix} \hat{\boldsymbol{f}}_2 + m_{\mathrm{cyl}} \mathbf{g} \\ -\mathbf{a}_{\mathrm{i}_\times} \hat{\boldsymbol{f}}_2 + m_{\mathrm{cyl}} (\mathbf{d}_{\mathrm{i}_\times} \mathbf{g}) \end{bmatrix} $$



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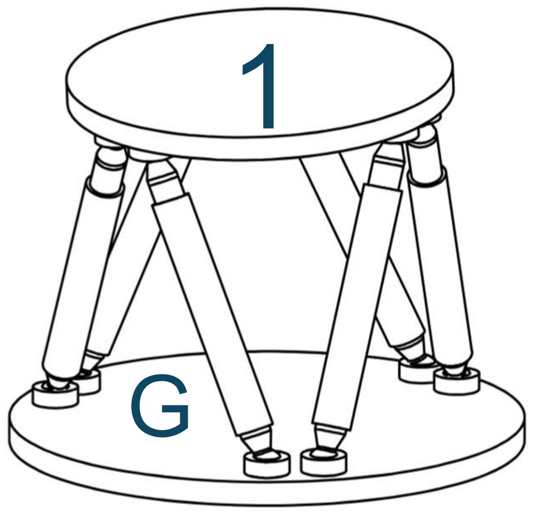
Dynamic Modeling

Solving IDP for Lower Segment (N=1)



Jacobian

$$^{G}\mathbf{J}_{1}$$

Fictitious Wrenches

$$\mathbf{f}_p = \begin{bmatrix} \mathbf{f} \\ \mathbf{n} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{\mathrm{d}} + M(\mathbf{g} - \mathbf{a}_{\mathrm{p}}) \\ \mathbf{n}_{\mathrm{d}} - \mathbf{I}_{\mathrm{p}}^{\mathrm{A}} \dot{\boldsymbol{\omega}}_{\mathrm{p}} - \boldsymbol{\omega}_{\mathrm{p}} \times \mathbf{I}_{\mathrm{p}}^{\mathrm{A}} \boldsymbol{\omega}_{\mathrm{p}} \end{bmatrix}$$

Jacobian for Limbs

$$\mathbf{J}_{\mathrm{i},cyl} = \frac{1}{l_{\mathrm{i}}} \begin{bmatrix} -c_{i,cyl} \mathbf{s}_{\mathrm{i}\times}^{2} & c_{i,cyl} \mathbf{s}_{\mathrm{i}\times}^{2} \mathbf{b}_{\mathrm{i}\times} \\ \mathbf{s}_{\mathrm{i}\times} & -\mathbf{s}_{\mathrm{i}\times} \mathbf{b}_{\mathrm{i}\times} \end{bmatrix}$$

$$\mathbf{J}_{\mathrm{i},pis} = \frac{1}{l_{\mathrm{i}}} \begin{bmatrix} -c_{i,pis} \mathbf{s}_{\mathrm{i}\times}^{2} + l_{\mathrm{i}} \mathbf{s}_{\mathrm{i}} \mathbf{s}_{\mathrm{i}}^{T} & c_{i,pis} \mathbf{s}_{\mathrm{i}\times}^{2} \mathbf{b}_{\mathrm{i}_\times} - l_{\mathrm{i}} \mathbf{s}_{\mathrm{i}} \mathbf{s}_{\mathrm{i}}^{T} \mathbf{b}_{\mathrm{i}_\times} \\ \mathbf{s}_{\mathrm{i}\times} & -\mathbf{s}_{\mathrm{i}\times} \mathbf{b}_{\mathrm{i}_\times} \end{bmatrix}$$

