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Review

Current development on using Rotary Inverted Pendulum as a benchmark for testing linear and nonlinear control algorithms

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ABSTRACT

Rotary Inverted Pendulum (RIP) is an under-actuated mechanical system which is inherently nonlinear and unstable. For decades, it has been widely used as an experimental setup to explain and test different kinds of control algorithms. The main control objectives of RIP are: Swing-up control, stabilization control, switching control and trajectory tracking control. All these control objectives are described in this study. State-of-the art works proposed for each control objective have also been reviewed. These comprise the linear, nonlinear time invariant, self-learning and adaptive nonlinear controllers. Moreover, different kinds of nonlinear dynamic models of the RIP together with the developed linear models in the literature have been analyzed. This is because most of the proposed controllers applied on RIP are found to be model dependent since they are mainly based on integral and/or invariant motion. Other types of RIP are also reported along with their advantages. Future research opportunities and challenges of the previous approaches in this area of research are presented. We believe that expert researchers can use this paper as starting point for further advancement while graduate scholars can use it as an initial point.

Contents

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1. Introduction

The initial motivation of studies of inverted pendulum (IP) arose based on the need to design balance controllers for rockets during vertical take-off. The rocket is highly unstable at the instant of launching. Thus, there is a need for a continuous alteration mechanism to stay at upright position in the open loop configuration [1]. IP is an important member of nonlinear unstable under-actuated mechanical systems. It is a suitable benchmark system that can be used for training and experimental validation of new control strategies in robotics and control theory. The most common types of IP are the single arm Rotary Inverted Pendulum (RIP), the cart IP and the double IP. The less familiar versions of IP are the two-link RIP, the triple IP, the parallel type dual IP, the 3D or spherical pendulum and the quadruple IP. RIP is one of the most available versions of IP that can be found in most control laboratories. This paper focuses on the RIP which inherits under-actuated, unstable, nonlinear and non-minimum phase system dynamics [2]. RIP was proposed in 1992 [3], and since then it has been investigated by many researchers [4–6]. The experimental setup of the RIP produced by Quanser is shown in Fig. 1(a) and (b). Other companies that are producing IP or RIP include Aimil, sisgeo. This setup consists of two optical encoders for measuring the angles of the pendulum and arm, respectively. It also comprises the data acquisition device for collecting the information from the encoders and feeding it to the computer. The data acquisition device receives the control signal from the computer and gives it to the power amplifier for amplification before feeding it to the motor.

The RIP systems perform in an extensive range of real life applications, such as aerospace systems, robotics, marine systems, mobile systems, flexible systems, pointing control, and locomotive systems [7]. In addition, the study of dynamic model and control algorithms in controlling the RIP plays an important role in controlling spacecraft and rockets, maintaining the equilibrium state for two legs robots and skyscraping buildings [8]. Moreover, when the pendulum of RIP is at hanging position, it represents real model of the simplified industry crane application [9].

The control objectives of the RIP can be categorized into four [10]: 1) Controlling the pendulum from downward stable position to upward unstable position, i.e., swing-up control [11]; 2) Regulating the pendulum to remain at the unstable position, i.e., stabilization control [12]; 3) Switching between swing-up control and stabilization control, i.e., switching control [13]; and 4) Controlling the RIP in such a way that the arm tracks a desired time varying trajectory while the pendulum

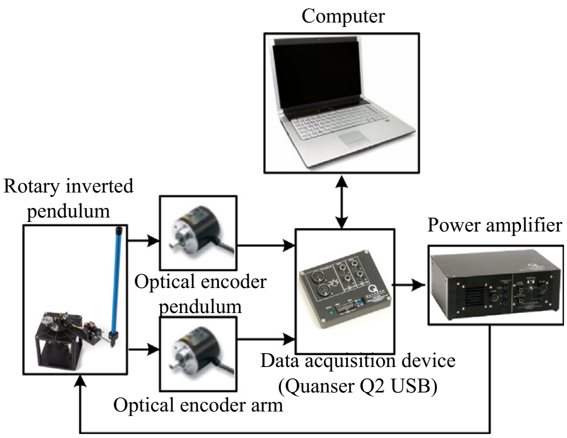

Fig. 1. Schematic diagram of the experimental setup of the RIP.

remains at unstable position, i.e., trajectory tracking control [5]. In this context, different kinds of control algorithms have been applied for these control objectives.

Other control problems associated with RIP that were investigated by researchers include the friction compensation [14,15], system identification [16], synchronization problem [7,17], decoupling of RIP to eliminate the under actuated problem [12,18] and stability analysis [19].

Despite the importance of RIP and the increasing number of research that uses it as a benchmark, only one review has been conducted on RIP. Acosta [4] studied RIP as a conventional nonlinear system for theory and practise of control strategies. He considered two control objectives of RIP [4], namely, swing up and stabilization controls. The autonomous oscillations are also studied. The model of RIP described in Ref. [4] is based on Newton-Euler Lagrange. The publications considered in Ref. [4] were up to the year 2009. However, from 2009 to date, there are many control investigations that used RIP for testing. There is also a need to include the trajectory tracking control. In addition, system identification, synchronization problem, decoupling of RIP to eliminate the under actuated problem and stability analysis need to be discussed. The overall historical picture and trend developments in nonlinear control strategies, based on different types of inverted pendulum are reviewed by Boubaker [20].

This paper describes all the four control objectives of RIP and also provide a review of the recent (i.e. from the year 2011 to date), state-of-the art work proposed for each control objective. Moreover, other types of RIP were reported along with their possible advantages. Future research work was proposed, and limitations of the previous approaches in this area of research were presented. The aim of this study is to summarize the state-of-the-art literature, point out unresolved problems, and then suggest future research. This is to prevent repetition of the study that has been conducted.

2. A review on the mathematical modeling of RIP

Most of the studies have used either Euler-Lagrange method [1,5,7,13–15,21–38] or Newton's laws of motion method [10–12,39–46] for the development of their dynamic model of RIP. Detailed dynamic equations of RIP were developed based on Lagrangian tensors formulation and iterative Newton-Euler formulation in Ref. [47]. Port-Hamiltonian representation of the RIP was done in Ref. [44]. The general model of RIP with arbitrary number of pendulums was developed in Ref. [31]. A dynamic model of RIP with dead zone was developed in Ref. [48]. The RIP dynamic model in Ref. [48] was rewritten as robotic dynamic model such that it can guarantee the stability. Rodriguez method for modelling was used in Refs. [49,50] for dynamic model. Ryalat and Laila [51] developed a technique that simplifies the partial differential equations related to the potential energy for interconnection and damping assignment passivity based control of under-actuated mechanical systems. This method avoids the elimination of nonlinearities and ease the difficulties in application of non-separable and separable port-controlled Hamiltonian systems.

The advantages of Newton-Euler, Lagrage-Euler and Lagrange methods are: for simple systems these methods are computationally efficient, quite intuitive and give a normal extension from quasi-static analysis. The major drawback of these methods is that for complex configurations, a significant effort is required to obtain a minimal (reduced) set of equations.

Some researchers argued that, in practice, it is difficult to obtain exact physical mathematical model of complex systems [52]. Thus, some other ways of finding the systems model has been tested on RIP. For example, the Takagi Sugeno Kang (TSK) fuzzy model was employed in Ref. [2] to find the nonlinear model of RIP. Subsequently, the nonlinear model is transformed into local linear time-invariant model. In Ref. [53] a fault detection method for nonlinear RIP was developed based on TSK fuzzy model. The TSK descriptor with a reduced number of rules was used in Ref. [54] to simplify practical implementation of the controller. The fuzzy model of RIP in Ref. [55] was constructed based on a variable universe of discourse. Artificial Neural Network (ANN) based system identification approach was employed to determine the nonlinear mathematical model of RIP in Refs. [8,16]. Moreover, a Matlab Graphical User Interface (GUI) was developed in Ref. [56,57] for automatic derivation and evaluation of the mathematical model of RIP. The adaptive state estimator and output controller was developed in Ref. [58] using Attractive Ellipsoid Method for the stabilization of the RIP without the use of a mathematical model.

3. Mathematical modelling of RIP

Most of the proposed controllers applied for RIP control are model dependent since most of them are based on integral and/or invariant motion. So, it is essential to have the accurate mathematical model of RIP. In this context, many types of RIP have been developed together with their mathematical models [25,31,59]. Newton-Euler, Lagrange-Euler and Lagrange methods were used to develop the nonlinear mathematical model of RIP [23,59,60]. Lagrange multiplier was used in the derivation using Lagrange method, while the Newton method involved the calculation of redundant forces [61,62]. As a result, Newton-Euler, Lagrange-Euler and Lagrange methods require complex formulation for a large multi-body system [63]. Consequently, they could lead to an inefficient computation. Kane's method [64] can be regarded as an alternative method of modelling. This method does not require the calculation of multipliers or redundant forces. It is based on the partial velocities of the constituents of the system [65]. Hence, Kane's method is likely to be more efficient than Lagrange and Newton-Euler methods in terms of computation. On the other hand, it should be stated that most literatures simplify the generated model by assuming that the pendulum is rotating in a constant plane. But in actual sense, the rotary motion of the arm should be considered together with the pendulum. The frictional force effect in the pendulum and arm joint are

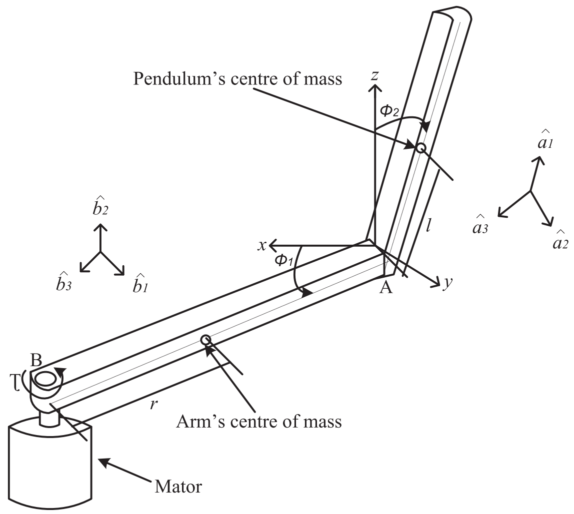

Fig. 2. Free body diagram of RIP.

normally neglected in previous works. However, the effect of friction in RIP is clearly visible. It has been shown in Ref. [14] that the friction in the driven arm might cause LCs with high amplitude. This can have a significant impact on the performance of the proposed controllers.

The RIP consists of two connected rigid bodies actuated by a servomotor system as shown in Fig. 2. These bodies are the rotational arm and the pendulum. Both the arm and pendulum have one degree of freedom (DOF). The arm is attached to the output gear of the motor, and it can rotate around the fixed point B. As shown in Ref. Fig. 2, the angle ϕ_1 is the generalized coordinate for arm, which is the angle between the arm and the horizontal x-axis (arm angle). The vectors $\hat{b}_1, \hat{b}_2, \hat{b}_3$ are the inertial earth fixed reference frame, in which $\hat{b}_3$ is away and perpendicular to the earth surface. The vectors $\hat{b}_r, \hat{b}_{\phi_1}, \hat{b}_3$ are the arm fixed reference in which $\bar{b}_r$ is away from fixed point B along the arm length. The pendulum is attached to the free end of the rotating arm and has its mass center at point C. The angle ϕ_2 is the generalized coordinate for the pendulum, which is the angle between the pendulum and the vertical z-axis (pendulum angle). The pendulum has two planes of symmetry through $\hat{a}_1$ axis with normal direction $\hat{a}_2$ and $\hat{a}_3$, and it can rotate in a plane perpendicular to $\hat{b}_r = \hat{b}_3$. The pendulum fixed references are $\hat{a}_1, \hat{a}_2, a_3$ in which $\hat{a}_1$ point is away from point A. The torque τ is applied at the fixed end of the arm by the motor and the direction of the torque depends on the direction of the voltage applied to the motor. The pendulum can move from *0* degree to *360* degrees about the arm axis in clockwise or anticlockwise directions.

Before designing a controller for a RIP, the equations that characterize the behavior of the RIP system have to be developed as correctly as possible, irrespective of the complexity of the equations. Normally, the nonlinear dynamic equation of a RIP is derived using the Euler-Lagrange method [25] or Newton's laws of motion method [11]. Usually the following five assumptions are made: 1) The system consists of two ideal rigid bodies (arm and pendulum), 2) the position of the whole system is fixed in a horizontal and flat surface, 3) the motor inductance and friction on the armature are neglected, 4) the equivalent frictional force of motor/arm is neglected, and 5) the pendulum is rotating in a constant plane. Subsequently, the nonlinear dynamic equations are linearized so as to simplify the design and analysis of the proposed controllers [4].

The mathematical model of RIP is derived using Kane's method in this study. This is based on the stated general assumptions. In addition, contrary to most literature in which the generated model is simplified by assuming that the pendulum is rotating in a constant plane [66,67]. The model in this study considered the rotary motion of the arms together. That is, it is assumed that the actual plane in which the pendulum is rotating is different at every instant. Also, equivalent frictional force of motor/arm is considered. This makes the developed model more complex, but with high accuracy. The detailed application of kane's method for developing the dynamic equation of control systems can found in Refs. [68,69].

