Machine Learning

HW 2 – Linear Regression
Spring 2025

1. Consider the following supervised *training* dataset:

$$X = \begin{bmatrix} -2 \\ -5 \\ -3 \end{bmatrix}, \quad Y = \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}$$

(a) Compute the coefficients for closed-form linear regression using least squares estimate (LSE). Show your work and remember to add a bias feature. (6pts).

We add a bias column to get

$$\widetilde{X} = \begin{pmatrix} 1 & -2 \\ 1 & -5 \\ 1 & -3 \end{pmatrix}.$$

Compute

$$\widetilde{X}^T\widetilde{X} = \begin{pmatrix} 3 & -10 \\ -10 & 38 \end{pmatrix}, \quad (\widetilde{X}^T\widetilde{X})^{-1} = \frac{1}{14}\begin{pmatrix} 38 & 10 \\ 10 & 3 \end{pmatrix}, \quad \widetilde{X}^T Y = \begin{pmatrix} -2 \\ 15 \end{pmatrix}.$$

So the weight vector $w = (b, m)^T$ is

$$w = (\widetilde{X}^T\widetilde{X})^{-1}\widetilde{X}^T Y = \frac{1}{14}\begin{pmatrix} 38 & 10 \\ 10 & 3 \end{pmatrix}\begin{pmatrix} -2 \\ 15 \end{pmatrix} = \begin{pmatrix} 37/7 \\ 25/14 \end{pmatrix}.$$

(b) Using your learned model in the previous part, what are your predictions, $\hat{Y}$, for the training data (2pts)?

For each x_i, $\hat{y}_i = b + m\,x_i$. Hence

$$\hat{Y} = \begin{pmatrix} 37/7 + \frac{25}{14}(-2) \\ 37/7 + \frac{25}{14}(-5) \\ 37/7 + \frac{25}{14}(-3) \end{pmatrix} = \begin{pmatrix} 12/7 \\ -51/14 \\ -1/14 \end{pmatrix}.$$

(c) What is the RMSE and SMAPE for this training set based on the model you learned in the previous part (2pts)? You might find different equations from SMAPE online, but use the one provided in the slides.

$$\text{RMSE} = \sqrt{\frac{1}{3}\sum_{i=1}^{3}(y_i - \hat{y}_i)^2} = \sqrt{\frac{25}{42}} \approx 0.771.$$

$$\text{SMAPE} = \frac{1}{3}\sum_{i=1}^{3}\frac{|y_i - \hat{y}_i|}{|y_i| + |\hat{y}_i|} \approx 0.437.$$