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The Double Auction Market Institution: A Survey

1. INTRODUCTION

Trade in actual markets (in the field or in the laboratory) is governed by a *market institution*, that is, a set of rules specifying which sorts of bids and other messages are legitimate, and how and when specific traders transact, given their chosen messages. The continuous double auction (DA) is an important type of market institution. It allows traders to make offers to buy or sell and to accept other traders' offers at any moment during a trading period.

Interest in the DA institution arises from two facts:

1. For more than 100 years, trade in the most important field markets for homogeneous goods has been governed primarily by DA rules. For example, the New York Stock Exchange and the major Chicago exchanges all use versions of the DA institution, and the traded goods include stocks, bonds, agricultural commodities, metals, and derivative securities. Laboratory markets also have relied primarily on variants of the DA since Smith.[79]

The Double Auction Market, Eds. D. Friedman and J. Rust,
SFI Studies in the Sciences of Complexity,
Proc. Vol. XIV, Addison-Wesley, 1993

2. The laboratory experiments demonstrate that even with rather small numbers of traders who have very imperfect information on supply and demand, the DA institution consistently produces very efficient allocations and prices, much more so than traditional theory would suggest. Smith[83] calls this finding a "scientific mystery." The mystery remains unsolved to this day, despite the central importance of the issues it raises for theorists and practitioners.

The purpose of this chapter is to survey recent developments in academic research which may help us better understand the double auction institution. (The practitioner's perspective is equally important and will be emphasized in the next chapter.) Here I report work by theorists and by experimentalists which addresses questions such as:

- How do outcomes in laboratory DA markets compare to relevant theoretical benchmarks?
- How does the DA institution fare in different laboratory environments and in different field environments?
- How sensitive are the results to apparently minor variations in the DA institution?
- How does the DA compare to alternative market institutions?

Section 2 standardizes terminology, places the DA institution and its variants in the taxonomy of market institutions, and distinguishes the different sorts of trading environments. Section 3 surveys several strands of the theoretical literature relevant to the DA. Section 4 surveys some of the relevant empirical literature. Concluding remarks are offered in Section 5.

2. MARKET INSTITUTIONS AND MARKET ENVIRONMENTS

Unfortunately no standard terminology for market institutions has yet emerged in the literature. For example, the DA institution sometimes is called a double oral auction or a bid-ask market, while a quite different institution (referred to below as the CH) is sometimes called a call market and sometimes called a (static) double auction! Sometimes a market institution is called dynamic because it is repeated over time, and sometimes because it allows traders to chose the timing of their messages continuously. To avoid confusion, I begin by standardizing my terminology.

Institutions I consider deal with at least two agents, called *traders*, and at least two commodities allocated to (owned by) the traders. Unless otherwise noted, I assume exactly two commodities, one of which is called the *good* and the other of which is called *money*. Money is the numeraire and is assumed divisible. *Exchange* refers to any non-coercive process whereby traders alter the allocation of commodities, total quantities of all commodities remaining constant (i.e., abstracting from production and consumption). A *market institution* defines an exchange process by specifying the set of admissible *messages* (i.e., traders' actions, usually price and/or quantity offers), and by specifying a final commodity allocation given any combination of messages chosen by the traders and any initial allocation.

A *net trade* vector is the difference between a final and an initial allocation. A *bilateral transaction* is a minimal net trade, involving non-zero components of money and the good for two traders. For any trader whose net trade component for the good is non-zero, the *transaction price* is the absolute value of her money component divided by her good component. If the good component is positive (negative), I refer to her as a *buyer* (*seller*). For example, if trader A's allocation of the good increases by 2 units and his allocation of money (in dollars) decreases by \$1.50 while trader B's allocation of the good decreases by 2 and her money increases by \$1.50, other traders' allocations constant, then we have a 2-unit bilateral transaction between buyer A and seller B at transaction price \$0.75. Sometimes the institution specializes all traders, either as buyers who can never sell or as sellers who can never buy; I refer to this as a case of *one-way traders.*

2.1 A PARTIAL TAXONOMY OF MARKET INSTITUTIONS

Bilateral search processes are minimal sorts of market institutions in which traders seek partners for mutually beneficial bilateral transactions. Of greater interest for present purposes are *unified* or consolidated market institutions which provide more information on trading opportunities and preclude simultaneous transactions at different prices. A (non-discriminatory) *auction* is a unified market institution in which traders' messages include an offered price (called a *bid* for an offer to buy and an *ask* for an offer to sell), and which gives higher priority in transactions to better offers (higher bids and lower asks). An auction is *one-sided* if only bids or only asks are permitted, and *two-sided* if both are permitted. Institutions with most but not all features of an auction will be called *quasi-auctions.*

For example, the (sellers') *posted offer* institution, in which traders simultaneously announce ask prices and then each trader can choose quantity in a bilateral transaction at his partner's announced ask price, is a one-sided quasi-auction; it is not a proper auction because it does not guarantee priority to lower asks. The primary market for U.S. Treasury securitites is conducted (at least through 1991–reform is under serious consideration) as a one-sided quasi-auction; it is not a proper auction, according to present terminology, because successful bidders can, and do, buy simultaneously at different prices.

Auctions (and other trading institutions) can be either *one-shot* or *repeated.* A repeated auction consists of several *trading periods*: agents receive new initial allocations at the beginning of each trading period, presumably engaging in production and consumption between trading periods.