3.1. Kinematics of RIP

There are two generalized speeds, defined as:

$$u_1 = \dot{\phi}_1 \tag{1}$$

and

$$u_2 = \dot{\phi}_2 \tag{2}$$

The pendulum angular velocity with respect to the earth fixed frame vectors is as follows [64]:

$$\bar{\omega}_{A/B} = u_1\hat{b}_3 + u_2\hat{a}_1 = u_1(\cos\phi_2\hat{a}_1 - \sin\phi_2\hat{a}_2) + \phi_2\hat{a}_3 \tag{3}$$

The position vector of pendulum mass center is:

$$\bar{r}_{C/B} = r\hat{b}_r + l\hat{a}_1 \tag{4}$$

Therefore, the linear velocity of pendulum's center of mass with respect to the earth fixed reference can be derived as [70]:

$$V_{C/B} = ru_1\hat{b}_{\phi_2} + \bar{\omega}_{A/B} \times l\hat{a}_1 = l\sin\phi_2(u_1\hat{b}_r - u_2\hat{b}_3) + (ru_1 - lu_2\cos\phi_2)\hat{b}_{\phi_1} \tag{5}$$

3.2. Dynamics of RIP

The pendulum fixed frame is associated with some major point directions on its axis. This is due to its symmetry. So, the angular momentum of the center of mass is:

$$\boldsymbol{H_C} = \bar{I}_{11}\omega_{\hat{a}_1}\hat{a}_1 + \bar{I}_{22}\omega_{\hat{a}_2}\hat{a}_2 + \bar{I}_{33}\omega_{\hat{a}_3}\hat{a}_3 \tag{6}$$

In which, the angular velocities $\omega_{\hat{a}1}, \omega_{\hat{a}_2}, \omega_{\hat{a}_3}$ are the coefficients of $\hat{a}_1, \hat{a}_2, \hat{a}_3$ in Eq. (3), respectively. Or simply

$$\boldsymbol{H_C} = \bar{I}_{11}u_1\cos\phi_2\hat{a}_1 - \bar{I}_{22}u_1\sin\phi_2\hat{a}_2 + \bar{I}_{33}\phi_2\hat{a}_3 \tag{7}$$

were $\bar{I}_{ii}$ is moment of inertia of mass centre.

For thin rod with $2l$ length, the following are proved [70]:

$$\bar{I}_{33} = \bar{I}_{22} = \frac{ml^2}{3}, \quad \text{and} \quad \bar{I}_{11} = 0 \tag{8}$$

Based on Euler's equation for center of mass and Newton's law, the external force $\boldsymbol{F}$ and the moment, $\boldsymbol{M}_c$, about the center of mass that act on the rigid body can be expressed as follows [18]:

$$\boldsymbol{F} = \frac{m\dot{V}_{C/B}}{3} \tag{9}$$

$$\boldsymbol{M}_c = \dot{\boldsymbol{H}}_C \tag{10}$$

This includes the sum of reaction force and gravitational force in the directions $\hat{b}_{\phi_1}$, $\hat{b}_r$ and $\hat{b}_3$ as well as the couple in the directions $\hat{a}_1$ and $\hat{a}_2$. Note, it is assumed that there is no friction in the pin. Therefore, there is no couple component in the $\hat{a}_3$ direction. The force equilibrium in Eq. (9) has the following components:

$$F_r(\hat{b}_r \text{ direction}) = ml\dot{u}_1\sin\phi_2 + 2mlu_1u_2\cos\phi_2 - mru_1^2 \tag{11}$$

$$F_{\phi_1}(\hat{b}_{\phi_1} \text{ direction}) = mr\dot{u}_1 - ml\dot{u}_2\cos\phi_2 + mlu_2^2\sin\phi_2 + mlu_1^2\sin\phi_2 \tag{12}$$

$$F_g(\hat{b}_3 \text{ direction}) = mg - ml\dot{u}_2\sin\phi_2 - mlu_2^2\cos\phi_2 \tag{13}$$

The components of the moment equilibrium for Eq. (10) are:

$$T_1(\hat{a}_1 \text{ direction}) = (\dot{u}_1\cos\phi_2 - u_1u_2\sin\phi_2)I_{11} + (I_{22} - I_{33})u_1u_2\sin\phi_2 \tag{14}$$

$$T_2(\hat{a}_2 \text{ direction}) = mrlu_1^2 - I_{22}\dot{u}_1\sin\phi_2 - (I_{22} + I_{33} - I_{11})u_1u_2\cos\phi_2 \tag{15}$$

$$T_3(\hat{a}_3 \text{ direction}) = ml(r\dot{u}_1\cos\phi_2 + g\sin\phi_2) - I_{33}\dot{u}_2 + (I_{22} - I_{11})u_1^2\cos\phi_2\sin\phi_2 \tag{16}$$

3.3. Pendulum dynamic equation

The equation for $\hat{a}_3$ direction is the pendulum equation of motion, and it can be expressed as follows:

$$ml(r\dot{u}_1\cos\phi_2 + g\sin\phi_2) = I_{33}\dot{u}_2 - (I_{22} - I_{11})u_1^2\cos\phi_2\sin\phi_2 \tag{17}$$

Eq. (17) can be written in the following form

$$\alpha u_1^2\cos\phi_2\sin\phi_2 + \beta\dot{u}_1\cos\phi_2 = \dot{u}_2 - \omega_n^2\sin\phi_2 \tag{18}$$

where $\alpha = \frac{I_{22}-I_{11}}{I_{33}}$, $\beta = \frac{mrl}{I_{33}}$ and $\omega_n^2 = \frac{mgl}{I_{33}}$ in which the parameter is approximately equal to one for thin rod, and ω_n is the natural frequency of the pendulum. Based on parallel axis theorem, $I_{33} = \bar{I}_{33} + ml^2$, $I_{22} = \bar{I}_{22} + ml^2$ and $I_{11} \equiv \bar{I}_{11}$.

The right hand side of Eq. (18) is the expression of standard pendulum on cart while left hand side is due to the additional rotation of the base on the circular arc.

3.4. Arm dynamic equation

The dynamic equation of the arm can be found from its $\hat{b}_3$ component of moment equilibrium about the fixed point B as follows:

$$\tau = \bar{b}u_1 + I_0\dot{u}_i + T_1\cos\phi_2 - T_2\sin\phi_2 + rF_{\phi_1} \tag{19}$$

where $\bar{b}u_1$ is the equivalent frictional force of motor/arm, I_0 is the transverse moment of inertia for arm at the fixed point B, τ is the torque applied to the arm by the motor. Substituting for T_1, T_2 and F_{ϕ_1} at their directions defined in Eqs. (14), (15) and (12) respectively, we have the dynamic equation for arm as follows:

$$\tau_0 = \left(\alpha\sin^2\phi_2 + I\right)\dot{u}_1 - \beta\dot{u}_2\cos\phi_2 + \beta u_2^2\sin\phi_2 + \alpha u_1u_2\sin 2\phi_2 + bu_1 \tag{20}$$

Where $\tau_0 = \frac{\tau}{I_{33}}$, $I = \frac{I_0+\bar{I}_{11}+mr^2}{I_{33}}$ and $b = \frac{\bar{b}}{I_{33}}$

The torque (τ) at the load gear is generated by servo motor, and it is described by Eq. (21) [10].

$$\tau = \frac{\eta_g K_g \eta_m k_t (V_m - K_g k_m \dot{\phi})}{R_m} \tag{21}$$

The nonlinear model of RIP in Eqs. (18) and (20) can be written in matrix form in Eq. (22) which can be used to find the nonlinear model in state space. The linear version is obtained using Taylor series expansion and is given in Eq. (24)

$$\begin{bmatrix} \ddot{\phi}_1 \\ \ddot{\phi}_2 \end{bmatrix} = \frac{1}{J}\begin{bmatrix} -\beta\cos\phi_2 & -1 \\ -(\alpha\sin^2\phi_2 + I) & -\beta\cos\phi_2 \end{bmatrix}\begin{bmatrix} \alpha u_1^2\cos\phi_2\sin\phi_2 + \omega_n^2\sin\phi_2 \\ -\alpha u_1u_2\sin 2\phi_2 - bu_1 - \beta u_2^2\sin\phi_2 + \tau_0 \end{bmatrix} \tag{22}$$

$$\text{Where } J = \beta^2 cos^2\phi_2 - \alpha\sin^2\phi_2 - I \tag{23}$$

$$\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \ddot{\phi}_1 \\ \ddot{\phi}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{\bar{b}(\bar{I}_{33}+ml^2)}{(I_0+\bar{I}_{11}+mr^2)(\bar{I}_{33}+ml^2)-m^2r^2L^2} & 0 & 0 & 0 \\ -\frac{mrl\bar{b}}{(I_0+\bar{I}_{11}+mr^2)(\bar{I}_{33}+ml^2)-m^2r^2L^2} & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \phi_1 \\ \phi_2 \\ \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}\tau \tag{24}$$

3.5. Friction and friction compensation in RIP system

The investigation of friction and friction compensation is very important in control engineering community. Friction can be seen as a highly nonlinear component which can result in limit cycles (LCs), steady state error and poor system performance. The RIP can be used to study the effect of friction and friction compensation since the frictional effects in RIP are so clearly visible. It can be shown that the friction in the driven arm might cause the LCs with high amplitude. The LCs can be predicted using widely known friction model such as LuGre model [4], Coulomb friction model [14] and Coulomb friction with stiction model [71]. The amplitudes of the LCs and all other effects due to friction can be reduced using friction compensator based on these friction models [14].The friction phenomena in RIP happens in the joints (i.e. the pendulum and arm joints). The friction in both joints can be demonstrated as follows:

3.5.1. Pendulum joint

The friction in this joint can be demonstrated using a damping constant through a small ball bearing. Based on the Rayleigh's dissipation function [72]:

$$D = \frac{1}{2}\left(b_1\xi_1^2 + b_2\xi_2^2 + \cdots + b_i\xi_i^2\right) \tag{25}$$

where b_i is the ith viscous damper coefficient and ξ_i is the velocity difference across the ith viscous damper which can be expressed as a function of the generalized velocity. Therefore, for the RIP pendulum,

$$D(\dot{\phi}_1) = \frac{1}{2}b_1\dot{\phi}_1^2 \tag{26}$$

The non-conservative torque τ_n is given by:

$$\tau_n = \frac{dD(\dot{\phi}_1)}{d\bar{\phi}_1} = \frac{d}{d\bar{\phi}_1}\left(\frac{1}{2}b_1\dot{\phi}_1^2\right) = b_1\dot{\phi}_1^2 \tag{27}$$

The constant b_i is found to be 226×10^{-7} based on the free motion experiment [4].

3.5.2. Arm joint

The frictional torque in the motor shaft comprised of the dynamic friction, static friction and natural damping [4]. The Coulomb friction model can be considered as a simple model given as:

$$\tau = \tau_c \mathrm{sgn}\left(\frac{d\phi_1}{dt}\right) \tag{28}$$

This model does not describe the friction at zero velocity [71]. The Lund Institute and Grenoble Laboratory (LuGre) model can be used to describe the friction at all points. The dynamic equation of LuGre model is as follows [73]:

$$\frac{dz}{dt} = v - \sigma_0\frac{|v|}{g(v)}z \tag{29}$$

$$g(v) = \tau_c + (\tau_s - \tau_c)e^{-\left(\frac{v}{v_s}\right)^2} \tag{30}$$

$$\tau_r = \sigma_0 z + \sigma_1\frac{dz}{dt} + \sigma_2 v \tag{31}$$

where z is the initial state of the model, v is the velocity at $\dot{\alpha} = \dot{\phi}$, v_s is the Stribek's velocity, σ_0 and σ_1 are the internal model parameters, σ_2 is the dynamic friction constant, τ_c is the Coulomb friction torque, τ_s is the static friction torque and τ_r is the estimated friction torque.

3.6. System identification

The physical meaning of the parameters in Eqs. (29)–(31) is explained in detail in Ref. [74]. Also, the detailed explanation and steps for identification of the parameters in Eqs. (29)–(31) for RIP were presented in Ref. [4]. Thus, this section briefly describes how these parameters can be identified. It can be stated that for the identification of all these parameters, only the arm and the motor is needed. Thus, the pendulum is decoupled from the system. The least square algorithm can be used to identify σ_2 and τ_c by considering the first order system as follows:

$$J_a\frac{dv}{dt} + \sigma_2 = \tau - \tau_c\mathrm{sgn}(v) \tag{32}$$

The moment of inertia of the arm and motor J_a can be estimated experimentally. To identify τ_s, the angle of the motor is feedback with a small PI action given in the form:

$$u = g_1 e + g_2\int_0^t e(\tau)d\tau \tag{33}$$

where e is the error between the desired reference and measured state. $g_{1,2}$ are PI gains. The parameters τ_s,J_a and τ_c are required to be known for identification of v_s which can be known based on the above identification steps. By introducing a very small speed in the shaft of the motor (i.e. $\frac{dz}{dt} \approx 0$), the fictitious state z in Eq. (29) can be approximated as:

$$z \approx \frac{v}{|v|}\frac{g(v)}{\sigma_0} \tag{34}$$

Also, τ_r from Eq. (42) can be approximated as:

$$\tau_r = \left(\tau_c + (\tau_s - \tau_c)e^{-(v/v_s)^2}\right)\mathrm{sgn}(v) \tag{35}$$

The equation

$$J_a\frac{dv}{dt} = \tau - \tau_r \tag{36}$$

can be linearized to

$$v(t) = v_s\sqrt{\ln \varepsilon(t)} \tag{37}$$

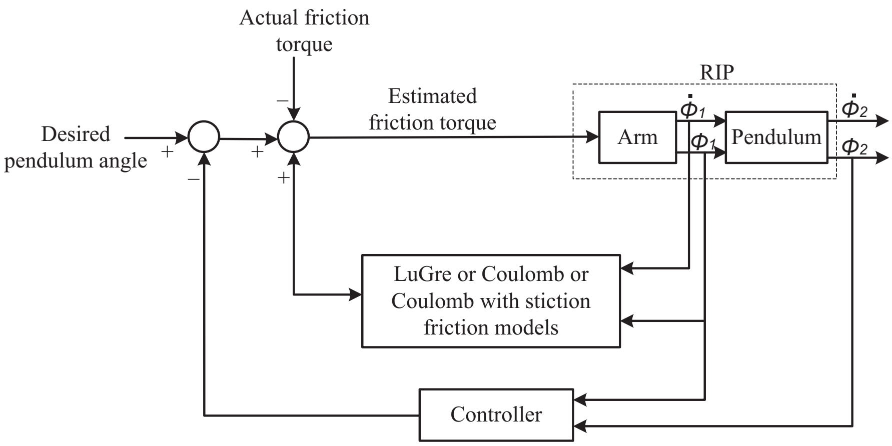

Fig. 3. Friction compensation model for RIP.

