BIOENG 1320 – Biological Signals and Systems (Spring 2022)

MATLAB Project 2

Issued: February 16, 2022 Due: 1:00p, March 23, 2022 (via Canvas)

Analyzing a biological system modeled by an LCCDE: As described in the lecture, the relationship between cardiac output and aortic blood pressure can be represented with a second-order LCCDE model of the arterial system. However, blood pressure is more easily measured in a peripheral artery than the central aorta, and the peripheral blood pressure signal ($p_p(t)$) differs substantially from the central blood pressure signal ($p_c(t)$). A third-order LCCDE model of the arterial system may be more useful in that it can represent the relationship between the left ventricular blood flow rate signal ($\dot{q}_l(t)$ whose average gives the cardiac output) and both $p_p(t)$ and $p_c(t)$. Figure 1 shows the model in electrical analog form, where voltage corresponds to blood pressure; charge, to blood volume; and current, to blood flow rate.

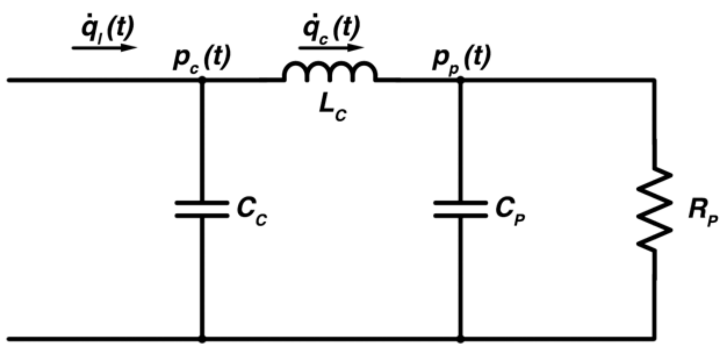

Figure 1: Third-order LCCDE model of arterial system.

The parameters of the model are C_c and C_p, which represent the central and peripheral "compliance" or blood volume storage ability; L_c, which represents the central "inertance" and indicates the pressure required to accelerate blood; and R_p, which represents the peripheral resistance to blood flow. This circuit model is governed by the following two third-order LCCDEs.

$$L_c R_p C_c C_p \frac{d^3 p_p(t)}{dt^3} + L_c C_c \frac{d^2 p_p(t)}{dt^2} + R_p\left(C_p + C_c\right)\frac{dp_p(t)}{dt} + p_p(t) = R_p \dot{q}_l(t)$$

$$\begin{aligned} L_c R_p C_c C_p \frac{d^3 p_c(t)}{dt^3} + L_c C_c \frac{d^2 p_c(t)}{dt^2} + R_p\left(C_p + C_c\right)\frac{dp_c(t)}{dt} + p_c(t) \\ = L_c R_p C_p \frac{d^2 \dot{q}_l(t)}{dt^2} + L_c \frac{d\dot{q}_l(t)}{dt} + R_p \dot{q}_l(t) \end{aligned}$$

1. Apply Laplace Transform techniques with "paper and pencil" to find the system functions, $H_p(s) = P_p(s)/\dot{Q}_l(s)$ and $H_c(s) = P_c(s)/\dot{Q}_l(s)$.
2. Let $C_c = 1.6$ ml/mmHg, $C_p = 0.2$ ml/mmHg, $L_c = 0.015$ mmHg/(ml/s^2), and $R_p = 1.1$ mmHg/(ml/s). Use the built-in functions, **freqs, tf**, **bode**, and **impulse**, to plot the frequency response, Bode plot, and impulse response for $H_p(s)$. Does this system show overdamped, underdamped, or critically damped characteristics? What MATLAB function can you use to verify your answer to this question?
3. Figure 2 shows a simple sine model of one beat of the input signal $\dot{q}_l(t)$. Create a vector to define the sine beat signal at a sampling interval of 0.001 sec. Set SV (stroke volume, which is the amount of blood ejected by the left ventricle per beat) to 80 ml, and T (the beat length) to 1 sec. Now create another vector to define a train of at least 20 unit-impulses spaced apart by T with zeros inserted in between. Use the same sampling interval for this "impulse train". Finally, form a periodic or "pulsatile" input signal by convolving the sine beat signal with the impulse train. Plot the input signal. Why does this method work?

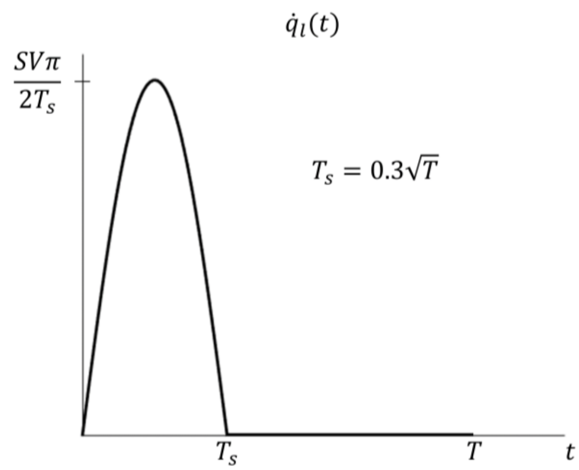

Figure 2: Sine model of one beat of the left ventricular blood flow rate.

4. Use the built-in function **lsim** to determine the output $p_p(t)$ of the system with system function $H_p(s)$ in response to the periodic sine flow rate waveform. Plot the output signal. How do you think this built-in function works? What is the cause of the transient part of the output?
5. Determine the output $p_c(t)$ of the system with system function $H_c(s)$ to the same input. Plot the output signal. Compare the two steady-state outputs. Are the results what you expected?
6. Vary the model parameters (SV, T, C_c, and R_p) individually by $\pm 50\%$ and compute the output for each case. How do the cardiac output, mean blood pressure, and peripheral pulse pressure (systolic minus diastolic blood pressure) change in response to each parametric variation? Show exemplary plots to support your answers. What are biological models good for?

Deliverables: Submit a single pdf file containing answers to the questions, properly labeled plots, and the source code.