In a *discrete-time* auction, all traders move in a single step from initial allocation to final allocation. By contrast, a *continuous-time* auction permits exchange at any moment during a trading period, and the overall net trade typically is composed of many bilateral transactions. The clearinghouse (CH) is the prime example of a discrete-time two-sided auction. Its key feature is that bid and ask messages are collected or "batched" during the trading period, and then "cleared" at the end of the period. Given the supply and demand revealed in the messages, a maximal trade vector is selected subject to a unified price constraint.

The focus in this chapter is the basic *continuous double auction* (DA). It is a continuous-time, two-sided auction in which messages consist of bids and asks for single units of the good, and of acceptances of the current best bid or ask. Net trades consist of the bilateral transactions triggered by an acceptance of the best bid or ask. Messages are admissible whenever the resulting transaction would not violate any specified non-negativity constraint on traders' money or good allocation.

2.2 DA VARIETIES AND HYBRID INSTITUTIONS

Many variants of the basic continuous double auction have been employed in field and laboratory environments. For example, inferior offers (e.g., bids lower than the current best bid) may or may not be made public or even be admissible. The identity of traders making offers may or may not be revealed. Bids and asks may be for stated quantities and not necessarily for a single unit. Numerous details must be specified; e.g., are unaccepted offers queued or must they be renewed following displacement by (and/or acceptance of) a better offer? Laboratory studies suggest that DA market performance can be affected by whether traders are one- or two-way, and whether the institution implemented on a computer or as oral "open outcry."

Computerized implementations of the DA are rapidly displacing oral implementations in the laboratory primarily because of the more direct control they offer over traders' information and their greater efficiency in data capture and data analysis. The main disadvantage (besides higher laboratory set-up costs) is that subjects seem to adapt to the environment more slowly in the computerized DA.[90] As Domowitz shows in the next chapter, there is also a trend towards the computerized DA in field markets, spurred by the promise of broader service at lower transactions cost and a better audit trail, but the trend is strongly resisted by traders whose "open outcry" skills would lose value in a computerized DA.

One can imagine auction market institutions with some, but not all, of the DA features. Several such hybrid institutions recently have been tested in the laboratory. In some respects a DA is like a long series of very short period repeated CH markets, most of which have zero transactions, and almost none with two or more. This observation (plus some arbitrary tie-breaking conventions, etc.) leads fairly directly to the "synchronized double auction" institution used in the double auction tournament hosted by the Santa Fe Institute, reported in chapter 6 of this volume. From another perspective, the key aspect of the DA is the continuous information on the "order flow" (traders' offers or messages). The Uniform Price Double Auction (UPDA) institution (presented in Chapter 11) also provides such information, allows recontracting (improvement of orders) during the trading period, and ends in a single unified price for all transactions. Friedman[26] reports versions of the CH which allow continuous order flow (or "order book") information. The UPDA institution is conceptually equivalent to some versions although there are some differences in implementation.

At the University of Arizona, work is also underway on a series of hybrid institutions which feature a "price clock" reminiscent of the clock used in Dutch flower auctions.[10] For example, in the Double Dutch institution,[55] a buyer price clock starts at a prohibitively high price and ticks down until a trader stops it, thereby purchasing a unit at a price at least as favorable to him as the price displayed on the clock. At this point, a seller price clock ticks upward from a very low initial price until stopped by a trader who thus sells a unit. Then the buyer clock resumes its descent until stopped again, etc. The trading period is over when the buyer clock price crosses the seller clock price, at that point all purchases and sales are transacted at the crossing price.

One observes a bewildering variety of market institutions in the field. In well-developed markets for homogeneous commodities and financial assets, variants of the DA seem most prevalent and CH variants are not unusual. DA field markets appear to differ largely in their assignment of specialized trader roles. Few, if any, field DAs have one-way traders, but most give some traders *privileges* such as a bigger message space and/or better access to other traders' messages than available to unprivileged traders. For example, the New York Stock Exchange (NYSE) offers at least three levels of privilege: for each good (traded security), there is a single "specialist" trader who alone has immediate access to all bids and asks (limit orders); there are "floor traders" who can periodically check the current list of unaccepted bids and asks ("the specialist's order book"); and there are unprivileged traders ("the public") with slightly delayed access only to the current best bid and ask. At the major Chicago exchanges, full participants in the DA (the "pit" traders) are usually agents for unprivileged traders who do not have direct access to current offers on the trading floor. Privileges are typically transferrable property rights—you can "buy a seat" at an exchange—and these property rights may help cope with the public-goods problem inherent in maintaining an organized market. See Friedman[29] for further discussion.

Field markets sometimes combine different institutions over time. For example, the NYSE and many financial markets begin trading (to start a new day or after a suspension of trade) with a CH and continue until the close with a DA. The newly launched Wunsch auction for after-hours trading in NYSE-listed securities is a hybid institution—in essence an UPDA or CH variant. See chapter 2 of Schwartz[75] and chapter 2 of the present volume for a more extended discussions of current financial market institutions.