The ε is given by:

$$\varepsilon = \frac{\tau_s - \tau_c}{(\tau - J_a \frac{dv}{dt})\operatorname{sgn}(v) - \tau_c} \tag{38}$$

This approximation is valid for $\varepsilon \geqslant 1$ and $\tau_s > \tau_c$. The values of ε and v_s can be found by inputting the sinusoidal torque to the system. The parameter σ_0 can be estimated by inputting the ramp signal in such a way that the angle cross zero. The range of values of the ramp signal should be around $\pm\frac{4}{5}\tau_s$. Therefore $\sigma_0 \approx \frac{\Delta\tau}{\Delta\phi_1}$. For identification of σ_1, the small displacement is assumed and $g(v) \gg |z|$. Hence, $v = \dot{z} = \dot{\phi}_1$ from Eq. (29).

Consequently, the linear second order can be used as:

$$\tau = J_a\ddot{\phi}_1 + (\sigma_1 + \sigma_2)\dot{\phi}_1 + \sigma_0(\phi_1 + \phi_1(0)) \tag{39}$$

Thus, the value of σ_1 can be estimated easily based on Eq. (40) since it is a linear second order system

$$\sigma_1 = 2\xi\sqrt{J_a\sigma_0} - \sigma_2 \tag{40}$$

where ξ is the linear model damping constant.

The identification result found in Ref. [4] indicates that the LuGre model can give a precise estimation of friction. This gives a chance to design a good friction compensator in the arm joint. Based on this, the control method can be designed considering the RIP as a conservative system. The generic diagram for friction compensation model is presented in Fig. 3.

3.7. Linearization of nonlinear model of RIP

The linearization of nonlinear dynamic equation is a common practice in control engineering design, especially for the design of linear controllers. This is to ease the complication of the dynamic equations. The nonlinearities in the equation of motion of RIP arises from the trigonometric function present in their dynamic equations. The nonlinear model can be approximated to linear model via the Taylor series expansion around an operating point and discarding the nonlinear terms [10]. Several other types of linearizations have been employed in order to linearize the nonlinear model of RIP. These include Jacobian linearization, input-output feedback linearization and optimal linearization. The small angle formula (at the equilibrium position of pendulum) was used for linearization in Refs. [1,11,40,75,76]. The Jacobian linearization was employed in Refs. [33,37,57]. Feedback linearization method was used in Refs. [5,15,28,77]. The linear model of RIP was found using partial feedback linearization in Refs. [49,50]. The resulting linear model of all the mentioned linearization methods have similar dynamics to nonlinear model around the specific operating point it was linearized. The weakness of the local linearization can be overcome using the optimal linearization method. This method gives a linear model that has exact dynamic of nonlinear model at any operating point with minimum approximation error. The optimal linearization of RIP was discussed in Ref. [37].

4. Control objectives of the RIP

As stated in the introduction, there are four main control objectives of RIP, which are: Swing-up control, stabilization control, switching control and trajectory tracking control. These control objectives will be discussed in this section. Normally the swing up control objective is implemented together with the stabilization control. Because when the pendulum swing-up to the neighborhood of the upright equilibrium position, it fails if there is no stabilization controller to stabilize it at that position [78]. Therefore, when the swing up controller swings the pendulum near the upright equilibrium position, the switching controller disables the swing up controller and at the same time enables the stabilization controller. Subsequently, the trajectory tracking controller is switched on simultaneously with stabilization controller so that the arm would follow the desired trajectory while the pendulum is stabilized at upright position [5,79]. New method was implemented wherein the switching control was eliminated. In this method, the swing up and stabilization control problems for RIP system are solved in which the swing up control is designed by using trajectory planning and inertia effect such that the pendulum can be swung to a desired position to activate the stabilization controller [80].

However, the stabilization controller can be implemented separately by manually putting the pendulum near the upright unstable equilibrium position [18,81,82]. Mostly, linear controllers are used for stabilization control objectives [27,57]. The overall objectives of RIP can be described using the block diagram in Fig. 4.

Many approaches of swing-up control were proposed in the literature [3,4,83]. The basic swing-up control was explained in detail in Ref. [3]. The non-linear mathematical model of RIP was used to develop a feedforward control law to swing the pendulum from the suspended to the upturned state using the steepest-descent method in Ref. [3]. Mainly, the resultant control law was derived in the form of a bang-bang control sequence as shown in Fig. 5, in which the set of switching instances are:

$$t^{(n)} = t_1, t_2, \ldots, t_n \tag{41}$$

The criteria function is minimizing

$$J = \phi_{\text{end}}^T Q \phi_{\text{end}} \tag{42}$$

where

$$\phi_{\text{end}} = [\phi_1(t_n), \phi_2(t_n), \phi_1(\dot{t}_n), \phi_2(\dot{t}_n)]^T \tag{43}$$

with the following restrictions:

$$Gx(k) = [-t_1(k), \{-t_2(k) - t_1(k)\}, \{-t_3(k) - t_2(k)\}, \{-t_4(k) - t_3(k)\}]^T \leqslant 0 \tag{44}$$

Eq. (40) can be minimized with the constraint of Eq. (45) using a Lagrangian, given by:

$$L(x) = J(x)^2 + \delta^T G \tag{45}$$

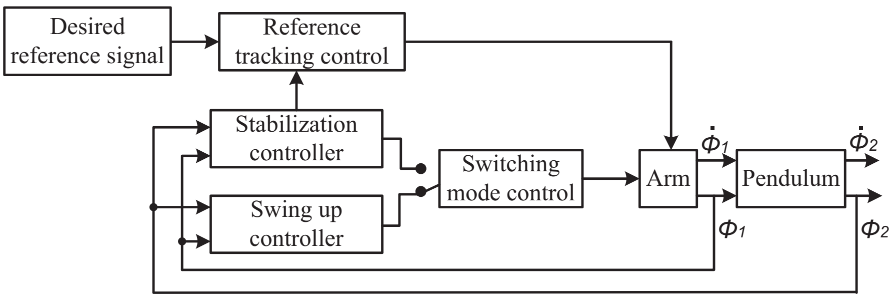

Fig. 4. Block diagram of main control objectives of RIP.

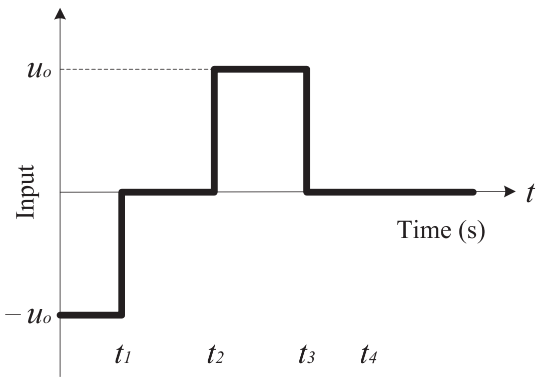

Fig. 5. Shape of the control input for pendulum swing-up (adapted from Ref. [3]).

For the Lagrangian, the modified steepest method is employed to evaluate its minimum value. The algorithm is given as follows [3]:

$$\delta = [\delta_1\ \delta_2\ \delta_3\ \delta_4]^T \geqslant 0 \tag{46}$$

$$L(x+\epsilon z) = \left(J(x) + \epsilon\frac{\partial J}{\partial z}z\right)^2 + \delta^T\left(\frac{\partial G}{\partial x}\epsilon z + G\right) \tag{47}$$

$$L(x+\epsilon z) - L(x) = \left(2J(x)\frac{\partial J}{\partial z}z + \delta^T\frac{\partial G}{\partial x}z\right)\epsilon + \epsilon^2\left(\frac{\partial J}{\partial z}z\right)^2 \tag{48}$$

$$z = \left(2J(x)\frac{\partial J}{\partial z} + \delta^T\frac{\partial G}{\partial x}\right)^T \tag{49}$$

$$\epsilon = -\frac{z^T z}{2\left(\frac{\partial J}{\partial z}\right)^2} \tag{50}$$

This method needs correct knowledge of the plant parameters before it could be implemented. Alternatively, to overcome this disadvantage, Bang-bang psuedo-state feedback control method was also considered in Ref. [3]. This was done by considering the nonlinear pendulum's equation and assuming that the pendulum is very light such that it can be swung-up with small $\dot{\phi}_1$. In this method the arm acceleration $\ddot{\phi}_1$ is considered as the system input. The assumption here is that there is no error in motor model such that the relation between τ and $\ddot{\phi}_1$ can be accurately derived. Thus, the required control law to find the switching law can be obtained, which makes the pendulum to swing up to the upright position.

The energy based swing-up control was investigated in Ref. [83]. The result obtained indicated that the swing up global behavior entirely depends on the ratio of the maximum acceleration of the pivot to the acceleration of gravity. It has been shown that the swing-up is sufficient if the ratio is greater than 1.33.

The linear and nonlinear control strategies can be used to stabilize the pendulum at upright position. Normally, the swing-up method is designed such that the pendulum passes through the origin with $(\phi_1, \dot{\phi}_1, \dot{\phi}_2, u) = (0,0,0,0)$. At neighborhood of this point, the stabilization controller is enabled by switching strategy.

The switching control is the criterion that is used to change the control action from one mode to another. Energy threshold and/or pendulum angle threshold are mostly used for this purpose in the literature [1,10,11,50,84]. For the sake of simplicity, both pendulum angle and energy criteria are used in this study for switching between swing up and stabilization control.

$$\text{switching criteria} = \begin{cases} \text{Stabilization} & \begin{cases} |\phi_2| < \frac{\pi}{9}\text{rad} & \text{and} \quad \dot{\phi}_2 < 2.62\text{rad/sec} \\ |E - E_r| < 0.04 \text{ Joule} & \text{and} \quad \dot{\phi}_2 < 2.62\text{rad/sec} \end{cases} \\ \text{swing up control} & \qquad\qquad\qquad\qquad \text{otherwise} \end{cases} \tag{51}$$

where EandE_r are pendulum energy and reference pendulum energy respectively, and $\dot{\phi}_2$ is the angular speed of the pendulum.

Trajectory tracking control is one of the important control objectives of the RIP. It shows the ability of the controller to control the arm of RIP in such a way that the arm tracks a desired time varying trajectory while the pendulum remains at unstable position. This problem includes the controller design $\tau \in \mathbb{R}$ in such a way that the trajectory error $\tilde{\phi}_1$ and the pendulum angle are uniformly ultimately bounded [85]. That is, the design controller guarantees the following:

$$\left\| \begin{matrix} \tilde{\phi}_1(t_0) \\ \phi_2(t_0) \end{matrix} \right\| \leqslant a \Rightarrow \left\| \begin{matrix} \tilde{\phi}_1(t) \\ \phi_2(t) \end{matrix} \right\| \leqslant b, \ \forall t \geqslant t_0 + T \tag{52}$$

where $a$and$b$ are constant, $T > 0$ and $\tilde{\phi}_1 = \phi_d - \phi_1$. The desired trajectory function ϕ_d used in this study is the time varying square function. This function is continuous and differentiable. Besides, $\phi_d, \dot{\phi}_d,$ and $\ddot{\phi}_d$ are bound for all $t \geqslant 0$. The output function is defined as:

$$y = \tilde{\phi}_1 - \phi_2 \tag{53}$$

The detailed explanation and formulation of RIP trajectory tracking control can be found in [5].

5. Control of RIP

The main goal of RIP control is to swing up the pendulum near the upright unstable equilibrium position and balance it there. Afterward, the arm is controlled to track a desired time varying trajectory while the pendulum remains at unstable position. Different kinds of controllers have been used for these purposes. These include the linear controllers, nonlinear time invariant controllers, self-learning and adaptive nonlinear controllers.

5.1. Cascade control method

The RIP is a single input multiple output (SIMO) system. In SIMO systems, a change of one of the outputs by some disturbances affects the control of the other outputs [86]. Thus, considering nonlinear behavior of RIP system, it is difficult to achieve a desired settling time. Also, it has high level of disturbances and large time constant. Therefore, the best control strategy is the cascade control one, since it has the advantage of attenuating the effect of disturbances and improving the dynamics of entire control loop [87]. The structure of the general cascade control for RIP system is shown in Fig. 6, where α_r and α are the desired pendulum angle and actual pendulum angle, respectively. The blocks G_1, G_2, G_3, G_4, G_5 and G_6 represent the scaling gains which are essential in some controllers, such as PID and Fuzzy Logic Controllers. In this study, the two-input-one output controllers are used for easy illustration. The output of the inner controller is the control voltage to the servomotor $V_m(t)$.

These scaling gains G_1, G_2, G_3, G_4, G_5 and G_6 are adjustable parameters just like in any PID controller. They are used to calibrate the input and output. The excellent performance of these type of controllers depends on the values of these gains [86]. Trial-and-error can be used to find appropriate values for the gains, but it is not feasible. Thus, there is a need for using a systematic procedure which is easier for finding the optimized values of these gains. In case of PID controller, the Ziegler–Nichols method can be used. Also, Meta-heuristic optimization methods can be used to find the optimal values of these controllers. Numerous researchers implemented different kinds of control of RIP system in cascade form. Some of them used two similar controllers in cascade form as in Refs. [10,11,35,46,88,89], while others used different controllers in cascade form as in Ref. [12,88,90].

