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BIOENG 1320 Project 2

Problem 1:

1. $H_p(s) = \frac{P_p(s)}{\dot{Q}_1(s)}$ $\quad H_c(s) = \frac{P_c(s)}{\dot{Q}_1(s)}$

$$L_c R_p C_c C_p \frac{d^3 P_p(t)}{dt^3} + L_c C_c \frac{d^2 P_p(t)}{dt^2} + R_p(C_p + C_c)\frac{dP_p(t)}{dt} + p_p(t) = R_p \dot{q}_1(t)$$

$$\frac{d^3 P_p(t)}{dt^3} \leftrightarrow s^3 P_p(s) - s^2 \cancel{p_p(0^-)}^{0} - s\cancel{p_p'(0^-)}^{0} - \cancel{P_p''(0^-)}^{0}$$

$$\frac{d^2 P_p(t)}{dt^2} \leftrightarrow s^2 P_p(s) - s\cancel{p_p(0^-)}^{0} - \cancel{P_p'(0^-)}^{0}$$

$$\frac{dP_p(t)}{dt} \leftrightarrow sP_p(s) - \cancel{p_p(0^-)}^{0}$$

$$\dot{q}_1(t) \leftrightarrow \dot{Q}_1(s) \qquad P_p(t) \leftrightarrow P_p(s)$$

$$L_c R_p C_c C_p s^3 P_p(s) + L_c C_c s^2 P_p(s) + R_p(C_p + C_c) s P_p(s) + P_p(s) = R_p \dot{Q}_1(s)$$

$$(L_c R_p C_c C_p s^3 + L_c C_c s^2 + R_p(C_p + C_c)s + 1) P_p(s) = R_p \dot{Q}_1(s)$$

$$* \quad \frac{R_p}{L_c R_p C_c C_p s^3 + L_c C_c s^2 + R_p(C_p + C_c)s + 1} = \frac{P_p(s)}{\dot{Q}_1(s)} = H_p(s)$$

$$L_c R_p C_c C_p \frac{d^3 P_c(t)}{dt^3} + L_c C_c \frac{d^2 P_c(t)}{dt^2} + R_p(C_p + C_c)\frac{dP_c(t)}{dt} + p_c(t) = L_c R_p C_p \frac{d^2 \dot{q}_1(t)}{dt^2} + L_c \frac{d\dot{q}_1(t)}{dt} + R_p \dot{q}_1(t)$$

$$\frac{d^2 \dot{q}_1(t)}{dt^2} = s^2 \dot{Q}_1(s) - s\cancel{\dot{q}_1(0^-)}^{0} - \cancel{\dot{q}_1'(0^-)}^{0}$$

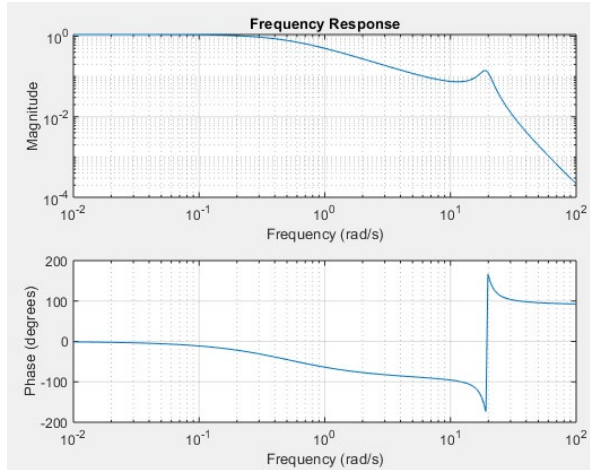
$$\frac{d\dot{q}_1(t)}{dt} = s\dot{Q}_1(s) - \cancel{\dot{q}_1(0^-)}^{0}$$

