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## NUMERICAL MODELLING OF MACROSEGREGATION FORMED DURING SOLIDIFICATION WITH SHRINKAGE USING A LEVEL SET APPROACH

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## **Acknowledgement**

Dedicated to humanity ...



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## List of Acronyms

Acronym	Standing for
ALE	Arbitrary Lagrangian-Eulerian
BTR	Brittleness temperature range
CAFD	Cellular Automata Finite Difference
CAFE	Cellular Automata Finite Element
CBB	Circumventing Babuška-Brezzi
CCEMLCC	Chill Cooling for the Electro-Magnetic Levitator in relation with Continuous Casting of steel
CEMEF	Center for Material Forming
CFL	Courant–Friedrichs–Lewy
C.FL	Computing and FLuids
CSF	Continuum Surface Force
DLR	Deutsches Zentrum für Luft- und Raumfahrt
DSPG	Darcy-Stabilising/Petrov-Galerkin
EML	Electromagnetic levitation
ESA	European Space Agency
FEM	Finite Element Method
FVM	Finite Volume Method
GMAW	Gas Metal Arc Welding
ISS	International Space Station
IWT	Institut für Werkstofftechnik
LHS	Left-hand side
LSIC	Least squares on incompressibility constraint
LSM	Level set method
MAC	Marker-and-cell
PF	Phase field
PSPG	Pressure-Stabilising/Petrov-Galerkin
RHS	Right-hand side
RUB	Ruhr Universität Bochum
RVE	Representative Elementary Volume
SBB	Satisfying Babuška-Brezzi
SUPG	Streamline-Upwind/Petrov-Galerkin
VMS	Variational MultiScale
VOF	Volume Of Fluid

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# Chapter 1

## Macrosegregation with solidification shrinkage

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## 1.1 Solidification shrinkage

Solidification shrinkage is, by definition, the effect of relative density change between the liquid and solid phases. In general, it results in a progressive volume change during solidification, until the phase change has finished. The four stages in [figs. 1.1a](#) to [1.1d](#) depict the volume change with respect to solidification time. First, at the level of the first solid crust, near the local solidus temperature, the solid forms with a density greater than the liquid's. The subsequent volume decrease creates voids with a negative pressure, forcing the fluid to be sucked in the direction of the volume change (cf. [fig. 1.1b](#)). As a direct result of the inward feeding flow, the ingot surface tends to gradually deform in the feeding direction, forming the so-called *shrinkage pipe*, shown in [fig. 1.2](#). Since the mass of the alloy and its chemical species is conserved, a density difference between the phases ( $\rho^l < \rho^s \implies \frac{\rho^l}{\rho^s} < 1$ ) eventually leads to a different overall volume ( $V^s < V^l$ ) once solidification is complete, as confirm the following equations:

$$\rho^l V^l = \rho^s V^s \quad (1.1a)$$

$$V^s = \frac{\rho^l}{\rho^s} V^l \quad (1.1b)$$

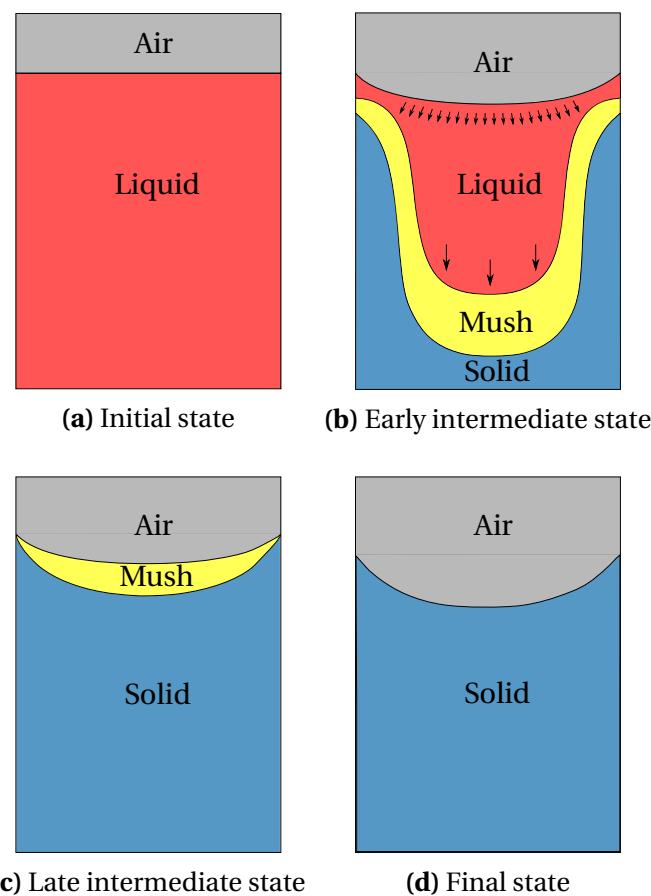
Solidification shrinkage is not the only factor responsible for volume decrease. Thermal shrinkage in both solid and liquid phases, as well as solutal shrinkage in the liquid phase are also common causes in a casting process. However, thermal shrinkage is very important to apprehend, as temperature decreases in steel casting, usually exceeding a 1000 °C, thus causing substantial density variations.

## 1.2 Choice of interface tracking

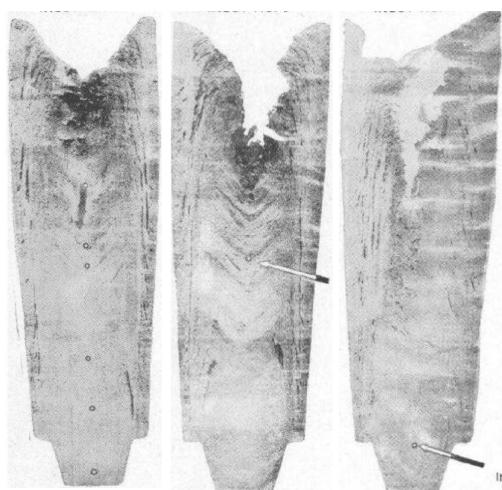
In chapter 2, several methods of interface tracking/capturing methods were presented along with their similarities and differences. In the case of solidification shrinkage, the metal-air interface can be tracked with any method from the previously mentioned. However, several reasons motivate us to settle on the level set method. First, the easiest solution is testing a method which already exists in *CimLib* library. The level set method was implemented by HERE as a framework for monolithic resolution. Since this work, the method has been extensively used and improved in several projects mainly for multiphase flows, which is the main competence of the Computing and FLuids (C.FL) group at CEMEF. Another motivation is the compatibility between *CimLib* and *Thercast*®, where the latter is the final destination of the code developed

## Chapter 1. Macrosegregation with solidification shrinkage

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**Fig. 1.1** – Schematic of the main cooling stages of an ingot against side and bottom mould walls (not shown)



**Fig. 1.2** – Sulphur prints of three ingots showing pipe formation at the top as a result of solidification shrinkage with various ingot inclination during casting [Onodera et al. 1959]. Positive macrosegregation is clearly seen in this area, while A-shape and V-shape positive mesosegregates are detected at the ingot's tips and center respectively.

during for the Ph.D. thesis. In its recent versions, *Thercast*<sup>®</sup> handles laminar and turbulent ingot filling where the level set method is used to capture the free surface of the molten metal. Aside from the practical motivations, some technical aspects of the level set method make it very attractive to apply it macroscopic surface tracking (in contrast to microscopic interface tracking, for instance the solid-liquid interface), such as topological properties that are readily available (e.g. curvature) and accurate position compared to volume-based methods like VOF.

## 1.3 Multidomain formalism

In the previous chapters, we considered in our simulations the metal as a saturated mixture of solid and liquid during solidification. It means that no gas phase may appear during the process, and this this chapter. The reason is we chose to describe our model in Eulerian description, for which we have considered a fixed grid to discretise the averaged conservation equations governing the phase change between the liquid and solid phases. Furthermore, with the introduction of shrinkage, an increase in global density means that a gas phase should enter the domain to replace the shrunk volume. At this point, several interfaces may be distinguished: liquid-solid ( $l-s$ ), liquid-air ( $l-a$ ) and solid-air ( $s-a$ ), where we defined 2 phases ( $l$  and  $s$ ) belonging to the "Metal" domain denoted  $M$ , while the "Air" domain, denoted  $A$ , is made up of a unique phase, ( $a$ ), with the same name. As a standard for this formalism, we consider that uppercase letters are used for domains, while lowercase letters are used for phases.

The main idea behind the multidomain formalism, is to go from the classic conservations equations introduced by volume averaging in chapter 2, in the context of a solidifying two-phase system to generalise it by taking into account a third gas phase, such as:

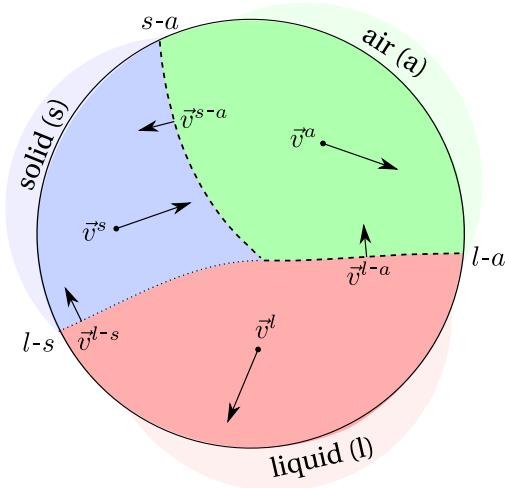
$$V^l + V^s + V^a = V_E \quad (1.2)$$

$$g^l + g^s + g^a = 1 \quad (1.3)$$

while keeping a physical integrity with the former monodomain model. Then, one is free to choose a suitable numerical method to track the interfaces between the several phases. In our applications, we are particularly interested in keeping an indirect representation of the  $l-s$  interface (dotted line in [fig. 1.3](#)) using the volume averaging theory, while employing a different method to track the  $l-a$  and  $s-a$  interfaces (dashed lines in [fig. 1.3](#)) with the level set method. This allows switching to the latter method

in a physically representative manner.

In this context, each domain can be seen as a material having a physical interface with the other domains. As a consequence of our interpretation, the gas phase should not exist in the metal, which may naturally occur if the thermodynamic conditions are in favour of nucleating and growing a new phase, or in the case of a gas that was trapped inside mould grooves.



**Fig. 1.3** – Schematic of a representative volume element containing 3 phases with distinct velocities, separated by 3 interfaces. The dotted line is the indirectly tracked solid-liquid interface while the other dashed lines, air-liquid and air-solid interfaces, are directly tracked.

### 1.3.1 Assumptions

Each phase in the system has its own velocity,  $\vec{v}^l$ ,  $\vec{v}^s$  and  $\vec{v}^a$ , while the respective interfaces  $l-s$ ,  $l-a$  and  $s-a$  have different and independent velocities, represented by  $\vec{v}^{l-s}$ ,  $\vec{v}^{l-a}$  and  $\vec{v}^{s-a}$ . Note that the solid-liquid interface velocity was denoted  $\vec{v}^*$  in the previous chapters as no more than two phases were considered. The first major assumption is that the solid phase, once formed from the liquid, is fixed and rigid. It means that no subsequent deformation may occur and therefore  $\vec{v}^{s-a}$  reduces to vector zero. Moreover, we use the already introduced volume averaging principles to write locally for any quantity  $\psi$ :

$$\langle \psi \rangle = \left\langle \psi^l \right\rangle + \langle \psi^s \rangle + \langle \psi^a \rangle \quad (1.4a)$$

$$= g^l \psi^l + g^s \psi^s + g^a \psi^a \quad (1.4b)$$

## 1.4. FE partitioned model

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where volume fractions,  $g^\phi$ , for each phase  $\phi$  were used. Rappaz et al. [2003] define the volume fraction by writing a general expression inside the representative volume  $V_E$ :

$$g^\phi = \frac{1}{V_E} \int_{V_E} \chi^\phi(x, t) d\Omega = \langle \chi^\phi \rangle \quad (1.5)$$

where the integrated quantity is an indicator (or presence) function relative to phase  $\phi$ , which defines the volume of this phase in the system,  $\Omega^\phi$ , as follows:

$$\chi^\phi(x, t) = \begin{cases} 1 & \text{if } x \in \Omega^\phi \\ 0 & \text{otherwise} \end{cases} \quad (1.6)$$

Any phenomenon that may displace an interface, whether by phase change or a phase motion, is mathematically translated by variations of the presence function, such that its total derivative for each phase satisfies the following:

$$\frac{d\chi^\phi}{dt} = \frac{\partial \chi^\phi}{\partial t} + \vec{v}^* \cdot \vec{\nabla} \chi^\phi = 0 \quad (1.7)$$

If we consider the liquid phase, the variations of any quantity, named  $\psi$ , are given by:

$$\left\langle \frac{\partial \psi^l}{\partial t} \right\rangle = \frac{\partial \langle \psi^l \rangle}{\partial t} - \frac{1}{V_E} \int_{l-a} \psi^l \vec{v}^{l-a} \cdot \vec{n}^{l-a} d\Gamma - \frac{1}{V_E} \int_{l-s} \psi^l \vec{v}^{l-s} \cdot \vec{n}^{l-s} d\Gamma \quad (1.8)$$

$$\left\langle \vec{\nabla} \psi^l \right\rangle = \vec{\nabla} \langle \psi^l \rangle + \frac{1}{V_E} \int_{l-a} \psi^l \vec{n}^{l-a} d\Gamma + \frac{1}{V_E} \int_{l-s} \psi^l \vec{n}^{l-s} d\Gamma \quad (1.9)$$

$$\left\langle \vec{\nabla} \cdot \vec{\psi}^l \right\rangle = \vec{\nabla} \cdot \langle \vec{\psi}^l \rangle + \frac{1}{V_E} \int_{l-a} \vec{\psi}^l \cdot \vec{n}^{l-a} d\Gamma + \frac{1}{V_E} \int_{l-s} \vec{\psi}^l \cdot \vec{n}^{l-s} d\Gamma \quad (1.10)$$

[Equation \(1.7\)](#) can be recast with the level set method by using the smoothed Heaviside function in the metal. For the metal, this function is equal to one and decreases to zero in the air in a smooth way across both interfaces, solid-air and liquid-air. Since the solid phase is assumed fixed without possible deformation, and knowing that air is assumed incompressible, the solid-air interface does not move, leading to the following equation:

$$\frac{dH^M}{dt} = \frac{\partial H^M}{\partial t} + \vec{v}^{l-a} \cdot \vec{\nabla} H^M = 0 \quad (1.11)$$

## 1.4 FE partitioned model

In this section, we start from a the monodomain finite element model presented in [??](#) that was relevant to the metal only, referred to by the superscript  $M$ , then present

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the essential assumptions and formulations that allow predicting solidification shrinkage in a Eulerian context that introduces another domain, the air, referred to by the superscript  $A$ .

### 1.4.1 In the metal

#### Mass and momentum conservation

By assuming a fixed solid phase ( $\vec{v}^s = \vec{0}$ ), the average velocity in the metal reduces only to liquid's average velocity. Therefore, we can write:

$$\langle \vec{v} \rangle^M = \left\langle \vec{v}^l \right\rangle = g^l \vec{v}^l \quad (1.12)$$

With eq. (1.12), the mass balance in the metal writes:

$$\frac{\partial \langle \rho \rangle^M}{\partial t} + \nabla \cdot \langle \rho \vec{v} \rangle^M = 0 \quad (1.13a)$$

$$\frac{\partial \langle \rho \rangle^M}{\partial t} + \nabla \cdot (g^l \rho^l \vec{v}^l) = 0 \quad (1.13b)$$

$$\frac{\partial \langle \rho \rangle^M}{\partial t} + \rho^l \nabla \cdot (g^l \vec{v}^l) + g^l \vec{v}^l \cdot \vec{\nabla} \rho^l = 0 \quad (1.13c)$$

$$\nabla \cdot \left\langle \vec{v}^l \right\rangle = -\frac{1}{\rho^l} \left( \frac{\partial \langle \rho \rangle^M}{\partial t} + \left\langle \vec{v}^l \right\rangle \cdot \vec{\nabla} \rho^l \right) \quad (1.13d)$$

Equation (1.13d) explains the flow due to shrinkage. A negative divergence term means that a liquid feeding is necessary to compensate for the density difference, hence acting as a flow driving force in the melt. Additional terms should appear in the other conservation equations, balancing the volume change in the heat and species transport.

When the metal's density was considered constant during solidification, the assumption of an incompressible system made it possible to use the Boussinesq approximation. However, in the case of solidification shrinkage, the average density  $\langle \rho \rangle^M$  varies, as it depends on the solidification path as well as on  $\rho^s$  and  $\rho^l$  which are not equal nor constant. Therefore, the incompressibility condition may not be applicable. In such case, the earlier given system ?? is reformulated without any reference value for

density:

$$\left\{ \begin{array}{l} \rho^l \left( \frac{\partial \langle \vec{v}^l \rangle}{\partial t} + \frac{1}{g^l} \vec{\nabla} \cdot (\langle \vec{v}^l \rangle \times \langle \vec{v}^l \rangle) \right) = \\ - g^l \vec{\nabla} p^l - 2\mu^l \vec{\nabla} \cdot (\bar{\nabla} \langle \vec{v}^l \rangle + \bar{\nabla}^t \langle \vec{v}^l \rangle) - g^l \mu^l \mathbb{K}^{-1} \langle \vec{v}^l \rangle + g^l \rho^l \vec{g} \\ \nabla \cdot \langle \vec{v}^l \rangle = - \frac{1}{\rho^l} \left( \frac{\partial \langle \rho \rangle^M}{\partial t} + \langle \vec{v}^l \rangle \cdot \vec{\nabla} \rho^l \right) \end{array} \right. \quad (1.14)$$

### Energy conservation

In the energy equation, a volumetric source term accounts for the heat dissipation caused by the shrinking metal volume. Before writing the new equation, we make the following assumptions:

- consequence of the static solid phase:  $\langle \rho h \vec{v} \rangle = g^l \rho^l h^l \vec{v}^l + g^s \rho^s h^s \vec{v}^s = g^l \rho^l h^l \vec{v}^l$
- the heat generated by mechanical deformation,  $\mathbb{S} : \dot{\epsilon}$ , is neglected

The unknowns in the energy conservation are the average volumetric enthalpy  $\langle \rho h \rangle^M$  and temperature  $T$ . The energy conservation equation writes:

$$\frac{\partial \langle \rho h \rangle^M}{\partial t} + \nabla \cdot \langle \rho h \vec{v} \rangle^M = \nabla \cdot (\langle \kappa \rangle^M \vec{\nabla} T) \quad (1.15a)$$

$$\frac{\partial \langle \rho h \rangle^M}{\partial t} + \nabla \cdot (g^l \rho^l h^l \vec{v}^l) = \nabla \cdot (\langle \kappa \rangle^M \vec{\nabla} T) \quad (1.15b)$$

$$\frac{\partial \langle \rho h \rangle^M}{\partial t} + \langle \vec{v}^l \rangle \cdot \vec{\nabla} (\rho^l h^l) = \nabla \cdot (\langle \kappa \rangle^M \vec{\nabla} T) - \rho^l h^l \nabla \cdot \langle \vec{v}^l \rangle \quad (1.15c)$$

$$\frac{\partial \langle \rho h \rangle^M}{\partial t} + \langle \vec{v}^l \rangle \cdot \vec{\nabla} (\rho^l h^l) = \nabla \cdot (\langle \kappa \rangle^M \vec{\nabla} T) + h^l \left( \frac{\partial \langle \rho \rangle^M}{\partial t} + \langle \vec{v}^l \rangle \cdot \vec{\nabla} \rho^l \right) \quad (1.15d)$$

The second term in the RHS of eq. (1.15d) is a heat power (of unit  $W m^{-3}$ ) that adds to the system in the mushy zone. This term is proportional to the solidification rate and expresses the heat generated in regions where the average density is changing and/or a gradient of liquid density is being advected.

