Machine Learning

HW 5 - Probabilistic Models
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1. Consider the following set of training examples for an unknown target function: $(x_1, x_2) \rightarrow y$:

Y	x_1	x_2	Count
+	T	T	3
+	T	F	4
+	F	T	4
+	F	F	1
-	T	T	0
-	T	F	1
-	F	T	3
-	F	F	5

(a) Computer the posteriors for the observation $x = [T, T]$ using:

i. Inference (5pts)

Count$(X_1 = T, X_2 = T) = 3 + 0 = 3$.

$$P(Y = +|X_1 = T, X_2 = T) = \frac{\text{Count}(Y = +, X_1 = T, X_2 = T)}{\text{Count}(X_1 = T, X_2 = T)} = \frac{3}{3} = 1$$

$$P(Y = -|X_1 = T, X_2 = T) = \frac{\text{Count}(Y = -, X_1 = T, X_2 = T)}{\text{Count}(X_1 = T, X_2 = T)} = \frac{0}{3} = 0$$

Results: $P(Y = +|T, T) = 1$, $P(Y = -|T, T) = 0$.

ii. Naive Bayes (5pts)

$$P(Y|X_1, X_2) \propto P(X_1|Y)P(X_2|Y)P(Y)$$

Priors: $P(Y = +) = \frac{12}{21} = \frac{4}{7}$ $P(Y = -) = \frac{9}{21} = \frac{3}{7}$

Class-conditional probabilities for $X_1 = T, X_2 = T$: For $Y = +$ (12 instances): $P(X_1 = T|Y = +) = \frac{7}{12}$ $P(X_2 = T|Y = +) = \frac{7}{12}$

For $Y = -$ (9 instances): $P(X_1 = T|Y = -) = \frac{1}{9}$ $P(X_2 = T|Y = -) = \frac{3}{9} = \frac{1}{3}$

Unnormalized posteriors for $x = [T, T]$: $P(Y = +)P(X_1 = T|Y = +)P(X_2 = T|Y = +) = \left(\frac{4}{7}\right)\left(\frac{7}{12}\right)\left(\frac{7}{12}\right) = \frac{7}{36}$ $P(Y = -)P(X_1 = T|Y = -)P(X_2 = T|Y = -) = \left(\frac{3}{7}\right)\left(\frac{1}{9}\right)\left(\frac{1}{3}\right) = \frac{1}{63}$

Normalization constant α: $\alpha = \frac{7}{36} + \frac{1}{63} = \frac{49}{252} + \frac{4}{252} = \frac{53}{252}$

Normalized posteriors for $x = [T, T]$: $P(Y = +|X_1 = T, X_2 = T) = \frac{7/36}{53/252} = \frac{49}{53}$ $P(Y = -|X_1 = T, X_2 = T) = \frac{1/63}{53/252} = \frac{4}{53}$

Results: $P(Y = +|T, T) = \frac{49}{53} \approx 0.9245$, $P(Y = -|T, T) = \frac{4}{53} \approx 0.0755$.