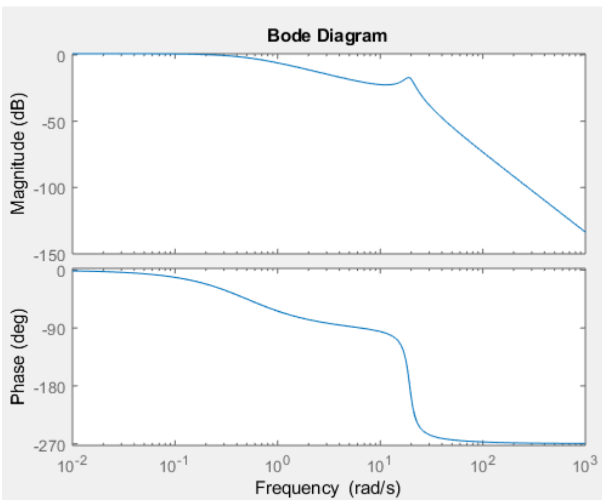
$$L_c R_p C_c C_p s^3 P_c(s) + L_c C_c s^2 P_c(s) + R_p(C_p + C_c) s P_c(s) + P_c(s) = L_c R_p C_p s^2 \dot{Q}_1(s) + L_c s \dot{Q}_1(s) + R_p \dot{Q}_1(s)$$

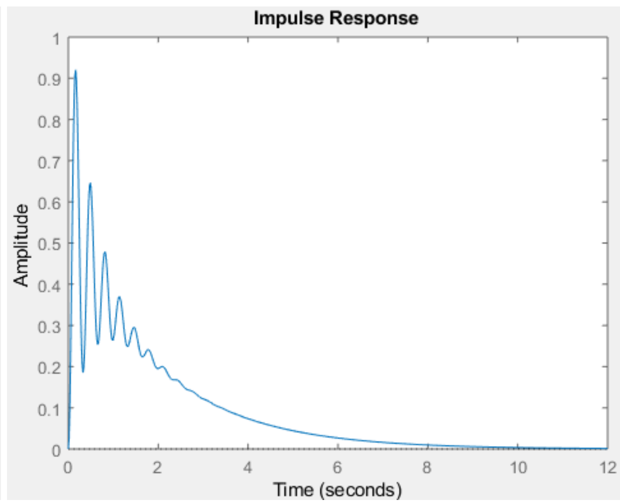
$$(L_c R_p C_c C_p s^3 + L_c C_c s^2 + R_p(C_p + C_c)s + 1) P_c(s) = (L_c R_p C_p s^2 + L_c s + R_p)\dot{Q}_1(s)$$

$$* \quad \frac{L_c R_p C_p s^2 + L_c s + R_p}{L_c R_p C_c C_p s^3 + L_c C_c s^2 + R_p(C_p + C_c)s + 1} = \frac{P_c(s)}{\dot{Q}_1(s)} = H_c(s)$$

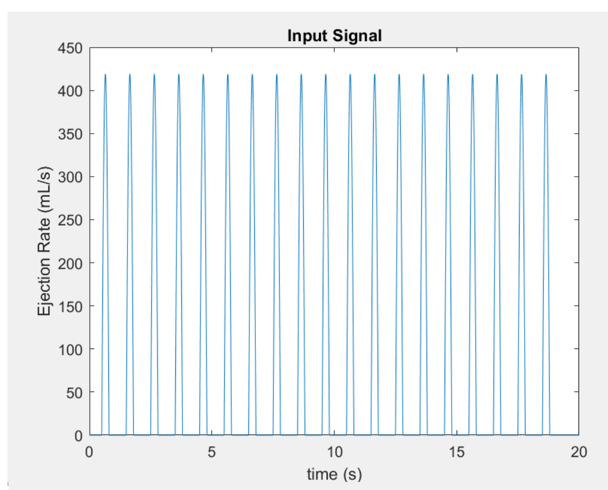
Problem 2: The system shows underdamped characteristics. In the bode magnitude plot, the spike before the decrease is indicative of a zeta <1 (underdamped), and in the bode phase plot the steep slope of the drop indicates an underdamped system (critically damped systems have more gradually changing slope). Use the MATLAB function damp(Hp) to see if the damping ratio zeta <1 for a pair of complex conjugate poles.



