# 1 General monodomain model: without level set

$$\frac{\partial \langle \rho h \rangle}{\partial t} + \langle \vec{v^l} \rangle \cdot \nabla \left( \rho^l h^l \right) + \nabla \cdot \left( \langle \kappa \rangle \vec{\nabla} T \right) = 0$$

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$$W^l = w^{l^*} = kw^{s^*} = kw^s$$

$$w^s = w^{s^*} = \frac{w_0}{k(1 - f^s) + f^s}$$

$$\frac{\partial \langle \rho w_i \rangle}{\partial t} + \nabla \cdot \langle \rho w_i \vec{v^l} \rangle + \nabla \cdot \left( g^l D^l \vec{\nabla} w_i^l \right) = 0$$

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$$\frac{\partial \langle \rho w_i \rangle}{\partial t} + \nabla \cdot \langle \rho^l \langle \vec{v^l} \rangle \times \langle \vec{v^l} \rangle = 0$$

$$\frac{\partial \langle \rho w_i \rangle}{\partial t} + \nabla \cdot \langle \rho^l \langle \vec{v^l} \rangle + \nabla \cdot \langle \rho^l \langle \vec{v^l} \rangle \times \langle \vec{v^l} \rangle = 0$$

$$\frac{\partial \langle \rho w_i \rangle}{\partial t} + \nabla \cdot \langle \rho^l \langle \vec{v^l} \rangle + \rho^l \rho^l \vec{g}$$

$$\nabla \cdot \langle \vec{v^l} \rangle = 0$$

#### $\mathbf{2}$ General multidomain model: with level set

$$\frac{\partial \widehat{\left\langle \rho h \right\rangle}}{\partial t} + \langle \vec{v^F} \rangle \cdot \nabla \left( \rho^F h^F \right) + \nabla \cdot \left( \widehat{\kappa} \vec{\nabla} T \right) = 0$$

 $T^t$ 

### Microsegregation

$$\begin{split} & \underbrace{\left(g^{\phi}, \langle w_i^{\phi} \rangle^{\phi}\right)} = f\left(\langle w_i \rangle, T\right) \\ & \underbrace{\langle \rho h \rangle} = H^M \langle \rho h \rangle + H^A \rho^A h^A \\ & g^F = H^M g^l + H^A g^A = H^M g^l + H^A \end{split}$$

 $(g^F)^t, (w_i^l)^t, (w_i^s)^t$ 

### Conservation of chemical species (Macrosegregation)

$$\frac{\partial \langle \rho w_i \rangle}{\partial t} + \nabla \cdot \langle \rho w_i \vec{v}^F \rangle + \nabla \cdot \left( g^F \widehat{D} \vec{\nabla} w_i^l \right) = 0$$

 $\langle w_i \rangle^t$ 

## Conservation of liquid momentum (Navier Stokes)

$$\begin{cases} \frac{\partial}{\partial t} \left( \rho^F \langle \vec{v^F} \rangle \right) + \frac{1}{g^F} \vec{\nabla} \cdot \left( \rho^F \langle \vec{v^F} \rangle \times \langle \vec{v^F} \rangle \right) = \\ -g^F \vec{\nabla} p^F - 2 \hat{\mu} \vec{\nabla} \cdot \left( \overline{\overline{\nabla}} \langle \vec{v^F} \rangle + \overline{\overline{\nabla^t}} \langle \vec{v^F} \rangle \right) - g^F \hat{\mu} \widetilde{\mathbb{K}}^{-1} \langle \vec{v^F} \rangle + g^F \rho^l \vec{g} \\ \nabla \cdot \langle \vec{v^F} \rangle = H^M \nabla \cdot \langle \vec{v^l} \rangle + \underline{H^A \nabla \cdot \langle \vec{v^A} \rangle} \end{cases}$$

 $\langle \vec{v^F} \rangle^t, (p^F)^t$ 

#### Level set transport and reinitialisation

$$\frac{d\alpha}{dt} = \frac{\partial \alpha}{\partial t} + \vec{v} \cdot \vec{\nabla} \alpha = 0$$

 $\alpha^t_{\text{regular}}$ 

#### Interface remeshing

$$\begin{array}{ll} \textbf{Property mixing in the diffuse interface} \\ \widehat{\rho} = H^M \rho^l + H^A \rho^A & \widehat{\mu} = H^M \mu^l + H^A \mu^A \\ \rho^F h^F = H^M \rho^l h^l + H^A \rho^A h^A & \widehat{D} = H^M D^l + H^A D^A \\ \widetilde{\mathbb{K}} = \lambda_2^2 g^{F^3} / 180 \left(1 - g^F\right)^2 & \widehat{\kappa} = H^M \langle \kappa \rangle + H^A \kappa^A \\ \end{array}$$