Linear Control Project

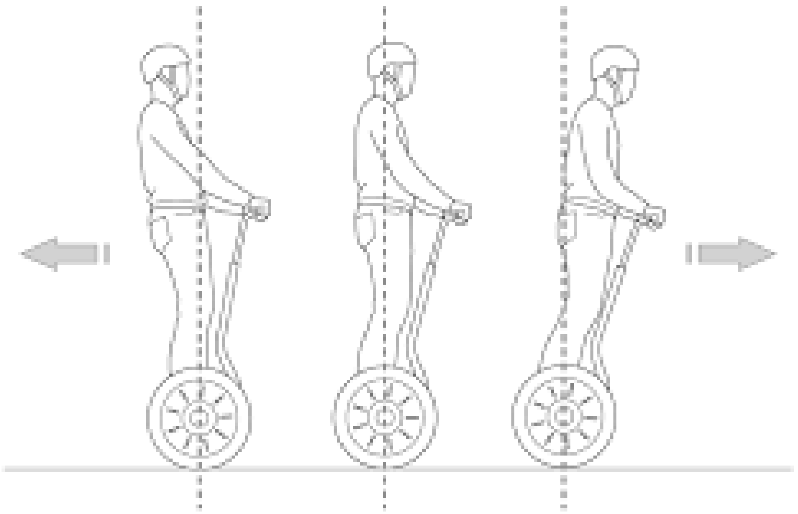
OUR TEAM:

Mohammad Ismail baki (9723007)
Arman Elyasi (9723011)
Maryam Barazandeh (9723016)
AliReza Tabatabaeian (9723052)

Amirkabir University of Technology
(Tehran Polytechnic)

Professor
Iman Sharifi
&
Dr. Elahe
Radmanesh

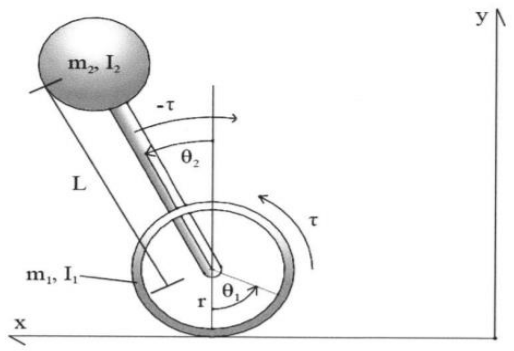
THE PRESENT IS THEIRS, THE FUTURE, FOR WHICH I REALLY WORK, IS MINE.



Project Title: Segway

Segway is a robotic system uses for transportation which is self-balancing and is classified as safe transportation robots. Inside & Outside of this robot, there are two Wheels, Gyroscope Sensors, a DC motor, Gearbox, Batteries, Microchips, Handlebar and Pedals for Standing on the Segway.

First of all, in order to analyze a system, we have to find the dynamical equations of that system. Since our robot is moving on a straight line, we can model it on a 2Dimentional paper.



$\tau + b\,(\dot{\theta}_2 - \dot{\theta}_1) = H_1\ddot{\theta}_1 + H_3\ddot{\theta}_2 - m_2 r L \sin(\theta_2)$

$-\tau - b\,(\dot{\theta}_2 - \dot{\theta}_1) = H_3\ddot{\theta}_1 + H_2\ddot{\theta}_2 - m_2 g L \sin(\theta_2)$

$$H = \begin{bmatrix} H_1 & H_3 \\ H_3 & H_2 \end{bmatrix} = \begin{bmatrix} (m_1 + m_2).r^2 + I_1 & m_2 r L \cos\theta_2 \\ m_2 r L \cos\theta_2 & m_2 L^2 + I_2 \end{bmatrix}$$

The user applies a torque by his/her weight toward then the robot corresponds to the users torque and apply another torque to both wheels and make it move. The robot corresponds using its dc motor. Then we have fractional losses which is modeled by ((b)) coefficient corresponding with angular velocity. Next, we see I_1 & I_2 which are inertia of masses. These equations are obtained by applying the second rule of newton.

Reference of the dynamical equations: (Page 22)

http://dspace.mit.edu/bitstream/handle/1721.1/69500/775672333-MIT.pdf?sequence=2

Alright now we need to linearization these equations near their quiescent point in order to write them in the state space model.
Step 1: users apply a torque slowly. So derivative is small and second order is smaller such a way that it is ignorable.
So $(\dot{\theta}_2)^2 \approx 0$
Step 2: according to first order McLaurin series (because its quiescent point is 0), $\cos\theta_2 \approx 1$ and $\sin(\theta_2) \approx \theta_2$
Now we have:

$$\tau + b\,(\dot{\theta}_2 - \dot{\theta}_1) = H_1\ddot{\theta}_1 + H_3\ddot{\theta}_2$$

$$-\tau - b\,(\dot{\theta}_2 - \dot{\theta}_1) = H_3\ddot{\theta}_1 + H_2\ddot{\theta}_2 - m_2 g L\theta_2$$

$$H = \begin{bmatrix} H_1 & H_3 \\ H_3 & H_2 \end{bmatrix} = \begin{bmatrix} (m_1 + m_2).r^2 + I_1 & m_2 r L \\ m_2 r L & m_2 L^2 + I_2 \end{bmatrix}$$

Now we're gonna write these equations in a normalized form which is called State Space form. We need to consider some assumptions:

$X_1 = \theta_1$

$X_2 = \dot{\theta}_1$

$X_3 = \theta_2$

$X_4 = \dot{\theta}_2$

Now we apply these assumptions into our equations:

$\tau + b\,(X_4 - X_2) = H_1\dot{X}_2 + H_3\dot{X}_4$

$-\tau - b\,(X_4 - X_2) = H_3\dot{X}_2 + H_2\dot{X}_4 - m_2 g L X_3$

$\dot{X}_1 = X_2$

$\dot{X}_3 = X_4$

From these equations we conclude that:

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\\ \dot{X}_4\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & \frac{-b(H_2+H_3)}{H_1H_2-H_3{}^2} & \frac{-H_3m_2gL}{H_1H_2-H_3{}^2} & \frac{b(H_2+H_3)}{H_1H_2-H_3{}^2}\\ 0 & 0 & 0 & 1\\ 0 & \frac{b(H_1+H_3)}{H_1H_2-H_3{}^2} & \frac{H_1m_2gL}{H_1H_2-H_3{}^2} & \frac{-b(H_1+H_3)}{H_1H_2-H_3{}^2}\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ X_3\\ X_4\end{bmatrix}+\begin{bmatrix}0\\ \frac{(H_2+H_3)}{H_1H_2-H_3{}^2}\\ 0\\ \frac{-(H_1+H_3)}{H_1H_2-H_3{}^2}\end{bmatrix}[\tau]$$

$$\begin{bmatrix}Y_1\\ Y_2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ X_3\\ X_4\end{bmatrix}$$

$\dot{X}$ = AX + BU

Y = CX

$$\emptyset(s) = |SI - A| = \begin{vmatrix} S & -1 & 0 & 0 \\ 0 & S + \frac{b(H_2+H_3)}{H_1H_2-H_3{}^2} & \frac{H_3m_2gL}{H_1H_2-H_3{}^2} & \frac{-b(H_2+H_3)}{H_1H_2-H_3{}^2} \\ 0 & 0 & S & -1 \\ 0 & \frac{-b(H_1+H_3)}{H_1H_2-H_3{}^2} & \frac{-H_1m_2gL}{H_1H_2-H_3{}^2} & S + \frac{b(H_1+H_3)}{H_1H_2-H_3{}^2} \end{vmatrix} =$$

$$S[\,(S)\left(\left(S + \frac{b(H_2+H_3)}{H_1H_2-H_3{}^2}\right)\left(S + \frac{b(H_1+H_3)}{H_1H_2-H_3{}^2}\right) - \left(\frac{-b(H_2+H_3)}{H_1H_2-H_3{}^2}\right)\left(\frac{-b(H_1+H_3)}{H_1H_2-H_3{}^2}\right)\right) +$$

$$-(-1)\left(\left(S + \frac{b(H_2+H_3)}{H_1H_2-H_3{}^2}\right)\left(\frac{-H_1m_2gL}{H_1H_2-H_3{}^2}\right) - \left(\frac{H_3m_2gL}{H_1H_2-H_3{}^2}\right)\left(\frac{-b(H_1+H_3)}{H_1H_2-H_3{}^2}\right)\right)\,]=$$

$$(S)\left(S^3 + \frac{b(H_1+H_2+2H_3)}{H_1H_2-H_3{}^2}S^2 - \frac{H_1m_2gL}{H_1H_2-H_3{}^2}S - \frac{bm_2gL}{H_1H_2-H_3{}^2}\right)=0$$