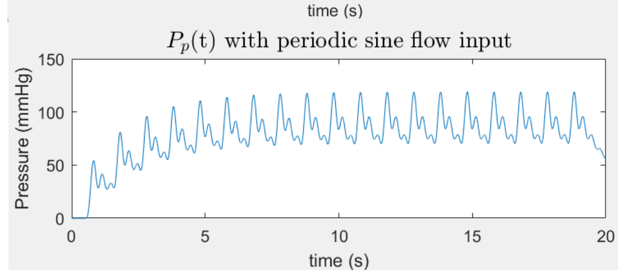




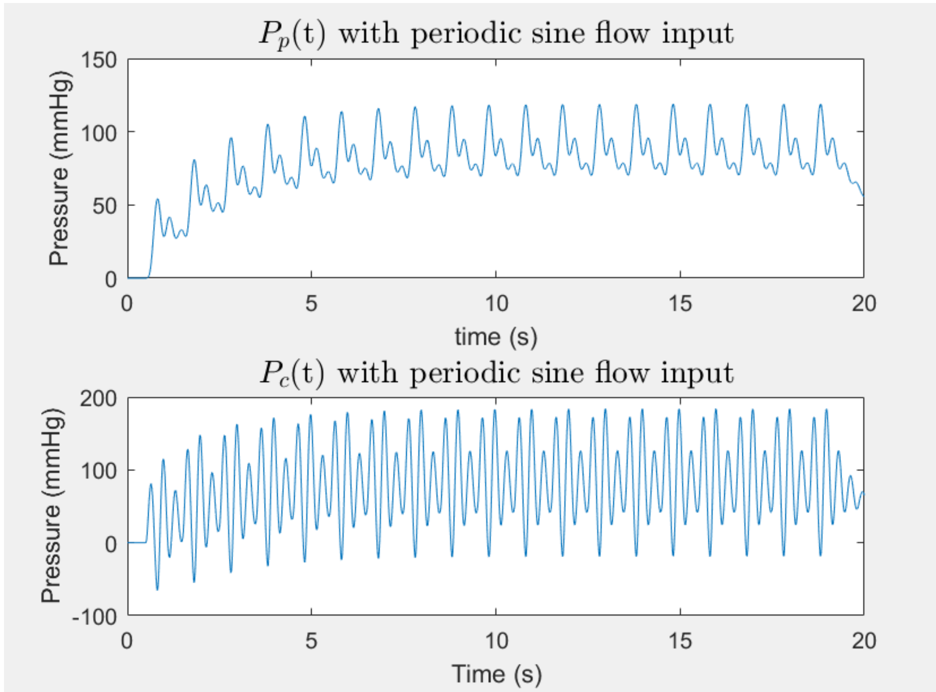
Problem 3: Convolving 20 unit impulses with the same period as the single sine beat results in the 20 sine beats seen in the input signal as the 20 impulses are “flipped and slid” across the singular sine beat and the shared area is the result of convolution. This works because the unit impulses were spaced out T apart, convolved with a sine wave with a period T. If they had different periods, it wouldn’t work.



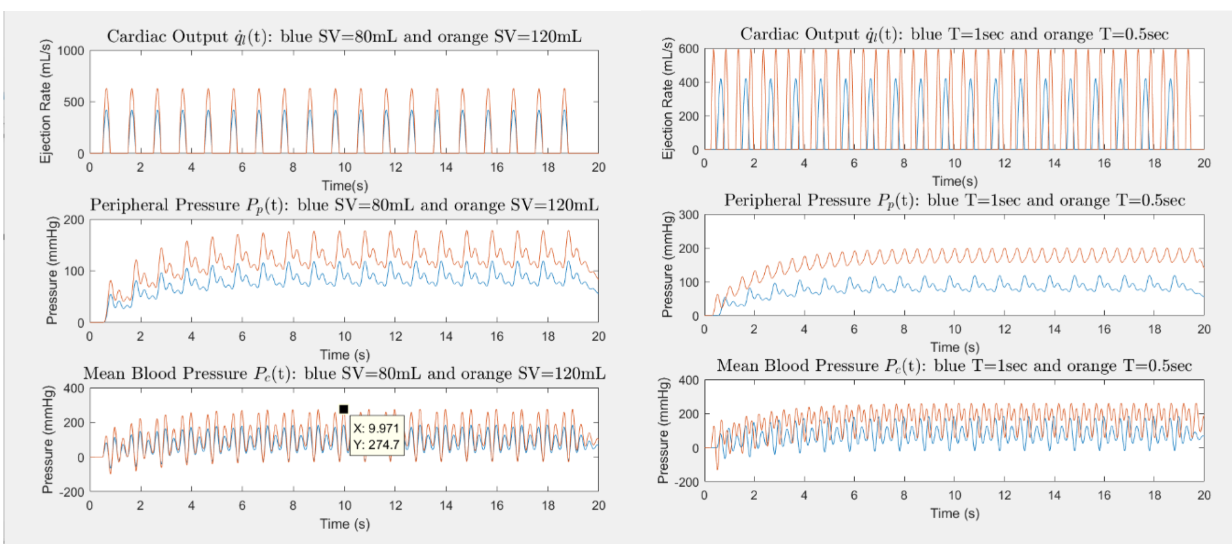
Problem 4: The lsim function uses the transfer function and the input signal to convolve the signal with the transfer function Hp for the output of the system. MATLAB assumes that the peripheral response starts from rest, when in reality you would never see that in a biological function, which causes the transient response observed in the plot.



