Norwegian University of Science and Technology

Artificial Neuron & PLR

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NTNU in Ålesund

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In this lecture

- Perceptron
- PLR
- Adaline
- Feature scaling
- examples

Perceptron

- Basic unit of artificial neuron.
- Can classify a task into two classes: 1 (positive class) and -1/0 (negative class)
- Decision function can be defined as a linear combination of certain input values x_i and a corresponding weight values w_i.where i =1, 2, …, n

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad w = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$$

$$net = w_1 x_1 + \ldots + w_m x_m + b = w^T x + b$$

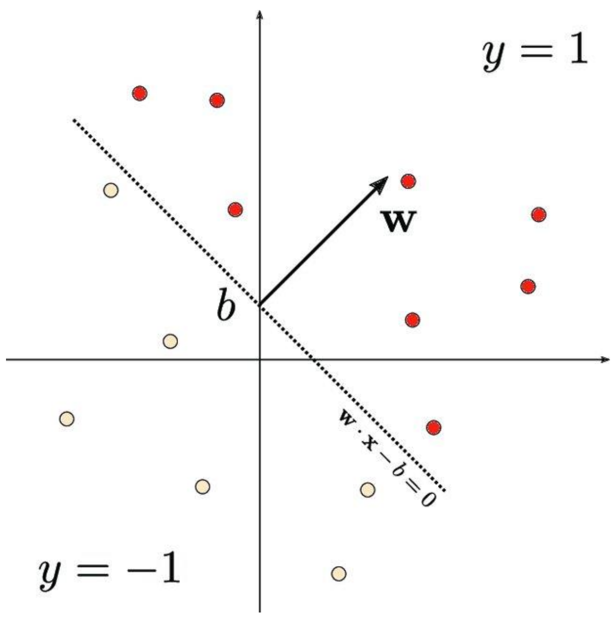
$$f(net) = \begin{cases} 1 & if \;\; net \geq 0 \\ -1 & otherwise \end{cases}$$

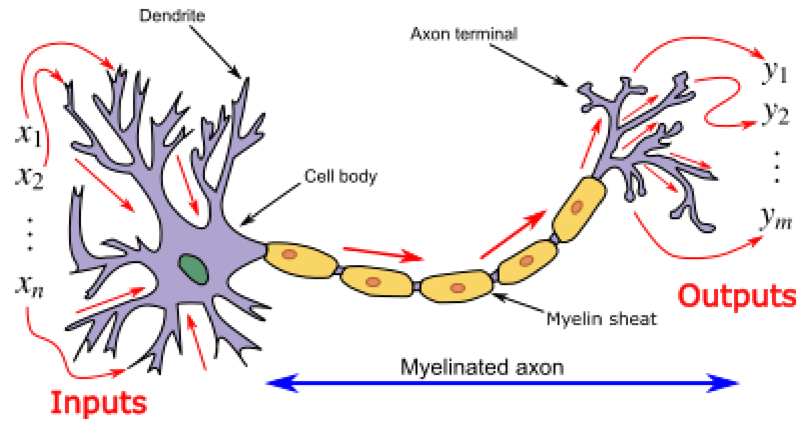
Notes:

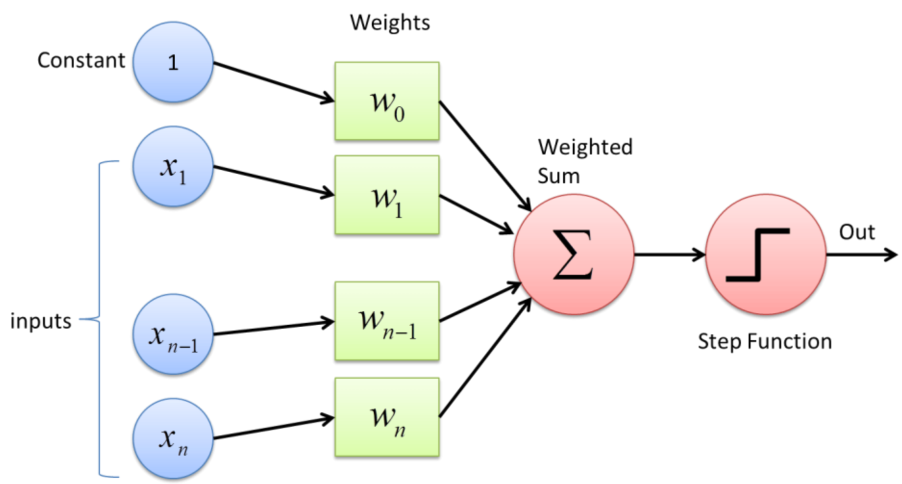
- Weights cause rotation of the decision line/boundary
- Bias cause translation of the decision line /boundary

Graphical model

- Inspired by Biological Neuron







The Perceptron learning Rule PLR

1. Initialize the weights to 0 or to small random numbers
2. For each training sample $x^{(i)}$
   a) Compute the output value
   b) Update the weights
   c) Repeat it until converge

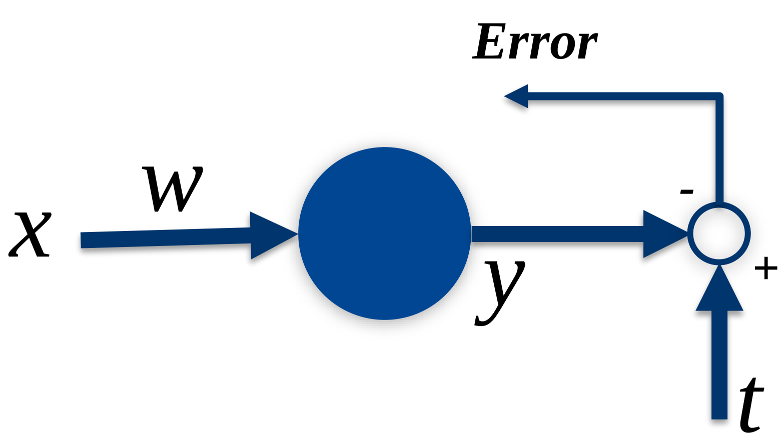
Δw = learning rate/step size * error * input

$$\Delta w_j = \eta\left(t^{(i)} - y^{(i)}\right)x_j^{(i)}$$

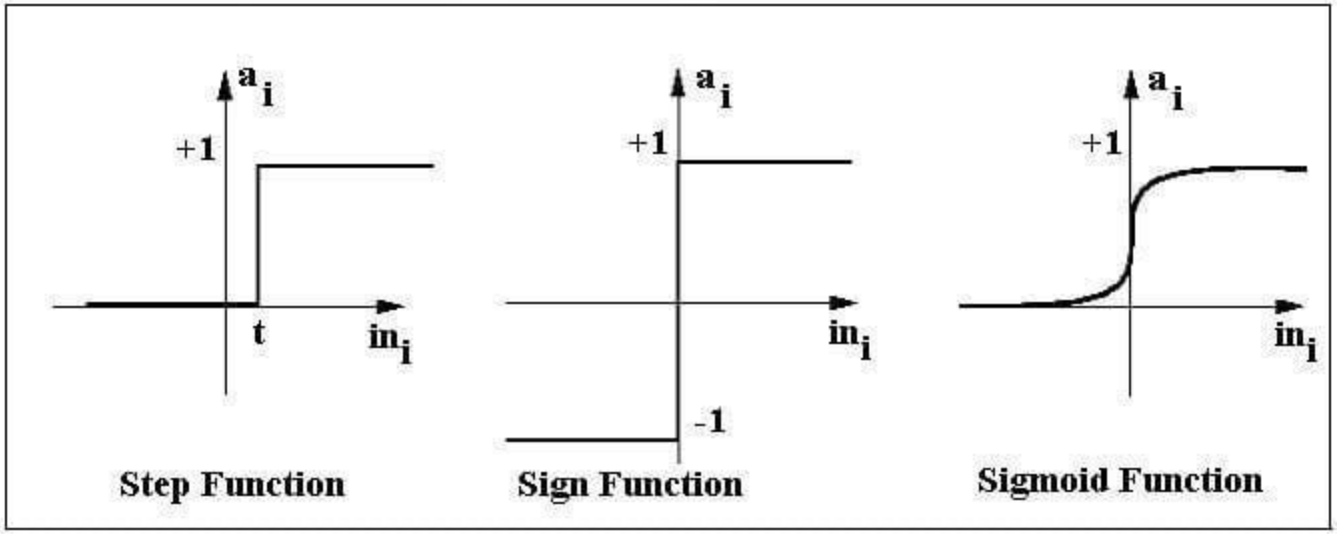
$$w_j = w_j + \Delta w_j$$

$$\Delta b = \eta\left(t^{(i)} - y^{(i)}\right)$$

$$b = b + \Delta b$$



Activation functions of a perceptron



Implementation

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

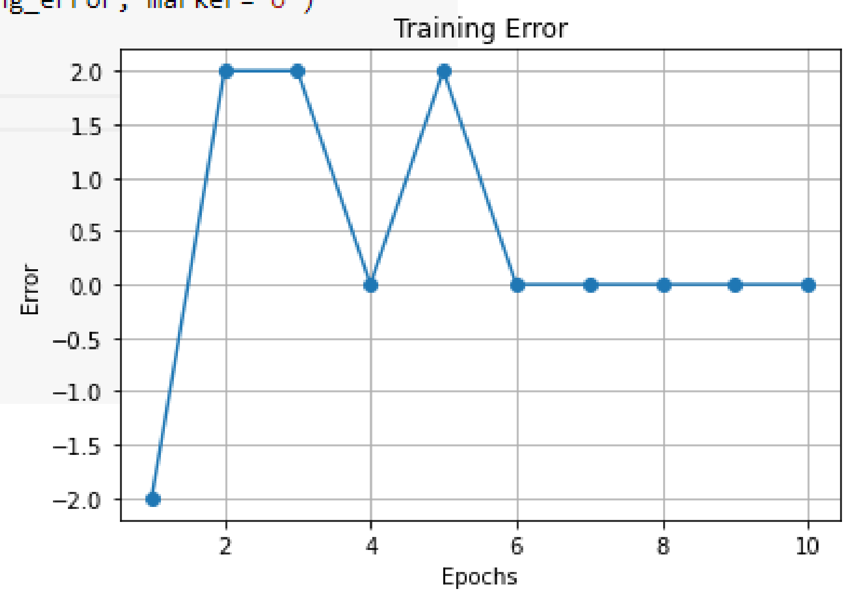
```
class Perceptron(object):

  def __init__(self, no_of_inputs, epochs=100, learning_rate=0.01):
        self.epochs = epochs
        self.learning_rate = learning_rate
        self.weights = np.zeros(no_of_inputs + 1)
        self.training_error = []

  def predict(self, inputs):
    net = np.dot(inputs, self.weights[1:])+self.weights[0]
    # step activation function
    if net >= 0:
      activation = 1
    else:
      activation = -1
    return activation

  def train(self, training_inputs, training_labels):
    for _ in range(self.epochs):
      error = 0
      for inputs, label in zip(training_inputs, training_labels):
        prediction = self.predict(inputs)
        self.weights[1:] += self.learning_rate * (label - prediction) * inputs
        self.weights[0] += self.learning_rate * (label - prediction)
        error += label - prediction
      self.training_error.append(error.mean())
```

```python
# test perceptron with IRIS data
df = pd.read_csv('https://archive.ics.uci.edu/ml/machine-learning-databases/iris/iris.data', header=None)
print(df.tail())
```

```
       0    1    2    3               4
145  6.7  3.0  5.2  2.3  Iris-virginica
146  6.3  2.5  5.0  1.9  Iris-virginica
147  6.5  3.0  5.2  2.0  Iris-virginica
148  6.2  3.4  5.4  2.3  Iris-virginica
149  5.9  3.0  5.1  1.8  Iris-virginica
```