The fact that so many DA variants and hybrids coexist in the field might suggest to some economists that market efficiency is insensitive to institutional details, because otherwise all markets would have adopted the most efficient variant. An equally plausible explanation with very different implications is that efficiency is very sensitive to institutional details, but also is very sensitive to environmental

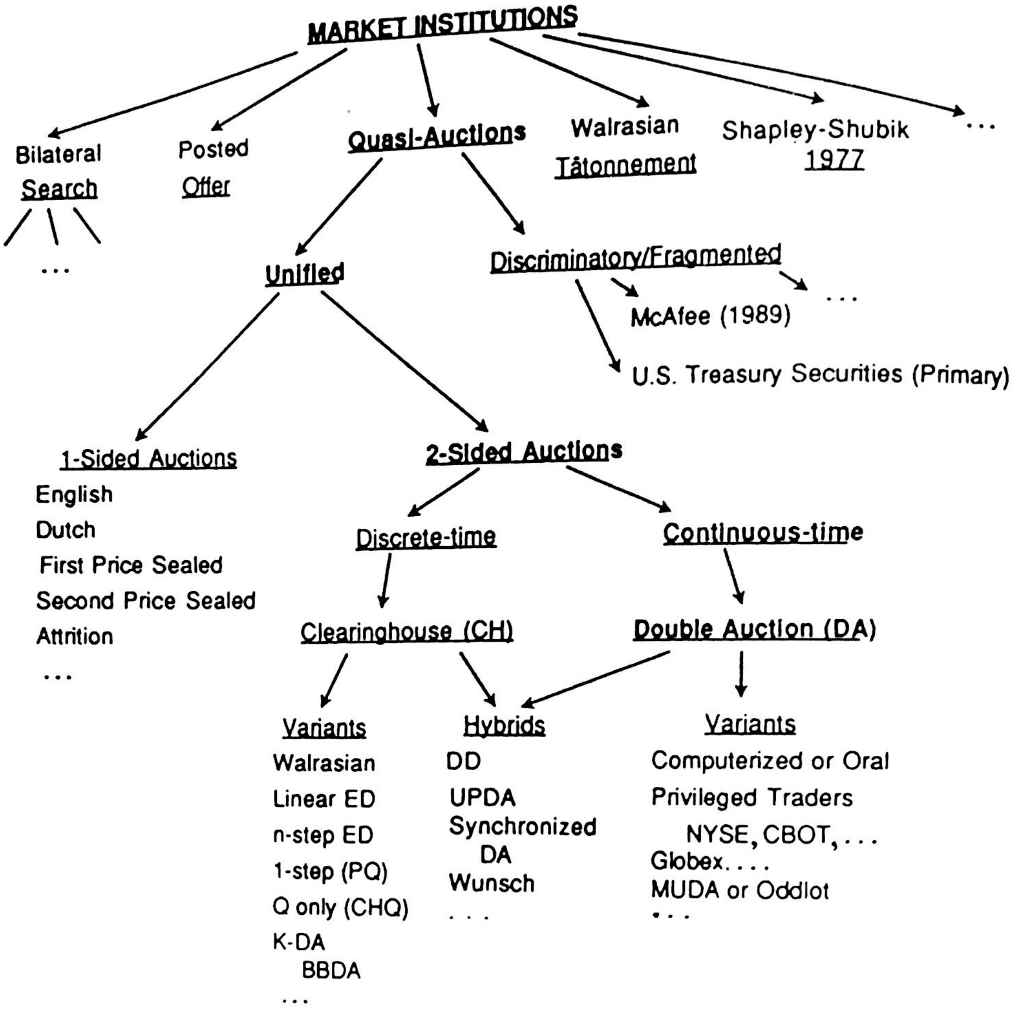

FIGURE 1 A Family Tree for the Double Auction.

TABLE 1 Some Trading Environments[1]

1	Goods	2 or >2
2	Traders	One-way or *two-way* 2 (bargaining) or *# buyers, # sellers* ≥ 2 or *n* $\geq$ *3 traders* or "large numbers"
3	Endowments	*single* or multiple units divisible or *indivisible* units (tokens)
4	Preferences	*reservation prices* (value or cost) for indivisible units, or smooth classical preferences for divisible units
5	Information	complete symmetric incomplete: private values, common values, affiliated values asymmetric ("insiders," . . .)

[1] Note: Italics indicate the defaults; e.g., unless otherwise mentioned, 2 goods and at least 3 two-way traders are assumed, and efficiency refers to *ex post* (no remaining gains from trade).

details (e.g., to the particular type of good traded, the national payments system, etc.), and the efficient variant is different in different circumstances. Another alternative explanation is that liquidity considerations induce strong increasing returns to market size ("order flow attracts order flow") so that the first institution to become viable in some market will become entrenched. In this case, the relative efficiency and the prevalence of an institution need not be closely linked.[4]

Evidently informal theorizing is insufficient to understand the prevalence of the DA and its variants. Therefore, I turn next to the relevant formal theory and empirical research. The literature is quite diverse in its maintained assumptions as well as in its terminology, but Table 1 and Figure 1 may provide some guidance. Table 1 lists the main dimensions of the assumed environment; italicized entries indicate the most commonly encountered cases. Figure 1 indicates the relation of the DA and its variants to most other market institutions I will discuss.

3. THEORETICAL APPROACHES TO THE DOUBLE AUCTION

3.1 WALRASIAN TRADITIONS

Leon Walras' discussion of price formation in 1874 was the first influential treatment of a market institution. In the modern theoretical literature, one can distinguish two different interpretations of Walras' institution. The tâtonnement interpretation regards the institution as a repeated quasi-auction in which some disinterested agent (or automaton) called the Walrasian auctioneer sends a price message, and all traders respond with quantity messages specifying their desired net trades at the given price. The auctioneer adjusts the price according to some algorithm (the tâtonnement or "groping") until the quantities sum to zero. At this point, traders announced net trades are transacted at the "market clearing" price. Although inspired by the trading arrangements on the Paris bourse of Walras' day, this tâtonnement institution is now very rare in the field; to the best of my knowledge, it resembles only the procedures used for many years in the London daily gold price "fixing" and in some Japanese wholesale rice markets.[73] The institution has occasionally been studied in the laboratory[6] with mixed results. Arrow and Hahn[3] summarize an extensive (and largely negative) literature regarding the stability properties of this institution under the assumption that traders are numerous and are non-strategic pricetakers. One gets the impression that the theoretical properties of the tâtonnement institution are even worse when strategic behavior is allowed, but relevant theoretical studies are rare.

