In the Name of Allah

Assignment 5

Deadline : 1401 / 03 / 16

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1. Calculate the discrete Fourier transform of the following signals.

a) $x[n] = \left(\frac{1}{2}\right)^n u[n-2] + n\left(\frac{1}{3}\right)^{|n|}$

b) $x[n] = 2\left(\frac{3}{4}\right)^n u[n]$

2. Find x[n] if it has the following Fourier transform as:

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} (-1)^k \pi \delta\left(\omega - \frac{k\pi}{2}\right)$$

3. The response of a discrete LTI system to input $x_1[n]$ is $y_1[n]$. Find the response of this system to $x_2[n]$.

$$x1[n] = 1 + \cos\frac{2\pi n}{3}$$

$$y1[n] = 2 + \sin\frac{2\pi n}{3}$$

$$x2[n] = \sum_{m=-\infty}^{\infty} \delta[n - 3m]$$

4. A system with the following frequency response is given.

a) Derive the Impulse response of the system.
b) Derive the differential equation of the system.
c) Find the response of the system to unit step in time domain.

$$H(\Omega) = \frac{1}{(1 - 0.3e^{-j\Omega})(1 - 0.5e^{-j\Omega})(1 - 0.7e^{-j\Omega})},$$

5. Find the Laplace transform and ROC of the following signals.

a) $\sum_{k=0}^{\infty} a_k \delta(t + kT)\,, a_k = cte$
b) $t\sin(t)u(t)$
c) $t^2 u(t-2)$
d) $te^{-at}u(t+1)$

6. Find the inverse Laplace transform of the following signals.

$$X(s) = \frac{e^{-2s}}{(s-1)^2}, \Re e[s] > 1$$
$$a > 0, X(s) = \frac{s+a}{s-a}, \Re e[s] < a$$
$$a > 0, X(s) = \frac{s-a}{s+a}, \Re e[s] > -a$$
$$X(s) = 1 + \frac{s^2 - 1}{s^2 + 1}, \Re e[s] < 0$$

7. Consider a signal $y(t)$ which is related to two signals $x_1(t)$ and $x_2(t)$ by

$$y(t) = x_1(t-2) * x_2(-t+3)$$

where

$$x_1(t) = e^{-2t}u(t) \text{ and } x_2(t) = e^{-3t}u(t).$$

use properties of the Laplace transform to determine the Laplace transform $Y(s)$ of $y(t)$.

8. The input $x(t) = \begin{cases} 4, t \geq 0 \\ 1, t < 0 \end{cases}$ is applied to a LTI, stable system with $H(s) = \frac{s}{s^2-1}$.

Derive the output of the system in time domain ($y(t)$).

9. The following information about a signal is given. Derive the Laplace transform and ROC of the signal.

X(s) has two poles and X(t) is real.

X(s) has no zeros in S-plane

X(s) has a pole in s=-1+j

$\int_{-\infty}^{+\infty} |e^{2t}x(t)|dt < \infty$

X(0) = 8

Good Luck