

Q1

Let's name the angles

$$\angle BAC = \alpha$$

$$\angle BCA = \beta$$

$$\angle DCE = \gamma$$

$$\angle DEC = \phi$$

$$\angle GEH = \delta$$

$$\angle GAE = \theta$$

length of BI, DJ, GK calculate

Consider the similar triangles ADJ and AEF

$$\frac{BI}{EF} = \frac{AI}{AE}$$

$$\frac{BI}{6m} = \frac{4m}{12m}$$

$$BI = 2m$$

Length of GK, similar triangles HEG and HEF

$$\frac{GK}{EF} = \frac{HK}{HE} \quad \frac{GK}{6m} = \frac{3m}{6m} \quad GK = 3m$$

similar triangles ABI and AEF

length of DJ

$$\frac{DJ}{EF} = \frac{AI}{AE}$$

$$\frac{DJ}{6m} = \frac{8}{12m}$$

$$DJ = 4m$$

required angles

$$\tan \gamma = \frac{DJ}{CI}$$

$$\gamma = \tan^{-1} \left( \frac{DJ}{AI - AC} \right) = \tan^{-1} \left( \frac{4}{8-6} \right) = 63.43^\circ$$

In triangle ABI,

$$\tan \alpha = \frac{BI}{AI}$$

$$\alpha = \tan^{-1} \left( \frac{2m}{4m} \right) = 26.57^\circ$$

In the triangle CBI

$$\tan \beta = \frac{BI}{CI}$$

$$\beta = \tan^{-1} \left( \frac{BI}{AC - AI} \right) = \tan^{-1} \left( \frac{2}{6-4} \right) = 45^\circ$$

HGK

$$\tan \theta = \frac{GK}{HK}$$

$$\theta = \tan^{-1} \left( \frac{3}{3} \right) = 45^\circ$$

EDJ

$$\tan \phi = \frac{DJ}{EJ}$$

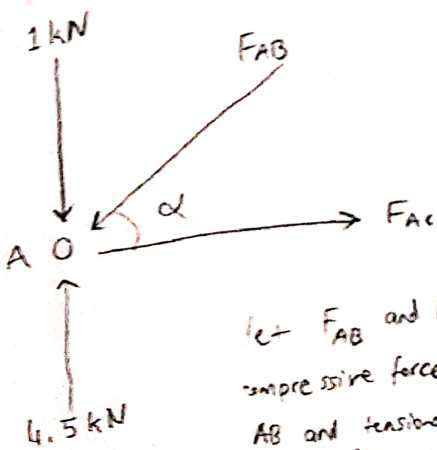
$$\phi = \tan^{-1} \left( \frac{4}{4} \right) = 45^\circ$$

Take the moments about the point A.

$$\sum M_A = 0$$

$$\begin{aligned} &-(2kN)(4m) - (2kN)(8m) \\ &-(1.75kN)(12m) - (1.5kN)(15m) \\ &-(0.75kN)(18m) + (H_y)(18m) = 0 \end{aligned}$$

Free body diagram of joint A,



Let  $F_{AB}$  and  $F_{AC}$  be the compressive force in the member AB and tensile force in the member AC respectively

$$H_y = \frac{1}{18} (8+16+21+22.5+13.5)$$

$$H_y = 4.5kN$$

Apply force equilibrium equation along the y direction

$$\sum F_y = 0$$

$$+A_y + H_y - (1kN) - (2kN) - (2kN) - (1.75kN) - (1.5kN) - (0.75kN) = 0$$

$$A_y + H_y = 9kN$$

$$A_y = (9kN) - H_y$$

Substitute 4.5 kN for  $H_y$

$$A_y = (9kN) - (4.5kN) = 4.5kN$$

Apply force equilibrium equation along vertical direction.

$$4.5 \text{ kN} = (1 \text{ kN}) + F_{AB} \sin \alpha$$

$$F_{AB} = \frac{3.5 \text{ kN}}{\sin \alpha}$$

Substitute  $26.57^\circ$  for  $\alpha$

$$F_{AB} = \frac{3.5 \text{ kN}}{\sin 26.57^\circ} = 7.83 \text{ kN}$$

Hence, the force in the member AB is  $7.83 \text{ kN (compressive)}$

apply force equilibrium equation along horizontal direction.