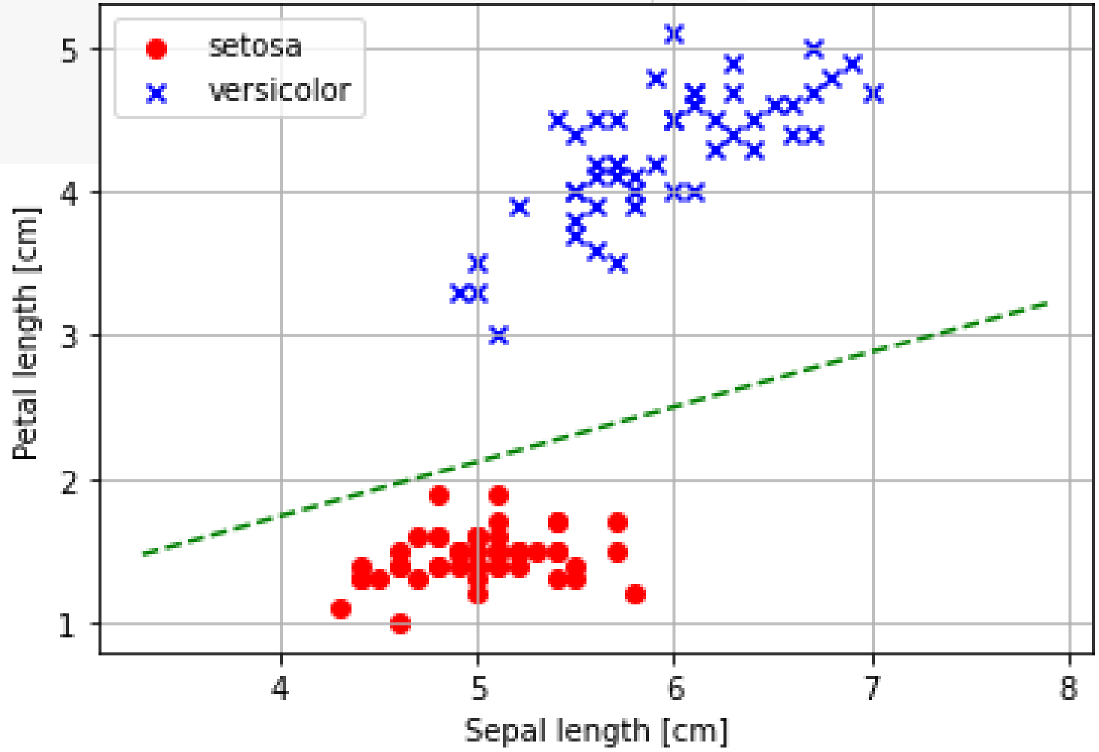
```python
# extract first 100 class labels (50 iris-setosa and 50 iris-versicolor)
x = df.iloc[0:100, [0, 2]].values
y = df.iloc[0:100,4].values
y =np.where(y=='Iris-setosa',1, -1)
print(y)
```

```python
# use a peceptron to find a decsion boundary to separate the two classe
p = Perceptron(no_of_inputs=2, epochs=10, learning_rate=0.1)
p.train(x, y)

plt.plot(range(1,len(p.training_error)+1), p.training_error, marker='o')
plt.xlabel('Epochs')
plt.ylabel('Error')
plt.title('Training Error')
plt.grid()
plt.show()

print(p.weights)
pred = []
for t in x:
  pred.append(p.predict(t))
```



```
# plot decicion line
x1_min, x1_max = x[:,0].min()-1, x[:,0].max()+1
xx1 = np.arange(x1_min, x1_max, 0.1)
xx2 = -p.weights[1]/p.weights[2] * xx1 - p.weights[0]/p.weights[2]
```

```
# scatter plot
plt.scatter(x[0:50,0], x[0:50,1], color='red', marker='o', label='setosa')
plt.scatter(x[50:100,0], x[50:100,1], color='blue', marker='x', label='versicolor')
plt.plot(xx1, xx2, 'g--')
plt.xlabel('Sepal length [cm]')
plt.ylabel('Petal length [cm]')
plt.legend(loc='upper left')
plt.grid()
plt.show()
```

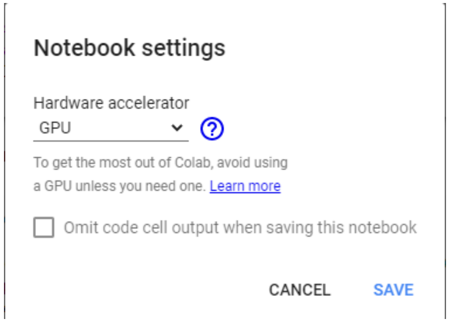



```
error = sum(y - pred)/len(y)
print(error)
```


https://colab.research.google.com/github/ibribr/DT8807/blob/master/perceptron.ipynb

Notes

- The perceptron learned a decision boundary that can classify iris training subset perfectly.
- The PLR converges iff (if and only if) the two classes can be separated perfectly by a linear hyperplane.
- If the classes cannot be separated perfectly by such a linear decision boundary, the weights will never stop updating unless we set a maximum number of epochs (i.e., iterations).
- To enable GPU in Google Colab, go to Runtime, select runtime type and then select your hardware accelerator.
- The types of GPUs that are available in Colab vary over time. The GPUs available in Colab often include Nvidia K80s, T4s, P4s and P100s.
- There is no way to choose what type of GPU you can connect to in Colab at any given time unless you subscribe to Colab Pro.



Gradient descent

- GD is a first-order iterative optimization algorithm for finding a local minimum of a differentiable function.
- The idea is to take repeated steps in the opposite direction of the gradient (or approximate gradient) of the function at the current point, because this is the direction of steepest descent.
- Iterative GD is called stochastic (on-line) gradient descent.
- For a loss/objective function $Q(w)$:

$$Q(w) = \frac{1}{n}\sum_{i=1}^{n} Q_i(w)$$

- where $Q_i(w)$ is the loss value associated with the i-th observation/example:

$$w = w - \eta \nabla Q_i(w) \qquad (SGD)$$

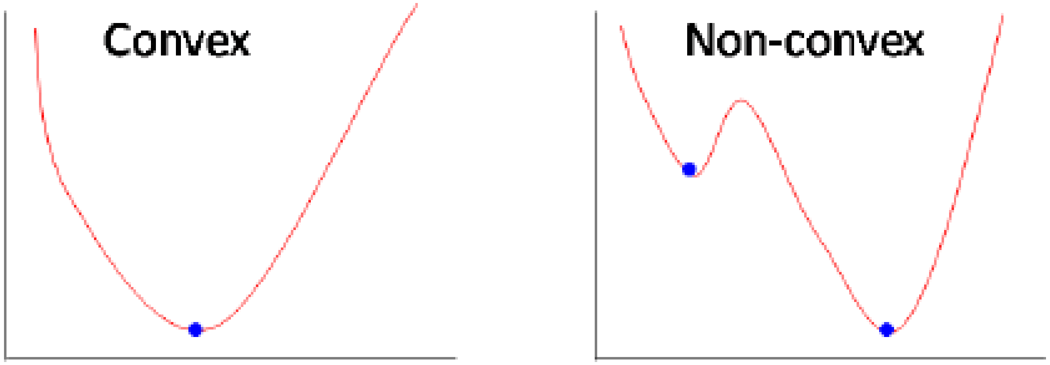
$$w = w - \eta \nabla Q(w) = w - \frac{\eta}{n}\sum_{i=1}^{n} \nabla Q_i(w) \qquad (BGD)$$

The basic intuition behind gradient descent

- Can be illustrated by a hypothetical scenario.
- A person is stuck in the mountains and is trying to get down (i.e. trying to find the global minimum).
- There is heavy fog such that visibility is extremely low. Therefore, the path down the mountain is not visible, so they must use local information to find the minimum.
- They can use the method of gradient descent, which involves looking at the steepness of the hill at their current position, then proceeding in the direction with the steepest descent (i.e. downhill).
- If they were trying to find the top of the mountain (i.e. the maximum), then they would proceed in the direction of steepest ascent (i.e. uphill).
- Using this method, they would eventually find their way down the mountain or possibly get stuck in some hole (i.e. local minimum or saddle point), like a mountain lake (i.e., flat area where derivative/slope equals zero).

Cost functions

- To get the gradient of the cost function, you need to derive/determine its first derivative.
- Square the function (e.g., SSE) to be a convex.
- Example:

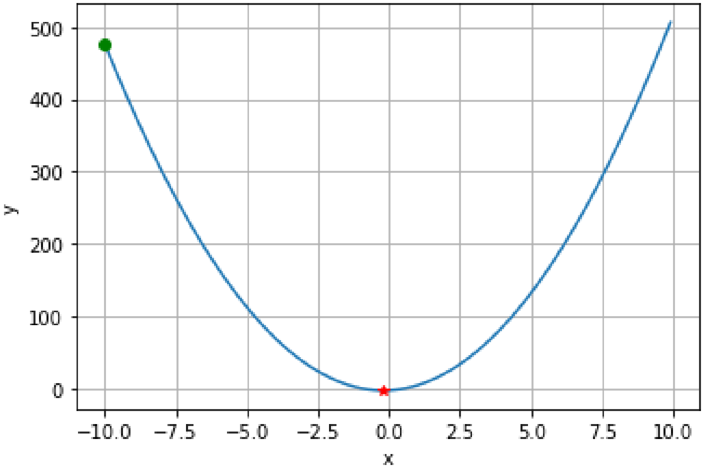


$$f(x) = 5x^2 + 2x - 4$$

$$\Delta(x) = f'(x) = 10x + 2$$

$$x = x + \Delta(x)$$

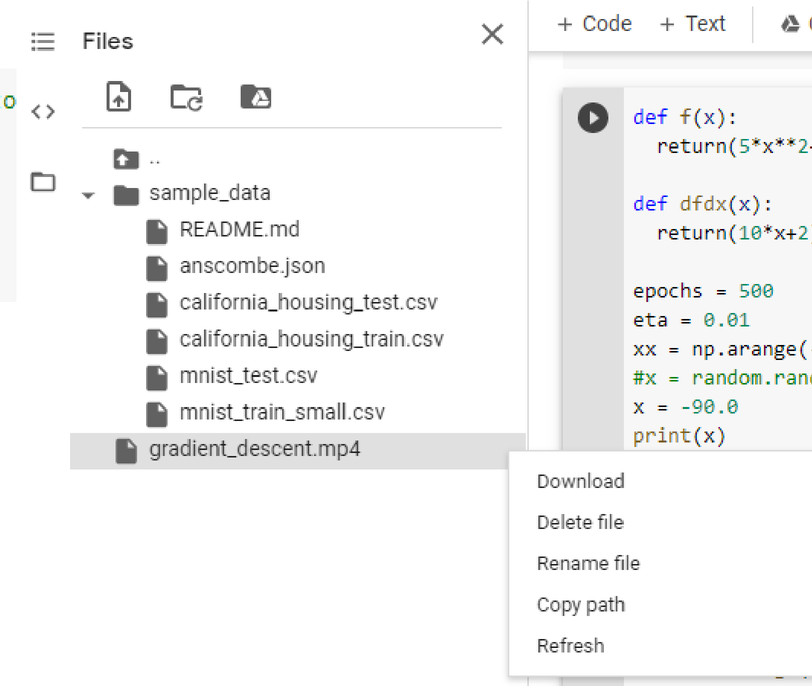
$$\overset{*}{x} = -0.2 \rightarrow f(\overset{*}{x}) = -4.2$$

