93,75

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Middle East Technical University
Department of Computer Engineering

CENG 477

Fall '18

Final Exam

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- **Duration:** **120** minutes.
- **Grading:**
  - Each of the **15** TRUE-FALSE questions is worth **2** points.
  - Each of the **10** Multiple-choice questions is worth **5** points.
  - Each of the **2** Classical-type questions is worth **10** points.
  - For TRUE-FALSE and multiple-choice questions **4** wrong points cancel out **1** correct point.
- **Asking questions:** is not allowed. If you decide that a question is wrong:
  - DO NOT ask the proctor about a clarification.
  - Indicate clearly your objection and your proposed answer on the **first page of the question booklet**.
  - For all transformation questions, assume that points are multiplied from the right with the matrix (as in $p' = Mp$)
- **Mark** your group ID (as A or B) on your answer sheet.
- **Turn in** your question booklet (this booklet) together with the answer sheet. Otherwise your answer sheet will not be evaluated, and you will receive a zero from this exam.
- GOOD LUCK !

26:10
27:10

We start with TRUE-FALSE questions, mark $\boxed{A}$ box for TRUE, $\boxed{B}$ box for FALSE on your <u>answer sheet</u>

T $\boxed{1}$ If a vertex shader is used to draw a primitive, OpenGL will not automatically transform the vertices with the model-view-projection matrix.

F $\boxed{2}$ Assuming a large number of triangles, using immediate mode (begin/end) will be generally faster than using vertex arrays in OpenGL.

T $\boxed{3}$ C2 continuity can be guaranteed by using natural cubic splines.

T $\boxed{4}$ It is possible to model orthographic projection in ray tracing by generating initial rays that are parallel to each other and orthogonal to the image plane.

F $\boxed{5}$ The depth buffer algorithm sorts the objects from front to back and then draws them in back-to-front order.

F $\boxed{6}$ The number of processed vertices in a vertex shader is always smaller than the number of processed fragments in a fragment shader. Think about a model with no triangles / empty model or clipping - culling

T $\boxed{7}$ A Hermite curve can be used to draw the same curve that a Bezier curve can draw.

F $\boxed{8}$ In the shadow mapping algorithm, the stencil buffer is used to create a shadow map from the light's perspective.

F $\boxed{9}$ The contents of the depth buffer does not change if the objects are static and only the camera is moving in the scene.

T $\boxed{10}$ A single Hermite curve can be used to draw a straight line segment. ?

F $\boxed{11}$ Mipmapping has a significant memory overhead. If a texture without mipmapping occupies A bytes of memory, it will occupy at least $3A$ bytes if mipmapping is used for that texture.

T $\boxed{12}$ In perspective projection, as the field of view gets smaller the screen size of the objects get larger.

T $\boxed{13}$ In OpenGL shaders, the values of *uniform* variables are not interpolated across the primitive.

T $\boxed{14}$ Interpolating normals across a primitive and computing shading for each fragment gives a more realistic result than computing shading at each vertex and then interpolating the colors.

F $\boxed{15}$ During rasterization, it is sufficient to know the color value at two vertices of a triangle. The color at the third vertex can be interpolated.

The TRUE-FALSE questions END here.

$\boxed{16}$ Which parameters are needed at minimum for performing world-to-camera coordinate transformations in 3D (excluding projection and viewport)?

5 8 8

8 12 13
+3 4 5
11 16 18

A) Camera gaze, camera up vector
B) Camera position
C) Camera position, camera up vector
D) Camera position, image plane, camera gaze, camera up vector
E) Camera position, camera gaze, camera up vector

u_A = 0, 0, −1 v_A = 0, −1, 0

w 1, 0, 0
u = 0, 0, −1
v = 0, −1, 0

17 What is the correct order for the following stages of the forward rendering pipeline? Note that $A \to B$ indicates that stage A occurs before stage B:

A) Vertex processing $\to$ Primitive Assembly $\to$ Clipping $\to$ Fragment Processing
B) Primitive assembly $\to$ Clipping $\to$ Fragment Processing $\to$ Vertex Processing
C) Clipping $\to$ Primitive Assembly $\to$ Vertex Processing $\to$ Fragment Processing
D) Vertex processing $\to$ Fragment Processing $\to$ Primitive Assembly $\to$ Clipping
E) Fragment Processing $\to$ Primitive Assembly $\to$ Clipping $\to$ Vertex Processing

$\begin{bmatrix} -10 \\ -8 \\ -3 \\ 1 \end{bmatrix}$

v = w × u

(−10, −8, −3)

18 The coordinates of a point is given as $(8, 12, 13)$ in the coordinate system of camera A, which is located at $(3, 4, 5)$, looking at $+x$ direction with an up-vector along $+y$ direction. What would be coordinates of the same point according to camera B, which is located at $(-1, 2, -4)$, looking at $-y$ direction with an up-vector along $-z$ direction? Assume that both cameras have a right-handed coordinate system.