5.2. Linear controllers applied to RIP

Having a linear model of RIP, a linear controller can simply be linked for state reference tracking [91]. The design of gain matrix K is the only requirement of this controller as shown in Fig. 7.

The assumptions made are: the system is full state controllable, the state variables can be measured and they are available for feedback, and the control input are unconstrained. Pole placement (PP) method can be used in this case to obtain suitable performance for control of overshoot and rise time. Given a linear closed loop system [13]:

$$\dot{X} = AX + BU \tag{54}$$

$$Y = CX \tag{55}$$

The control vector U can be designed in a state feedback form as follows [92]:

$$U = -KX \tag{56}$$

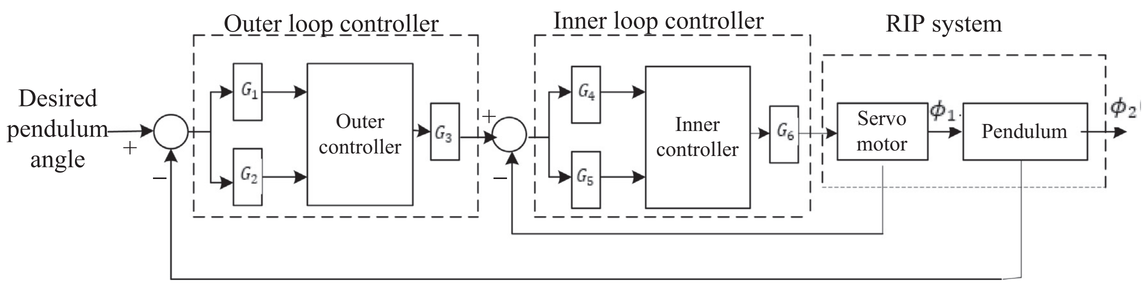

Fig. 6. General cascade control method for RIP.

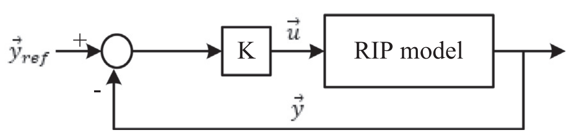

Fig. 7. Linear controller for reference tracking.

Therefore, Eq. (56) become

$$\dot{X} = (A - BK)X = A_{CL}X \tag{57}$$

The PP comprises evaluating a gain matrix K in such a way that the desired poles in linear model of RIP system are the eigenvalues of A_{CL}. These solutions have been given for real pole assignment. Nevertheless, the solutions for complex conjugate poles are very restrictive or the determination of shifted poles must be found by successive choices to get an optimal control [93]. Second-order linear Lyapunov equation is to be solved for shifting two complex conjugate poles, the details on how to shift the complex conjugate pole is presented in Ref. [94].

The PP was used for stabilization control of RIP in Refs. [27,57], also Nath and Mitra [13] presented the counter based controller and PP with integrator for swing up and stabilization controls respectively. They used energy criteria for the switching control.

Linear quadratic regulation (LQR) is slightly more complex than PP. The LQR provides an optimal control law for a linear system with a quadratic performance index J. The design problem is that of finding a control input U, which minimizes the performance index given by Ref. [95]:

$$J = \int_0^\infty (X^T QX + U^T RU)dt \tag{58}$$

where R and Q are positive definite square matrices. The matrix R and Q are used to scale the relative contributions of the terms of the quadratic forms $U^T QU$, and $X^T QX$, respectively. In this case, the K in Eq. (56) is given by:

$$K = R^{-1}B^T P \tag{59}$$

and the symmetric definite matrix P is the solution to the algebraic Riccati equation given by

$$PA + A^T P + Q - PBR^{-1}B^T P = 0 \tag{60}$$

Many researchers employed the LQR approach to design controllers for RIP. This is likely because the LQR guarantees the optimal control law. For example, the stabilization problem of RIP was solved using LQR in Refs. [1,7,22,29,75,76,96]. The stabilization and swing-up controls are implemented via a unified LQR controller in Ref. [37] which can efficiently evade switching control between the two stages. A feedback control system was used for stabilization of RIP using a Fractional Order based LQR controller. Improved stabilization result was found compared to conventional LQR [97]. Some researchers compared the performance of LQR with other linear controllers. The findings from this comparison are contradictory. This is because the LQR method is shown to have lower overshoot and settling time than PP in Ref. [27]. However, the results reported in Ref. [57] shows that the PP has superior robustness than LQR. Also, the mix of H_2/H_∞ control is shown to be better than the LQR control. All these evaluations cannot be generalised because they are peculiar to the specific model of RIP and specific tuning of the corresponding controller. The controllers that can be designed using LQR or PP are considerably different. This is because LQR depends on the weighting matrices Q and R [98] while PP depends on selected pole [99]. Generally, a higher gain and better convergence can be achieved by selecting bigger Q gains for the LQR or poles that are more to the left half plane for PP controller. But this produced more vibrations at equilibrium [91]. When faster settling time is required, the LQR is preferred in choosing those states, which is impossible with PP. Thus, LQR can be used to manage the trade-offs between actuators and states more accurately [91].

Linear Quadratic Gaussian (LQG) controller is another type of LQR that comprises the Kalman filter observer and optimal LQR controller as shown in Fig. 8. The Kalman filter is used for noise immunity and state estimation. The detailed explanation on LQG is presented in Ref. [100]. Measured states are used in LQR without a filter. Though, LQR is a sufficient design approach that holds its phase margin assurances, this is so in cases whereby the observer dynamics are much faster than the system dynamics. The LQG was proposed in Ref. [101] to ensure stabilization of pendulum with minimal deviation of arm and pendulum angles. This is based on applying large penalties on the arm and pendulum angles in the cost function of the optimal control law.

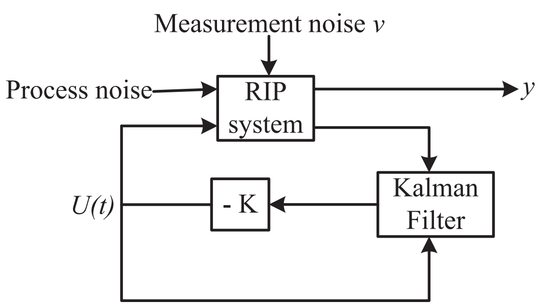

Fig. 8. Linear quadratic Gaussian controller.

The H_∞ and H_2 controllers are alternative robust and optimal controllers. These controllers are using different cost functions, and they are less sensitive to disturbances and model errors [102]. Al-Jodah et al. [33] proposed the energy based controller for swing up and the mix of H_2 and H_∞ state feedback for stabilization control. The proposed stabilization controller was compared with the conventional full state feedback controller and LQR controller. The results showed that the proposed stabilization method is better than its comparatives.

The Proportional Integral Derivative (PID) controller shows an excellent performance for the linear system of RIP. The PID controller is commonly used in industries because of its simple structure and robustness in different operation conditions [89]. The PID controller alone, or in hybrid form with other controller, or PI controller (when derivative gain is zero), or PD controller (when integral gain is zero) have been widely applied to different control objectives of RIP in literature. In Ref. [50], The PID controller was employed for the swing up control, partial feedback linearization for stabilization and the angle threshold for switching control. The Genetic algorithm (GA) based PID controller, particle swarm optimization (PSO) based PID controller, and ant colony optimization based PID controller are proposed for stabilization and reference tracking controls of RIP in Ref. [89]. Kharitnove polynomial based PI controller was designed for the same purpose in Ref. [19]. The PD controller and fuzzy PD controller were used for swing up and stabilization, respectively in Ref. [11]. The energy threshold method was employed for switching control while the fractional order PD was applied for stabilization control of RIP in Ref. [49]. The performances of PD and PD with dead zone for RIP stabilization were compared in Ref. [48]. The 2-DOF fractional control method for RIP, which is a configuration of feed-forward and feedback paths is proposed in Ref. [103]. The perturbation attenuation was relating with the primary controller, while the secondary controller is accountable for set point tracking. The PP technique is employed for the design of 2-DOF PID controller. The proposed controller was used to solve the problem of stabilization and trajectory tracking control of RIP.

The main issue in PID, PI and PD controllers is finding the appropriate gains. The acceptable gains can be found simply from the experimentation via trial and error. Else, Ziegler–Nichols method can be employed to set the initial gains. However meta-heuristic algorithms can be used to find the best gains with respect to an objective function [89].

Chou and Chen [28] investigated the new swing-up control method for RIP using energy based control and the feedback linearization method. The energy function is chosen to design the control law in which the gain k is selected based on the trajectories in the phase-plane. The result shows that the fast energy rate depends on the value of gain k. Seman et al. [29] proposed two ways of swinging up control of RIP based on energy. The first way is the conventional method based on comparing the present total energy of the RIP system with the energy in its up-right position. The second way is by using an exponentiation operation over the pendulum position. The stabilization control is tackled using LQR. Mathew et al. [1] studied the performances of energy based and PD controllers for swing up control of RIP and sliding mode controller and LQR controllers for stabilization control. The simulation results indicate that the energy-based controller is better than PD controller with less number of oscillation to reach the equilibrium position. Also, for stabilization, the LQR has less overshoot and a faster settling time but the SMC is more robust in the presence of parameter uncertainties. Nguyen and Shen [76] proposed a hybrid control scheme for swing up and LQR control for the stabilization of RIP. The flexibility of the hybrid control scheme is based on the choice of energy-based swing up controller and heuristic swing up controller at the different pendulum's positions. A novel composite control method for tracking control of RIP is reported in Ref. [5]. This scheme comprises of an energy-based compensation and a feedback-linearization-based controller. The analysis of the closed loop system indicated that the system is uniformly ultimately bounded to the trajectories of error. The real-time experiments and numerical simulations show the viability of the proposed method. The proposed control scheme is compared with the hybrid output tracking controller consisting of a differential flatness, nonlinear back stepping and small gain theorem proposed in Ref. [104].

The trajectory tracking control of RIP using active disturbance rejection control (ADRC) method was studied in Ref. [105]. A linear controller of the ADRC was designed based on linear observer. This is done on the origin of the flat tangent system linearization around an arbitrary equilibrium. The proposed method was compared with the SMC in real time. The results show that the proposed method has superior performance over the SMC. Aguilar-Avelar and Moreno-Valenzuela [15] compared a feedback linearization control method and output tracking nonlinear controller for tracking control of RIP. The results indicated the superiority of the feedback linearization method based on regulation and tracking error with slightly higher control torque.

Naturally, most real-world control systems are nonlinear. These systems are traditionally linearized in order to apply the control techniques. But since the system dynamics will perform differently in different operating condition, several controllers will have to be designed and "scheduled" with respect to the operating conditions to produce acceptable control system performance. In most cases, developing a satisfactory controller schedule can take more human resources than the linearization and linear system design tasks. Furthermore, the stability of the resulting system cannot be guaranteed.

Nonlinear control techniques take advantage of the given nonlinear system dynamics to generate high performance designs. No linearization or gain scheduling is needed for their implementation. Also, the stability of the resulting system can be guaranteed. These features allow the designer to pay attention to the control system design aspects of the problem with no tedious model manipulations. This is the main advantage of nonlinear control method over linear control.

5.3. Nonlinear time invariant controllers

Sliding mode control (SMC), fuzzy logic control and back-stepping are the most commonly employed nonlinear control strategies applied on RIP. It is widely known that SMC effectively provides robust control for nonlinear system even in the presence of uncertainties and disturbances [106]. This method has been applied successfully for RIP control due to its attractive features [23]. These features include easy realization, compatibility to Multiple Input and Multiple Output (MIMO) systems, good control performance for nonlinear system such as good transient response, fast response and insensitivity to external disturbance and/or plant parameter variation [42]. In addition, it is possible to guarantee the stability of SMC because it benefits from the merit of switching control law [107]. However, the switching control law introduced chattering in the system. This is due to its alternating switching, which occurs from its digital implementation [108]. The stabilization problem of RIP was solved using different kind of SMC. For example, Linear Matrix Inequalities (LMI) based multi objective integral SMC is proposed in Ref. [23]. The proposed controller indicates the superior performance compared with the LQR. The SMC based on higher order differentiator observer was used for stabilization control of RIP in Ref. [42] and conventional SMC was proposed in Ref. [70] for the same purpose. Stabilization problem of RIP was solved using SMC with time delay in Ref. [107]. Cascade optimal SMC was proposed in Ref. [90] for RIP stabilization control. The parameters of SMC were optimally updated using a discrete-time, nonlinear model predictive control structure. Kurode et al. [78] proposed the SMC for both swing up and stabilization controls of RIP. The proposed method was compared with the PD swing up and LQR stabilization controllers. The results show that the SMC can swing up within a small period of time and it is more robust than PD and LQR controllers, respectively.

The proposed controller in Ref. [78] was shown to be better than PD controller swing-up and LQR stabilization. The integral SMC based on the observed values of the state variables is proposed in Ref. [6]. The novel algorithm outperformed its comparative in both pendulum stabilization and trajectory tracking control.