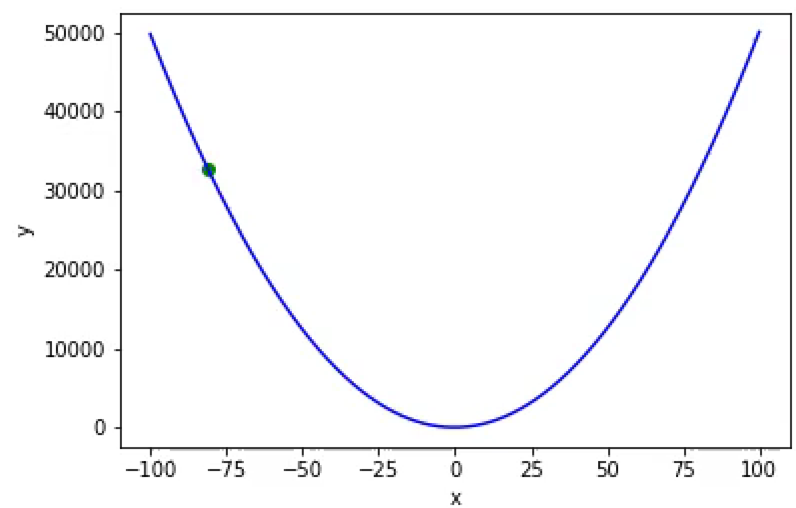


Gradient descent

```
#animate GD algorithm: code from Rasmus <openrasmus@gmail.co
!pip install celluloid
import matplotlib
import matplotlib.pyplot as plt
import random
from celluloid import Camera
```

```
def f(x):
  return(5*x**2+2*x-4)

def dfdx(x):
  return(10*x+2)

epochs = 500
eta = 0.01
xx = np.arange(-100, 100, 0.1)
#x = random.randrange(-100, 100, 2)
x = -90.0
print(x)

plt.close('all')
fig = plt.figure()
plt.xlabel('x')
plt.ylabel('y')
camera = Camera(fig)

for i in range(epochs):
  x += -eta * dfdx(x);    #try: eta/(i+1)
  plt.plot(x,f(x),'go')
  plt.plot(xx, f(xx), 'b-')
  camera.snap()
  if np.abs(dfdx(x)) <= 10**-1:
    break

animation = camera.animate(blit=False)
animation.save('gradient_descent.mp4',fps=15)
print('animation is ready')
```



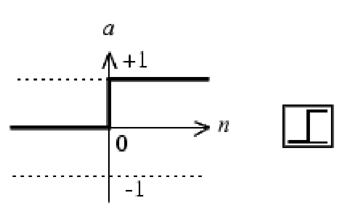


https://colab.research.google.com/github/ibribr/DT8807/blob/master/gradient_desent_simple_function.ipynb

Adaline algorithm

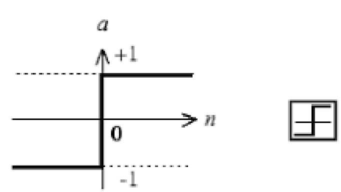
- Stands for **ada**ptive **line**ar neuron.
- In PLR weights are updated based on a unit step function.
- Adaline introduces the concept of minimizing a continuous (convex) cost function – the sum of squared errors (SSE) between the calculated outcome and the true class label:.

$$J(w) = \frac{1}{2}\sum_i (t^{(i)} - y^{(i)})^2$$



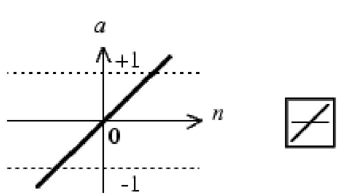
$a = hardlim(n)$

Hard-Limit Transfer Function



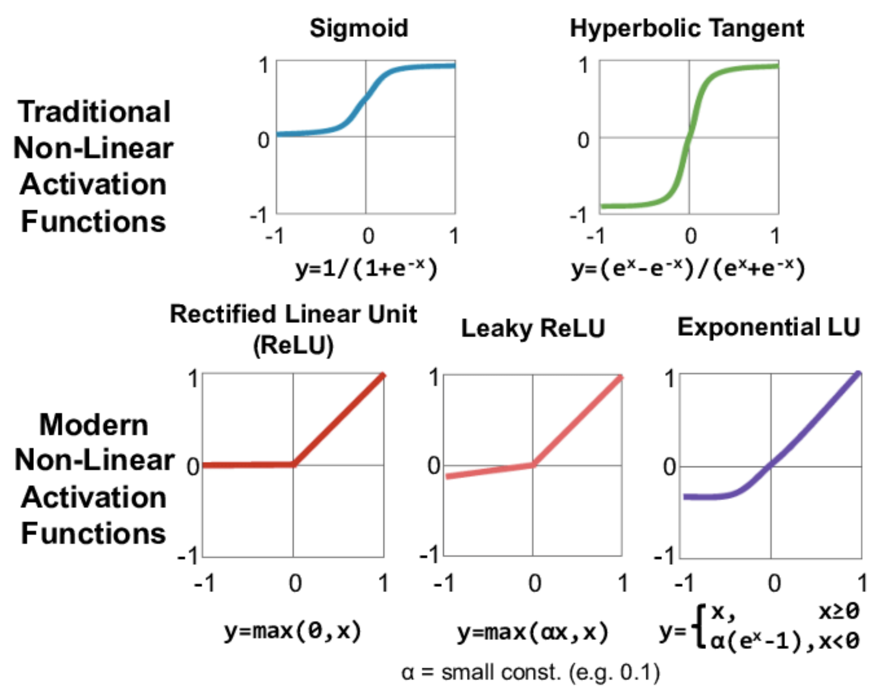
$a = hardlims(n)$

Symmetric Hard-Limit Trans. Funct.

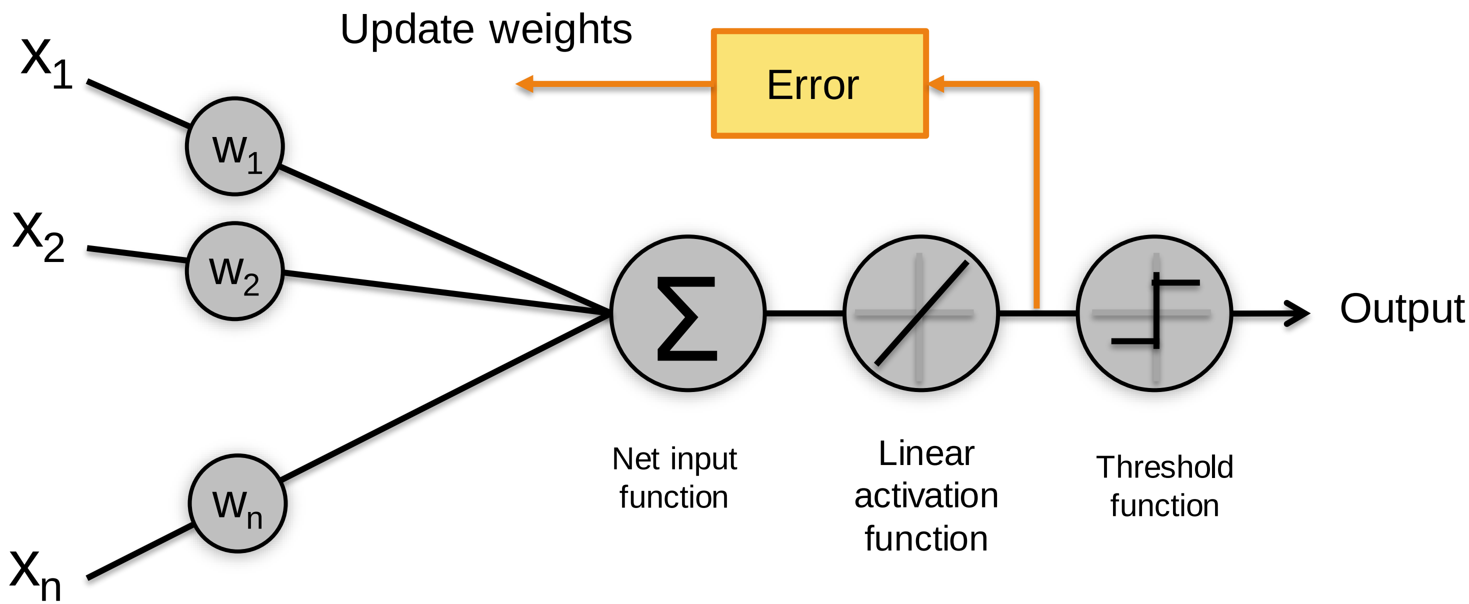


$a = purelin(n)$

Linear Transfer Function



Adaptive Linear Neuron



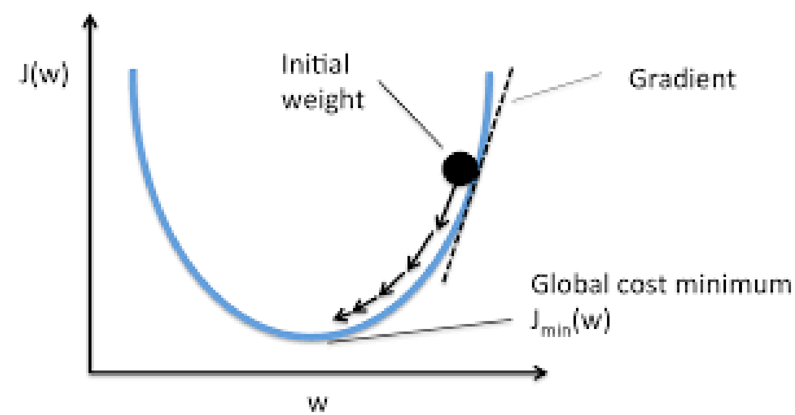
$$J(w) = \frac{1}{2}\sum_i (t^{(i)} - y^{(i)})^2$$

$$w = w + \Delta w$$

$$\Delta w = -\eta \nabla J(w) = -\eta \frac{\partial J}{\partial w}$$

$$\Delta w_j = -\eta \frac{\partial J}{\partial w_j} = -\eta \frac{\partial J}{\partial y}\frac{\partial y}{\partial w_j}$$

$$= -\eta\left(-\sum_i \left(t^{(i)} - y^{(i)}\right)\right) x_j^{(i)} = \eta\left(\sum_i \left(t^{(i)} - y^{(i)}\right)\right) x_j^{(i)}$$

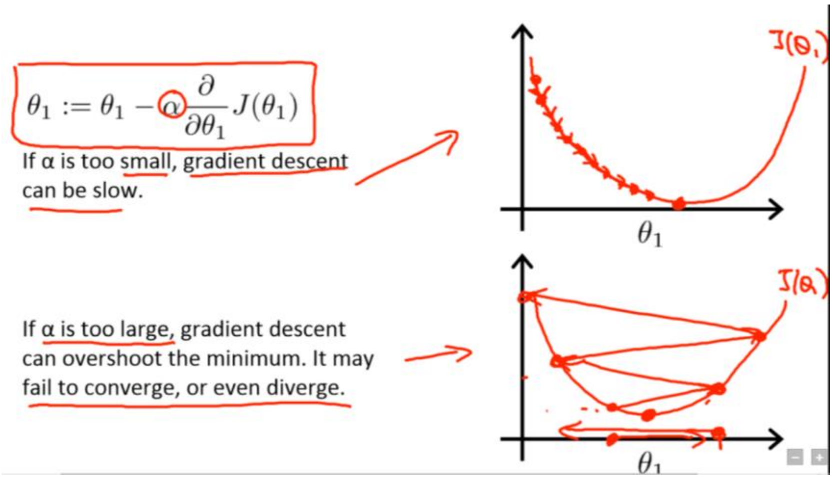


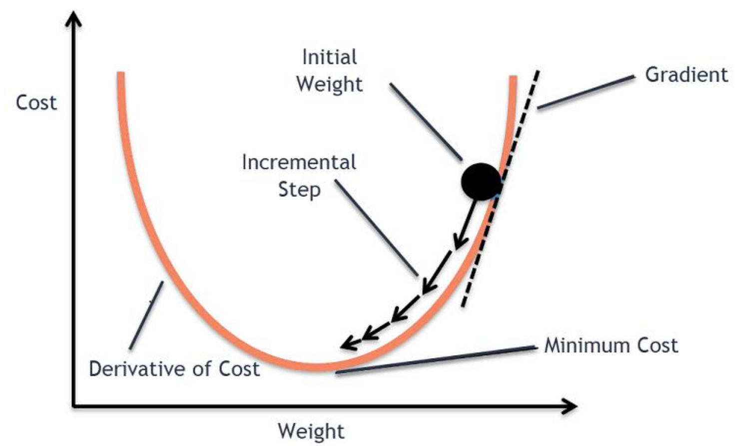
Notes:

- In a perceptron we update weights after each individual training sample.
- Here, you can still calculate the gradient and update the weights after each individual training sample (stochastic gradient descent).
- You can also calculate the gradient and update the weights based on the whole training dataset (each epoch) [batch gradient descent).