### Species conservation

The last conservation principle is applied to the chemical species or solutes. This principle allows predicting macrosegregation when applied to a solidification system, along with the mass, momentum and energy balances. However, the conservation

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equation should be reformulated in the case of a melt flow driven by shrinkage. Assumptions

- the solidification path is tabulated using thermodynamic data at equilibrium
- the macroscopic solute diffusion coefficient  $D^s$  in the solid phase is neglected in the mass diffusive flux term.
- consequence of the static solid phase:  $\langle \rho w \vec{v} \rangle^M = g^l \rho^l \langle w \rangle^l \vec{v}^l + \cancel{g^s \rho^s \langle w \rangle^s \vec{v}^s} = g^l \rho^l \langle w \rangle^l \vec{v}^l$

The species conservation is pretty similar the energy conservation formulated in the previous section. The main difference is the breakup of the volumetric variable  $\langle \rho w \rangle^M$  into a product of density  $\langle \rho \rangle^M$  and the mass concentration  $\langle w \rangle^M$ . For a binary alloy, we write:

$$\frac{\partial \langle \rho w \rangle^M}{\partial t} + \nabla \cdot \langle \rho w \vec{v} \rangle^M - \nabla \cdot \left( \langle D^l \rangle \vec{\nabla} (\rho^l \langle w \rangle^l) \right) = 0 \quad (1.16a)$$

$$\langle \rho \rangle^M \frac{\partial \langle w \rangle^M}{\partial t} + \langle w \rangle^M \frac{\partial \langle \rho \rangle^M}{\partial t} + \nabla \cdot \left( g^l \rho^l \langle w \rangle^l \vec{v}^l \right) - \nabla \cdot \left( g^l D^l \vec{\nabla} (\rho^l \langle w \rangle^l) \right) = 0 \quad (1.16b)$$

$$\begin{aligned} & \langle \rho \rangle^M \frac{\partial \langle w \rangle^M}{\partial t} + \langle w \rangle^M \frac{\partial \langle \rho \rangle^M}{\partial t} + (\rho^l \langle w \rangle^l) \nabla \cdot \langle \vec{v}^l \rangle + \langle \vec{v}^l \rangle \cdot \vec{\nabla} (\rho^l \langle w \rangle^l) \\ & - \nabla \cdot (g^l D^l \vec{\nabla} (\rho^l \langle w \rangle^l)) = 0 \end{aligned} \quad (1.16c)$$

The mass balance gives the following relation when the liquid density is constant:

$$\nabla \cdot \langle \vec{v}^l \rangle = -\frac{1}{\rho^l} \left( \frac{\partial \langle \rho \rangle^M}{\partial t} \right) \quad (1.17)$$

If we use the result of eq. (1.17) in eq. (1.16c), then we get the following equation:

$$\langle \rho \rangle^M \frac{\partial \langle w \rangle^M}{\partial t} + \langle w \rangle^M \frac{\partial \langle \rho \rangle^M}{\partial t} = \langle w \rangle^l \frac{\partial \langle \rho \rangle^M}{\partial t} - \langle \vec{v}^l \rangle \cdot \vec{\nabla} (\rho^l \langle w \rangle^l) + \nabla \cdot (g^l D^l \vec{\nabla} (\rho^l \langle w \rangle^l)) \quad (1.18)$$

Applying Voller-Prakash [Voller et al. 1989] variable splitting, the system ends up with only one variable,  $\langle w \rangle^M$ . The splitting is done as follows:

$$\langle w \rangle^l = (\langle w \rangle^l)^t + \langle w \rangle^M - (\langle w \rangle^M)^t \quad (1.19)$$

where the superscript  $t$  refers to the previous time step. The chemical species conservation writes:

$$\begin{aligned} \langle \rho \rangle^M \frac{\partial \langle w \rangle^M}{\partial t} + \cancel{\langle w \rangle^M \frac{\partial \langle \rho \rangle^M}{\partial t}} = \\ \cancel{\langle w \rangle^M \frac{\partial \langle \rho \rangle^M}{\partial t}} - \rho^l \langle \vec{v}^l \rangle \cdot \vec{\nabla} \langle w \rangle^M + \nabla \cdot (g^l \rho^l D^l \vec{\nabla} \langle w \rangle^M) \\ + \frac{\partial \langle \rho \rangle^M}{\partial t} \left[ (\langle w \rangle^l)^t - (\langle w \rangle^M)^t \right] - \rho^l \langle \vec{v}^l \rangle \cdot \vec{\nabla} \left( (\langle w \rangle^M)^t - (\langle w \rangle^l)^t \right) \end{aligned} \quad (1.20a)$$

$$\begin{aligned} \langle \rho \rangle^M \frac{\partial \langle w \rangle^M}{\partial t} + \rho^l \langle \vec{v}^l \rangle \cdot \vec{\nabla} \langle w \rangle^M - \nabla \cdot (g^l \rho^l D^l \vec{\nabla} \langle w \rangle^M) = \\ - \frac{\partial \langle \rho \rangle^M}{\partial t} \left[ (\langle w \rangle^M)^t - (\langle w \rangle^l)^t \right] + \rho^l \langle \vec{v}^l \rangle \cdot \vec{\nabla} \left( (\langle w \rangle^M)^t - (\langle w \rangle^l)^t \right) \end{aligned} \quad (1.20b)$$

$$\begin{aligned} \langle \rho \rangle^M \frac{\partial \langle w \rangle^M}{\partial t} + \rho^l \langle \vec{v}^l \rangle \cdot \vec{\nabla} \langle w \rangle^M - \nabla \cdot (g^l \rho^l D^l \vec{\nabla} \langle w \rangle^M) = \\ - \frac{\partial \langle \rho \rangle^M}{\partial t} \left[ (\langle w \rangle^M)^t - (\langle w \rangle^l)^t \right] \\ + \rho^l \langle \vec{v}^l \rangle \cdot \vec{\nabla} \left( (\langle w \rangle^M)^t - (\langle w \rangle^l)^t \right) - \nabla \cdot \left[ g^l \rho^l D^l \vec{\nabla} \left( (\langle w \rangle^M)^t - (\langle w \rangle^l)^t \right) \right] \end{aligned} \quad (1.21)$$

It is noted that eq. (1.21) is valid only if both densities  $\rho^l$  and  $\rho^s$  are constant but have different values. Since density changes are incorporated in this equation, inverse segregation following solidification shrinkage is predicted. For the case where macrosegregation is solely due to fluid flow generated by natural or forced convection, i.e. no shrinkage occurs whether due to thermal-solutal contraction or phase change, the overall volume remains constant, hence density is constant. In this situation,  $\rho^s = \rho^l = \langle \rho \rangle$  and the term  $\partial \langle \rho \rangle / \partial t$  therefore vanishes. After dividing both sides by  $\langle \rho \rangle = \rho^l$ , eq. (1.21) reduces to:

$$\begin{aligned} \frac{\partial \langle w \rangle^M}{\partial t} + \langle \vec{v}^l \rangle \cdot \vec{\nabla} \langle w \rangle^M - \nabla \cdot (g^l D^l \vec{\nabla} \langle w \rangle^M) \\ = \langle \vec{v}^l \rangle \cdot \vec{\nabla} \left( (\langle w \rangle^M)^t - (\langle w \rangle^l)^t \right) - \nabla \cdot \left[ g^l D^l \vec{\nabla} \left( (\langle w \rangle^M)^t - (\langle w \rangle^l)^t \right) \right] \end{aligned} \quad (1.22)$$

### 1.4.2 In the air

The presence of an air domain in our approach is important to follow the free surface of the solidifying metal. For this particular reason, some assumptions are introduced

## **Chapter 1. Macrosegregation with solidification shrinkage**

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and explained in this section in order to limit unnecessary treatment within the air, since it does not undergo phase change. It should be reminded that we consider air as a single-phase system, hence superscripts  $A$  and  $a$  are interchangeably used.

### **Mass and momentum conservation**

To simplify fluid flow resolution in the air, we consider it as incompressible. This assumption is acceptable in the context of casting processes where air velocity has an insignificant order of magnitude. Therefore, the free metal surface is not disturbed by air flow in its vicinity. With the incompressibility of air, we are saying that any deformation of the free surface is solely due to an air mass increase, coming from the system boundaries. The mass balance hence writes:

$$\nabla \cdot \langle \vec{v} \rangle^A = \nabla \cdot \vec{v}^a = 0 \quad (1.23)$$

The air flow is governed by time-dependent incompressible Navier-Stokes equations, as previously done for the metal:

$$\left\{ \begin{array}{l} \rho^a \left( \frac{\partial \vec{v}^a}{\partial t} + \vec{\nabla} \cdot (\vec{v}^a \times \vec{v}^a) \right) = \\ - \vec{\nabla} p^a - 2\mu^a \vec{\nabla} \cdot (\vec{\nabla} \vec{v}^a + \vec{\nabla}^t \vec{v}^a) + \rho^a \vec{g} \\ \nabla \cdot \vec{v}^a = 0 \end{array} \right. \quad (1.24)$$

The air density  $\rho^a$  is considered constant along the casting process, therefore thermal gradients in the air that arise due to the low thermal conductivity, do not generate any flow, i.e. no Boussinesq approximation is made on the term  $\rho^a \vec{g}$  in eq. (1.24).

### **Energy conservation**

It was mentioned in the introduction of the current section that air is a single-phase system that cannot undergo any phase change. Therefore, heat transfer in this domain simplifies to pure thermal conduction with a low thermal conductivity coefficient,  $\langle \kappa \rangle^a$ . The energy balance governs the air enthalpy  $\langle \rho h \rangle^A$  (which is equal to  $\rho^a h^a$  in the

current context) as follows:

$$\frac{\partial \langle \rho h \rangle^A}{\partial t} + \nabla \cdot (\langle \rho h \vec{v} \rangle^A) = \nabla \cdot (\langle \kappa \rangle^A \vec{\nabla} T) \quad (1.25a)$$

$$\frac{\partial \langle \rho h \rangle^A}{\partial t} + \nabla \cdot (\rho^a h^a \vec{v}^a) = \nabla \cdot (\langle \kappa \rangle^a \vec{\nabla} T) \quad (1.25b)$$

$$\frac{\partial \langle \rho h \rangle^A}{\partial t} + \vec{v}^a \cdot \vec{\nabla} (\rho^a h^a) = \nabla \cdot (\langle \kappa \rangle^a \vec{\nabla} T) \quad (1.25c)$$

### Species conservation

The composition of alloying elements is crucial quantity to predict in this work. Nevertheless, such prediction is only relevant in the metallic alloy, even if the air is also made up of other chemical species (nitrogen, oxygen ...). For this obvious reason, the species conservation equation should not be solved in the air, but that of course is contradictory to the principle of a monolithic approach. Consequently, we should compute the conservation of chemical species in both the metal and the air, considering the latter as a *fictitious metal*, thus having initially the same solute mass composition,  $\langle w_0 \rangle^M = \langle w_0 \rangle^A$ . Then, we limit as much as possible solute transport between these domains by limiting solute advection and diffusion. The computed air velocity,  $\vec{v}^a$ , will not be used but rather a zero-velocity vector instead, thus suppressing the solute advection term influence. As for solute diffusion, a very low macroscopic solute diffusion coefficient is used, ensuring that its order of magnitude is at most a thousand times less than that in the melt,  $D^a \ll D^l$ . The low artificial diffusion in the air may slightly violate the wanted no-exchange condition at the air-liquid interface, but it is acceptable since suppressing the diffusion term in the air would result in a numerically stiff partial differential equation.

$$\langle \rho \rangle^A \frac{\partial \langle w \rangle^A}{\partial t} - \nabla \cdot (D^a \vec{\nabla} (\rho^a \langle w \rangle^a)) = 0 \quad (1.26)$$

In contrast to [eq. \(1.21\)](#) for the metal, solute balance in the air, given by [eq. \(1.26\)](#), provides a linear equation as we consider a special case where the domain is monophase, therefore:  $\langle w \rangle^A = \langle w \rangle^a$  at all times. Otherwise, we would have applied the variable decomposition done earlier in [eq. \(1.19\)](#) to linearise the equation.

## 1.5 FE monolithic model

### 1.5.1 Monolithic equations

The monolithic model combines all conservation equations derived for metal and air in a unique set of equations, to be solved on a fixed mesh. This can be accomplished by using the Heaviside function (defined in ??) relative to each domain, creating a mixture of properties that vary across the interface according to a specified mixing law. However, one of the main technical difficulties of the monolithic resolution is that the obtained equation should be consistent with each domain's original equation regarding its shape and terms, making its resolution easier. While for energy and solute balances the procedure is straightforward, the presence of the Darcy dissipative term in the metal's Navier-Stokes system makes it more difficult to formulate a single monolithic equation, which is discussed after writing the equations of the monolithic model.

#### Mass and momentum conservation

For the system's velocity,  $\langle \vec{v}^F \rangle$ , is given by an arithmetic mixing between each domain's relative average fluid velocity, i.e. we need the relative fluid velocity with respect to other fixed/rigid phases in each domain. In the present context, the metal domain consists of a single fluid phase (liquid) and solid phases that form in fixed and rigid structures (assuming that solidification results in an undeformable columnar dendritic and eutectic structures, without any free equiaxed dendritic structure), while the latter domain entirely consists of a fluid phase (air). With this notation, we express the monolithic mass balance as:

$$\nabla \cdot \langle \vec{v}^F \rangle = \nabla \cdot (H^M \langle \vec{v} \rangle^M + H^A \langle \vec{v} \rangle^A) \quad (1.27)$$

$$\nabla \cdot \langle \vec{v}^F \rangle = H^M \nabla \cdot \langle \vec{v} \rangle^M + H^A \nabla \cdot \langle \vec{v} \rangle^A + \vec{\nabla} H^M \cdot (\langle \vec{v} \rangle^M - \langle \vec{v} \rangle^A) \quad (1.28)$$

$$\nabla \cdot \langle \vec{v}^F \rangle = H^M \nabla \cdot \langle \vec{v}^l \rangle \quad (1.29)$$

where we used the relation eq. (1.12) in the case of a fixed rigid solid to obtain eq. (1.29). As for the second term in eq. (1.28), we have made the assumption that air is incompressible. Therefore any volume variation of the metal domain, will trigger an air inflow or suction effect through the surface boundaries of the air domain. The third and last term in the same equation expresses a velocity jump at the interface. In our case, we neglect this contribution by assuming that both velocities tend to be equal when the interface thickness tends to zero. Finally, the monolithic mass balance

writes:

$$\nabla \cdot \langle \vec{v}^F \rangle = H^M \left( -\frac{1}{\rho^l} \left( \frac{\partial \langle \rho \rangle^M}{\partial t} + \langle \vec{v}^l \rangle \cdot \vec{\nabla} \rho^l \right) \right) \quad (1.30)$$

The momentum balance looks like the one derived for the metal, but using level set mixed properties, we get the following:

$$\begin{cases} \hat{\rho} \left( \frac{\partial \langle \vec{v}^F \rangle}{\partial t} + \frac{1}{g^F} \vec{\nabla} \cdot (\langle \vec{v}^F \rangle \times \langle \vec{v}^F \rangle) \right) = \\ - g^F \vec{\nabla} p - 2\hat{\mu} \vec{\nabla} \cdot (\vec{\nabla} \langle \vec{v}^F \rangle + \vec{\nabla}^t \langle \vec{v}^F \rangle) - g^F \hat{\mu} \tilde{\mathbb{K}}^{-1} \langle \vec{v}^F \rangle + \hat{\rho} g \vec{g} \\ \nabla \cdot \langle \vec{v}^F \rangle = H^M \left( -\frac{1}{\rho^l} \left( \frac{\partial \langle \rho \rangle^M}{\partial t} + \langle \vec{v}^l \rangle \cdot \vec{\nabla} \rho^l \right) \right) \end{cases} \quad (1.31)$$

Note that the Darcy term has a special treatment explained in the next section. The mechanical properties are mixed as follows:

$$\text{Fluid fraction : } g^F = H^M g^l + H^A g^a = H^M g^l + H^A \quad (1.32)$$

$$\text{Density : } \hat{\rho} = H^M \rho^l + H^A \rho^a \quad (1.33)$$

$$\text{Dynamic viscosity : } \hat{\mu} = H^M \mu^l + H^A \mu^a \quad (1.34)$$

$$\text{Weight force : } \hat{\rho} g \vec{g} = H^M g^l \rho^l \vec{g} + H^A g^a \rho^a \vec{g} = H^M g^l \rho^l \vec{g} + H^A \rho^a \vec{g} \quad (1.35)$$

We defined a fluid fraction,  $g^F$ , as an arithmetic mixing between liquid and air fractions across the interface. This quantity will be essential for the monolithic Darcy term. As for the weight force in both domains, it is taken into account via eq. (1.35). The phase densities may vary as functions of other parameters such as temperature or phase composition ( $\rho^l$  depends on both), creating buoyancy forces of convection inside the fluid. In our approach, since we are only interested in liquid's flow, we keep the air phase density  $\rho^a$  constant, so as to prevent a mixture of forces around the level set, which helps stabilise the fluid flow resolution.

## Chapter 1. Macrosegregation with solidification shrinkage

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### Energy conservation

Deriving the monolithic energy conservation equation is straightforward. The monolithic system writes:

$$\frac{\partial \widehat{\langle \rho h \rangle}}{\partial t} + \left\langle \vec{v}^F \right\rangle \cdot \vec{\nabla} \widehat{\langle \rho h \rangle^F} = \nabla \cdot \left( \widehat{\langle \kappa \rangle} \vec{\nabla} T \right) + \widehat{\Phi} \quad (1.36)$$

The solution of [eq. \(1.36\)](#) is  $\widehat{\langle \rho h \rangle}$ , a mixed field between both domains average volumetric enthalpies. The other parameters are  $\widehat{\langle \rho h \rangle^F}$  and  $\widehat{\langle \kappa \rangle}$  which denote respectively the fluids' mixture volume enthalpy and the mixture of average thermal conductivities. The last term,  $\widehat{\Phi}$ , is an average heat source accounting for energy change caused by the alloy's shrinking volume. As no volume change was considered for the air,  $\widehat{\Phi}$  is present only in the metal's energy balance. These quantities are defined in the following equations:

$$\text{Total enthalpy : } \widehat{\langle \rho h \rangle} = H^M \langle \rho h \rangle^M + H^A \langle \rho h \rangle^A \quad (1.37)$$

$$\text{Fluid phases enthalpy : } \widehat{\langle \rho h \rangle^F} = H^M \rho^l h^l + H^A \rho^a h^a \quad (1.38)$$

$$\text{Average thermal conductivity : } \widehat{\langle \kappa \rangle} = H^M \langle \kappa \rangle^M + H^A \langle \kappa \rangle^A \quad (1.39)$$

$$\text{Average heat change : } \widehat{\Phi} = H^M \rho^l h^l \nabla \cdot \left\langle \vec{v}^l \right\rangle \quad (1.40)$$

### Species conservation

As previously explained in [section 1.4.2](#), we consider that species conservation is solved over both domains, as follows:

$$\begin{aligned} & \langle \rho \rangle^M \frac{\partial \widehat{\langle w \rangle}}{\partial t} + \rho^l \left\langle \vec{v}^l \right\rangle \cdot \vec{\nabla} \widehat{\langle w \rangle} - \nabla \cdot \left( g^l \rho^l D^l \vec{\nabla} \widehat{\langle w \rangle} \right) = \\ & - \frac{\partial \langle \rho \rangle^M}{\partial t} \left[ \left( \langle w \rangle^M \right)^t - \left( \langle w \rangle^l \right)^t \right] \\ & + \rho^l \left\langle \vec{v}^l \right\rangle \cdot \vec{\nabla} \left( \left( \langle w \rangle^M \right)^t - \left( \langle w \rangle^l \right)^t \right) - \nabla \cdot \left[ g^l \rho^l D^l \vec{\nabla} \left( \left( \langle w \rangle^M \right)^t - \left( \langle w \rangle^l \right)^t \right) \right] \end{aligned} \quad (1.41)$$

then we deduce the metal's solute composition by postprocessing the obtained solution.

i should multiply solid fraction by HeavisideM before Sortie to correct the visu. no problem for resolution as in Tsolver HeavisideA is taken into account then next time step's T2H will determine a raw solid fraction in the air and the metal

i should create a new FeC tabulation with concentration variation. Once done, revert to 5 times the Concentration parameter in the ChampParametre of Tsolver and T2H

### 1.5.2 Darcy term in the air

We have seen in the previous chapter that adding the Darcy term into Navier-Stokes system modifies the shape of the equation, dividing all terms by the liquid fraction,  $g^l$  (REF VMS SOLVER CHAPTER 4). The presence of this dissipation term in one domain, obliges us to keep it in both domains but "deactivate" it where it is useless, i.e. in the air. This is done by computing a fictitious permeability in the air as function of the air fraction using the Carman-Kozeny model, as used previously for the metal in ???. We may speak of level set mixing for the Darcy term. It has a double advantage:

1. the consistency in shape is kept between both domains equations, thus easily deriving the monolithic system;
2. since the monolithic system retains the shape of the monodomain equation, the VMS solver does not require further implementation updates and subsequent validation.

$$\tilde{K} = \frac{\lambda_2^2 g^F}{180 (1 - g^F)^2} \quad (1.42)$$

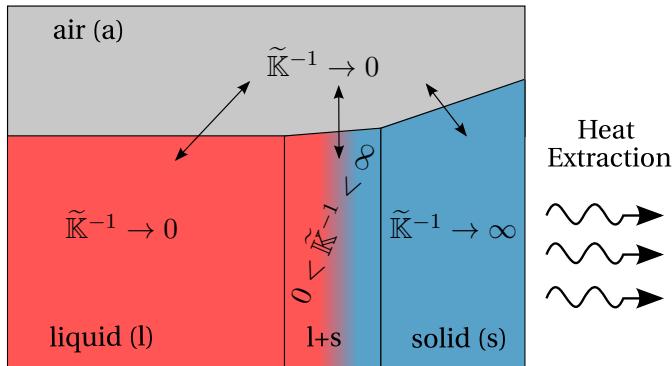
then from the fluid fraction (eq. (1.32)), we deduce a modified permeability,  $\tilde{K}$ . Depending on the values of this quantity, the extent to which the Darcy becomes imposing in Navier-Stokes varies as follows:

- $\tilde{K}^{-1} \rightarrow 0$  (completely permeable), then Darcy's term is negligible in Navier-Stokes resolution,
- $\tilde{K}^{-1} > 0$  (slightly permeable), then fluid flow is greatly dissipated due to a decreasing permeability,
- $\tilde{K}^{-1} \rightarrow \infty$  (non permeable), then no fluid flow may exist.