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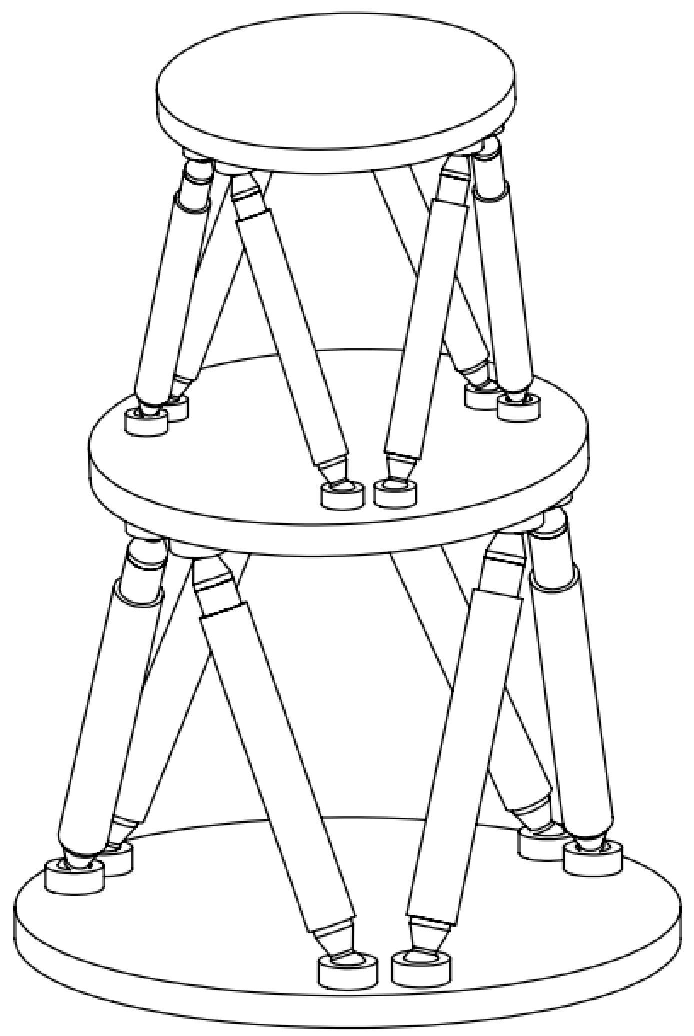
Overview of Methodology