Using wolfram, we solved the upper equation:

$$x = 0\,, \quad h1\,h2 - h3^2 \neq 0$$

$$\begin{aligned}
& h1\,h2 - h3^2 \neq 0\,,\\
& x = -\frac{b\,h1 + b\,h2 + 2\,b\,h3}{3\,(h1\,h2 - h3^2)} + \Big(\big(27\,b\,g\,L\,m2\,h3^4 - 16\,b^3\,h3^3 + 18\,b\,g\,h1\,L\,m2\,h3^3 - 24\,b^3\,h1\,h3^2 - 24\,b^3\,h2\,h3^2 + 9\,b\,g\,h1^2\,L\,m2\,h3^2 - 45\,b\,g\,h1\,h2\,L\,m2\,h3^2 - 12\,b^3\,h1^2\,h3 - 12\,b^3\,h2^2\,h3 -\\
& \quad 24\,b^3\,h1\,h2\,h3 - 18\,b\,g\,h1^2\,h2\,L\,m2\,h3 - 2\,b^3\,h1^3 - 2\,b^3\,h2^3 - 6\,b^3\,h1\,h2^2 - 6\,b^3\,h1^2\,h2 + 18\,b\,g\,h1^2\,h2^2\,L\,m2 - 9\,b\,g\,h1^3\,h2\,L\,m2 +\\
& \quad \sqrt{\big(4\,(-(b\,h1 + b\,h2 + 2\,b\,h3)^2 - 3\,g\,h1\,(h1\,h2 - h3^2)\,L\,m2)^3 + (27\,b\,g\,L\,m2\,h3^4 - 16\,b^3\,h3^3 + 18\,b\,g\,h1\,L\,m2\,h3^3 - 24\,b^3\,h1\,h3^2 - 24\,b^3\,h2\,h3^2 + 9\,b\,g\,h1^2\,L\,m2\,h3^2 - 45\,b\,g\,h1\,h2\,L\,m2\,h3^2 - 12\,b^3\,h1^2\,h3 - 12\,b^3\,h2^2}\\
& \quad \overline{h3 - 24\,b^3\,h1\,h2\,h3 - 18\,b\,g\,h1^2\,h2\,L\,m2\,h3 - 2\,b^3\,h1^3 - 2\,b^3\,h2^3 - 6\,b^3\,h1\,h2^2 - 6\,b^3\,h1^2\,h2 + 18\,b\,g\,h1^2\,h2^2\,L\,m2 - 9\,b\,g\,h1^3\,h2\,L\,m2)^2\big)}\big)\,{}^\wedge(1/3)\Big)\Big/\\
& \quad \big(3\,\sqrt[3]{2}\,(h1\,h2 - h3^2)\big) - \big(\sqrt[3]{2}\,(-(b\,h1 + b\,h2 + 2\,b\,h3)^2 - 3\,g\,h1\,(h1\,h2 - h3^2)\,L\,m2)\big)\Big/\\
& \quad \Big(3\,(h1\,h2 - h3^2)\,\big(27\,b\,g\,L\,m2\,h3^4 - 16\,b^3\,h3^3 + 18\,b\,g\,h1\,L\,m2\,h3^3 - 24\,b^3\,h1\,h3^2 - 24\,b^3\,h2\,h3^2 + 9\,b\,g\,h1^2\,L\,m2\,h3^2 - 45\,b\,g\,h1\,h2\,L\,m2\,h3^2 - 12\,b^3\,h1^2\,h3 - 12\,b^3\,h2^2\,h3 -\\
& \quad 24\,b^3\,h1\,h2\,h3 - 18\,b\,g\,h1^2\,h2\,L\,m2\,h3 - 2\,b^3\,h1^3 - 2\,b^3\,h2^3 - 6\,b^3\,h1\,h2^2 - 6\,b^3\,h1^2\,h2 + 18\,b\,g\,h1^2\,h2^2\,L\,m2 - 9\,b\,g\,h1^3\,h2\,L\,m2 +\\
& \quad \sqrt{\big(4\,(-(b\,h1 + b\,h2 + 2\,b\,h3)^2 - 3\,g\,h1\,(h1\,h2 - h3^2)\,L\,m2)^3 + (27\,b\,g\,L\,m2\,h3^4 - 16\,b^3\,h3^3 + 18\,b\,g\,h1\,L\,m2\,h3^3 - 24\,b^3\,h1\,h3^2 - 24\,b^3\,h2\,h3^2 + 9\,b\,g\,h1^2\,L\,m2\,h3^2 - 45\,b\,g\,h1\,h2\,L\,m2\,h3^2 - 12\,b^3\,h1^2\,h3 - 12\,b^3\,h2^2}\\
& \quad \overline{h3 - 24\,b^3\,h1\,h2\,h3 - 18\,b\,g\,h1^2\,h2\,L\,m2\,h3 - 2\,b^3\,h1^3 - 2\,b^3\,h2^3 - 6\,b^3\,h1\,h2^2 - 6\,b^3\,h1^2\,h2 + 18\,b\,g\,h1^2\,h2^2\,L\,m2 - 9\,b\,g\,h1^3\,h2\,L\,m2)^2\big)}\big)\,{}^\wedge(1/3)\Big)
\end{aligned}$$

$$\begin{aligned}
& h1\,h2 - h3^2 \neq 0\,, \quad x = -\frac{b\,h1 + b\,h2 + 2\,b\,h3}{3\,(h1\,h2 - h3^2)} -\\
& \quad \Big((1 - i\sqrt{3})\,\big(27\,b\,g\,L\,m2\,h3^4 - 16\,b^3\,h3^3 + 18\,b\,g\,h1\,L\,m2\,h3^3 - 24\,b^3\,h1\,h3^2 - 24\,b^3\,h2\,h3^2 + 9\,b\,g\,h1^2\,L\,m2\,h3^2 - 45\,b\,g\,h1\,h2\,L\,m2\,h3^2 - 12\,b^3\,h1^2\,h3 - 12\,b^3\,h2^2\,h3 - 24\,b^3\,h1\,h2\,h3 -\\
& \quad 18\,b\,g\,h1^2\,h2\,L\,m2\,h3 - 2\,b^3\,h1^3 - 2\,b^3\,h2^3 - 6\,b^3\,h1\,h2^2 - 6\,b^3\,h1^2\,h2 + 18\,b\,g\,h1^2\,h2^2\,L\,m2 - 9\,b\,g\,h1^3\,h2\,L\,m2 +\\
& \quad \sqrt{\big(4\,(-(b\,h1 + b\,h2 + 2\,b\,h3)^2 - 3\,g\,h1\,(h1\,h2 - h3^2)\,L\,m2)^3 + (27\,b\,g\,L\,m2\,h3^4 - 16\,b^3\,h3^3 + 18\,b\,g\,h1\,L\,m2\,h3^3 - 24\,b^3\,h1\,h3^2 - 24\,b^3\,h2\,h3^2 + 9\,b\,g\,h1^2\,L\,m2\,h3^2 - 45\,b}\\
& \quad \overline{g\,h1\,h2\,L\,m2\,h3^2 - 12\,b^3\,h1^2\,h3 - 12\,b^3\,h2^2\,h3 - 24\,b^3\,h1\,h2\,h3 - 18\,b\,g\,h1^2\,h2\,L\,m2\,h3 - 2\,b^3\,h1^3 - 2\,b^3\,h2^3 - 6\,b^3\,h1\,h2^2 - 6\,b^3\,h1^2\,h2 + 18\,b\,g\,h1^2\,h2^2\,L\,m2 - 9\,b\,g\,h1^3\,h2\,L\,m2)^2\big)}\big)\,{}^\wedge(1/3)\Big)\Big/\\
& \quad \big(6\,\sqrt[3]{2}\,(h1\,h2 - h3^2)\big) + \big((1 + i\sqrt{3})\,(-(b\,h1 + b\,h2 + 2\,b\,h3)^2 - 3\,g\,h1\,(h1\,h2 - h3^2)\,L\,m2)\big)\Big/\\
& \quad \Big(3 \times 2^{2/3}\,(h1\,h2 - h3^2)\,\big(27\,b\,g\,L\,m2\,h3^4 - 16\,b^3\,h3^3 + 18\,b\,g\,h1\,L\,m2\,h3^3 - 24\,b^3\,h1\,h3^2 - 24\,b^3\,h2\,h3^2 + 9\,b\,g\,h1^2\,L\,m2\,h3^2 - 45\,b\,g\,h1\,h2\,L\,m2\,h3^2 - 12\,b^3\,h1^2\,h3 - 12\,b^3\,h2^2\,h3 -\\
& \quad 24\,b^3\,h1\,h2\,h3 - 18\,b\,g\,h1^2\,h2\,L\,m2\,h3 - 2\,b^3\,h1^3 - 2\,b^3\,h2^3 - 6\,b^3\,h1\,h2^2 - 6\,b^3\,h1^2\,h2 + 18\,b\,g\,h1^2\,h2^2\,L\,m2 - 9\,b\,g\,h1^3\,h2\,L\,m2 +\\
& \quad \sqrt{\big(4\,(-(b\,h1 + b\,h2 + 2\,b\,h3)^2 - 3\,g\,h1\,(h1\,h2 - h3^2)\,L\,m2)^3 + (27\,b\,g\,L\,m2\,h3^4 - 16\,b^3\,h3^3 + 18\,b\,g\,h1\,L\,m2\,h3^3 - 24\,b^3\,h1\,h3^2 - 24\,b^3\,h2\,h3^2 + 9\,b\,g\,h1^2\,L\,m2\,h3^2 - 45\,b\,g\,h1\,h2\,L\,m2\,h3^2 - 12\,b^3\,h1^2\,h3 - 12\,b^3\,h2^2}\\
& \quad \overline{h3 - 24\,b^3\,h1\,h2\,h3 - 18\,b\,g\,h1^2\,h2\,L\,m2\,h3 - 2\,b^3\,h1^3 - 2\,b^3\,h2^3 - 6\,b^3\,h1\,h2^2 - 6\,b^3\,h1^2\,h2 + 18\,b\,g\,h1^2\,h2^2\,L\,m2 - 9\,b\,g\,h1^3\,h2\,L\,m2)^2\big)}\big)\,{}^\wedge(1/3)\Big)
\end{aligned}$$