These 3 cases are graphically represented in fig. 1.4, showing the different values along with the transitions with respect to phases and domains distribution. It is clear that neither in liquid nor air, the flow is dissipated by the presence of the Darcy term in the monolithic system, which is confirmed in REF FIGURE DRAW DARCY/PERMEABILITY FROM LIQUID FRACTION RESULT.

### 1.5.3 Interface motion and stability

In chapter 2, we have presented the transport equation of the distance function using the velocity solution provided by solving momentum conservation equations. In this



**Fig. 1.4** – Schematic representation of an ingot undergoing solidification while shrinking. The inverse of the modified permeability,  $\tilde{K}^{-1}$ , falls to zero in the air and liquid phases, indicating that the Darcy term is only activated in the solid and liquid+solid regions. The arrows indicate three different transitions of the Darcy term between the air and metal domains.

section, we discuss some important points regarding the stability of the interface while being transported. When real values are assigned for the mechanical properties, especially density, a ratio of three orders of magnitude exists in the diffuse interface. This high ratio may badly influence the stability of the interface, unless we pay attention to the order of the integration method as well as the time step used in Navier-Stokes resolution.

### Integration order influence

#### Time step influence

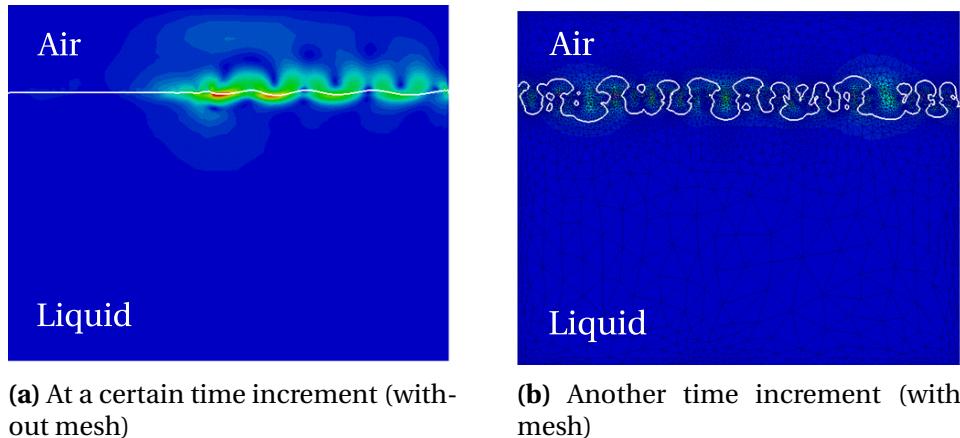
In the first tests, we noticed

TRY TIME STEP 0.01 sec (stable) and 0.02 sec (unstable): is CFL the cause ?

#### Classical coupling

A "classical" coupling comes in the sense of "unmodified" coupling. This approach consists of taking the output of the fluid mechanics solver, then feed it as raw input to level set transport solver. The physical translation would be that the interface motion is dictated by the fluids flow in its vicinity. No treatment whatsoever is done between the two mentioned steps. While conservation principles are best satisfied with this approach, the latter yields some drawbacks, preventing its application in a generic way. For instance, the free liquid surface is not necessarily horizontal at all times and that can lead to the wrong shrinkage profile when solidification is complete.

present the example of unstable interface when the ratio between fluids properties became greater than some value+discussion



**Fig. 1.5 – Interface destabilisation under the effect of high properties ratio across the interface.**

### Modified coupling

In contrast to a classic coupling, here we attempt to modify the velocity field before feeding to the transport solver. The main motivation for considering this approach is the lack of stability that we observed whenever the mechanical properties of the fluids were different by several orders of magnitude. The algorithm should simultaneously fulfil these requirements:

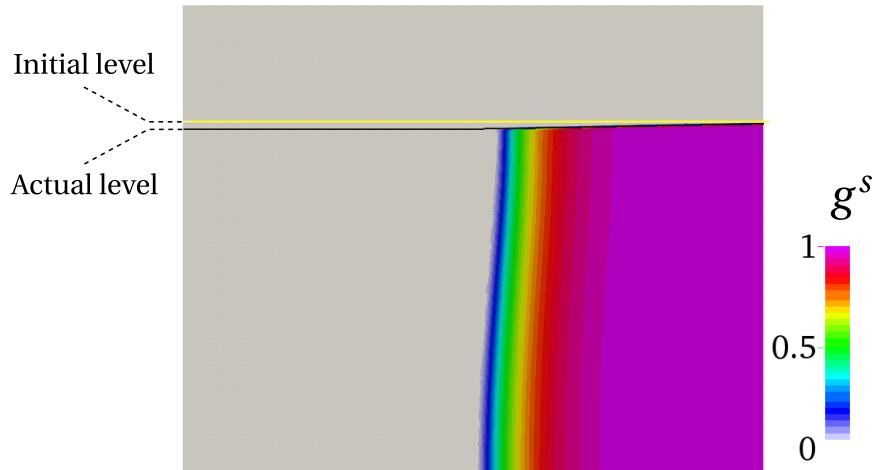
- support high ratios of fluids density with close viscosities by preserving a non-oscillating interface,
- maintain a horizontal level at the free surface of the melt,
- follow shrinking metal surface profile in solidifying regions,
- satisfy the mass conservation principle, essentially in the metal.

We want to process the original transport velocity by imposing a uniform motion (speed and direction) at the nodes of the free surface, and at the same time, be able to follow the pipe formation at the surface as a result of solidification shrinkage, as shown in [fig. 1.6](#).

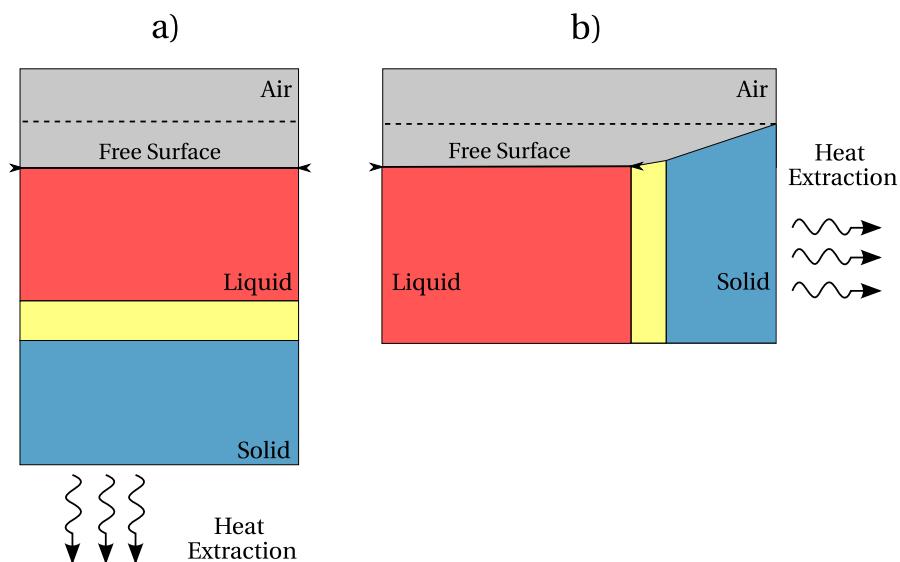
How to transport level set using velocity from momentum conservation DIRECTLY or AVERAGED PER ELEMENTS, show examples of instability/stability when using false/nominal air properties

Validation of LS transport: perform test case simulation of buoyancy driven air droplet in water by 2005Nagrath that I also have seen in Shyamprasad's masters report). => I didnt notice: what time step  $\delta t$  did they use ?

The general idea is read the velocity around the interface up to a certain thickness, which may be the same thickness as the diffuse interface defined in ??, then compute a volumetric average from all the elements in the thickness. This average is then



**Fig. 1.6** – Snapshot of a solidifying ingot by a cooling flux from the side. The profile of the actual surface changes in solid and mushy regions to adapt the new density while staying perfectly horizontal in the liquid phase.



**Fig. 1.7** – Treatment of liquid free surface in a) bottom and b) side heat extraction configurations. The dashed line represents the initial level of the free liquid surface.

## 1.6. 1D application: solidification with inverse segregation

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given to the transport solver, which will apply the same magnitude and direction to transport the interface. However, as we only need the transport velocity to be uniform within the "100% liquid" elements, it should not be the case for the other elements that belong either to the mushy zone or the solid region, where shrinkage is taking place. Therefore, depending on the heat extraction configuration, two scenarios are possible. If heat extraction is far from the interface, i.e. there is not direct contact as in [fig. 1.7a](#), the surface area remains unchanged at any time, hence all the elements around the interface are "100% liquid". This happens when a bottom cooling is applied to the ingot. In contrast, if a side cooling is applied as shown in [fig. 1.7b](#), the surface area of the interface will be reduced over time as a consequence of the solid front progression. In this case, the average transport velocity should be computed only from the elements belonging to the free surface. The remaining part of the interface which belongs to partial or full solid regions, is transported with Navier-Stokes output, which should be some orders of magnitude less than the velocity imposed at the free surface, as a result of a decreasing permeability.

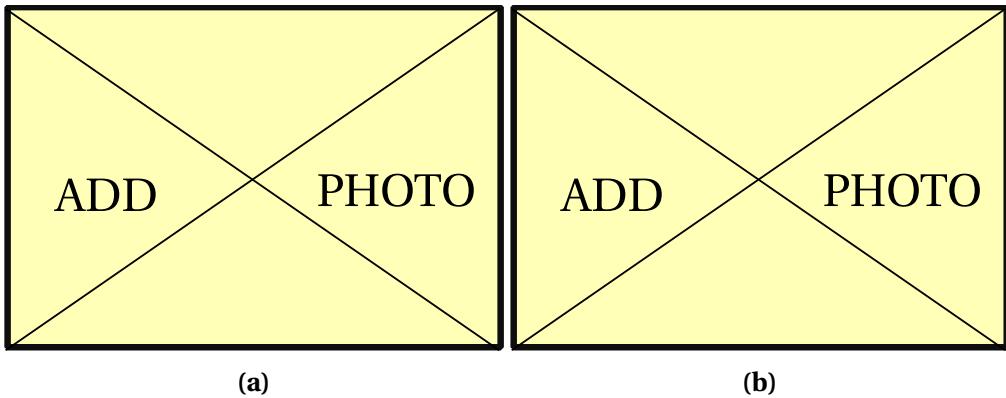
## 1.6 1D application: solidification with inverse segregation

### 1.6.1 Geometry and boundary conditions

A simple but very efficient way of analysing the model is to test it through a 1D flow configuration with energy and species conservation. For this purpose, we take an aluminium-silicon alloy with the same properties used for the thermal solver validation in [??](#). The major difference with respect to the former validation setup is the air domain that should also be included in the mesh. We consider a 2D rod having as dimensions  $0.14\text{ m} \times 0.001\text{ m}$ , where initially the air column's height is only  $0.04\text{ m}$  and the remainder of the length is for the metal. **FIGURE** shows the geometry, mesh used for this section simulations, while **FIGURE** shows the thermal and mechanical boundary conditions. In the latter, velocity-slip conditions were imposed on the lateral boundaries while a no-slip was used at the bottom where heat is extracted, to ensure a 1D air flow from the free air inlet at the top.

In this case, imposing slip conditions on lateral sides is two-fold: on one hand, we need to ensure that the fluid flow solution remain one-dimensional, hence symmetry on the boundaries solves the issue, while on the other hand during solidification, the resulting feeding flow should be able to transport the interface intersecting with boundary nodes. If boundary velocities are zero, then the interface transport will face problems at these boundary nodes. This is indeed an important and relevant point in

the next 2D test case.



**Fig. 1.8 – Geo and boundary conditions**

### 1.6.2 Shrinkage without macrosegregation

The first simulation is the solidification without any segregation, hence a unique solidification path is considered at  $w_0 = \text{Al-7 wt.\% Si}$ , shown in [fig. 1.10](#). This case is interesting as a reference case, where we can study volume shrinkage and level set behaviour in a simple segregation-free configuration. We first use a homogeneous isotropic mesh of constant size  $h = 200 \mu\text{m}$ . The liquid and solid phase densities are assumed constant and respectively equal to  $2600 \text{ kg m}^{-3}$  and  $2800 \text{ kg m}^{-3}$ . This density difference is equivalent to a ratio of  $\beta_{SS} = (\rho^s - \rho^l) / \rho^s$ , also termed as *solidification shrinkage* by [Flemings et al. \[1967\]](#). In the current conditions,  $\beta_{SS}$  is constant and equal to 7.14 %. The domain is initially meshed before any resolution, as shown in [fig. 1.9](#), in a way to reduce interpolation errors that may cause coarse elements within the transition of fluids density and dynamic viscosity, reminding that these parameters are crucial for the stability of the velocity solution used later in the transport step. The chosen mixing laws are arithmetic for the density and logarithmic for the viscosity. The choice is based on tests done in former Ph.D. projects at CEMEF. The initial mesh sizes are given in [table 1.1](#).

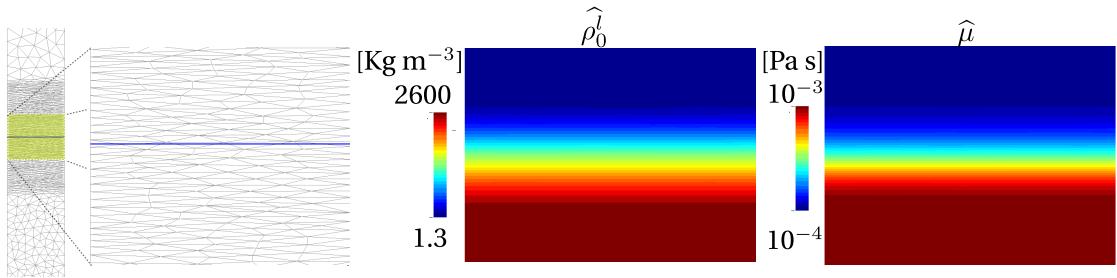
At a first glance, results show that the interface stability is compromised by a chosen time step for a given mesh size, and that the interface dynamics require attention even before investigating the feeding flow created by solidification. As a demonstration, [fig. 1.11](#) shows different time steps with the same adaptive meshing parameters. For time steps greater than 0.01 s, Navier-Stokes computations did not converge resulting in an high artificial flow quickly destabilising the interface. It should be noted that the frame corresponding to 0.02 s was taken at an earlier time than the two other frames.

## 1.6. 1D application: solidification with inverse segregation

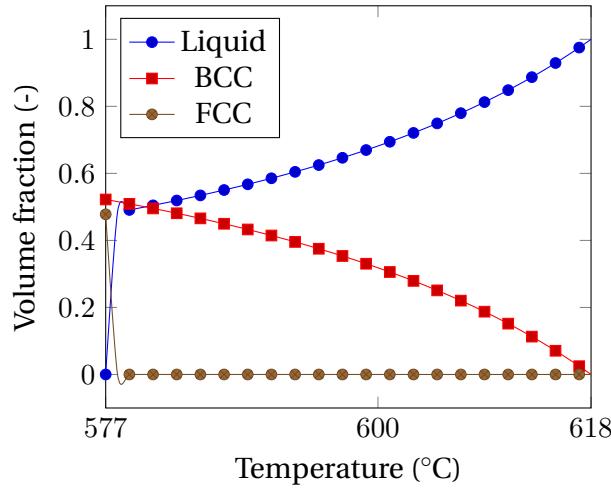
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**Table 1.1** – Summary of the different mesh sizes used to generate an adaptive anisotropic mesh, along with the level mixing thickness,  $\varepsilon$ . Refer to ?? for the definition of each mesh parameter.

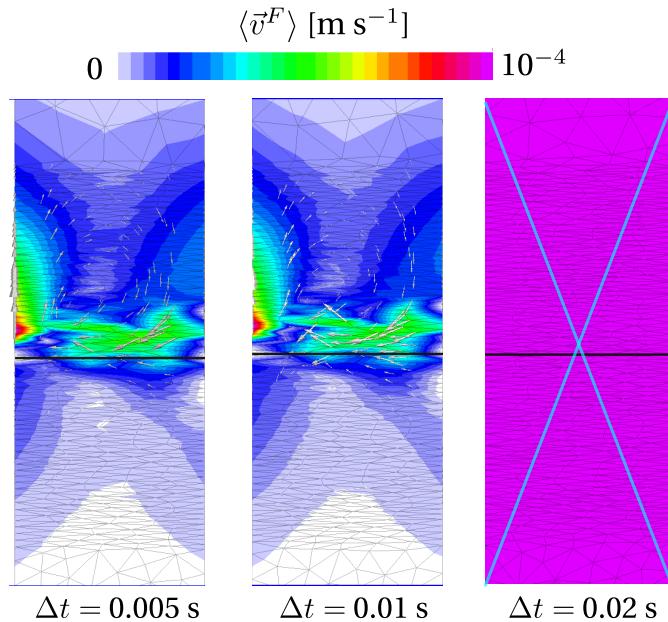
Mesh parameter	Size [m]
$\varepsilon$	$2.5 \times 10^{-4}$
$h_{\vec{n}}$	$2.5 \times 10^{-5}$
$h_{\vec{\tau}}$	$2 \times 10^{-4}$
$h_M$	$1.5 \times 10^{-4}$
$h_A$	$2.5 \times 10^{-4}$



**Fig. 1.9** – Snapshots of the initial adapted mesh around the interface with different mesh sizes in the air and the metal. The adapted region is stretched beyond the level set mixing thickness to ensure better interpolation around the interface, in case of emergence of diffusion instabilities. To the right, the reference fluid density and viscosity are plotted in the transition zone, showing a symmetric mixing for the former and a shifted mixing for the latter, as a consequence of the mixing laws. The thick blue line represents the zero iso-value of the distance function.



**Fig. 1.10** – Unique solidification path at nominal composition for the shrinkage case without macrosegregation.



**Fig. 1.11** – Three average fluid velocity frames at different time steps: 0.005 s, 0.01 s and 0.02 s. The first and second frame are taken after 50 seconds of cooling while for the last frame, the frame was taken after only 1 second of cooling, thus it is crossed to show non-convergence. The thick black line represents the zero iso-value of the distance function.

Although no solidification has yet started at 100 s, a two-dimensional flow is observed around the interface, while tends to  $10^{-8} \text{ m s}^{-1}$  elsewhere in the ingot. Figure 1.11 confirms that this flow is still predicted at smaller time steps. This flow seems like a pure numerical response to the properties jump across the interface, namely density and dynamic viscosity. It is also noted that the interface position is not modified by the neighbouring currents, that reach a maximum magnitude of  $10^{-4} \text{ m s}^{-1}$ . Therefore, the optimal time step for this simulation is set to 0.01 s, and we refer to it as case R, which stands for "real" air.

The fact that properties transition is crucial in the solution stability, is investigated by 2 reference cases, having equal properties (density and dynamic viscosity) but with different time steps, 0.01 (case A1) and 0.1 s (case A2), where "A" stands for artificial. All simulation cases are grouped in table 1.2.

**Table 1.2** – Summary of the comparative shrinkage simulations without macrosegregation.

Case	Air viscosity [Pa s]	Air density [ $\text{kg m}^{-3}$ ]	Time step [s]
A1	$10^{-3}$	2600	0.01
A2	$10^{-3}$	2600	0.1
R	$10^{-4}$	1.3	0.01

## 1.6. 1D application: solidification with inverse segregation

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When the air subdomain is given the metal's properties, it becomes denser and more viscous by several orders of magnitude. [Figure 1.12](#), in which cases A1 and A2 are compared, shows no noticeable sign of velocity instability near the interface before 200 s. It can be explained by the fact that the air behaves mechanically like a fluid metal given similar properties, therefore no steep transitions are computed at the interface. However, it is interesting to compare results of [fig. 1.12a](#) and [fig. 1.12b](#) at 600 s. For case A1, the interfaces is slightly skewed due to slower flow at the left side of the interface, while for case A2, the flow disturbs the interface deforming it until the end of solidification, as seen at 1000 s. This shows the importance of the chosen time step in both the Navier-Stokes solver. We can also notice a common issue for all three case, A1, A2 and R, approximately at 400 s, the mushy zone enters within the diffuse interface, affecting the level set transport and thus causing mass conservation problems. These problems are discussed in the next section.