$\boxed{c = -3}$

5 − c = 8

A) $(9, -14, -17)$
B) $(14, -9, 17)$
C) $(17, 9, -14)$
D) $(-9, -17, 14)$
E) $(14, 9, -17)$

u = −t × w
w =
u = 1, 0, 0
v = w × u

$\begin{pmatrix} -5 \\ -8 \\ -8 \end{pmatrix}$

5d − c = 8
4d − b = 12
3d − a = 13
d = 1
$\boxed{b = -8}$
$\boxed{a = -10}$

19 Assume that you are asked to design a new curve with the following properties:

- The curve will be represeted by *quadratic* (i.e. second degree) polynomials
- The curve will be controlled by three control points:
  - P_1: starting point ($t = 0$ or $u = 0$)
  - P_2: middle point ($t = 0.5$ or $u = 0.5$)
  - P_3: ending point ($t = 1.0$ or $u = 1.0$)

  The curve must pass through all of these control points.
- The geometry (or the constraints) matrix will be of the form:

t = (0, 0, −1)
w = (0, 1, 0)
u = t × w
u = 1, 0, 0

u = t × w
w = (0, 1, 0)
t = (0, 0, −1)
u = (1, 0, 0)
v = (0, 0, −1)

0 0 1
−3 4 −1
1 0 0

~~3 4 2~~
2 −4 2
−3 4 −1
1 0 0

$$G = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

v = w × u
v = (0, 0, −1)

Which of the following is the correct basis matrix, M, for this curve such that all the points on this curve, $Q(t)$, can be computed by:

$$Q(t) = [t^2 \quad t \quad 1]MG$$

−12 −4
3 −4
8 −2

$$\begin{bmatrix} Q(0) \\ Q(0.5) \\ Q(1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1/4 & 1/2 & 1 \\ 1 & 1 & 1 \end{bmatrix} M \begin{bmatrix} Q(0) \\ Q(0.5) \\ Q(1) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 11 \\ 16 \\ 17 \\ 1 \end{bmatrix}$$

12 4

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 1/4 & 1/2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} -9 \\ -12 \\ \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 2 & 4 & 0 & 4 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & -1 & -3 & 0 & -4 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right]$$

0,32
0,312
0,12
0,2

A) $\begin{bmatrix} -3 & 4 & -1 \\ 2 & -4 & 2 \\ 1 & 0 & 0 \end{bmatrix}$

~~D)~~ $\begin{bmatrix} 2 & -4 & 2 \\ -3 & 4 & -1 \\ 1 & 0 & 0 \end{bmatrix}$

B) $\begin{bmatrix} -3 & 4 & -1 \\ 1 & 0 & 0 \\ 2 & -4 & 2 \end{bmatrix}$

E) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & -4 & 2 \\ -3 & 4 & -1 \end{bmatrix}$

$\frac{-8}{1000} + \frac{12}{100} + \frac{2}{10}$

C) $\begin{bmatrix} 1 & 0 & 0 \\ -3 & 4 & -1 \\ 2 & -4 & 2 \end{bmatrix}$

-12

18 - 4 - 2

20 Consider the two adjacent control points as $(0,0)$ and $(4,6)$ and the derivatives at these control points as $(1,2)$ and $(0,2)$, respectively. The Hermite basis matrix M_H is given as:

TMG

$\begin{bmatrix} \frac{1}{1000} & \frac{1}{100} & \frac{1}{10} & 1 \end{bmatrix}$

0 4 1 0

2	-2	1	1
-3	3	-2	-1
0	0	1	0
1	0	0	0

$\begin{bmatrix} 0 & 0 \\ 4 & 6 \\ 1 & 2 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -7 & -8 \\ 10 & 12 \\ 1 & 2 \\ 0 & 0 \end{bmatrix}$

$\frac{-7}{1000} + \frac{10}{100} + \frac{1}{10}$

0,1 0,2
0,1

-0,007

If the curve is at $(0,0)$ for $u = 0.0$ $(t = 0.0)$ and at $(4,6)$ for $u = 1.0$ $(t = 1.0)$, what is the coordinate of the point at $u = 0.1$ $(t = 0.1)$ using Hermite curves?