Training mode

- Stochastic Gradient Descent (SGD) – online learning (example by example)
- Batch gradient (calculate the gradient based on the whole dataset)
- Mini Batch gradient (divide the data into mini-batches due to memory restrictions)
- Large learning rate may overshoot the global minimum and fail converge or even diverge





Implementation

https://colab.research.google.com/github/ibribr/DT8807/blob/master/Adaline_v2.ipynb

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from numpy import random
```

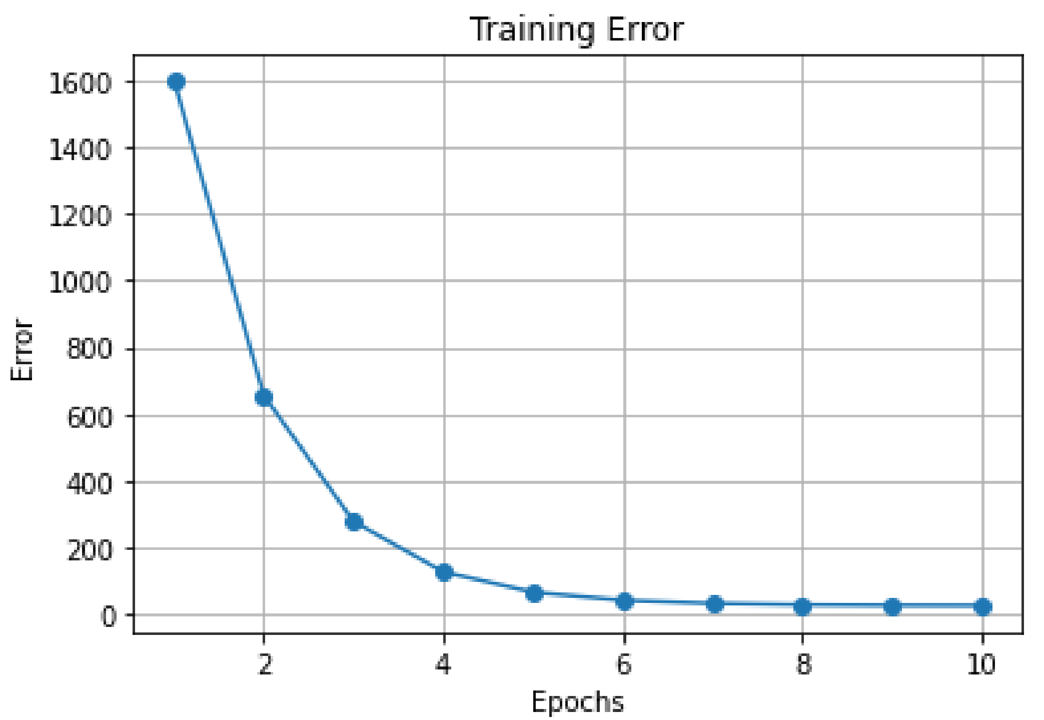
```
# Load IRIS data
df = pd.read_csv('https://archive.ics.uci.edu/ml/machine-learning-databases/iris/iris.data', header=None)
print(df.tail())
```

```
# extract first 100 class labels (50 iris-setosa and 50 iris-versicolor)
x = df.iloc[0:100, [0, 2]].values
y = df.iloc[0:100,4].values
y =np.where(y=='Iris-setosa',1, -1)
print(y)
print(len(x))
```

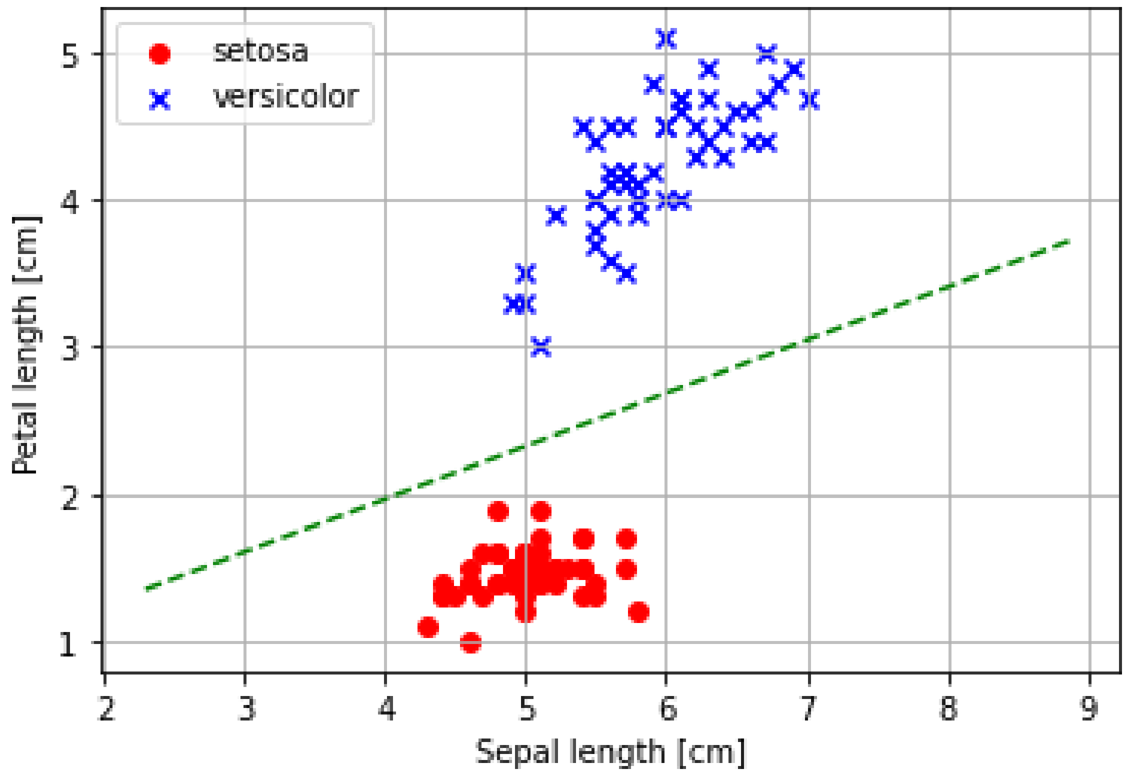
NTNU

```
class adaline(object):
  def __init__(self, epochs=100, eta=0.1):
    self.epochs = epochs
    self.eta = eta

  def train(self, training_inputs, training_labels):
    x = training_inputs
    t = training_labels
    self.cost = []   # to plot cost function over epochs
    self.w = random.rand(training_inputs.ndim+1)
    for i in range(self.epochs):
      net = np.dot(x, self.w[1:])+self.w[0]
      y = net  #linear activation function
      error = (t - y)   # this is vector
      #update weights using sum of gradients
      self.w[1:] += self.eta *  (np.dot(error, x)).mean()
      self.w[0] += self.eta * error.mean()
      cost = 0.5 * (error**2).sum()
      self.cost.append(cost)
    return self

  def predict(self, inputs):
    net=np.dot(inputs, self.w[1:])+self.w[0]
    return(np.where(net>=0, 1, -1))
```

```python
model=adaline(epochs=10, eta=0.0001)
model.train(x, y)
predictions = model.predict(x)

print(model.cost)
print(model.w)
print(predictions)