A second interpretation, more commonly used in the literature but less often explicitly discussed, is that the Walrasian institution is really a one-shot clearinghouse. Traders' messages are their announced excess demand correspondences (or just their announced demand or supply curves), and the final allocation arises from maximal net trade at a single market-clearing price. Perhaps because of its familiarity and because of its "direct revelation" nature, this Walrasian CH institution seems very natural to most theorists—so natural indeed, that work employing it is often called "institution free"! Unfortunately, truthtelling in the Walrasian CH is not a Nash equilibrium strategy, except in the "large numbers" limit in which each trader's transactions are always a negligible fraction of aggregate transactions; it follows that the instition is theoretically inefficient.[69] Its presumed inefficiency together with its huge information requirements (the dimension of each trader's message space potentially is infinite) probably account for the non-existence of the Walrasian CH in field environments. Only greatly simplified versions have been tested in the laboratory. (For example, McCabe et al.[53] restrict excess demand messages to two-parameter linear functions.) Still, the large numbers limit of the Walrasian CH institution defines the benchmark of perfect competition in mainstream theoretical literature.

3.2 GAMES OF INCOMPLETE INFORMATION

Perhaps the most natural way for a modern theorist to think about a market institution is as a game of incomplete information, since traders generally do not know each others' preferences (e.g., sellers' costs and buyers' true valuations). Vickrey[88] pioneered this approach in his classic analysis of auctions. He considers a first-price sealed-bid auction, a one-sided one-shot buyer's auction for a single indivisible unit. Traders' messages are simultaneously chosen bids, and the resulting net trade is a bilateral transaction for the unit between the (passive) seller and the highest bidder at her bid price.

Vickrey's analysis assumes each trader i has prior knowledge regarding (a) the IID uniform $[0,1]$ distribution of reservation prices ν_j and (b) the bid function $b(\nu_j) = (n-1)\nu_j/n$ of the other $n-1$ traders. He verifies that a risk-neutral trader with private knowledge of her own reservation price ν_i will maximize expected utility by bidding $b(\nu_i)$, i.e., by using the same bid function. Vickrey's analysis anticipates *Bayesian Nash equilibrium* (BNE), the general solution concept for games of incomplete information introduced by Harsanyi.[38] In BNE, players use Bayes theorem to form expectations given individual private signals and given common prior information regarding (a) the distribution of players' (buyers') relevant characteristics ("type") and (b) players', (buyer's) type-contingent strategies. Relative to these expectations each player's (buyers') strategy maximizes expected utility; i.e., the strategies are simultaneous best responses.

As documented in recent surveys such as McAfee and McMillan[50] and Wilson,[92] the BNE approach has provided very satisfying analyses of one-sided one-shot auctions which extend Vickrey in allowing risk aversion, general distributions of reservation prices, and multiple units. Two other sorts of extensions require brief discussion at this juncture. First, Vickrey assumed *private values* in that each trader's private signal is precisely her own reservation value ν_i. One can also consider private signals which provide each trader useful but imperfect information regarding ν_i, and perhaps also regarding other traders' values ν_j. The most general useful case assumes a condition called *affiliated values*. Another very important special case, in some sense polar to private values, is called *common values.* Here all traders have the same true reservation value and receive independent, unbiased signals regarding that uncertain value. See the surveys cited above for more precise definitions and results.

The second sort of extension is to different auction institutions. Vickrey himself discussed at least four institutions: single unit, first- and second-price sealed-bid auctions, and English (ascending) and Dutch (descending) continuous auctions. The literature now covers many other one-sided auction institutions, not all of which yield efficient outcomes even *ex ante*, e.g., "the war of attrition."[68] Some that are *ex ante* efficient have surprising transactions patterns, e.g., the one-sided multi-unit Dutch seller's auction.[7]

The extension of the Bayesian Nash equilibrium (BNE) approach to two-sided auctions begins with Chatterjee and Samuelson,[12] who consider the bilateral monopoly (1 buyer + 1 seller) single-unit case with private values drawn independently from known uniform distributions. The institution is a CH with the transaction price at the midpoint of the interval of market-clearing prices when that interval is non-empty; otherwise, no transaction occurs. They find linear BNE bidding strategies which miss mutually beneficial transactions with probability 1/6.

A series of papers by Satterthwaite and Williams, summarized in chapter 4 of this volume, extend the results to an environment of one-way traders (usually m sellers and m buyers) with independent private values for single indivisible units. They first analyzed a version of the CH institution, called BBDA, which takes the upper endpoint of the interval of market-clearing prices as the transaction price in order to give one side, the sellers, a dominant truthtelling strategy. Later papers analyze versions of the CH, called k-DA's, which use other points along the interval of market-clearing prices. They show that in BNE the difference between buyers' bids and true values is $O(1/m)$ and foregone gains from trade are $O(1/m^2)$, so *ex post* inefficiency vanishes reasonably fast as the market gets larger. See also McAfee[51] for a slightly stronger result concerning two-sided quasi-auctions which give lower transaction prices to sellers than to buyers.