$$\begin{aligned}
& h1\,h2 - h3^2 \neq 0\,, \quad x = -\frac{b\,h1 + b\,h2 + 2\,b\,h3}{3\,(h1\,h2 - h3^2)} -\\
& \quad \Big((1 + i\sqrt{3})\,\big(27\,b\,g\,L\,m2\,h3^4 - 16\,b^3\,h3^3 + 18\,b\,g\,h1\,L\,m2\,h3^3 - 24\,b^3\,h1\,h3^2 - 24\,b^3\,h2\,h3^2 + 9\,b\,g\,h1^2\,L\,m2\,h3^2 - 45\,b\,g\,h1\,h2\,L\,m2\,h3^2 - 12\,b^3\,h1^2\,h3 - 12\,b^3\,h2^2\,h3 - 24\,b^3\,h1\,h2\,h3 -\\
& \quad 18\,b\,g\,h1^2\,h2\,L\,m2\,h3 - 2\,b^3\,h1^3 - 2\,b^3\,h2^3 - 6\,b^3\,h1\,h2^2 - 6\,b^3\,h1^2\,h2 + 18\,b\,g\,h1^2\,h2^2\,L\,m2 - 9\,b\,g\,h1^3\,h2\,L\,m2 +\\
& \quad \sqrt{\big(4\,(-(b\,h1 + b\,h2 + 2\,b\,h3)^2 - 3\,g\,h1\,(h1\,h2 - h3^2)\,L\,m2)^3 + (27\,b\,g\,L\,m2\,h3^4 - 16\,b^3\,h3^3 + 18\,b\,g\,h1\,L\,m2\,h3^3 - 24\,b^3\,h1\,h3^2 - 24\,b^3\,h2\,h3^2 + 9\,b\,g\,h1^2\,L\,m2\,h3^2 - 45\,b}\\
& \quad \overline{g\,h1\,h2\,L\,m2\,h3^2 - 12\,b^3\,h1^2\,h3 - 12\,b^3\,h2^2\,h3 - 24\,b^3\,h1\,h2\,h3 - 18\,b\,g\,h1^2\,h2\,L\,m2\,h3 - 2\,b^3\,h1^3 - 2\,b^3\,h2^3 - 6\,b^3\,h1\,h2^2 - 6\,b^3\,h1^2\,h2 + 18\,b\,g\,h1^2\,h2^2\,L\,m2 - 9\,b\,g\,h1^3\,h2\,L\,m2)^2\big)}\big)\,{}^\wedge(1/3)\Big)\Big/\\
& \quad \big(6\,\sqrt[3]{2}\,(h1\,h2 - h3^2)\big) + \big((1 - i\sqrt{3})\,(-(b\,h1 + b\,h2 + 2\,b\,h3)^2 - 3\,g\,h1\,(h1\,h2 - h3^2)\,L\,m2)\big)\Big/\\
& \quad \Big(3 \times 2^{2/3}\,(h1\,h2 - h3^2)\,\big(27\,b\,g\,L\,m2\,h3^4 - 16\,b^3\,h3^3 + 18\,b\,g\,h1\,L\,m2\,h3^3 - 24\,b^3\,h1\,h3^2 - 24\,b^3\,h2\,h3^2 + 9\,b\,g\,h1^2\,L\,m2\,h3^2 - 45\,b\,g\,h1\,h2\,L\,m2\,h3^2 - 12\,b^3\,h1^2\,h3 - 12\,b^3\,h2^2\,h3 -\\
& \quad 24\,b^3\,h1\,h2\,h3 - 18\,b\,g\,h1^2\,h2\,L\,m2\,h3 - 2\,b^3\,h1^3 - 2\,b^3\,h2^3 - 6\,b^3\,h1\,h2^2 - 6\,b^3\,h1^2\,h2 + 18\,b\,g\,h1^2\,h2^2\,L\,m2 - 9\,b\,g\,h1^3\,h2\,L\,m2 +\\
& \quad \sqrt{\big(4\,(-(b\,h1 + b\,h2 + 2\,b\,h3)^2 - 3\,g\,h1\,(h1\,h2 - h3^2)\,L\,m2)^3 + (27\,b\,g\,L\,m2\,h3^4 - 16\,b^3\,h3^3 + 18\,b\,g\,h1\,L\,m2\,h3^3 - 24\,b^3\,h1\,h3^2 - 24\,b^3\,h2\,h3^2 + 9\,b\,g\,h1^2\,L\,m2\,h3^2 - 45\,b\,g\,h1\,h2\,L\,m2\,h3^2 - 12\,b^3\,h1^2\,h3 - 12\,b^3\,h2^2}\\
& \quad \overline{h3 - 24\,b^3\,h1\,h2\,h3 - 18\,b\,g\,h1^2\,h2\,L\,m2\,h3 - 2\,b^3\,h1^3 - 2\,b^3\,h2^3 - 6\,b^3\,h1\,h2^2 - 6\,b^3\,h1^2\,h2 + 18\,b\,g\,h1^2\,h2^2\,L\,m2 - 9\,b\,g\,h1^3\,h2\,L\,m2)^2\big)}\big)\,{}^\wedge(1/3)\Big)
\end{aligned}$$

Now we are going to find the transfer functions but their calculations are too difficult so we're going to substitute the real values which are true and useable in the real world (we used manual of the manufacturer):

Table 2: Segway PT Specifications (cont.)

Model Specification	i2 SE	x2 SE
Dimensions		
Machine Weight Without Batteries	82 lbs (37 kg)	96 lbs (44 kg)
Battery Weight	See **Table 11** on page 86.	
Ground Clearance (Unloaded)	3.4 in (8.5 cm)	4.4 in (11.2 cm)
Machine Length and Width	25.5 x 25 in (65 x 63 cm)	26.5 x 33 in (67 x 84 cm)
Handlebar Height from Mat	38 - 43 in (97 - 109 cm)	38 - 43 in (97 - 109 cm)
Handlebar Height from Ground (Unloaded)	46 - 51 in (117 - 130 cm)	47 - 52 in (119 - 132 cm)
Powerbase Height (Unloaded)	8 in (20 cm)	9 in (22.9 cm)
Tire Diameter	19 in (48.3 cm)	21 in (53.3 cm)
Tire Type	Standard	All-terrain
InfoKey Controller		
Battery	CR2430 replacement batteries are available at electronics stores.	

Intro

NOTE:

* See "Weight Limits for Riders and Cargo" (p. 14) for more information on weight limits.

** See "Maximizing Range" (p. 16) for more information on maximizing the distance you can travel on your Segway PT, and factors that can increase or reduce your range.

Download link of the manual paper: (Page 13)

https://static4.segway.com/wp-content/uploads/2019/09/24010-00001_ab_pr_se_user_manual.pdf

13

M1 = 30 kg
M2 = 10 kg
R = 25 cm = 0.25 m
L = 1 m
I_1 = 7.5
I_2 = 10
b (Normal Friction Coefficient) = 0.7
g ≈ 10

So:

H = $\begin{bmatrix} H_1 & H_3 \\ H_3 & H_2 \end{bmatrix} = \begin{bmatrix} 10 & 2.5 \\ 2.5 & 20 \end{bmatrix}$ B = $\begin{bmatrix} 0 \\ 0.116 \\ 0 \\ -0.064 \end{bmatrix}$

$[SI - A] = \begin{bmatrix} S & -1 & 0 & 0 \\ 0 & S + 0.0812 & 1.29 & -0.0812 \\ 0 & 0 & S & -1 \\ 0 & -0.045 & -5.161 & S + 0.045 \end{bmatrix}$

$\emptyset(S) = S(S^3 + 0.126S^2 - 5.161S - 0.361) = 0$

$S_1 = -2.30087$ / $S_2 = -0.0698946$ / $S_3 = 0$ / $S_4 = 2.24477$

Here, we have two TF cause we have two different output which are θ_1 & θ_2 :

$Y = C[SI - A]^{-1} B\, \tau$

$H = C[SI - A]^{-1} B$

$[SI - A]^{-1}$ **(picture below):**

Result:

$$\begin{pmatrix} \frac{1}{s} & \frac{s^2+0.045\,s-5.161}{s\,(s^3+0.1262\,s^2-5.161\,s-0.361023)} & \frac{0.361023-1.29\,s}{s\,(s^3+0.1262\,s^2-5.161\,s-0.361023)} & \frac{0.0812\,s-1.29}{s\,(s^3+0.1262\,s^2-5.161\,s-0.361023)} \\ 0 & \frac{s^2+0.045\,s-5.161}{s^3+0.1262\,s^2-5.161\,s-0.361023} & \frac{0.361023-1.29\,s}{s^3+0.1262\,s^2-5.161\,s-0.361023} & \frac{0.0812\,s-1.29}{s^3+0.1262\,s^2-5.161\,s-0.361023} \\ 0 & \frac{0.045}{s^3+0.1262\,s^2-5.161\,s-0.361023} & \frac{s^3+0.1262\,s^2}{s\,(s^3+0.1262\,s^2-5.161\,s-0.361023)} & \frac{s+0.0812}{s^3+0.1262\,s^2-5.161\,s-0.361023} \\ 0 & \frac{0.045\,s}{s^3+0.1262\,s^2-5.161\,s-0.361023} & \frac{5.161\,s+0.361023}{s^3+0.1262\,s^2-5.161\,s-0.361023} & \frac{s\,(s+0.0812)}{s^3+0.1262\,s^2-5.161\,s-0.361023} \end{pmatrix}$$

$$H = \begin{bmatrix} \frac{\theta_1}{\tau} \\ \frac{\theta_2}{\tau} \end{bmatrix} = \begin{bmatrix} \frac{0.116(S^2 + 0.0002S - 4.449)}{S(S^3+0.126S^2-5.161S-0.361)} \\ \frac{-0.064(S-0.0003)}{S^3+0.126S^2-5.161S-0.361} \end{bmatrix}$$

As inverse of (SI-A) is too long to write, we calculate H by symbolab.com online.
We also used wolfram using code written bellow(for inversing and multiplying):
{{1,0,0,0},{0,0,1,0}}*inversematrix{{s,-1,0,0},{0,s+0.0812,1.29,-0.0812},{0,0,s,-1},{0,-0.045,-5.161,s+0.045}}*{{0},{0.116},{0},{-0.064}}
till now, we were working on both outputs but for achieve balancing stability, the desired output is θ_2 and we just analyze θ_2 .
Poles of TF: $S_1 = -2.30087$ / $S_2 = -0.0698946$ / $S_4 = 2.24477$
Zeros of TF: $s=0.0003\approx 0$
Our system is of zero kind. (actually, θ_1 is first kind and θ_2 is zero kind but as we said, we just talk about θ_2)
The step response is unstable and it seems that is doesn't have any delay because it started from zero on time axis but it might have a Padé approximation of time delay which is ignorable.