In contrast, [fig. 1.13b](#) shows more viable results as far as the level set transport is concerned. From 200 s to 800 s, the local flow instability (discussed earlier in [fig. 1.11](#)) is sustained, even until after solidification is complete. However, in regions of 100% metal and 100% air the computed velocity is nearly the same order of magnitude as predicted for all three simulations. Finally, in [fig. 1.13b](#), we notice a recirculating air flow in the vicinity of the interface as no metal shrinkage may further occur once solidification is complete, thus air flows freely in and out of the upper boundary with a very low magnitude ( $\sim 10^{-7} \text{ m s}^{-1}$ ), while impinging on the air-metal surface. Regarding the CPU times, cases A1 ran for 14 hours, case A2 took only 2 hours while case R ran for 23.3 hours.

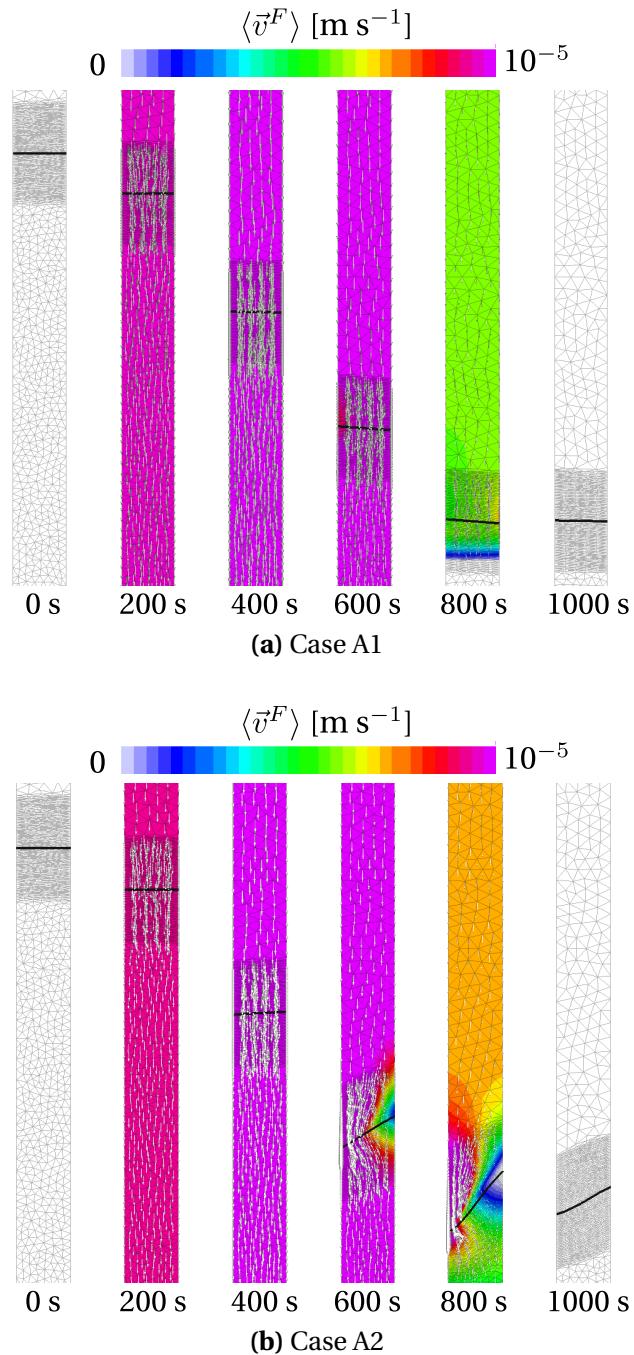
### Mass conservation study

The previous computations showed that the mass conservation, which is the consequence of the level set transport, can be lost. In order to better understand the process, we define the metal's mass as a function of the metal's average density and the Heaviside function relative to the metal subdomain, as follows:

$$m^M = \int_{\Omega} H^M \langle \rho \rangle^M \, d\Omega \quad (1.43)$$

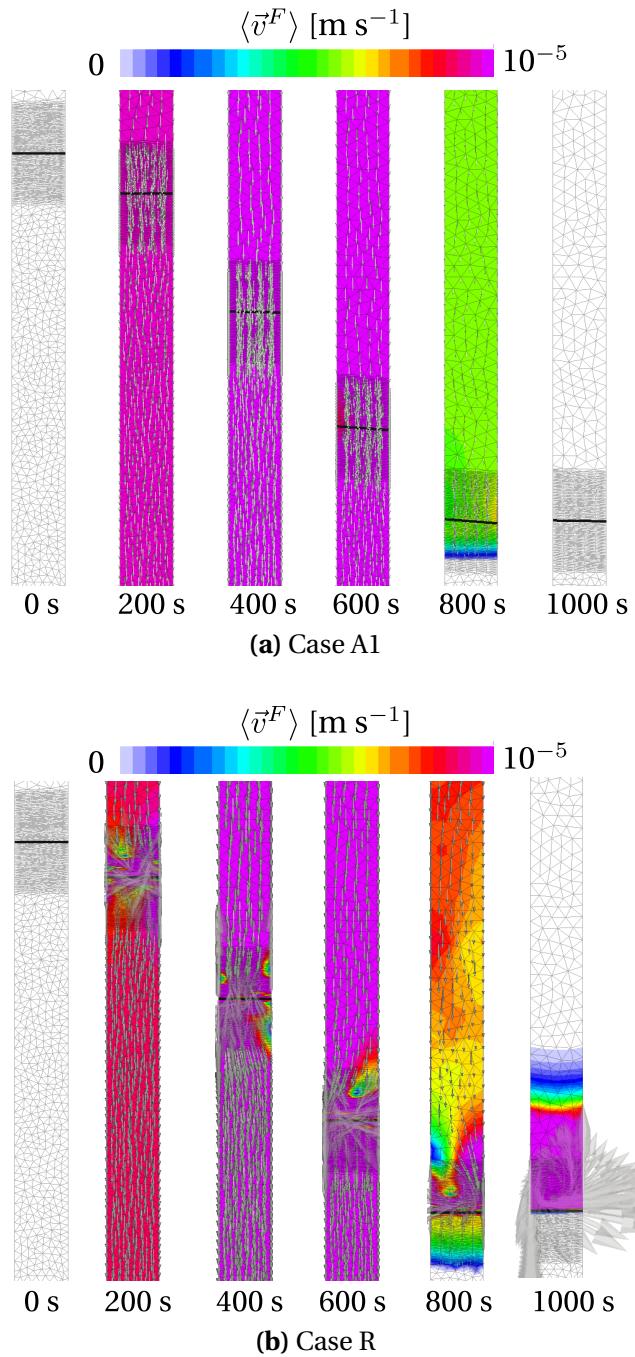
Then, the mass conservation can be monitored by processing [eq. \(1.43\)](#) at each time step, and computing the relative mass change by writing:

$$m_{\%}^M = \frac{m^M - m_i^M}{m_i^M} \times 100 \quad (1.44)$$



**Fig. 1.12** – Comparison of two simulations in several stages of solidification ending shortly after 800 s. The results shows the influence of density and viscosity properties across the level set interface. The plotted field is the average fluid velocity, on which the corresponding nodal vectors are superimposed, pointing towards the solidification front. The thick black line represents the zero iso-value of the distance function.

## 1.6. 1D application: solidification with inverse segregation

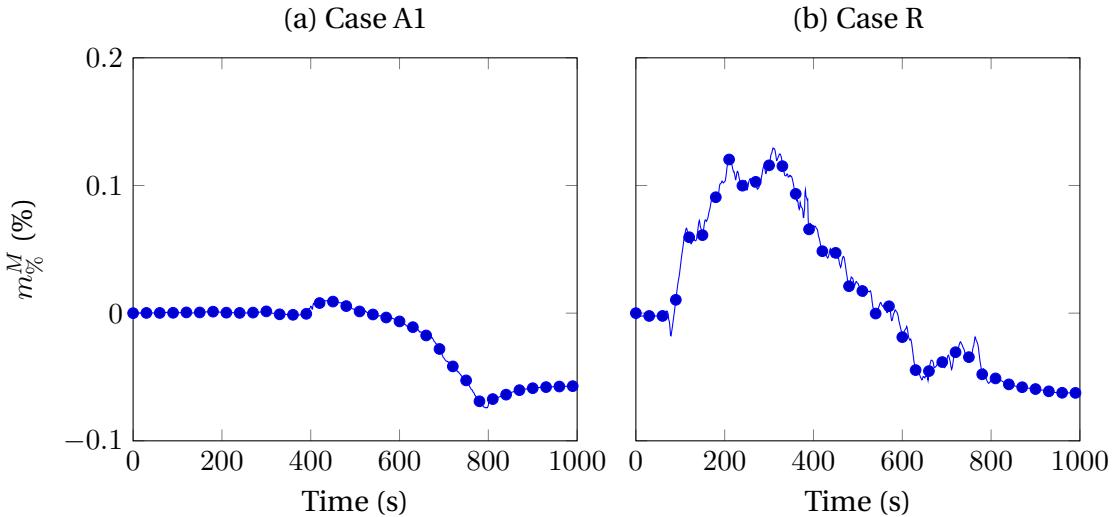


**Fig. 1.13** – Comparison of two simulations in several stages of solidification ending shortly after 800 s. The results shows the influence of density and viscosity properties across the level set interface. The plotted field is the average fluid velocity, on which the corresponding nodal vectors are superimposed, pointing towards the solidification front. The thick black line represents the zero iso-value of the distance function. The vectors were made transparent in the transition zone so as not to hide the interface level.

## Chapter 1. Macrosegregation with solidification shrinkage

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The relative mass change gives us information on the level set transport. As the current case is 1D and phase densities are constant throughout the simulation, mass conservation can be checked by a much simpler than by checking eq. (1.44). Since we know the initial metal's column length,  $l_0^M$ , and the expected solidification shrinkage is 7.14 %, we should expect a final length of  $l_f^M = (1 - \beta_{SS}) l_0^M = 0.09286$  m. However, this equation is very useful and applicable in 2D and 3D cases of solidification shrinkage.



**Fig. 1.14** – Variation of the metal's mass versus solidification time in cases A1 and R.

In the previous section, we observed transport problems taking place around 400 s of cooling. Although it was clearly seen in fig. 1.12b, it applies for both cases, whether air properties are equal or different than the liquid's properties across the interface. The mass variation plots in fig. 1.14 confirm these observations. Therefore, we can deduce that regardless of the time step and the level set mixing of properties, transport problems may occur.

The level set method is known to have poor mass conservation properties. However, the mass variation we see in the previous plots is more related to a physical problem: **at which velocity does the interface move?** In our simulations, we systematically considered the Navier-Stokes solution  $\langle \vec{v}^F \rangle$ , to transport the metal-air interface. This solution is equal to  $\langle \vec{v}^l \rangle = g^l \vec{v}^l$  in the metal subdomain. However in reality, between the metal and the air, two interfaces exist: the liquid-air (*l-a*) and solid-air (*s-a*) interfaces, and these interfaces move at different velocities as solidification proceeds. The liquid-air interface exists at early stages of solidification where only liquid is in contact with the air. In later stages, the mushy zone delimited by dendrite tips, reaches the free liquid surface, creating hence these distinct interfaces: a first interface separating

## 1.6. 1D application: solidification with inverse segregation

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interdendritic liquid from the air, and a second interface that separates the dendrites also from the air. The  $l-a$  interface is driven by solidification shrinkage and therefore by the real microscopic velocity of the interdendritic liquid,  $\vec{v}^l$ . According to [Dantzig et al. \[2009\]](#), this velocity is constant when the solidification shrinkage and the isotherms velocity  $\vec{v}_T$  are constant, as states the equation:

$$\vec{v}^l = -\beta_{SS} \vec{v}_T \quad (1.45)$$

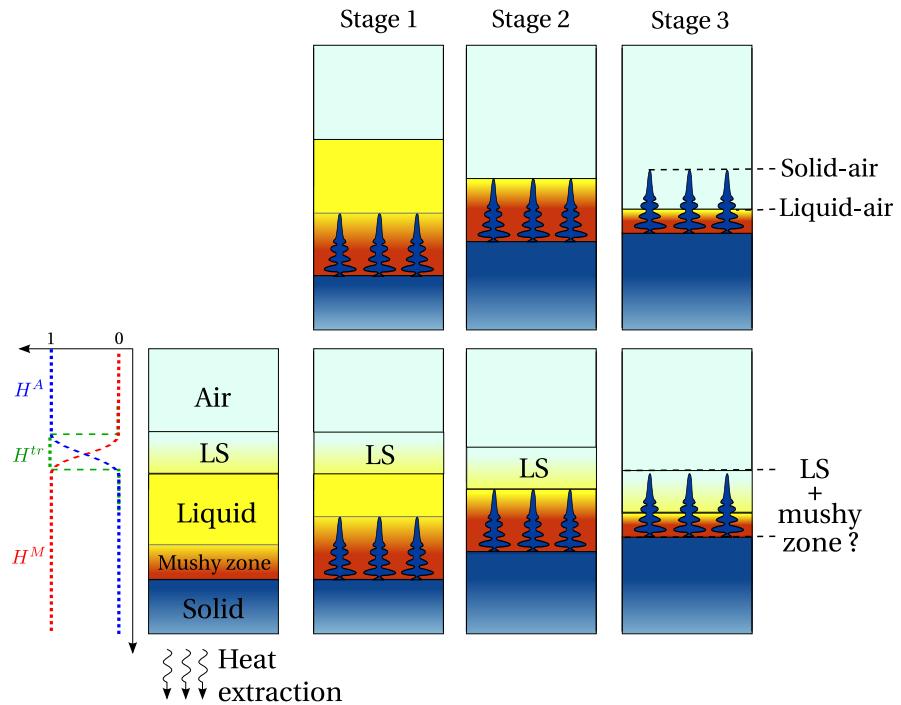
As for the  $s-a$  interface, its motion is induced by a mechanical deformation of the solid phase either due to thermal shrinkage or external mechanical stresses. The first factor is ubiquitous in any solidification process, while the second factor is process-dependent. In the present work, we remind that the solid phase is assumed fixed and rigid, therefore we consider dendrites to be undeformable during their growth. Unfortunately, this assumption is contradictory with our current situation where the metal keeps shrinking, until an overlap between the level set's diffuse interface and the mushy zone overlap, as shown in [fig. 1.15](#)). At this stage, tracking a single interface induces concept errors, because we are not sure which of the previously mentioned interfaces is being tracked, and hence failing to determine an adequate transport velocity field.

In the light of these facts, we will try to limit as much as possible the interface motion, once the mushy zone has reached the diffuse interface. To do so, we firstly advise to keep a very small thickness interface, in order to delay the previously explained overlap. Moreover, we suggest computing the transport velocity, used in ??, at each node  $j$  as follows:

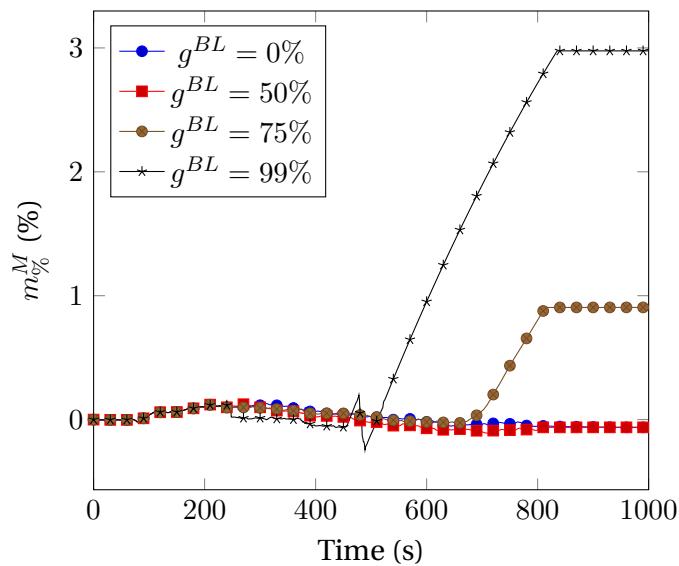
$$\vec{v} = \begin{cases} \left\langle \vec{v}^l \right\rangle_j & \text{if } g_j^l > g^{BL} \\ 0 & \text{otherwise} \end{cases} \quad (1.46)$$

where  $g^{BL}$  is the threshold for the liquid fraction, below which we consider that the interface should not be transported. [Figure 1.16](#) shows the mass variation for three blocking fractions: 0, 50, 75 and 99 percent. The first value corresponds the case where the Navier-Stokes solution is directly passed to the transport solver. For the last value, the portion of the interface in contact with the low solid fraction part of the mushy zone becomes immobile. It is clearly seen that the consequence on the mass conservation is not very good, as the mass increases up to 3% while the eutectic front is consuming the liquid within the mushy zone, while no further shrinkage is allowed.

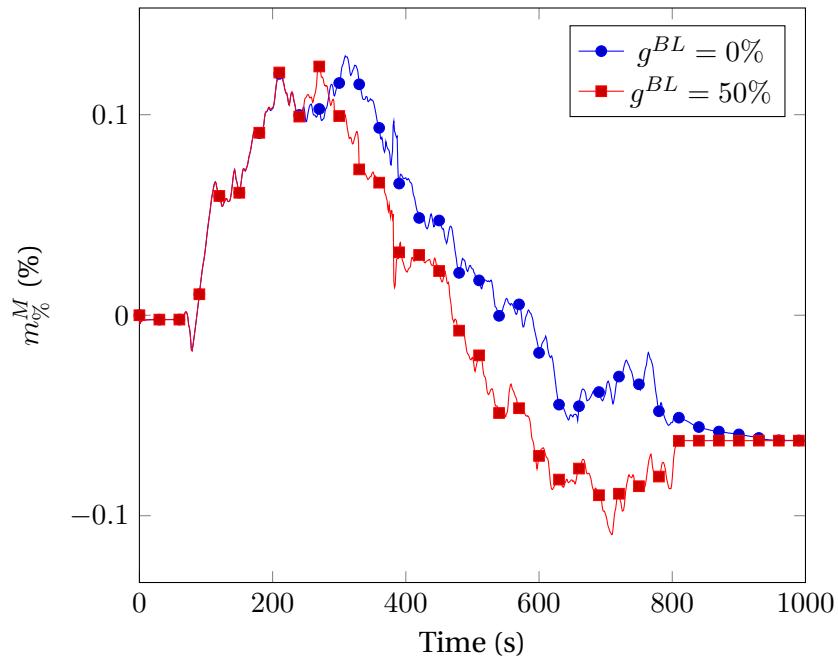
In [fig. 1.17](#), we plot again the same curves as in [fig. 1.16](#), but keeping the values of



**Fig. 1.15** – Schematic describing the physical (first row) and numerical (second row) process of moving an interface between air and metal subdomains, in three intermediate stages of solidification.



**Fig. 1.16** – Relative mass change versus time for different blocking fractions  $g^{BL}$  in the transport solver.



**Fig. 1.17** – Relative mass change versus time only for 0% and 50% blocking fractions.

$g^{BL}=0\%$  and  $g^{BL}=50\%$ .. We notice that both values produce the same results until about 300 s. Then, when the mushy zone reaches the interface region, differences appear as a consequence of the reduced transport for the higher blocking fraction. However, it should be pointed out that the differences between 300 s and 800 s are not important because the permeability predicted by the Carman-Kozeny model, falls to zero quickly for liquid fractions less than about 60%.

In the current application, it is not clear whether the idea of the blocking fraction is useful or not, since the feeding flow occurs in a single direction and solidification takes place far from the interface. Therefore, it is interesting to test again in the coming 2D and 3D applications to see if it yields advantages on the final shape of the interface.

### 1.6.3 Shrinkage with macrosegregation

in this section, we consider species conservation equation, in addition to energy conservation and fluid momentum conservation equations, used in the previous section to predict solidification shrinkage. The interesting point here is to study the formation of macrosegregation in a one-dimensional configuration and the effect of solidification shrinkage on it. As shown in chapter 2, our approach to solve the energy equation relies on tabulations of various solidification paths. In this case, we will generate a simple tabulation based on a phase diagram with linear liquidus and solidus lines,

## Chapter 1. Macrosegregation with solidification shrinkage

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whose properties are reminded in [table 1.3](#)

**Table 1.3** – Main properties of the linearised phase diagram for Al-Si alloys.

Parameter	Symbol	Value	Unit
Nominal composition	$\langle w \rangle_0$	7	wt.%
Liquidus temperature	$T_l$	618	°C
Eutectic temperature	$T_E$	577	°C
Segregation coefficient	$k$	0.13	—
Liquidus slope	$m_l$	-6.5	K wt.% <sup>-1</sup>

Using the values from [table 1.3](#), a python program generates a *CimLib* compatible tabulation assuming lever rule as microsegregation law, with a 0.1 wt.% step for average composition within an offset of 20% around the nominal value. For temperature, a cooling range between  $T_E$  and 630 °C is considered with a step of 1 °C. It is noted that for this application, the phase enthalpies are deduced from constant specific heat of each phase as well as constant latent heat.