Incomplete information game theory indicates that some *ex post* inefficiency is inevitable. For example, Myerson and Satterthwaite,[59] and Makowski and Ostroy[48] use the revelation principle to show that BNE of *any* one-shot market institution cannot be both individually rational and *ex post* efficient for a finite number of players with general preferences. Since a dynamic trading process may gradually disseminate private information, it is unclear whether these inefficiency results apply to a continuous trading institution such as the DA. See Myerson[60] for an explanation of some difficulties in applying the revelation principle to extensive form games with communication.

Wilson[91] is the only serious attempt of which I am aware to analyze the basic DA (or any other continuous auction) as a game of incomplete information. He proposes a strategy selection in which, roughly speaking, traders play a waiting game for making serious bids and asks, with each buyer's (seller's) impatience arising from possible preemption of gains by other buyers (sellers). Once a serious bid (ask) is made, that buyer (seller), in essence, conducts a Dutch auction until a seller (buyer) accepts. The waiting game resumes after the resulting transaction. These messages fully reveal traders' private values and produce final allocations that are nearly *ex post* efficient—at worst, a few of the least valuable trades are missed. Wilson verifies that the proposed strategy selection satisfies the necessary conditions for BNE.

3.3 PROBLEMS WITH THE INCOMPLETE INFORMATION APPROACH

Modeling the DA as a game of incomplete information is natural, since in both laboratory and field applications, traders do not know each others' reservation prices, and often do not even know their own final valuations (e.g., in markets for risky assets). Moreover, this approach has been very fruitful in analyzing one-sided auction institutions. Nevertheless, it may be appropriate to note some problems in applying it to the DA, and to suggest some alternative approaches.

On the theoretical side, the incomplete information approach relies heavily on prior common-knowledge assumptions. Even very simple auctions involve type-contingent strategies—your bid in a first-price auction depends on your reservation price. In continuous auctions, such as the DA, one must also take history-contingent and time-contingent actions—you care about the sequence of bids, asks, and acceptances observed so far as well as about the time remaining when you decide on your next bid, ask, or acceptance message. In BNE each trader in a DA must have prior knowledge of other traders' type-contingent strategies (themselves time- and history-contingent messages) and must be able to compute expected utility-maximizing messages at every moment as the DA unfolds. Such prior knowledge and computational ability is literally incredible. As Vernon Smith has remarked in private correspondence, this approach pushes the real action off stage: one leaves unformalized the acquisition of prior knowledge and computational algorithms. It is possible to model information acquisition itself as a game of incomplete information, but this seems only to compound the problem.[24]

On the empirical side, the evidence so far does not seem favorable. As I will note in the next section, DA outcomes in the laboratory seem quite insensitive to the number of traders beyond a minimal two or three active buyers and two or three active sellers. Moreover, as discussed at length in Friedman and Ostroy[27] parameter choices which, according to an incomplete information analysis, should greatly reduce efficiency in CH (and presumably in DA) markets had no such effect in recent laboratory tests. This empirical issue is discussed more broadly in McCabe, Rassenti, and Smith.[52]

3.4 ALTERNATIVE APPROACHES

Models of trading institutions as games of incomplete information have dominated the recent theoretical literature, but there are alternative approaches. In no particular order, I will review the market microstructure literature, some other literature based primarily on decision theory (i.e., ignoring strategic interaction), and some models using games of complete information.

A large body of finance literature studies two-sided markets for a common-value good. Unfortunately, most of it assumes a Walrasian CH and "large numbers" (e.g., Grossman and Stiglitz[36]). Other market institutions are studied in the market microstructure literature, so named by Garman.[32] The classic microstructure articles largely ignore game-theoretic considerations, but the more recent articles often implictly or explicitly model games of incomplete information. I am unaware of any survey of this literature more recent than Cohen et al.[13]

The market microstructure literature begins with Stigler[86] and Demsetz,[17] who use a simple utility-maximizing model to analyze a monopoly specialist DA institution: only one trader can send bids and asks, the other traders being restricted to acceptances. Recent analyses of this institution, in Copland and Galai[14] and Glosten and Milgrom,[34] impose a zero-profit condition on (competitive) specialists, and study how the bid-ask spread in BNE responds to the presence of traders with superior information and to liquidity-motivated ("noise") traders. In essentially the same setting, Kyle[43] finds an optimal strategy for a trader with sole access to superior information, and shows that the strategy gradually but profitably disseminates the information.

Ho and Stoll[39] study the DA institution using dynamic programming techniques. They seek to characterize equilibrium bid-ask spreads as a function of order-unit size, risk tolerance, and uncertainty. They argue that the bid-ask spread is essentially independent of the number n of active traders as long as $n \geq 2$; the intuition is that the best price wins in a form of Bertrand competition which requires only two traders.

The microstructure literature also considers CH institutions. For example, Mendelson[56] analyzes the performance of a repeated CH market given an exogenous stochastic flow of offers. More recently, Kyle[44] adopts the game of incomplete information framework to study the effect of inside information in a one-shot CH market. Under his parametric assumptions, traders with heterogeneous superior information (i.e., less noisy signals about the common value of the good) optimally will submit linear excess demand functions which in BNE will reveal some but not all of their information.

Apart from an obscure precursor (Garcia[31]), Easley and Ledyard[20] were the first theorists to attempt to explain the efficiency of laboratory DA markets. They emphasize convergence to competitive equilibrium across successive trading periods in a DA experiment. Easley and Ledyard postulate plausible behavioral rules on how specialized traders set and adjust "reservation prices" (the links to true value/cost parameters left unspecified) and bids or asks across and within trading periods. They also test three implications of the model against some laboratory data, with favorable results. The final version of their paper appears as chapter 3 of this volume.

