Resource Allocation Game Under Double-Sided Auction Mechanism: Efficiency and Convergence

Suli Zou, Zhongjing Ma, and Xiangdong Liu

***Abstract*—With an effort to allocate divisible resources among suppliers and consumers, a double-sided auction model is designed to decide strategies for individual players. Under the auction mechanism with the Vickrey–Clarke–Groves-type payment, the incentive compatibility holds, and the efficient bid profile is a Nash equilibrium (NE). However, it brings difficulties for players to implement the efficient solution due to the fact that there exist infinite number of NEs in the underlying double-sided auction game. To overcome this challenge, we formulate the double-sided auction game as a pair of single-sided auction games which are coupled via a joint potential quantity of the resource. A decentralized iteration procedure is then designed to achieve the efficient solution, where a single player, a buyer, or a seller implements his best strategy with respect to a given potential quantity and a constraint on his bid strategy. Accordingly, the potential quantity is updated with respect to iteration steps as well. It is verified that the system converges to the efficient NE within finite iteration steps in the order of $\mathcal{O}(\ln(1/\varepsilon))$ with ε representing the termination criterion of the algorithm.**

***Index Terms*—Convergence, decentralized process, double-sided auction, efficiency, Nash equilibrium (NE), resource allocation.**

Nomenclature

Indices, numbers, and sets

n, i	Indices for individual buyers.
m, j	Indices for individual sellers.
N, M	Population size of buyers and sellers respectively (resp. for short).
$\mathcal{N}, \mathcal{M}$	Set of buyers and sellers resp.
$\mathcal{Z}$	Set of admissible allocations of all players.
$\mathcal{B}_n, \mathcal{S}_m$	Set of admissible bids of buyer n and seller m resp.
$\mathcal{B}_n^t, \mathcal{S}_m^t$	Set of admissible truth-telling bids of buyer n and seller m resp.
$\mathcal{T}_n^b, \mathcal{T}_m^s$	Constrained set of buyer n and seller m resp.

Variables

x_n, y_m	Allocation of buyer n and seller m resp.
$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$	Allocation profile of buyers, sellers and all players resp., s.t. $\boldsymbol{z} \equiv (\boldsymbol{x}, \boldsymbol{y})$.
b_n	Buyer n's bid composed of bid price and bid demand, s.t. $b_n \equiv (\beta_n, d_n)$.
s_m	Seller m's bid composed of bid price and bid supply, s.t. $s_m \equiv (\alpha_m, h_m)$.
$\boldsymbol{b}, \boldsymbol{s}, \boldsymbol{r}$	Bid profile of buyers, sellers and all the players resp. s.t. $\boldsymbol{r} \equiv (\boldsymbol{b}, \boldsymbol{s})$.
$\boldsymbol{r}_{-i}$	Bid profile of all the players except player i.
$\Gamma, \widehat{\Gamma}$	Potential quantities of the auction games.
p_b, p_s	Matched price of buyers and sellers resp.
D_n	Upper demand constraint for buyer n.
H_m	Upper supply constraint for seller m.

Constant valued parameters

$\bar{\rho}, \underline{\rho}$	Upper bound and lower bound resp. of the Lipschitz constants of buyers' marginal valuations.
$\bar{\sigma}, \underline{\sigma}$	Upper bound and lower bound resp. of the Lipschitz constants of sellers' marginal costs.

Functions

$v_n(x_n)$	Valuation function of buyer n on his allocation x_n.
$c_m(y_m)$	Cost function of seller m on his allocation y_m.
$W(\boldsymbol{z})$	System social welfare on players' allocation profile $\boldsymbol{z}$.
$U(\boldsymbol{z}, \boldsymbol{r})$	Total system income on $\boldsymbol{z}$ and $\boldsymbol{r}$.
$\tau_n(\boldsymbol{r}), \tau_m(\boldsymbol{r})$	Money transfer of buyer n and seller m on $\boldsymbol{r}$ resp.
$f_n(\boldsymbol{r}), f_m(\boldsymbol{r})$	Payoff function of buyer n and seller m on $\boldsymbol{r}$ resp.

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I. Introduction

THE allocation of resources among a group of entities, including suppliers and consumers, has emerged to be a hot issue in many fields, such as telecommunication networks, power systems, and cloud computing, see [1]–[5] and references therein. Due to heavy burdens on the communication and computation via centralized methods, decentralized schemes have gradually become the mainstream for the evolution of resource allocation problems. In case the entities are autonomous and selfish, they do not desire to share their private information and would like to pursue their own benefits, respectively, instead of the whole system objective. Hence, it is challenging to design decentralized methods to motivate the entities to achieve a global objective. To consider the system efficiency, Vickrey–Clarke–Groves (VCG) mechanisms have been widely applied to decentralized systems [6]–[9], due to the fact that 1) the incentive compatibility holds, i.e., a truth-telling bid is a dominant strategy, and 2) there exists an efficient equilibrium. However, it may be impractical to apply the VCG mechanism in the allocation of infinitesimally divisible resources since the exchange of the whole infinite-dimension preference functions of players is costly and may violate the privacy of players.

Particularly, the auction is effective for decentralized resource allocations among rational players, e.g., [2], [4], [10]–[12], such that players submit their individual bids, respectively, to the system which determines the resource allocation with respect to the collected bid profile. How to design the auction mechanism that can achieve the efficient resource allocation, despite of the strategic behaviors and unawareness of revealing the private information of players, has become a fundamental issue. In view of the advantages of the VCG and auction mechanisms, many researchers have worked on how to design VCG auctions by allowing players to submit low-dimensional bid strategies, which are used to represent the partial valuations of individuals, e.g., [2], [7], [8], [13]–[15]. An auction game with one-dimensional (1-D) bids to construct a surrogate valuation function for each player was proposed in [2], [7], [8], and [13], while a progressive second price (PSP) auction mechanism with 2-D bids each of which is composed of a per unit price and a maximum quantity of the demand was proposed in [14] and [15]. Specifically, in [13], it achieved a unique and efficient NE while the mechanisms traded off the dominant-strategy implementation for ease of implementation. The mechanisms proposed in [7] and [8] also achieved the uniqueness and efficiency, but they required the surrogate valuation function to be twice continuously differentiable. Iosifidis and Koutsopoulos [2] formulated a double-sided auction game for resource allocations in autonomous networks. As an extension of PSP mechanism, Jain and Walrand [1] generalized the PSP mechanism to double-sided auctions in telecommunication networks with multilink routes. There are other works to study the market efficiency and properties of the NE for auction games based on the PSP mechanism in different application fields, e.g., [1], [16]–[20], and references therein.

In this paper, we study the resource allocation among suppliers and consumers in a double-sided auction framework, wherein suppliers and consumers are regarded as sellers and buyers, respectively. In the auction mechanism, each player (a buyer or a seller) submits a bid to the auctioneer, who then determines a resource allocation and a payment on the basis of players' submitted bid profile. In our formulation, a 2-D message is employed as each buyer (seller) is asked to reveal a bid signal including a unit price that he wants to pay (charge) and a maximum amount of the resource that he demands (supplies). With respect to these bid information, players' resource allocations are determined by maximizing a revealed system profit and their payments following the VCG mechanisms. The incentive compatibility holds, and the efficient strategy that maximizes the social welfare is a Nash equilibrium (NE).

As shown in the paper, besides the efficient NE, there exist infinite number of other NEs for the underlying double-sided auction game. This brings difficulties to implement the efficient NE in a decentralized way. In our work, we divide the double-sided auction into a pair of single-sided auction games, say a buyer-sided auction game and a seller-sided one, which are coupled via a joint potential quantity. Thus, each player competes with his opponents in the corresponding single-sided auction under a given potential quantity which represents a common quantity of the resource traded in the system, i.e., a quantity of the resource allocated to all the buyers or generated by all the sellers. Based on this formulation, we propose a novel dynamic process to implement the efficient NE, under which players update their strategies in the corresponding single-sided auction game with respect to a given potential quantity, and then the potential quantity is updated with respect to the bid profile of players as well.

In each of the single-sided auction games, each player implements his best response, with respect to a bid profile of his opponents, by maximizing his payoff. In this case, each player is greedy and asks for as many resource units as possible on condition that he wins the auction. Due to this, it may take too many steps for the system to reach an NE solution, which may not be efficient, or even the system may oscillate and hence cannot converge to any NE. Alternatively, in order to improve the convergence performance, we first introduce a pair of parameters, say upper bounds of the gradients of buyers' marginal valuation and sellers' marginal cost, which reveal some rough information related to the (infinite-dimension) valuation or cost functions of players; then, assisted with the given parameters, at each iteration step, a buyer and a seller update their best responses under a constraint on the bid demand and that on the bid supply, respectively. The underlying auction system is guaranteed to converge to the efficient NE by applying the proposed method. Furthermore, it is verified that the system can converge to the efficient NE within finite iteration steps in the order of $\mathcal{O}(\ln(1/\varepsilon))$, where ε is the termination parameter of the algorithm.

In the literature, some research works have been dedicated to the implementation of the efficient NE of resource allocation auction games [15], [21]–[28]. The algorithms have been presented in [21]–[23] for single-sided auctions to implement the NE. In [24] and [25], an efficient protocol was designed to conduct combinatorial auctions, where the auctioneer computes an optimal resource allocation at each stage, and then the losers at this stage are allowed to increase their bids, respectively. As stated in [24], the convergence rate is shown to be the order of $\mathcal{O}(B/\varepsilon)$, where B and ε represent the number of bundles and the termination criterion, respectively. In [15], the players are allowed to sequentially update their own bid strategies with respect to the submitted bid profile to maximize their own individual payoffs. At each iteration step, the resource may be reallocated from the players with lower prices to the player, who updates his bid at this step. Due to the deficiency of the enough information related to players' marginal valuations in a single bid, the computational complexity for the implementation of the NE is the order of $\mathcal{O}(1/\varepsilon)$. Alternatively, an auction mechanism with multibid profiles was adopted by Tuffin *et al.* in [16], [26], and [27], such that the efficient solution can be implemented in a single step asymptotically as the dimension of players' submitted bid strategy goes to infinity. Compared with the multistep implementation before the system can converge to the equilibrium by applying our method, the one-step implementation of the equilibrium proposed by Tuffin *et al.* is based upon the infinite-dimension bid submitted by each player. However, in practice, the individuals may not be willing to share their full private information with others, and the transmission of the complete information may create heavy communication burdens. In [28], Jia *et al.* presented a quantized auction algorithm under which the system converges to a quantized NE by applying 2-D bids. It was shown under certain conditions that the system may converge very fast or oscillates indefinitely. This work was extended to double-sided auction games in [29] and to multilevel cooperative network systems in [30], such that the auction system can converge to a quantized NE near to the efficient one. To ensure the convergence to the efficient NE with the method proposed in [29], the collection of inefficient NEs is eliminated in advance. This kind of elimination is not required for our proposed method.

In summary, compared with the main research works discussed above, this paper proposes a dynamic algorithm for the double-sided auction games, such that the system converges to the efficient NE, based upon 2-D bids submitted by individual players, within an amount of iteration steps which is bounded from above with a certain value.

The rest of the paper is organized as follows. In Section II, we design a double-sided auction to formulate the resource allocation problems among a group of suppliers and customers. In Section III, we present a novel update algorithm by adopting which underlying auction system converges to the efficient NE. Numerical simulations are studied in Section IV to demonstrate the main results developed in the paper. Moreover, in Section V, the developed work in this paper is applied in the field of the optimal scheduling in power electricity systems and cellular telecommunication networks, respectively. Finally, we summarize our work in Section VI.

II. Resource Allocation Under Auction Mechanism

A. Resource Allocation Problems

We consider a class of resource allocation problems, where a collection of consumers $\mathcal{N}$ requires an amount of a divisible resource supplied by a collection of suppliers $\mathcal{M}$.

We denote by x_n and y_m the demand of consumer n and the supply of supplier m, respectively, such that

$$x_n \geq 0, \quad y_m \geq 0. \tag{1}$$

Denote by $\boldsymbol{z} \equiv (\boldsymbol{x}, \boldsymbol{y})$, with $\boldsymbol{x} \equiv (x_n, n \in \mathcal{N})$ and $\boldsymbol{y} = (y_m, m \in \mathcal{M})$, a resource allocation of all the individuals. $\boldsymbol{z}$ is called *admissible* if it satisfies (1) and the equality constraint of $\sum_{n \in \mathcal{N}} x_n = \sum_{m \in \mathcal{M}} y_m$. The set of admissible resource allocations is denoted by $\mathcal{Z}$.

