Black-Litterman Model with Evolutionary Views

[1] RiskLab, Graduate School of Management and Economics | Sharif University of Technology

[2] RiskLab, University of Toronto

Alireza Mousakhani

Hamidreza Arian

Housein Naderian

RiskLab

Outline

Motivation

- constructing a desired, profitable portfolio has been a significant concern for asset managers, dealers, and financial decision-makers.
- The Black-Litterman model enables investors to incorporate market equilibrium weights with their expectations of assets future behavior as a view vector someways that leads to the construction of intuitive and well-diversified portfolios.
- Although the term "investors' view" is a novel, creative aspect of the Black-Litterman theory, due to the complexity of the market, this human views must be precisely scrutinized day by day and be revised based on newly inferred information afterward.
- The main objective of this paper is to incorporate computerized meta-heuristic algorithms with the original Black-Litterman theory to provide better forecasting and investment results.
- We propose a novel approach based on coordinating the Black-Litterman model and evolutionary algorithms to provide more efficient views which can be implemented alongside the human forecasts to capture exceptional investment opportunities.

Theoretical Background

- we require human assistance to complete the Black-Litterman model for portfolio management purposes, which means that we are not dealing with a stand-alone approach.
- It might be too costly to take advantage of experts' opinions for constructing our portfolio management model.
- We should also mention that human interventions may consist of errors that can be irrecoverable in particular and sensitive cases.
- We intend to eliminate the human intervention section in the Black-Litterman model to reduce probable heavy costs and diminish plausible errors, which is inevitable in any human intervention.
- We decided to replace the experts' opinions with a machine-learning algorithm to generate the vector Q by customizing the conventional GA to consider our required elements to solve the portfolio optimization problem and generate our desired results.

Innovations

- **Initial Population** Considering n stocks; we obtain k distinct numbers as

$$k = \binom{n}{2} = \frac{n!}{2! \times (n-2)!} = \frac{n(n-1)}{2}$$

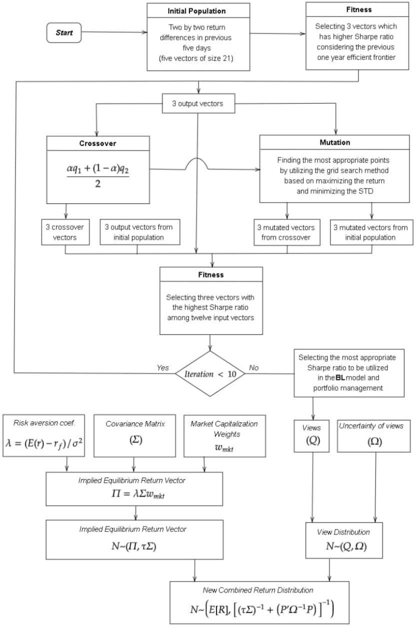
- **Selection**

$$P = \begin{pmatrix} p_{1,1} & \cdots & p_{1,n} \\ \vdots & \ddots & \vdots \\ p_{k,1} & \cdots & p_{k,n} \end{pmatrix}, \qquad Q = \begin{pmatrix} q_1 \\ \vdots \\ q_k \end{pmatrix}$$

- **Crossover**

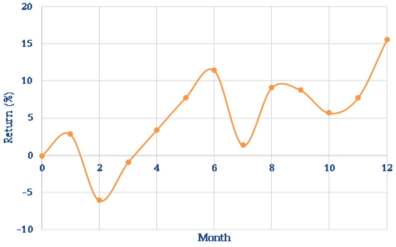
$$\text{Crossover} = \frac{(\alpha \times q_1) + ((1-\alpha) \times q_2)}{2}$$

- **Mutation** Performing the *grid search analysis*

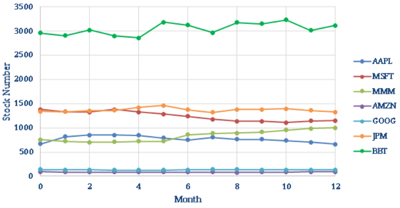


Naive portfolio Diversification

- As the first strategy, we have simply assigned the weight equal to $\frac{1}{7}$ to each stock.
- At the end of 1-year period, we have the maximum return of 15.5432%.