plt.plot(range(1,len(model.cost)+1), model.cost, marker='o')
plt.xlabel('Epochs')
plt.ylabel('Error')
plt.title('Training Error')
plt.grid()
plt.show()
```

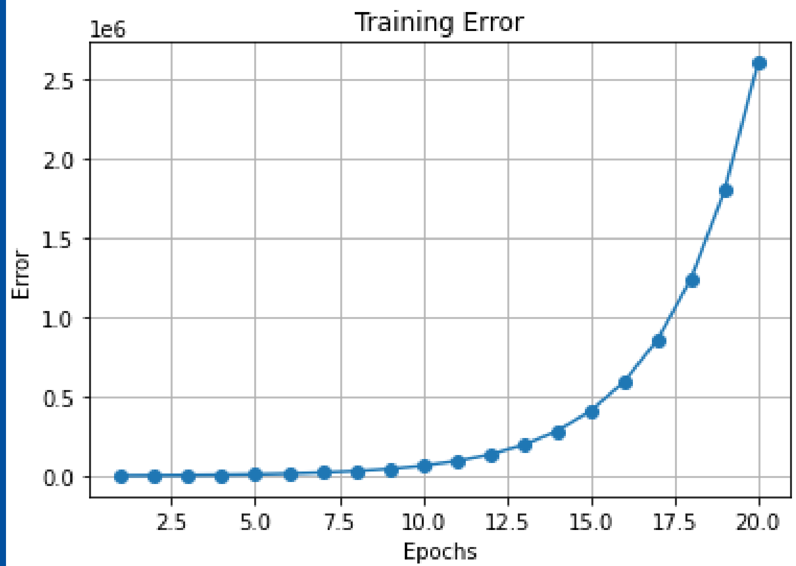


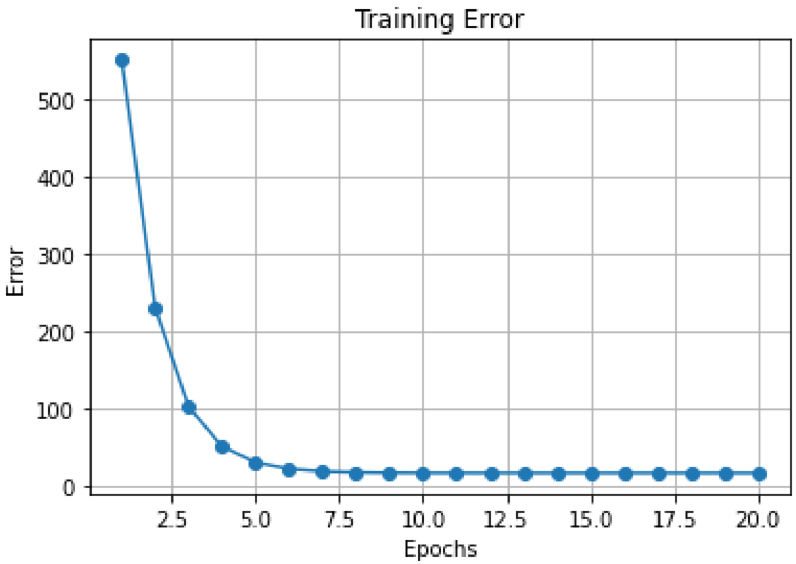
```python
# scatter plot of predictions
plt.scatter(x[predictions==1,0], x[predictions==1,1], color='red', marker='o', label='setosa')
plt.scatter(x[predictions==-1,0], x[predictions==-1,1], color='blue', marker='x', label='versicolor')
# decsion line
xx1 = np.arange(x[:,0].min()-2, x[:,0].max()+2, 0.1)
xx2 = -model.w[1]/model.w[2] * xx1 - model.w[0]/model.w[2]
plt.plot(xx1, xx2, 'g--')
plt.xlabel('Sepal length [cm]')
plt.ylabel('Petal length [cm]')
plt.legend(loc='upper left')
plt.grid()
plt.show()
```



Learning rate

- 0.0006 vs 0.0001



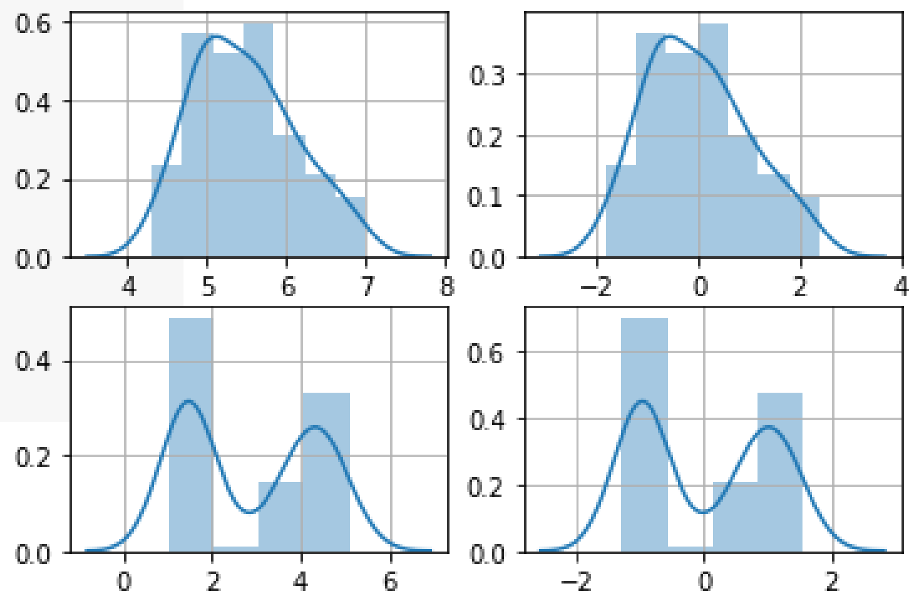


How to improve gradient descent through feature scaling?

- Standardization shifts the mean and variance of each feature so that it is centered at zero and has a feature standard deviation of 1 (i.e., standard normal distribution).

$$x_j = \frac{x_j - \mu_j}{\sigma_j}$$

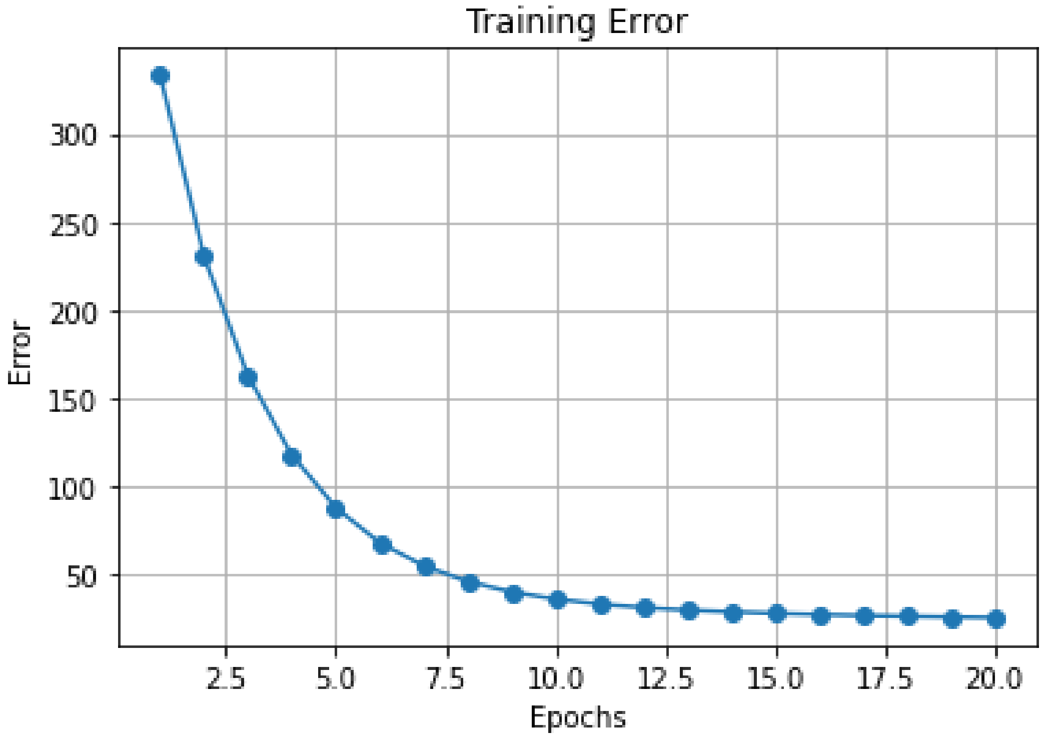
```
# visualization distribution of the feature inputs
xn = (x-x.mean(axis=0))/x.std(axis=0)
print(x.mean(axis=0))
print(x.std(axis=0))
#print([x, xn])
fig, axs = plt.subplots(2, 2)
sns.distplot(x[:,0],ax=axs[0,0]);
axs[0,0].grid()
sns.distplot(xn[:,0],ax=axs[0,1]);
axs[0,1].grid()
sns.distplot(x[:,1],ax=axs[1,0]);
axs[1,0].grid()
sns.distplot(xn[:,1],ax=axs[1,1]);
axs[1,1].grid()
```



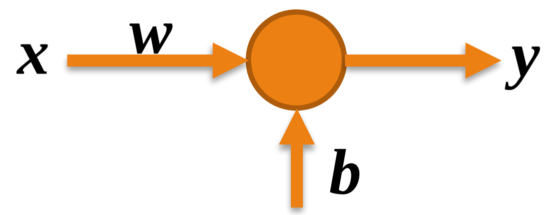
https://colab.research.google.com/github/ibribr/DT8807/blob/master/Adaline_v2.ipynb

Train Adaline with scaled features

```
# test training with scalled features
model1=adaline(epochs=20, eta=0.01)
model1.train(xn, y)
plt.plot(range(1,len(model1.cost)+1), model1.cost, marker='o')
plt.xlabel('Epochs')
plt.ylabel('Error')
plt.title('Training Error')
plt.grid()
plt.show()
```

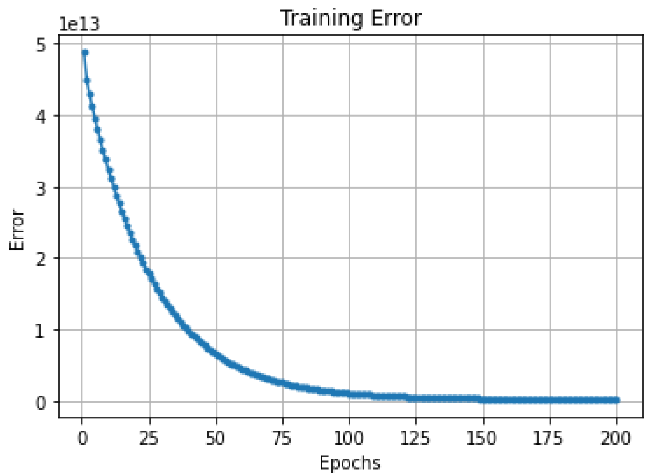


Use Adaline to predict annual salary in terms of number of years of experience

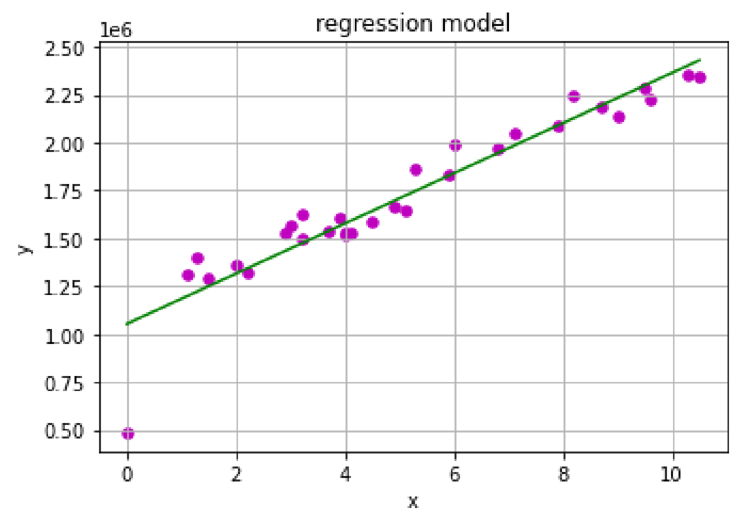


- Annual salary dataset

$$y = wx + b$$



	w	b
Adaline	380044.39	1697073.26
regression	131008.22	1053819.82

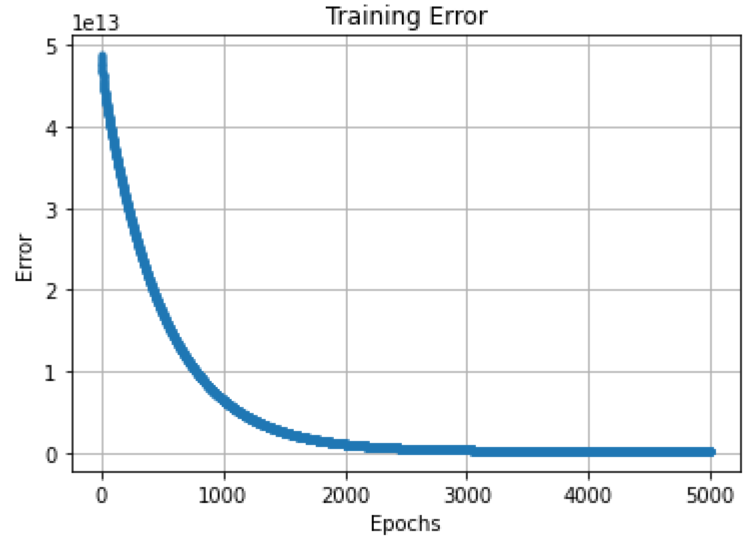


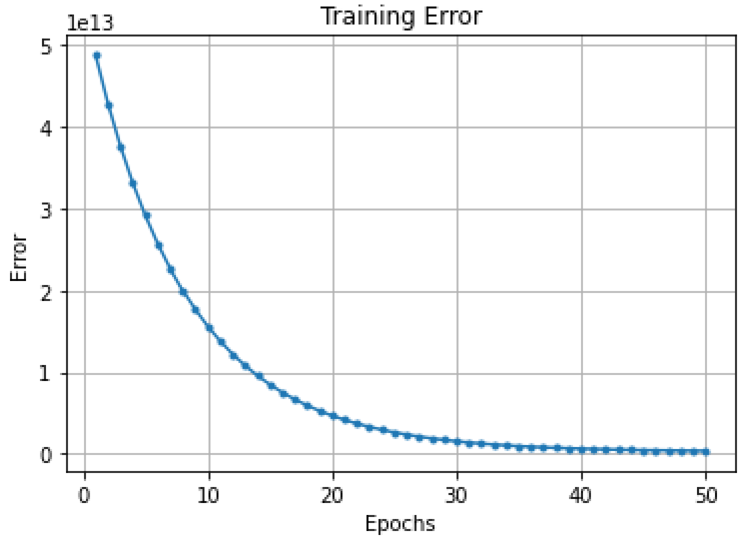
https://github.com/ibribr/ML/blob/master/adaline_regression_salary_dataset.ipynb

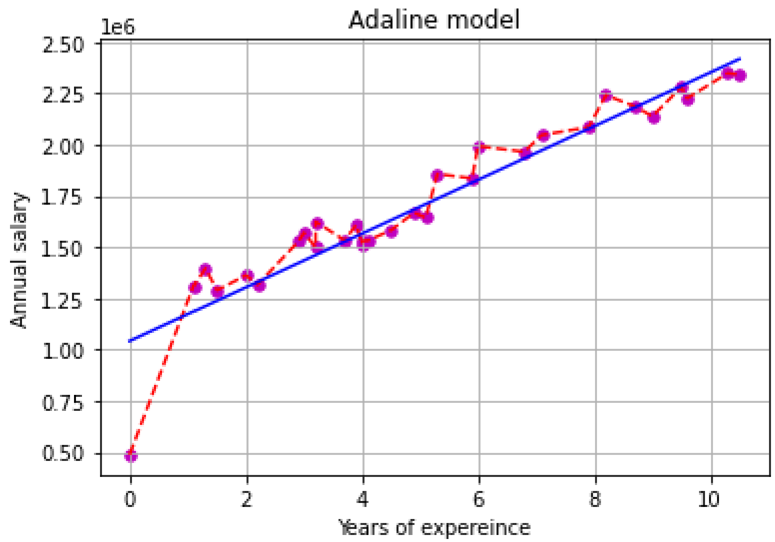
Learning rate:

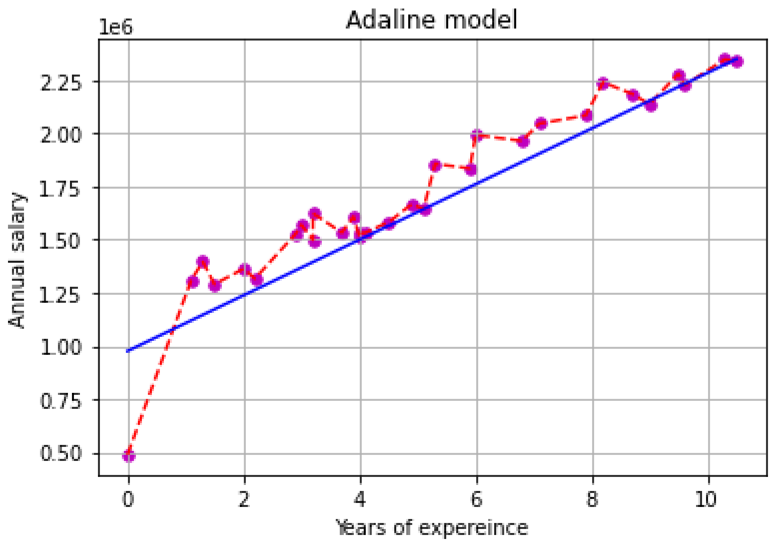
0.001

0.06







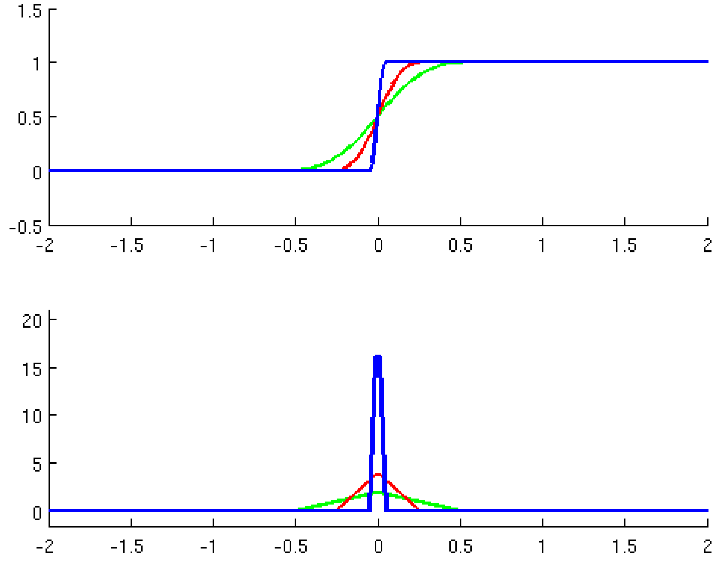


Activation functions

- The Activation Functions can be basically divided into 2 types:
  1. Linear Activation Function
  2. Non-linear Activation Functions

Unit step (hardlim/symmetric hardlim) activation function

- Used for binary classification
- Derivative of step is delta (i.e., impulse) function



$$h(t) = \begin{cases} 1 & t \geq 0 \\ 0 & otherwise \end{cases}$$

$$\delta(t) = \frac{dh(t)}{d(t)} \quad or \quad \int_{-\infty}^{\infty} \delta(t)dt = \int_{-\infty}^{\infty} dh(t) = h(t)\Big|_{-\infty}^{\infty} = h(\infty) - h(-\infty) = 1 - 0 = 1 \ \ (or\ \ 2)$$

Sigmoid

- It is used as a squashing function that limits the output to a range between 0 and 1.
- It predicts probability of a binary classification problem.

$$sig(x) = \frac{1}{1+e^{-x}} = \frac{e^{x}}{e^{x}+1}$$

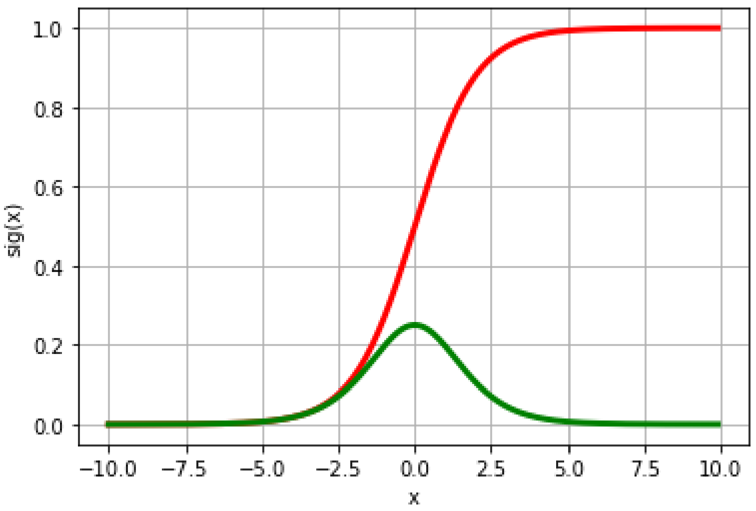
$$\frac{ds(x)}{dx} = \frac{d}{dx}\left(1+e^{-x}\right)^{-2}$$

$$= -\left(1+e^{-x}\right)^{-2} \times \frac{d}{dx}\left(1+e^{-x}\right)$$

$$= \frac{e^{-x}}{(1+e^{-x})^{2}} = \frac{1+e^{-x}-1}{(1+e^{-x})^{2}}$$

$$= \frac{1}{1+e^{-x}} - \frac{1}{(1+e^{-x})^{2}}$$

$$= s(x) - s^{2}(x) = sig(x)(1-sig(x))$$



Softmax

- Softmax is known as normalized exponential function
- Softmax function calculates the probabilities distribution of the event over 'n' different events (i.e., it calculates the probabilities of each target class over all possible target classes)
- The range will be from 0 to 1, and the sum of all the probabilities will sum to one.
- If the softmax function used for multi-classification model it returns the probabilities of each class where the target class will have the highest probability.
- Softmax is a multivariable function, generally.
- You wouldn't take a softmax of a single variable just like you wouldn't take a maximum of a single variable.

$$soft(x_i) = \frac{e^{x_i}}{\sum_{j=1}^{k} e^{x_j}} \forall i = 1, 2, \ldots, k$$