FLCs are often employed for nonlinear systems control. This is due to its attractive features which include easy incorporation of expert knowledge into the control law, less model dependent, robustness and ease of use to model linguistic rules [10]. FLC can approximate any nonlinear control law based on the number of fuzzy set. Yet, the stability of a general FLC is difficult to confirm. This is due to the piecewise nature of the control law which makes it difficult to confirm the Lyapunov stability. Different types of FLC have been used for different control objective of RIP in literature. A stabilization control of RIP was tackled using robust FLC in cascade form [46]. Both inner and outer FLCs in Ref. [46] are based on a uniform fuzzy set with a minimum number of rules. The Swing-up control, stabilization control, and trajectory tracking controls problems were solved using intelligent optimized cascade fuzzy-PD controller based on GA and differential evolution in Ref. [35]. A parallel distributed compensation based FLC using LQR method is reported in Ref. [2] for stabilization control of RIP system. The composite FLC (CFLC) for swing up and stabilization of RIP system is presented in Ref. [109]. This is essentially state feedback control by fuzzy summation of FLC and PD controls. In Ref. [110], the performance of FLC was shown to be better in comparison with LQR for stabilizing the RIP system. The variable universe of discourse FLC (VUDFLC) was used for stabilization control of RIP in Ref. [55]. The simulation results indicate the robustness and effectiveness of VUDFLC over the general FLC. Fuzzy PD controller was used for stabilization control in Ref. [11,41]. The swing-up stabilization and trajectory tracking controls problems were solved using intelligent optimized cascade interval type-2 fuzzy-PD controller based on PSO and GA in Ref. [10]. Nearly the same strategy was presented in Ref. [111] using cuckoo search algorithm. In both research, the same controller was employed for all the control objectives. The controller only changes the mode from one objective to another based on the selected threshold of energy.

The SMC have been hybridised with other controllers, especially for suppressing chattering. For example, SMC and FLC were hybridised in Ref. [112] for stabilization control of RIP. This control scheme has the advantages of both constituents and deviates from the limitations of each constituent.

Nonlinear LQR was applied in stabilization of nonlinear control of RIP called State-Dependent Riccati Equation (SDRE) method [56]. This is based on the transformation of nonlinear system into its equivalent form through extended linearization. The results obtained by SDRE indicate a better performance than the standard LQR in terms of settling time and overshoot. A novel swing up control of RIP by means of the Speed-Gradient method, which is based on the measured coordinates is reported in Ref. [79]. A nonlinear stabilization based on the Forwarding method was presented, and shown to be better than LQR stabilization. Lyapunov based switching method was used. In the work reported in Ref. [88], the researchers investigated the real time application of an event-based control method for tracking control of RIP. The communication between the plant and the controller is achieved via Ethernet, i.e. TCP/IP. This reduced the bandwidth used by control loop and leads to a Networked Control System. The experimental results indicate how the proposed method can reach a substantial reduction of the bandwidth consumed with an insignificant worsening of the performance.

Aguilar-Avelar and Moreno-Valenzuela [5] hybridized a feedback-linearization-based controller with an energy-based compensation for reference tracking control of RIP. The proposed method ensured that the closed-loop system is uniformly ultimately bounded. The proposed method demonstrate a better tracking performance than the combination of backstepping, small gain theorem and differential flatness proposed in Ref. [104]. Tsuge et al. [113] developed nonlinear controller based on polynomial and non-polynomial representations using PSO and sum of squares methods for stabilization of RIP. The stability condition of polynomial and non-polynomial systems was derived by approximating the domain of attraction with input magnitude constraints. The tensor product model transformation based swing-up control of RIP was proposed using the LMI based control in Ref. [9]. The stabilization control of RIP was tackled using interconnection and damping assignment passivity- based control in Ref. [44]. The energy shaping where the reference energy function to the passive map was allocated, and injection of damping to guarantee asymptotic stability. Fabbri et al. [32] applied the Packet-Based Control method with dynamic controller for swing-up and stabilization control of RIP. Ethernet network was used to implement the network communication channel. The proposed controller was compared with the conventional local controller for divers time-varying actuation delays which indicates its effectiveness. Aracil et al. [79] investigated a new nonlinear method for RIP control. The Speed-Gradient approach based on directly measured coordinates was used to swing-up the RIP. The nonlinear controller based on the Forwarding method was proposed for stabilization of RIP. Türker et al. [34] proposed the stabilization control of RIP using a static feedback controller based on direct Lyapunov method and partial feedback linearization. The asymptotic stability of the system has region of attraction containing nearly all points in the upper half-plane of the pendulum independent of the physical parameters. The direct Lyapunov stability method based on a set of transformations for stabilization of RIP is proposed in Ref. [43]. Fabbri et al. [32] investigated the packet-based control (PBC) method for swing up and stabilization controls of RIP in real time. The PBC was compared with a local micro-controller for diverse time-varying actuation delays. The results indicate the validity and robustness of PBC method in the presence of actuation delays. A novel 2-DOF fractional control strategy based on 2-loop topology for RIP system was proposed in Ref. [114]. This method comprises feed-forward and feedback paths. The primary controller recounts the perturbation attenuation while the secondary controller is accountable for set point tracking. The proposed controller was found to be better than SMC.

5.4. Self-learning and adaptive nonlinear controllers

Quyen et al. [8] proposed a hybridization of a PID controller and ANN for the RIP. Here, the ANN was trained by using the input and output data via a supervised learning method. The training data is derived from the model of RIP with the PID controller of two variables. The output controller based on Attractive Ellipsoid Method (AEM) and adaptive state estimator was developed in Ref. [58] for the stabilization of RIP. This method guaranties the stabilization of the controlled system trajectories within an ellipsoid of a "minimal size". The same method was proposed in Ref. [115] with some modification of the AEM idea that allows the use of online information acquired during the process. Azar and Serrano [24] investigated the adaptive SMC, second order SMC and PD + SMC for stabilization control of RIP. The variable structure design procedure and Lyapunov stability theorem are used to develop these control methods to obtain asymptotically stable system trajectories. The results obtained indicate the superiority of adaptive SMC over second order SMC and PD + SMC. A novel adaptive NN-based control method for RIP was proposed in Ref. [116]. Based on the input and output weights adaptation laws, the proposed method can guarantee reference tracking of signal for the arm while the pendulum remains at unstable position. Three different reference trackings were tested, namely, sinusoid trajectory with torque disturbances, complex trajectory and sinusoid trajectory with additive Gaussian noise in measured feedback signals torque. The proposed method was compared with linear PID controller, adaptive NN + PID and adaptive NN + PD controller. The proposed control method was found to be more robust than the comparative methods. Lyapunov based adaptive controller was proposed in Ref. [12] to stabilize the RIP with time-varying uncertainties. The hierarchical adaptive back-stepping SMC was proposed in Ref. [18] for balancing control of RIP subjected to external disturbances. The RIP model was decoupled into two subsystems. To derive each subsystem to a desired sliding surface, an adaptive back-stepping based control law was designed for each subsystem. The balancing control of RIP using a Temporal Based NN (TBNN) model was proposed in Ref. [117,118]. The online training ability of TBNN controller makes it possible to control the RIP without the need of its exact model. The Adaptive Neuro Fuzzy Inference (ANFIS) controller was presented in Ref. [45] for balancing of RIP. The simulation results demonstrate that ANFIS controller is better when compared with FLC and PID controller in terms of overshoot, settling time and parameter variation. Singh et al. [119] studied the linear fusion function based on LQR mapping, and tuning of controller parameters adaptively using ANFIS for stabilization of RIP. The weights tuning and number of rule explosion which are the main issues in FLC are eliminated by the proposed method. The proposed method is shown to be better in disturbances rejection, tracking performance and robustness against parameters variation than the classical LQR controller. A model free discrete time NN control is designed for the trajectory tracking of a RIP in Ref. [120]. The proposed method has three main characteristics: 1) the tracking error is used in place of the estimation error in the weights learning equations, evading the need of a good behavior estimation of the unknown elements in the nonlinear model, 2) the projection method is proposed to avoid over-fitting in the control law, and 3) the Lyapunov technique is utilized to assure uniform stability of the tracking and weights errors.

5.5. Model-free controller

Model-free controller design has broken the limitation of parametric adaptive and realized structural adaptive. Function combination design of model-free controller is guided by controlled system's functional requirement on controller, as an alternative of relying on controlled system's mathematical model for the design of the controller. Its design is based on following universal model [121]:

$$y(k+1) - y(k) = \gamma(k)^{\tau}[u(k) - u(k-1)] \tag{61}$$

Where $y(k)$ is the controlled system's output, $u(k-1)$ is the input and $\gamma(k)$ is the characteristic parameter of universal model (41), $\{u(k-1), y(k)\}$ and $\{u(k), y(k+1)\}$ are two groups of observation data of the system in adjoining moments.

assuming $u(k-1) \neq u(k)$,

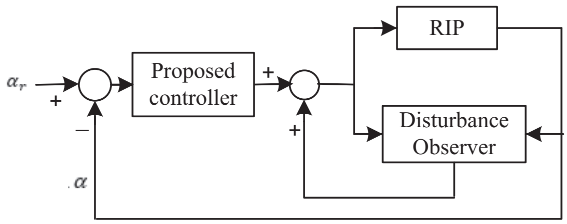

Fig. 9. RIP control include the disturbance observer.

The universal model is required to be adjusted in such a way that it coexists with the basic form of control law without model, which is given as follows [122]:

$$u(k) = u(k-1) + \frac{\lambda_k}{\alpha + ||\hat{\gamma}(k)||^2}\hat{\gamma}(k)[y_0 - y(k)] \tag{62}$$

Where α is a small positive number, $\hat{\gamma}(k)$ is the estimated value of characteristic parameter of $\gamma(k)$, λ_k is a control parameter and y_0 is the set value of controller.

The algorithm of model free control law is composed of online interaction of identification algorithm of universal model of Eq. (61) and basic control algorithm of Eq. (62). Control algorithm in Eq. (62) can be used to feedback the control system after obtaining the estimated value $\hat{\gamma}(k)$.

$y(k+1)$ can be identified by getting a group of new observation data from the control result and adding it to the existing data.

The design method of model-free control law is composed of three parts, namely: 1) Basic Form of Model-free Control Law, 2) Use Direct Design Method, Obtain Nonlinear Structure of Model-free Controller and 3) the Application of Function Combination Method in Model-free Controller Design. The detailed explanation of these parts can be found in Ref. [121].

5.6. Disturbance observer

The external disturbances or unmodelled system dynamics can cause low frequency disturbances in RIP control. The low frequency disturbances can be rejected by disturbance observer [123]. The disturbance observer aids in improving the control action of the proposed controller. A disturbance observer is employed to estimate state of the system. Then, the estimation of disturbance can be accomplished by comparing the measured and predicted system states. The resulting estimated disturbance is added to the proposed controller's output, as shown in Fig. 9. The generalized PI disturbance observer based control was used for stabilization of RIP in Ref. [40]. Stamnes et al. [38] confirms the presence of a globally exponentially convergent speed observer in closed loop form for general Euler–Lagrange systems. The complexity of the observer is reduced compared to the one proposed in Ref. [124]. The proposed observer is used to approximate the pendulum's velocities.

6. Other types of RIP

Some studies have introduced additional complexity to the common single RIP. This is to test the effectiveness and robustness of the proposed controllers on more complex RIP. For example, Fujita et al. [96] investigated the swinging up and stabilizing control for twin RIP pendulums. The arm of twin RIP pendulums is connected to the motor shaft at its center and the pendulum is positioned at each ends of the arm, as shown in Fig. 10. Energy-based controller was used for swing up control while the LQR was used for stabilization when the two pendulums simultaneously reach the neighborhood of upright position. The energy-based controller was also proposed in Ref. [125], which concurrently achieves the swing up of the twin RIP and the minimum of the mechanical energy when the rotary pendulums are in their respective vertical positions. ANN was used in system identification of twin RIP in Ref. [16].

Some researchers implement their controllers on another type of double RIP, as shown in Fig. 11. In this case, the first pendulum is connected to the arm and the second pendulum is connected to the free end of the first pendulum. The multiple feedback delay was used to control this kind of RIP in Ref. [126].

Pujol et al. [30] generated a disturbance to the RIP by incorporating another inverted pendulum to the major inverted pendulum. This is done using semi-rigid spring as shown in Fig. 12. The system center of mass was changing when the motion was induced in the second pendulum. This creates a perturbation analogous to that of transportation units of mobile inverted pendulum. LMI controller was designed considering only the main pendulum. Consequently, the whole closed-loop system's behavior and the performance of the controller were analyzed experimentally [30].

The stabilization problem for triple RIP was addressed in Ref. [127]. The triple RIP has three inverted pendulums positioned at direct drive motor, as shown in Fig. 13. The SMC was used for stabilization control.

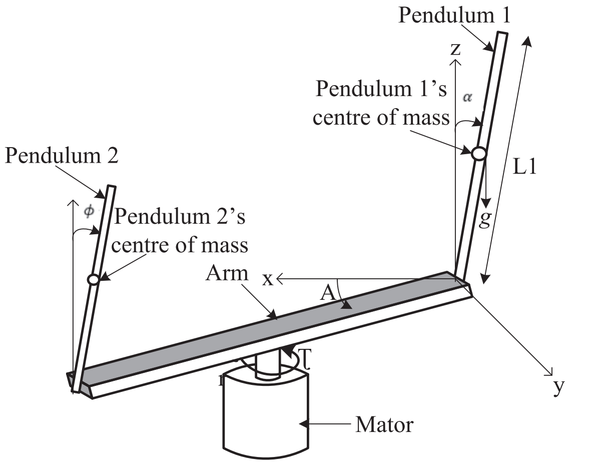

Fig. 10. Twin RIP.

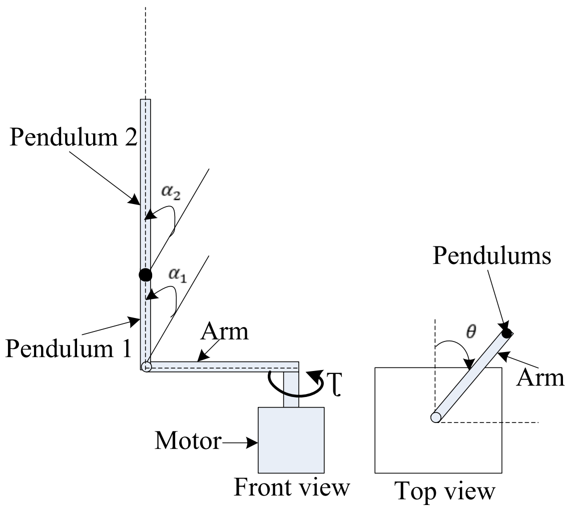

Fig. 11. Double RIP.