The system is unstable so it's better to not talk about its minimum phase but there are 2 reasons that we can have a nonminimum phase system: we have a zero on imaginary axis so it can be nonminimum phase. And also, the gain is negative.

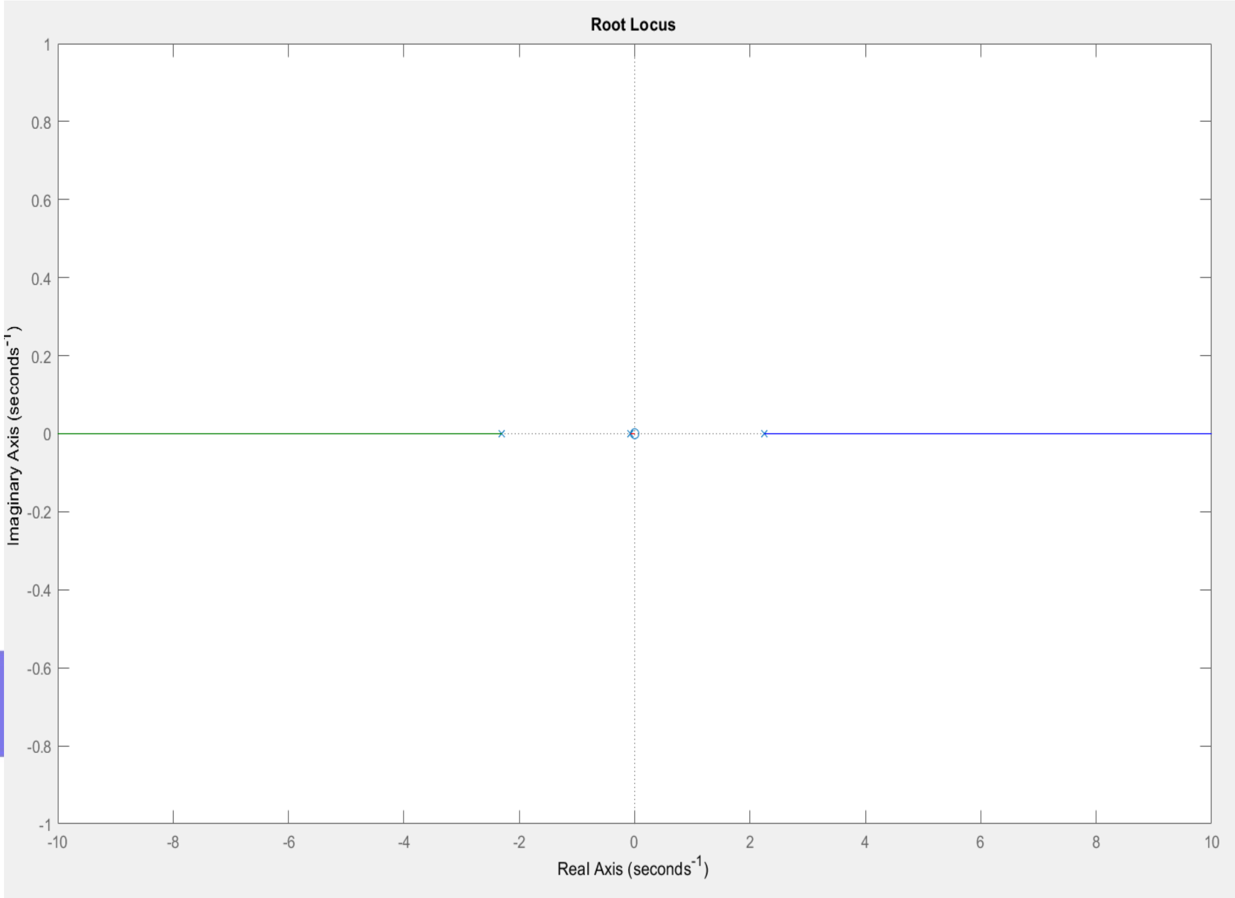
The steady state error is infinite for step, ramp and parabolic response.
We have no dominant pole.(actually S_1 can be dominant but because our system is unstable, again it's better to not talk about dominant poles.

Of course, we will find these again when we stabilized our system.

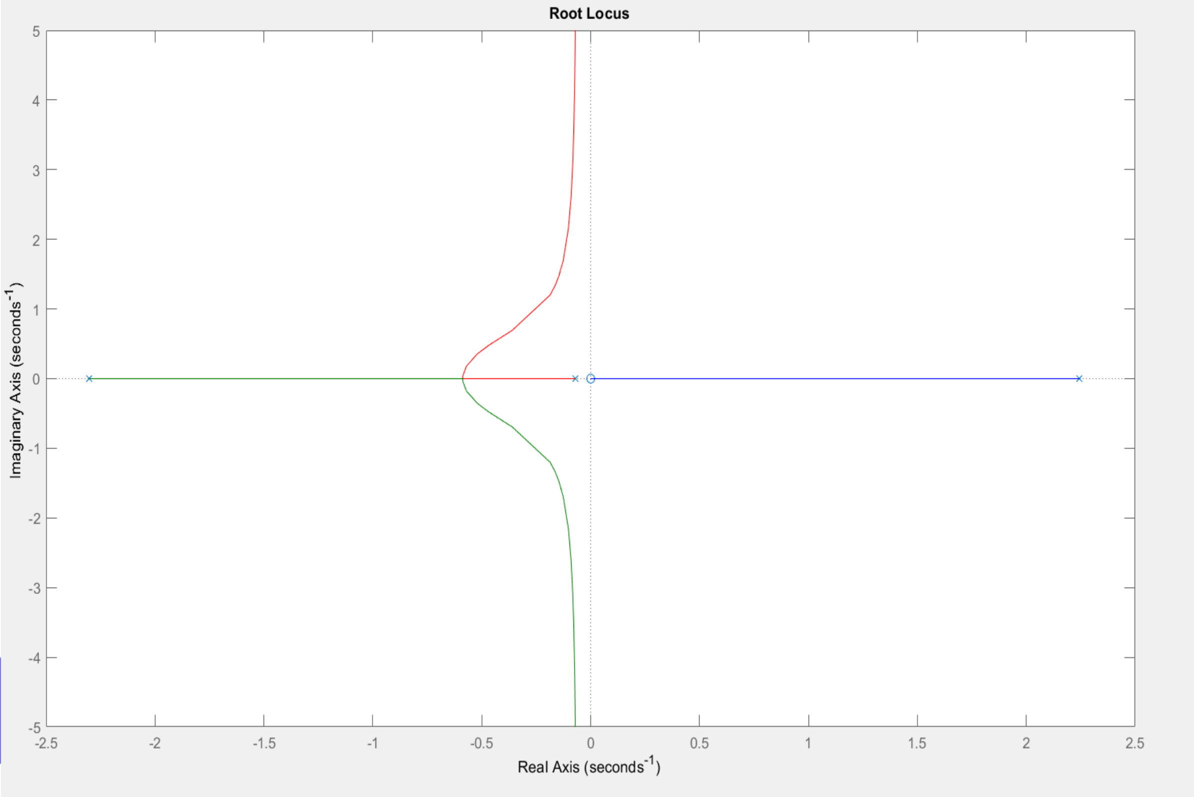
Part 2:

First of all, we will plot root locus for K>0 and K<0

For K>0:



Actually, for no positive K, our system is stable.
Because for all positive K, we have a pole in RHB.
For K<0:



Again, for no K<0, we can obtain a stable system.

So, we need to use a PID controller in order to achieve stability.

PI: K$\frac{(S+A)(S+B)}{S}$

We have to cancel zero at origin because it is the reason of instability. Also, we can omit the unstable pole of the main system with (S+A) in which, A must be small and inside LHB.

