۹۹/۱۱/۱۸

$\boxed{7}$ $$y = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_1^2 + \epsilon$$

where $\epsilon \sim N(0, \sigma^2)$

$$P(y \mid x_1, x_2) = P(x_1, x_2, w_0, w_1, w_2, w_3)$$

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y^i - w_0 + w_1 x_1^i + w_2 x_2^i + w_3 x_3^i)}{2\sigma^2}\right)$$

B. log-likelihood

$$L(w_0, w_1, w_3, \sigma^2) =$$

$$\log \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y^i - w_0 + w_1 x_1^i + w_2 x_2^i + w_3 x_3^i)}{2\sigma^2}\right)$$

$$= \sum_{i=1}^{n} \log p(y_i \mid x_i, w_0, w_1, w_2, w_3, \sigma^2)$$

$$= -\frac{n}{2} \log 2\pi - n \log \sigma - \frac{1}{2\sigma^2} \sum_{i=1}^{n} \left(y^i - (w_0 + w_1 x_1^i + w_2 x_2^i + w_3 x_3^i)\right)^2$$

۱۸

بهمن

شنبه

6. Feb. 2021

۲۳ جمادی الثانیه ۱۴۴۲

ش ی د س چ پ ج

$$f(w_0, w_1, w_2, w_3) = +\frac{n}{2}\log 2\pi + n\log \sigma$$

$$+\frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(y^i - (w_0 + w_1 x_1^i + w_2 x_2^i + w_3 x_1^{i\,2})\right)^2$$

$$\rightarrow \frac{\partial f}{\partial w_0} = \frac{\partial f}{\partial w_1} = \frac{\partial f}{\partial w_2} = \frac{\partial f}{\partial w_3} = 0$$

$$\frac{\partial f}{\partial w_0} = \frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(y_i - (w_0 + w_1 x_1^i + w_2 x_2^i + w_3 x_1^{i\,2})\right) = 0 \quad (۱)$$

$$\frac{\partial f}{\partial w_1} = \frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(y_i - (w_0 + w_1 x_1^i + w_2 x_2^i + w_3 x_1^{i\,2})\right)x_1^i = 0 \quad (۲)$$

$$\frac{\partial f}{\partial w_2} = \frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(y_i - (w_0 + w_1 x_1^i + w_2 x_2^i + w_3 x_1^{i\,2})\right)x_2^i = 0 \quad (۳)$$

$$\frac{\partial f}{\partial w_3} = \frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(y_i - (w_0 + w_1 x_1^i + w_2 x_2^i + w_3 x_1^{i\,2})\right)(x_1^{i\,2}) = 0 \quad (۴)$$

$$(۱) \rightarrow n\bar{y} = \sum_{i=1}^{n} y_i = \sum w_0 + w_1 x_1^i + w_2 x_2^i + w_3 x_1^{i\,2}$$

$$\cancel{(۲) \rightarrow n\bar{y} = n\bar{y}}$$

$$(۲) \quad \sum y^i x_1^i = \sum\left(w_0 + w_1 x_1^i + w_2 x_2^i + w_3 x_1^{i\,2}\right) x_1^i$$

$$(۳) \quad \sum y^i x_2^i \quad \sum\left(w_0 + w_1 x_1^i + w_2 x_2^i + w_3 x_1^{i\,2}\right) x_2^i$$

۹۹/۱۱/۱۶

④ $\sum y^i x_1^{i\,2} = \sum (w_0 + w_1 x_1^i + w_2 x_2^i + w_3 x_1^{i\,2}) x_1^{i\,2}$

→ in statistical literature we have

① $\cancel{E(y) = \frac{w_0}{n} + w_1 E(x_1) + w_2 E(x_2) + w_3 E(x_1^2)}$

② $\cancel{E(X_1 Y) = w_0 E(x_1) + w_1 E(x_1^2) + w_2 E(x_1 x_2)}$

① $\bar{y} = \frac{w_0}{n} + w_1 \overline{x_1} + w_2 \overline{x_2} + w_3 \overline{x_1^2}$

② $\overline{x_1 y} = w_0 \overline{x_1} + w_1 \overline{x_1^2} + w_2 \overline{x_1 x_2} + w_3 \overline{x_1^2}$

③ $\overline{x_2 y} = w_0 \overline{x_2} + w_1 \overline{x_1 x_2} + w_2 \overline{x_2^2} + w_3 \overline{x_1^2 x_2}$

④ $\overline{x_1^2 y} = w_0 \overline{x_1^2} + w_1 \overline{x_1^3} + w_2 \overline{x_1^2 x_2} + w_3 \overline{x_1^4}$

$\Rightarrow W = [w_0 \quad w_1 \quad w_2 \quad w_3]$

$Z = [\bar{y} \quad \overline{x_1 y} \quad \overline{x_2 y} \quad \overline{x_1^2 y}]$

$$V = \begin{bmatrix} 1 & \overline{x_1} & \overline{x_2} & \overline{x_1^2} \\ \overline{x_1} & \overline{x_1^2} & \overline{x_1 x_2} & \overline{x_1^3} \\ \overline{x_2} & \overline{x_2 x_1} & \overline{x_2^2} & \overline{x_1^2 x_2} \\ \overline{x_1^2} & \overline{x_1^3} & \overline{x_1^2 x_2} & \overline{x_1^4} \end{bmatrix}$$

۱۶ بهمن
شنبه
4. Feb. 202
جمادی الثانیه ۱۴۴۲

ش ی د س چ پ ج

So $Z = W \cdot V$

$\rightarrow \boxed{W = Z \cdot V^{-1}}$