A recent decision-theoretic approach to laboratory DA markets, focussing on behavior within a single trading period, appears in Friedman.[25] A special case of the NCE model discussed below, it makes the strong simplifying assumption that traders disregard the impact that their current bid or ask may have on other traders subsequent offers. Thus traders act as if they are playing a game against nature, but otherwise are good Bayesian decision makers. Such traders are shown to engage in Bertrand competition with respect to reservation prices which decay to true cost and value as trading time runs out. Therefore final outcomes are nearly 100% efficient.

Perhaps the most obvious alternative to the incomplete information approach is to regard the DA and other auction institutions as games of complete information. This terminology need not be taken literally; it is only in the last 20 years or so that theorists have supposed that ordinary NE requires each agent to know (as common knowledge) the preferences of all agents. Luce and Raiffa[47] and probably most other theorists of that time preferred to regard NE as the rest point of some unformalized groping process. Of course, theorists will require a satisfactory formal model of the process before accepting this interpretation again, but it is possible that current work (e.g., Fudenberg and Kreps,[30] Gilboa and Matsui,[33] and Friedman[28]) could lead to such a model. In the meantime, one can at least entertain the thought that stationary repetition in the laboratory, or long experience in the field, may be a practical substitute for direct knowledge of other traders' preferences and strategies and, therefore, might lead traders to choose messages which constitute an NE in the complete information game.

Shapley and Shubik[76] were probably the first to use explicit complete information game-theoretic models for exchange institutions. Like Roberts and Postlewaite,[69] they find that only in the large numbers limit do NE strategies produce efficient outcomes for their (non-auction) institution. Much different theoretical results are reported in Dubey,[19] Simon,[77] and Benassy[5] for multi-commodity CH auctions and in Friedman and Ostroy[27] for simple CH auctions. Here each trader's message consists of a single price-quantity (PQ) pair. (Recall that Satterthwaite and Williams[72] assume price-only messages for single indivisible units, so their results do not apply.) In the PQ version of the CH institution, we have the surprising result that "active" NE (at least two buyers and two sellers) coincide with Walrasian equilibria. (See also Schmeidler.[74]) Thus small numbers are compatible with 100% efficiency in this institution. Intuitively, we have a form of Bertrand competition.

I am not aware of any satisfactory model of the continuous DA as a game of complete information. Friedman[24] contains a partial model, where NE for the extensive game is replaced by a renegotiation-proofness concept called NCE (for no-congestion equilibrium; the idea is that traders are not failing to transact at the close of trade because they all waited to the last second and the market got congested). The main result is that if strategies satisfy NCE, then the DA will produce (a) 100% efficient outcomes in smooth economies (i.e., divisible goods and classical preferences) with at least three traders, and (b) outcomes just one (least valuable) trade shy of 100% efficiency in economies with indivisible units and at least two "buyers" and two "sellers." Again, it is a form of Bertrand competition (plus time pressure) that is responsible for small-numbers efficiency.

My conjecture that games of complete information will be useful in solving Smith's "scientific mystery" now can be broken into two parts. First, that competitive (Walrasian) equilibrium coincides with ordinary (complete information) NE in interesting environments for the DA institution. Second, that the DA promotes some plausible sort of learning process which eventually guides both clever and not-so-clever traders to behavior which constitutes an "as-if" complete-information NE. The work cited in the previous three paragraphs lends some plausibility to these conjectures but does not begin to prove them.

4. EMPIRICAL WORK

In studying double auction markets, empirical researchers can tap three general data sources: field data from large-scale ongoing markets, laboratory data from small-scale DA and comparison markets, and computer simulation data. The strengths of field data are its availability and its relevance. Its main weakness is that unobserved and/or uncontrolled variables may preclude valid inferences.[45] Laboratory data permits control of many variables and renders others observable, but it can be expensive (in time and money) to acquire. Computer simulations now are relatively cheap and are completely controllable and observable; e.g., the researcher can confidently detect a mixed strategy in a computer simulation but not in laboratory data. The main disadvantage is that traders' strategies are not endogenously chosen, but rather must be specified exogenously in a computer simulation.

A moment's reflection discloses strong complementarities among the three data sources. This section, and the volume as a whole, will draw on all three. Here I emphasize comparisons of DA market outcomes to theoretical benchmarks and to outcomes under alternative market institutions. For these purposes, it turns out that the laboratory data are most informative, so my treatments of field data and computer simulation data will be a bit cursory.

4.1 FIELD DATA

The primary theoretical benchmark for the DA (or for any market institution) is competitive equilibrium; to what extent do actual allocations and transactions prices resemble competitive equilibrium allocations and prices? When some traders have private information, the efficient benchmark becomes fully revealing rational expectations equilibrium (FRREE): to what extent do actual allocations and transactions prices resemble those of a competitive equilibrium in which all private information was made public?

Preferences and private information are not observable in field data, so these questions can't be answered directly. The largest body of empirical field literature follows Fama[22] in testing indirect consequences of FRREE in asset markets: do (suitably adjusted) transactions prices follow a martingale? Do they immediately and fully respond to new information? LeRoy[46] provides a skeptical survey of martingale tests and concludes that, although most previous studies reached favorable conclusions, the support for martingales (relative to interesting alternatives) actually is quite weak. The arrival of private information precludes direct answers to the second question, but most of the numerous event studies following Fama et al.[21] seem favorable. A more controversial approach to this question is to compare transaction price volatility to price volatility in some appropriate FRREE model—again see LeRoy,[46] for a recent account. It may also be worth mentioning that O'Brien and Srivastava[61] conclude that their laboratory asset markets are quite inefficient even though they pass the standard martingale and volatility tests.

