



$$\hat{\mu} = \underset{\mu}{\operatorname{argmax}} \prod_{i=1}^n \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x_i - \mu)^2}{2\sigma^2}}$$

$$= \underset{\theta}{\operatorname{argmax}} \operatorname{Log}(\operatorname{Likelihood}(x; \theta)) =$$

$$\underset{\mu}{\operatorname{argmax}} \sum_{i=1}^n \underbrace{\log\left(\frac{1}{\sqrt{2\pi}\sigma}\right)}_{\text{const}} + \sum_{i=1}^n \underbrace{-\frac{(x_i - \mu)^2}{2\sigma^2}}_{\text{const}}$$

$$= \underset{\mu}{\operatorname{argmax}} \underbrace{-\sum (x_i - \mu)^2}_{\mathcal{L}}$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = \sum_{i=1}^n 2(x_i - \mu) = 0 \rightarrow \left[\sum x_i \right] - \left[\frac{n}{\cancel{\mu}} \right] = 0$$

$$\Rightarrow \hat{\mu} = \frac{\sum x_i}{n} = \bar{x}$$