```
#softmax: normalized exponential function
# it  Calculate the softmax for the give inputs (array)
def softmax(x):
  return(np.exp(x)/np.exp(x).sum())

x = [2,3,5,6]
print(softmax(x))
print(softmax(x).sum())
```

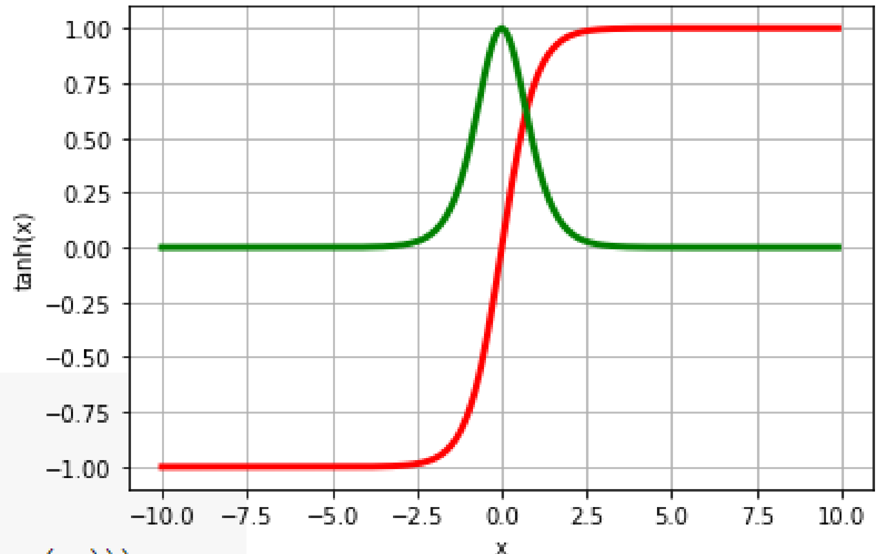
```
[0.01275478 0.03467109 0.25618664 0.69638749]
1.0
```

Hyperbolic tangent function: $\tanh(x)$

- The range of the tanh function is from (-1 to 1).
- tanh is also sigmoidal (s - shaped) symmetric function.

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\frac{d}{dx}\tanh(x) = 1 - \tanh^2(x)$$




```python
# tanh(x)
x = np.arange(-10,10,0.1)

def tanh(x):
  return((np.exp(x)-np.exp(-x))/(np.exp(x)+np.exp(-x)))

def tanh_derivative(x):
  return(1-tanh(x)**2)

plt.plot(x,tanh(x),'r-', linewidth=3.0)
plt.plot(x,tanh_derivative(x),'g-', linewidth=3.0)
plt.xlabel('x')
plt.ylabel('tanh(x)')
plt.grid()
plt.show()
```

NTNU

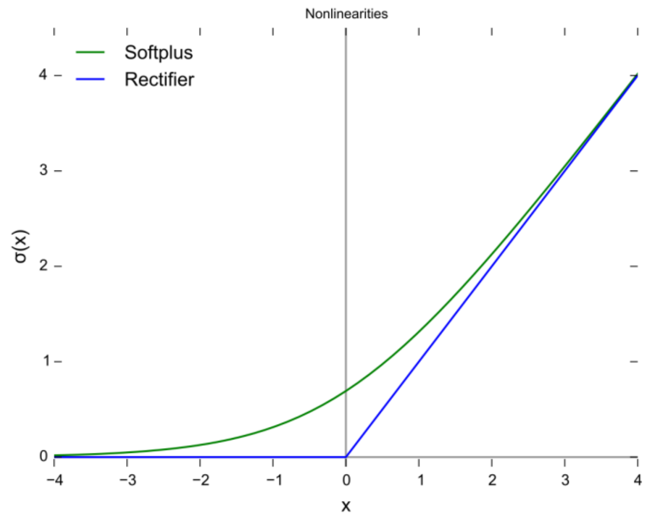
Rectified linear unit: ReLU

- The ReLU is the most used activation function in the world right now in almost all the convolutional neural networks or deep learning.
- Has a range from 0 to infinity
- ReLU activation function turns the negative input values into zero immediately in the graph, which in turns affects the resulting graph by not mapping the negative values appropriately
- Softplus is a smooth approximation to the rectifier.
- First derivative of ReLu is a step.
- First derivative of a softplus is a sigmoid.

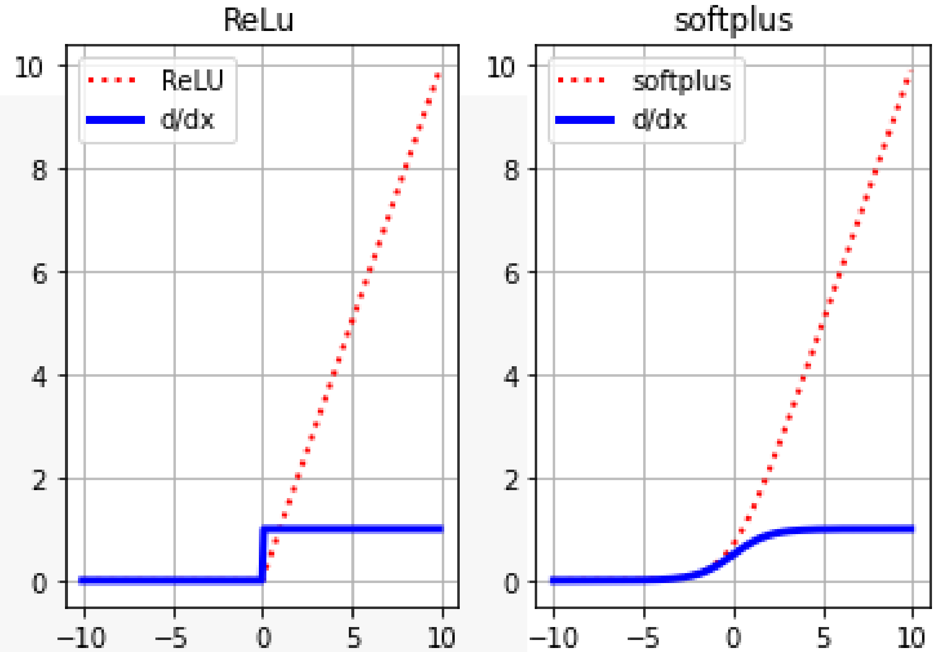
$$R(x) = \max(0, x)$$

$$sp(x) = \ln(1+e^{x}) \ \ or \ \ \frac{\ln(1+e^{kx})}{k}$$

$$sp'(x) = \frac{1}{1+e^{-kx}}$$



How to implement ReLU and Softplus activation functions?