Field comparisons of market institutions are much less common than efficiency studies. For example, Cohen et al.[13] compare two DA markets with monopoly "specialists" (NYSE and AMEX) to two non-specialist DA markets in Tokyo and Rio de Janeiro. Their results are consistent with reduced price volatility in specialist markets for thinly traded issues. Amihud and Mendelson[2] point out that conclusions are ambiguous in this sort of study because it is "...hard to discern differences [in market performance] resulting from the trading mechanism itself from differences due to dissimilarities of securities and environments." They try to finesse the problem by comparing NYSE close-to-close price changes with NYSE open-to-open changes, noting that opening price is set in a CH market. They find that close-to-close returns have greater variance and kurtosis, and greater deviations from a random walk in ARMA (1,1) estimates. Stoll and Whaley[87] reach similar conclusions in a more recent and thorough study of the NYSE data. Neither study considers the alternative hypothesis that, given the difference in information conditions between opening and mid-day, the CH institution was chosen to reduce returns variance at opening which otherwise might be even greater. In the absence of a clean institutional comparison, one must make a leap of faith to attribute measured differences market performance to differences in the market institution. Field data unfortunately rarely permit such clean comparisons.

4.2 EFFICIENCY IN LABORATORY MARKETS

Laboratory data are especially useful for present purposes because the experimenter can observe private information and preferences and can control the market institution. Consequently, one can directly test efficiency and directly compare market institutions. The small scale and simplicity of laboratory markets suggests some caution in generalizing results to large, complex field markets, but small scale and simplicity are actually advantages in testing explicit theory.[64,83]

Chamberlin[11] is the first published report of (not very well controlled) laboratory market experiments. He induced cost and value parameters for one-way traders of single indivisible units, and employed a bilateral search institution with public announcement of transaction prices. His outcomes were not very efficient. Smith,[79] by contrast, reported highly efficient outcomes in a set of otherwise similar repeated DA experiments. Since then, numerous other studies have abundantly confirmed what Smith[82] calls the Hayek Hypothesis: "Strict privacy [i.e., induced private values] together with the trading rules of a market institution [viz., the DA repeated several times] suffice to produce competitive outcomes at or near 100% efficiency" (page 167). This result even seems to hold with as few as two or three buyers and two or three sellers[64,83] and even when very unfavorable value parameters are chosen, such as the box and swastika designs in Smith and Williams.[85]

Asset-market experiments as introduced by Plott and Sunder,[65] and Forsythe, Palfrey, and Plott[23] feature DA markets with two-way traders and a good (the "asset") whose marginal value is constant for each trader and state. In these experiments, states are uncertain, private information is present, and/or the asset pays a dividend over several trading periods. Traders' endowments and (more importantly) contingent valuations and information typically differ across trader types, an environment of affiliated values rather than private values. In some asset-market experiments, there is only one trader type, in which case we have common values and all allocations are vacuously *ex post* efficient.

The main question addressed in asset-market experiments is usually informational efficiency—to what extent do transactions prices approach the fully revealing rational expectations equilibrium (FRREE) benchmark? In DA experiments with assets that live only one period and two or more trader types, the results generally support FRREE. Even when traders have differential information or some have superior information, the asset price does a surprisingly good job of aggregating or disseminating the information. I should note that Camerer and Weigelt,[8] Plott and Sunder,[66] and Copeland and Friedman[15] report some interesting anomalies, but in my view the basic lesson is that DA prices are very informationally efficient in this environment.

The lesson from the DA asset-market experiments of Smith, Suchanek, and Williams[84] seems rather different. They report frequent large bubbles—episodes where transaction price rises well above the fundamental (the FRREE value) for an extended time period, usually ending in a sudden price crash to or below the fundamental. Besides using common values, their experiments differ from most other asset-market studies in that they use very long lived assets ("dividends" paid over 15 or 30 periods rather than the usual 1, 2, or 3) and much less stationary repetition (only 1 to 4 reinitializations with a given groups of traders, rather than the usual 5 to 20). Despite some useful follow-up work (e.g., Porter and Smith[67]), it is not yet clear whether the observed market inefficiencies should be attributed to common values, to learning and experience effects, or to other differences in design. Camerer and Weigelt present new results and a fuller discussion in chapter 13 of this volume.

A few remarks on one-sided auctions may also be in order. Large numbers of one-sided private values auctions have been conducted in the laboratory in the last ten years or so, e.g., Cox, Smith, and Walker.[16] My view is that the results are generally consistent with the incomplete information theory, although there are some apparent anomalies whose interpretation recently has become controversial.[37] Of greater relevance to present concerns are one-sided auction experiments with induced common values by Kagel and Levin.[42] Their outcomes are less consistent with BNE and they conclude that the winner's curse (overbidding) is a relatively persistent phenomenon which is eventually overcome by slow and environment-specific learning.

In chapter 10 of this volume, Kagel and Vogt extend the same basic setup—independent draws from known uniform distributions—to the BBDA and a variant. Again they find substantial deviations from BNE. Their results suggest (to me at least) that these institutions do not encourage rapid learning of equilibrium strategies.