Each consumer $n \in \mathcal{N}$ has a valuation on his allocation x_n, denoted by $v_n(x_n)$, and each supplier $m \in \mathcal{M}$ has a cost on his allocation y_m, denoted by $c_m(y_m)$.

Assumption 1: We consider the following assumptions:

1) The valuation function v_n, for all $n \in \mathcal{N}$, is differentiable, increasing and strictly concave.
2) The cost function c_m, for all $m \in \mathcal{M}$, is differentiable, increasing and strictly convex.

■

We define $W(\boldsymbol{z})$ as the system social welfare, such that

$$W(\boldsymbol{z}) \triangleq \sum_{n \in \mathcal{N}} v_n(x_n) - \sum_{m \in \mathcal{M}} c_m(y_m). \tag{2}$$

The objective of the system is to implement an efficient allocation to maximize $W(\boldsymbol{z})$, i.e., to determine an allocation $\boldsymbol{z}^{**}$, such that

$$\boldsymbol{z}^{**} = \underset{\boldsymbol{z} \in \mathcal{Z}}{\operatorname{argmax}}\ W(\boldsymbol{z}). \tag{3}$$

Under Assumption 1, by the Karush–Kuhn–Tucker (KKT) optimality conditions and [31], there exists a unique efficient solution $\boldsymbol{z}^{**}$ for the optimization problem, such that

$$v_n'(x_n^{**}) \begin{cases} = \lambda, & \text{if } x_n^{**} > 0 \\ \leq \lambda, & \text{otherwise} \end{cases} \tag{4a}$$

$$c_m'(y_m^{**}) \begin{cases} = \lambda, & \text{if } y_m^{**} > 0 \\ \geq \lambda, & \text{otherwise} \end{cases} \tag{4b}$$

$$\sum_{n \in \mathcal{N}} x_n^{**} = \sum_{m \in \mathcal{M}} y_m^{**} \tag{4c}$$

with a constant value $\lambda > 0$.

B. Double-Sided Auction Mechanism for Resource Allocation Problems

We study the resource allocation among resource suppliers and consumers in a double-sided auction framework, wherein these resource suppliers and consumers are regarded as sellers and buyers, respectively.

Buyer n submits a 2-D bid, denoted by b_n, to the system

$$b_n \equiv (\beta_n, d_n) \in \mathcal{B}_n = [0, \infty) \times [0, \infty), \forall n \in \mathcal{N}$$

where β_n and d_n represent a buying price per unit that buyer n would like to pay, and the maximum units of the resource he demands, respectively, while seller m submits a bid, denoted by s_m, to the system

$$s_m \equiv (\alpha_m, h_m) \in \mathcal{S}_m = [0, \infty) \times [0, \infty), \forall m \in \mathcal{M}$$

where α_m and h_m represent a selling price per unit that seller m would like to sell and the maximum units of the resource he would like to supply, respectively.

The bid profile of buyers is $\boldsymbol{b} \equiv (b_n, n \in \mathcal{N})$, and the bid profile of sellers is $\boldsymbol{s} \equiv (s_m, m \in \mathcal{M})$. Denote by $\boldsymbol{r} \equiv (\boldsymbol{b}, \boldsymbol{s})$ a bid profile of all the players in the auction game. Let $\boldsymbol{r}_{-n} \equiv \boldsymbol{r}/\{b_n\}$ and $\boldsymbol{r}_{-m} \equiv \boldsymbol{r}/\{s_m\}$ represent the bid profile of all the players except buyer n and seller m, respectively.

Thus, considering a bid of buyer n, say $b_n \in \mathcal{B}_n$, a bid profile $\boldsymbol{r}$ can be represented as $\boldsymbol{r} = (b_n, \boldsymbol{r}_{-n})$, and similarly considering a bid of seller m, $s_m \in \mathcal{S}_m$, $\boldsymbol{r}$ can be represented as $\boldsymbol{r} = (s_m, \boldsymbol{r}_{-m})$ as well.

We further define a function $U(\cdot)$ on an admissible allocation $\boldsymbol{z}$ with respect to a bid profile $\boldsymbol{r}$ as the following:

$$U(\boldsymbol{z}, \boldsymbol{r}) \triangleq \sum_{n\in\mathcal{N}} \beta_n x_n - \sum_{m\in\mathcal{M}} \alpha_m y_m$$

which can be interpreted as the total income of the system with respect to $\boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{y})$. The auctioneer assigns an optimal allocation $\boldsymbol{z}^*(\boldsymbol{r}) \equiv (\boldsymbol{x}^*(\boldsymbol{r}), \boldsymbol{y}^*(\boldsymbol{r}))$, such that

$$\boldsymbol{z}^*(\boldsymbol{r}) = \underset{\substack{\boldsymbol{z}\in\mathcal{Z} \\ \boldsymbol{x}\leq\boldsymbol{d}, \boldsymbol{y}\leq\boldsymbol{h}}}{\operatorname{argmax}}\ U(\boldsymbol{z}, \boldsymbol{r}). \tag{5}$$

We will specify the transfer money for each player, denoted by $\tau_l, l \in \mathcal{N} \cup \mathcal{M}$, following the so-called VCG mechanism introduced in [14], [16], such that

$$\tau_n(\boldsymbol{r}) \triangleq U(\boldsymbol{z}^*(\boldsymbol{r}_{(n)}), \boldsymbol{r}_{(n)}) - \big(U(\boldsymbol{z}^*(\boldsymbol{r}), \boldsymbol{r}) - \beta_n x_n^*(\boldsymbol{r})\big), \tag{6}$$

$$\tau_m(\boldsymbol{r}) \triangleq U(\boldsymbol{z}^*(\boldsymbol{r}_{(m)}), \boldsymbol{r}_{(m)}) - \big(U(\boldsymbol{z}^*(\boldsymbol{r}), \boldsymbol{r}) + \alpha_m y_m^*(\boldsymbol{r})\big) \tag{7}$$

where $\boldsymbol{r}_{(n)} \equiv ((\beta_n, 0); \boldsymbol{r}_{-n})$ is the bid profile when buyer n is absent from the auction, and $\boldsymbol{r}_{(m)} \equiv ((\alpha_m, 0); \boldsymbol{r}_{-m})$ is the bid profile when seller m is absent from the auction. That is, under the VCG mechanism, the money transfer made by an individual player is the externality he imposes on others through his participation, and is defined as the difference of the summation of other players' revenue in the situation that he is absent from the auction and that he joins in the auction.

Note: The transfer money made by the buyer n and the seller m can be interpreted as the payment that the buyer n should pay and the opportunity cost that is caused to the seller m, respectively, [1]. Also, the opposite of the opportunity cost of a seller can be considered as the income of this seller.

The payoff functions of the players are then specified in the following:

$$f_n(\boldsymbol{r}) \triangleq v_n(x_n^*(\boldsymbol{r})) - \tau_n(\boldsymbol{r}), \quad \forall\, n \in \mathcal{N},$$

$$f_m(\boldsymbol{r}) \triangleq -c_m(y_m^*(\boldsymbol{r})) - \tau_m(\boldsymbol{r}), \quad \forall\, m \in \mathcal{M}.$$

Definition 1: A bid profile $\boldsymbol{r}^0$ is an NE of the auction game if the following holds:

$$f_n(b_n^0, \boldsymbol{r}_{-n}^0) \geq f_n(b_n, \boldsymbol{r}_{-n}^0), \forall\, b_n \in \mathcal{B}_n,$$

$$f_m(s_m^0, \boldsymbol{r}_{-m}^0) \geq f_m(s_m, \boldsymbol{r}_{-m}^0), \forall\, s_m \in \mathcal{S}_m$$

for each $n \in \mathcal{N}$ and $m \in \mathcal{M}$.

In Lemma 2.2, we will analyze some properties of the NE. Before that, we first specify a set of the truth-telling bids of buyers and sellers, respectively, and show the so-called incentive compatibility of the underlying auction game in Lemma 2.1.

We define a specific set of (truth-telling) bids of buyer n, denoted by $\mathcal{B}_n^t$, such that

$$\mathcal{B}_n^t \triangleq \{b_n \equiv (\beta_n, d_n) \in \mathcal{B}_n,\ \text{s.t. } \beta_n = v_n'(d_n)\}, \tag{8}$$

i.e., $\mathcal{B}_n^t$ is composed of all those bids, of buyer n, the bid price of each of which is exact the marginal valuation of his bid demand.

Similarly, we define a specific set of (truth-telling) bids of seller m, denoted by $\mathcal{S}_m^t$, such that

$$\mathcal{S}_m^t \triangleq \{s_m \equiv (\alpha_m, h_m) \in \mathcal{S}_m,\ \text{s.t. } \alpha_m = c_m'(h_m)\} \tag{9}$$

i.e., $\mathcal{S}_m^t$ is composed of all those bids, of seller m, the bid price of each of which is exact the marginal cost of his bid supply.

Lemma 2.1 (Incentive Compatibility): Under Assumption 1, there always exists at least one truth-telling bid that is weakly dominant for each buyer or seller.

Proof: See Appendix A for the proof. ■

Consider a specific truth-telling bid profile, denoted by $\boldsymbol{r}^* \equiv (\boldsymbol{b}^*, \boldsymbol{s}^*)$, such that

$$b_n^* \equiv (\beta_n^*, d_n^*) = (v_n'(x_n^{**}), x_n^{**}), \quad \forall\, n \in \mathcal{N}, \tag{10a}$$

$$s_m^* \equiv (\alpha_m^*, h_m^*) = (c_m'(y_m^{**}), y_m^{**}), \quad \forall\, m \in \mathcal{M} \tag{10b}$$

where $\boldsymbol{z}^{**} = (\boldsymbol{x}^{**}, \boldsymbol{y}^{**})$ represents the efficient resource allocation as specified in (3). By applying the KKT conditions, we can verify that $\boldsymbol{z}^*(\boldsymbol{r}^*) = \boldsymbol{z}^{**}$, i.e., the resource allocation with respect to $\boldsymbol{r}^*$ is efficient. We call $\boldsymbol{r}^*$ the efficient bid profile, and it satisfies

$$x_n^* = d_n^*,\ \beta_n^* \begin{cases} = \lambda, & \text{if } d_n^* > 0 \\ \leq \lambda, & \text{otherwise} \end{cases}, \forall n \in \mathcal{N} \tag{11a}$$

$$y_m^* = h_m^*,\ \alpha_m^* \begin{cases} = \lambda, & \text{if } h_m^* > 0 \\ \geq \lambda, & \text{otherwise} \end{cases}, \forall m \in \mathcal{M} \tag{11b}$$

$$\sum_{n\in\mathcal{N}} d_n^* = \sum_{m\in\mathcal{M}} h_m^*. \tag{11c}$$

Lemma 2.2: Consider a bid profile $\boldsymbol{r}^0$, such that

$$\beta_n^0 = \beta,\ \alpha_m^0 = \alpha,\ \forall n \in \mathcal{N}, m \in \mathcal{M},\ \text{ with } \beta \geq \alpha; \tag{12a}$$

$$\sum_{n\in\mathcal{N}} d_n^0 = \sum_{m\in\mathcal{M}} h_m^0 = q^0 \tag{12b}$$

then $\boldsymbol{r}^0$ is an NE for the underlying double-sided auction game under Assumption 1. Moreover, we have $\boldsymbol{r}^0 = \boldsymbol{r}^*$, in case $\beta = \alpha$, i.e., $\boldsymbol{r}^*$ is the efficient NE. Also, for any NE $\boldsymbol{r}^0$ specified in (12), $q^* \geq q^0$.

Proof: Lemma 2.2 can be verified by applying the similar technique adopted in [19] and [20]. ■

We give an example of a double-sided auction with multiple NEs. For the purpose of demonstration, we consider a simple case involving $N = 2$ buyers and $M = 2$ sellers. Suppose that the valuations of buyers share a common form of $v_n(x_n) = a_n(x_n + 1)^{0.5}$ with $a_1 = 2, a_2 = 1$, and the costs of sellers share a common form of $c_m(y_m) = a_{N+m} y_m^2$ with $a_3 = 0.05, a_4 = 0.03$. Fig. 1 displays the efficient NE and an inefficient one.