Generated Trajectory → IKP → IDP

$$\hat{f} = \begin{bmatrix} \hat{f}_1 \\ \hat{f}_2 \end{bmatrix}$$



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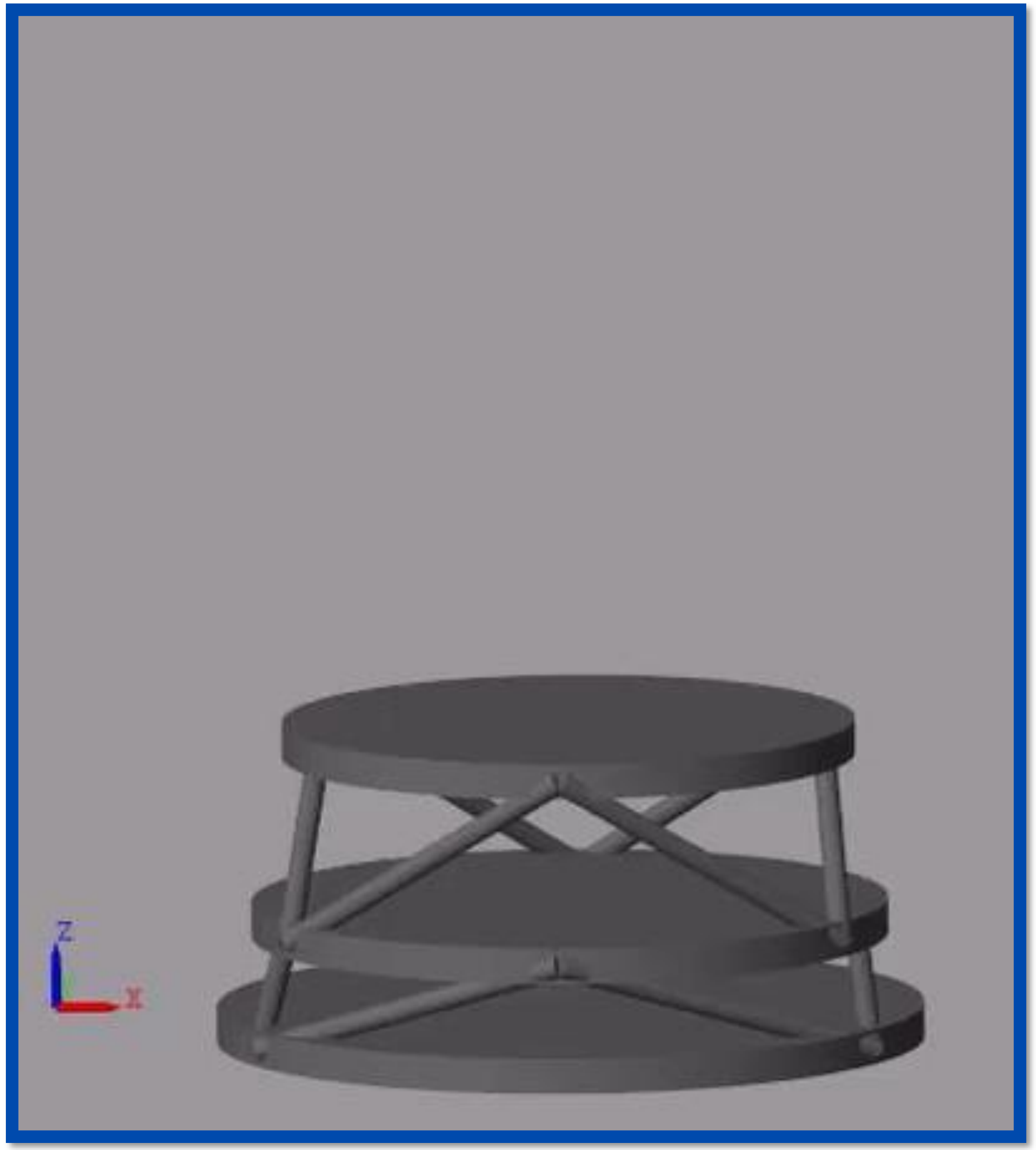
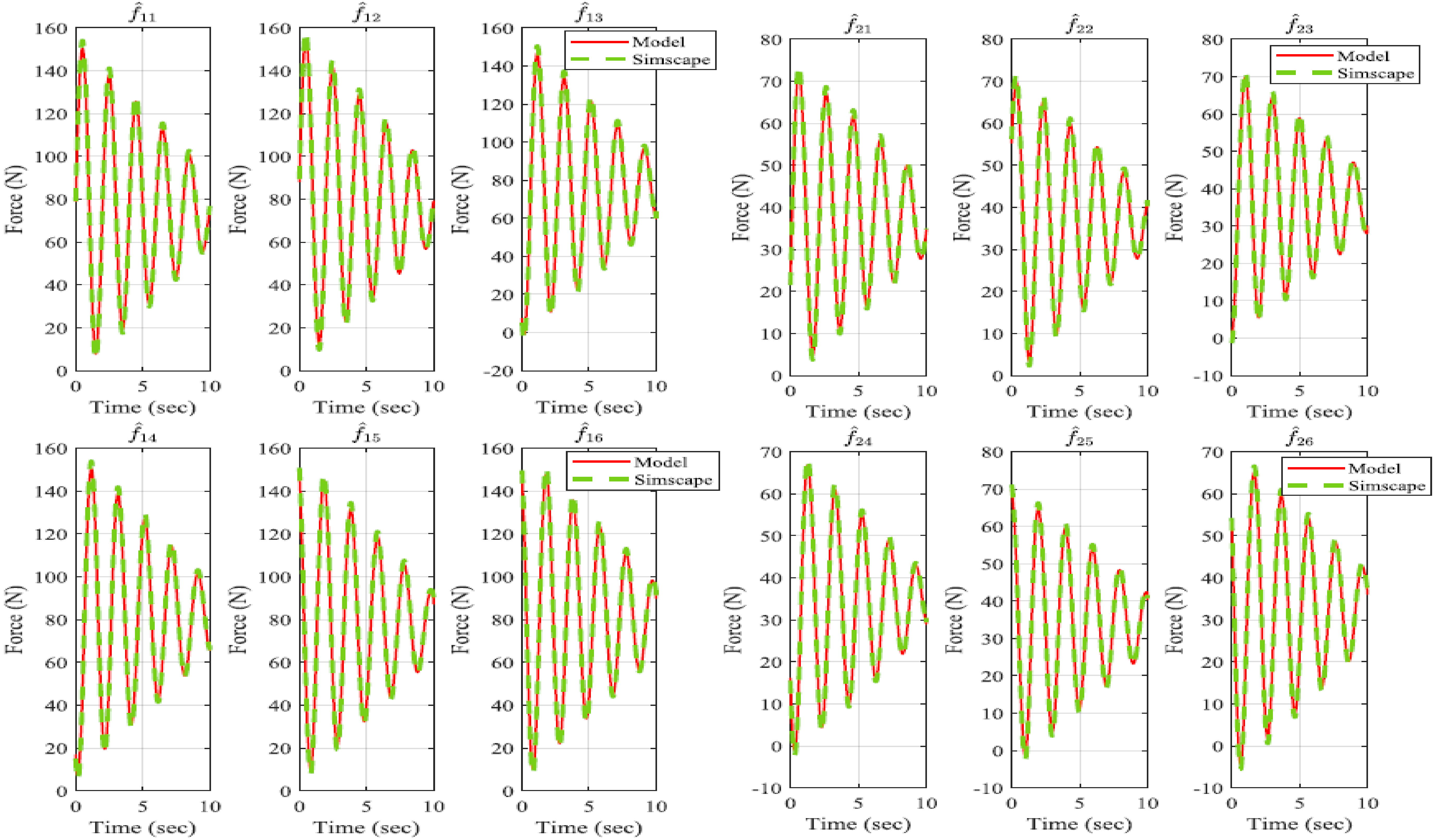
Simulation for Double GSM