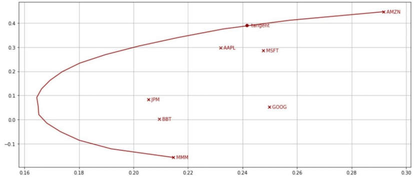
(a) Naive Diversification Returns



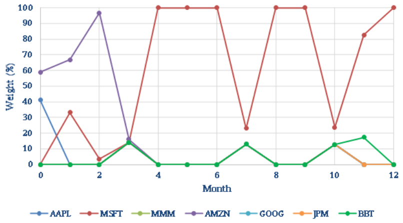
(b) Naive Diversification Stocks

Markowitz Theory

- We implement the Markowitz theory on the provided dataset based on the average historical returns for the first year.
- The efficient frontier of the Markowitz approach has been plotted and can be observed in the first figure.
- In order to trade based on Markowitz's framework, we calculate the variance-covariance matrix monthly based on the past 252 days, then utilize it for portfolio management purposes.
- The corresponding weights of stocks are plotted, which can be observed in the second figure.

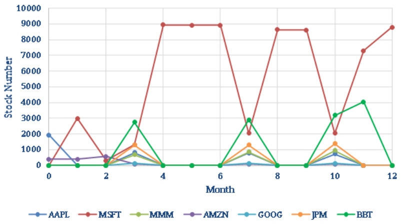


(c) The Efficient Frontier

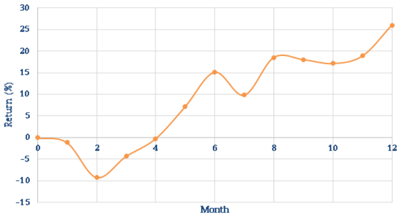


(d) Weights

Markowitz Theory



(e) Stock Numbers

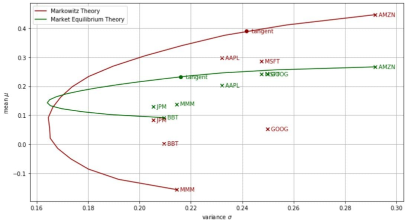


(f) Returns

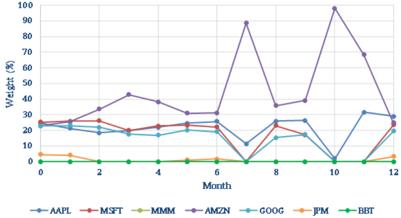
- For each month, we obtain the best weights and invest based on them, and for the next month, we re implement the entire process for the past 252 days, which ultimately leads to buying or selling some stocks to adjust our portfolio for the new weights.
- We trade monthly according to the Markowitz theory by 0.1% of transaction cost and calculate the obtained return from this theory.
- After one year of trading, we get a maximum profit of 25.9548%.
- The diagram of Markowitz's monthly returns is depicted in the last figure.

Market Equilibrium Theory

- In addition to the variance-covariance matrix, we calculate the implied equilibrium excess return with the market weights.
- The efficient frontiers corresponding to the Markowitz and market equilibrium theory are both plotted in the first figure.
- we create a loop and calculate the variance-covariance matrix for the past 252 days and the market equilibrium weights to find the efficient frontier and the tangent point for the best monthly weights and invest in them.
- The corresponding weights of stocks are plotted,which can be observed in the second figure.

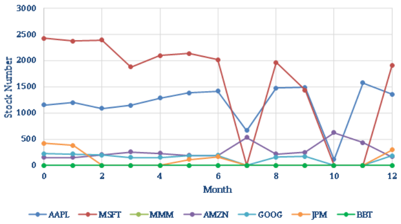


(g) The Efficient Frontier

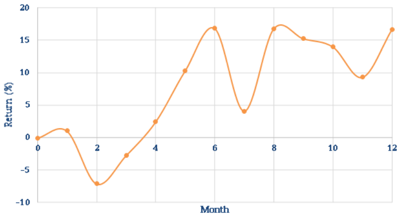


(h) Weights

Market Equilibrium Theory



(i) Stock Numbers

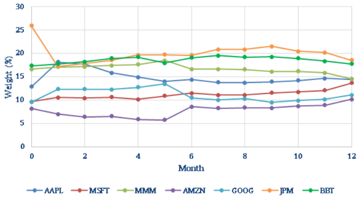


(j) Returns

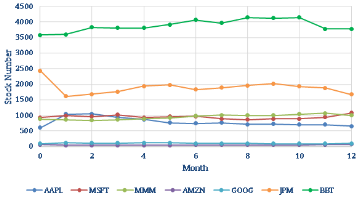
- Considering the new information, if our portfolio changes in the next month, we buy or sell the extra stock we invested.
- By considering 0.1% of transaction cost, we calculate the return of this theory and compare it with the S&P 500 index (assuming that the investor rebuilds his portfolio every month, with 0.1% of transaction cost, as well.
- We trade monthly according to the Market Equilibrium theory by 0.1% transaction cost and calculate the return of this theory. After one year of trading, we achieve a maximum return equal to 16.814%.
- We plotted the diagram of market equilibrium monthly returns in the last figure.