By solving the optimization problem (3), we can obtain that the efficient solution $\boldsymbol{z}^{**}$ that maximizes the social welfare given in (2) is $(7.8, 1.2, 3.4, 5.6)$. The corresponding efficient bid

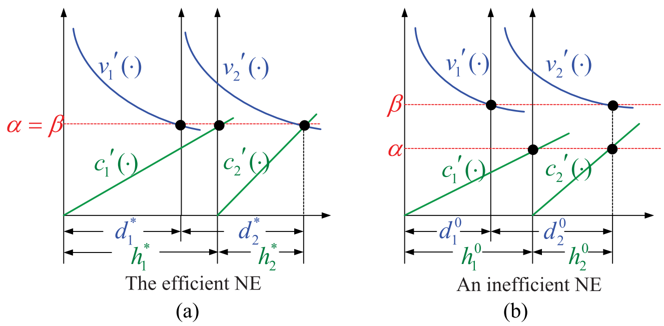

Fig. 1. Illustration of NEs for the underlying double-sided auction game.

profile $\boldsymbol{r}^*$ is $\big((0.34, 7.8), (0.34, 1.2), (0.34, 3.4), (0.34, 5.6)\big)$, which satisfies (12) with $\beta^* = \alpha^* = 0.34$. By Definition 1, we have that $\boldsymbol{r}^*$ is an NE. Hence, $\boldsymbol{r}^*$ is the efficient NE.

Consider another bid profile $\boldsymbol{r}^\dagger = ((0.35, 7), (0.35, 1), (0.3, 3), (0.3, 5))$, which also satisfies (12) with $\beta^\dagger = 0.35 > \alpha^\dagger = 0.3$. By Definition 1, it can be verified that $\boldsymbol{r}^\dagger$ is an NE, and $q^* = 9 > q^\dagger = 8$ which is consistent with Lemma 2.2.

In Section III, we propose a novel method to implement the efficient NE in a decentralized way.

III. Decentralized Efficient NE Implementation

In order to achieve the efficient NE in a decentralized way, we design an update process to implement the efficient NE in Algorithm 1. Basically, the double-sided auction game is formulated as a pair of single-sided auction games coupled via a joint potential quantity, such that the best responses of the players in each single-sided auction and the potential quantity are updated under certain regulations, respectively. Here, we outline the organization of this section here. In Section III-A, we design the resource allocation and payment rules for the players in each single-sided auction game with respect to a given potential quantity, and in Section III-B, we present a method to update the potential quantity with respect to a bid profile of all the players. In Section III-C, we formalize the algorithm, show that the underlying double-sided auction game converges to the efficient NE, and further quantify the iteration steps within which the system can converge to the efficient NE.

A. Allocation and Money Transfer in Single-Sided Auctions With Respect to a Given Potential Quantity

We denote by Γ the joint potential quantity of the resource for the pair of coupled single-sided auction games, such that all buyers share Γ units of the resource in the buyer-sided auction, and all sellers supply Γ units of the resource in the seller-sided auction.

We design the *resource allocation* and *payment rules* for the players of the single-sided auction games, respectively, with respect to their bid profile and a given potential quantity.

The resource allocation and the payment of buyer n, denoted by x_n^* and τ_n, respectively, are specified with respect to a bid profile of all the buyers $\boldsymbol{b}$ and a potential quantity Γ as below:

$$x_n^*(\boldsymbol{b}, \Gamma) = \min\left\{d_n, \left[\Gamma - \sum_{i \in \mathcal{N}_n(\boldsymbol{b})} d_i\right]^+\right\}, \tag{13a}$$

$$\tau_n(\boldsymbol{b}, \Gamma) = \sum_{i \neq n} \beta_i \left(x_i^*(\boldsymbol{b}_{(n)}, \Gamma) - x_i^*(\boldsymbol{b}, \Gamma)\right) \tag{13b}$$

where $\mathcal{N}_n(\boldsymbol{b}) \triangleq \{i \in \mathcal{N}; \text{s.t.} \beta_i > \beta_n\} \cup \{i \in \mathcal{N}; \text{ s.t. } \beta_i = \beta_n \text{ and } i < n\}$, and $[u]^+$ represents $\max\{0, u\}$; $\boldsymbol{b}_{(n)} \equiv ((\beta_n, 0); \boldsymbol{b}_{-n})$ is the bid profile when buyer n is absent from the auction.

The resource allocation and the opportunity cost of seller m, denoted by y_m^* and τ_m, respectively, are specified with respect to a bid profile of all sellers $\boldsymbol{s}$ and a potential quantity Γ as below:

$$y_m^*(\boldsymbol{s}, \Gamma) = \min\left\{h_m, \left[\Gamma - \sum_{j \in \mathcal{M}_m(\boldsymbol{s})} h_j\right]^+\right\}, \tag{14a}$$

$$\tau_m(\boldsymbol{s}, \Gamma) = \sum_{j \neq m} (-\alpha_j) \left(y_j^*(\boldsymbol{s}_{(m)}) - y_j^*(\boldsymbol{s})\right) \tag{14b}$$

where $\mathcal{M}_m(\boldsymbol{s}) \triangleq \{j \in \mathcal{M}; \text{ s.t. } \alpha_j < \alpha_m\} \cup \{j \in \mathcal{M}; \text{ s.t. } \alpha_j = \alpha_m \text{ and } j < m\}$, and $\boldsymbol{s}_{(m)} \equiv ((\alpha_m, 0); \boldsymbol{s}_{-m})$ represents the bid profile when seller n is absent.

Note: The payment of a player, defined in (13b) and (14b) for a buyer and a seller, respectively, is the externality he imposes on his opponents in the corresponding single-sided auction through his participation, hence is the VCG-style money transfer made in the buyer-sided and seller-sided auction systems with a common potential quantity.

Lemma 3.1: Given a bid profile $\boldsymbol{r} \equiv (\boldsymbol{b}, \boldsymbol{s})$ and a potential quantity Γ, the admissible allocation of buyers $\boldsymbol{x}^*(\boldsymbol{b}, \Gamma)$ obtained by (13a) satisfies the following:

1) $x_n^* = d_n$ for all $n \in \mathcal{N}$, if $\sum_{i \in \mathcal{N}} d_i \leq \Gamma$; otherwise, there only exists at most one buyer $n \in \mathcal{N}$ such that $x_n^* \in (0, d_n)$;

and that of sellers $\boldsymbol{y}^*(\boldsymbol{s}, \Gamma)$ obtained by (14a) satisfies the following:

1) $y_m^* = h_m$ for all $m \in \mathcal{M}$, if $\sum_{m \in \mathcal{M}} h_m \leq \Gamma$; otherwise, there only exists at most one seller $m \in \mathcal{M}$ such that $y_m^* \in (0, h_m)$.

Proof: It is straightforward to verify Lemma 3.1 by the allocation rules given in (13a) and (14a). ∎

B. Updates of the Potential Quantity

We present a method to update the potential quantity with respect to a given bid profile of buyers, a bid profile of sellers and a potential quantity Γ.

First, we define a pair of scalar valued parameters, $\bar{\rho}$ and $\bar{\sigma}$, related to the valuation functions of buyers and the cost functions

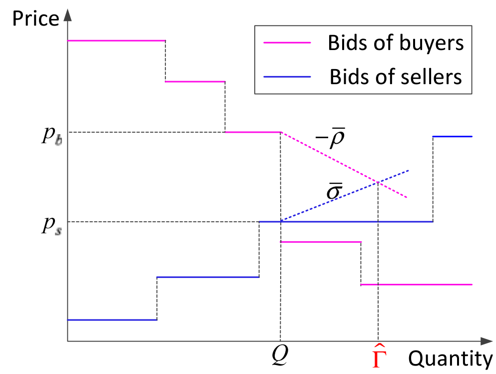

Fig. 2. Matched prices and the updated potential quantity with respect to bid profile.

of sellers, such that

$$\bar{\rho} \geq \max_{i\in\mathcal{N}} \sup_{x,\delta} \left\{ \frac{1}{\delta} \left| v_i'(x+\delta) - v_i'(x) \right| \right\}, \tag{15a}$$

$$\bar{\sigma} \geq \max_{j\in\mathcal{M}} \sup_{y,\delta} \left\{ \frac{1}{\delta} \left| c_j'(y+\delta) - c_j'(y) \right| \right\} \tag{15b}$$

i.e., $\bar{\rho}$ denotes an upper bound of the Lipschitz constants of buyers' marginal valuations, and $\bar{\sigma}$ denotes an upper bound of the Lipschitz constants of sellers' marginal costs.

Note: A pair of scale-valued parameters, $\bar{\rho}$ and $\bar{\sigma}$ specified in (15a) and (15b), reveals pretty rough information of the valuation and cost functions of individual players.

We then define the matched prices $p_b(\boldsymbol{r}, \Gamma)$ and $p_s(\boldsymbol{r}, \Gamma)$ with respect to $\boldsymbol{r}$, with $\boldsymbol{r} \equiv (\boldsymbol{b}, \boldsymbol{s})$, and Γ, such that

$$p_b(\boldsymbol{r}, \Gamma) \triangleq \min_{i\in\mathcal{N}} \{\beta_i, \text{ s.t. } x_i > 0\}, \tag{16a}$$

$$p_s(\boldsymbol{r}, \Gamma) \triangleq \max_{j\in\mathcal{M}} \{\alpha_j, \text{ s.t. } y_j > 0\}, \tag{16b}$$

where $x_i \equiv x_i^*(\boldsymbol{b}, \Gamma)$ and $y_j \equiv y_j^*(\boldsymbol{s}, \Gamma)$ are the allocations to buyer i and seller j, with respect to $(\boldsymbol{r}, \Gamma)$, implemented in (13a) and (14a), respectively.

The updated potential quantity, denoted by $\widehat{\Gamma}(\boldsymbol{r}, \Gamma)$, with respect to $\boldsymbol{r}$ and Γ, is specified as the following:

$$\widehat{\Gamma}(\boldsymbol{r}, \Gamma) = Q + \frac{p_b - p_s}{\bar{\rho} + \bar{\sigma}} \tag{17}$$

where $Q \equiv Q(\boldsymbol{r}, \Gamma) \triangleq \min\{\sum_{i\in\mathcal{N}} x_i, \sum_{j\in\mathcal{M}} y_j\}$, with $x_i \equiv x_i^*(\boldsymbol{b}, \Gamma)$ and $y_j \equiv y_j^*(\boldsymbol{s}, \Gamma)$, and $p_b \equiv p_b(\boldsymbol{r}, \Gamma)$ and $p_s \equiv p_s(\boldsymbol{r}, \Gamma)$ are given in (16), respectively.

Note: The potential quantity represents a common quantity of the resource, for each of the single-sided auction games, which is updated in (17) with respect to the submitted bid profile of players and the given pair of parameters $(\bar{\rho}, \bar{\sigma})$.

The matched prices and the update of the potential quantity are illustrated in Fig. 2, respectively. As shown in Fig. 2, p_b represents the lowest bid price of buyers with positive allocations, while p_s represents the highest bid price of sellers with positive allocations. The potential quantity $\widehat{\Gamma}$ is updated with respect to $\bar{\rho}$ and $\bar{\sigma}$, an upper bound of the Lipschitz constants of the marginal valuation function and the marginal cost function, respectively.

C. Implementation of the Efficient NE

Given a bid profile $\boldsymbol{r} \equiv (\boldsymbol{b}, \boldsymbol{s})$ and a pair of the potential quantities $(\Gamma, \widehat{\Gamma})$, an upper demand constraint for buyer n and an upper supply constraint for seller m, denoted by D_n and H_m, respectively, are defined as follows:

$$D_n(\boldsymbol{b}, \Gamma, \widehat{\Gamma}) \triangleq x_n^*(\boldsymbol{b}, \Gamma) + \left[\widehat{\Gamma} - \sum_{i\in\mathcal{N}} x_i \right]^+, \tag{18a}$$

$$H_m(\boldsymbol{s}, \Gamma, \widehat{\Gamma}) \triangleq y_m^*(\boldsymbol{s}, \Gamma) + \left[\widehat{\Gamma} - \sum_{j\in\mathcal{M}} y_j \right]^+ \tag{18b}$$

with $x_i \equiv x_i^*(\boldsymbol{b}, \Gamma)$ and $y_j \equiv y_j^*(\boldsymbol{s}, \Gamma)$ specified in (13a) and (14a), respectively.