~~A)~~ (0.193,0.312) ✓
B) (1.103,2.152)
C) (1.103,1.152)
D) (0.207,0.328)
E) none of the above

$\frac{-7}{1000} + \frac{10}{100} + \frac{1}{10}$

0,103

21 Assume that you are given the following normal vectors for different faces of a unit cube centered at origin:

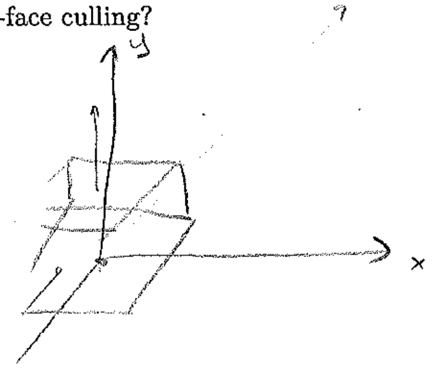
- Front (A): $(0,0,1)$
- Back (B): $(0,0,-1)$
- Right (C): $(1,0,0)$
- Left (D): $(-1,0,0)$
- Top (E): $(0,1,0)$
- Bottom (F): $(0,-1,0)$

$\left[\begin{array}{cccc|cccc} 0 & 0 & -1 & 5 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 4 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$

Imagine that we have back-face culling enabled and we are rendering with a camera that is also located at the origin and looking at the negative z-direction. Which option most accurately represents the faces that will be culled due to back-face culling?

A) C, D
B) E, F
C) B, A
D) B
~~E)~~ All faces

$\begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 11 \\ 13 \\ 1 \end{bmatrix}$



-5
-8
-8

-8+3

-13+5=-8

$$M_{orth} = \begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 1/48 & \frac{13}{12} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 5 \\ -104 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -4 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -104 & -400 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 35 \\ 25 \\ -20 \\ 1 \end{bmatrix} = \begin{bmatrix} -140 \\ -100 \\ 2080 \\ -20 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ -104 \\ 1 \end{bmatrix}$$

proj

2 is not same

22 Find the projected coordinates of the point, $P = (35, 25, -20)$, on the image plane, if a symmetric perspective projection is setup with the following function call:

$-\frac{n+f}{n-f} = \frac{+104}{96}$ $n - f = 96$ $\frac{2}{96} = \frac{1}{48}$ $f = -100$ $n = -4$

gluPerspective(90,1,4,100);

where 90 is the vertical field of view angle, in degrees, in the y direction, 1 is the aspect ratio which means that the horizontal field of view is also 90 degrees, 4 is the distance from the viewer to the near clipping plane, and 100 is the distance from the viewer to the far clipping plane. The camera (i.e., viewer) is at (0,0,0) looking down the negative z-axis and its up vector is the positive y-axis.

$r = 4$, $l = -4$, $t = 4$, $b = -4$

$e = 0,0,0$
$w = 0,0,1$
$u = t \times w$
$u = 1,0,0$
$v = w \times u$
$v = 0,1,0$

$\frac{7}{4}, \frac{5}{4}$

A) (7,5,-4)
B) (1.75,1.25,-4) ✓
C) (35,25,-4)
D) (3.5,2.5,-4)
E) none of the above

$\frac{+104}{96}$ $2080 - 400$ $2080 + 1680$

23 Consider the 2D ray equation $r(t) = (2,3) + (4,-1)t$. At which of the following points, the ray will intersect the line $x = 14$?

A) (14,-3)
B) (14,-2)
C) (14,-1)
D) (14,0)
E) (14,1)

$2 + 4t = 14$
$t = 3$

24 Assume that the screen coordinates of the corners of a triangle are at $v_0 = (10, 20)$, $v_1 = (30, 10)$, and $v_2 = (20, 30)$. The colors at these vertices are $c_0 = (255, 0, 0)$, $c_1 = (0, 255, 0)$, and $c_2 = (0, 0, 255)$, respectively. What will be the color value of the fragment at coordinate (18,22) assuming that barycentric interpolation is used during rasterization (you can round to the nearest integer)?

A) (100,49,100)
B) (101,50,101)
C) (102,51,102) ✓ $204 + 51 = 255$
D) (103,52,103)
E) (104,53,104)

$60 - 36 - 22 + 10$

25 Which one of the following is the correct composite transformation matrix to rotate an object 70 degrees counter-clockwise along the axis from (5,0,3) to (5,5,3).

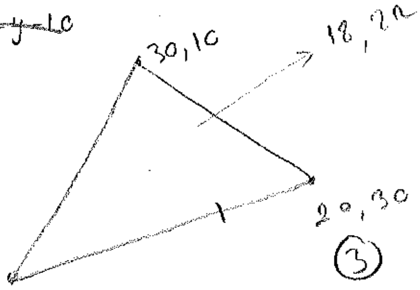
on y axis

A) $T(5,0,3)R_x(70)T(-5,0,-3)$
B) $T(5,0,3)R_y(70)T(-5,0,-3)$
C) $T(-5,0,-3)R_x(70)T(5,0,3)$
D) $T(-5,0,-3)R_y(70)T(5,0,3)$
E) none of the above

$60 - 2x - y + 10 = 0$

$\frac{x_2 - x_1}{y_2 - y_1} = \frac{x_2 - x}{y_2 - y}$

The Multiple-choice questions END here.