6.1. Opportunities for future research

There are many other intelligent controllers that are yet to be applied to the RIP, among which is Type-2 fuzzy logic controller (T2FLC). Interval T2FLC was proposed by Ref. [10], but there is no study on general T2FLC. Also, the hybrid T2FLC and SMC are yet to be tested on the RIP. In addition, only few works addressed the problem of trajectory tracking control. There is a need to have more research in this domain since it is one of the most important control objectives of RIP. Furthermore, there is a need to have a comprehensive work that deals with all the control objectives with detailed explanation of their relationship. Most of the proposed schemes used different controllers to achieve a single control objective (i.e. a controller for stabilization and another controller for swing up). It has been shown that only one smooth state-feedback controller can be used to solve this problem experimentally [4,128]. However, proving it mathematically remains a challenging issue.

Based on the present study, it can be stated that researchers focus on the application of single RIP with 2-DOF to test their proposed controllers. Only few control algorithms were applied on twin RIP [16,96,125], double RIP [126] and triple RIP [127] despite the fact that the twin RIP, double RIP and triple RIP are more challenging than single RIP. Researchers are expected to test and explain their control algorithms on these types of RIP.

Furthermore, there are other control schemes that are yet to be applied using RIP. These include the reinforcement learning control. This control algorithm can make the whole control process somewhat robust [129]. Nevertheless, lots of trials are required in reinforcement learning before finding a desirable control law. The three essential ways for solving the reinforcement learning problem based should be applied to RIP. These are dynamic programming (DP), temporal-difference learning (TDL) and Monte Carlo methods (MC). The DP methods are mathematically efficient but need a knowledge of

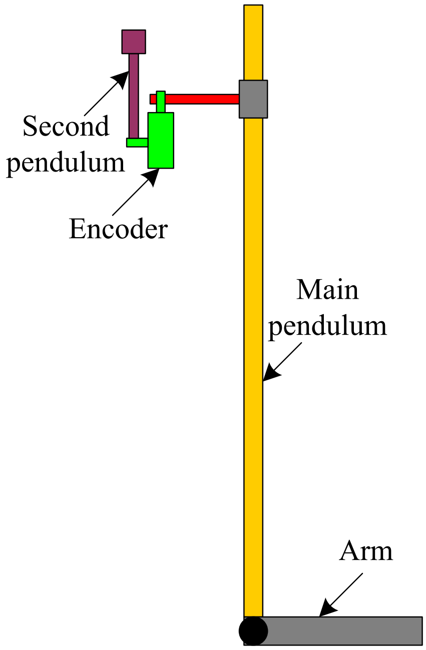

Fig. 12. RIP with additional pendulum.

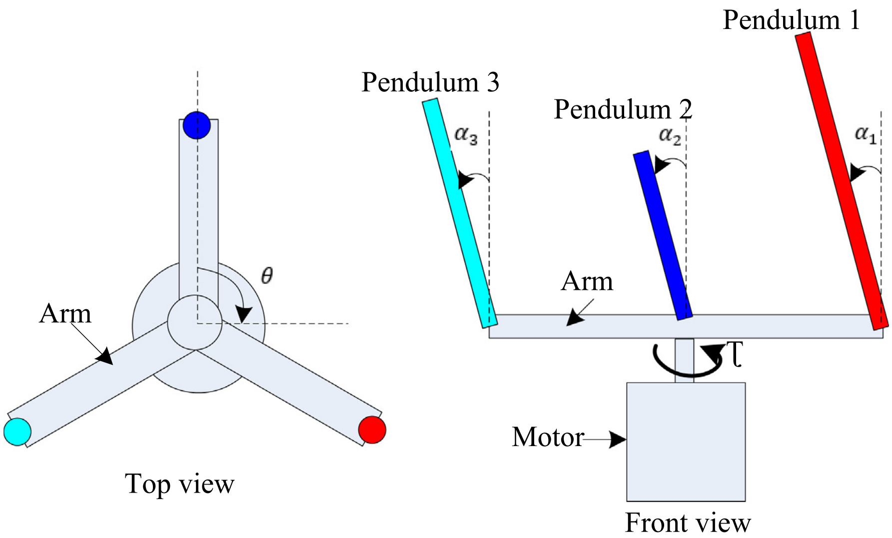

Fig. 13. Triple RIP.

accurate and complete model of the plant. The MC methods are conceptually simple and don't require a complete model, but are not suited for step-by-step incremental computation. The TDL methods are fully incremental and require no model, but are more complex to analyze. These techniques also vary in some ways with respect to their speed of convergence and efficiency [130]. Therefore, the DP, TDL and MC are good opportunity for future research in the ROP application.

We would like to propose the more general inverted pendulum named Two Wheeled Rotary Inverted Pendulum (TWRIP). The TWRIP has all the futures in RIP with a two-wheeled robot. It has 5 DOF consisting of all what is in RIP and all what is in two wheeled robot, as shown in Fig. 14. For the stabilization, the TWRIP will need two controllers; one to stabilize the RIP pendulum and the other to stabilize the whole body of TWRIP on the two tires. Also, the two trajectory tracking controls may be applied; one for the navigation of the whole TWRIP and the other for the RIP arm. The TWRIP can be used for testing intelligent and complex controllers. The TWRIP have many real-life applications, particularly in industries where a robot that is able to move and transport load in form of crane is needed. It can also find application in the military sector where transportation and pointing are needed. The development of TWRIP model is an open research in this field.

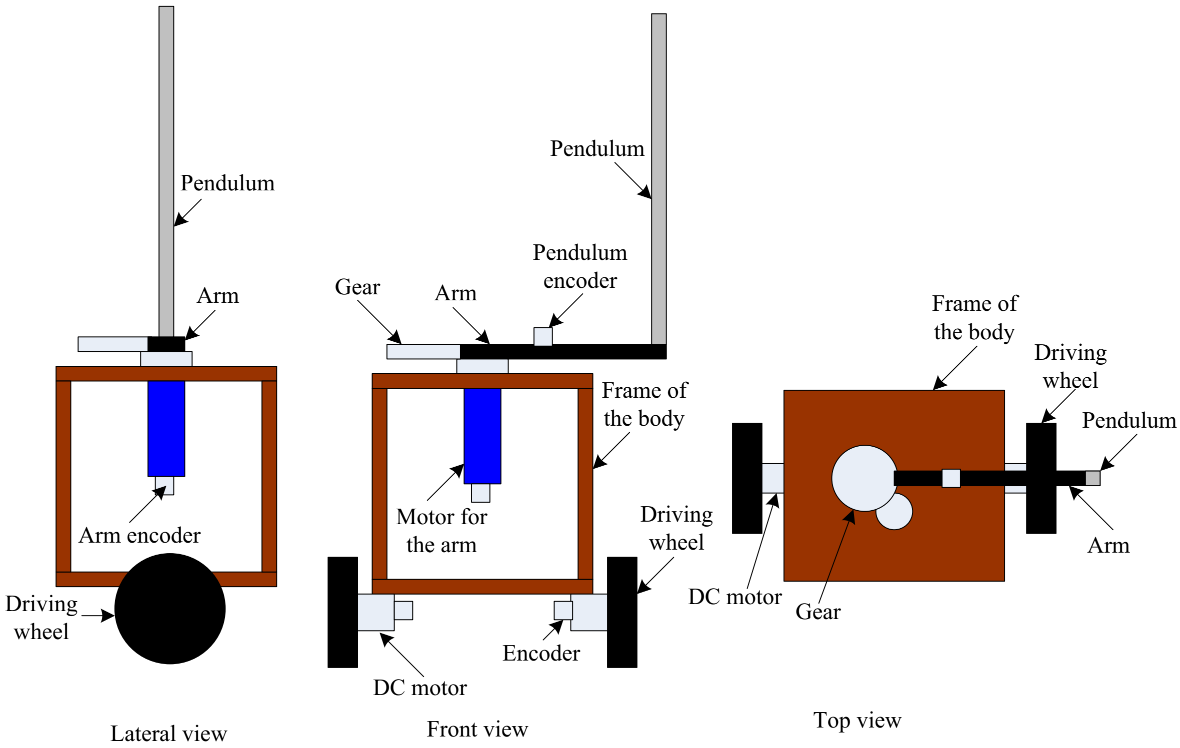

Fig. 14. Proposed TWRIP.

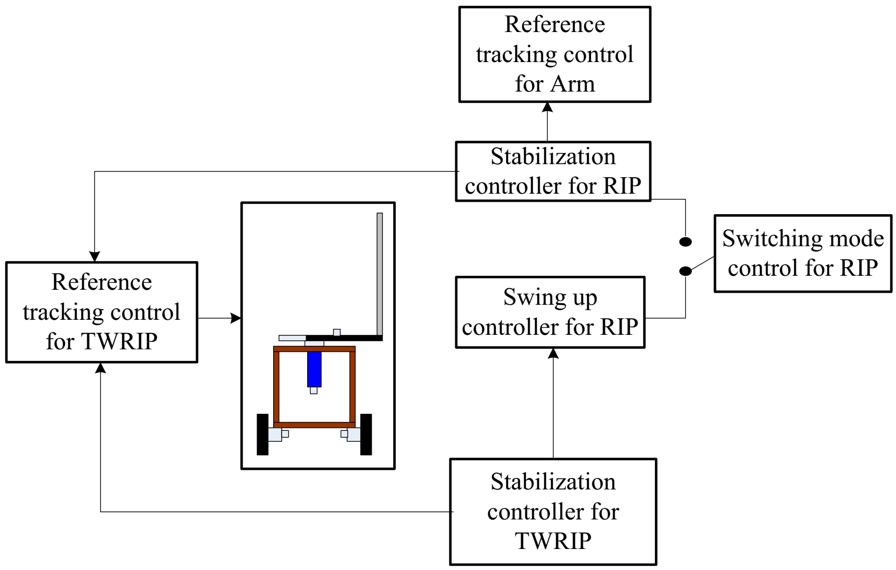

Fig. 15. Illustrative scheme of Control of TWRIP.

The control of TWRIP is schematically illustrated in Fig. 15. It consists of six blocks of control. First, the whole TWRIP is stabilized through the stabilization controller for TWRIP, after it stabilizes at upright position. Then it communicates with the swing-up controller to swing the pendulum to the neighborhood of upright equilibrium position. At this point, the stabilization controller takes over the control action through the switching mode control. After the pendulum stabilizes, the reference tracking controller for the arm starts in such a way that the arm tracks a desired time varying trajectory while the pendulum remains at unstable position. The TWRIP will start following the desired trajectory when both the pendulum and whole TWRIP are stabilized.

7. Limitation of previous methods

Most of the controllers proposed on RIP are for stabilization control objective of RIP. Though some other works deal with the stabilization control together with the swing up control objective which necessitate the introduction of switching control. However, some of these works did not explain their switching control methods. Another limitation is that most of the researchers applied their proposed intelligent self-learning or adaptive nonlinear controllers only on simulations. Although simulations studies can be used to demonstrate robustness to model uncertainties and disturbances, there may be some difficulties in practical implementation of such controllers. These include the selection of the sampling time, sensor noise, and the total lag time for real system. Experimental results would be preferred in order to show the effectiveness and robustness of the controllers due to the real system effects mentioned. Also, the training of unstable ANN controller in real time is uncertain. It is expected that the simulation results should be validated with experiment in real-life settings. Many studies do not compare the performance of their proposed controllers with the state of the art controllers. Therefore, it is difficult or even impossible to measure the effectiveness and robustness of the proposed controllers over the state of the art controllers. Even for those who made the comparison, they could not make a general conclusion. Regarding the switching mode control, some researchers used pendulum angle threshold while others used the energy threshold. All these have the disadvantage of causing some oscillations in the system.