The constrained sets of bid profiles of buyer n and seller m, denoted by $\mathcal{T}_n^b(\boldsymbol{b}, \Gamma, \widehat{\Gamma})$ and $\mathcal{T}_n^s(\boldsymbol{s}, \Gamma, \widehat{\Gamma})$, respectively, are defined as below:

$$\mathcal{T}_n^b(\boldsymbol{b}, \Gamma, \widehat{\Gamma}) \triangleq \{\widehat{b}_n \equiv (v_n'(\widehat{d}_n), \widehat{d}_n) \in \mathcal{B}_n ; \widehat{d}_n \leq D_n\}, \tag{19a}$$

$$\mathcal{T}_m^s(\boldsymbol{s}, \Gamma, \widehat{\Gamma}) \triangleq \{\widehat{s}_m \equiv (c_m'(\widehat{h}_m), \widehat{h}_m) \in \mathcal{S}_m ; \widehat{h}_m \leq H_m\}. \tag{19b}$$

Thus, the best responses of buyer n and seller m, denoted by $b_n^*(\boldsymbol{r}, \Gamma, \widehat{\Gamma})$ and $s_n^*(\boldsymbol{r}, \Gamma, \widehat{\Gamma})$, respectively, subject to the constrained sets of bid profiles $\mathcal{T}_n^b(\boldsymbol{b}, \Gamma, \widehat{\Gamma})$ and $\mathcal{T}_n^s(\boldsymbol{s}, \Gamma, \widehat{\Gamma})$, are specified in the following:

$$b_n^*(\boldsymbol{r}, \Gamma, \widehat{\Gamma}) \triangleq \underset{b_n \in \mathcal{T}_n^b(\boldsymbol{b}, \Gamma, \widehat{\Gamma})}{\operatorname{argmax}} \left\{ f_n(b_n, \boldsymbol{b}_{-n}) \right\}, \tag{20a}$$

$$s_m^*(\boldsymbol{r}, \Gamma, \widehat{\Gamma}) \triangleq \underset{s_m \in \mathcal{T}_m^s(\boldsymbol{s}, \Gamma, \widehat{\Gamma})}{\operatorname{argmax}} \left\{ f_m(s_m, \boldsymbol{s}_{-m}) \right\}. \tag{20b}$$

We develop an update procedure in Algorithm 1 to implement the efficient NE. Before that, we first specify an assignment of a buyer and a seller, denoted by $\widehat{n}(\boldsymbol{b}, \boldsymbol{x})$ and $\widehat{m}(\boldsymbol{s}, \boldsymbol{y})$, with respect to $(\boldsymbol{b}, \boldsymbol{x})$ and $(\boldsymbol{s}, \boldsymbol{y})$, respectively. By Lemma 3.1, $\widehat{n}(\boldsymbol{b}, \boldsymbol{x})$ and $\widehat{m}(\boldsymbol{s}, \boldsymbol{y})$ can be defined in the following:

$$\widehat{n}(\boldsymbol{b}, \boldsymbol{x}) = \begin{cases} \nu, & \text{if } \exists \nu \in \mathcal{N}, \text{ s.t. } x_\nu \in (0, d_\nu) \\ \nu, & \text{else if } \exists \nu \in \mathcal{N}, \text{ s.t. } x_\nu = 0 < d_\nu \\ \nu, & \text{else if } \nu \in \operatorname{argmax}_{l\in\mathcal{N}} \{\beta_l\} \end{cases} \tag{21a}$$

$$\widehat{m}(\boldsymbol{s}, \boldsymbol{y}) = \begin{cases} \mu, & \text{if } \exists \mu \in \mathcal{M}, \text{ s.t. } y_\mu \in (0, h_\mu) \\ \mu, & \text{else if } \exists \mu \in \mathcal{M}, \text{ s.t. } y_\mu = 0 < h_\mu \\ \mu, & \text{else if } \mu \in \operatorname{argmin}_{l\in\mathcal{M}} \{\alpha_l\} \end{cases}. \tag{21b}$$

Note: By Lemma 3.1, we obtain that there only exists at most one buyer ν, such that $x_\nu \in (0, d_\nu)$. Hence, if there is such a ν, $\widehat{n}(\boldsymbol{b}, \boldsymbol{x})$ is assigned to be ν; else, find a buyer ν satisfying $x_\nu = 0 < d_\nu$, i.e., with zero allocation and positive bid demand, and assign $\widehat{n}(\boldsymbol{b}, \boldsymbol{x}) = \nu$. However, the buyer satisfying $x_\nu = 0 < d_\nu$ may not be unique; then, in this case, a buyer will be arbitrarily chosen from the buyers satisfying $x_\nu = 0 < d_\nu$.

If there is no buyer satisfying the above two cases, $\widehat{n}(\boldsymbol{b}, \boldsymbol{x})$ is assigned to be ν such that $\nu \in \operatorname{argmax}_{l\in\mathcal{N}}$. Also, in this case, there may also exist multiple buyers, and then $\widehat{n}(\boldsymbol{b}, \boldsymbol{x})$ will be arbitrarily assigned among those buyers with the highest bid price.

Similarly, the assignment of $\widehat{m}(\boldsymbol{s}, \boldsymbol{y})$ follows that of $\widehat{n}(\boldsymbol{b}, \boldsymbol{x})$.

Algorithm 1: Implementation of the efficient NE.

Require:

Given an initial bid profile $\boldsymbol{r}^0$ s.t. $\max_{i\in\mathcal{N}}\{\beta_i^0\} > \min_{j\in\mathcal{M}}\{\alpha_j^0\}$;

Set an initial potential quantity s.t. $\Gamma^0 < \sum_{i\in\mathcal{N}} d_i^0$ and $\Gamma^0 < \sum_{j\in\mathcal{M}} h_j^0$;

Set $k = 0$ and $\varepsilon^0 > \varepsilon$;

Ensure:

A bid profile $\boldsymbol{r} \equiv (\boldsymbol{b}, \boldsymbol{s})$;

```
while ε^k > ε do
  Update z^k ≡ (x^k, y^k) w.r.t. r^k and Γ^k by (13a) and (14a);
  Assign a single buyer n ≡ n̂(b^k, x^k) and a single seller m ≡ m̂(s^k, y^k) by (21) respectively;
  if x_n^k = 0 < d_n^k or y_m^k = 0 < h_m^k then
    Update Γ^{k+1} = Γ^k;
  else
    Update Γ^{k+1} = Γ̂(r^k, Γ^k) by (17);
  end if
  Update b_n^{k+1} and s_m^{k+1} w.r.t (r^k, Γ^k, Γ^{k+1}) by (20);
  Update b^{k+1} = (b_n^{k+1}, b_{-n}^k) and s^{k+1} = (s_m^{k+1}, s_{-m}^k);
  Set ε^{k+1} = |Γ^{k+1} − Γ^k| + ||r^{k+1} − r^k||_1;
  Set k = k + 1;
end while
```

In Algorithm 1, denote by $\boldsymbol{r}^k \equiv (\boldsymbol{b}^k, \boldsymbol{s}^k)$ the bid profile at iteration step k. Also, we consider $\boldsymbol{z}^k \equiv \boldsymbol{z}^k(\boldsymbol{r}^k, \Gamma^k)$, i.e., $\boldsymbol{z}^k$ represents the allocation of the players at step k, and Γ^k denotes the potential quantity at k. We denote by buyer n and seller m the pair of players who are assigned to implement their best responses at iteration step k, and b_n^{k+1}, s_m^{k+1} represent the best responses of buyer n and seller m, respectively.

Note on Algorithm 1: Following the allocation rule of (13a) and (14a) adopted in Algorithm 1, for those buyers with an identical bid price, the one with a lower index will be allocated first. Nevertheless, in order to improve the fairness of the proposed method, this allocation rule could be properly revised as follows.

At iteration step k, before the allocation among players w.r.t. the submitted bid profile, the system randomly sets a unique number for each of the players; then, for those buyers with an identical bid price, the one, with a (randomly set) lower number, will be allocated first. Thus, due to the randomness of the allocation sequence of players set at each iteration step, the proposed allocation method ensures fairness to some extent.

Furthermore, it is worth to note that the main results developed in our work, like the convergence of Algorithm 1 and the efficiency of the implemented NE, are not affected by the above given allocation rule.

D. Main Results of the Proposed Algorithm

In this section, we study the convergence, computational complexity, and global optimality of the proposed algorithm in Theorems 3.1, 3.2, and 3.3, respectively. Before these, we first specify a property of the best response of players by applying Algorithm 1 in Lemma 3.2 below.

Lemma 3.2: Under Assumption 1 and by applying Algorithm 1, suppose that buyer n and seller m are assigned to implement their best responses at iteration step k, respectively, then there exist truth-telling bids, $\widetilde{b}_n \equiv (\widetilde{\beta}_n, \widetilde{d}_n) \in \mathcal{B}_n^t$ and $\widetilde{s}_m \equiv (\widetilde{\alpha}_m, \widetilde{h}_m) \in \mathcal{S}_m^t$, such that

$$f_n(\widetilde{b}_n, \boldsymbol{b}_{-n}^k) = \min_{b_n\in\mathcal{B}_n} f_n(b_n, \boldsymbol{b}_{-n}^k), \text{ and } x_n = \widetilde{d}_n, \tag{22a}$$

$$f_m(\widetilde{s}_m, \boldsymbol{s}_{-m}^k) = \min_{s_n\in\mathcal{S}_m} f_m(s_m, \boldsymbol{s}_{-m}^k), \text{ and } y_m = \widetilde{h}_m \tag{22b}$$

with $x_n \equiv x_n^*(\widetilde{b}_n, \boldsymbol{b}_{-n}^k, \Gamma^{k+1})$ and $y_m \equiv y_m^*(\widetilde{s}_m, \boldsymbol{s}_{-m}^k, \Gamma^{k+1})$.

Proof: We can verify the conclusion of (22a) by showing that, given $\boldsymbol{b}_{-n}^k$ and Γ^{k+1}, for any bid b_n, we can always find another bid $\widetilde{b}_n$ under which the allocation equals the bid demand, such that the payoff under $\widetilde{b}_n$ is larger than or equal to that under b_n.

By Algorithm 1, b_n^{k+1} is the best response of buyer n; then, suppose that $\widetilde{d}_n = x_n^{k+1}$, and consider $x_n \equiv x_n^*(\widetilde{b}_n, \boldsymbol{b}_{-n}^k, \Gamma^{k+1})$. By the concavity of $v_n(\cdot)$ under Assumption 1 and $x_n^{k+1} \le d_n^{k+1}$, we have $\widetilde{\beta}_n = v_n'(x_n^{k+1}) \ge v_n'(d_n^{k+1}) = \beta_n^{k+1}$; then, by $\widetilde{d}_n = x_n^{k+1}$ and (13a), we have $\widetilde{x}_n = x_n^{k+1} = \widetilde{d}_n$, i.e., $\widetilde{b}_n$ is a bid with a full allocation. Moreover, under $\boldsymbol{b}_{-n}^k$ and Γ^{k+1}, the allocation of $\widetilde{b}_n$ is the same as that of b_n^{k+1}; then, the payoffs subject to $(\widetilde{b}_n, \boldsymbol{b}_{-n}^k)$ and $\boldsymbol{b}^{k+1}$ are the same. It implies that $\widetilde{b}_n$ is also a best response. ■

Similarly, we can verify that (22b) holds for seller m as well.

Note: At each iteration step k, the truth-telling bid strategies $\widetilde{b}_n$ and $\widetilde{s}_n$ specified in (22) represents a best response of buyer n and that of seller m, respectively. As shown in Lemma 3.2, buyer n and seller m are fully allocated under the bid profiles $(\widetilde{b}_n, \boldsymbol{b}_{-n}^k)$ and $(\widetilde{s}_n, \boldsymbol{s}_{-m}^k)$, respectively.

Theorem 3.1: Under Assumption 1, the auction system converges to an equilibrium by adopting Algorithm 1.

Proof: By Algorithm 1, the termination criterion ε^{k+1} is set to be

$$\varepsilon^{k+1} = |\Gamma^{k+1} - \Gamma^k| + \|\boldsymbol{r}^{k+1} - \boldsymbol{r}^k\|_1.$$

In order to verify Theorem 3.1, it needs to verify that both Γ^k and $\boldsymbol{r}^k$ converge. By Lemma 3.2, a best response of each player is truthful and with a full allocation. That is to say, at each iteration step k, $\boldsymbol{r}^k \equiv (b_n^k, s_m^k; n\in\mathcal{N}, m\in\mathcal{M})$, s.t. $b_n^k = (v_n'(d_n^k), d_n^k)$, $s_m^k = (c_m'(h_m^k), h_m^k)$, and the associated collection of allocations, $\boldsymbol{z}^k \equiv (\boldsymbol{x}^k, \boldsymbol{y}^k)$, is given as $\boldsymbol{x}^k = \boldsymbol{d}^k$, $\boldsymbol{x}^k = \boldsymbol{h}^k$. Hence, to show this theorem is equivalent to show that both Γ^k and $\boldsymbol{z}^k$ converge to an equilibrium respectively. The details are specified in (I) and (II) below, respectively.