In order to understand better the effect of shrinkage combined with macrosegregation, we plot in [fig. 1.18](#), the cooling curves from 4 different simulations:

- Gray curve - case G0: pure diffusion solidification with  $\rho^l = \rho^s$  (no level set) used previously in chapter 2 for validation; we use it as a reference case,
- Green curve - case G: convection-diffusion solidification with  $\rho^l = \rho^s$  (with level set) at a constant average composition
- Blue curve - case B: convection-diffusion solidification with  $\rho^l \neq \rho^s$  (with level set) at a constant average composition; this curve is plotted in [fig. 1.18a](#) and [fig. 1.18b](#),
- Red curve - case R (not to be confused with case R defined in the previous section): convection-diffusion solidification with  $\rho^l \neq \rho^s$  (with level set) and macrosegregation.

### Shrinkage effect on temperature

If we focus first on [fig. 1.18a](#), we first compare solidification cases G and G0, both with equal phase densities, hence no shrinkage. This first comparison shows that the introduction of the level set method, compared to a monodomain configuration, heats up the overall sample temperature by about 5 °C, causing solidification to finish a few seconds later than predicted in case G0. This is because we set a very high initial temperature in the air, 800 °C, to prevent a brutal diffusive flux that may lead

## 1.6. 1D application: solidification with inverse segregation

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to surface solidification in the metal. As the sample cools down, the air conductivity ( $10^{-2} \text{ W m}^{-1} \text{ K}^{-1}$ ) is not low enough to prevent a small diffusion flux in the metal's direction. However, since in both cases the cooling trend is predicted, we will keep the same thermal diffusion properties in the air, so as not to use unreal conductivity values, but we keep in mind that the current approach delays the solidification.

The second comparison is done between cases G and B, both using the level set approach but only case B considers solidification shrinkage. We notice that blue curve temperature of the sixth eulerian sensor rises steadily from 180 s to 600 s reaching a constant temperature of  $800^\circ\text{C}$ , the air's temperature. This rise confirms the metal has shrunk in length (volume in 3D), becoming less than 10 cm, hence replaced by air that entered through the open top boundary. The sensors at 12 cm and 14 cm are not shown in this figure as the simulation done for the pure diffusion without level set, the air subdomain does not exist. Another interesting difference resulting from shrinkage is that solidification ends sooner by about 70 s, compared to the pure diffusion case. As mass is almost perfectly conserved in both cases, cooling flux is the only factor that may accelerate the cooling. The imposed cooling boundary condition is a Fourier-type with the same heat transfer coefficient  $h_{\text{ext}}$  in both cases. However, a shrinkage flow transports energy in its direction, i.e. towards the solidification front, and thus raising slightly the temperature in regions close to the cool wall. Therefore, the Fourier flux proportional to the temperature difference  $(T - T_{\text{ext}})$  increases and the sample solidifies earlier.

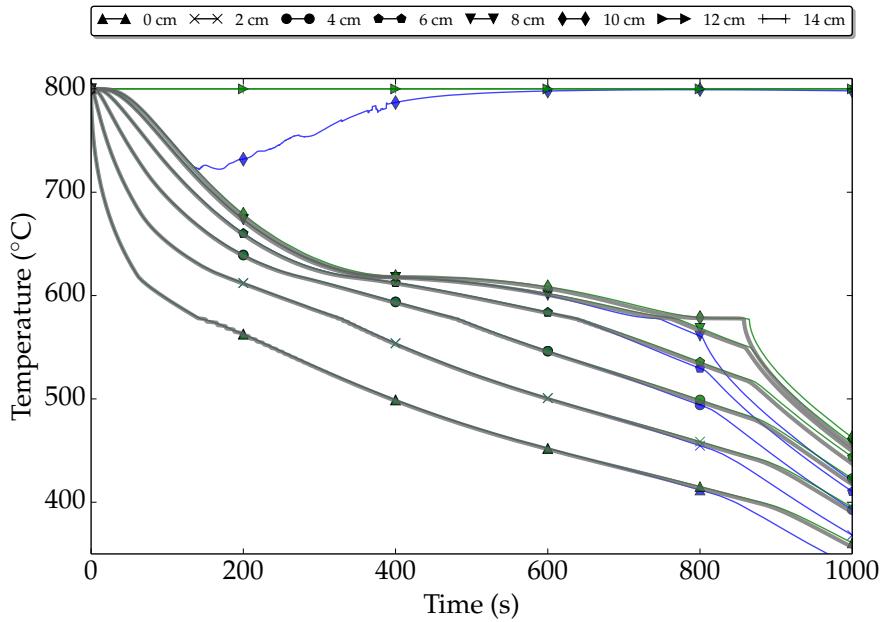
Finally, [fig. 1.18b](#) compares cases B and R, both with unequal phase densities but only predicting macrosegregation in the latter case. Differences are not striking, as temperatures along the metal sample are the same. We spot however a difference at the 10 cm probe, where a slight rise in temperature is observed with respect to the case without macrosegregation.

### Shrinkage effect on average composition

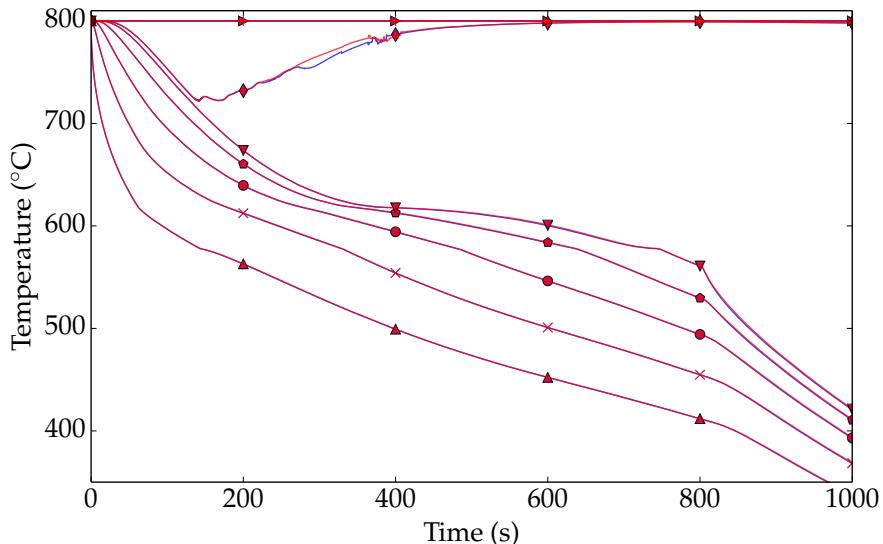
[Figure 1.19](#) shows snapshots at different times of the shrinkage flow caused by the density difference between liquid and solid phases. By solid phases, we mean the primary solid phase assumed in a dendritic structure, then the primary and secondary solid phases form together the eutectic that we see starting at 150 s in [fig. 1.19](#). We can see that some solute segregation has already initiated in the mushy zone at 100 s. However, the average composition reaches a positive peak of  $\langle w \rangle = \text{Al-7.5 wt.\% Si}$  (CHANGE VALUE ACCORDING TO PLOT) with the initiation and progression of the eutectic front. The sudden transformation of the remaining liquid in the mushy zone into eutectic solid, triggers a local velocity increase (real intrinsic velocity) as each

## Chapter 1. Macrosegregation with solidification shrinkage

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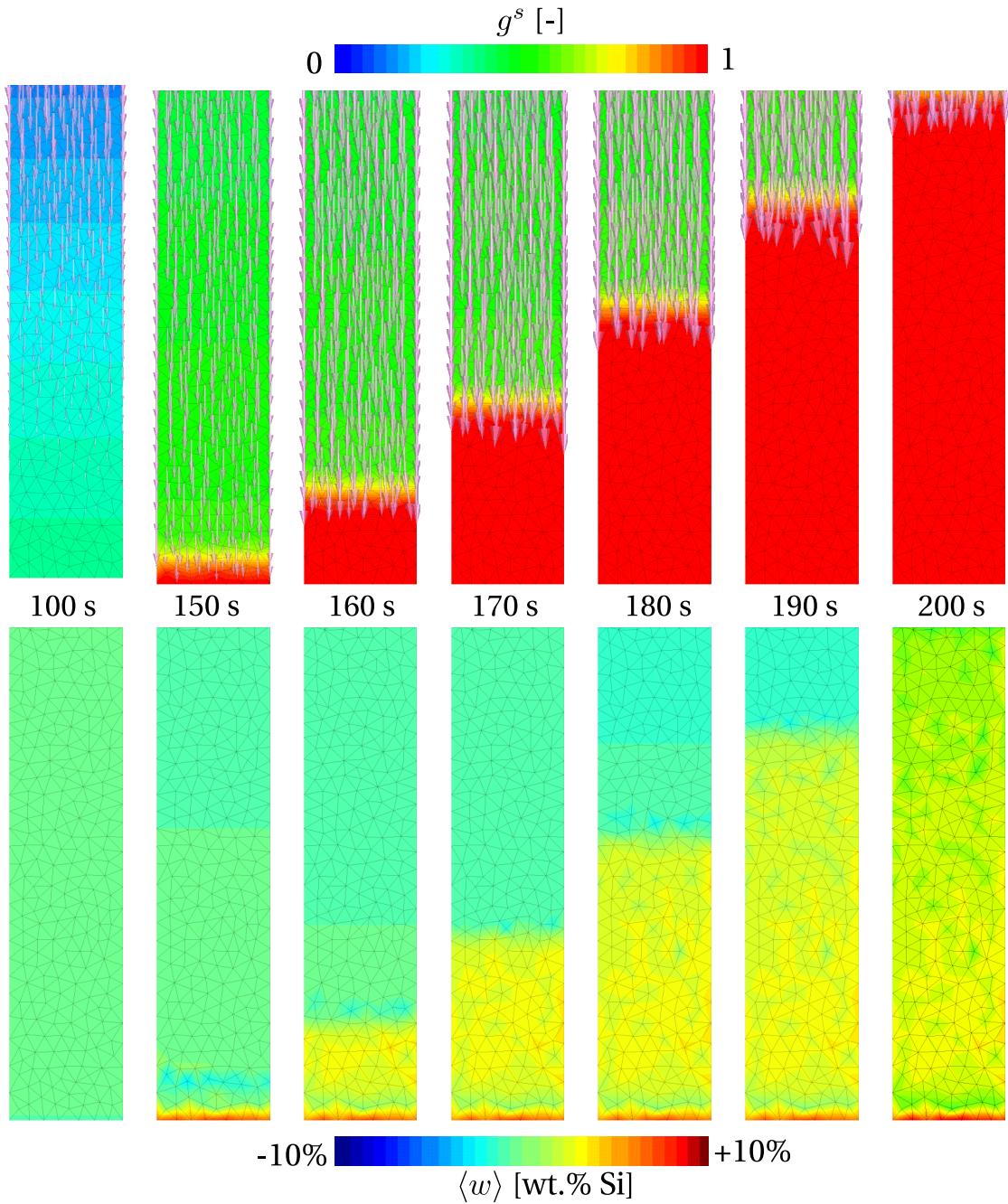
**(a)** Solidification shrinkage effect: grey curves correspond to a pure diffusion in a metal monodomain case, green curves consider the latter case but with level set (metal and air subdomains) while blue curves correspond to a shrinkage-driven flow case. All cases are solved without macrosegregation.



**(b)** Macrosegregation effect: blue curves represent the same simulation corresponding to the shrinkage-driven flow without macrosegregation, while the red curves correspond to a simulation of shrinkage-driven flow with macrosegregation.

**Fig. 1.18** – Cooling curves at different fixed positions from 0 to 10 cm (initial metal length) and from 10 cm to 14 cm (initial air length) where we show (a) the effect of solidification shrinkage on temperature history without any macrosegregation then we show (b) the effect of macrosegregation on temperature in the presence of solidification shrinkage.

## 1.6. 1D application: solidification with inverse segregation



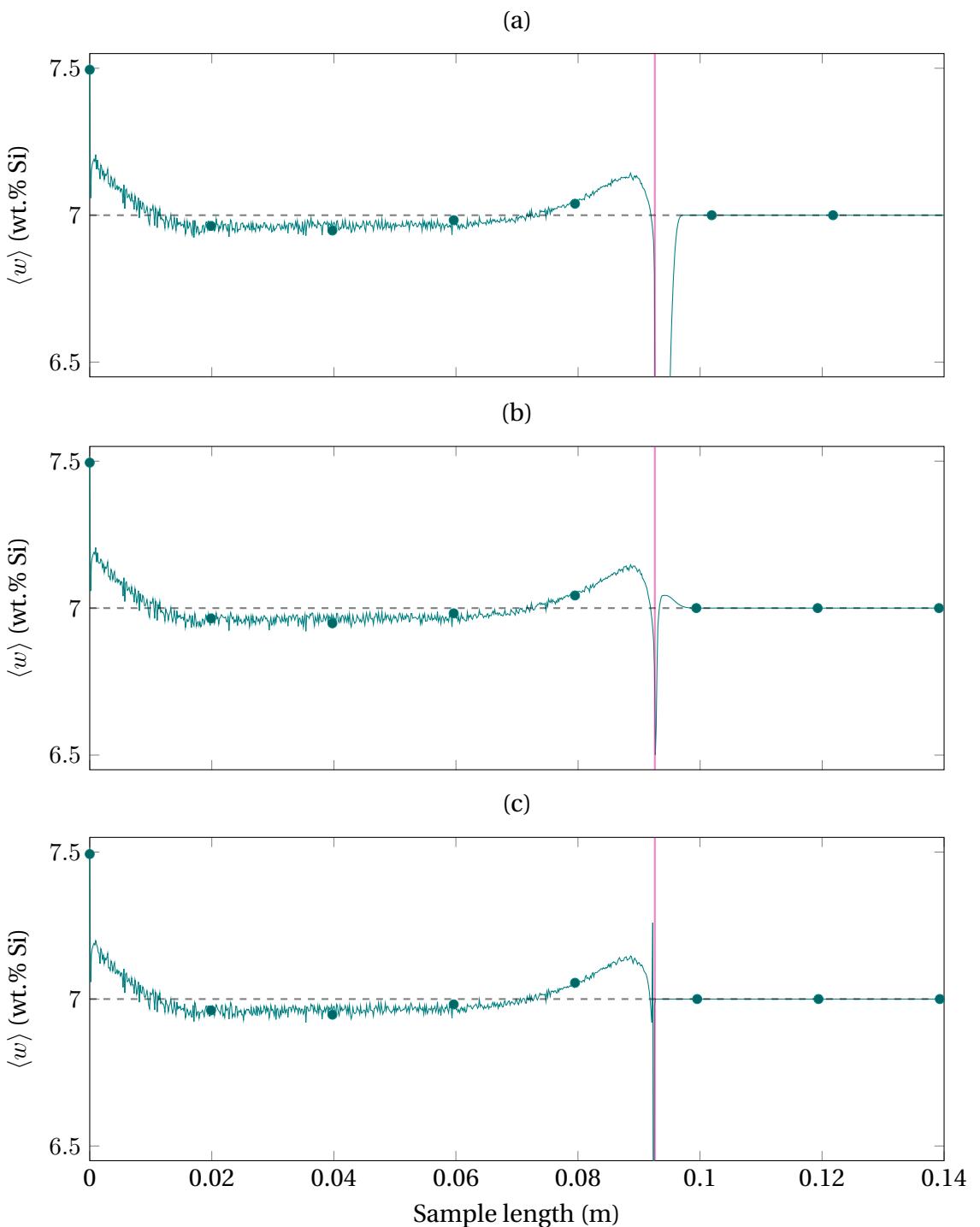
**Fig. 1.19** – Zoom on the lower part, approximately 1.5 cm of the alloy close to the cooling boundary condition. The upper row of figures show the evolution of volume solid fraction, when eutectic transformation takes place. The vectors represent the direction of the average velocity field, with a length proportional to the magnitude. The lower row of figures show for the same time increments, the solute redistribution, clearly changing behind the eutectic front.

## Chapter 1. Macrosegregation with solidification shrinkage

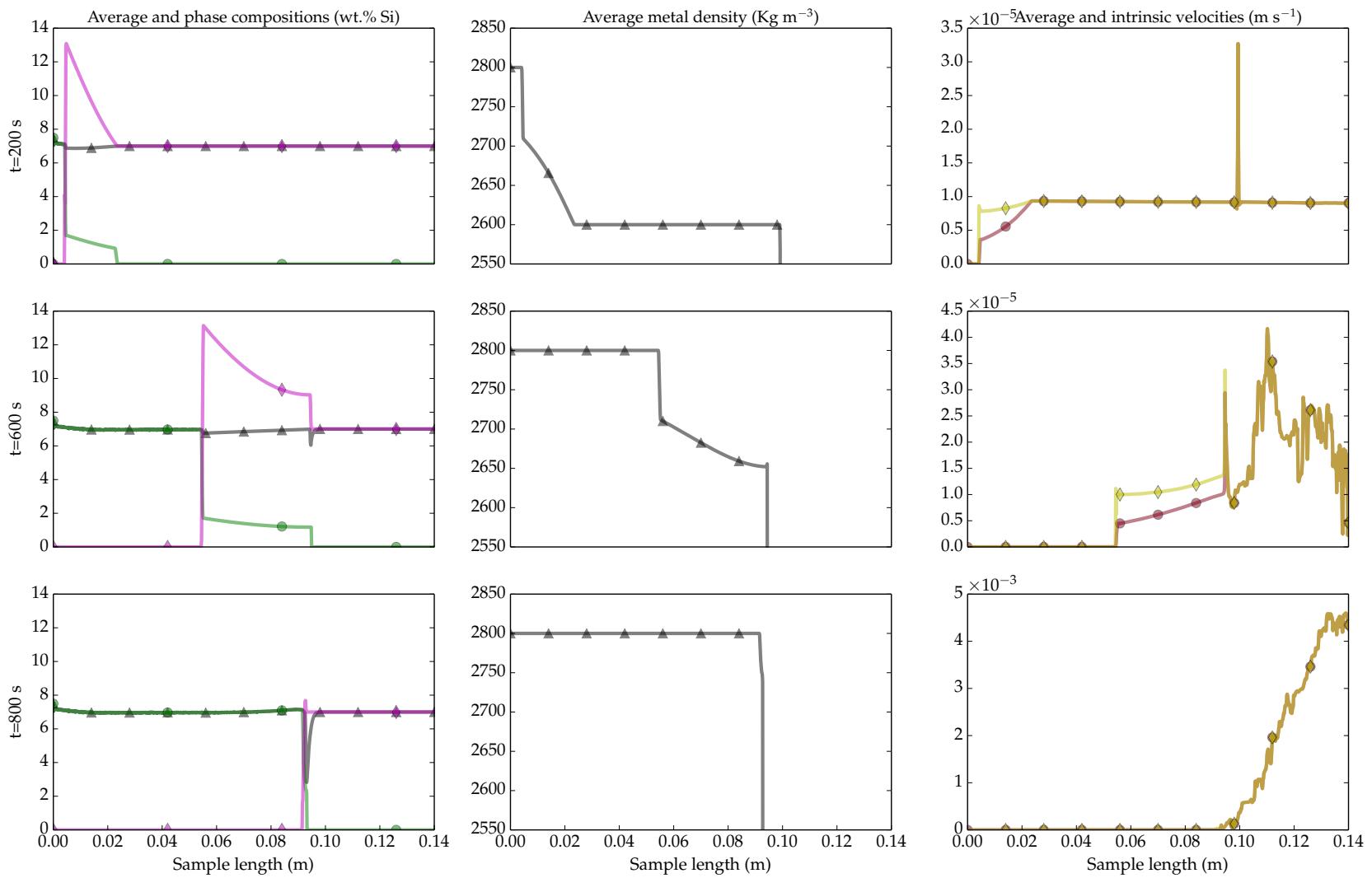
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node's density varies from  $\rho^l$  to  $\rho^s$  in a single time step, at eutectic temperature. The velocity increase, shown later in [fig. 1.21](#), causes species transport in the opposite direction of solidification, hence solute "freezes" in the eutectic structure leading to positive macrosegregation in the first solidified nodes. This phenomenon is better known as inverse segregation. As the transport continues in the same direction, solute is progressively depleted in the remaining liquid, causing negative macrosegregation at nodes located between 2 cm and 7 cm from the cold wall. [Figure 1.19](#) shows only the first 1.5 cm of the solidifying sample, therefore a complete segregation profile is plotted along the sample length in [fig. 1.20a](#), showing thus the negative macrosegregation as previously explained. However, from 7 cm to the metal-air interface, the average composition rises as clearly shown in [fig. 1.20a](#). This looks more like a numerical instability than a physical rise in concentration. Bearing in mind that a reduced solute diffusion coefficient is imposed in the air subdomain  $1.5 \times 10^{-12} \text{ m}^2 \text{ s}^{-1}$  compared to  $1.5 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$  in the metal, we investigate into this instability by proposing two test cases: first, we try to limit the solute advection in the transition zone where flow instabilities may form as seen previously in the segregation-free sample but keeping equal solute diffusion coefficients in both domains to  $1.5 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$ , while for the second test case, we combine both low solute diffusion and low solute advection. The average composition results of these two test cases are respectively plotted in [fig. 1.20b](#) and [fig. 1.20c](#). These results (second plot in [fig. 1.20](#)) clearly show that just by reducing solute advection in the mixing zone reduce the amplitude of the composition instability. In contrast, changing solute diffusion properties for the air coupled with reduced solute advection (third plot in [fig. 1.20](#)) offers no further stability in the segregation profile, which should normally stay below the nominal value near the interface.