8. Conclusion

Numerous types of controllers with varying performance have been applied to the RIP, even though it has been demonstrated that some simple linear controllers can control the RIP even in the presence of huge un-modelled system. It has also been seen that the research in RIP control is increasing rapidly in the last decade. Theoretically, different models of RIP have been proposed without any experimental validation. Although it has been seen that some advanced controllers have superior robustness or performance, the level to which it is enhanced is not well enumerated. The RIP with more complexity than the standard single RIP was also studied. Based on research conducted over the last decade, several limitations of the proposed control methods were pointed out. Furthermore, the opportunities for further investigations were presented, particularly for experiment validation for the controllers which have only been tested in simulation studies. It has also been shown that there is a need for comparison between proposed controllers and state of the art controllers. Finally, a new, more general system, which comprises the features of RIP and two-wheeled robot, called TWRIP, has been proposed for testing of higher order intelligent controllers

References

[1] N.J. Mathew, K.K. Rao, N. Sivakumaran, Swing up and stabilization control of a rotary inverted pendulum, IFAC Proceedings Volumes (IFAC-PapersOnline (2013) 654–659.
[2] M. Fairus, Z. Mohamed, M. Ahmad, Fuzzy modeling and control of rotary inverted pendulum system using LQR technique, IOP Conf. Ser.: Mater. Sci. Eng. (2013), IOP Publishing 012009.
[3] K. Furuta, M. Yamakita, S. Kobayashi, Swing-up control of inverted pendulum using pseudo-state feedback, Proc. Inst. Mech. Eng. Part I: J. Syst. Control Eng. 206 (1992) 263–269.
[4] J. Acosta, Furuta's pendulum: a conservative nonlinear model for theory and practise, Math. Probl. Eng. 2010 (2010).
[5] C. Aguilar-Avelar, J. Moreno-Valenzuela, A composite controller for trajectory tracking applied to the Furuta pendulum, ISA Trans. (2015).
[6] A. Ferreira, F.J. Bejarano, L.M. Fridman, Robust control with exact uncertainties compensation: with or without chattering?, IEEE Trans Contr. Syst. Technol. 19 (2011) 969–975.
[7] F. Ghorbani, M.A. Shooredeli, M. Teshnehlab, Fault tolerant improvement with chaos synchronization using Fuzzy-PID control, 13th Iranian Conference on Fuzzy Systems, IFSC 2013, 2013.
[8] N. Quyen, N. Thuyen, N.Q. Hoc, N. Hien, Rotary inverted pendulum and control of rotary inverted pendulum by artificial neural network, Proc. Natl. Conf. Theory Phys. (2012) 243–249.
[9] Š. Ileš, J. Matuško, F. Kolonic, TP transformation based control of rotary pendulum, in: MIPRO, 2011 Proceedings of the 34th International Convention, IEEE, 2011, pp. 833–839.
[10] M.F. Hamza, H.J. Yap, I.A. Choudhury, Genetic algorithm and particle swarm optimization based cascade interval type 2 fuzzy PD controller for rotary inverted pendulum system, Math. Probl. Eng. 2015 (2015).
[11] S.-E. Oltean, Swing-up and stabilization of the rotational inverted pendulum using PD and fuzzy-PD controllers, Procedia Technol. 12 (2014) 57–64.
[12] Y.-F. Chen, A.-C. Huang, Adaptive control of rotary inverted pendulum system with time-varying uncertainties, Nonlinear Dyn. 76 (2014) 95–102.
[13] V. Nath, R. Mitra, Swing-up and control of rotary inverted pendulum using pole placement with integrator, 2014 Recent Advances in Engineering and Computational Sciences (RAECS), 2014.
[14] M. Gafvert, J. Svensson, K.J. Astrom, Friction and friction compensation in the Furuta pendulum, in: European Control Conference, ECC 1999 – Conference Proceedings, 2015, pp. 3154–3159.
[15] C. Aguilar-Avelar, J. Moreno-Valenzuela, A feedback linearization controller for trajectory tracking of the Furuta pendulum, in: Proceedings of the American Control Conference, 2014, pp. 4543–4548.
[16] D. Chandran, B. Krishna, V.I. George, I. Thirunavukkarasu, System identification of rotary double inverted pendulum using artificial neural networks, in: 2015 International Conference on Industrial Instrumentation and Control, ICIC 2015, 2015, pp. 612–617.
[17] S. Liu, Stable synchronization of two Furuta pendulums network based on the method of controlled Lagrangian, in: Proceedings of the 33rd Chinese Control Conference, CCC 2014, 2014, pp. 2144–2149.
[18] S. Rudra, R. Barai, M. Maitra, D. Mandal, S. Ghosh, S. Dam, A. Dutta, P. Bhattacharyya, Stabilization of Furuta pendulum: a backstepping based hierarchical sliding Mode approach with disturbance estimation, in: Intelligent Systems and Control (ISCO), 2013 7th International Conference on, IEEE, 2013, pp. 99–105.
[19] J. George, B. Krishna, V.I. George, C. Shreesha, M.K. Menon, Stability analysis and design of PI controller using Kharitnov polynomial for rotary inverted pendulum, Sens. Transducers 138 (2012) 104–113.
[20] O. Boubaker, The inverted pendulum benchmark in nonlinear control theory: a survey, Int. J. Adv. Robot. Syst. 10 (2013).

[21] Y. Tsuge, T. Jennawasin, T. Narikiyo, M. Kawanishi, Nonlinear control of partially known systems based on polynomial representation and reinforcement learning, IEEJ Trans. Electron. Inf. Syst. 135 (2015) 215–224.
[22] V. Petchithai, V.S. Chitra, M. Manimaran, T. Senthilrajan, Design of multivariable systems controlled by novelty based techniques, Indian J. Sci. Technol. 8 (2015) 407–412.
[23] M. Fairus, Z. Mohamed, M. Ahmad, W. Loi, LMI-based multiobjective integral sliding mode control for rotary inverted pendulum system under load variations, Jurnal Teknologi 73 (2015).
[24] A.T. Azar, F.E. Serrano, Adaptive sliding mode control of the furuta pendulum, Stud. Comput. Intell. (2015) 1–42.
[25] M. Antonio-Cruz, R. Silva-Ortigoza, J. Sandoval-Gutierrez, C.A. Merlo-Zapata, H. Taud, C. Marquez-Sanchez, V.M. Hernandez-Guzman, Modeling, simulation, and construction of a furuta pendulum test-bed, in: Electronics, Communications and Computers (CONIELECOMP), 2015 International Conference on, IEEE, 2015, pp. 72–79.
[26] H. Qin, W. Shao, L. Guo, Research and verification on swing-up control algorithm of rotary inverted pendulum, in: 26th Chinese Control and Decision Conference, CCDC 2014, 2014, pp. 4941–4945.
[27] W.-c. Lee, Y.-Y. Lyu, T.-Y. Hsu, C.-c. Wei, Using the Taguchi methods to study the balance control of a rotary inverted pendulum, in: Advanced Robotics and Intelligent Systems (ARIS), 2014 International Conference on, IEEE, 2014, pp. 171–175.
[28] K. Chou, Y. Chen, Energy based swing-up controller design using phase plane method for rotary inverted pendulum, in: Control Automation Robotics & Vision (ICARCV), 2014 13th International Conference on, IEEE, 2014, pp. 975–979.
[29] P. Seman, B. Rohal'-Ilkiv, M. Salaj, Swinging up the Furuta pendulum and its stabilization via model predictive control, J. Electr. Eng. 64 (2013) 152–158.
[30] G. Pujol, L. Acho, A. Ńapoles, J. Estrada, D. Santos, V. Ṕerez-Garćia, A new device laboratory: experimental validation, IFAC Proceedings Volumes (IFAC-PapersOnline) (2013) 226–231.
[31] S. Jadlovský, J. Sarnovský, Modelling of classical and rotary inverted pendulum systems–a generalized approach, J. Electr. Eng. 64 (2013) 12–19.
[32] T. Fabbri, D. Fenucci, S. Falasca, M. Gamba, A. Bicchi, Packet-based dynamic control of a Furuta Pendulum over Ethernet, in: Control & Automation (MED), 2013 21st Mediterranean Conference on, 2013, pp. 464–470.
[33] A. Al-Jodah, H. Zargarzadeh, M.K. Abbas, Experimental verification and comparison of different stabilizing controllers for a rotary inverted pendulum, in: 2013 IEEE International Conference on Control System, Computing and Engineering (ICCSCE 2013), 2013, pp. 417–423.
[34] T. Türker, H. Görgün, G. Cansever, Lyapunov's direct method for stabilization of the Furuta pendulum, Turk. J. Electr. Eng. Comput. Sci. 20 (2012) 99–110.
[35] S.K. Oh, W.D. Kim, W. Pedrycz, Design of optimized cascade fuzzy controller based on differential evolution: simulation studies and practical insights, Eng. Appl. Artif. Intell. 25 (2012) 520–532.
[36] Q. Gao, Y. Li, Generalized predictive control for rotary inverted pendulum system, Appl. Mech. Mater. (2012) 4256–4260.
[37] J. Zhang, Y.P. Zhang, Optimal linear modeling and its applications on swing-up and stabilization control for rotary inverted pendulum, in: 2011 30th Chinese Control Conference (CCC), 2011, pp. 493–500.
[38] Ø.N. Stamnes, O.M. Aamo, G.O. Kaasa, A constructive speed observer design for general EulerLagrange systems, Automatica 47 (2011) 2233–2238.
[39] M. Ramírez-Neria, H. Sira-Ramírez, R. Garrido-Moctezuma, A. Luviano-Juárez, Linear active disturbance rejection control of underactuated systems: the case of the Furuta pendulum, ISA Trans. 53 (2014) 920–928.
[40] M. Ramírez-Neria, H. Sira-Ramírez, R. Garrido-Moctezuma, A. Luviano-Juarez, On the linear active disturbance rejection control of the Furuta pendulum, in: Proceedings of the American Control Conference, 2014, pp. 317–322.
[41] S.-E. Oltean, A.-V. Duka, Balance control system using microcontrollers for a rotational inverted pendulum, Procedia Technol. 12 (2014) 11–19.
[42] P. Faradja, G. Qi, M. Tatchum, Sliding mode control of a Rotary Inverted Pendulum using higher order differential observer, in: International Conference on Control, Automation and Systems, 2014, pp. 1123–1127.
[43] T. Turker, H. Gorgun, G. Cansever, Stabilisation of a class of 2-DOF underactuated mechanical systems via direct Lyapunov approach, Int. J. Control 86 (2013) 1137–1148.
[44] M. Ryalat, D.S. Laila, IDA-PBC for a class of underactuated mechanical systems with application to a rotary inverted pendulum, in: 2013 IEEE 52nd Annual Conference on Decision and Control (CDC), 2013, pp. 5240–5245.
[45] R. Agrawal, R. Mitra, Adaptive neuro fuzzy inference structure controller for rotary inverted pendulum, Adv. Intell. Syst. Comput. (2013) 1163–1170.
[46] B. Alt, C. Hartung, F. Svaricek, Robust fuzzy cascade control revised: application to the rotary inverted pendulum, in: Control & Automation (MED), 2011 19th Mediterranean Conference on, IEEE, 2011, pp. 1472–1477.
[47] B.S. Cazzolato, Z. Prime, On the dynamics of the Furuta pendulum, J. Control Sci. Eng. 2011 (2011) 3.
[48] J.d.J. Rubio, Z. Zamudio, J. Pacheco, D. Mújica Vargas, Proportional derivative control with inverse dead-zone for pendulum systems, Math. Probl. Eng. 2013 (2013).
[49] P. Mandic, M.P. Lazarevic, T.B. Sekara, Fractional order PD control of Furuta pendulum: D-decomposition approach, 2014 International Conference on Fractional Differentiation and Its Applications, ICFDA 2014, 2014.
[50] P. Mandić, M. Lazarević, S. Stojanović, M. Ristanović, Real time control of rotary inverted pendulum, Ann. Faculty Eng. Hunedoara-Int. J. Eng. 12 (2014).
[51] M. Ryalat, D.S. Laila, A simplified IDA-PBC design for underactuated mechanical systems with applications, Eur. J. Control 27 (2016) 1–16.
[52] S. Boonto, H. Werner, Closed-loop identification of LPV models using cubic splines with application to an arm-driven inverted pendulum, in: American Control Conference (ACC). 2010, IEEE, 2010, pp. 3100–3105.
[53] M.G. Zarch, M.A. Shoorehdeli, M. Teshnehlab, Fault detection of nonlinear systems based on Takagi-Sugeno fuzzy models by parity relations, Int. Rev. Autom. Control 5 (2012) 309–315.
[54] Q.V. Dang, B. Allouche, L. Vermeiren, A. Dequidt, M. Dambrine, Design and implementation of a robust fuzzy controller for a rotary inverted pendulum using the Takagi-Sugeno descriptor representation, IEEE SSCI 2014 - 2014 IEEE Symposium Series on Computational Intelligence - CICA 2014: 2014 IEEE Symposium on Computational Intelligence in Control and Automation, Proceedings, 2015.
[55] Z.Q. Long, Y. Yuan, W. Long, Designing fuzzy controllers with variable universes of discourse using input-output data, Eng. Appl. Artif. Intell. 36 (2014) 215–221.
[56] S. Jadlovská, J. Sarnovský, Application of the state-dependent Riccati equation method in nonlinear control design for inverted pendulum systems, in: SISY 2013 - IEEE 11th International Symposium on Intelligent Systems and Informatics, Proceedings, 2013, pp. 209–214.
[57] S. Jadlovska, J. Sarnovsky, A complex overview of modeling and control of the rotary single inverted pendulum system, Adv. Electr. Electron. Eng. 11 (2013) 73–85.
[58] P. Ordaz, A. Poznyak, The Furuta's pendulum stabilization without the use of a mathematical model: attractive ellipsoid method with KL-adaptation, in: Proceedings of the IEEE Conference on Decision and Control, 2012, pp. 7285–7290.
[59] J. Driver, D. Thorpe, Design, Build and Control of a Single/Double Rotational Inverted Pendulum, The University of Adelaide, School of Mechanical Engineering, Australia, 2004, p. 4.
[60] M.A. Shamsudin, R. Mamat, S.W. Nawawi, Dynamic modelling and optimal control scheme of wheel inverted pendulum for mobile robot application, Int. J. Control Theory Comput. Model. 3 (2013) 1–20.
[61] A.I. Isa, M.F. Hamza, Effect of sampling time on PID controller design for a heat exchanger system, in: Adaptive Science & Technology (ICAST), 2014 IEEE 6th International Conference on, IEEE, 2014, pp. 1–8.
[62] A.I. Isa, N. Magaji, M.F. Hamza, Development of hybrid fuzzy PD Plus I controller for voltage stability, 16th International Conference on Mathematical and Computational Methods in Science and Engineering (MACMESE '14), World Scientific and Engineering Academy and Society, Kuala Lumpur Malaysia, 2014. http://www.wseas.org.