$\alpha = f_{31}(18,22) / f_{31}(10,20)$

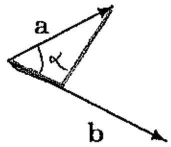
$\beta =$

$\gamma =$

$f_{31} = \frac{10}{-20} = \boxed{\frac{30 - x}{10 - y} = \frac{-1}{2}}$ $f_{12} = \frac{-20}{10} = \boxed{\frac{10 - x}{20 - y} = -2}$ $f_{23} = \frac{10}{10} = \boxed{\frac{20 - x}{30 - y} = 1}$

Classical questions BEGIN here. You must show your work with a clear writing. You can use the back of the page if needed.

26 Given two arbitrary 3D vectors a and b as shown in the figure,



$\frac{a}{|a|} \bullet \frac{b}{|b|} \bullet a$

- (8 pts) Find the vector which is the projection of **a** onto **b**.

cos α   lenght of projection → that lenght times unit vec on b's direction

$$\mathbf{a}' = \frac{a}{\|a\|} \bullet \frac{b}{\|b\|} \cdot \|a\| \cdot \frac{b}{\|b\|} = \boxed{\left(\frac{a \bullet b}{\|b\|^2}\right) \cdot b}$$

where $\|v\| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

- (2 pts) Express your solution as a matrix multiplication of the form:

$$\begin{bmatrix} a'_x \\ a'_y \\ a'_z \end{bmatrix} = \overset{3\times3}{M} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

10

$$M = \begin{bmatrix} \frac{b_x}{\|b\|^2} \cdot b_x & \frac{b_y}{\|b\|^2} \cdot b_x & \frac{b_z}{\|b\|^2} \cdot b_x \\ \frac{b_x}{\|b\|^2} \cdot b_y & \frac{b_y}{\|b\|^2} \cdot b_y & \frac{b_z}{\|b\|^2} \cdot b_y \\ \frac{b_x}{\|b\|^2} \cdot b_z & \frac{b_y}{\|b\|^2} \cdot b_z & \frac{b_z}{\|b\|^2} \cdot b_z \end{bmatrix}$$

$b \cdot \frac{a_x b_x + a_y b_y + a_z b_z}{b_x^2 + b_y^2 + b_z^2}$

5,6

27 Consider a scene viewed with an orthographic camera with the following transformation matrix.

0,56
−0,1

$$C \begin{bmatrix} 10 & 0 & 0 & 40 \\ 0 & 10 & 0 & 40 \\ 0 & 0 & -0.1 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1,6 \\ -5,6 \\ 1 \end{bmatrix} = \begin{bmatrix} 40 \\ 24 \\ 0,46 \\ 1 \end{bmatrix}$$

This matrix is used to transform a vertex given in world coordinates all the way to the viewport coordinates. (Note that no perspective division is necessary due to the orthographic view.) Imagine that we have a directional light source in the scene and if we were to view the scene from the viewpoint of the light source, we would have to use the following transformation matrix (again from world to viewport coordinates as in the camera view. Furthermore, in both the camera and the light view, the depth values within the viewing volume are transformed to values between 0.0 and 1.0 using these matrices):

−16+40
56−10
−16+40
= 24
−10+56

No need to multiply with bias

$$L \begin{bmatrix} 0 & 0 & -10 & -10 \\ 0 & 10 & 0 & 40 \\ -0.1 & 0 & 0 & 0.4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1,6 \\ -5,6 \\ 1 \end{bmatrix} = \begin{bmatrix} 46 \\ 24 \\ 0,4 \\ 1 \end{bmatrix}$$

If we are given the following part of the depth buffer, which is generated when the scene is from the light's viewpoint, determine whether the point $P = (0, -1.6, -5.6)$ given in world coordinates is in shadow or not. Show your calculations and reasoning. (The first row and the first column show the viewport coordinates of the pixels in the buffer.)

	40	41	42	43	44	45	46	47
27	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
26	1.0	1.0	1.0	1.0	0.4	0.37	0.37	0.37
25	1.0	1.0	1.0	1.0	0.4	0.37	0.37	0.37
24	0.46	0.46	0.46	0.46	0.4	0.37	0.37	0.37
23	0.46	0.46	0.46	0.46	0.4	0.37	0.37	0.37
22	1.0	1.0	1.0	1.0	0.4	0.4	0.4	0.4
21	1.0	1.0	1.0	1.0	0.4	0.4	0.4	0.4
20	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

for cam $= \begin{bmatrix} 40 \\ 24 \\ 0,46 \\ 1 \end{bmatrix} \rightarrow$ No use

for light $= \begin{bmatrix} 46 \\ 24 \\ 0,4 \\ 1 \end{bmatrix} \rightarrow$ It is in shadow since the z value from the matrix multiplication is 0,4, which is greater than 0,37. This means that another point with closer to the light source blocks that point