## 1.6. 1D application: solidification with inverse segregation



**Fig. 1.20** – Plot of the average composition as function of length, along a vertical line passing through the center of the sample at 1000 s, with (a) low solute solute diffusion in the air but unmodified solute advection (minimum composition reaches 2.2 wt.% Si but not shown), (b) reduced solute advection in the level set transition zone or (c) a combination of the previous techniques. The solid magenta line shows the position of the interface after solidification while the black dashed line shows the nominal average composition of the alloy.



**Fig. 1.21** – Group of plots where each row is a given time increment ( $t=200\text{ s}$ ,  $t=600\text{ s}$  and  $t=800\text{ s}$ ) of the 1D solidification shrinkage simulation with macrosegregation, while each column one or more physical quantities are plotted along a vertical line passing through the center of the sample. Plots are titled with the corresponding plotted quantities.

## 1.7. 2D application: controlled solidification benchmark

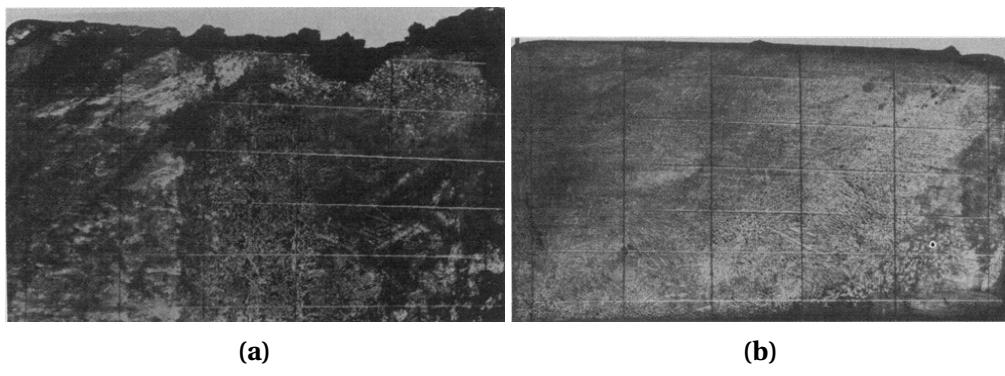
The plots in [fig. 1.21](#) give important information on the segregation shown in [fig. 1.19](#).  
CONTINUE COMMENTING ON [fig. 1.21](#).

### Solute mass conservation

Show solute mass conservation with equal or low solute diffusion in air

## 1.7 2D application: controlled solidification benchmark

This type of applications is very known to the solidification literature and very important in understanding and predicting macrosegregation produced by liquid convection, solid grains settling and volume shrinkage. An molten alloy is put in a rectangular crucible with controlled cooling flux on one or two sides to the crucible. The importance of the experiment lies in the thermal convection forces arising from temperature gradients, but also solutal buoyancy forces arising from liquid concentration gradients. Depending on the density of each chemical species, these convection forces may either be in the same or opposite direction, changing the final macrosegregation pattern. [Hebditch et al. \[1974\]](#) suggested one of the first experiments working on Sn-Zn and Sn-Pb alloys. A part of their results is shown in [fig. 1.22](#). More recently, an experimental benchmark was performed by [Hachani et al. \[2012\]](#) to obtain more accurate composition results with different Sn-Pb and Pb-Sn alloys. In the current section, we are interested in the latter experiment, especially in the prediction of the metal's shrunk surface together with the final macrosegregation.



**Fig. 1.22** – Final macrosegregation patterns in (a) Sn-5 wt.% Zn and (b) Sn-5 wt.% Pb castings, with a visible sign of volume shrinkage at the top. The rectangular grid on both alloys is used to measure concentration at grid points [[Hebditch et al. 1974](#)].

### 1.7.1 Absence of convection

We first try to simulate the solidification process by neglecting thermasolutal dependence of phase densities in the Navier-Stokes solver. By doing so, no convection forces should arise during solidification, hence the generated flow is only due to the difference between solid and liquid densities, that is the shrinkage driving force.

## 1.8 3D application: reduced-gravity solidification

As presented in the introductory chapter, the aim of the CCEMLCC project is to "reach a better understanding of surface defects formed during processing of steels from the liquid state" [Gandin 2014]. Among the several scientific topics being studied, the interaction between skin macrosegregation and thermomechanical deformation is investigated through chill cooling experiments. The idea is to have the molten steel in a containerless environment, which could be done by several ways: electromagnetic levitation, onboard parabolic flights or sounding rockets and finally in a real microgravity context as in the ISS. Heat is extracted from the sample by contact with a ceramic ( $\text{Si}_3\text{N}_4$ ) substrate at room temperature (hence the term "chill cooling"), that collides into the alloy at a controlled speed. This contact situation generating high thermal gradients is comparable to casting processes between the molten alloy and the moulds. For ground-based experiments, EML is the exclusive technique to achieve a chill cooling experiment. However, it is technically difficult to achieve levitation without currents in the spherical sample, generated by means of electromagnetic stirring (Lorentz forces) on the one hand but also by thermal convection on the other hand. In reduced-gravity conditions, the dynamics of the phenomena behind fluid motion are less significant. The current application is therefore compared to chill cooling experiments performed in parabolic flights and sounding rockets with reduced gravitational forces ( $\|\vec{g}\| \in [10^{-1}; 10^{-5}] \text{ m s}^{-2}$ ).

### 1.8.1 Previous work

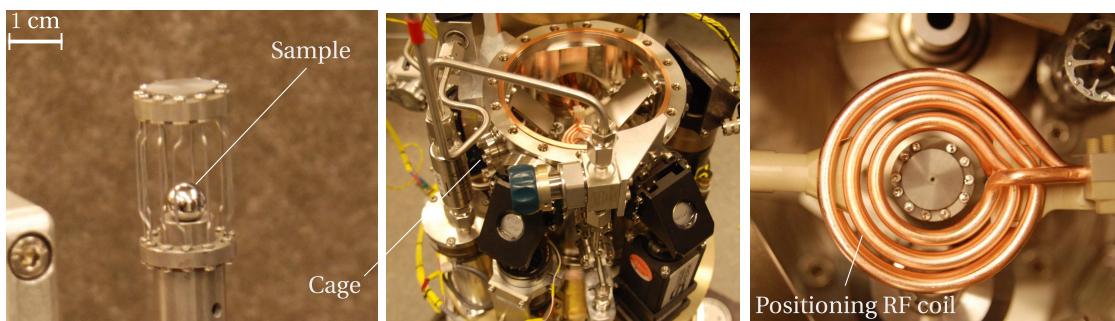
#### TEMPUS experiment on parabolic flight

The TEMPUS experiment came as a first alternative for EML experiments in which accurate thermophysical and rheological characterisation were difficult to achieve. The flight consists of several cycles of free fall, a reduced-gravity environment is hence created, allowing to use only a single RF coil to stabilise the position of the droplet, while the substrate comes into contact with the molten sample from above it. An axial

pyrometer measures the sample temperature during the process. Also, a high-speed camera records the solidification process, producing frames as shown in [fig. 1.24](#). This is useful to measure the front growth speed. Each parabola cycle lasts for 50 s, offering an effective low gravity ( $\|\vec{g}\| \approx 10^{-1} \text{ m s}^{-2}$ ) for about 20 s.

### TEXUS sounding rocket

TEXUS-46 is the name of the sounding rocket mission that carries the experimental setup, but for simplicity we will refer to the latter as being the TEXUS experiment. The setup is shown in [fig. 1.23](#). The main difference with respect to parabolic flight experiments, TEXUS features solidification in near-zero gravitational fields and for extended periods of time (3 minutes). We do not have an exact measurement of the gravitational field magnitude, but it is several orders less than Earth's gravity magnitude ( $\|\vec{g}\| \in [10^{-5}; 10^{-8}] \text{ m s}^{-2}$ ).



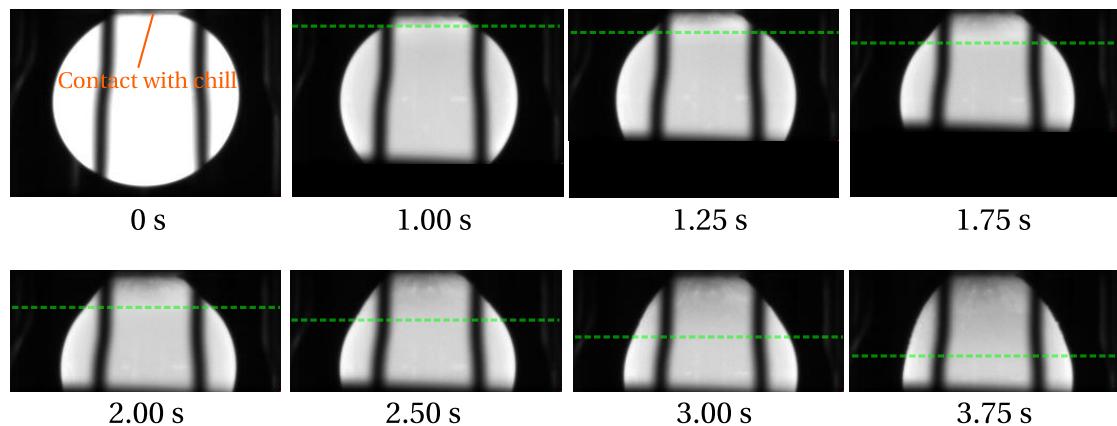
**Fig. 1.23** – Three frames describing the experimental setup used to achieve reduced-gravity solidification onboard a sounding rocket flight, showing the initial alloy sample, the cage and the positioning coil. The setup is similar to the one used for TEMPUS experiments.

### Numerical contribution

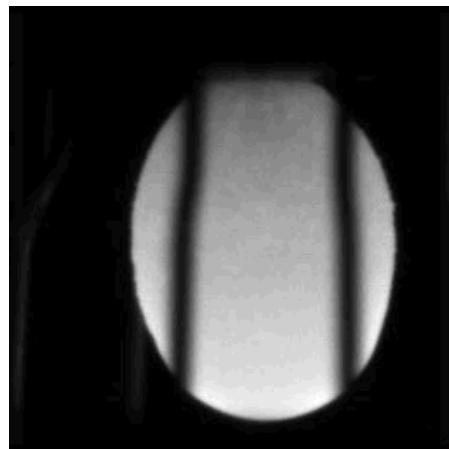
A former numerical contribution was done by [Rivaux \[2011\]](#) at CEMEF, as mentioned in the first chapter. His model considered both the steel droplet and the ceramic chill in a Lagrangian formulation, i.e. each object is modelled using a separate deformable mesh. Conservation equations of mass, energy, chemical species and momentum were solved in the metal domain, while the energy conservation was the sole equation solved on the chill mesh. The mechanical problem was divided into two parts: fluid and mechanics and solid mechanics. For the first part, the momentum conservation in the liquid phase was solved using an incompressible P1/P1 SUPG-PSPG formulation of Navier-Stokes equations, i.e. without any contraction for the liquid phase neither

## Chapter 1. Macrosegregation with solidification shrinkage

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**Fig. 1.24** – Image sequence given by a high speed camera onboard a TEMPUS parabolic flight, showing the solidification progress between 0 s (when contact with the chill is initiated) to 3.75 s in a Fe-0.9 wt.% C-0.2 wt.% Si steel droplet. The progress of the solidification front is marked by the green dashed line. In some frames, the droplet is partially hidden by the narrow opening of the sample holder facing the camera.



**Fig. 1.25** – Camera image from the TEMPUS 2014 experiment, showing the fully solidified droplet with a deformed shape after 10 s.

solidification shrinkage at the solid-liquid interface. The second part, solid mechanics, was solved using P1+/P1 formulation to predict solid deformation caused by the solid's thermal contraction as well as solidification shrinkage, using an elastic-viscoplastic behaviour. The simulation results showed that the total droplet deformation that has been observed in the experiments is not primarily due to solid deformation. The density jump between the solid and liquid phases at the solidification front is actually predominant. High speed camera images endorse this observation, where the droplet underwent a continuous spherical-to-elliptic shape change while the solidification front travelled away from the contact point. Another interesting point to comment is the computation of solidification shrinkage in the solid resolution, although this type of shrinkage does not generate stresses in the solidifying alloy, compared to thermal shrinkage for instance.

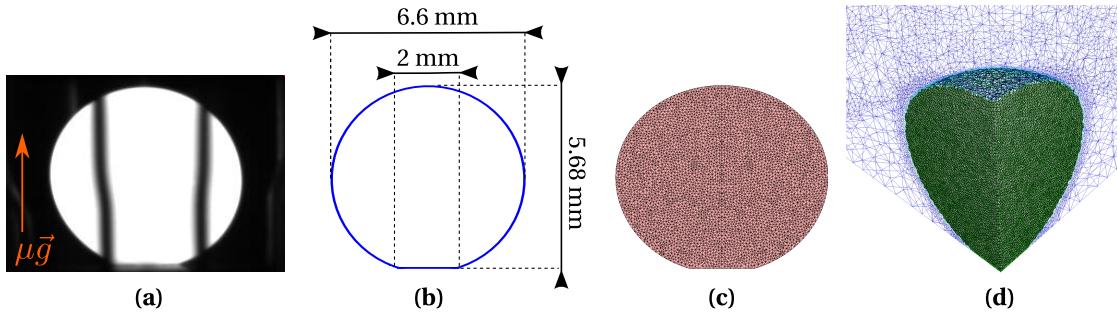
### 1.8.2 Computational configuration

#### Geometry and mesh

The simulation considers only 1/4 of the droplet-substrate-gas system, given the axial symmetry of the problem. Furthermore, the substrate is implicitly taken into account via a boundary condition, explained in the next section. This is sufficient in the current context, because we are only interested in the energy transfer from the droplet to the substrate.

The steel sample is not perfectly spherical initially as surface oscillations perturb the equilibrium shape. Such perturbations may be attributed to Lorentz forces creating by the positioning coil. The droplet hence is compared to an ellipsoid having a vertical minor axis of 5.68 mm and a horizontal major axis of 6.6 mm, as shown in [fig. 1.26](#). The bottom is a planar surface (diameter of 2 mm, where the contact is initiated). Also in [fig. 1.26](#), the alloy is immersed in a gas medium (argon), such that both subdomains form together 1/4 of a cylinder having 8 mm in radius and 8 mm in height (not fully shown in the figure).

The mesh is then automatically adapted to the moving interface using *Remesh2*. We adopt the same remeshing strategy applied for 1D cases, whereby a fixed mesh size is imposed in the metal subdomain to limit as much as possible information diffusion due to remeshing, this is specially important for the average composition field. The corresponding parameters are given by [table 1.4](#). Remeshing is performed each second.



**Fig. 1.26** – (a) The camera frame before the onset of solidification gives the essential information to (b) rebuild the droplet geometry then (c) a standalone 2D mesh used to obtain (d) the final immersed 3D mesh.

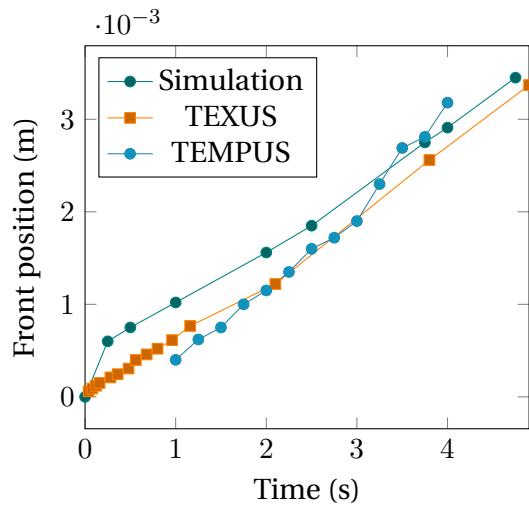
**Table 1.4** – Summary of the different mesh sizes used to generate an adaptive isotropic mesh, along with the level mixing thickness,  $\varepsilon$ . Refer to ?? for the definition of each mesh parameter.

Mesh parameter	Size [m]
$\varepsilon$	$1.5 \times 10^{-4}$
$h_{\vec{n}} = h_{\vec{\tau}}$	$2 \times 10^{-5}$
$h_M$	$1 \times 10^{-4}$
$h_A$	$6 \times 10^{-4}$

### Initial and boundary conditions

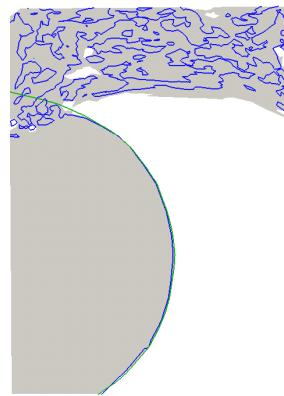
The thermal boundary conditions are set as follows: Heat loss by radiation is experimentally avoided, therefore it is not considered in our model, hence all boundaries are considered adiabatic, except for the metal-substrate contact area, as previously mentioned. This surface is modelled by a Fourier condition with  $T_{\text{ext}}=25\text{ }^{\circ}\text{C}$  and an effective exchange coefficient  $h_{\text{ext}}$  of  $6 \times 10^4\text{ W m}^{-2}\text{ K}^{-1}$ . The  $h_{\text{ext}}$  coefficient's value has been determined by running multiple simulations with different values in the aim of predicting a front speed as closer as possible to the experimental measurements plotted in fig. 1.27, as explained in the coming sections.

For the velocity-pressure boundary conditions, fig. 1.29 shows that a no-slip condition is imposed on the droplet-substrate surface, since this area solidifies in the first place without further fluid motion, as shows fig. 1.24. It is noted that the first solidified shell may experimentally deform under thermal contraction stresses, but we do not consider it hereafter. For the rest of the domain, we impose the normal velocity component to zero on both symmetry faces, while keeping free tangential components. The remaining boundaries, namely the top and the outer surface of the argon gas (cylinder generatrix), have free velocity components. However, such condition may cause instability in the level set transport solver, as seen in fig. 1.28. This problem



**Fig. 1.27** – Position of solidification front versus time for the binary alloy simulation compared to the experimental findings of the TEXUS-46 flight in 2009 and TEMPUS 2014 measurements [Gandin 2014].

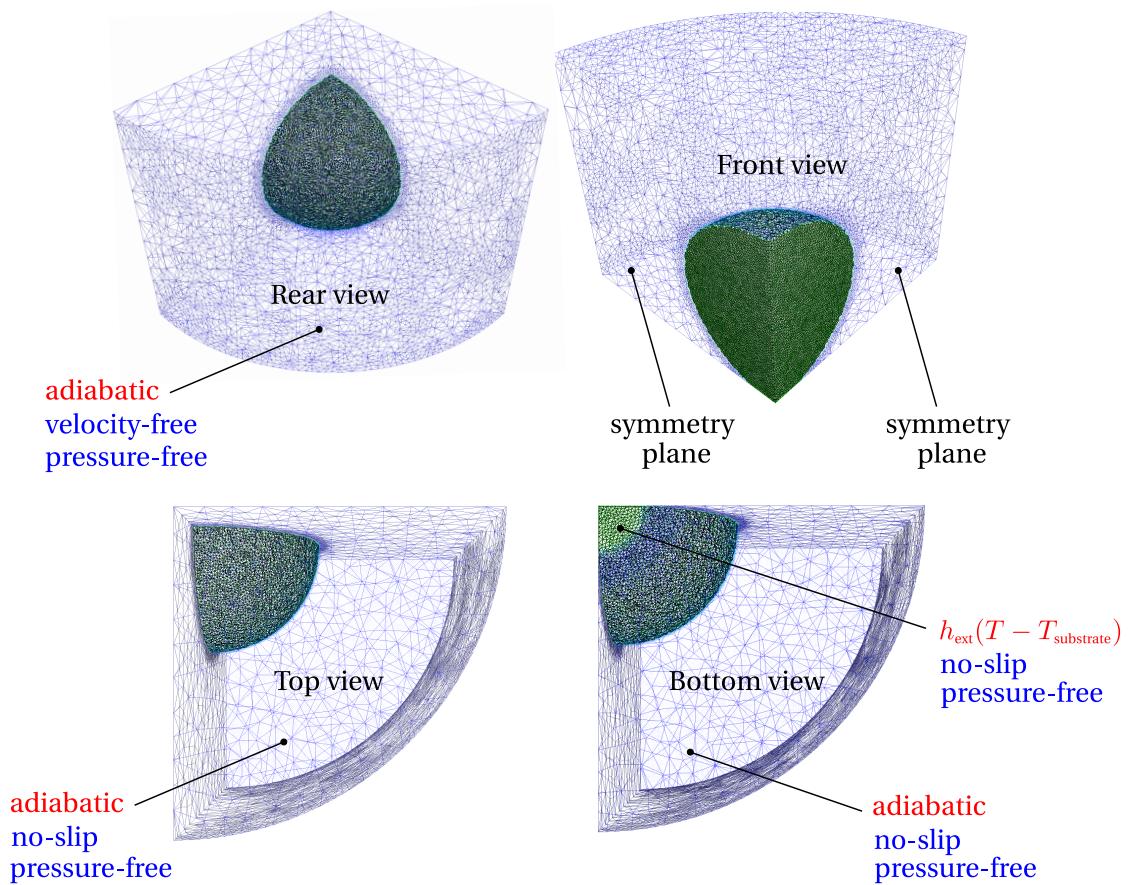
has been reported by [Basset 2006], showing a limitation in the imposed boundary conditions between Navier-Stokes solver and level set transport. Therefore, we limit these instabilities by imposing a no-slip condition, thus allowing the argon to flow in the computational domain through the generatrix. The cylinder height was taken big enough to prevent any flow damping near the droplet's north pole, which may spuriously alter its final shape. The pressure condition for the argon gas is left free for all boundaries. The adopted time step is 0.01 s.



