

# 1 Monodomain model: without level set

**Conservation of energy (Nonlinear Heat Transfer)**

$$\frac{\partial \langle \rho h \rangle}{\partial t} + \nabla \cdot \langle \rho h \vec{v} \rangle + \nabla \cdot \left( \langle \kappa \rangle \vec{\nabla} T \right) = 0$$

**Conservation of liquid momentum (Navier Stokes)**

$$\frac{\partial}{\partial t} (\rho^l g^l \vec{v}^l) + \vec{\nabla} \cdot (\rho^l g^l \vec{v}^l \times \vec{v}^l) = \vec{\nabla} \cdot (g^l \overline{\vec{\sigma}^l}) + g^l \vec{F}_v^l + \vec{\Gamma}^l$$

**Conservation of chemical species (Macrosegregation)**

$$\frac{\partial \langle \rho w_i \rangle}{\partial t} + \nabla \cdot \langle \rho w_i \vec{v} \rangle + \nabla \cdot (g^l D^l \vec{\nabla} w_i^l) = 0$$

**Microsegregation**

$$\begin{aligned} (g^\phi, \langle w_i^\phi \rangle^\phi) &= f(\langle w_i \rangle, T) \\ \frac{\partial \langle \rho h \rangle}{\partial T} &= \frac{\partial}{\partial T} \left( \sum_\phi g^\phi \langle \rho h \rangle^\phi \right) \end{aligned}$$

## 2 Multidomain model: with level set

**Property mixing in the diffuse interface**

$$\hat{\kappa} = H^M \langle \kappa \rangle + H^A \kappa^A$$

$$\hat{\mu} = H^M \mu^l + H^A \mu^A$$

**Conservation of energy (Nonlinear Heat Transfer)**

$$\frac{\partial \langle \rho h \rangle}{\partial t} + \nabla \cdot \langle \rho h \vec{v} \rangle + \nabla \cdot (\hat{\kappa} \vec{\nabla} T) = 0$$

**Conservation of liquid momentum (Navier Stokes)**

$$\frac{\partial}{\partial t} (\rho^l g^l \vec{v}^l) + \vec{\nabla} \cdot (\rho^l g^l \vec{v}^l \times \vec{v}^l) = \vec{\nabla} \cdot (g^l \overline{\sigma^l}) + g^l \vec{F}_v^l + \vec{\Gamma}^l$$

**Conservation of chemical species (Macrosegregation)**

$$\frac{\partial \langle \rho w_i \rangle}{\partial t} + \nabla \cdot \langle \rho w_i \vec{v} \rangle + \nabla \cdot (g^l D^l \vec{\nabla} w_i^l) = 0$$

**Microsegregation**

$$(g^\phi, \langle w_i^\phi \rangle^\phi) = f(\langle w_i \rangle, T)$$

$$\frac{\partial \langle \rho h \rangle}{\partial T} = \frac{\partial}{\partial T} \left( \sum_\phi g^\phi \langle \rho h \rangle^\phi \right)$$