In the Name of Allah

Assignment 4

Deadline : 1401 / 02 / 18

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1. Each of the two sequences $x_1[n]$ and $x_2[n]$ has a period $N = 4$, and the corresponding Fourier series coefficients are specified as

$$x_1[n] \leftrightarrow a_k,\ x_2[n] \leftrightarrow b_k,$$

where

$$a_0 = a_3 = \frac{1}{2}a_1 = \frac{1}{2}a_2 = 1 \text{ and } b_0 = b_1 = b_2 = b_3 = 1.$$

Using the multiplication property in Table 3.1 (Oppenheim), determine the Fourier series coefficients c_k for the signal $g[n] = x_1[n]x_2[n]$.

2. Let $x[n]$ be a periodic signal with period $N = 8$ and Fourier series coefficients $a_k = -a_{k-4}$. A signal

$$y[n] = \left(\frac{1 + (-1)^n}{2}\right) x[n-1]$$

with period $N = 8$ is generated. Denoting the Fourier series coefficients of $y[n]$ by b_k, find a function $f[k]$ such that

$$b_k = f[k]a_k.$$

3. Let $x[n]$ be a periodic sequence with period N and Fourier series representation

$$x[n] = \sum_{k=<N>} a_k e^{jk(2\pi/N)n}.$$

(a) Suppose that N is even and that $x[n]$ in eq. (P3.49-1) satisfies

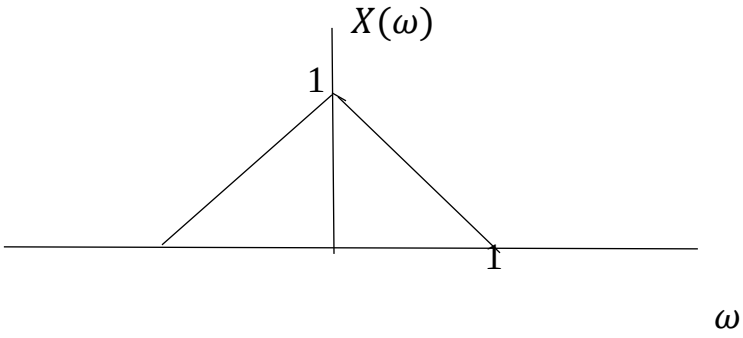
$$x[n] = -x\left[n + \frac{N}{2}\right] \text{ for all } n$$

Show that $a_k = 0$ for all even integers k.
(b) Suppose that N is divisible by 4 . Show that if

$$x[n] = -x\left[n + \frac{N}{4}\right] \text{ for all } n$$

then $a_k = 0$ for every value of k that is a multiple of 4 .
(c) More generally, suppose that N is divisible by an integer M. Show that if $\sum_{r=0}^{(N/M)-1} x\left[n + r\frac{N}{M}\right] = 0$ for all n then $a_k = 0$ for every value of k that is a multiple of M.

4. Calculate the fourier transform of $y(t) = x(t)p(t)$, if $X(\omega)$ is as the following figure and $p(t)$ is given as equations (a) and (b).