I) *To show the convergence of* Γ^k.

By adopting Algorithm 1 and (17), for all k, the potential quantity is updated as

$$\Gamma^{k+1} = \begin{cases} \Gamma^k, \text{ if } x_n^k = 0 < d_n^k \text{ or } y_m^k = 0 < h_m^k \\ \Gamma^k + \dfrac{p_b^k - p_s^k}{\bar{\rho} + \bar{\sigma}}, \text{ otherwise} \end{cases} \tag{23}$$

since $Q^k = \Gamma^k$ for all k by Appendix B, and where $p_b^k \equiv p_b(\boldsymbol{r}^k, \Gamma^k)$ and $p_s^k \equiv p_s(\boldsymbol{r}^k, \Gamma^k)$.

As verified in Appendix C, $p_b^k \geq p_s^k$ holds, by which, together with (23), we have

$$\Gamma^{k+1} \geq \Gamma^k \tag{24}$$

which implies that Γ^k increases under Algorithm 1 with respect to iteration steps.

Next, we would like to show that Γ^k is bounded from above. Define a notion Γ^*, such that

$$\Gamma^* = \sum_{n \in \mathcal{N}} d_n^* = \sum_{m \in \mathcal{M}} h_m^*$$

i.e., Γ^* represents the total amount of the resource under the efficient bid profile $\boldsymbol{r}^*$. Then, we will show, by proof of contradiction, that for any iteration step k, $\Gamma^k \leq \Gamma^*$.

Suppose that $\Gamma^k > \Gamma^*$ at some step k. By Appendix B, we have $Q^k = \Gamma^k$, with $Q^k = \sum_{n \in \mathcal{N}} x_n^k = \sum_{m \in \mathcal{M}} y_m^k$; then, by Lemma 2.1 and (11), there must exist a buyer i with $x_i^k > 0$, such that $\beta_i^k < \lambda$ and a seller j with $y_j^k > 0$, such that $\alpha_j^k > \lambda$, where λ is a positive constant specified in (11). This, together with (16), implies that $p_b^k < p_s^k$, which is contradicted with $p_b^k \geq p_s^k$ shown in Appendix C.

Hence, $\Gamma^k \leq \Gamma^*$ always holds for all k, by which together with (24), we obtain that Γ^k converges to a certain value $\Gamma^\dagger$.

II) *To show the convergence of* $\boldsymbol{z}^k$*:*

As shown in Appendix D, under Assumption 1, the following holds by adopting Algorithm 1:

$$x_i^{k+1} \geq x_i^k \text{ and } y_j^{k+1} \geq y_j^k \tag{25}$$

for all $i \in \mathcal{N}$ and all $j \in \mathcal{M}$.

By the allocation rule given in (13a) and (14a), we have $x_n^k \leq \Gamma^k$ for all $n \in \mathcal{N}$ and $y_m^k \leq \Gamma^k$ for all $m \in \mathcal{M}$. It implies that, $\boldsymbol{z}^k \equiv (\boldsymbol{x}^k, \boldsymbol{y}^k)$ is bounded from above as well. Thus, $\boldsymbol{z}^k$ converges to a certain value $\boldsymbol{z}^\dagger$.

In conclusion, the auction system converges to an equilibrium by adopting Algorithm 1. ■

Furthermore, in Theorem 3.2 below, we give an upper bound of the convergence iteration steps. We first define a pair of parameters in the following:

$$\underline{\rho} \leq \min_{i \in \mathcal{N}} \inf_{x,\delta} \left\{ \frac{1}{\delta} \left| v_i'(x+\delta) - v_i'(x) \right| \right\}, \tag{26a}$$

$$\underline{\sigma} \leq \min_{j \in \mathcal{M}} \inf_{y,\delta} \left\{ \frac{1}{\delta} \left| c_j'(y+\delta) - c_j'(y) \right| \right\}. \tag{26b}$$

Theorem 3.2: Under Algorithm 1, the system converges to a bid profile, denoted by $\boldsymbol{r}^\epsilon$, within $K(\varepsilon)$ iteration steps which is the order of $\mathcal{O}(\ln(1/\varepsilon))$.

Proof: As verified in Appendix E, we show that the auction system converges in $K(\varepsilon)$ iterations steps, such that

$$K(\varepsilon) \leq \widehat{k} + \frac{1}{\ln(\theta)} \left\lceil \max\left\{ \ln(N\theta(\bar{\rho}+1)/\varepsilon), \ln(M\Lambda(\bar{\sigma}+1)/\varepsilon) \right\} \right\rceil \tag{27}$$

where $\widehat{k} \leq \max\{N, M\}$, $\theta \equiv \frac{\bar{\rho}+\bar{\sigma}}{\bar{\rho}+\bar{\sigma}-\underline{\rho}-\underline{\sigma}}$, Λ represents a maximum value of the bid demand or bid supply of all the players, and $\lceil a \rceil$ is the smallest integer larger than or equal to a. ■

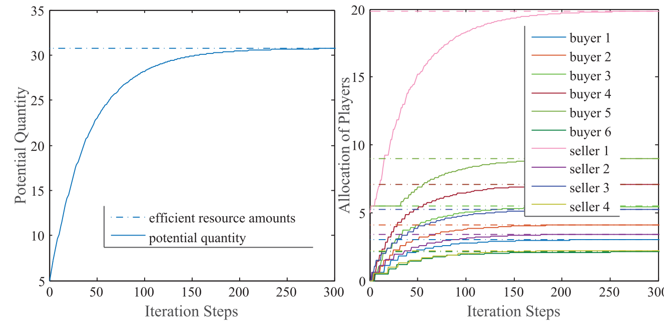

Fig. 3. Updates of the potential quantity and players' allocations under Algorithm 1.

Theorem 3.3: Under Algorithm 1, the double-sided auction system converges to the efficient NE in case the termination parameter ε vanishes.

Proof: We suppose that, the system converges to a bid profile, denoted by $\boldsymbol{r}^{k+1}$, at iteration step k.

In Appendix F, we can verify that the bid profile at the converged step k satisfies the following properties:

$$\beta_i^{k+1} \begin{cases} = \lambda, & \text{if } d_i^{k+1} > 0 \\ \leq \lambda, & \text{otherwise} \end{cases}, \forall i \in \mathcal{N}, \tag{28}$$

$$\alpha_j^{k+1} \begin{cases} = \lambda, & \text{if } h_j^{k+1} > 0 \\ \geq \lambda, & \text{otherwise} \end{cases}, \forall j \in \mathcal{M} \tag{29}$$

and $\sum_{i \in \mathcal{N}} d_i^{k+1} = \sum_{j \in \mathcal{M}} h_j^{k+1} = \Gamma^{k+1}$; then by (4), (10), (28), and (29), the implemented bid profile under Algorithm 1 is exactly the efficient NE for the underlying auction system. ■

IV. Numerical Examples

As a numerical example, we consider a resource allocation problem among six consumers and four suppliers. Suppose that all the consumers share a common form of the valuation function $v_n(x_n) = 2a_n(x_n+1)^{0.8}$, such that a_n, with $n = 1, \ldots, 6$, equals 2, 2.1, 2.2, 2.3, 2.4, and 1.9, respectively, and all the suppliers possess a common form of the cost function $c_m(y_m) = b_m(y_m+1)^{1.2}$, such that a_m, with $m = 1, \ldots, 4$, is equal to 1.1, 1.5, 1.4, and 1.6, respectively. The objective is to implement an allocation among all the consumers and suppliers to maximize the social welfare as defined in (2). By solving the optimization problem (3), we can obtain that the efficient consumption is $\boldsymbol{x}^{**} = [3.012\ 4.122\ 5.463\ 7.068\ 8.986\ 2.104]$ and the efficient supply is $\boldsymbol{y}^{**} = [19.873\ 3.427\ 5.250\ 2.205]$, with identical marginal valuation for consumers and marginal cost for suppliers, i.e., $v_n'(x_n^{**}) = c_m'(y_m^{**}) = 2.424$, for all $n \in \mathcal{N}$ and $m \in \mathcal{M}$. The corresponding maximum social welfare $W_{\max}$ equals 41.15.

By adopting Algorithm 1, we first consider an initial bid profile of players, $\boldsymbol{b}^0$ and $\boldsymbol{s}^0$, such that $\beta_i^0 = v_i'(d_i^0)$ and $\alpha_j^0 = c_j'(h_j^0)$, with $d_i^0 = h_j^0 = 1$ for all $i \in \mathcal{N}$ and $j \in \mathcal{M}$.

Fig. 3 displays the evolution of the potential quantity and the players' allocation with respect to iteration steps under Algorithm 1, respectively. As illustrated, the potential quantity Γ and the allocation of players $\boldsymbol{z}$ increase with respect to

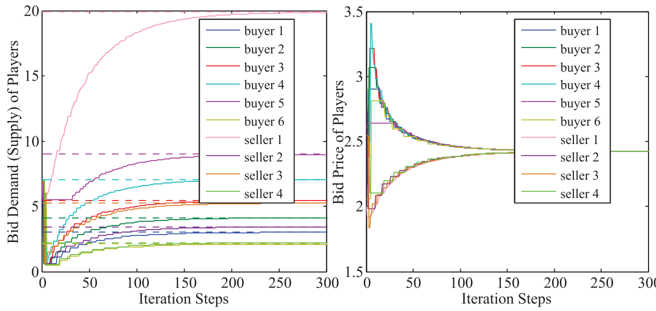

Fig. 4. Updates of players' bid profile under Algorithm 1.

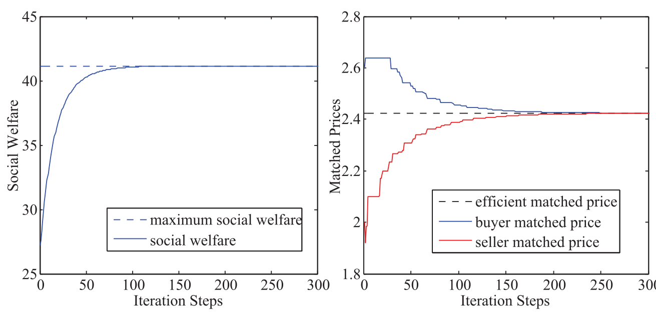

Fig. 5. Updates of the social welfare and the matched prices under Algorithm 1.

iteration steps, respectively. This is consistent with the results developed in Theorem 3.1.

As observed, the system converges in about 300 iteration steps less than 500 which is an upper bound convergence steps specified in (27). $\boldsymbol{z}^k$ converges to the efficient allocation, and Γ^k converges to the total quantity associated with the efficient allocation. These simulation results are consistent with the analysis developed in Theorems 3.2 and 3.3.

In Fig. 4, it displays the updates of individual players' bid profile under Algorithm 1, while Fig. 5 demonstrates the corresponding evolutions of the system social welfare and the matched prices p_b and p_s subject to the updated bid profile of players. As observed, like the evolution of the potential quantity, the system social welfare increases with respect to iteration steps as well. The matched price p_b of the buyers become identical with the matched prices p_s of the sellers when the dynamic process terminates. The convergence behavior of the pair of matched prices (p_b^k, p_s^k) is consistent with the convergence of the potential quantity illustrated in Fig. 3.

In summary, as demonstrated in the simulation results studied above, we can state that the bid profile of players, including buyers and sellers, converges to the efficient bid profile of the underlying double-sided auction games, and the system reaches the maximum social welfare.

V. Applications of the Proposed Auction Mechanism

Auction mechanisms have been widely adopted as effective ways to implement the divisible resource allocation problems in different fields, like power electricity markets [32]–[37], cloud markets [38], financial markets [39], and wireless networks [40]–[42].

More specifically, we will briefly introduce the application of our proposed double-sided auction mechanism in power electricity markets and cellular telecommunication networks in Sections V-A and V-B, respectively.