Hierarchical Risk Parity (HRP)

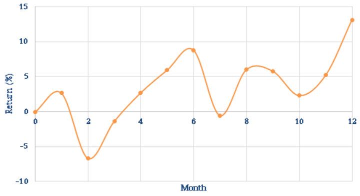
- In this method, for each month the 3 steps of the algorithm has been implemented based on its previous 1-year dataset; every month we step forward, its related dataset also moves forward monthly.
- Similar to the previous model results, by considering a 0.1% of transaction cost, we trade on the portfolio and calculate the return obtained from this theory. After one year of trading, we obtain a maximum profit of 13.1041%.
- The clusters associated with the first and last weeks and their corresponding unclustered and clustered heat maps are plotted and can be observed in the following frame.



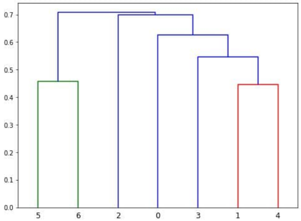
(k) Weights



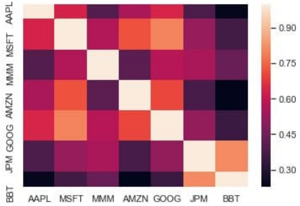
(l) Stock Numbers



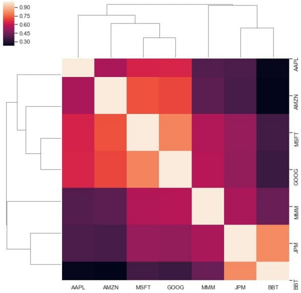
(m) Returns



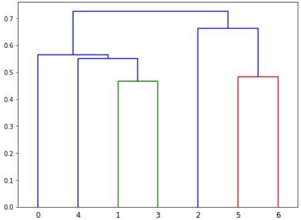
(n) Cluster of The First Week



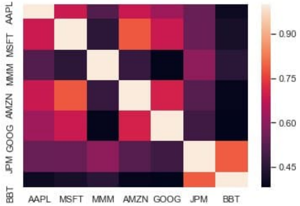
(o) Unclustered Heat Map



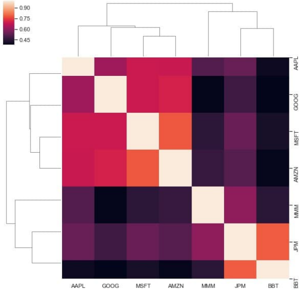
(p) Clustered Heat Map



(q) Cluster of The Last Week



(r) Unclustered Heat Map



(s) Clustered Heat Map

The Entire Returns Plotted in One Figure

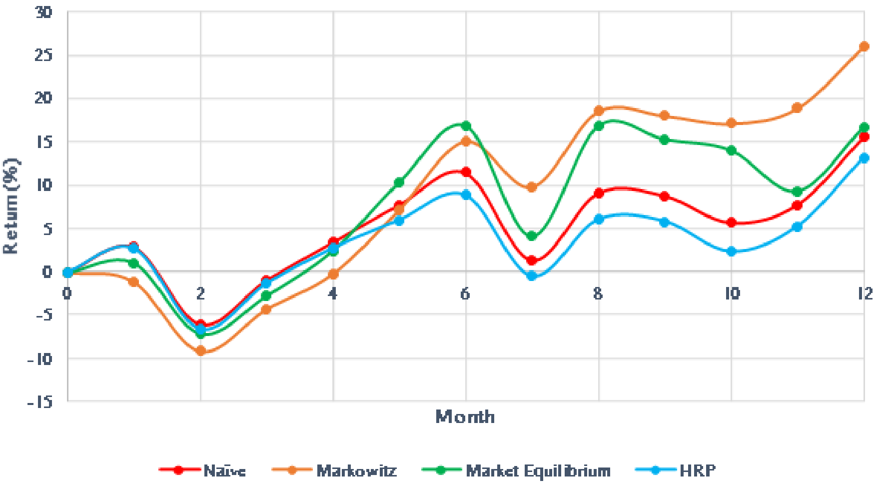
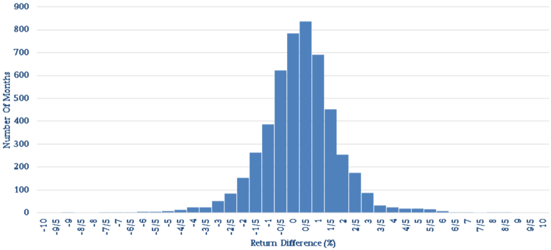