$$F_{AC} = F_{AB} \cos \alpha$$

Substitute  $7.83 \text{ kN}$  for  $F_{AB}$  and  $26.57^\circ$

$$\text{for } \alpha \quad F_{AC} = (7.83 \text{ kN}) \cos 26.57^\circ = 7 \text{ kN}$$

hence, the force in the member AC is  $7 \text{ kN (tensile)}$

Add equations (1) and (2)

$$0.447 F_{BD} - 0.707 F_{BC} + 0.894 F_{BD} + 0.707 F_{BC} = 1.5 + 7$$

$$= 1.5 + 7$$

$$1.341 F_{BD} = 8.5$$

$$F_{BD} = 6.34 \text{ kN}$$

Hence, the force in the member BD is

$$\underline{6.34 \text{ kN (compressive)}}$$

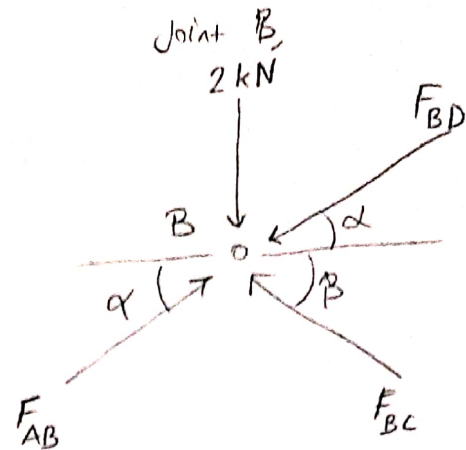
Substitute  $6.34 \text{ kN}$  for  $F_{BD}$  in the equation (2),

$$0.894 F_{BD} + 0.707 F_{BC} = 7$$

$$0.707 F_{BC} = 7 - 0.894(6.34)$$

$$\underline{F_{BC} = 1.89 \text{ kN}}$$

Free body diagram of the



Here,  $F_{BD}$  and  $F_{BC}$  are the compressive forces in the members BD and BC respectively.

Balance the forces in the vertical direction,

$$F_{AB} \sin \alpha + F_{BC} \sin \beta = F_{BD} \sin \alpha + (2 \text{ kN})$$

Substitute  $26.57^\circ$  for  $\alpha$ ,  $45^\circ$  for  $\beta$

and  $7.83 \text{ kN}$  for  $F_{AB}$

$$(7.83 \text{ kN}) \sin 26.57^\circ + F_{BC} \sin 45^\circ =$$

$$F_{BD} \sin 26.57^\circ + (2 \text{ kN})$$

$$3.5 + 0.707 F_{BC} = 0.447 F_{BD} + 2$$

$$0.447 F_{BD} - 0.707 F_{BC} = 1.5 \dots (1)$$

Balance the forces in the horizontal direction,

$$F_{BD} \cos \alpha + F_{BC} \cos \beta = F_{AB} \cos \alpha$$

Substitute  $26.57^\circ$  for  $\alpha$ ,  $45^\circ$  for  $\beta$ ,

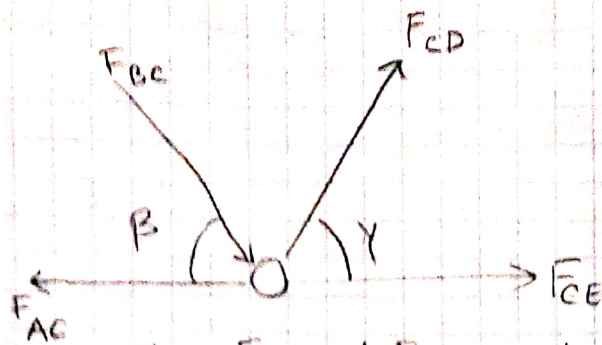
and  $7.83 \text{ kN}$  for  $F_{AB}$

$$F_{BD} \cos 26.57^\circ + F_{BC} \cos 45^\circ = (7.83 \text{ kN}) \cos 26.57^\circ$$

$$0.894 F_{BD} + 0.707 F_{BC} = 7 \dots (2)$$



Free body diagram of the joint C,



Here,  $F_{CD}$  and  $F_{CE}$  are the tensile forces in the members CD and CE respectively.