A. Application in Power Electricity Markets

The auction has been adopted to schedule the electricity generation in conventional electricity whole-sale markets in many regions, e.g., [32] and [33]. More recently, with the increasing penetration of distributed generators, like wind turbines and photovoltaics (PVs), the demand response is involved to improve the utilizations of these renewable energy resources. As a consequence, certain double-sided auction mechanisms have been designed to effectively allocate electricity among generators and load units, [34]–[37]. Note that in these works, the specifications, such as the ramping rates of conventional generators, the topology of power systems and the capacities of transmission/feeder lines, etc., are not considered.

In order to apply our proposed method to the underlying electricity allocation problems, we will give the necessary notions and specifications in the following.

Denote by $\mathcal{N}$ and $\mathcal{M}$ the sets of load units and generators, respectively. Consider the valuation of load n on his demand x_n, denoted by $v_n(x_n)$, is in the form of $v_n(x_n) = \xi_n \log(x_n+1)$, [36], with ξ_n denoting a positive-valued parameter. Suppose that the generation cost of generator m, denoted by $c_m(y_m)$, is in a quadratic form on its supply y_m, say $c_m(y_m) = \mathsf{a}_m y_m^2 + \mathsf{b}_m y_m$, which has been widely adopted in the literature, e.g., [43], [44], and references therein.

Denote by $b_n \equiv (\beta_n, d_n)$ the bid strategy of load n, with β_n (\$/MWh) and d_n (MWh) representing, respectively, the per unit bid price load n is willing to pay and the maximum bid demand of the electricity. Denote by $s_m \equiv (\alpha_m, h_m)$ the bid strategy of generator m, with α_m (\$/MWh) and h_m (MWh) representing the per unit price that generator m is willing to sell and the maximum bid supply it can generate, respectively. Following (13a) and (13b), we can specify the allocations and the payments of each load n with respect to a bid profile $\boldsymbol{b} \equiv (\boldsymbol{b}_n; n \in \mathcal{N})$ and a potential quantity Γ, respectively. Similarly, the allocation and the payment of each generator m are obtained following (14a) and (14b), respectively.

Following the above specifications of $v_n(\cdot)$ and $c_m(\cdot)$, we have that Assumption 1 is satisfied; then, by Theorems 3.1 and 3.3, we can conclude that the formulated auction system converges to the efficient solution by applying Algorithm 1.

For the purpose of demonstration, we give a numerical simulation here. Consider there are three generators and three load units, and the parameters are given as $\boldsymbol{\xi} \equiv (\xi_1, \xi_2, \xi_3) = [50, 55, 56]$, $\mathsf{a} = [30, 33, 35]$ and $\mathsf{b} = [0, 0, 0]$. Fig. 6 displays the updates of individual strategies and the converged solution by applying Algorithm 1. This is consistent with the developed results.

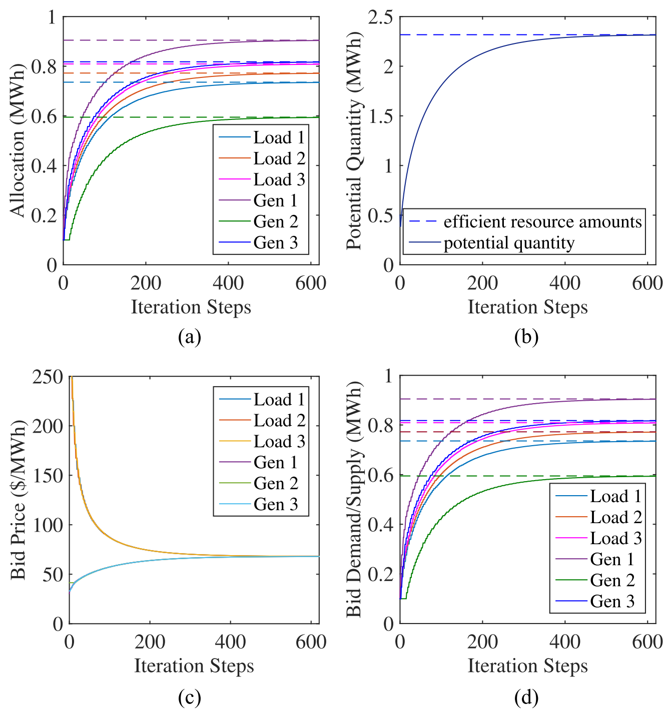

Fig. 6. Updates of individual strategies of generators and load units by applying Algorithm 1. (a) Allocation. (b) Potential quantity. (c) Bid price. (d) Bid demand/supply.

B. Application in Cellular Telecommunication Networks

With the tremendous growth of traffic and services in cellular networks, it is emerging to allocate wireless bandwidth resources properly in the forthcoming fifth generation cellular networks [45], [46], improving resource usage efficiency and reducing the capital expenses and operation expenses. Under the technique of wireless virtualization, it can be modeled as a double-sided resource allocation among infrastructures providers (InPs) and mobile virtual network operators (MVNOs).

Parallel with Section V-A, to implement the efficient solution for the underlying wireless resource allocation problems by applying our proposed method, we will give the necessary notions and specifications as well.

Denote by $\mathcal{N}$ and $\mathcal{M}$ the sets of MVNOs and InPs operating within the same geographical area, respectively. As given in [45] and [46], the utility function of MVNO n is specified as $v_n(x_n) = \xi_n \ln(x_n)$, with x_n representing MVNO n's desired bandwidth, while the cost function of InP m is given as $c_m(y_m) = \mathsf{a}_m e^{y_m}$, with y_n representing the bandwidth it can supply and a_m as a properly valued parameter. By the above specifications of $v_n(\cdot)$ and $c_m(\cdot)$, we have that Assumption 1 is satisfied.

More specifically, to apply the proposed method, we consider that each MVNO $n \in \mathcal{N}$ submits a bid $b_n \equiv (\beta_n, d_n)$ representing the per unit bid price n is willing to pay and the maximum demand of the wireless resource; and denote by $s_m \equiv (\alpha_m, h_m)$ the bid strategy of InP m, with α_m and h_m representing the per unit price that m is willing to sell and the maximum supply it can supply, respectively.

Similarly, we can claim that the formulated auction system converges to the efficient solution by applying Algorithm 1.

VI. Conclusion

In this paper, we studied the efficient resource allocation problem among suppliers and consumers. We formulated this problem as a VCG-type double-sided auction game, which possesses the incentive compatibility and the existence of the efficient NE. To implement the efficient NE, we proposed a novel dynamic process for the underlying auction game. More specifically, the double-sided auction game is decomposed into two single-sided ones coupled via a so-called potential quantity, which represents the total trade quantity of the resource in the system. An iterative update scheme is then designed to implement the efficient NE based on the mechanisms designed in the single-sided auctions. At each iteration step, the auctioneer assigns a specific buyer and a specific seller to allow them update their strategies, respectively, and update the potential quantity. Assisted with the given extra system information, a certain constraint is set on the bid demand of the assigned buyer and the bid supply of the assigned seller, respectively. Under the proposed method, the potential quantity and the social welfare increase with respect to iteration steps, respectively. And we show that the underlying auction system converges to the efficient NE at which the system reaches the maximal social welfare. Furthermore, we verify that the convergence iteration steps are within a certain value which is the order of $\mathcal{O}(\ln(1/\varepsilon))$.

Appendix

A. Proof of Lemma 2.1

It is equivalent to show the incentive compatibility for any buyer $n \in \mathcal{N}$ by verifying that, for any bid $b_n \equiv (\beta_n, d_n) \in \mathcal{B}_n$, there exists a truth-telling bid $b_n^t \equiv (\beta_n^t, d_n^t) \in \mathcal{B}_n^t$, say $\beta_n^t = v_n'(d_n^t)$, such that

$$f_n(b_n^t, \boldsymbol{r}_{-n}) \geq f_n(b_n, \boldsymbol{r}_{-n}) \tag{30}$$

for any given bid profile $\boldsymbol{r}_{-n}$ of other players.

Denote by x_n and x_n^t the allocations of buyer n with respect to $(b_n, \boldsymbol{r}_{-n})$ and $(b_n^t, \boldsymbol{r}_{-n})$, respectively. We will show (30) in the following:

1) *In case $\beta_n < v_n'(d_n)$:* Consider a bid b_n^t such that $d_n^t = x_n \leq d_n$. By Assumption 1, we have $\beta_n^t \geq v_n'(d_n) > \beta_n$. Then, by (5), we have $x_n^t \geq x_n$. Also by $x_n^t \leq d_n^t = x_n$, we have $x_n^t = x_n$. Hence, by the specification of the payoff functions of players, we have $f_n(b_n^t, \boldsymbol{r}_{-n}) = f_n(b_n, \boldsymbol{r}_{-n})$.
2) *In case $\beta_n > v_n'(d_n)$:* Consider a bid b_n^t, such that $d_n^t = d_n$; then, $\beta_n > v_n'(d_n) = v_n'(d_n^t) = \beta_n^t$. By (5), we have $x_n^t \leq x_n$. When $x_n^t = x_n$, the payoffs $f_n(b_n^t, \boldsymbol{r}_{-n}) = f_n(b_n, \boldsymbol{r}_{-n})$. Next we consider the case $x_n^t < x_n$. The

following holds:

$$\begin{aligned}f_n(b_n,\boldsymbol{r}_{-n}) - f_n(b_n^t,\boldsymbol{r}_{-n}) &= v_n(x_n) - v_n(x_n^t) + \tau_n(b_n^t,\boldsymbol{r}_{-n}) - \tau_n(b_n,\boldsymbol{r}_{-n})\\ &\le \beta_n^t(x_n - x_n^t) + U(\boldsymbol{z}) - \beta_n x_n - U(\boldsymbol{z}^t) + \beta_n^t x_n^t\\ &\le \beta_n^t(x_n - x_n^t) - \beta_n^t(x_n - x_n^t) = 0.\end{aligned}$$

In conclusion, by 1) and 2) given above, we obtain that (30) holds.

Following the same technique applied for buyers, we can also verify the incentive compatibility for any seller $m \in \mathcal{M}$.

B. Proof of $Q^k = \Gamma^k$ in Theorem 3.1

By adopting Algorithm 1, we have $\Gamma^0 < \sum_{i\in\mathcal{N}} d_i^0$ and $\Gamma^0 < \sum_{j\in\mathcal{M}} h_j^0$, by which together with (13a) and (14a), we have $\Gamma^0 = \sum_{i\in\mathcal{N}} x_i^0 = \sum_{j\in\mathcal{M}} y_j^0$; then, $Q^0 \equiv \min\{\sum_{i\in\mathcal{N}} x_i^0, \sum_{j\in\mathcal{M}} y_j^0\} = \Gamma^0$.

At iteration step $k \ge 1$, a buyer n asks to implement his best response in the buyer-sided auction. Denote by $\boldsymbol{b}^k = (b_n^k, \boldsymbol{b}_{-n}^k)$ the updated bid profile at step $k-1$, where $\boldsymbol{b}_{-n}^k = \boldsymbol{b}_{-n}^{k-1}$, and $b_n^k = (\beta_n^k, d_n^k)$ represents the best response of buyer n. Denote by $\boldsymbol{x}^k = \boldsymbol{x}(\boldsymbol{b}^k, \Gamma^k)$, i.e., $\boldsymbol{x}^k$ represents the allocation of buyers.

We will show $Q^k = \Gamma^k$ by verifying that $\sum_{i\in\mathcal{N}} x_i^k = \Gamma^k$ and $\sum_{j\in\mathcal{M}} y_j^k = \Gamma^k$ in 1) and 2), respectively.

1) To show $\sum_{i\in\mathcal{N}} x_i^k = \Gamma^k$ by proof of contradiction below.

First, since Γ^k is the total allocated quantity, we have $\sum_{i\in\mathcal{N}} x_i^k \le \Gamma^k$; then suppose that $\sum_{i\in\mathcal{N}} x_i^k < \Gamma^k$, by (13a), we have $x_i^k = d_i^k$ for all $i \in \mathcal{N}$, i.e., all the buyers are fully allocated. Hence, $\sum_{i\in\mathcal{N}} x_i^k = \sum_{i\in\mathcal{N}} d_i^k < \Gamma^k$.

By (18a), we have

$$D_n^{k-1} \equiv D_n(\boldsymbol{b}^{k-1}, \Gamma^{k-1}, \Gamma^k) = x_n^{k-1} + \left[\Gamma^k - \sum_{i\in\mathcal{N}} x_i^{k-1}\right]^+. \tag{31}$$

We will show that $d_n^k < D_n^{k-1}$ in (i)–(ii) below.

i) In case $\Gamma^k - \sum_{i\in\mathcal{N}} x_i^{k-1} < 0$.