$$\mathbf{p}(t) = \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix} = \begin{bmatrix} r(t)\cos(\omega t) \\ r(t)\sin(\omega t) \\ h + 0.02t \end{bmatrix}$$

$$r(t) = 0.125 - 0.01t$$
$$\omega = \pi$$



Results for Double GSM



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Results for Double GSM

Trajectory	RMSE (%)	Max Error (%)
Helical	2.13	3.30
Infinity	0.943	1.5

$$ {}^{1}\mathbf{J}_{2}^{\mathrm{T}}\,\hat{\boldsymbol{f}}_{2} + \mathbf{f}_{b_2} + \mathbf{f}_{e_2} + \sum_{\mathrm{i}=7}^{12} \left(\mathbf{J}_{\mathrm{i,cyl}}^{\mathrm{T}}\,\mathbf{f}_{\mathrm{cyl}} + \mathbf{J}_{\mathrm{i,pis}}^{\mathrm{T}}\,\mathbf{f}_{\mathrm{pis}} \right) = 0 $$

$$ {}^{G}\mathbf{J}_{1}^{\mathrm{T}}\,\hat{\boldsymbol{f}}_{1} + \mathbf{f}_{\mathrm{Tr}} + \mathbf{f}_{e_1} + \sum_{i=1}^{6} \left(\mathbf{J}_{\mathrm{i,cyl}}^{\mathrm{T}}\,\mathbf{f}_{\mathrm{cyl}} + \mathbf{J}_{\mathrm{i,pis}}^{\mathrm{T}}\,\mathbf{f}_{\mathrm{pis}} \right) = 0 $$

Conclusion

- Review on Kinematics
- Verification of Dynamics for Single GSM
- Develop method for redundant Double GSM.
- Validation of Model with SimScape
- Minimal Dynamic Error

