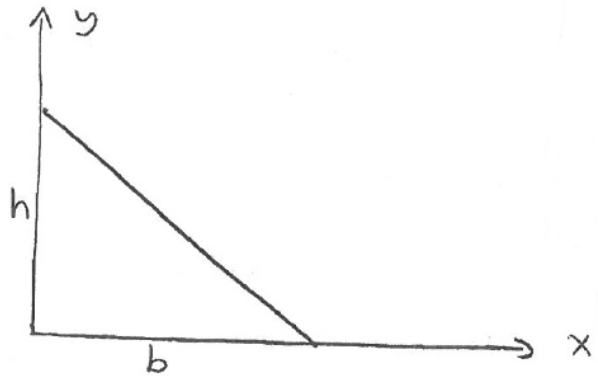
07/01/2021

ES 221 MECHANICS I (STATICS) RECITATION XII

Q1)

Calculate the moment of inertia of the triangle about the x and y axes.



Solution to Q1

$$y = \frac{-h}{b}x + h$$

$$I_x = \int y^2 dA$$

$$I_x = \int y^2 x dy$$

$$I_x = \int_0^h y^2 (y - h)\frac{-b}{h} dy$$

$$I_x = \frac{-b}{h}\int_0^h (y^3 - y^2 h) dy = \frac{bh^3}{12}$$

$$I_y = \int x^2 dA$$

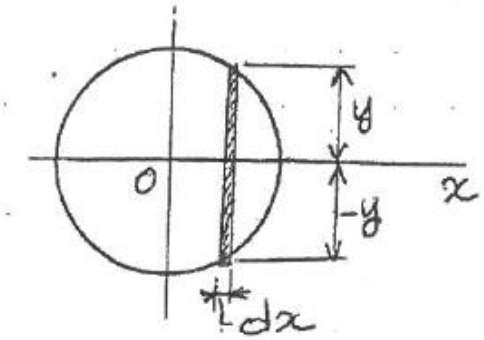
$$I_y = \int x^2 y dx$$

$$I_y = \int_0^b x^2 \left(\frac{-h}{b}x + h\right) dx$$

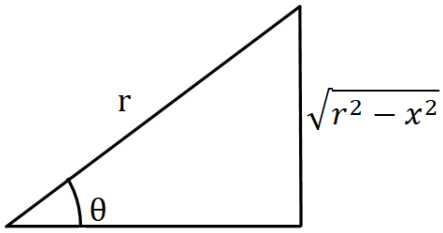
$$I_y = \int_0^b \left(\frac{-hx^3}{b} + x^2 h\right) dx = \frac{b^3 h}{12}$$

Q2)

Calculate the moment of inertia of the circle with respect to the x axis.



Solution to Q2



$$dI_x = \frac{dx(2y)^3}{12}$$

$$\int dI_x = \frac{2}{3}\int y^3 dx = \frac{2}{3}\int (r^2 - x^2)^{3/2} dx\,;$$

$$I_x = \frac{2}{3}\int r^3 sin^3\theta(-rsin\theta)d\theta = \frac{-2r^4}{3}\int sin^4\theta d\theta\,;$$

$$I_x = \frac{-2r^4}{3}\int \left(\frac{1}{4} - \frac{cos2\theta}{2} + \frac{cos^2 2\theta}{4}\right) d\theta$$

$$sin^2\theta + cos^2\theta = 1$$

$$sin^2\theta - cos^2\theta = -cos2\theta$$

$$sin^2\theta = \frac{1 - cos2\theta}{2}$$

$$I_x = \frac{-2r^4}{3}\int \left[\frac{1}{4} - \frac{cos2\theta}{2} + \frac{(1 + cos4\theta)/2}{4}\right] d\theta$$

$$sin^2 2\theta + cos^2 2\theta = 1$$

$$-sin^2 2\theta + cos^2 2\theta = cos4\theta$$

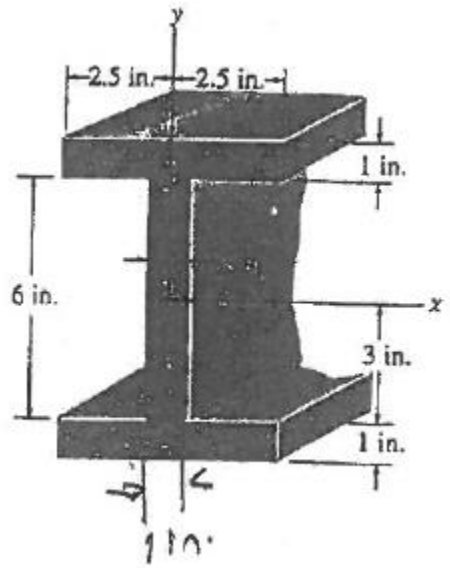
$$I_x = \frac{-2r^4}{3}\left(\frac{\theta}{4} - \frac{sin2\theta}{4} + \frac{\theta}{8} + \frac{sin4\theta}{32}\right)$$

$$I_x = \frac{-2r^4}{3}\left[\frac{3cos^{-1}\left(\frac{x}{r}\right)}{8} - \frac{sin2\left(cos^{-1}\left(\frac{x}{r}\right)\right)}{4} + \frac{sin4\left(cos^{-1}\left(\frac{x}{r}\right)\right)}{32}\right]_{-r}^{r}$$

$$I_x = \frac{-2r^4}{3}\left(\frac{-3\pi}{8}\right) = \frac{\pi r^4}{4}$$

Q3)

Determine the moments of inertia of the shaded area with respect to the x and y centroidal axes.



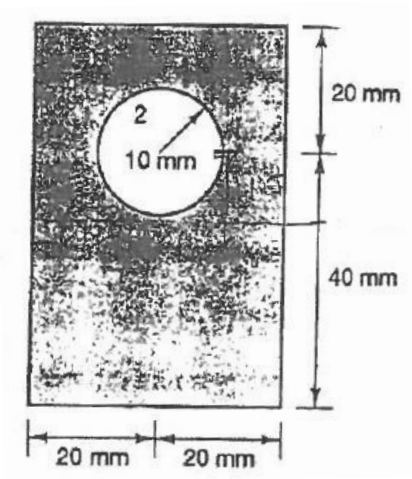
Solution to Q3

$$I_x = \frac{5 \times 1^3}{12} + 5 \times 1 \times (4 - 0.5)^2 + \frac{1 \times 6^3}{12} + \frac{5 \times 1^3}{12} + 5 \times 1 \times (7.5 - 4)^2 = 141.3\ m^4$$

$$I_y = \frac{1 \times 5^3}{12} + \frac{6 \times 1^3}{12} + \frac{1 \times 5^3}{12} = 21.3\ m^4$$

Q4)

Determine the second moment of area about the x-centroidal axis of the area shown.



$$\bar{y} = \frac{40 \times 60 \times 30 - \pi \times 10^2 \times 40}{40 \times 60 - \pi \times 10^2} \cong 28.5\ mm\ \uparrow$$

$$I_x = \frac{40 \times 60^3}{12} + 40 \times 60 \times (30 - 28.5)^2 - \frac{\pi \times 10^4}{4} - \pi \times 10^2 \times (40 - 28.5)^2 = 676 \times 10^3\ mm^4$$