The following inequalities hold:

$$\begin{aligned}\Gamma^k &< \sum_{i\in\mathcal{N}} x_i^{k-1} \le x_n^{k-1} + \sum_{i\in\mathcal{N}/\{n\}} d_i^{k-1}\\ &= x_n^{k-1} + \sum_{i\in\mathcal{N}/\{n\}} d_i^k,\\ \Gamma^k &> \sum_{i\in\mathcal{N}} d_i^k = d_n^k + \sum_{i\in\mathcal{N}/\{n\}} d_i^k\end{aligned}$$

then it implies that $d_n^k < x_n^{k-1}$. Also by (31), we have $D_n^{k-1} = x_n^{k-1}$ in case $\Gamma^k - \sum_{i\in\mathcal{N}} x_i^{k-1} < 0$; then we have $d_n^k < D_n^{k-1}$.

ii) In case $\Gamma^k - \sum_{i\in\mathcal{N}} x_i^{k-1} \ge 0$.

We have the following:

$$\Gamma^k > \sum_{i\in\mathcal{N}} d_i^k = d_n^k + \sum_{i\in\mathcal{N}/\{n\}} d_i^{k-1} \ge d_n^k + \sum_{i\in\mathcal{N}/\{n\}} x_i^{k-1}$$

which implies that $d_n^k < \Gamma^k - \sum_{i\in\mathcal{N}/\{n\}} x_i^{k-1}$. Also by (31), we have

$$D_n^{k-1} = x_n^{k-1} + \Gamma^k - \sum_{i\in\mathcal{N}} x_i^{k-1} = \Gamma^k - \sum_{i\in\mathcal{N}/\{n\}} x_i^{k-1} \tag{32}$$

in case $\Gamma^k - \sum_{i\in\mathcal{N}} x_i^{k-1} \ge 0$; then, we have $d_n^k < D_n^{k-1}$.

By *a) and b)* above, we have $d_n^k < D_n^{k-1}$. Also by $\sum_{i\in\mathcal{N}} d_i^k < \Gamma^k$, we can define another bid profile for player n, denoted by $\widehat{b}_n \equiv (\widehat{\beta}_n, \widehat{d}_n)$, such that

$$\widehat{d}_n = d_n^k + \varepsilon, \text{ with } 0 < \varepsilon < \min\left\{D_n^{k-1} - d_n^k, \Gamma^k - \sum_{i\in\mathcal{N}} d_i^k\right\}.$$

By $\varepsilon < \Gamma^k - \sum_{i\in\mathcal{N}} d_i^k$, we have $\sum_{i\in\mathcal{N}} d_i^k + \varepsilon = \sum_{i\in\mathcal{N}/\{n\}} d_i^k + \widehat{d}_n < \Gamma^k$; then, by (13a), we have $\widehat{x}_n \equiv x_n((\widehat{b}_n, \boldsymbol{b}_{-n}^k), \Gamma^k) = \widehat{d}_n$, and $x_i((\widehat{b}_n, \boldsymbol{b}_{-n}^k), \Gamma^k) = d_i^k = x_i^k$ for all $i \in \mathcal{N}/\{n\}$.

Hence, by the payment of buyer n specified in (13b), we have $\tau_n(\boldsymbol{b}^k, \Gamma^k) = \tau_n\left((\widehat{b}_n, \boldsymbol{b}_{-n}^k), \Gamma^k\right)$. Thus, we have

$$\begin{aligned}f_n(\boldsymbol{b}^k) - f_n(\widehat{b}_n, \boldsymbol{b}_{-n}^k) &= v_n(x_n^k) - v_n(\widehat{x}_n)\\ &= v_n(d_n^k) - v_n(\widehat{d}_n)\\ &< 0\end{aligned}$$

where the last inequality holds by Assumption 1, which is contradicted with the consideration that b_n^k is the best response of buyer n. It implies that $\sum_{i\in\mathcal{N}} x_i^k < \Gamma^k$ cannot be held. Hence, $\sum_{i\in\mathcal{N}} x_i^k = \Gamma^k$.

2) By the same technique applied in the proof of $\sum_{i\in\mathcal{N}} x_i^k = \Gamma^k$ given in (I) above, we can show $\sum_{j\in\mathcal{M}} y_j^k = \Gamma^k$ as well.

In summary, by the specification of Q^k given in (17), we have

$$Q^k \triangleq \min\left\{\sum_{i\in\mathcal{N}} x_i^k, \sum_{j\in\mathcal{M}} y_j^k\right\} = \Gamma^k.$$

C. Proof of $p_b^k \ge p_s^k$ for all $k \ge 0$ in Theorem 3.1

Under Algorithm 1, the matched prices p_b^0 and p_s^0 subject to the initial bid profile are supposed to satisfy the inequality of $p_b^0 \ge p_s^0$; then, to show $p_b^k \ge p_s^k$ holds for all $k \ge 0$, it is equivalent to verify that $p_b^{k+1} \ge p_s^{k+1}$ holds in case $p_b^k \ge p_s^k$ with $k \ge 0$.

By (23) and the assumed $p_b^k \ge p_s^k$ for some $k \ge 0$, it can be shown that

$$\Gamma^{k+1} \ge \Gamma^k. \tag{33}$$

Also suppose that at iteration step k, in the buyer-sided auction, buyer n updates his best response, denoted by b_n^{k+1}.

By Appendix B, we can have $\Gamma^k = \sum_{i\in\mathcal{N}} x_i^k$, by which together with (18a) and (33), the following holds:

$$\begin{aligned}D_n^k \equiv D_n\left(\boldsymbol{b}^k, \Gamma^k, \Gamma^{k+1}\right) &= x_n^k + \left[\Gamma^{k+1} - \sum_{i\in\mathcal{N}} x_i^k\right]^+\\ &= x_n^k + \Gamma^{k+1} - \Gamma^k.\end{aligned} \tag{34}$$

By the concavity of v_n under Assumption 1 and (15a), we have

$$v_n'(D_n^k) = v_n'\left(x_n^k + \Gamma^{k+1} - \Gamma^k\right) \ge v_n'(x_n^k) - \bar{\rho}(\Gamma^{k+1} - \Gamma^k). \tag{35}$$

Moreover, by (20a), the best response of buyer n satisfies $d_n^{k+1} \leq D_n^k$; then, by which together with Assumption 1, we have $\beta_n^{k+1} \geq v_n'(D_n^k)$.

In the following, we will verify that

$$p_b^{k+1} \geq p_b^k - \bar{\rho}(\Gamma^{k+1} - \Gamma^k) \tag{36}$$

in 1)–3) below.

1) In case $x_n^k \in (0, d_n^k)$.

By (13a) and (16a), we have $p_b^k = \beta_n^k$. Also by (13a) we have

$$x_i^k = \begin{cases} d_i^k, & \text{in case } \{\beta_i^k > \beta_n^k\} \text{ or } \{\beta_i^k = \beta_n^k, i < n\} \\ 0, & \text{otherwise} \end{cases}$$

for all $i \in \mathcal{N}/\{n\}$, and by which, together with $\beta_n^{k+1} \geq v_n'(D_n^k)$, we can obtain that those buyers $i \in \mathcal{N}/\{n\}$, such that $x_i^k = 0$ and $\beta_i^k \geq v_n'(D_n^k)$, can increase their own allocations, respectively. Thus, by (16a), we have $p_b^{k+1} \geq v_n'(D_n^k)$. Hence, by (35), we get that

$$\begin{aligned} v_n'(D_n^k) &> v_n'(d_n^k) - \bar{\rho}(\Gamma^{k+1} - \Gamma^k) \\ &= \beta_n^k - \bar{\rho}(\Gamma^{k+1} - \Gamma^k) \\ &= p_b^k - \bar{\rho}(\Gamma^{k+1} - \Gamma^k) \end{aligned}$$

which implies that the inequality (36) holds by $p_b^{k+1} \geq v_n'(D_n^k)$, and where the inequality holds by $x_n^k < d_n^k$, and the second equality holds by $p_b^k = \beta_n^k$.

2) In case $x_n^k = 0 < d_n^k$. By (23), we have $\Gamma^{k+1} = \Gamma^k$.

By (34) and $\Gamma^{k+1} = \Gamma^k$, we have $D_n^k = 0$ by which together with $d_n^{k+1} \leq D_n^k$, we have $x_n^{k+1} = d_n^{k+1} = 0 = x_n^k$. Hence, we have $p_b^{k+1} = p_b^k$ which implies (36).

3) Other cases besides those considered in (I) and (II).

By adopting Algorithm 1, we have $x_i^k = d_i^k$ for all $i \in \mathcal{N}$, i.e., all buyers are fully allocated, and $\beta_n^k = \max_{i\in\mathcal{N}}\{\beta_i^k\}$; then $p_b^k = \min_{i\in\mathcal{N}}\{\beta_i^k\} \leq \beta_n^k$.

By $\beta_n^{k+1} \geq v_n'(D_n^k)$, if $v_n'(D_n^k) > p_b^k$, we have $p_b^{k+1} = p_b^k$; then, $p_b^{k+1} \geq p_b^k - \bar{\rho}(\Gamma^{k+1} - \Gamma^k)$ by $\Gamma^{k+1} \geq \Gamma^k$; else, we have $p_b^{k+1} \geq v_n'(D_n^k)$. Hence, by (35) and $p_b^k \leq \beta_n^k$, the inequality (36) holds.

By adopting the same technique to verify (36) for the matched price on the buyer-sided auction game in 1)–3) above, we can show the following inequality property for the matched price on the seller-sided auction game as well:

$$p_s^{k+1} \leq p_s^k + \bar{\sigma}(\Gamma^{k+1} - \Gamma^k). \tag{37}$$

By (23), $\Gamma^{k+1} = \Gamma^k$ or $\Gamma^{k+1} = \Gamma^k + \frac{p_b^k - p_s^k}{\bar{\rho}+\bar{\sigma}}$; then, we have the following analysis:

1) *In case* $\Gamma^{k+1} = \Gamma^k$. By (36), (37) and $p_b^k \geq p_s^k$, we have $p_b^{k+1} \geq p_s^{k+1}$.
2) *In case* $\Gamma^{k+1} = \Gamma^k + \frac{p_b^k - p_s^k}{\bar{\rho}+\bar{\sigma}}$. We have $p_b^k - \bar{\rho}(\Gamma^{k+1} - \Gamma^k) = p_s^k + \bar{\sigma}(\Gamma^{k+1} - \Gamma^k)$; then, by (36) and (37), we have $p_b^{k+1} \geq p_s^{k+1}$.

In summary, we have $p_b^{k+1} \geq p_s^{k+1}$ on the condition that $p_b^k \geq p_s^k$, i.e., the conclusion holds.

D. Proof of (25) in Theorem 3.1

We will verify (25) in 1) and 2) below.

1) To show $x_i^{k+1} \geq x_i^k$ for all $i \in \mathcal{N}$: Suppose that $x_n^{k+1} < x_n^k$. Consider another bid $\widehat{b}_n = (\widehat{\beta}_n, \widehat{d}_n)$ with $\widehat{\beta}_n = v_n'(\widehat{d}_n)$ and $\widehat{d}_n = x_n^k$, and denote by $\widehat{\boldsymbol{x}}$ the allocation with respect to the bid profile $\widehat{\boldsymbol{b}} \equiv (\widehat{b}_n, \boldsymbol{b}_{-n}^{k+1})$.