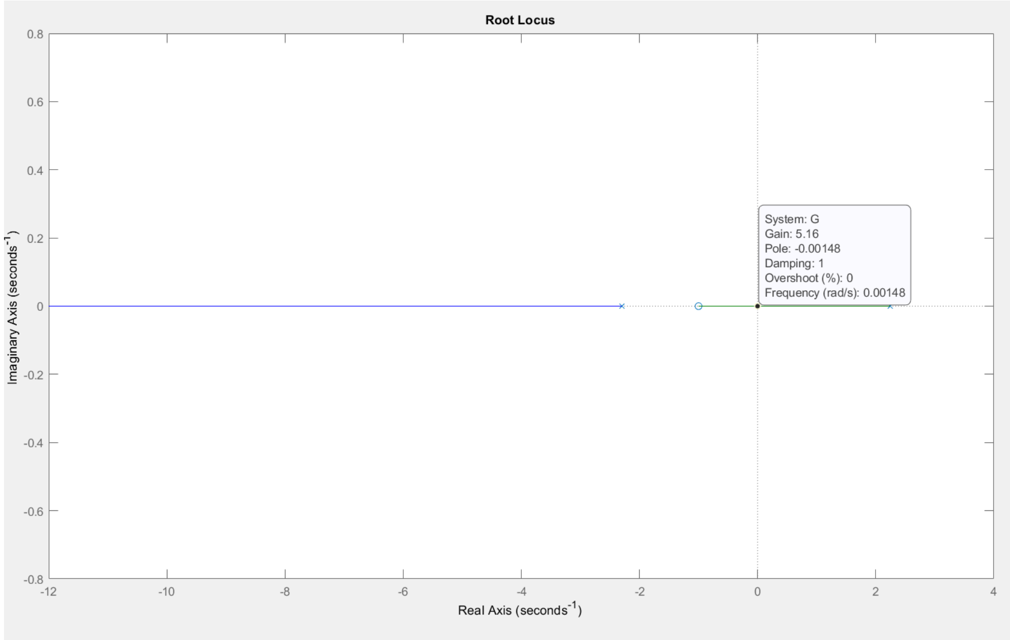
Y=-0.064$\frac{S}{(S+2.3)(S+0.069)(S-2.24)}$

So, we choose A=+0.069 & B=1

So, our new open loop system, is what we have to consider as our new system:

New open loop System(S):

$$G(s) = \frac{(S+1)}{(S+2.3)(S-2.24)}$$

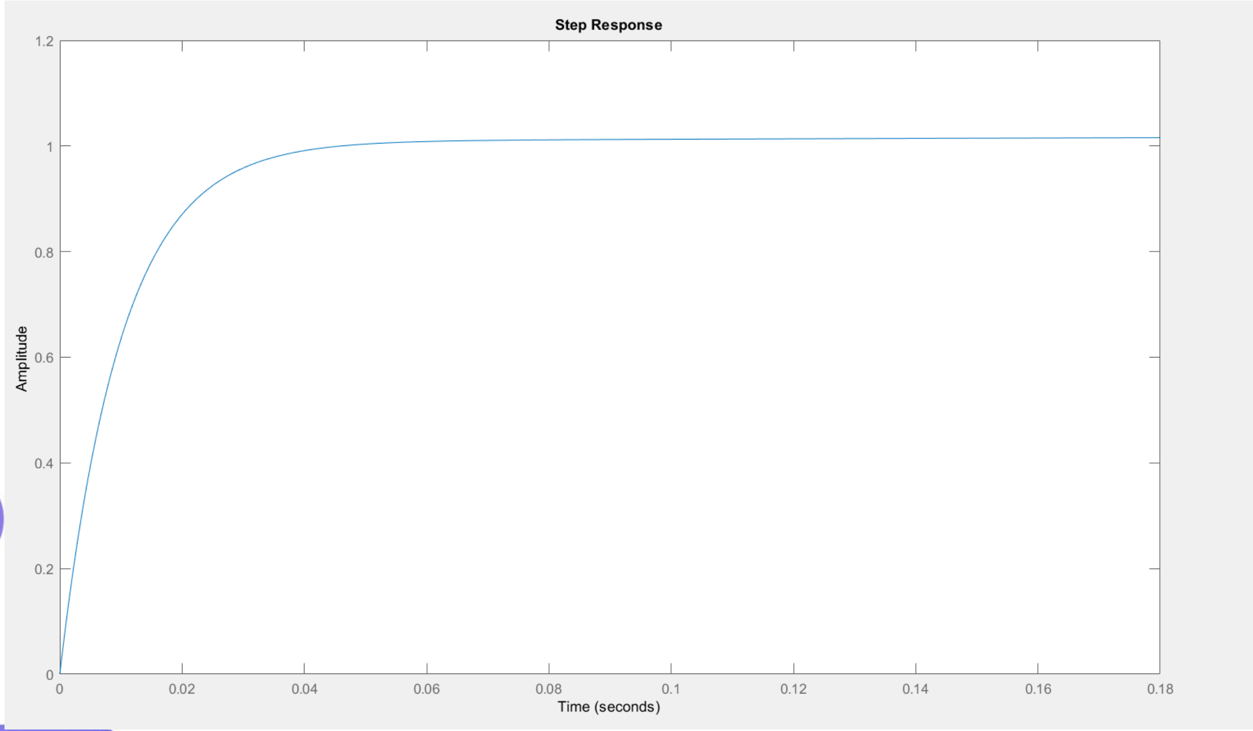


By a gain of about 5 or more, we can have a stable system.

By choosing K=100, we have:

$$H(s)=\frac{100G}{1+100G}$$

Step Response of H(s):



Fortunately, T_S is less than 0.8s and don't need any other K for this.

Third Part:

We need to design a compensator for our unstable system in state feedback which can make it stable and achieve T_S<0.8 .

Remember:(Reference: Faradars.ir)

If a system be controllable, we can use state feedback.

Our system is controllable because its rank is 4 so it is full rank.

MATLAB code:

Z=ctrb(A,B);

Rank(Z);

```
>> A=[0,1,0,0;0,-0.0812,-1.29,0.0812;0,0,0,1;0,0.045,5.161,-0.045];
>> B=[0;0.116;0;-0.064];
>> z=ctrb(A,B);
>> z

z =

         0    0.1160   -0.0146    0.0844
    0.1160   -0.0146    0.0844   -0.0442
         0   -0.0640    0.0081   -0.3313
   -0.0640    0.0081   -0.3313    0.0605

>> r=rank(z);
>> r

r =

     4
```

We know:

$T_{S\,(\%2)} = \frac{4}{\xi\omega_n} < 0.8$ **So** $-\xi\omega_n < -5$ **So the real part of our poles should be less than -5**

We want to place our new poles at:

S=-6,-7,-8,-9

So we have:

$|SI - A + BK|$**=(S+6) (S+7) (S+8) (S+9)**

$$\begin{vmatrix} \mathrm{S} & -1 & 0 & 0 \\ 0.116\mathrm{K}_1 & \mathrm{S} + 0.0812 + 0.116\mathrm{K}_2 & 1.29 + 0.116\mathrm{K}_3 & -0.0812 + 0.116\mathrm{K}_4 \\ 0 & 0 & \mathrm{S} & -1 \\ -0.064\mathrm{K}_1 & -0.045 - 0.064\mathrm{K}_2 & -5.161 - 0.064\mathrm{K}_3 & \mathrm{S} + 0.045 - 0.064\mathrm{K}_4 \end{vmatrix}$$

=(S+6) (S+7) (S+8) (S+9)

K=$[K_1 \quad K_2 \quad K_3 \quad K_4] = 10^4[-.58 \quad -0.32 \quad -1.6 \quad -0.62]$

K can be found by this code in MATLAB:

```
A=[0,1,0,0;0,-0.0812,-
1.29,0.0812;0,0,0,1;0,0.045,5.161,-0.045];
B=[0;0.116;0;-0.064];
J=[-6,-7,-8,-9];
K=place(A,B,J);
```

A_{new} = A-BK

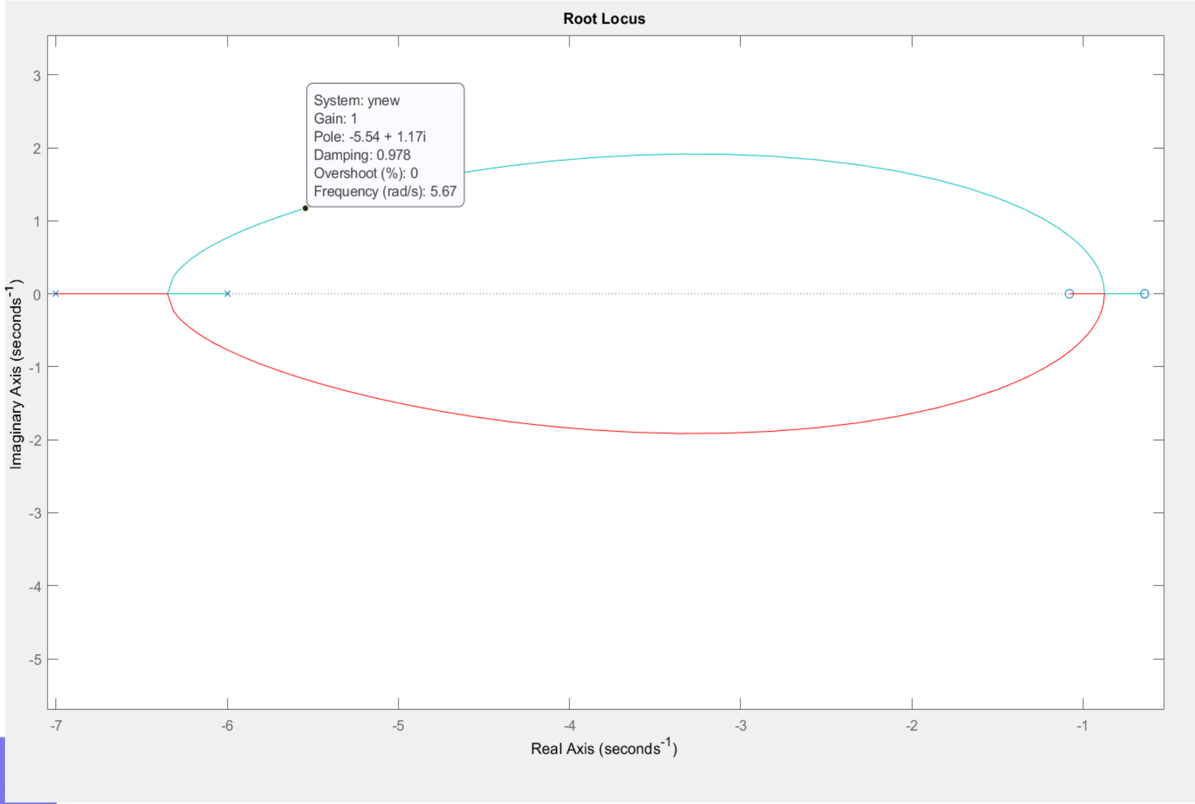
The new open loop transfer function:

$$Y_{new} = \frac{280(S+1.08)(S+0.64)}{(S+6)(S+7)(S+8)(S+9)}$$

Now we have a stable system.
We need lag compensator because phase margin and gain margin and settling time are OK.