**Fig. 1.28** – 2D view showing a transport instability at the top. The green line is the initial metal-gas interface, while the blue line represents the transported zero-level of the distance.

## Chapter 1. Macrosegregation with solidification shrinkage

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**Fig. 1.29** – 3D views showing the thermal (in red) and mechanical (in blue) boundary conditions used in reduced-gravity simulations.

**Table 1.5** – Nominal composition (wt.%) of the experimental *b1* steel and its simulation equivalent alloys.

Alloy	C	Si	Mn	Al	S	P
<i>b1</i>	0.105	0.268	0.636	0.0067	0.009	0.0189
Binary <i>b1</i>	0.105	-	-	-	-	-
Ternary <i>b1</i>	0.105	0.268	-	-	-	-
Quaternary <i>b1</i>	0.105	0.268	0.636	-	-	-

### Choice of alloy

Various steel grades were considered in the CCEMLCC project. Each grade was assigned to a specific experiment. We limit our study to the steel assigned for TEXUS missions, the grade is designated as "b1" alloy. Its nominal composition is given in [table 1.5](#). As our approach relies on thermodynamic tabulations, we show in the next section that we can take into account the multicomponent alloy to predict segregation, by considering first only one species, hence a binary Fe-C alloy. In a later step, we consider a ternary Fe-C-Si alloy. Finally, we consider a quaternary Fe-C-Mn-Si alloy.

By performing the same reduced-gravity simulation while varying the alloy from binary to quaternary, we can study how the varying solidification paths (as a consequence of macrosegregation) may affect the final droplet shape, as the shrinkage profile is directly related to the solid fraction and its evolution with time.

### Parametric study: final shape prediction

In this subsection, we focus on obtaining a comparable final shape of the droplet between the experiment and simulation. To do so, we vary 2 main important parameters: first, the heat transfer coefficient of the metal-substrate contact surface controls the heat extraction and hence the solidification rate. Second parameter is the magnitude of the gravitational field, which has a great influence on the fluid flow inside the molten droplet. The importance of this parametric study is two-fold:

1. in our model, the energy equation solved with the level set methodology considers only heat conduction and advection in the gas subdomain, hence no account for the heat dissipated by radiation (which is an ongoing PhD project at CEMEF). Therefore a trial-and-error strategy is necessary to determine an optimal value of  $h_{\text{ext}}$  to ensure that the solidification rate is the same as in the experiment,
2. from a hydrodynamics perspective, a containerless molten droplet levitated under reduced-gravity conditions is maintained nearly spherical under the action of surface tension forces. Other forces due to tangential surface tension gradients (Marangoni force) or Lorentz force may also exist. Although possible to implement by the CSF method, accounting numerically for surface tension adds complexity to the model by imposing a time step constraint. However, if we neglect this force, the droplet will tend to collapse if gravity acceleration is fast enough. Consequently, a parametric study helps us determine this gravity threshold, in the absence of surface tension.

## Chapter 1. Macrosegregation with solidification shrinkage

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A series of test simulations were launched in the aim of getting comparable results with the experiment. Several values of  $h_{\text{ext}}$  were tested in the interval  $[10^2; 10^6]$ , while the gravity acceleration influence was tested for values lying in the interval  $[10^{-6}; 10^{-2}]$ . The best match for the final shape while preserving a front propagation speed close to  $0.7 \text{ mm s}^{-1}$ , was obtained by setting simultaneously  $h_{\text{ext}} = 6 \times 10^4 \text{ W m}^{-2} \text{ K}^{-1}$  and  $\|\vec{g}\| = 5 \times 10^{-5} \text{ m s}^{-2}$ . To demonstrate the effect of varying these parameters, we present a parametric study in [table 1.6](#), where only the most relevant cases are studied with a binary alloy, Fe-0.105 wt.% C.

**Table 1.6** – Summary of the parametric study for the conductive heat transfer coefficient (H) and the magnitude of the gravity vector (G, not to be confused with thermal gradient). The cases are defined by fixing each parameter to a reference value then varying the latter parameter. The reference values,  $H_0=6 \times 10^4$  and  $G_0=5 \times 10^{-5}$ , ensure a good compromise when compared to the experimental solidification rate and final droplet shape.

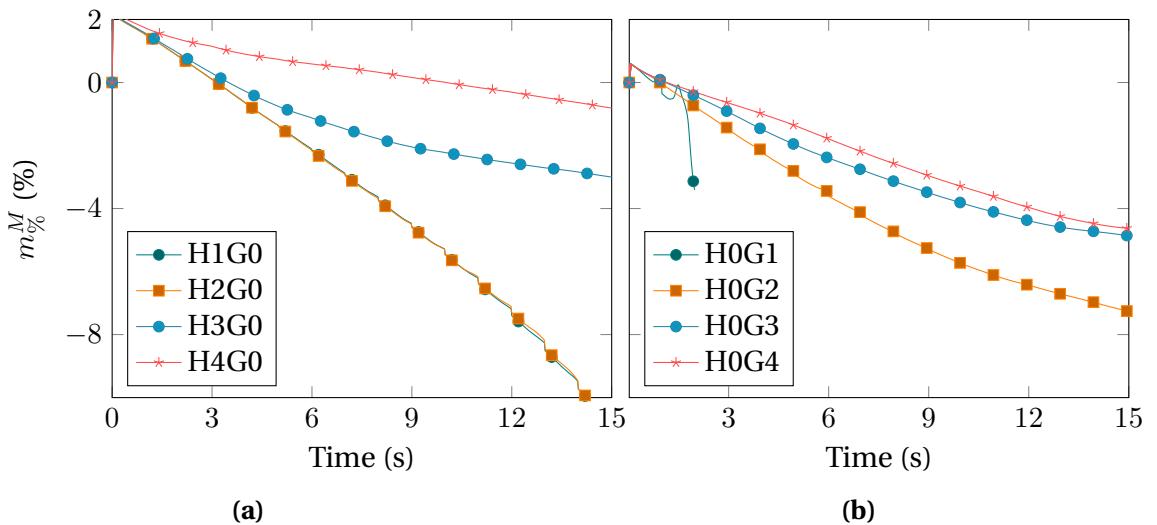
Case	$h_{\text{ext}} [\text{W m}^{-2} \text{ K}^{-1}]$	$\ \vec{g}\  [\text{m s}^{-2}]$
H1G0	$10^3$	$5 \times 10^{-5}$
H2G0	$10^4$	$5 \times 10^{-5}$
H3G0	$10^5$	$5 \times 10^{-5}$
H4G0	$10^6$	$5 \times 10^{-5}$
H0G1	$6 \times 10^4$	$10^{-3}$
H0G2	$6 \times 10^4$	$10^{-4}$
H0G3	$6 \times 10^4$	$10^{-5}$
H0G4	$6 \times 10^4$	$10^{-6}$

We start the analysis by observing the results in [fig. 1.31](#), where the parameter  $h_{\text{ext}}$  increases from case H1G0 to H4G0, while maintaining a constant gravity acceleration at  $5 \times 10^{-5} \text{ m s}^{-2}$ . In the first case, H1G0, the heat coefficient is at its lowest between the droplet and the chill. As this contact is the only way to dissipate heat from the droplet, a low heat exchange coefficient means a slow cooling. Therefore, contact area of the droplet solidifies first. As we consider a fixed solid in our model, any solidified part can no longer move or deform. As time passes, solidification is slow, such that the droplet starts collapsing at about 10 sec, undergoing a significant shape change under the gravity's action. In reality, such microgravity conditions are not sufficient to deform the droplet as seen in case H1G0. However, it should be noted that at such small gravity accelerations, surface tension forces play a central role in stabilising the sample shape, by minimising its surface energy. As we neglect it in our simulations, the droplet tends naturally in the direction of the gravity vector. We can make the same conclusion for case H2G0, while taking note of the smaller overall vertical deformation. We may also see that the solid shell base is thicker in the horizontal direction, featuring

also necking around the droplet axis mid-height, showing a competition between solidification shrinkage and gravity effect. It should be noted that in both cases H1G0 and H2G0, solidification is not complete at 15 s.

More interesting results are obtained in case H3G0 where the heat coefficient is two orders of magnitude higher than that in the first case. The high solidification rate allows the mushy front to capture liquid nodes before deformation occurs by gravity. We see a global deformation which is qualitatively comparable to the experimental results: an ellipsoid form with a longer vertical axis with respect to the initial shape, while the horizontal axis decreases compared to the original sample diameter. Finally, we observe the same deformation tendency if we compare cases H3G0 and H4G0. However, the latter shows less deformation on the sides, which is the direct result of the fast cooling rate.

In order to have a clear idea on the effect varying the cooling rate parameter on mass conservation, we plot in fig. 1.30a the mass variation versus time for all four cases. We can notice that mass variation for case H4G0 occurs between 2% and -1% near the solidification end, recording the least variations compared to other cases. On the other hand, cases H1G0 and H2G0 show an important mass loss before solidification comes to end, reaching -11%. We are particularly interested in case H3G0, which shows a good compromise between the deformation magnitude and mass loss, the latter being at -3% of mass metal.



**Fig. 1.30 –** Mass conservation analysis for (a) cases HxG0 ( $x=1,2,3,4$ ) and (b) cases H0Gx ( $x=1,2,3,4$ ).

Now, we study the effect of varying the gravity parameter and its influence on the final deformation. We observe first the results for case H0G1 where gravity magnitude is

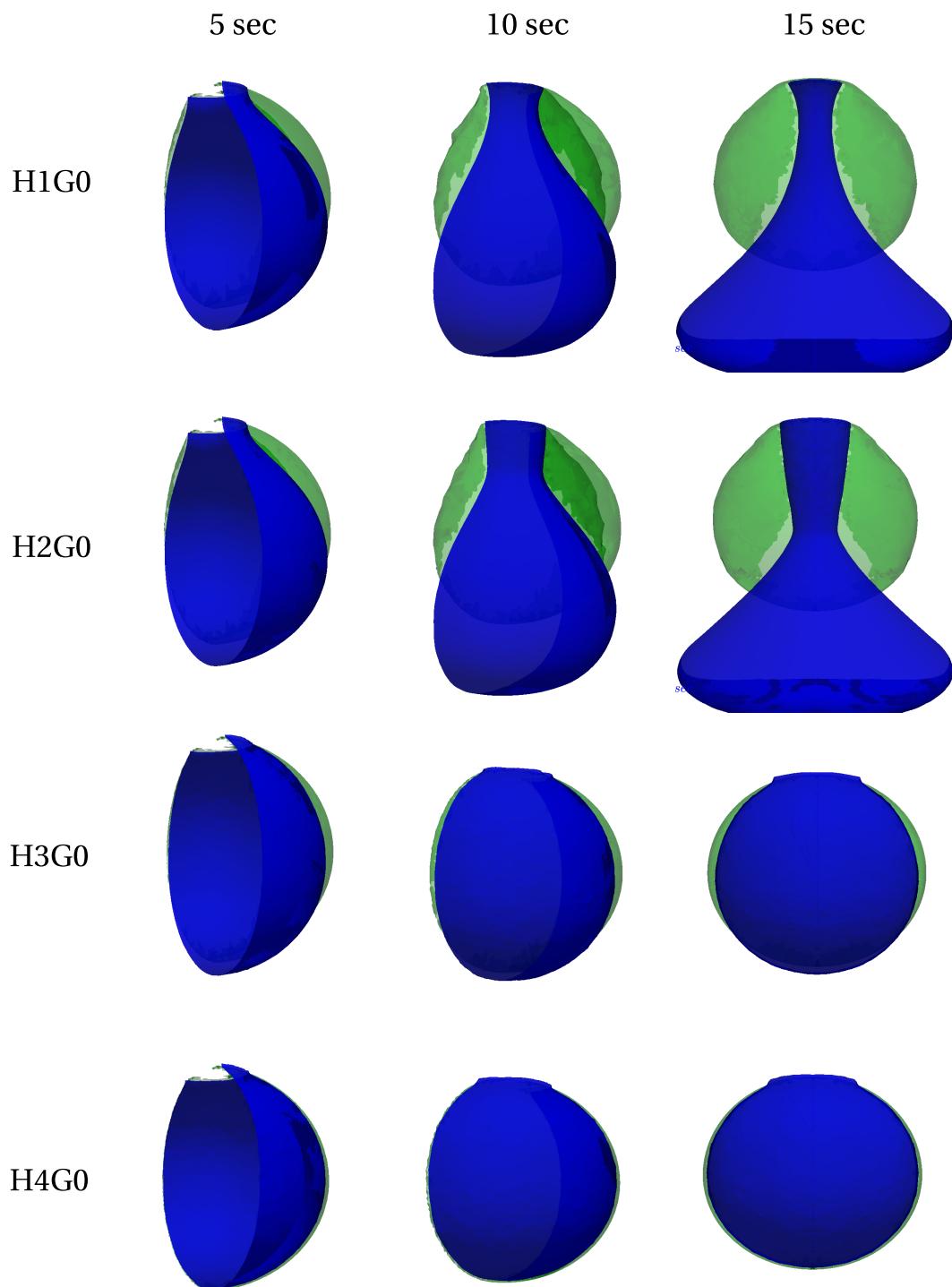
about four orders less than the Earth gravity at zero altitude. While the base solidifies, the remaining part falls down deforming severely by its weight and leading to a non-converging level set transport. The last recorded time is 1.75 s. For case H0G2, the droplet is less solicited by its weight, and therefore solidifies while having a vertically elongated shape. It should be reminded that in the current global numerical model does not account for the metal's surface tension, which may clearly have a drastic influence on the final shape, especially at higher gravity magnitudes, such as for cases H0G1 and H0G2. Moving on to cases H0G3 and H0G4, the weight driving force becomes negligible compared to the shrinkage driving force. Therefore, the sample shows significant lateral deformation, while in the central vertical plane of the droplet, the droplet has shrunk when compared to the initial profile. This is more visible in case H0G4, where the final shape is overall smaller than the initial volume, which is not the same as found in cases H3G0 and H4G0. The mass conservation analysis corresponding to the gravity magnitude variation are plotted in [fig. 1.30b](#). The plots show, as expected, better mass conservation for decreasing gravity acceleration, i.e. from case H0G1 to H0G4. We think however that surface tension would change this analysis, and non-convergence obtained in case H0G1 may be prevented.

### 1.8.3 Texus binary alloy

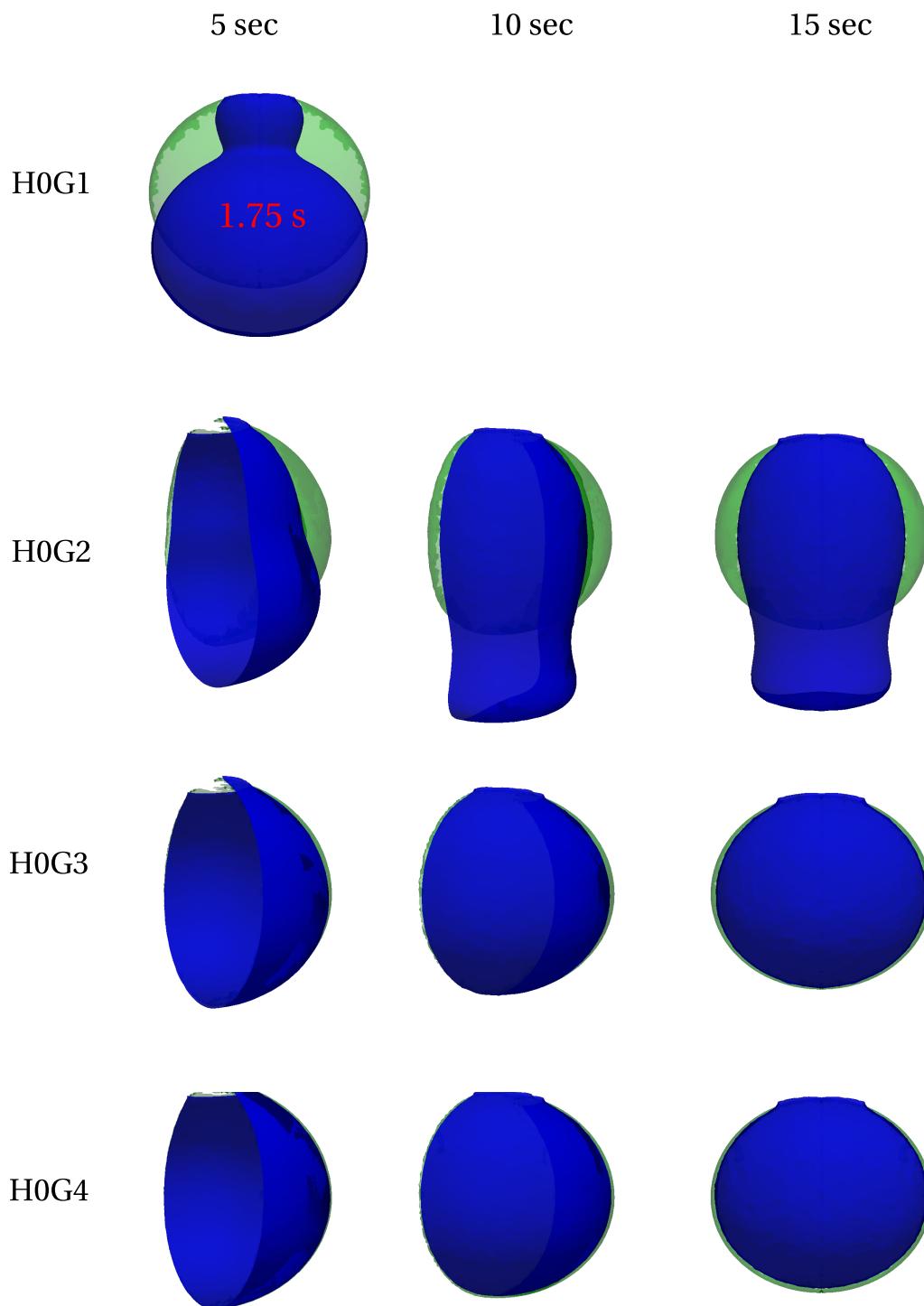
Now that the optimal computational configuration is known, we proceed to simulate the solidification of the binary alloy given previously in [table 1.5](#).

The nominal composition for this alloy is Fe-0.105 wt.% C. In order to obtain accurate segregation results, a fine resolution mapping was performed from equilibrium calculations, using 21 values of composition between a minimum of 0.01 wt.% and 1 wt.%. This is equivalent for a composition step of 0.0495 wt.%, with a temperature step of 1 °C varying in the interval [20 °C; 1600 °C]. The importance of choosing small steps in composition and temperature is to predict accurate solidification paths during mesosegregation (segregation length is limited to the droplet scale), which is the main input of solidification shrinkage. Therefore, less accurate mappings may result in false shrinkage profile prediction.