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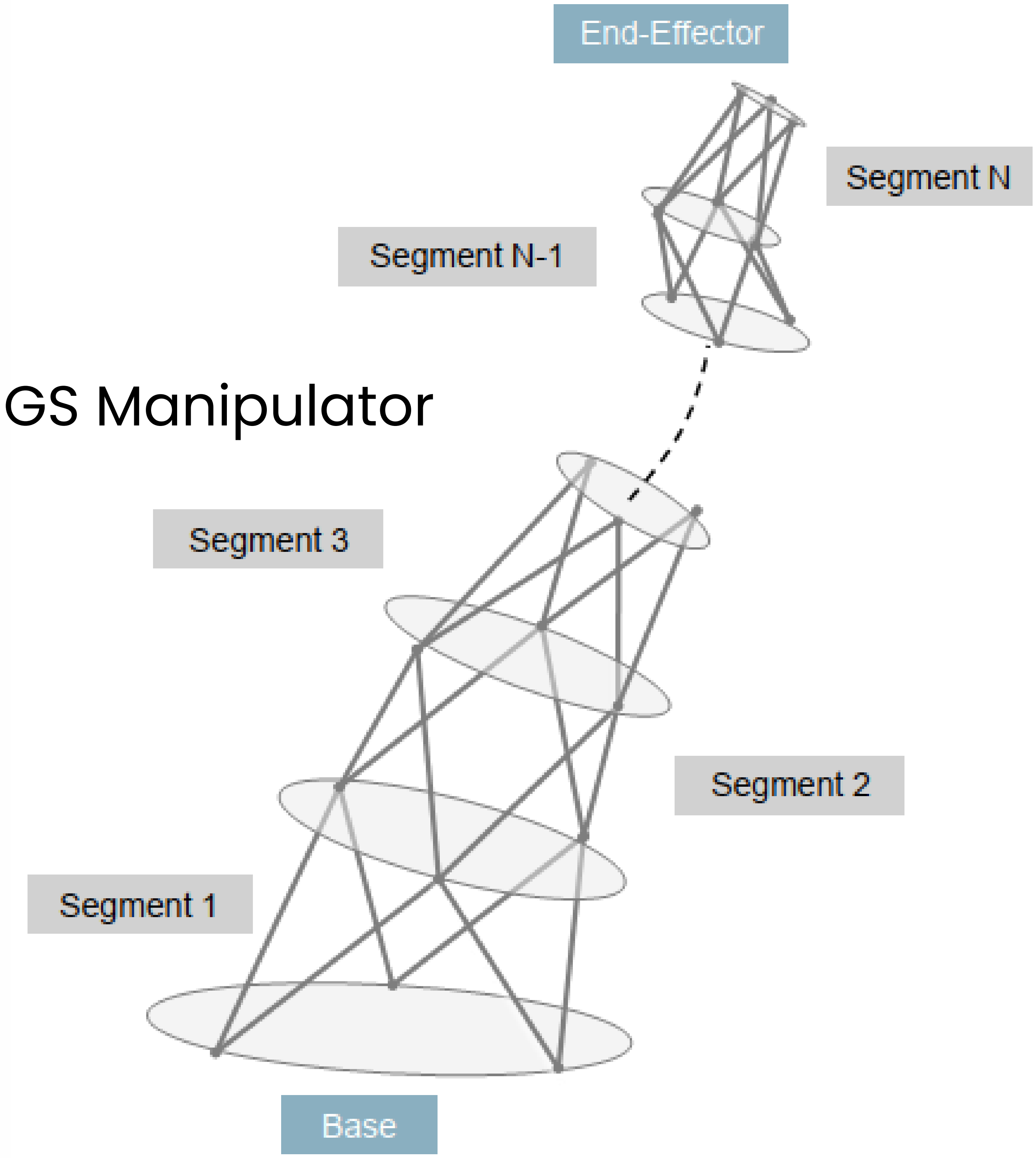
Future Works

- Minimal Set Parameters
- Dynamic Modeling of Hyper-Redundant GS Manipulator



THANK YOU

Any Question?

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