a) $p(t) = \cos(2t) - \cos(t)$

b) $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\pi)$

5. Calculate the following integrals using fourier transform.

a) $\int_{-\infty}^{\infty} \sin\ c^5(t)dt$

b) $\int_0^{\infty} e^{-\alpha t}\text{sinc}\,(t)dt,\ \alpha > 0$

c) $\int_0^{\infty} e^{-\alpha t}\cos\,(\beta t)dt,\ \alpha > 0$

6. Which of the following fourier transform properties are for the fourier transform of the given signals.

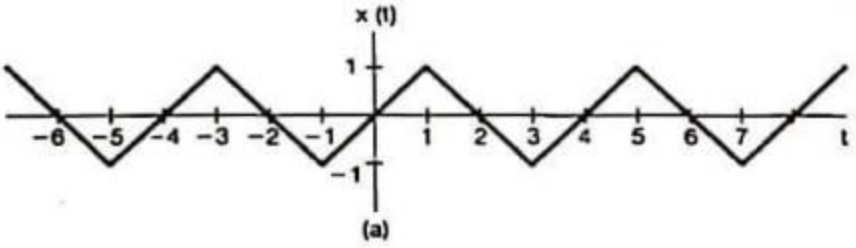
a) Re $[X(j\omega)] = 0$

b) Im $[X(j\omega)] = 0$

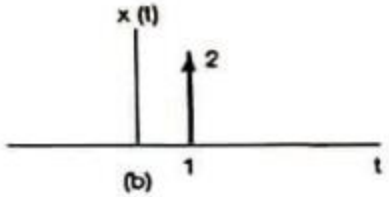
c) $\int_{-\infty}^{+\infty} X(j\omega)d\omega = 0$

d) $\int_{-\infty}^{+\infty} \omega X(j\omega)d\omega = 0$

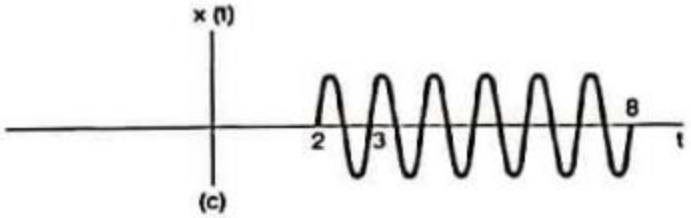
e) $X(j\omega)$ is periodic.


x (t)
1
−1
−6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7 t

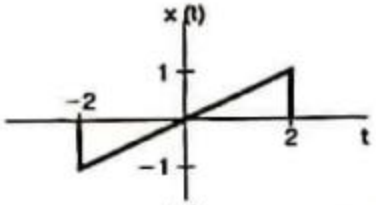
(a)


x (t)
2
1
t

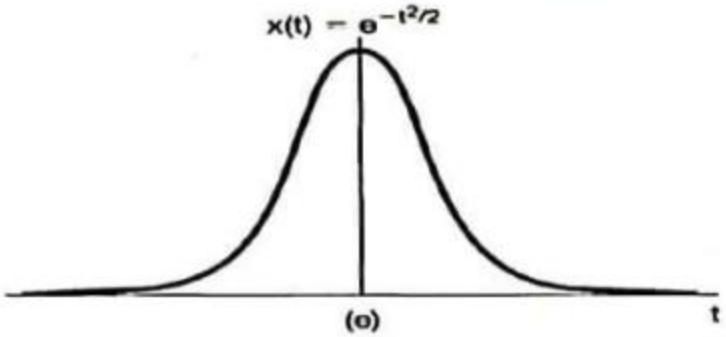
(b)


x (t)
2
3
8
t

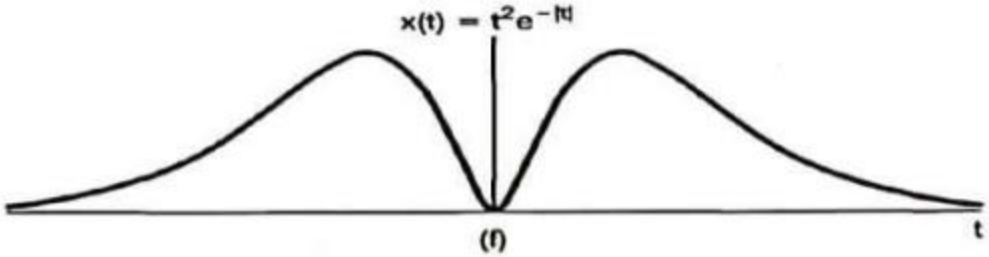
(c)


x (t)
1
−2
2
t
−1

(d)


x(t) = e^{-t^2/2}
t

(e)


x(t) = t^2 e^{-|t|}
t

(f)

7. Calculate the fourier transform for the following signals.

a) $\alpha > 0, [e^{-at}\cos\ \omega_0 t]u(t)$
b) $e^{2+t}u(-t+1)$
c) $x(t) = t\left(\frac{\sin t}{\pi t}\right)^2$
d) $x(t) = \frac{t^{n-1}}{(n-1)!}e^{-at}u(t), a > 0,$
e) $\delta(t) + 2\delta(3-2t)$

Good Luck