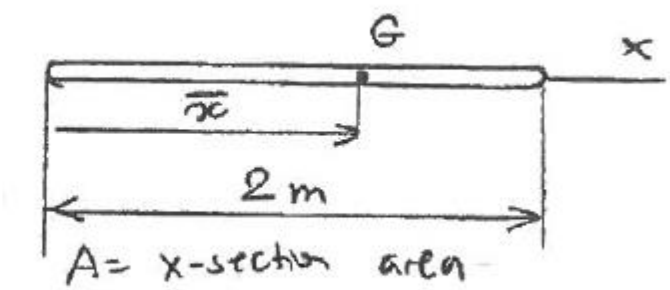
31/12/2020

ES 221 MECHANICS I (STATICS) RECITATION XI

Q1)

The density of the rod having a uniform cross-sectional area, A, can be expressed as $\rho = 2x^3 kg/m^3$. Determine the distance $\bar{x}$ to the rod's center of mass.



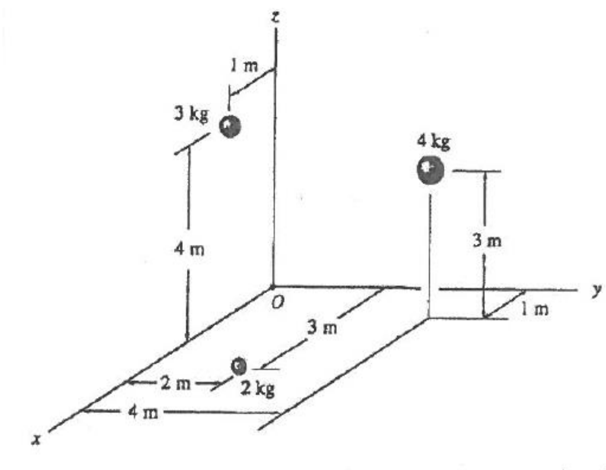
Solution to Q1

$dV = Adx$

$$\bar{x} = \frac{\int \tilde{x}\rho dV}{\int \rho dV} = \frac{\int_0^2 x(2x^3)Adx}{\int_0^2 (2x^3)Adx} = \frac{\int_0^2 x^4 dx}{\int_0^2 x^3 dx} = 1.6\ m$$

Q2)

Determine the location of the center of mass $(\bar{x}, \bar{y}, \bar{z})$ of the three particles.



Solution to Q2

$Let\ A, B, C\ be\ the\ coordinates\ of\ the\ 2, 3\ and\ 4\ kg\ mass, respectively$:

$A\ (3,2,0)$

$B\ (1,0,4)$

$C\ (1,4,3)$

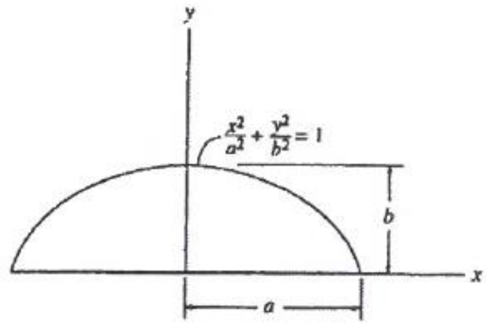
$$\bar{x} = \frac{2 \times 3 + 3 \times 1 + 4 \times 1}{2 + 3 + 4} = \frac{13}{9} \cong 1.44\ m$$

$$\bar{y} = \frac{2 \times 2 + 3 \times 0 + 4 \times 4}{9} = \frac{20}{9} \cong 2.22\ m$$

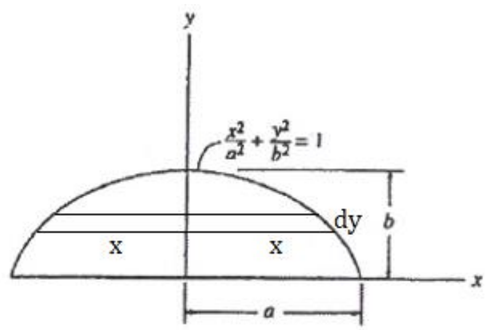
$$\bar{z} = \frac{2 \times 0 + 3 \times 4 + 4 \times 3}{9} = \frac{24}{9} \cong 2.67\ m$$

Q3)

Locate the centroid $(\bar{x}, \bar{y})$ of the semielliptical area.



Solution to Q3



$$\bar{y} = \frac{\int_0^b \tilde{y} dA}{\frac{\pi ab}{2}} = \frac{\int_0^b 2xydy}{\frac{\pi ab}{2}} = \frac{2\int_0^b y\sqrt{\left(a^2 - \frac{y^2a^2}{b^2}\right)}\, dy}{\frac{\pi ab}{2}};$$

$$Let\; u = a^2 - \frac{y^2a^2}{b^2};$$

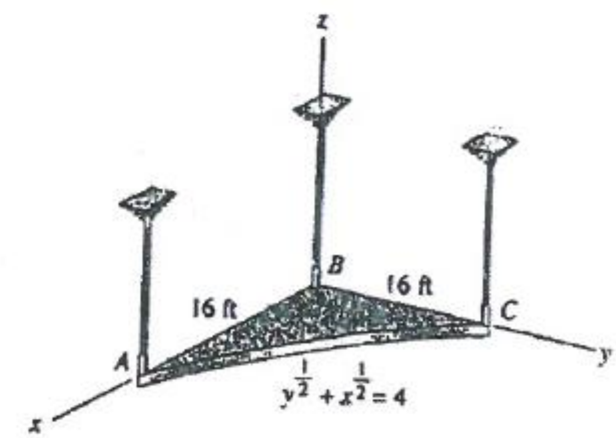
$$du = \frac{-2a^2}{b^2} ydy$$

$$\bar{y} = \frac{2\int_0^b \sqrt{u}\left(\frac{-b^2}{2a^2}\right) du}{\frac{\pi ab}{2}} = \frac{\frac{-b^2}{a^2}\int_0^b \sqrt{u}du}{\frac{\pi ab}{2}};$$

$$\bar{y} = \frac{\left[\frac{-b^2}{a^2} u^{3/2} \times \frac{2}{3}\right]_0^b}{\frac{\pi ab}{2}} = \frac{\left[\frac{-b^2}{a^2}\left(a^2 - \frac{y^2 a^2}{b^2}\right)^{3/2} \times \frac{2}{3}\right]_0^b}{\frac{\pi ab}{2}} = \frac{4b}{3\pi}$$

Q4)

The plate has a thickness of 0.25 ft and a specific weight of $\gamma = 180 lb/ft^3$. Determine the location of its center of gravity. Also, find the tension in each of the cords used to support it.



Solution to Q4

$$\bar{x} = \frac{\int \tilde{x} dA}{\int dA} = \frac{\int xydx}{\int ydx} = \frac{\int_0^{16} x\left(x - 8\sqrt{x} + 16\right)dx}{\int_0^{16}\left(x - 8\sqrt{x} + 16\right) dx} = 3.2\ ft$$

$$\bar{y} = \frac{\int \tilde{y} dA}{\int dA} = \frac{\int_0^{16} \frac{y}{2} ydx}{\int_0^{16} ydx} = \frac{\frac{1}{2}\int_0^{16} y^2 dx}{\int_0^{16} ydx} = 3.2\ ft$$

$$W = 180\ \frac{lb}{ft^3} \times 42.67 \times 0.25 = 1920\ lb;$$

$$\circlearrowleft \sum M_x = \sum M_y = 0;$$

$$T_C = T_A = 384\ lb$$

$$T_B = 1152\ lb$$