4.3 COMPARISONS OF LABORATORY MARKET INSTITUTIONS

Comparisons of market institutions prior to 1980 established that the DA produced more efficient outcomes than many other trading institutions such as bilateral search, posted offer, and one-sided auctions (e.g., Plott and Smith[63]; see also Williams[90]). Smith et al.[81] report that a computerized DA generally produces more efficient final allocations and more rapid price convergence than various versions of the CH, except that allocations are perhaps a bit more efficient in a recontracting version of the CH which allows each trader to send multiple price-quantity messages. Equally important, most of the CH variants yield more efficient allocations than incomplete information game theory would suggest. Friedman and Ostroy[27] confirm these findings for divisible goods and very unfavorable "box like" parameters, and also report inefficient allocations under a modified CH institution which permits underrevelation in quantities but not in prices.

Institutional comparisons are currently a very active area of laboratory research. Preliminary results at Arizona suggest that the double Dutch institution is more efficient than the DA in some simple environments.[13] Friedman[26] reports that some hybrid institutions (continuous-information versions of the CH) have efficiencies comparable to the DA in several asset-market environments. In chapter 11 of this volume, McCabe, Rassenti, and Smith present evidence that their UPDA institution also achieves efficiencies comparable to the DA in one-way trader environments. The patterns of inefficiencies differ across the two institutions; in the UPDA one occasionally sees substantial unrealized gains when a block of traders underbids a bit too much. In chapter 12 of this volume, Plott and Clauser compare institutions in an unusual laboratory environment which encourages seller conspiracies. They find that a DA variant called ISMDA shares the efficiency (or conspiracy-thwarting) properties of the DA.

Institutional comparisons of this sort eventually should produce well-documented stylized facts about the absolute and comparative advantage of many market institutions in various environments. Such facts will be invaluable to theorists and to practitioners.

4.4 COMPUTER SIMULATIONS AND TRADING STRATEGIES IN THE DA

Some empirical studies look at individual behavior as well as market-wide outcomes. Benchmarks provided by computer simulations are particularly useful for this purpose. For example, in chapter 8 of this volume, Domowitz and Bollerslev use computer-generated bidding to take a first look at the effect of a DA variant and maximum length of the order book queue. In Chapter 6 of this volume, Rust, Miller, and Palmer, look at both individual behavior and market outcomes in a tournament of computer-simulated trading strategies in a hybrid DA-CH institution called the synchronized double auction. Follow-up work will allow direct comparisons of computer-simulated traders and human traders.

At a 1990 Santa Fe Institute workshop, Sunder introduced another performance benchmark which already has become quite influential. His zero-intelligence (ZI) algorithm prescribes random advantageous bids and asks. When all (simulated) traders employ the ZI algorithm in a DA market, the outcomes are surprisingly efficient, comparable to those achieved by human subjects in early trading periods. See chapter 7 of this volume by Gode and Sunder for some follow-up work which raises the question of whether DA efficiency is attributable to trader rationality or inherent in the market institution itself.

The theoretical models of Wilson[91] and Friedman[25] as well as the ZI algorithm provide alternative characterizations of individual trader behavior and, therefore, make distinct predictions regarding transactions price sequences, bid and ask sequences, transactions partners, and efficiency within a DA trading period. In chapter 9 of this volume, Cason and Friedman begin the task of comparing the predictions to experimental data. The results so far are mixed; for example, the Friedman[25] model best predicts the bid and ask sequences, and ZI best predicts the price sequences, at least for inexperienced traders.

5. CONCLUDING REMARKS

The rest of this volume documents the acceleration of research in DA markets in recent years. Theoretical and empirical researchers both have recently developed promising new approaches. Evidence accumulates on which features of the DA are consequential and which are not, and which sorts of environments are conducive to efficient outcomes and which are not. Yet Smith's mystery remains: why is the DA (and some of its hybrids and variants) so efficient in most environments?

One can now distinguish three main lines of attack, in some respects complementary and in other respects rival.

1. The BNE approach extends the Vickrey-Harsanyi modeling strategy to increasingly complex market institutions, and tests the extensions empirically as they become available. This approach influences virtually all chapters in this volume and dominates chapters 4, 5, and 10.

2. The Nash equilibrium learning approach regards trader behavior as changing in response to experience with a market institution, and eventually settling down (if at all) in an as-if complete-information Nash equilibrium. The influence of this approach can be detected in several chapters, but it is explicit only in the present chapter.
3. The zero intelligence (ZI) approach uses computer simulations of very simple trader strategies to compare outcomes across market institutions. To the extent that simulated outcomes resemble outcomes with human traders, and to the extent that ZI strategies are not easily exploited by humans, more complex analysis becomes unnecessary. Many of the empirical chapters of this volume are influenced by the ZI approach.

All three approaches promise good returns to further work. I close with some conjectures about which activities will generate the highest returns in the next few years. In my view, the biggest gap in laboratory investigations of two-sided auctions is in environments which closely implement the prior knowledge assumptions of incomplete information games. Variants of the DA and CH institutions should be examined in environments with single-sided or with non-specialized traders whose value/cost parameters are drawn independently *each period* from known distributions. Such random value experiments should better establish the domains of applicability for incomplete versus complete information theories.

A second neglected area in the experimental literature is in comparing outcomes in two-sided auctions across common values and private values (and affiliated values) environments. Previous discussion foreshadows my conjecture that the lack of induced trading incentives and the "winner's curse" problem may make some or all market institutions less efficient in common values environments.

On the theoretical side, I hope to see more attempts to compare market institutions, with the goal of predicting which institutions will be most efficient in different environments. I suspect that in practice, the most efficient market institutions are those which best promote rapid learning of efficient equilibrium strategies. If so, we may have to supplement economists' usual equilibrium theories with positive theories of learning.

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