By (24), (13a), and $x_n^{k+1} < x_n^k = x_n^*(\boldsymbol{b}^k, \Gamma^k)$, buyer n does not grab other buyers' allocations under both $\boldsymbol{b}^{k+1}$ and $\widehat{\boldsymbol{b}}$. Since buyer n is assigned to implement his best response, he must satisfy one of the three cases specified in (21a) in sequence. Then, by (13a), we obtain that those buyers whose bid prices satisfy $\beta_i^k > \beta_n^k$, for some $i \in \mathcal{N}/\{n\}$, must be fully allocated. In other words, their allocation cannot be increased. Since for all $i \in \mathcal{N}/\{n\}$, $\beta_i^k = \beta_i^{k+1}$ holds, we obtain that only the buyers whose bid prices satisfy $\beta_i^{k+1} \leq \beta_n^k$ can increase their allocations. That is,

$$\begin{cases} x_i^{k+1} = x_i^k \text{ and } \widehat{x}_i = x_i^k, & \text{in case } \beta_i^{k+1} > \beta_n^k \\ x_i^{k+1} \geq x_i^k \text{ and } \widehat{x}_i \geq x_i^k, & \text{otherwise} \end{cases} \tag{38}$$

then by the definition of the payoff function f_n, the following holds:

$$\begin{aligned} & f_n(\boldsymbol{b}^{k+1}) - f_n(\widehat{\boldsymbol{b}}) \\ &= v_n(x_n^{k+1}) - v_n(x_n^k) - \sum_{i\in\mathcal{N}/\{n\}} \beta_i(x_i^{k+1} - \widehat{x}_i) \\ &< v_n'(x_n^k)(x_n^{k+1} - x_n^k) - \sum_{i\in\mathcal{N}/\{n\}} \beta_i(x_i^{k+1} - \widehat{x}_i) \\ &\leq v_n'(x_n^k)(x_n^{k+1} - x_n^k) - \beta_n^k \sum_{i\in\mathcal{N}/\{n\}} (x_i^{k+1} - \widehat{x}_i) \\ &\leq \beta_n^k(x_n^{k+1} - x_n^k) - \beta_n^k \sum_{i\in\mathcal{N}/\{n\}} (x_i^{k+1} - \widehat{x}_i) \\ &\leq 0 \end{aligned}$$

which is contradicted with the fact that b_n^{k+1} is the best response of buyer n, and where the first inequality holds by Assumption 1, the second inequality holds by (38), and the last one holds by $x_n^{k+1} < x_n^k \leq d_n^k$. Hence, $x_n^{k+1} \geq x_n^k$ holds.

By (24), we have $\Gamma^{k+1} \geq \Gamma^k = \sum_{i\in\mathcal{N}} x_i^k$, by which together with (18a), we have $D_n^k = x_n^k + \Gamma^{k+1} - \Gamma^k$. By (20a), the best response of buyer n satisfies $d_n^{k+1} \leq D_n^k$; then, by $x_n^{k+1} = d_n^{k+1}$ specified in (22a) in Lemma 3.2, we have

$$x_n^{k+1} \leq x_n^k + \Gamma^{k+1} - \Gamma^k$$

by which we can show that $x_i^{k+1} \geq x_i^k$ for all $i \in \mathcal{N}/\{n\}$, i.e., the allocations of other buyers will not decrease.

2) By the same technique applied in (I), we can show $y_j^{k+1} \geq y_j^k$ for all $j \in \mathcal{M}$.

E. Proof of Theorem 3.2

By Theorem 3.1 together with (22), we can obtain that after certain iteration steps $\widehat{k} \leq \max\{N, M\}$, the following holds:

1) The potential quantity of resource is completely distributed among players, i.e., $\Gamma^k = \sum_{i\in\mathcal{N}} x_i^k = \sum_{j\in\mathcal{M}} y_j^k$ for any $k \geq \widehat{k}$.

2) All the players are fully allocated, i.e., $x_i^k = d_i^k, \forall i$ and $y_j^k = h_j^k, \forall j$ for any $k \geq \widehat{k}$.

Suppose that buyer n and seller m are the players who are assigned to update their best responses, respectively, at iteration step k with $k > \widehat{k}$; then, $\|\boldsymbol{d}^{k+1} - \boldsymbol{d}^k\|_1 = |d_n^{k+1} - d_n^k|$, and $\|\boldsymbol{h}^{k+1} - \boldsymbol{h}^k\|_1 = |h_m^{k+1} - h_m^k|$.

First, in the buyer-sided auction, by (19a), and 1) and 2), it gives

$$\|\boldsymbol{d}^{k+1} - \boldsymbol{d}^k\|_1 = x_n^k + \Gamma^{k+1} - \sum_{i\in\mathcal{N}} x_i^k - d_n^k$$
$$= \Gamma^{k+1} - \Gamma^k = \frac{p_b^k - p_s^k}{\bar{\rho} + \bar{\sigma}}. \tag{39}$$

By the definition of p_b and p_s specified in (16), and 2), we have

$$p_b^k = \begin{cases} p_b^{k-1}, & \text{if } \beta_i^k \geq p_b^{k-1} \\ \beta_i^k, & \text{otherwise} \end{cases},$$
$$p_s^k = \begin{cases} p_s^{k-1}, & \text{if } \alpha_j^k \leq p_s^{k-1} \\ \alpha_j^k, & \text{otherwise} \end{cases}$$

where i, j represent the buyer and the seller who update their strategies at iteration step $k-1$, respectively.

By (39), we have

$$\|\boldsymbol{d}^{k+1} - \boldsymbol{d}^k\|_1 \begin{cases} = \frac{\beta_i^k - \alpha_j^k}{\bar{\rho}+\bar{\sigma}}, & \text{in case } \beta_i^k < p_b^{k-1}, \alpha_j^k > p_s^{k-1} \\ \geq \frac{\beta_i^k - \alpha_j^k}{\bar{\rho}+\bar{\sigma}} & \text{otherwise} \end{cases} \tag{40}$$

which implies that $\|\boldsymbol{d}^{k+1} - \boldsymbol{d}^k\|_1$ is bounded by $\frac{\beta_i^k - \alpha_j^k}{\bar{\rho}+\bar{\sigma}}$ from below.

By (25) and 2) in earlier part of this section, we have $\boldsymbol{d}^{k+1} \geq \boldsymbol{d}^k$ for any k with $k \geq \widehat{k}$; then, by which together with (40), at step k, we adopt

$$\|\boldsymbol{d}^{k+1} - \boldsymbol{d}^k\|_1 = \frac{\beta_i^k - \alpha_j^k}{\bar{\rho} + \bar{\sigma}} \tag{41}$$

to specify an upper bound for the convergence steps of the algorithm.

By the specifications of $\underline{\rho}$ and $\underline{\sigma}$ given in (26), and (39), we can obtain that

$$\beta_i^{k-1} - \beta_i^k > \underline{\rho}\frac{p_b^{k-1} - p_s^{k-1}}{\bar{\rho} + \bar{\sigma}},$$
$$\alpha_j^k - \alpha_j^{k-1} > \underline{\sigma}\frac{p_b^{k-1} - p_s^{k-1}}{\bar{\rho} + \bar{\sigma}}$$

by which, together with (39), we have

$$\beta_i^k - \alpha_j^k < \beta_i^{k-1} - \alpha_j^{k-1} - \left(\underline{\rho} + \underline{\sigma}\right)\frac{p_b^{k-1} - p_s^{k-1}}{\bar{\rho} + \bar{\sigma}}$$
$$= \beta_i^{k-1} - \alpha_j^{k-1} - \left(\underline{\rho} + \underline{\sigma}\right)\|\boldsymbol{d}^k - \boldsymbol{d}^{k-1}\|_1. \tag{42}$$

Also by (16), we have $\beta_i^{k-1} \geq p_b^{k-1}$ and $\alpha_j^{k-1} \leq p_s^{k-1}$ which and (39) imply that

$$\beta_i^{k-1} - \alpha_j^{k-1} \geq p_b^{k-1} - p_s^{k-1} = (\bar{\rho} + \bar{\sigma})\|\boldsymbol{d}^k - \boldsymbol{d}^{k-1}\|_1. \tag{43}$$

Then, by (41) and (42), we have

$$\|\boldsymbol{d}^{k+1} - \boldsymbol{d}^k\|_1$$
$$< \frac{1}{\bar{\rho} + \bar{\sigma}}\left(\frac{\beta_i^k - \alpha_j^k}{\|\boldsymbol{d}^k - \boldsymbol{d}^{k-1}\|_1} - (\underline{\rho} + \underline{\sigma})\right)\|\boldsymbol{d}^k - \boldsymbol{d}^{k-1}\|_1 \tag{44}$$

by which together with (43), we have

$$\|\boldsymbol{d}^{k+1} - \boldsymbol{d}^k\|_1 < \frac{1}{\theta}\|\boldsymbol{d}^k - \boldsymbol{d}^{k-1}\|_1, \text{ with } \theta \equiv \frac{\bar{\rho} + \bar{\sigma}}{\bar{\rho} + \bar{\sigma} - \underline{\rho} - \underline{\sigma}}.$$

By the same method used above, the following holds for the seller-sided auction:

$$\|\boldsymbol{h}^{k+1} - \boldsymbol{h}^k\|_1 < \frac{1}{\theta}\|\boldsymbol{h}^k - \boldsymbol{h}^{k-1}\|_1.$$

By $\beta_l = v_l'(d_l), l \in \mathcal{N}$ and $\alpha_l = c_l'(h_l), l \in \mathcal{M}$, together with (15a) and (15b)

$$\|\boldsymbol{r}^{k+1} - \boldsymbol{r}^k\|_1$$
$$= \|\boldsymbol{b}^{k+1} - \boldsymbol{b}^k\|_1 + \|\boldsymbol{s}^{k+1} - \boldsymbol{s}^k\|_1$$
$$\leq (\bar{\rho} + 1)\|\boldsymbol{d}^{k+1} - \boldsymbol{d}^k\|_1 + (\bar{\sigma} + 1)\|\boldsymbol{h}^{k+1} - \boldsymbol{h}^k\|_1.$$

By the termination condition in Algorithm 1, we can verify that the system converges within $K = \widehat{k} + \widehat{K}$ iteration steps, with $\widehat{K}$ given in (27).

F. Verification of (28) and (29) in Theorem 3.3

Suppose that, under Algorithm 1, the system converges at iteration step k, and at the equilibrium as specified in Algorithm 1, we have $\boldsymbol{r}^{k+1} = \boldsymbol{r}^k$ and $\Gamma^{k+1} = \Gamma^k$.

First, we analyze the properties of $\boldsymbol{b}^{k+1}$ in the buyer-sided auction. Suppose that the system assigns buyer n to update his best response with respect to the allocation $\boldsymbol{x}^k$ and $\boldsymbol{b}^k$, and b_n^{k+1} is the best response updated by buyer n at step k. By (22), we have $x_n^{k+1} = d_n^{k+1}$.

By $\boldsymbol{b}^{k+1} = \boldsymbol{b}^k$, $\Gamma^{k+1} = \Gamma^k$ and (13a), we have $\boldsymbol{x}^{k+1} = \boldsymbol{x}^k$; then, by $d_n^k = d_n^{k+1}$, we have $x_n^k = d_n^k$, i.e., buyer n assigned by the system is fully allocated before his best response is implemented. By the rule to assign buyer n in Algorithm 1, we have

$$x_i^k = d_i^k \text{ and } \beta_n^k \geq \beta_i^k, \forall i \in \mathcal{N}. \tag{45}$$

Suppose $\beta_n^k = \lambda$, for some positive valued λ; then, by (45), $\boldsymbol{x}^{k+1} = \boldsymbol{x}^k$ and $\boldsymbol{r}^{k+1} = \boldsymbol{r}^k$, the following properties hold, for all $i \in \mathcal{N}$:

$$x_i^k = x_i^{k+1} = d_i^k = d_i^{k+1}, \tag{46}$$

$$\beta_i^k = \beta_i^{k+1} \leq \beta_n^k = \beta_n^{k+1} = \lambda. \tag{47}$$

From $\Gamma^{k+1} = \Gamma^k$ and Appendix B, we have $\Gamma^{k+1} = \sum_{i\in\mathcal{N}} x_i^k$; then, $D_n^k \equiv D_n(\boldsymbol{b}^k, \Gamma^k, \Gamma^{k+1}) = d_n^{k+1}$ by (18a) and (46). By $\beta_n = v_n'(d_n)$, we have $v_n'(D_n^k) = \beta_n^{k+1}$; then, by (46), (47), and (16a), we can verify that $p_b^{k+1} = \lambda$.

Hence by (16a), we have $\beta_i^{k+1} \geq \lambda$ in case $x_i^{k+1} > 0$ for all $i \in \mathcal{N}$, by which together with (46) and (47), we have $\beta_i^{k+1} = \lambda$ in case $d_i^{k+1} > 0$ for all $i \in \mathcal{N}$.

In summary, we obtain that the updated bid profile $\boldsymbol{b}^{k+1}$ at step k satisfies (28) and $\sum_{i\in\mathcal{N}} d_i^{k+1} = \Gamma^{k+1}$, and the matched price of buyer is specified as $p_b^{k+1} = \lambda$.

Since $\Gamma^{k+1} = \Gamma^k$, we have $p_b^k = p_s^k$; then, by the same technique of the analysis of $\boldsymbol{b}^{k+1}$, we can verify (29) and $\sum_{j\in\mathcal{M}} s_j^{k+1} = \Gamma^{k+1}$ as well.

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