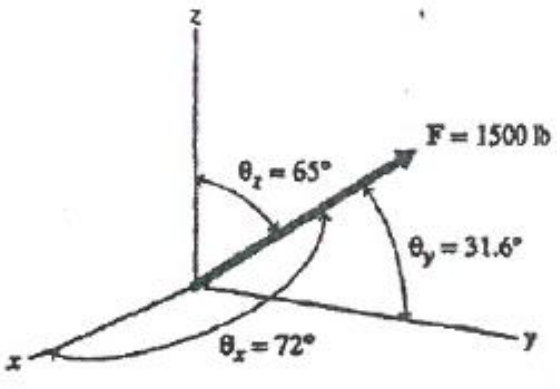
22/10/2020

ES 221 MECHANICS I (STATICS) RECITATION II

Q1)



A force F is applied at a point in a body as shown.

a) Determine the x,y and z scalar components of the force.
b) Express the force in Cartesian vector form.

Answer to Q1

a) $F_x = 1500 \times cos72^0 = 464\ lb$

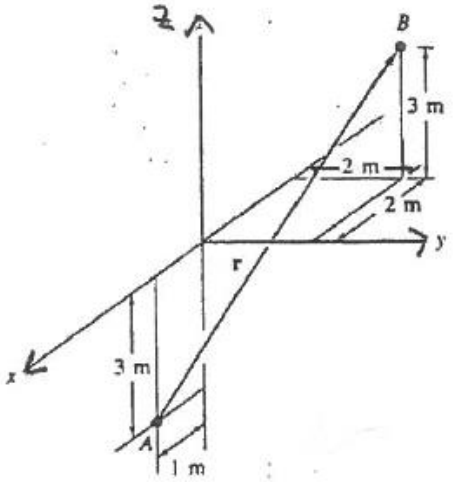
$F_y = 1500 \times cos31.6^0 = 1278\ lb$

$F_z = 1500 \times cos65^0 = 634\ lb$

b)

$\vec{F} = \{464\vec{\imath} + 1278\vec{\jmath} + 634\vec{k}\}\ lb$

Q2)



Determine the magnitude and coordinate direction angles of the position vector extending from A to B.

Answer to Q2

Point Coordinates:

$A(1,0,-3)$

$B(-2,2,3)$

$\vec{r}_{AB} = \{-2-1\}\vec{\imath} + \{2-0\}\vec{\jmath} + \{3-(-3)\}\vec{k}$

$\vec{r}_{AB} = \{-3\vec{\imath} + 2\vec{\jmath} + 6\vec{k}\}\, m$

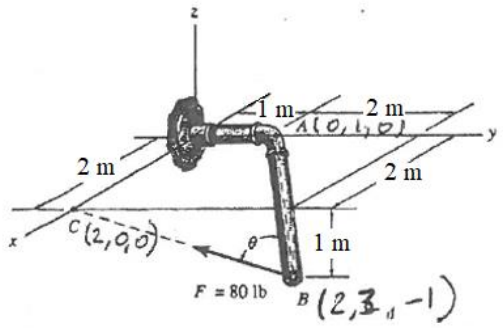
$|r_{AB}| = \sqrt{(-3)^2 + 2^2 + 6^2} = 7$

$cos^{-1}\left(\frac{r_x}{r}\right) = cos^{-1}\left(\frac{-3}{7}\right) = \theta_x = 115.4^0$

$cos^{-1}\left(\frac{2}{7}\right) = \theta_y = 73.4^0$

$cos^{-1}\left(\frac{6}{7}\right) = \theta_z = 31^0$

Q3)



The pipe shown is subjected to the force F=80 lb at its end B.

Find:

a) The angle θ between $\vec{F}$ and the pipe segment BA

b) Magnitudes of the components of $\vec{F}$ which are parallel and perpendicular to BA.

