UNIVERSAL LOW-RANK MATRIX RECOVERY FROM PAULI MEASUREMENTS: AN IDEA

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THE PROBLEM

- $M \in \mathbb{C}^{d\times d}$ is an unknown matrix of rank $r \ll d$
- $\{W_1, \dots, W_{d^2}\}$ is an orthonormal basis for $\mathbb{C}^{d\times d}$ with $\|W_i\|_{op} \le \frac{K}{\sqrt{d}}$
- $\{S_1, \dots, S_m\} \subset \{W_1, \dots, W_{d^2}\}$ is randomly uniformly chosen
- $[\mathcal{A}(X)]_i = \frac{d}{\sqrt{m}}\langle S_i, X\rangle = \frac{d}{\sqrt{m}}\mathrm{tr}(S_i^* X)$ are given
- If $\mathcal{A}$ has RIP on $\mathrm{U} = \{Z \in \mathbb{C}^{d\times d}\colon \|Z\|_F \le 1,\ \mathrm{rank}(Z) \le r\}$, reconstruction is possible
- Clearly, $\mathrm{U} \subset \mathrm{V} = \{Z \in \mathbb{C}^{d\times d}\colon \|Z\|_F \le 1,\ \|Z\|_* \le \sqrt{r}\|Z\|_F\} \subset \sqrt{r}B_*$
- The proof is essentially the same as (Rudelson, 2007) except for the technique used to bound $\mathcal{N}(B_*, \|\cdot\|_X, \epsilon)$

SOME DEFINITIONS

- $(E, \|\cdot\|)$ is said to be 2-convex, if for some finite λ we have for all $y, z \in E$

$$\|y\|^2 + \frac{\|z\|^2}{\lambda^2} \leq \frac{1}{2}\left(\|y+z\|^2 + \|y-z\|^2\right)$$

- The smallest such λ is denoted by $\lambda(E)$
- A Hilbert space is 2-convex with $\lambda(H) = 1$ (parallelogram identity)
- $(E, \|\cdot\|)$ is said to be type 2, if for some finite T_2 we have for all $(y_i)_{i=1}^N \subset E$

$$\mathbb{E}_\epsilon \left\| \sum_{i=1}^N \epsilon_i y_i \right\|^2 \leq T_2^2 \sum_{i=1}^N \|y_i\|^2$$

- The smallest such T_2 is denoted by $T_2(E)$
- A Hilbert space is type 2 with $T_2(H) = 1$ (contraction principle)

THE MAIN THEOREM

- Let E be a 2-convex Banach space with constant $\lambda(E)$. Let E^* be its dual space with the type 2 constant of $T_2(E^*)$. If $x_1, x_2, \ldots, x_m \in E^*$ such that $\|x_j\|_{op} \leq L$ for all $j \in [m]$, for every $y \in E$ define $\|y\|_X = \max_{j \in [m]} |\langle x_j, y \rangle|$. Then

$$\epsilon \sqrt{\log \mathcal{N}(B_E, \|\cdot\|_X, \epsilon)} \leq CL\lambda^2(E) T_2(E^*) \sqrt{\log m}\,.$$

APPLYING THE THEOREM

- To bound $\mathcal{N}(B_1, \|\cdot\|_X, \epsilon)$ [or $\mathcal{N}(B_*, \|\cdot\|_X, \epsilon)$] we cannot directly use the theorem because $(\mathbb{R}^n, \ \|\cdot\|_1)$ [or $\left(\mathbb{C}^{d\times d}, \ \|\cdot\|_*\right)$] is not 2-convex
- Instead, we bound $\mathcal{N}(B_p, \|\cdot\|_X, \epsilon)$ for some $1 < p \leq 2$ and use the fact that $B_1 \subset B_p$
- $\lambda\left(\mathbb{R}^n, \|\cdot\|_p\right) = \frac{1}{\sqrt{p-1}}$ for $p \in (1,2]$
- $T_2(E^*) \leq \lambda(E)$
- But then L means $\max_j \left\|x_j\right\|_q$, where $\frac{1}{p} + \frac{1}{q} = 1$
- So, for the best bound we need to find

$$\inf_{p\in(1,2]} \frac{\max_j \left\|x_j\right\|_q}{(p-1)^{3/2}}$$

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