Figure: The Returns Of All Investigated Methods Including Naive, Markowitz, Market Equilibrium and HRP

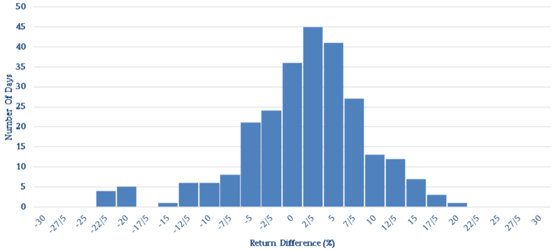
Black-Litterman Theory with Evolutionary Algorithm for Views Matri

- This theory has been renowned as a novel, optimal tool for portfolio management purposes which can be utilized to calculate the weights of a portfolio in order to make more efficient investments.
- We calculate the monthly return of each stock for the previous 5 days, and compare them in a 2 by 2 manner.
- We plotted the daily distribution of Q vectors for all 7 stocks; the obtained distribution is approximately normal with the $\mu = med \approx 0$, and the maximum difference of the stock returns lies within the interval $[-10\%; +10\%]$.
- Then, we plotted the monthly distribution of Q vectors for all 7 stocks which contains 252 data overall.
- The interval of changes is lies between the values $[-30\%; +20\%]$ with the $\mu = med \approx 0$, which indicates that we can consider deviations of $[-50\%; +50\%]$ in rebalancing our portfolio in the most extreme cases.
- The following three frames demonstrate the daily and monthly Q-distributions, backtesting **BL-GA** returns, and standard deviations obtained in one year of simulations.

Black-Litterman Theory with Evolutionary Algorithm for Views Matrix



(a) Q - Daily Distribution



(b) Q - Monthly Distribution

Black-Litterman Theory with Evolutionary Algorithm for Views Matrix

Returns											
		Probability									
	Mutation	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
random number	0.4 to 0.5	9.54%	8.92%	11.24%	14.99%	16.64%	17.33%	17.65%	18.12%	17.90%	17.15%
	0.3 to 0.4	8.82%	11.15%	13.45%	14.77%	16.80%	18.55%	18.81%	18.92%	18.81%	18.49%
	0.2 to 0.3	13.61%	10.70%	11.88%	11.12%	13.67%	16.19%	18.45%	20.76%	21.22%	20.43%
	0.1 to 0.2	16.00%	13.90%	12.93%	11.89%	12.74%	13.77%	15.38%	17.77%	20.19%	22.07%
	0 to 0.1	23.62%	21.99%	21.77%	19.90%	17.93%	16.89%	15.97%	15.55%	16.12%	16.19%
	-0.1 to 0	17.84%	15.61%	14.36%	14.53%	13.51%	13.68%	13.98%	12.01%	12.36%	12.04%
	-0.2 to -0.1	18.66%	15.99%	15.91%	14.98%	16.21%	16.02%	15.84%	17.52%	16.95%	15.28%
	-0.3 to -0.2	16.56%	16.01%	16.91%	18.19%	15.82%	16.21%	17.04%	14.51%	15.20%	14.09%
	-0.4 to -0.3	17.74%	16.33%	14.30%	14.46%	12.89%	13.38%	10.63%	8.35%	4.12%	5.10%
	-0.5 to -0.4	16.49%	15.23%	14.44%	14.06%	10.73%	9.80%	7.67%	7.65%	6.60%	6.35%

