

1 Monodomain model: without level set

Conservation of energy (Nonlinear Heat Transfer)

$$\frac{\partial \langle \rho h \rangle}{\partial t} + \nabla \cdot \langle \rho h \vec{v} \rangle + \nabla \cdot \left(\langle \kappa \rangle \vec{\nabla} T \right) = 0$$

Conservation of liquid momentum (Navier Stokes)

$$\begin{aligned} \frac{\partial}{\partial t} (\rho^l \langle \vec{v}^l \rangle) + \frac{1}{g^l} \vec{\nabla} \cdot (\rho^l \langle \vec{v}^l \rangle \times \langle \vec{v}^l \rangle) = \\ - g^l \vec{\nabla} p^l - 2\mu^l \vec{\nabla} \cdot \left(\overline{\overline{\nabla}} \langle \vec{v}^l \rangle + \overline{\overline{\nabla}^t} \langle \vec{v}^l \rangle \right) - g^l \mu^l \mathbb{K}^{-1} \langle \vec{v}^l \rangle + g^l \rho^l \vec{g} \end{aligned}$$

Conservation of chemical species (Macrosegregation)

$$\frac{\partial \langle \rho w_i \rangle}{\partial t} + \nabla \cdot \langle \rho w_i \vec{v} \rangle + \nabla \cdot \left(g^l D^l \vec{\nabla} w_i^l \right) = 0$$

Microsegregation

$$\begin{aligned} \left(g^\phi, \langle w_i^\phi \rangle^\phi \right) &= f(\langle w_i \rangle, T) \\ \frac{\partial \langle \rho h \rangle}{\partial T} &= \frac{\partial}{\partial T} \left(\sum_\phi g^\phi \langle \rho h \rangle^\phi \right) \end{aligned}$$

2 Multidomain model: with level set

Property mixing in the diffuse interface

$$\hat{\kappa} = H^M \langle \kappa \rangle + H^A \kappa^A$$

$$\hat{\mu} = H^M \mu^l + H^A \mu^A$$

Conservation of energy (Nonlinear Heat Transfer)

$$\frac{\partial \langle \rho h \rangle}{\partial t} + \nabla \cdot \langle \rho h \vec{v} \rangle + \nabla \cdot (\hat{\kappa} \vec{\nabla} T) = 0$$

Conservation of liquid momentum (Navier Stokes)

$$\frac{\partial}{\partial t} (\rho^l g^l \vec{v}^l) + \vec{\nabla} \cdot (\rho^l g^l \vec{v}^l \times \vec{v}^l) = \vec{\nabla} \cdot (g^l \overline{\sigma^l}) + g^l \vec{F}_v^l + \vec{\Gamma}^l$$

Conservation of chemical species (Macrosegregation)

$$\frac{\partial \langle \rho w_i \rangle}{\partial t} + \nabla \cdot \langle \rho w_i \vec{v} \rangle + \nabla \cdot (g^l D^l \vec{\nabla} w_i^l) = 0$$

Microsegregation

$$(g^\phi, \langle w_i^\phi \rangle^\phi) = f(\langle w_i \rangle, T)$$

$$\frac{\partial \langle \rho h \rangle}{\partial T} = \frac{\partial}{\partial T} \left(\sum_\phi g^\phi \langle \rho h \rangle^\phi \right)$$