[63] W. Nukulwuthiopas, S. Laowattana, T. Maneewarn, Dynamic modeling of a one-wheel robot by using Kane's method, in: Industrial Technology, 2002. IEEE ICIT'02. 2002 IEEE International Conference on, IEEE, 2002, pp. 524–529.
[64] M.F. Hamza, H.J. Yap, I.A. Choudhury, A.I. Isa, Application of Kane's method for dynamic modeling of rotary inverted pendulum system, in: MNTMSiM2016: First International Conference on Micro & Nano Technologies, Modelling & Simulation, IEEE, Malaysia, 2016, pp. 1–7.
[65] R.D. Komistek, J.B. Stiehl, D.A. Dennis, R.D. Paxson, R.W. Soutas-Little, Mathematical model of the lower extremity joint reaction forces using Kane's method of dynamics, J. Biomech. 31 (1997) 185–189.
[66] P. Ernest, P. Horacek, Algorithms for control of a rotating pendulum, Proceeding of the 19th IEEE Mediterranean Conference on Control and Automation (MED'11), National Technical University, Athens, 2011.
[67] C. van Kats, Nonlinear Control of a Furuta Rotary Inverted Pendulum, DCT Report, 2004.
[68] C.M. Roithmayr, D.H. Hodges, Dynamics: Theory and Application of Kane's Method, Cambridge University Press, 2016.
[69] Z. Hussain, N.Z. Azlan, KANE's method for dynamic modeling, in: Automatic Control and Intelligent Systems (I2CACIS), IEEE International Conference on, IEEE, 2016, pp. 174–179.
[70] H. Ashrafiuon, A.M. Whitman, Closed-loop dynamic analysis of a rotary inverted pendulum for control design, J. Dyn. Syst. Meas. Control Trans. ASME 134 (2012).
[71] M. Gäfvert, Dynamic model based friction compensation on the Furuta pendulum, control applications, 1999, in: Proceedings of the 1999 IEEE International Conference on, IEEE, 1999, pp. 1260–1265.
[72] K. Ogata, System Dynamics, Prentice Hall, New Jersey, 1998.
[73] S. Awtar, N. King, T. Allen, I. Bang, M. Hagan, D. Skidmore, K. Craig, Inverted pendulum systems: rotary and arm-driven-a mechatronic system design case study, Mechatronics 12 (2002) 357–370.
[74] F. Altpeter, Friction Modeling, Identification and Compensation, 1999.
[75] D.I. Barbosa, J.S. Castillo, L.F. Combita, Rotary inverted pendulum with real time control, in: Robotics Symposium, 2011 IEEE IX Latin American and IEEE Colombian Conference on Automatic Control and Industry Applications (LARC), IEEE, 2011, pp. 1–6.
[76] H.C.T. Nguyen, A.W. Shen, Using hybrid and LQR method control of a self-erecting rotary inverted pendulum system based on PIC 18F4431, Int. J. Comput. Sci. Inf. Technol. 2 (2011) 2548–2553.
[77] J. Moreno-Valenzuela, C. Aguilar-Avelar, Feedback Linearization Control of the Furuta Pendulum, Motion Control of Underactuated Mechanical Systems, Springer, 2018, pp. 69–92.
[78] S. Kurode, A. Chalanga, B. Bandyopadhyay, Swing-up and stabilization of rotary inverted pendulum using sliding modes, Preprints of the 18th IFAC World Congress Milano (Italy) August, 2011.
[79] J. Aracil, J.Á. Acosta, F. Gordillon, A nonlinear hybrid controller for swinging-up and stabilizing the Furuta pendulum, Control Eng. Pract. 21 (2013) 989–993.
[80] X. Yang, X. Zheng, Swing up and stabilization control design for an underactuated rotary inverted pendulum system: theory and experiments, IEEE Trans. Ind. Electron. (2018).
[81] A. Jabbar, F.M. Malik, S.A. Sheikh, Nonlinear stabilizing control of a rotary double inverted pendulum: a modified backstepping approach, Trans. Inst. Meas. Control 39 (2017) 1721–1734.
[82] B. Prakash, B.K. Roy, R.K. Biswas, Design, implementation and comparison of different controllers for a rotary Inverted Pendulum, Power Electronics, Intelligent Control and Energy Systems (ICPEICES), in: IEEE International Conference on, IEEE, 2016, pp. 1–6.
[83] K.J. Åström, K. Furuta, Swinging up a pendulum by energy control, Automatica 36 (2000) 287–295.
[84] C.C. Cheng, K.S. Yang, J.H. Yang, Robust finite time controller design for second order nonlinear underactuated mechanical systems, Trans. Can. Soc. Mech. Eng. 37 (2013) 549–557.
[85] H.K. Khalil, J. Grizzle, Nonlinear Systems, Prentice Hall, New Jersey, 1996.
[86] S.-K. Oh, S.-H. Jung, W. Pedrycz, Design of optimized fuzzy cascade controllers by means of Hierarchical Fair Competition-based Genetic Algorithms, Expert Syst. Appl. 36 (2009) 11641–11651.
[87] S.K. Oh, H.J. Jang, W. Pedrycz, A comparative experimental study of type-1/type-2 fuzzy cascade controller based on genetic algorithms and particle swarm optimization, Expert Syst. Appl. (2011) 11217–11229.
[88] V. Casanova, J. Alcaína, J. Salt, R. Pizá, Á. Cuenca, Control of the rotary inverted pendulum through threshold-based communication, ISA Trans. (2015).
[89] I. Hassanzadeh, S. Mobayen, Controller design for rotary inverted pendulum system using evolutionary algorithms, Math. Probl. Eng. 2011 (2011).
[90] K.R. Muske, H. Ashrafiuon, S. Nersesov, M. Nikkhah, Optimal sliding mode cascade control for stabilization of underactuated nonlinear systems, J. Dyn. Syst. Trans. ASME 134 (2012).
[91] R.P.M. Chan, K.A. Stol, C.R. Halkyard, Review of modelling and control of two-wheeled robots, Annu. Rev. Control 37 (2013) 89–103.
[92] A. Tewari, Modern Control Design, John Wiley & Sons, NY, 2002, pp. 283–308.
[93] S. Gomes, C. Guimarães, N. Martins, G. Taranto, Damped Nyquist Plot for a pole placement design of power system stabilizers, Electr. Power Syst. Res. 158 (2018) 158–169.
[94] M.H. Amin, Optimal pole shifting for continuous multivariable linear systems, Int. J. Control 41 (1985) 701–707.
[95] B.D. Anderson, J.B. Moore, Optimal Control: Linear Quadratic Methods, Courier Corporation, 2007.
[96] Y. Fujita, M. Izutsu, S. Hatakeyama, Swing-up and stabilization control of twin furuta pendulums by energy control, in: Proceedings, IECON 2014 - 40th Annual Conference of the IEEE Industrial Electronics Society, 2014, pp. 2878–2883.
[97] A. Rojas-Moreno, J. Hernandez-Garagatti, O.P.D.L. Vega, L. Lopez-Lozano, FO based-LQR stabilization of the rotary inverted pendulum, in: Proceedings of the 28th Chinese Control and Decision Conference, CCDC 2016, 2016, pp. 4292–4297.
[98] B. Ufnalski, A. Kaszewski, L.M. Grzesiak, Particle swarm optimization of the multioscillatory LQR for a three-phase four-wire voltage-source inverter with an output filter, Ind. Electron. IEEE Trans. 62 (2015) 484–493.
[99] X. Wei, J.E. Mottershead, Y.M. Ram, Partial pole placement by feedback control with inaccessible degrees of freedom, Mech. Syst. Sig. Process. 70 (2016) 334–344.
[100] E. Videcoq, M. Girault, K. Bouderbala, H. Nouira, J. Salgado, D. Petit, Parametric investigation of Linear Quadratic Gaussian and Model Predictive Control approaches for thermal regulation of a high precision geometric measurement machine, Appl. Therm. Eng. 78 (2015) 720–730.
[101] N.T. Chiluvuri, O.A. Harshe, C.D. Patterson, W.T. Baumann, Using heterogeneous computing to implement a trust isolated architecture for cyber-physical control systems, in: CPSS 2015 - Proceedings of the 1st ACM Workshop on Cyber-Physical System Security, Part of ASIACCS 2015, 2015, pp. 25–35.
[102] B. Hassibi, A.H. Sayed, T. Kailath, Indefinite-Quadratic Estimation and Control: A Unified Approach to H2 and H-Infinity Theories, SIAM, 1999.
[103] P. Dwivedi, S. Pandey, A. Junghare, Performance analysis and experimental validation of 2-DOF fractional-order controller for underactuated rotary inverted pendulum, Arab. J. Sci. Eng. 42 (2017) 5121–5145.
[104] Q. Yan, Output tracking of underactuated rotary inverted pendulum by nonlinear controller, Decision and Control, 2003, in: Proceedings. 42nd IEEE Conference on, IEEE, 2003, pp. 2395–2400.
[105] M. Ramírez-Neria, H. Sira-Ramírez, R. Garrido-Moctezuma, A. Luviano-Juarez, Linear active disturbance rejection control of underactuated systems: the case of the Furuta pendulum, ISA Trans. 53 (2014) 920–928.
[106] J.Y. Hung, W. Gao, J.C. Hung, Variable structure control: a survey, IEEE Trans. Ind. Electron. 40 (1993) 2–22.
[107] F. Pan, D. Xue, D. Chen, X. Jia, Sliding mode variation structure network controller design based on inverted pendulum, in: Proceedings of the 2012 24th Chinese Control and Decision Conference, CCDC 2012, 2012, pp. 960–964.
[108] G. Feng, A survey on analysis and design of model-based fuzzy control systems, IEEE Trans. Fuzzy Syst. 14 (2006) 676–697.
[109] J.H. Li, Composite fuzzy control of a rotary inverted pendulum, IEEE International Symposium on Industrial Electronics, 2013.

[110] M. Ul Hassan, M.B. Kadri, I. Amin, Proficiency of Fuzzy Logic Controller for Stabilization of Rotary Inverted Pendulum Based on LQR Mapping, Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), 2013, pp. 201–211.

[111] M.F. Hamza, H.J. Yap, I.A. Choudhury, Cuckoo search algorithm based design of interval Type-2 Fuzzy PID Controller for Furuta pendulum system, Eng. Appl. Artif. Intell. 62 (2017) 134–151.

[112] H. Ahangar-Asr, M. Teshnehlab, M. Mansouri, A. Pazoki, A hybrid strategy for the control of Rotary Inverted Pendulum, Electrical and Control Engineering (ICECE), in: 2011 International Conference on, IEEE, 2011, pp. 5656–5659.

[113] Y. Tsuge, M. Kawanishi, T. Narikiyo, T. Jennawasin, Nonlinear controller design based on polynomial and non-polynomial representation, in: IEEE International Conference on Control and Automation, ICCA, 2014, pp. 831–838.

[114] P. Dwivedi, S. Pandey, A. Junghare, Robust and novel two degree of freedom fractional controller based on two-loop topology for inverted pendulum, ISA Trans. (2018).

[115] P. Ordaz, A. Poznyak, Adaptive-robust stabilization of the Furuta's pendulum via attractive ellipsoid method, J. Dyn. Syst. Meas. Control Trans. ASME 138 (2016).

[116] J. Moreno-Valenzuela, C. Aguilar-Avelar, S.A. Puga-Guzman, V. Santibanez, Adaptive Neural network control for the trajectory tracking of the Furuta pendulum, IEEE Trans. Cybern. (2016).

[117] R. Hercus, K.Y. Wong, S.K. Shee, K.F. Ho, Control of an inverted pendulum using the NeuraBase network model, Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), 2013, pp. 605–615.

[118] R. Hercus, K.Y. Wong, K.F. Ho, Balancing of a simulated inverted pendulum using the neurabase network model, Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), 2013, pp. 527–536.

[119] Y. Singh, A. Kumar, R. Mitra, Design of ANFIS controller based on fusion function for rotary inverted pendulum, in: Advances in Power Conversion and Energy Technologies (APCET), 2012 International Conference on, IEEE, 2012, pp. 1–5.

[120] J. de Jesús Rubio, Discrete time control based in neural networks for pendulums, Appl. Soft Comput. (2017).

[121] G. Ma, W. Cai, Design and application of model free controller, in: Electric Information and Control Engineering (ICEICE), 2011 International Conference on, IEEE, 2011, pp. 2058–2061.

[122] Z. Han, Designing problem of model free controller, Control Eng. China 9 (2002) 19–22.

[123] W.-H. Chen, D.J. Ballance, P.J. Gawthrop, J. O'Reilly, A nonlinear disturbance observer for robotic manipulators, Ind. Electron. IEEE Trans. 47 (2000) 932–938.

[124] A. Astolfi, R. Ortega, A. Venkatraman, A globally exponentially convergent immersion and invariance speed observer for mechanical systems with non-holonomic constraints, Automatica 46 (2010) 182–189.

[125] X. Zhao, Z. Zhang, J. Huang, Energy-based swing up control of rotary parallel inverted pendulum, in: Intelligent Control and Automation (WCICA), 2016 12th World Congress on, IEEE, 2016, pp. 666–671.

[126] V. Casanova, J. Salt, R. Piza, A. Cuenca, Controlling the double rotary inverted pendulum with multiple feedback delays, Int. J. Comput. Commun. Control 7 (2012) 20–38.

[127] K. Sugawara, M. Izutsu, S. Hatakeyama, M. Iwase, Stabilization of Triple Furuta pendulum by sliding mode control, in: Proceedings, IECON 2014 - 40th Annual Conference of the IEEE Industrial Electronics Society, 2014, pp. 2872–2877.

[128] J. Aracil, F. Gordillo, A family of smooth controllers for swinging up a pendulum, Automatica 44 (2008) 1841–1848.

[129] C.G. Atkeson, J.C. Santamaria, A comparison of direct and model-based reinforcement learning, Robotics and Automation, 1997, in: Proceedings, 1997 IEEE International Conference on, IEEE, 1997, pp. 3557–3564.

[130] R.S. Sutton, A.G. Barto, Reinforcement Learning: An Introduction, MIT Press Cambridge, 1998.