Figure: Table of **BL-GA** Returns Obtained In The One-Year Time Period

Black-Litterman Theory With Evolutionary Algorithm For Views Matrix

Standard Deviation										
Mutation	Probability									
random number	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
0.4 to 0.5	10.27%	9.56%	7.54%	6.45%	5.04%	3.94%	3.24%	3.09%	2.82%	1.18%
0.3 to 0.4	10.99%	8.15%	6.98%	6.69%	5.39%	4.08%	3.19%	2.65%	2.63%	1.67%
0.2 to 0.3	8.24%	8.40%	7.11%	5.79%	6.40%	5.99%	5.32%	3.81%	2.64%	1.58%
0.1 to 0.2	8.38%	7.82%	6.88%	5.71%	4.74%	5.64%	5.92%	5.87%	4.95%	4.46%
0 to 0.1	7.66%	6.59%	7.13%	7.31%	6.58%	5.20%	5.15%	4.26%	4.72%	4.56%
-0.1 to 0	6.38%	6.28%	7.22%	7.81%	8.25%	8.04%	8.50%	8.73%	8.88%	9.71%
-0.2 to -0.1	9.27%	8.27%	9.20%	10.19%	9.45%	9.71%	11.21%	10.92%	11.54%	10.97%
-0.3 to -0.2	8.08%	8.53%	9.82%	10.70%	10.51%	10.40%	11.47%	12.08%	11.93%	10.52%
-0.4 to -0.3	9.81%	9.89%	9.50%	10.69%	10.99%	9.26%	11.56%	10.60%	9.49%	8.97%
-0.5 to -0.4	9.16%	11.20%	11.39%	11.45%	11.44%	9.87%	9.72%	9.76%	9.75%	9.41%

Figure: Table of **BL-GA** Standard Deviations Obtained In The One-Year Time Period

Backtesting

- We can utilize backtesting procedure to evaluate the competence of trading strategies by performing them on available historical datasets. Good performance of Backtesting grants analysts and traders to implement the examined strategy for future applications.
- the backtesting procedure is performed within the dates from 31/Oct/2019 to 30/Oct/2020 (a one-year interval). The market crash occurred on February 20th, 2020 due to the outbreak of the *COVID-19* virus.
- we select the ten highest returns; This set includes the first four values of the random number interval $[0, 0.1]$ which are $p = 10\%$, $p = 20\%$, $p = 30\%$, $p = 40\%$. For random numbers between 0 and 0.2, we have p = 80%, $p = 90\%$ and $p = 100\%$ of mutation probability. Eventually, for random numbers between 0.3 and 0.4, we only have one mutation probability which is $p = 100\%$.
- Within the mentioned intervals, we implement the algorithm for the entire next year in order to examine the results.
- The following frame depicts the entire returns obtained from previous approaches along with the S&P 500 index.

Backtesting

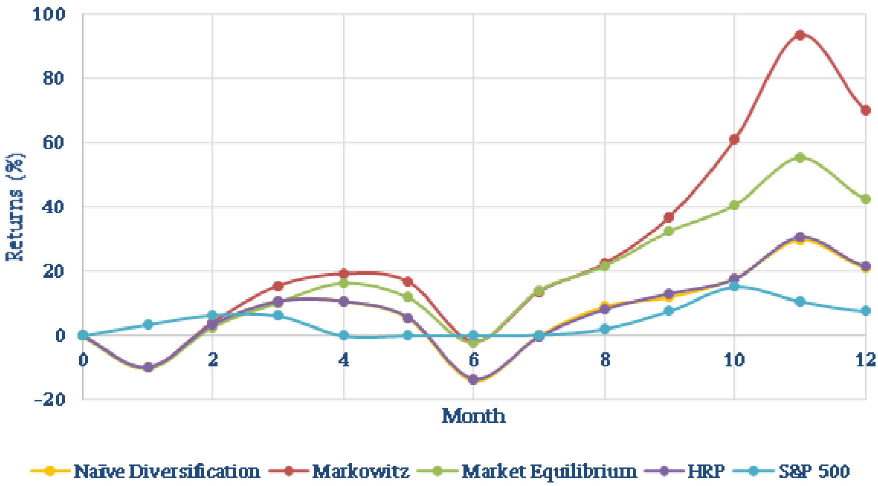


Figure: Backtesting Returns Corresponding To Each Method

Backtesting

- Each of the algorithm's portion was performed by Monte Carlo trials equal to 100. Considering 10 selections, we obtained overall 1000 numbers of outcomes with a distribution depicted in the following Figure.