```python
# ReLU: Rectifier
x = np.arange(-10,10,0.1)

def relu_(x):
  return(np.maximum(x,0))

def softplus_(x):
  return(np.log((1+np.exp(x))))

def relu_derivative(x):
  return(np.where(x>=0,1,0))

def softplus_derivative(x):
  return(1/(1+np.exp(-x)))

plt.subplot(1, 2, 1)
plt.plot(x, relu_(x),'r:', linewidth=2.0, label='ReLU')
plt.plot(x, relu_derivative(x),'b-', linewidth = 3.0, label='d/dx')
plt.legend(loc='upper left')
plt.grid()
plt.title('ReLu')

plt.subplot(1, 2, 2)
plt.plot(x, softplus_(x), 'r:', linewidth=2.0, label='softplus')
plt.plot(x, softplus_derivative(x),'b-',linewidth = 3.0, label='d/dx')
plt.legend(loc='upper left')
plt.grid()
plt.show()
```

Why differentiation/derivative?

- The main use for differentiation is to find the gradient of a function at any point on the graph.
- When updating the curve, to know in which direction and how much to change or update the curve depending upon the slope.

$$y = x^2 - 2x - 3$$

$$\frac{dy}{dx} = 2x - 2$$

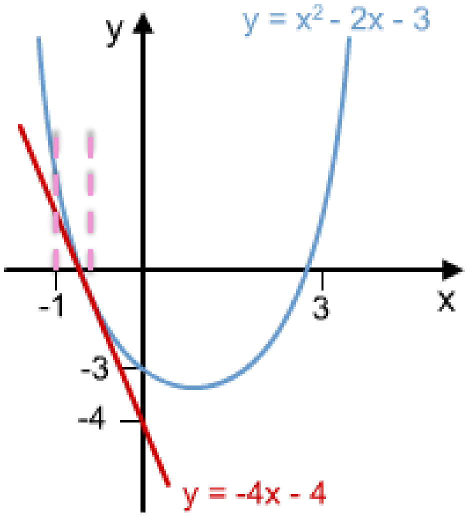
$$For\ \ x = -1 \rightarrow \frac{dy}{dx} = -2 - 2 = -4\ (slope)$$

-ve gradient means moving in the direction of increasing x

$$\rightarrow and\ \ y = (-1)^2 - 2(-1) - 3 = 0$$

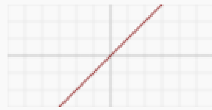
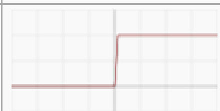
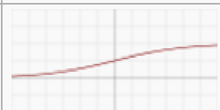
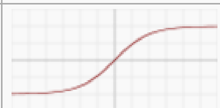
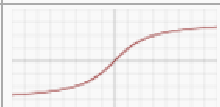
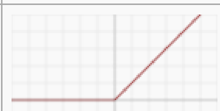
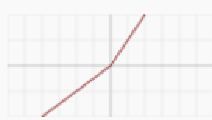
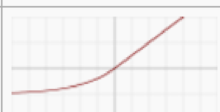
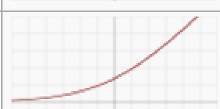
Equation of the tangent:

$$y - y_1 = m(x - x_1)$$
$$y - 0 = -4(x - (-1)) \Rightarrow y = -4x - 4$$

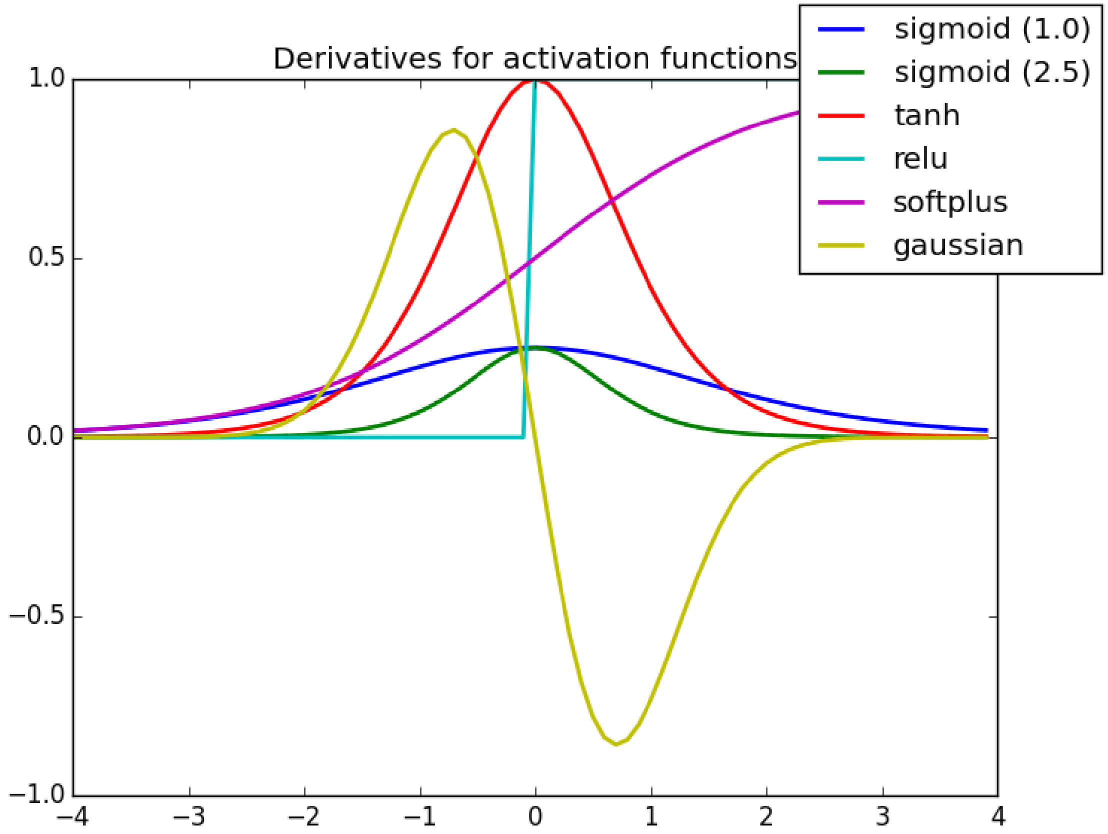


$$\Delta x = -\eta \frac{dy}{dx}$$
$$= -\eta(2x - 2) = 4\eta$$
$$x = x + \Delta x = x + 4\eta$$
$$= -1 + 4(0.1) = -0.6$$

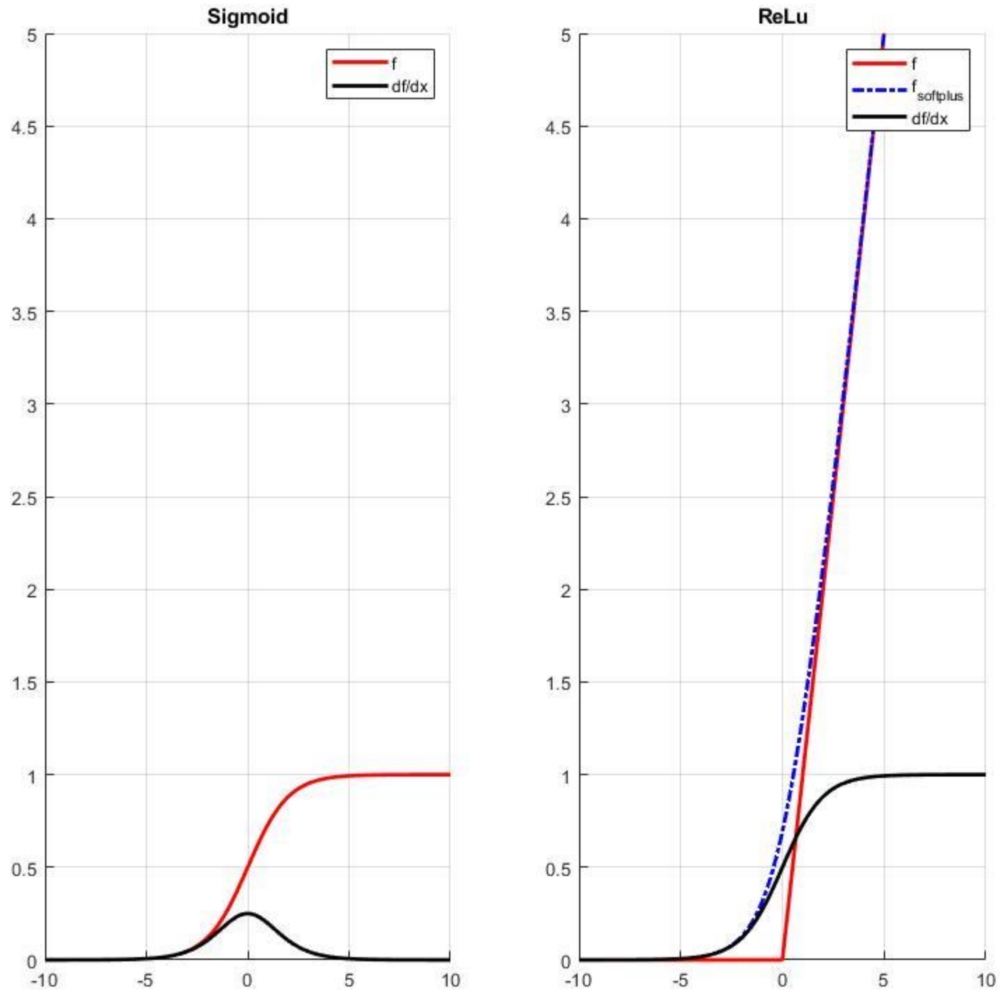
Derivatives & differentiations

Name	Plot	Equation	Derivative
Identity		$f(x) = x$	$f'(x) = 1$
Binary step		$f(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ 1 & \text{for} \quad x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for} \quad x \neq 0 \\ ? & \text{for} \quad x = 0 \end{cases}$
Logistic (a.k.a Soft step)		$f(x) = \frac{1}{1 + e^{-x}}$	$f'(x) = f(x)(1 - f(x))$
TanH		$f(x) = \tanh(x) = \frac{2}{1 + e^{-2x}} - 1$	$f'(x) = 1 - f(x)^2$
ArcTan		$f(x) = \tan^{-1}(x)$	$f'(x) = \frac{1}{x^2 + 1}$
Rectified Linear Unit (ReLU)		$f(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ x & \text{for} \quad x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ 1 & \text{for} \quad x \geq 0 \end{cases}$
Parameteric Rectified Linear Unit (PReLU) [2]		$f(x) = \begin{cases} \alpha x & \text{for} \quad x < 0 \\ x & \text{for} \quad x \geq 0 \end{cases}$	$f'(x) = \begin{cases} \alpha & \text{for} \quad x < 0 \\ 1 & \text{for} \quad x \geq 0 \end{cases}$
Exponential Linear Unit (ELU) [3]		$f(x) = \begin{cases} \alpha(e^x - 1) & \text{for} \quad x < 0 \\ x & \text{for} \quad x \geq 0 \end{cases}$	$f'(x) = \begin{cases} f(x) + \alpha & \text{for} \quad x < 0 \\ 1 & \text{for} \quad x \geq 0 \end{cases}$
SoftPlus		$f(x) = \log_e(1 + e^x)$	$f'(x) = \frac{1}{1 + e^{-x}}$

Derivatives of activation functions



ReLU & vanishing gradient problem



https://github.com/ibribr/DT8807/blob/master/activation_functions.ipynb

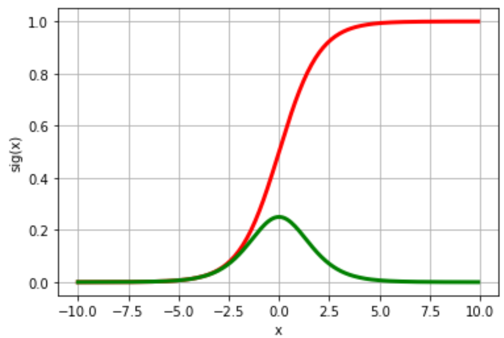
Vanishing gradient (VG) problem

- VG problem is encountered when training artificial neural networks with gradient-based learning methods and backpropagation.
- It describes the situation where a deep multilayer feedforward neural network or a recurrent neural network is unable to propagate useful gradient information from the output end of the model back to the layers near the input and hence the inability of the model with many layers to learn on a given dataset or to prematurely converge to a poor solution.
- In gradient-based learning, each of the neural network's weights receive an update proportional to the partial derivative of the error function with respect to the current weight in each iteration of training.

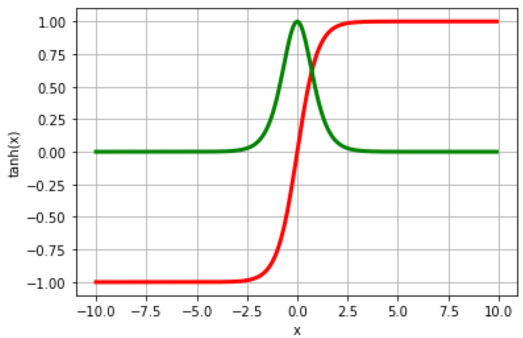
$$For \quad \frac{\partial E}{\partial w} \simeq 0 \Rightarrow \Delta w = -\eta \frac{\partial E}{\partial w} \simeq 0$$

Vanishing gradient (VG) problem

- In some cases, the gradient will be vanishingly small, effectively preventing the weight from changing its value. In the worst case, this may completely stop the neural network from further training.
- As one example of the problem cause, traditional activation functions such as the hyperbolic tangent function have gradients in the range (0, 1), and backpropagation computes gradients by the chain rule. This has the effect of multiplying n of these small numbers to compute gradients of the "front" layers in an n-layered network, meaning that the gradient (error signal) decreases exponentially with n while the front layers train very slowly or not train at all.
- When weights of the front layer do not train/change, front layer transfers random inputs to the proceeding layers.
- ReLu is used to improve the flow of gradients through the model.



Sigmoid



Hyperbolic tangent

Training/learning modes

- **Batch gradient descent (old man moving carefully and wisely):**
  - Minimize the cost function by taking a step in the opposite direction of the cost gradient from the whole training set.
  - Very computationally expensive specially for large datasets with millions of data points where we need to revaluate the whole training dataset each time we take one step towards the global minimum.
  - Update once each epoch

BGD	$\Delta w = \frac{\eta}{n} \sum_{i=1}^{n} (t^{(i)} - y^{(i)}) x^{(i)}$
SGD	$\Delta w = \eta (t^{(i)} - y^{(i)}) x^{(i)}$ $\quad \eta = \frac{c_1}{[\textit{number of iterations}] + c_2}$

Training/learning modes

- **Stochastic gradient descent (SGD) – iterative or online gradient descent (a drunk man moving down a hill almost in instability and powerlessness):**
  - Instead of updating the weights based on the sum of the accumulated errors over all samples, we updated the weights incrementally for each training sample (n updates each epoch).
  - It reaches convergence much faster because of the more frequent updates.
  - Gradient is based on a single training example and therefore the error surface is noisier and can escape shallow local minima.
  - To obtain satisfying results via SGD, it is important to shuffle the training set for every epoch to prevent cycles.
  - In SGD, learning rate η is often replaced by an adaptive learning rate that decreases over time (c_1 and c_2 are constants).
  - Can be used for online learning where our model is trained on the fly as new training data arrives. In online learning, the model can immediately adapt to changes and the training data can be discarded after updating the model if storage space is an issue.

BGD

$$\Delta w = \frac{\eta}{n}\sum_{i=1}^{n}(t^{(i)} - y^{(i)})x^{(i)}$$

SGD

$$\Delta w = \eta(t^{(i)} - y^{(i)})x^{(i)} \qquad \eta = \frac{c_1}{[number\ of\ iterations] + c_2}$$

Training/learning modes

- **Mini-batch learning (a young man actively moving down a hill):**
  - Is understood as applying batch gradient descent to smaller subsets of the training data, for example, 32 samples at a time.
  - The advantage over batch gradient descent is that convergence is reached faster via mini-batches because of the more frequent weight updates.
  - Update by (dataset size)/(mini-batches size) each epoch.

Home assignment

- **Go to:** https://github.com/ibribr/ML/blob/master/Adaline_v2.ipynb
  - Batch gradient is applied
  - Repeat it using SGD and mini-batch gradient
  - Compare results in terms of number of epochs, time, memory use, etc.

BGD vs SGD

- Check implementation at
- https://github.com/ibribr/DT8807/blob/master/Adaline_BGD_vs_SGD.ipynb

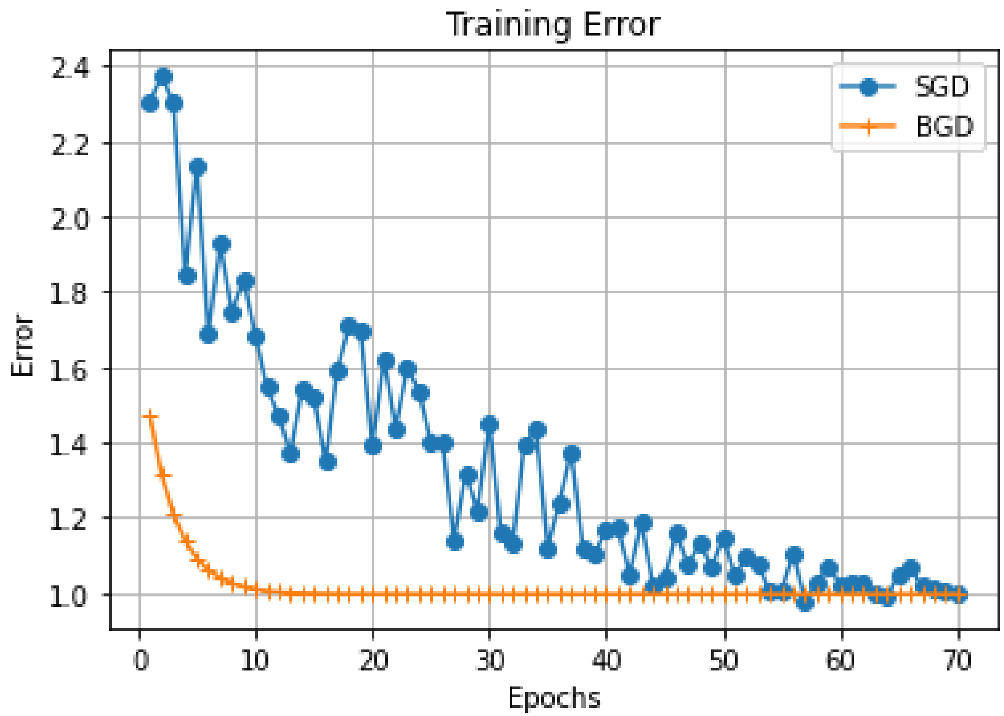
```
class adaline(object):
  def __init__(self, epochs=100, eta=0.1):
    self.epochs = epochs
    self.eta = eta