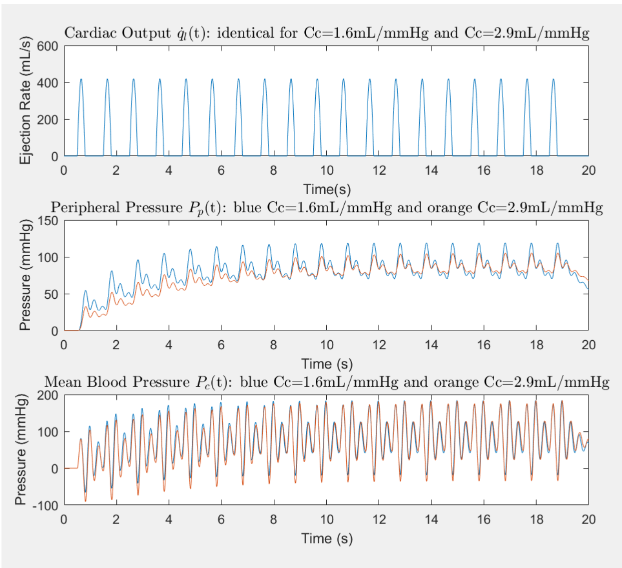
Problem 5: Yes, it is what we expected. The central aortic pressure should be higher than the peripheral arterial pressure because blood goes from the aorta to the arteries, and our plots reflect that.

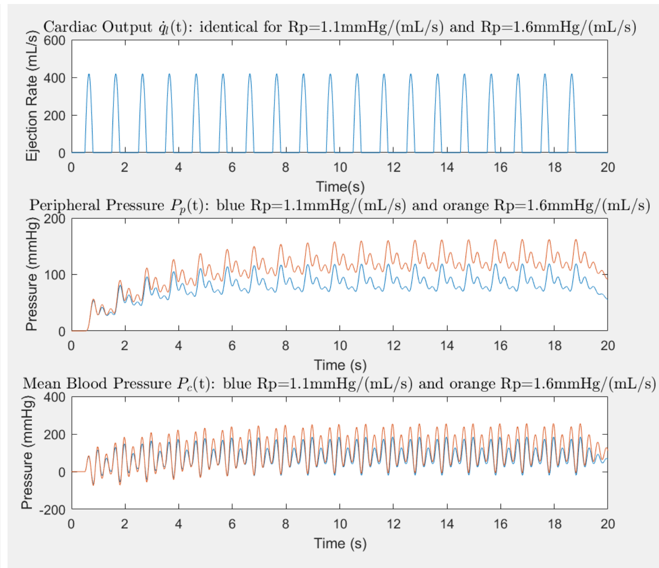


Problem 6: Increasing the stroke volume by 50% increased the amplitude of the cardiac output and the mean blood pressure and shifted the peripheral pressure up. Decreasing the beat length by 50% shortened the period and increased the values of the cardiac output, the mean blood pressure, and the peripheral pressure.



Increasing the central compliance by 50 % did not impact the cardiac output and shifted the mean blood pressure only down slightly. Increasing the central compliance decreased the amplitude of the peripheral pressure. Increasing the peripheral resistance to blood flow by 50% also did not change the cardiac output, slightly increased the amplitude of the mean blood pressure, and shifted the peripheral pressure higher.





Biological models allow you to observe how manipulating different variables might change an output or related outputs, like we did in the last problem of this project. It's very easy to see the impact of changing a single model parameter in these figures. You can control variables or parameters that you might not be able to in real systems, since the experimentation could put a test subject in critical condition. Additionally, you can know the effects of one variable on multiple others – for example, if a patient's stroke volume severely decreases due to an abnormality in ventricle function, we can know that it's important to increase peripheral pressure while figuring out how to restore the stroke volume to a normal level, allowing us to mitigate potential side effects of major complications.