Answer to Q3

a) Point Coordinates:

$A(0,1,0)$

$B(2,3,-1)$

$C(2,0,0)$

$\vec{r}_{BA} = \{-2\vec{\imath} - 2\vec{\jmath} + 1\vec{k}\}\, m, \qquad |r_{BA}| = \sqrt{(-2)^2 + (-2)^2 + 1^2} = 3\, m$

$\vec{r}_{BC} = \{-3\vec{\jmath} + 1\vec{k}\}\, m, \qquad |r_{BC}| = \sqrt{(-3)^2 + 1^2} = \sqrt{10}\, m$

$\boldsymbol{\vec{r}_{BA} \cdot \vec{r}_{BC} = |r_{BA}| \times |r_{BC}| \times cos\theta}$

$\vec{r}_{BA} \cdot \vec{r}_{BC} = 3\sqrt{10}cos\theta \ldots (1)$

$\boldsymbol{\vec{r}_{BA} \cdot \vec{r}_{BC} = r_{BA_x} r_{BC_x} + r_{BA_y} r_{BC_y} + r_{BA_z} r_{BC_z}}$

$\vec{r}_{BA} \cdot \vec{r}_{BC} = (-2) \times (-3) + (1)(1) = 7 \ldots (2)$

From equations (1) and (2);

$$3\sqrt{10}cos\theta = 7$$

$$cos^{-1}\left(\frac{7}{3\sqrt{10}}\right) = 42.5^0$$

b)

$$\vec{F} \cdot \vec{u}_{BA} = F_{\parallel,BA}$$

$$\vec{u}_{BA} = \frac{\vec{r}_{BA}}{r_{BA}} = \left\{\frac{-2\vec{\imath}-2\vec{\jmath}+1\vec{k}}{3}\right\}$$

$$\vec{F} = F\vec{u}_{BC} = F\frac{\vec{r}_{BC}}{r_{BC}} = 80\left\{\frac{-3\vec{\jmath}+1\vec{k}}{\sqrt{10}}\right\} = \{-75.9\vec{\jmath} + 25.3\vec{k}\}\ lb$$

$$\vec{F} \cdot \vec{u}_{BA} = \{-75.9\vec{\jmath} + 25.3\vec{k}\} \cdot \{(-2/3)\vec{\imath} - (2/3)\vec{\jmath} + (1/3)\vec{k}\} = 59\ lb$$

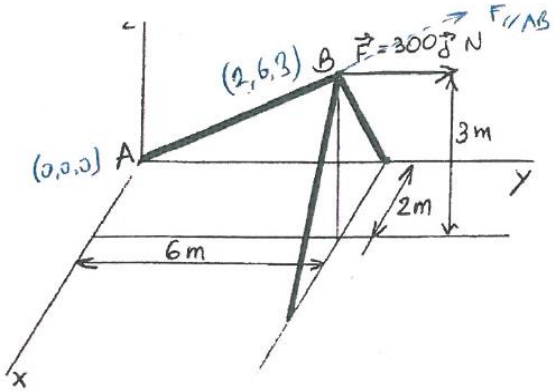
$$F_{\perp} = \sqrt{80^2 - 59^2} = 54\ lb$$

Alternative Method:

$$80 \times cos42.5^0 = 59\ lb$$

$$80 \times sin42.5^0 = 54\ lb$$

Q4)



The frame shown is subjected to a horizontal force $\vec{F} = 300\,\vec{j}\,N$. Find the magnitudes of the components of $\vec{F}$ parallel and perpendicular to AB.

Answer to Q4

$$F_{\parallel,AB} = \vec{F}_{AB} \cdot \vec{u}_{AB}$$

$$\vec{u}_{AB} = \left\{\frac{2\vec{\imath} + 6\vec{\jmath} + 3\vec{k}}{7}\right\}$$

$$F_{\parallel,AB} = \{300\,\vec{\jmath}\} \cdot \{(2/7)\vec{\imath} + (6/7)\vec{\jmath} + (3/7)\vec{k}\} = 257.1\ N$$

$$F_{\perp,AB} = \sqrt{300^2 - 257.1^2} = 155\ N$$