  def train(self, training_inputs, training_labels, mode='BGD'):
    x = training_inputs
    t = training_labels
    self.cost = []   # to plot cost function over epochs
    self.w = random.rand(training_inputs.ndim+1)
    for i in range(self.epochs):
      if mode=='BGD':
        net = np.dot(x, self.w[1:])+self.w[0]
        y = net  #linear activation function
        error = (t - y)   # this is vector
        #update weights using sum of gradients
        self.w[1:] += self.eta *  (np.dot(error, x)).mean()
        self.w[0] += self.eta * error.mean()
        cost = 0.5 * (error**2).sum()
        self.cost.append(cost)
      elif mode=='SGD':
        cost = 0
        for j in range(len(x)):
          net = np.dot(x[j,:],self.w[1:])+self.w[0]
          y = net
          error = (t[j]-y)
          #update the weights using SGD
          self.w[1:] += self.eta * np.dot(error, x[j,:])
          self.w[0] += self.eta * error
          cost += error**2;
        self.cost.append(cost/len(x))
    return self

  def predict(self, inputs):
    net=np.dot(inputs, self.w[1:])+self.w[0]
    return(np.where(net>=0, 1, -1))
```

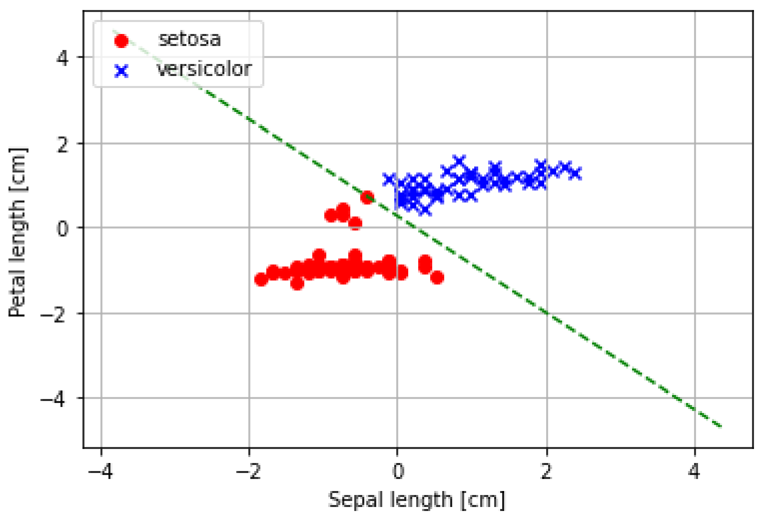
BGD

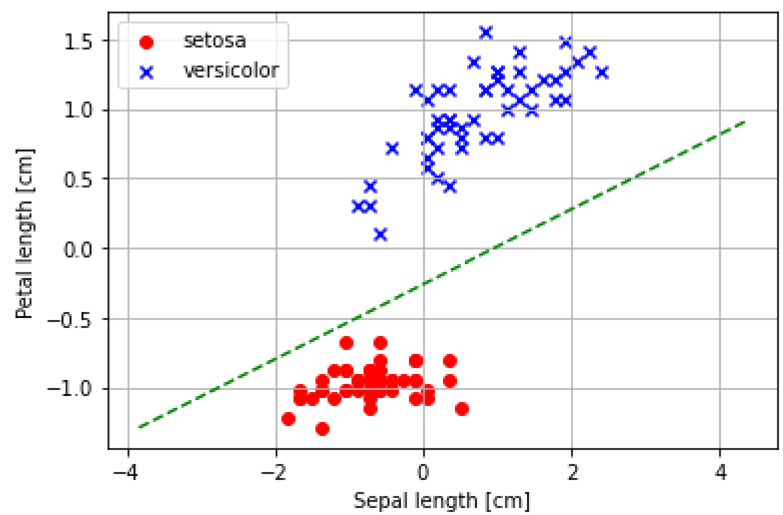
- Smooth error surface
- Requires more epochs
- Computationally expensive in terms of memory need

SGD

- Noisy error surface
- Fast convergence
- Online learning







Home assignment

- Plot the decision plane in 3D
- Implement mini-BGD.
- Make comparison between BGD, MBGD and SGD in terms of:
  - Running time
  - Memory use
  - Accuracy
  - Computational costs

	time	memory	accuracy	…
BGD				
MBGD				
SGD				