Using the initial and boundary conditions defined earlier, 15 seconds of simulation give the final shrinkage profile given in [fig. 1.33](#). We notice that the predicted overall deformation of the droplet is in a good agreement with the experimental shape after solidification. This agreement is still not perfect as some key input parameters are still missing in the model, namely the real gravity acceleration on-board the parabolic flight (which should be much greater than value used in the simulation), and the correct heat flux between the sample and the substrate. It is emphasized that for

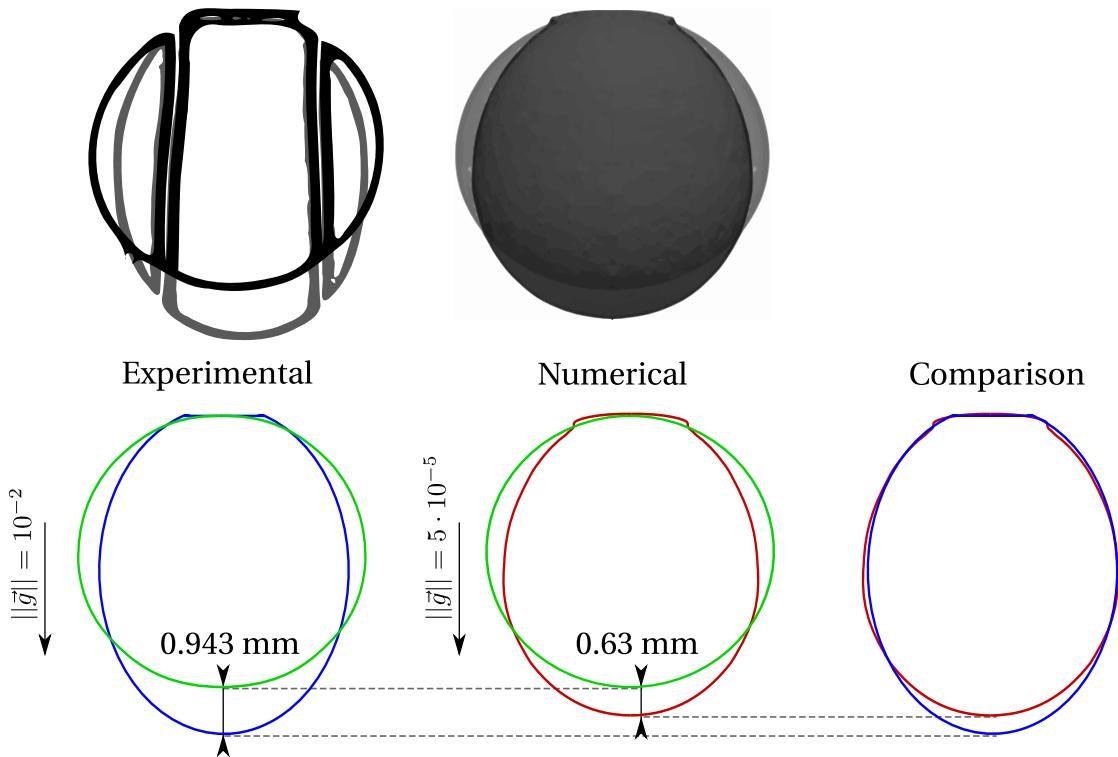


**Fig. 1.31** – 3D snapshots of a droplet (only half shown for symmetry) undergoing solidification shrinkage where the heat exchange coefficient increases from H1 to H4 according to [table 1.6](#). The green surface is the initial droplet profile while the blue surface is the deforming droplet profile. The camera rotation over time allows observing deformation from different angles. The gravity vector points downwards.



**Fig. 1.32** – 3D snapshots of a droplet (only half shown for symmetry) undergoing solidification shrinkage where the magnitude of the gravitational field decreases from G1 to G4 according to **table 1.6**. The green surface is the initial droplet profile while the blue surface is the deforming droplet profile. The camera rotation over time allows observing deformation from different angles. The gravity vector points downwards.

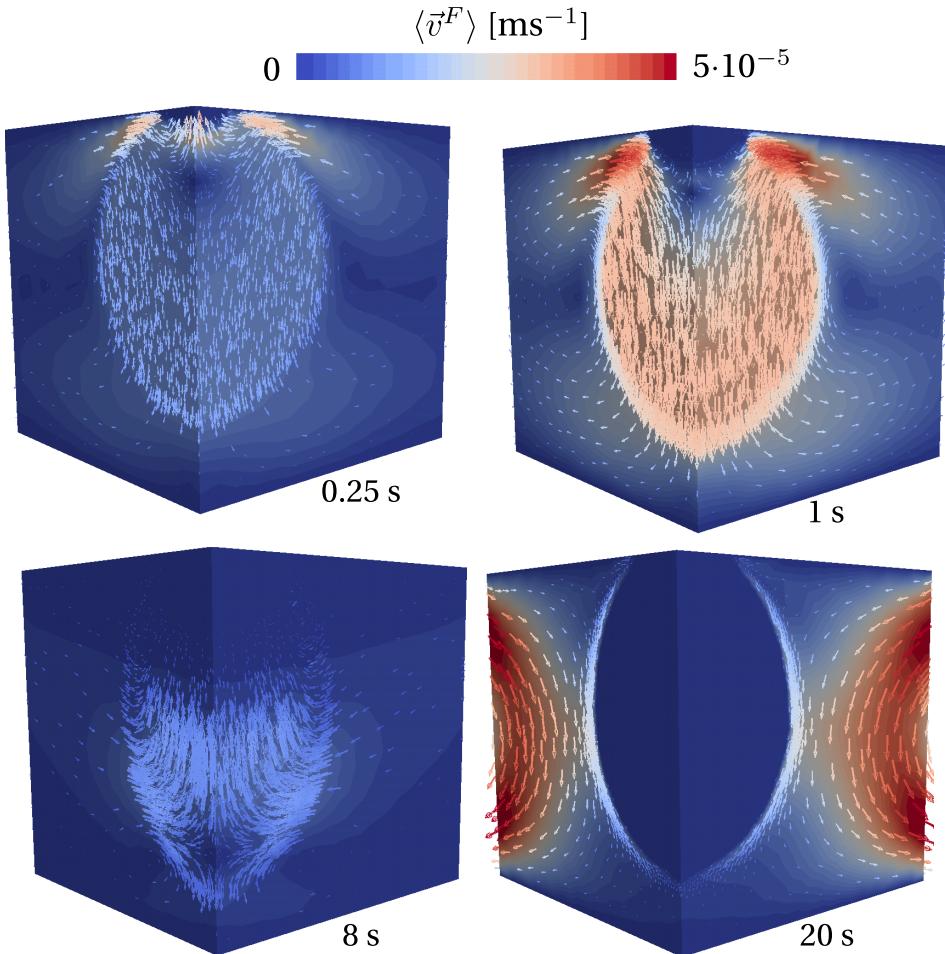
higher gravity accelerations, surface tension is of central importance since it counters the gravitational force by stabilising the air-metal interface.



**Fig. 1.33** – Comparison of experimental (blue) and numerical (red) shrinkage profiles, compared to their respective initial shapes (green). A vector image processing algorithm is used to extract the droplet outlines from the experimental images. The experimental displacement at the top of the droplet was estimated by scaling the initial numerical profile to the experimental one, and then comparing the final profiles. The direction of the gravitational fields points downwards, depicted by the arrow (note that the vector length is not scaled to its magnitude).

Three solid phases are considered for the binary steel, a primary BCC phase, a peritectic FCC phase and a cementite phase. The latter can be obtained by cooling the sample at low temperatures to achieve solid-state transformation. The next point to discuss is segregation and fluid flow. With the chosen gravity acceleration, the liquid metal moves in the downward direction when the ceramic substrate comes from above the droplet. As soon as solidification takes place right after the metal-substrate contact, a BCC-rich mushy zone forms near the contact surface. The abrupt phase change imposes a fast shrinkage rate, which tends to straighten the interface near the substrate, as we can observe in [fig. 1.33](#). A part of the flow thus deviates towards the solid front to compensate for the density increase, as shown in [fig. 1.34](#). This flow pattern in the sample shows distinct regions shown at 0.25 s and 1 s in the previous figure: upward flow driven by solidification shrinkage contributes to a slight enrichment by

inverse segregation, while a downward flow driven by gravity redistributes species in the containerless melt. Upon completely solidifying, the droplet forms a rigid and fixed solid, surrounded by natural argon flow.



**Fig. 1.34** – Flow patterns in reduced-gravity solidification with shrinkage: deviation towards the solidification front at 0.25 s and 1 s, contributing to solute transport in gravity's opposite direction. At 8 sec, the mushy zone reaches the droplet vertex marking a flow pattern change. At 20 s, the argon flows freely in the domain around the completely solidified and rigid sample. Please note that the scale of latter snapshot has a maximum magnitude of  $10^{-6} m s^{-1}$ , not shown for illustrative simplicity.

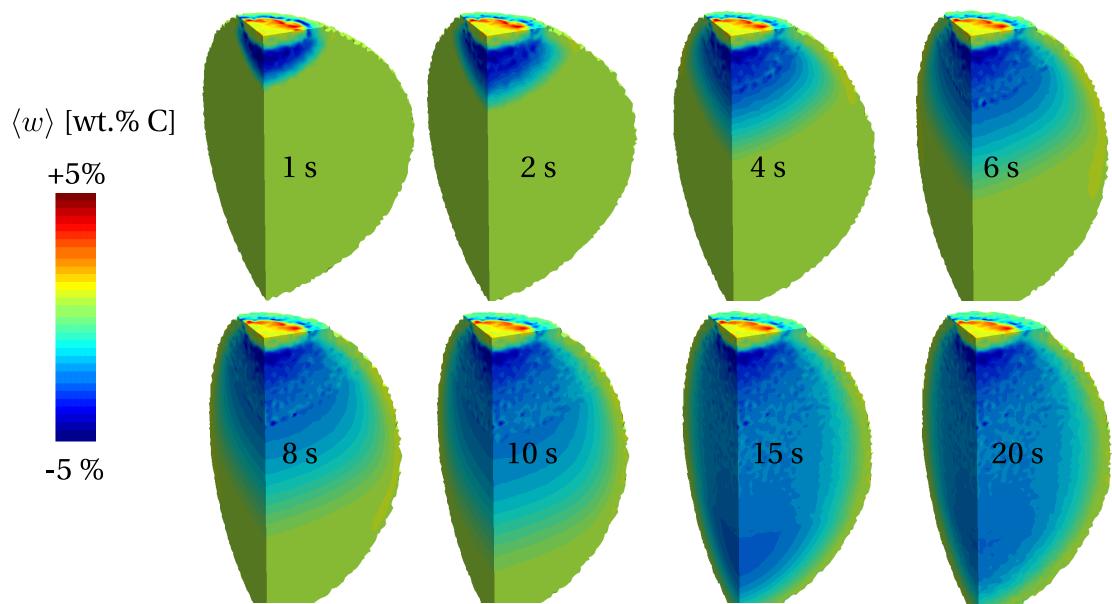
The fluid flow is behind the reduced-gravity segregation shown at different stages in [fig. 1.35](#). We may call it mesosegregation, as the droplet is small and considered as a laboratory-scale sample. As earlier mentioned, a restricted region of positive segregation settles at the contact area with the substrate, from the first second after the contact. Later, between 2 s and 8 s, the solid front advances in the melt, creating a noticeable negatively segregated area, about 4% less than the nominal composition, just below the positive segregation zone. Normally, we would expect that the compo-

sition decreases gradually once the solid front advances in time, as confirm the 1D segregation profiles in [fig. 1.20](#). To interpret this unusual observation, we refer to the fluid flow shown earlier in [fig. 1.34](#). At 0.25 s, a velocity zero-level isovolume (i.e. depicting a volume with null velocity magnitude) forms between the two distinct regions of upward and downward flow. The strong negative divergence that settles in this area results in solute depletion in the two directions and due to the various driving forces.

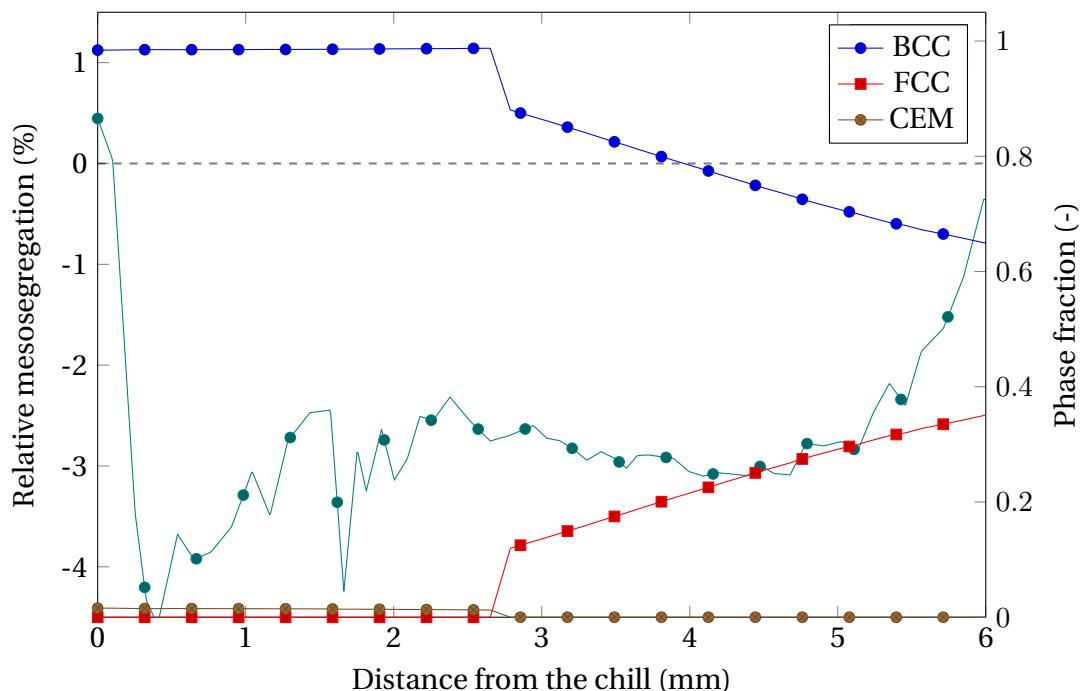
However, at 1 s, the zero isovolume clearly shrinks in a matter of only 0.75 s. That is because the initial temperature gradient is the highest during the process, then it decreases gradually. Since a higher temperature gradient produces a greater cooling flux according to the Fourier model, solidification is faster in the beginning and the volume shrinkage is fast, hence the shrinkage flow coexists with the gravity flow. As the transformation progresses, shrinkage flow becomes insignificant compared to the latter, therefore the negative segregation intensity decreases gradually from 2.2 mm to 4.3 mm from the chill, corresponding to the first seconds of contact ( $t < 8$  s). This result is also shown in [fig. 1.36](#) where we plot the relative segregation profile along the vertical rotation axis of the droplet. At 8 s, [fig. 1.34](#) shows the zero-velocity isovolume moved down the vertical revolution axis by following the solidification front, then vanishing at about 10 s. It means that from this point in time, the flow is so dissipated by the mushy zone's low permeability, hence the low-magnitude shrinkage flow dominates again. We may correlate this flow pattern once again to the segregation profile in [fig. 1.36](#): As of 4.3 mm and down to the tip of the deformed sample, we observe a steady rise in solute content caused by the shrinkage-dominated flow between dendrites compensating for density differences. This rise in solute content is however not strictly correct, says the species mass conservation study, shown in FIG.

In [fig. 1.36](#), the final phase distribution along the vertical revolution axis is plotted . The plots show that in the upper part of the droplet close to the chill, a eutectoid product (we may not speak of eutectoid microstructure as the current approach is only macroscopic, without information on the smaller scale) that results from the hypoeutectoid composition, consisting of 98% of  $\alpha$ -BCC phase together and 2% of CEM between 0 and 2.9 mm away from the substrate. Beyond this point, the austenitic  $\gamma$ -FCC phase is gradually replaced by  $\alpha$ -BCC, which represents the proeutectoid  $\alpha$  phase, taking place before temperature reaches the eutectoid isotherm at 727 °C.

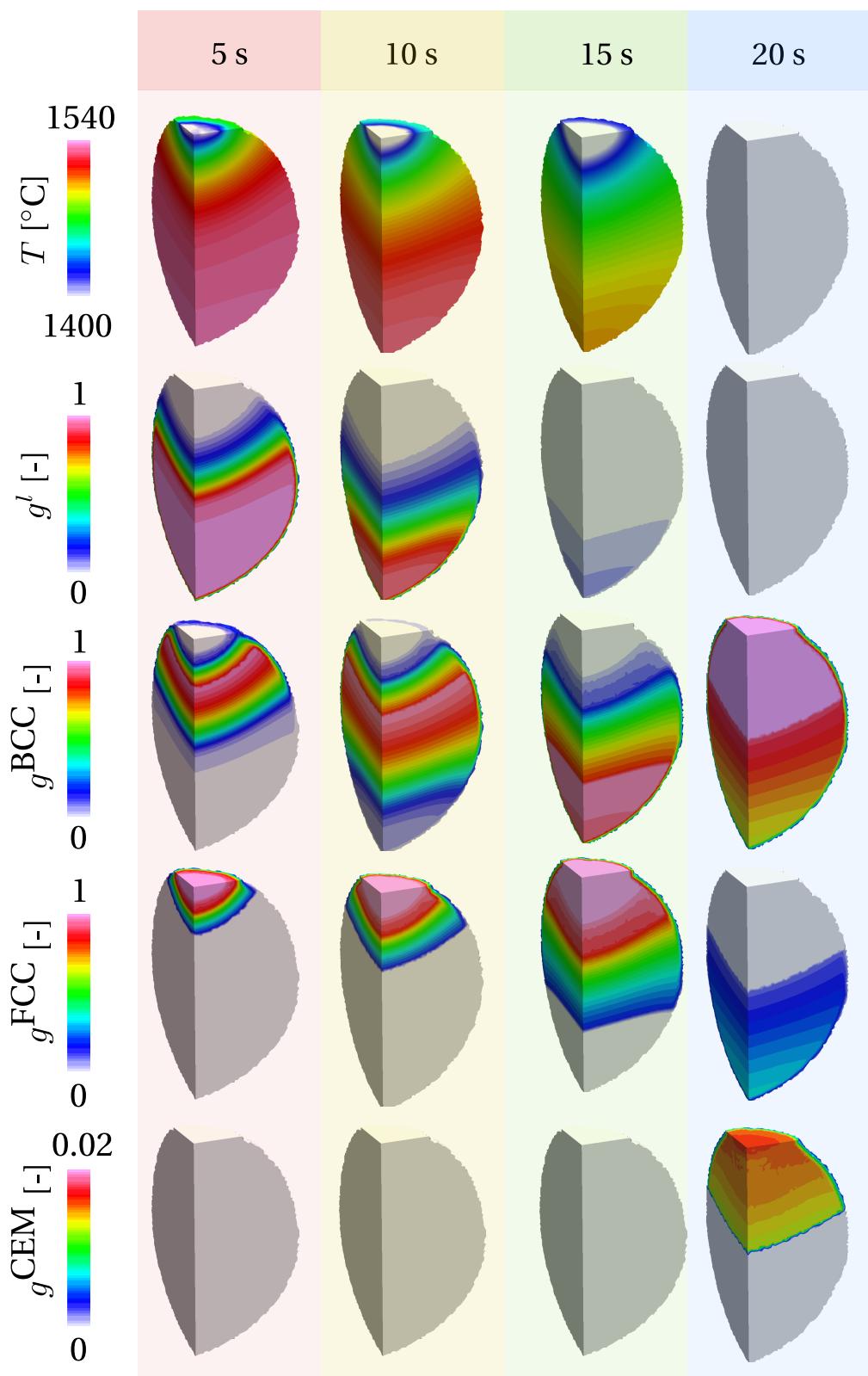
A better global visualisation of the transformation is given in [fig. 1.37](#), at different time stages. Each column depicts a definite time with temperature and phase distribution.



**Fig. 1.35** – Evolution of the average composition with solidification time, showing evidence of mesosegregation and shape deformation between 0s and 20s.



**Fig. 1.36** – Segregation profile in percent with respect to the nominal composition, along the vertical axis of the droplet.



**Fig. 1.37** – Solidification progress at 5, 10, 15 and 20 s showing the effect of segregation on the transformation paths, from liquid to solid and solid-state.

### 1.8.4 Texus ternary and quaternary alloys

In this section, we apply the thermodynamic-database-compatible energy solver to extend the previous binary alloy to a multicomponent alloy. we are mainly interested in showing that:

- the transformation path changes with the alloy, hence the shrinkage kinetics also,
- the mapping resolution in composition and temperature is important to get accurate shrinkage predictions, and
- that

### 1.8.5 TODO

Draw table of tabulation size on disk and computation time. Stress on the fact that shrinkage results from binary to quaternay are much more different because the tabulation is more reliable in terms of resolution. For the others the deformation may be spurious, driven by an unprecise solidification paths.

Compare mass conservation between anisotropic mesh (up to 20% loss) and isotropic fixed (up to 3% only)

Compare results of binary, ternary and quaternary tabulations then simplified quaternary

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