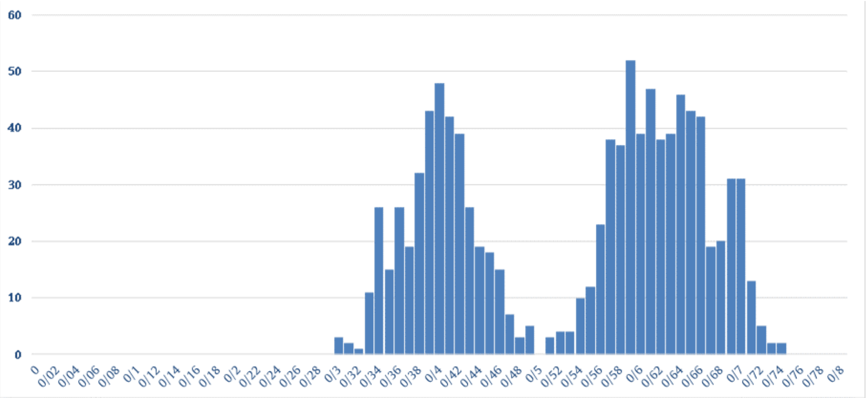


Figure: Distribution Of 1000 Generated Outcomes

Backtesting

- the above distribution has two peaks. One of them about 30% of return, the other is around 64%. It can be observed that the four values within the interval $[0, 0.1]$ have higher values of standard deviations (and, of course, higher variances) in comparison to the other six values.
- the left peak has a high standard deviation since We conclude that in addition to the mean value (μ), we should also consider the standard deviation (σ). In other words, we should select the scenarios which have high μ and low σ.
- Among ten chosen scenarios, we picked the cases with standard deviations below 5%; which means the 6 of them with high probability mutation. Hence we finally have 600 outcomes which their distribution is represented in the following Figure.
- As can be observed, the entire 600 outcomes were able to beat naive diversification, market equilibrium and HRP methods, and the *S&P 500* index. In other words, we obtain a probability of 100%. While 409 of 600 outcomes could beat the Markowitz method, which has a probability of 68.17%; on the other hand, 191 of 600 outcomes or about 31.82% of the entire selected scenarios could not beat Markowitz's approach.

Backtesting

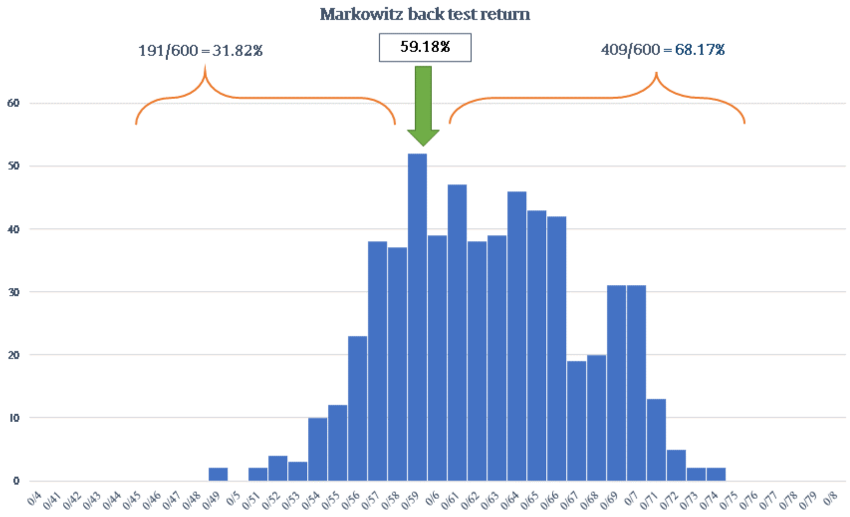


Figure: Distribution Of 600 Outcomes From The 6 Selected Scenarios In The Backtesting Procedure

Conclusion and Future Research

- In order to attain the highest possible profits, portfolio optimization has been a great concern among researchers, traders, and financial market activists.
- we mainly focused on the Black-Litterman method and tried to eliminate any human intervention in this approach by utilizing specific machine learning techniques.
- This paper has implemented the genetic algorithm to generate the views matrix instead of taking advantage of any human diagnosis, including trade specialists.
- by adjusting proper mutation values and their corresponding probabilities, our proposed approach was able to attain higher returns comparing to almost all of the other methods.
- Backtesting results which were obtained from the period of the market crash due to the *COVID-19* outspread, indicate that our novel approach beat the well-known Markowitz method with the probability value equal to almost 70%.
- Since our approach opens a window through the past 5 days and constructs the investor's views according to them, it leads to a more efficient response to various unexpected events, including crises.

Conclusion and Future Research

- Our approach updates itself yearly. This update process includes the past-opening window of five days and the daily and monthly Q-distributions, which impact crossover and mutation.
- For future investigation and research, we can impose restrictions over the crossover section in a three-by-three space. In other words, creating a 3×3 matrix and search in this environment to discover the optimum solution.
- We also can implement another machine learning algorithm among various available algorithms.
- It should be noted that we have to adopt the new optimizing engine tool to the original Black-Litterman framework.