# Machine Learning

HW 6 - Markov Models
Spring 2025
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1. In this assignment we will have a Markov Model that can be in one of three (3) states at any given time: $s_1, s_2, s_3$. We have observed the following four sequences (our training data):

   - $q_1 = (s_1, s_1, s_3, s_2, s_1)$
   - $q_2 = (s_3, s_1, s_1, s_2, s_2)$
   - $q_3 = (s_3, s_2, s_1, s_1, s_1)$
   - $q_4 = (s_2, s_3, s_3, s_3, s_1)$

   (a) What is the initial state vector, $\pi$ and the state transition matrix $A$, given these training samples? (6pts)

   The initial state vector $\pi$ is derived from the first state of the 4 sequences: $s_1$ appears once, $s_2$ once, and $s_3$ twice.

   $$\pi = [P(s_1), P(s_2), P(s_3)] = \left[\frac{1}{4}, \frac{1}{4}, \frac{2}{4}\right] = \left[\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right]$$

   The state transition matrix $A_{ij} = P(S_{t+1} = s_j | S_t = s_i)$ is found by counting all transitions:

   - From $s_1$ (6 total): $s_1 \to s_1$ (4), $s_1 \to s_2$ (1), $s_1 \to s_3$ (1)
   - From $s_2$ (4 total): $s_2 \to s_1$ (2), $s_2 \to s_2$ (1), $s_2 \to s_3$ (1)
   - From $s_3$ (6 total): $s_3 \to s_1$ (2), $s_3 \to s_2$ (2), $s_3 \to s_3$ (2)

   The matrix $A$:

   $$A = \begin{pmatrix} 4/6 & 1/6 & 1/6 \\ 2/4 & 1/4 & 1/4 \\ 2/6 & 2/6 & 2/6 \end{pmatrix} = \begin{pmatrix} 2/3 & 1/6 & 1/6 \\ 1/2 & 1/4 & 1/4 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

   (b) Given your answer in the previous part, what is the likelihood (evaluation problem) of observing the sequence $q = (s_3, s_2, s_2)$? (4pts)

   The likelihood is the product of the initial probability of $s_3$ and the transition probabilities.

   $$P(q) = P(s_3) \times P(s_2|s_3) \times P(s_2|s_2)$$

   Using the values from $\pi$ and $A$:

   $$P(q) = \left(\frac{1}{2}\right) \times \left(\frac{2}{6}\right) \times \left(\frac{1}{4}\right) = \left(\frac{1}{2}\right) \times \left(\frac{1}{3}\right) \times \left(\frac{1}{4}\right) = \frac{1}{24}$$

   So the likelihood is $\frac{1}{24} \approx 0.0417$.