BINOMIAL DISTRIBUTION

- If a random variable X is the result of a binomial experiment, it can be modelled by a binomial distribution with $X \sim B(n, p)$

- This function can be used to calculate $P(X=r)$:

$$\rightarrow X \sim B(n,p) \Rightarrow P(X=r) = \binom{n}{r} p^r (1-p)^{n-r}; \quad r = 0, 1, \ldots, n$$

where:

- n is number of trials
- r is number of success
- p is the probability of success

- If you don't want the precise number of successes ("less than", "more than", "at least")

Then you need to use a cumulative probability distribution

"Less than or equal to"

$$\Rightarrow P(X \leq r) = P(X=0) + P(X=1) + \ldots + P(X=r)$$

- Long calculation; calculator provides this option.

"More than" $\Rightarrow P(X > r) = 1 - P(X \leq r)$

"At least" $\Rightarrow P(X \geq r) = 1 - P(X \leq r-1)$

RULES

1 There can only be 2 outcomes : success P or failure Q

2: The outcome of 1 event must be independent of the next..

(does not affect next event)

3. The probability of success must remain the same through all trials

Example: Probability a person is left-handed is 13%

$\Rightarrow$ $P = 0.13$, $Q = 0.87$

Geometric

- Questions centre around 1^{st} success

Example :

1) What is the probability that the first left-handed person is the 5^{th} person we meet?

2) What is the probability that the first left-handed person we find is after the 4^{th} person?

3) How many people do we expect to see until we get our first left-handed person?

Solution :

1) $(0.87)^4 \times 0.13 = 0.0745$

2) $(0.87)^4 = 0.5729$

3) $\mu = \frac{1}{p} = \frac{1}{0.13} = 7.69$

↳ mean (average)

Binomial

- Given specific amount of trials n
- Find number of successes in those n trials

Example:

- Let x be the random variable for how many out of 5 people could be left-handed.

Solution:

A	B	C	D	E
O	O	O	O	O
↑	↑	↑	↑	↑

X	P
5	$(0.13)^5 = 0.000037$
4	$(0.13)^4 \times 1.87 \times 5C4 = 0.0012$
3	$(0.13)^3 \times (0.87)^2 \times 5C3 = 0.0166$
2	$(0.13)^2 \times (0.87)^3 \times 5C2 = 0.1113$
1	$0.13\,(0.87)^4 \times 5C1 = 0.3724$
0	$(0.87)^5 = 0.4984$

$P(X > 3) = 0.0012 + 0.000037$

$P(X \geq 1) = 1 - 0.4984 = 0.5016$

Standard Deviation.

$\mu_x = np = 5 \times 0.13 = 0.65$ $\rightarrow$ $E(X)$ expectation

$\sigma_x = \sqrt{npq} = \sqrt{5 \times 0.13 \times 0.87} = 0.75$ $\rightarrow$ $Var(X) = np(1-p) = npq$ variance

where $q = 1-p$

Example:

1) You have a 15% chance to win a game at a carnival. How many times do you expect to play until you get your first win?

2) If you play the game 60 times, how many times do you expect to win, and with what standard deviation?

Solution:

1) $\mu = \frac{1}{p} = \frac{1}{0.15} \approx 6.67$ times.

2). $\mu_x = 60 \times 0.15 = 9.$

$\sigma_x = \sqrt{60 \times 0.15 \times 0.85} = 2.77$