$$Ynew = \frac{(S+1.08)(S+0.64)}{(S+6)(S+7)(S+8)(S+9)}$$

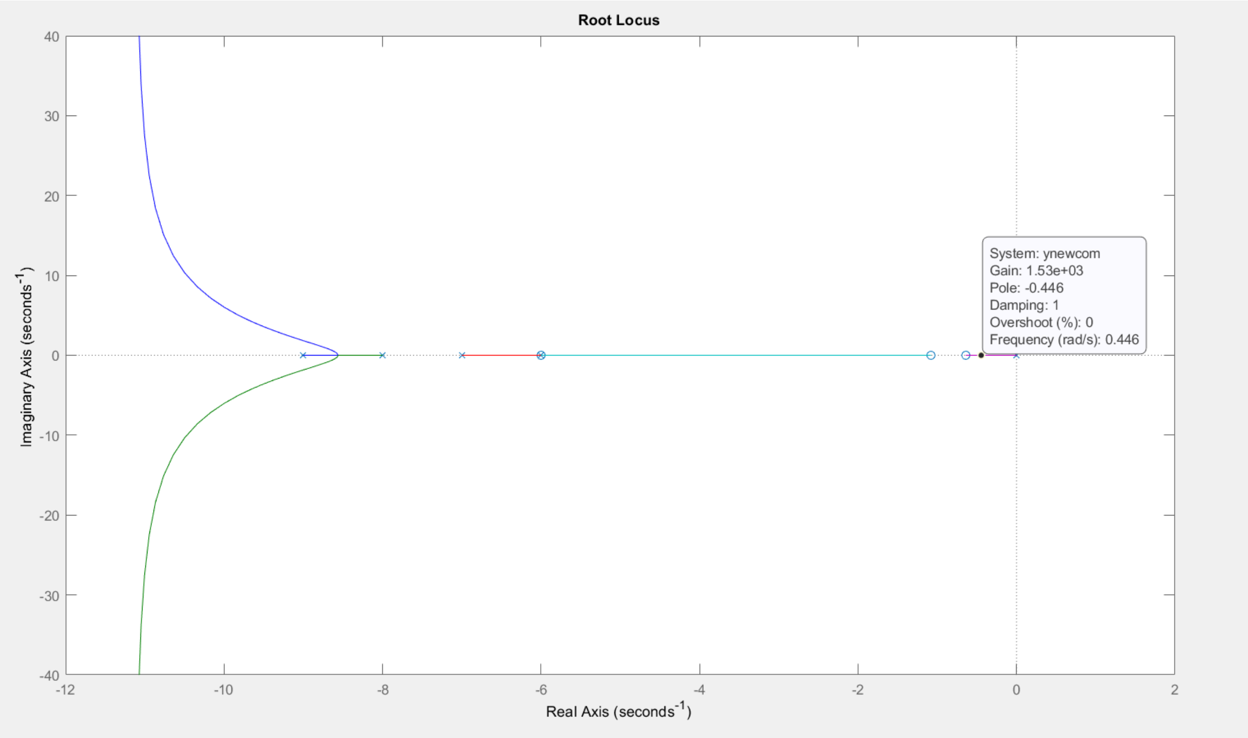
Root locus:



Root Locus
System: ynew
Gain: 1
Pole: -5.54 + 1.17i
Damping: 0.978
Overshoot (%): 0
Frequency (rad/s): 5.67
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
3
2
1
0
-1
-2
-3
-4
-5
-7
-6
-5
-4
-3
-2
-1

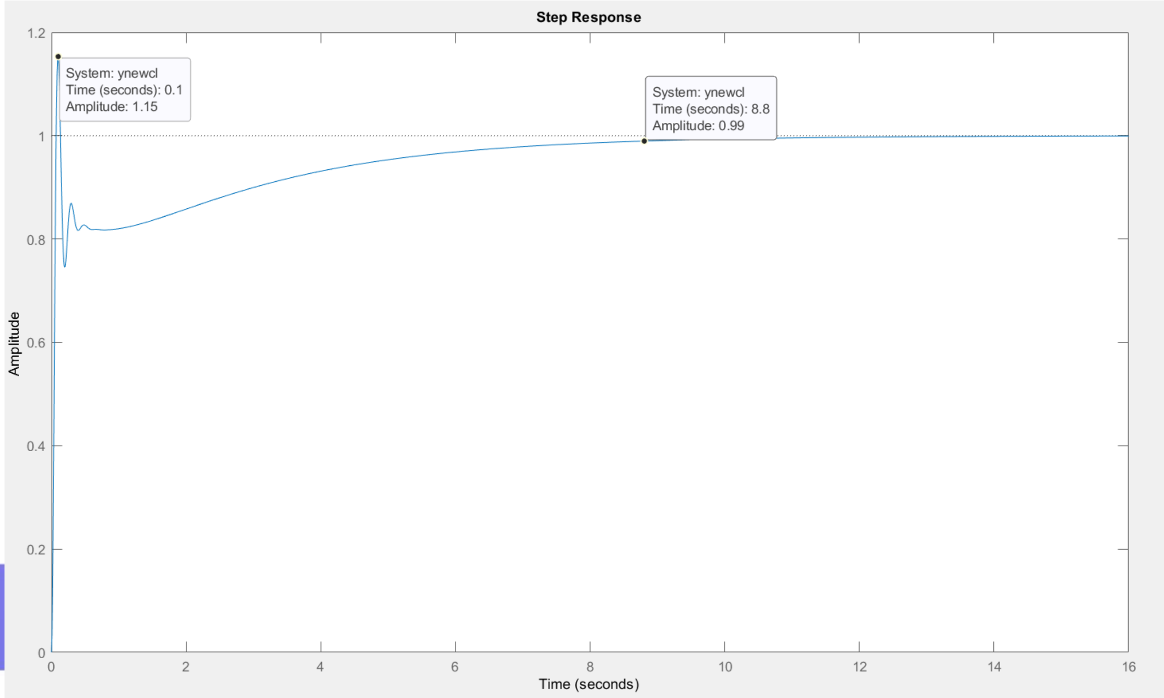
Lag compensator: we choose K_0=1 as you see in above picture.
Then we need ess<0.01 so K*H(0)>99
So K should be near 45,000,000
Then $K_c=\frac{K_0}{K}=\frac{1}{45000000}$
After 100 try, we observed we should choose Z_c at -6 so P_c=-6/45000000
So $G_c=1\frac{S+6}{s+0.000000133}$
It acts like a PI controller.

Root locus of the open loop system:

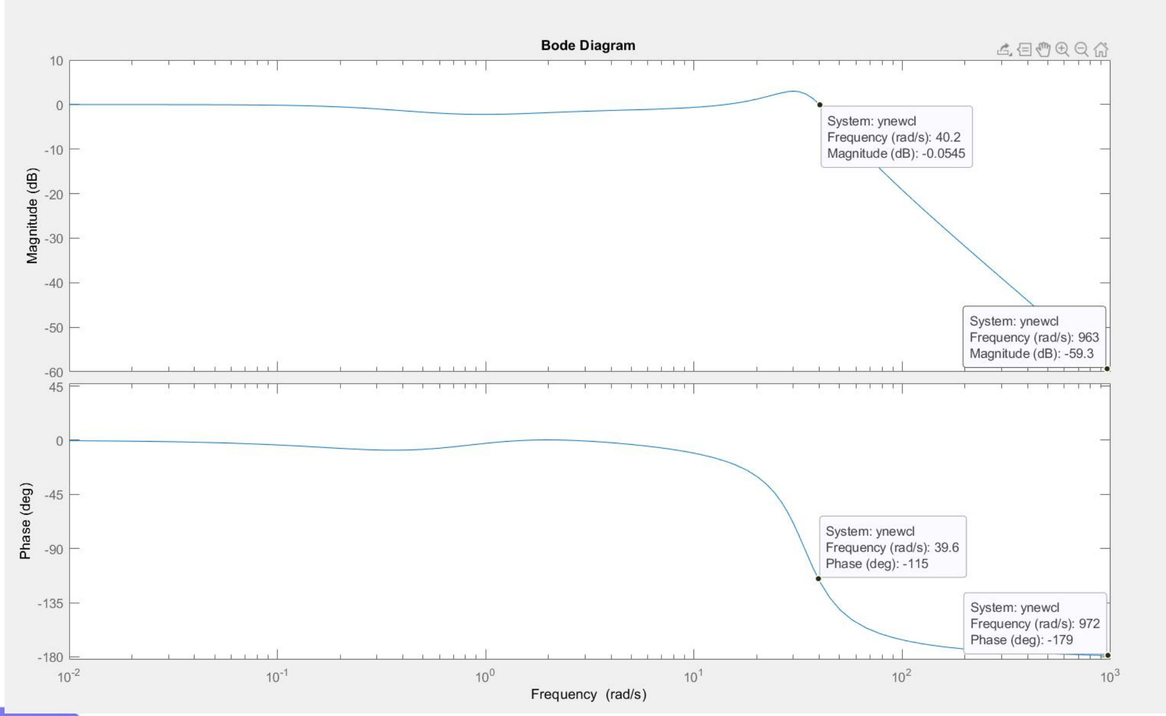


FINAL SYSTEM$=\dfrac{1000G_cY}{1+1000G_cY}$

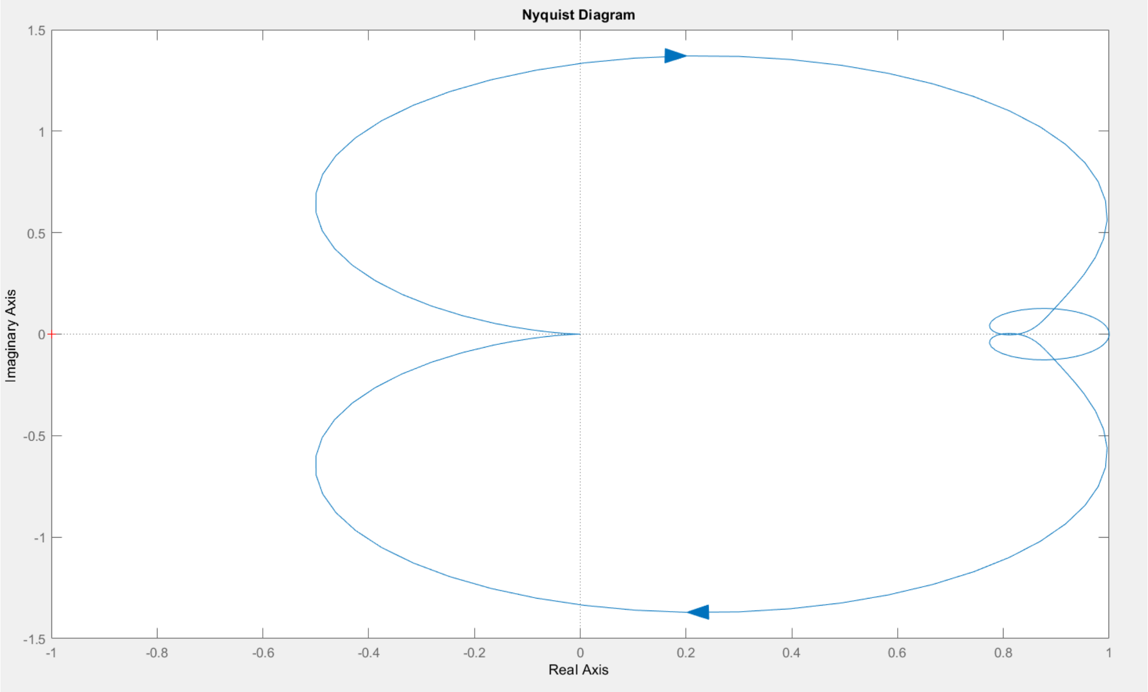
STEP RESPONSE:



BODE:


Bode Diagram
Magnitude (dB)
10
0
-10
-20
-30
-40
-50
-60
System: ynewcl
Frequency (rad/s): 40.2
Magnitude (dB): -0.0545
System: ynewcl
Frequency (rad/s): 963
Magnitude (dB): -59.3
Phase (deg)
45
0
-45
-90
-135
-180
System: ynewcl
Frequency (rad/s): 39.6
Phase (deg): -115
System: ynewcl
Frequency (rad/s): 972
Phase (deg): -179
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)

NYQUIST:



Nyquist Diagram
1.5
1
0.5
0
-0.5
-1
-1.5
Imaginary Axis
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Real Axis

Second Part for the final system:

P.O.=15%

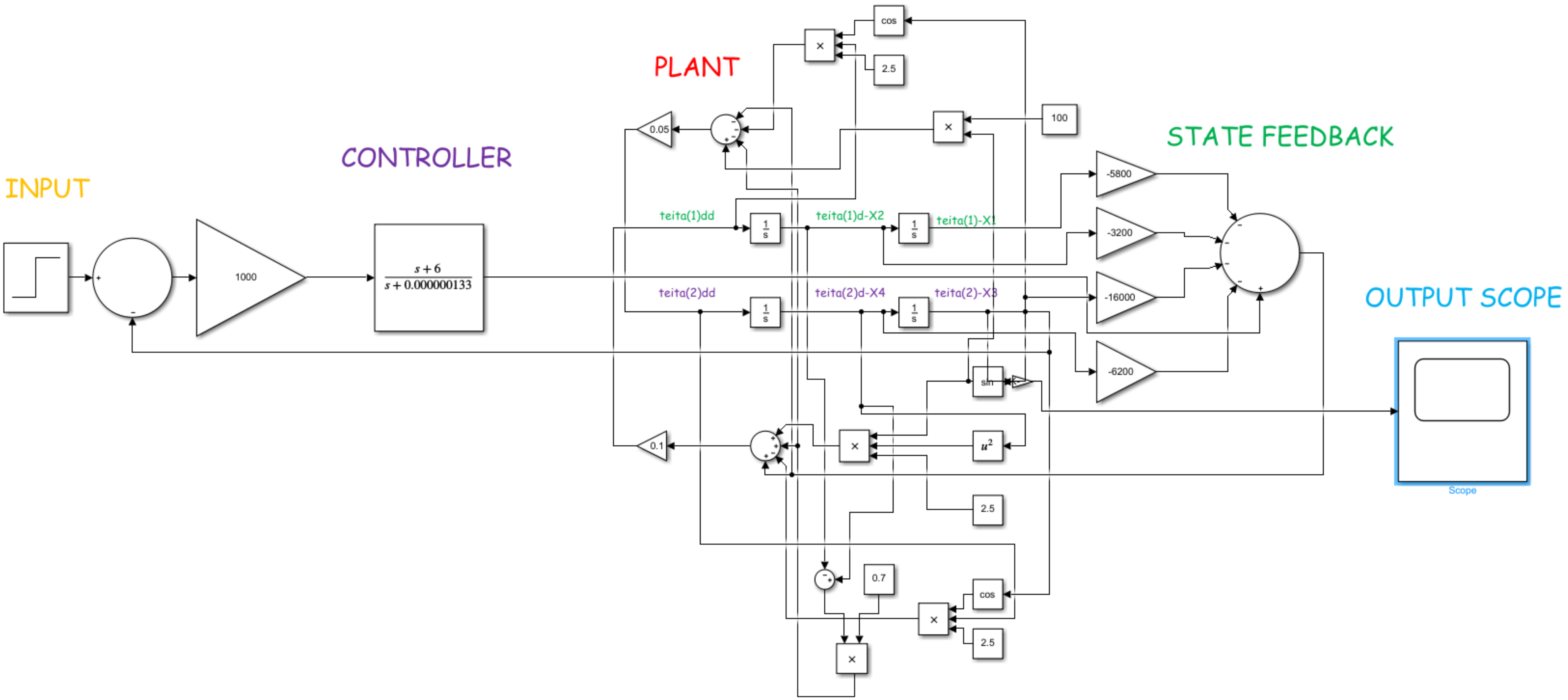
T_S=8.8 S

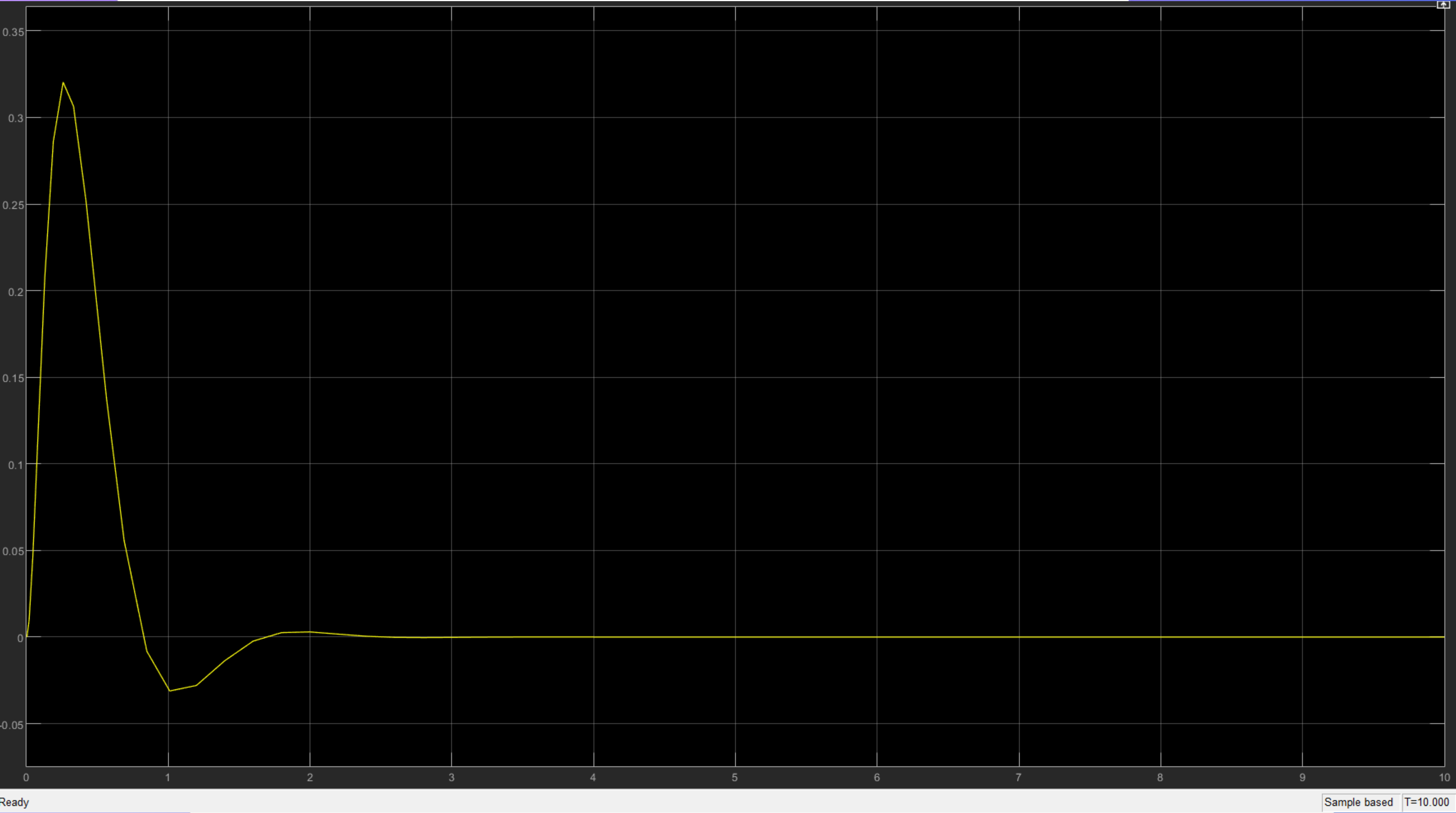
M_p=1.15

Phase margin=65 Deg

Gain margin=60 Db

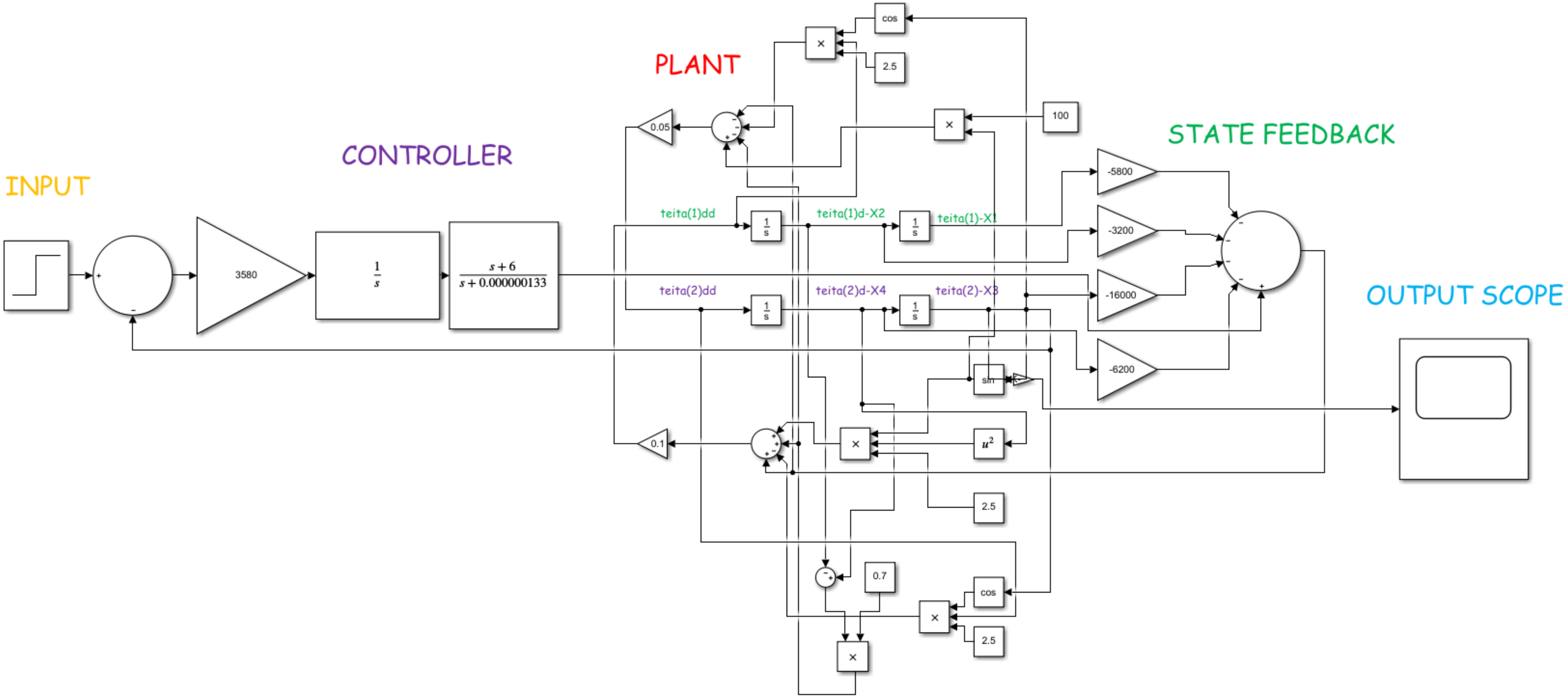
Applying Controller to Non-Linear System:

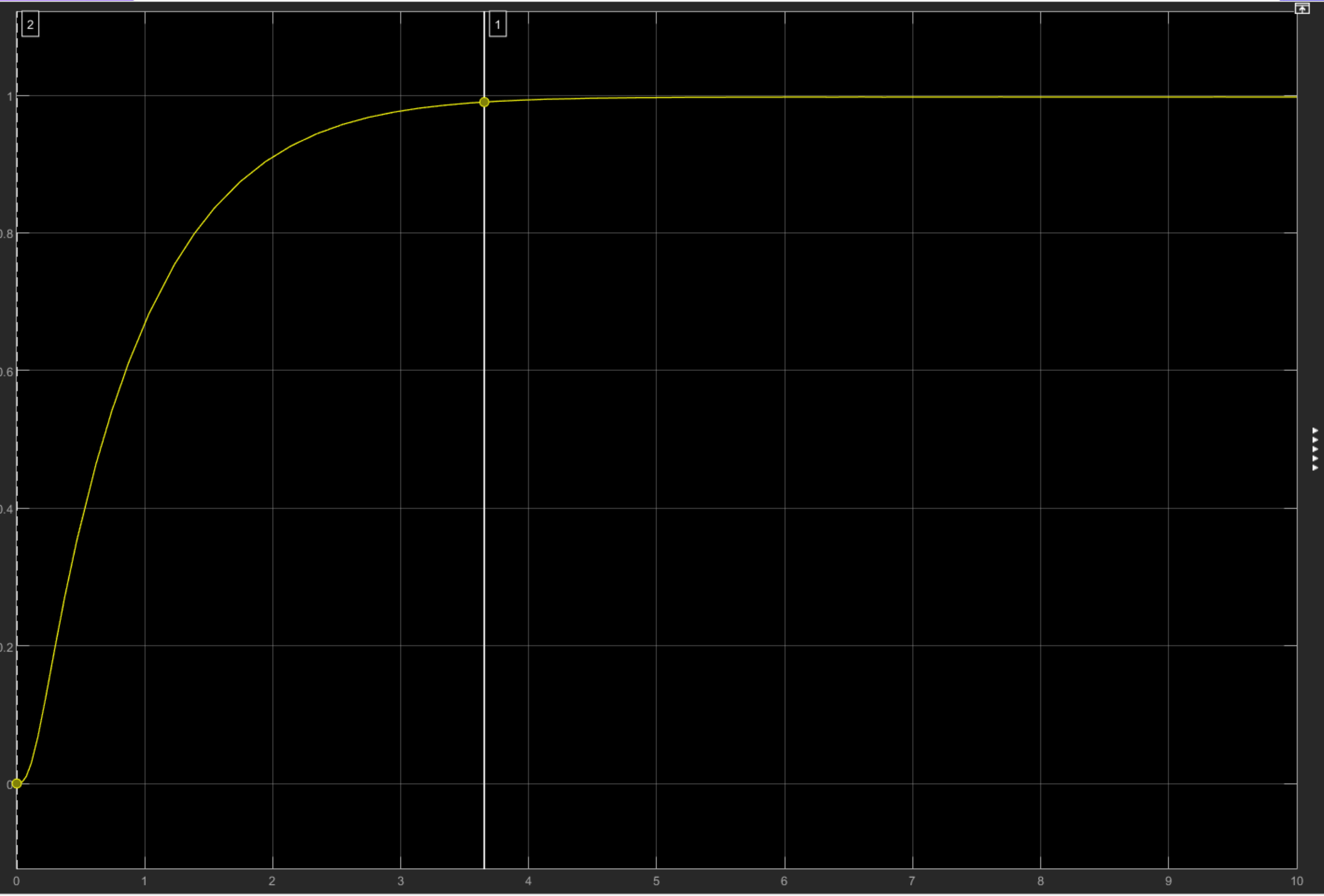



0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0
1
2
3
4
5
6
7
8
9
10
Ready
Sample based
T=10.000

For re-designing, we saw that the output became zero.
So it means we have a zero at the origin.
In order to compensate it, we put a pole at the origin and then change the gain in order to achieve the desired output.

Applying Controller to Non-Linear System After Redesigning:





Cursor Measurements

▶ Settings

▼ Measurements

	Time	Value
1	3.653	9.900e-01
2	0.000e+00	-0.000e+00
ΔT	3.653 s	ΔY 9.900e-01

1 / ΔT	273.779 mHz
ΔY / ΔT	271.049 (/ks)

Ready

Sample based T=10.000

THANK YOU!