In the Name of Allah

دانشگاه علم و صنعت ایران

Assignment 6

Deadline : 1401 / 04 / 01

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1. The Z-Transform of the impulse response of three discrete-time signals are given as follows. Determine whether the systems are causal or not.

a) $\frac{1-\frac{4}{3}z^{-1}+\frac{1}{2}z^{-2}}{z^{-1}\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}$

b) $\frac{z-\frac{1}{2}}{z^2+\frac{1}{2}z-\frac{3}{16}}$

c) $\frac{z+1}{z+\frac{4}{3}-\frac{1}{2}z^{-2}-\frac{2}{3}z^{-3}}$

2. If $y[n] = (3^n x_2[-n+1]) * x_1[n+3]$ and, $\mathrm{x}_1[\mathrm{n}] = (-1)^{\mathrm{n}}\cos\frac{\mathrm{n}\pi}{6}\mathrm{u}[\mathrm{n}]$ and $x_2[n] = 3^n u[n]$, then find the ROC of $\mathrm{Y}(\mathrm{z})$.

3.

a) Determine the Z-Transform of the following signal. Draw the zero-pole diagram and determine the ROC.

$$x[n] = \begin{cases} 0, & n < 0 \\ 1, & 0 \le n \le 9 \\ 0, & 9 < n \end{cases}$$

b) The Z-Transform of two signals are given as follows. Determine the corresponding time-domain signals.

(a) $X(z) = \log(1 - 2z)$,
$|z| < \frac{1}{2}$
(b) $X(z) = \log\left(1 - \frac{1}{2}z^{-1}\right)$,
$|z| > \frac{1}{2}$

4. The frequency which, under the sampling theorem, must be exceeded by the sampling frequency is called the Nyquist rate. Determine the Nyquist rate corresponding to each of the following signals:

(a) $x(t) = 1 + \cos(2{,}000\pi t) + \sin(4{,}000\pi t)$
(b) $x(t) = \frac{\sin(4{,}000\pi t)}{\pi t}$
(c) $x(t) = \left(\frac{\sin(4{,}000\pi t)}{\pi t}\right)^2$

5. Let $x_c(t)$ be a continuous-time signal whose Fourier transform has the property that $X_c(j\omega) = 0$ for $|\omega| \ge 2{,}000\pi$. A discrete-time signal

$$x_d[n] = x_c\left(n(0.5 \times 10^{-3})\right)$$

is obtained. For each of the following constraints on the Fourier transform $X_d\left(e^{j\omega}\right)$ of $x_d[n]$, determine the corresponding constraint on $X_c(j\omega)$:

(a) $X_d\left(e^{j\omega}\right)$ is real.
(b) The maximum value of $X_d\left(e^{j\omega}\right)$ over all ω is 1 .
(c) $X_d\left(e^{j\omega}\right) = 0$ for $\frac{3\pi}{4} \leq |\omega| \leq \pi$.
(d) $X_d\left(e^{j\omega}\right) = X_d\left(e^{j(\omega-\pi)}\right)$.

Good Luck