1 Monodomain model: without level set

Conservation of energy (Nonlinear Heat Transfer)

$$\frac{\partial \langle \rho h \rangle}{\partial t} + \langle \vec{v^l} \rangle \cdot \nabla \left(\rho^l h^l \right) + \nabla \cdot \left(\langle \kappa \rangle \vec{\nabla} T \right) = 0$$

T^t

Microsegregation (Direct lever rule or discrete mapping)

$$w^l = w^{l*} = k w^{s*} = k w^s$$
$$w^s = w^{s*} = \frac{w_0}{k(1 - f^s) + f^s}$$

$(g^l)^t, (w_i^l)^t, (w_i^s)^t$

Conservation of chemical species (Macrosegregation)

$$\frac{\partial \langle \rho w_i \rangle}{\partial t} + \nabla \cdot \langle \rho w_i \vec{v^l} \rangle + \nabla \cdot \left(g^l D^l \vec{\nabla} w_i^l \right) = 0$$

$\langle w_i \rangle^t$

Conservation of liquid momentum (Navier Stokes)

$$\begin{cases} \dfrac{\partial}{\partial t} \left(\rho^l \langle \vec{v^l} \rangle \right) + \dfrac{1}{g^l} \vec{\nabla} \cdot \left(\rho^l \langle \vec{v^l} \rangle \times \langle \vec{v^l} \rangle \right) = \\ - g^l \vec{\nabla} p^l - 2 \mu^l \vec{\nabla} \cdot \left(\overline{\overline{\nabla}} \langle \vec{v^l} \rangle + \overline{\overline{\nabla^{\mathrm{t}}}} \langle \vec{v^l} \rangle \right) - g^l \mu^l \mathbb{K}^{-1} \langle \vec{v^l} \rangle + g^l \rho^l \vec{g} \\ \\ \nabla \cdot \langle \vec{v^l} \rangle = 0 \end{cases}$$

$\langle \vec{v^l} \rangle^t, (p^l)^t$

2 Multidomain model: with level set

Conservation of energy (Nonlinear Heat Transfer)

$$\frac{\partial \widehat{\langle \rho h\rangle}}{\partial t} + \langle \vec{v^F}\rangle \cdot \nabla\left(\rho^F h^F\right) + \nabla\cdot\left(\widehat{\kappa}\vec{\nabla}T\right)=0$$

T^t

Microsegregation

$$\left(g^\phi, \langle w_i^\phi\rangle^\phi\right) = f\left(\langle w_i\rangle, T\right)$$
$$\widehat{\langle\rho h\rangle} = H^M\langle\rho h\rangle + H^A\rho^A h^A$$

$(g^l)^t, (w_i^l)^t, (w_i^s)^t$

Conservation of chemical species (Macrosegregation)

$$\frac{\partial\langle\rho w_i\rangle}{\partial t} + \nabla\cdot\langle\rho w_i \vec{v^F}\rangle + \nabla\cdot\left(g^F\widehat{D}\vec{\nabla}w_i^l\right)=0$$

$\langle w_i\rangle^t$

Conservation of liquid momentum (Navier Stokes)

$$\begin{cases} \dfrac{\partial}{\partial t}\left(\rho^l\langle\vec{v^F}\rangle\right) + \dfrac{1}{g^F}\vec{\nabla}\cdot\left(\rho^l\langle\vec{v^F}\rangle\times\langle\vec{v^F}\rangle\right) = \\ -g^F\vec{\nabla}p^F - 2\widehat{\mu}\vec{\nabla}\cdot\left(\overline{\overline{\nabla}}\langle\vec{v^F}\rangle + \overline{\overline{\nabla}}^{t}\langle\vec{v^F}\rangle\right) - g^F\widehat{\mu}\mathbb{K}^{-1}\langle\vec{v^l}\rangle + g^F\rho^l\vec{g} \\ \\ \nabla\cdot\langle\vec{v^l}\rangle = 0 \end{cases}$$

$\langle\vec{v^F}\rangle^t, (p^F)^t$

Level set transport and reinitialisation

$$\frac{d\alpha}{dt} = \frac{\partial\alpha}{\partial t} + \vec{v}\cdot\vec{\nabla}\alpha = 0$$

Interface remeshing

Property mixing in the diffuse interface

$\widehat{\kappa} = H^M\langle\kappa\rangle + H^A\kappa^A$ $\quad$ $\widehat{\mu} = H^M\mu^l + H^A\mu^A$

$\widehat{\kappa} = H^M\langle\kappa\rangle + H^A\kappa^A$ $\quad$ $\widehat{D} = H^M D^l + H^A D^A$