# 1 General monodomain model: without level set

$$\frac{\partial \langle \rho h \rangle}{\partial t} + \langle \vec{v^l} \rangle \cdot \nabla \left( \rho^l h^l \right) + \nabla \cdot \left( \langle \kappa \rangle \vec{\nabla} T \right) = 0$$

 $T^t$ 

#### Microsegregation

Discrete mapping:  $\left(g^{\phi}, \langle w_i^{\phi} \rangle^{\phi}\right) = f\left(\langle w_i \rangle, T\right)$ 

Direct lever rule:

$$w^{l} = w^{l^*} = kw^{s^*} = kw^{s}$$
  
 $w^{s} = w^{s^*} = \frac{w_0}{k(1 - f^s) + f^s}$ 

 $(g^{\phi})^t$  $(w_i^l)^t, (w_i^s)^t$ 

## Conservation of chemical species (Macrosegregation)

$$\frac{\partial \langle \rho w_i \rangle}{\partial t} + \langle \vec{v^l} \rangle \cdot \nabla \left( \rho^l w_i^l \right) + \nabla \cdot \left( g^l D^l \vec{\nabla} \left( \rho^l w_i^l \right) \right) = 0$$

 $\langle w_i \rangle^t$ 

## Conservation of liquid momentum (Navier Stokes)

$$\begin{cases} \frac{\partial}{\partial t} \left( \rho^l \langle \vec{v^l} \rangle \right) + \frac{1}{g^l} \vec{\nabla} \cdot \left( \rho^l \langle \vec{v^l} \rangle \times \langle \vec{v^l} \rangle \right) = \\ -g^l \vec{\nabla} p^l - 2\mu^l \vec{\nabla} \cdot \left( \overline{\overline{\nabla}} \langle \vec{v^l} \rangle + \overline{\overline{\nabla^t}} \langle \vec{v^l} \rangle \right) - g^l \mu^l \mathbb{K}^{-1} \langle \vec{v^l} \rangle + g^l \rho^l \vec{g} \\ \nabla \cdot \langle \vec{v^l} \rangle = 0 \end{cases}$$

 $\langle \vec{v^l} \rangle^t, (p^l)^t$ 

#### $\mathbf{2}$ General multidomain model: with level set

Conservation of energy (Nonlinear Heat Transfer)

$$\frac{\partial \widehat{\langle \rho h \rangle}}{\partial t} + \langle \vec{v^F} \rangle \cdot \nabla \left( \rho^F h^F \right) + \nabla \cdot \left( \widehat{\kappa} \vec{\nabla} T \right) = 0$$

 $T^t$ 

Microsegregation (Discrete mapping)

Metal: 
$$\left(g^{\phi}, \langle w_i^{\phi} \rangle^{\phi}\right) = f\left(\langle w_i \rangle, T\right)$$

Metal:  $\left(g^{\phi}, \langle w_i^{\phi} \rangle^{\phi}\right) = f\left(\langle w_i \rangle, T\right)$ Fluid fraction:  $g^F = H^M g^l + H^A g^A = H^M g^l + H^A$ Total volume enthalpy:  $\widehat{\langle \rho h \rangle} = H^M \langle \rho h \rangle + H^A \rho^A h^A$ 

$$=H^M\sum_{\phi\in\mathrm{metal}}^{\bullet}g^{\phi}h^{\phi}+H^A\rho^Ah^A$$

Conservation of chemical species (Macrosegregation)

$$\frac{\partial \langle \rho w_i \rangle}{\partial t} + \langle \vec{v}^F \rangle \cdot \nabla \left( \rho^l w_i^l \right) + \nabla \cdot \left( g^F \widehat{D} \vec{\nabla} \left( \rho^l w_i^l \right) \right) = 0$$

 $\langle w_i \rangle^t$ 

Conservation of liquid momentum (Navier Stokes)

$$\begin{cases} \frac{\partial}{\partial t} \left( \widehat{\rho} \langle \overrightarrow{v^F} \rangle \right) + \frac{1}{g^F} \overrightarrow{\nabla} \cdot \left( \widehat{\rho} \langle \overrightarrow{v^F} \rangle \times \langle \overrightarrow{v^F} \rangle \right) = \\ -g^F \overrightarrow{\nabla} p^F - 2\widehat{\mu} \overrightarrow{\nabla} \cdot \left( \overline{\overline{\nabla}} \langle \overrightarrow{v^F} \rangle + \overline{\overline{\nabla^t}} \langle \overrightarrow{v^F} \rangle \right) - g^F \widehat{\mu} \widetilde{\mathbb{K}}^{-1} \langle \overrightarrow{v^F} \rangle + g^F \rho^l \overrightarrow{g} \\ \nabla \cdot \langle \overrightarrow{v^F} \rangle = H^M \nabla \cdot \langle \overrightarrow{v^l} \rangle + \underline{H^A \nabla \cdot \langle \overrightarrow{v^A} \rangle} \end{cases}$$

$$\langle \vec{v^F} \rangle^t, (p^F)^t$$

Level set transport - reinitialisation - Interface remeshing

$$\frac{d\alpha}{dt} = \frac{\partial \alpha}{\partial t} + \vec{v} \cdot \vec{\nabla} \alpha = 0$$

 $\alpha_{\text{regular}}^t$ 

Property mixing in the diffuse interface

$$\widehat{\rho} = H^{M} \rho^{l} + H^{A} \rho^{A} \qquad \widehat{\mu} = H^{M} \mu^{l} + H^{A} \mu^{A}$$

$$\rho^{F} h^{F} = H^{M} \rho^{l} h^{l} + H^{A} \rho^{A} h^{A} \qquad \widehat{D} = H^{M} D^{l} + H^{A} D^{A}$$

$$\widetilde{\mathbb{K}} = \lambda_{2}^{2} g^{F^{3}} / 180 \left(1 - g^{F}\right)^{2} \qquad \widehat{\kappa} = H^{M} \langle \kappa \rangle + H^{A} \kappa^{A}$$

$$\widehat{\mu} = H^M \mu^l + H^A \mu^A$$

$$\widehat{D} = H^M D^l + H^A D^A$$

$$\widetilde{\mathbb{K}} = \lambda_2^2 g^{F^3} / 180 \left( 1 - g^F \right)^2$$

$$\widehat{\kappa} = H^M \langle \kappa \rangle + H^A \kappa$$