Balance the forces in the vertical direction,

$$F_{CD} \sin \gamma = F_{BC} \sin \beta$$

$$F_{CD} = \frac{F_{BC} \sin \beta}{\sin \gamma}$$

Substitute  $63.43^\circ$  for  $\gamma$ ,  $45^\circ$  for  $\beta$  and  $1.89 \text{ kN}$  for  $F_{BC}$

$$F_{CD} = \frac{(1.89 \text{ kN}) \sin 45^\circ}{\sin 63.43^\circ} = 1.49 \text{ kN}$$

Hence, the force in the member CD is  $\boxed{1.49 \text{ kN (tensile)}}$

Balance the forces in the horizontal direction,

$$F_{CE} + F_{CD} \cos \gamma + F_{BC} \cos \beta = F_{AC}$$

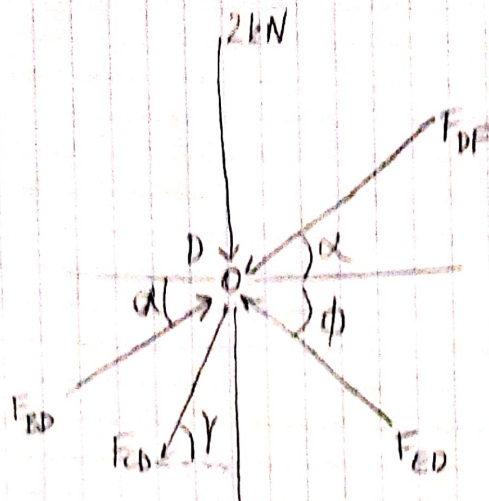
$$F_{CE} = F_{AC} - F_{CD} \cos \gamma - F_{BC} \cos \beta$$

Substitute  $63.43^\circ$  for  $\gamma$ ,  $45^\circ$  for  $\beta$ ,  $7 \text{ kN}$  for  $F_{AC}$ ,  $1.49 \text{ kN}$  for  $F_{CD}$ , and  $1.89 \text{ kN}$  for  $F_{BC}$

$$F_{CE} = (7 \text{ kN}) - (1.49 \text{ kN}) \cos 63.43^\circ - (1.89 \text{ kN}) \cos 45^\circ$$
$$= \boxed{5 \text{ kN (tensile)}}$$



Free body diagram of the Joint D.



$$F_{BD} \sin \alpha + F_{ED} \sin \phi = F_{DF} \sin \alpha + (2 \text{ kN}) + F_{CD} \sin \gamma$$

$$(6.34 \text{ kN}) \sin 26.57^\circ + F_{ED} \sin 45^\circ = F_{DF} \sin 26.57^\circ + (2 \text{ kN}) + (1.49 \text{ kN}) \sin 63.43^\circ$$

$$0.707 F_{ED} - 0.447 F_{DF} = 0.497 \dots (3)$$

$$F_{DF} \cos \alpha + F_{ED} \cos \phi + F_{CD} \cos \gamma = F_{BD} \cos \alpha$$

$$F_{DF} \cos 26.57^\circ + F_{ED} \cos 45^\circ + (1.49 \text{ kN}) \cos 63.43^\circ = (6.34 \text{ kN}) \cos 26.57^\circ$$

$$0.894 F_{DF} + 0.707 F_{ED} = 5 \dots (4)$$

Subtract equation (3) and (4)

$$0.894 F_{DF} + 0.707 F_{ED} - 0.707 F_{ED} + 0.447 F_{DF} = 5 - 0.497$$

$$1.341 F_{DF} = 4.503$$

$$\boxed{F_{DF} = 3.36 \text{ kN}}$$

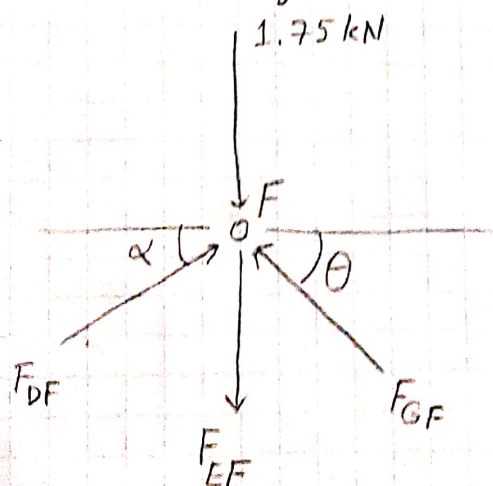
Substitute 3.36 kN for  $F_{DF}$  in the equation (3).

$$0.707 F_{ED} - 0.447 F_{DF} = 0.497$$

$$0.707 F_{ED} - 0.447 (3.36 \text{ kN}) = 0.497$$

$$\boxed{F_{ED} = 2.83 \text{ kN}}$$

Free body diagram of the Joint F



$$F_{GF} \cos \theta = F_{DF} \cos \alpha$$

$$F_{GF} = \frac{F_{DF} \cos \alpha}{\cos \theta}$$

$$F_{GF} = \frac{(3.36 \text{ kN}) \cos 26.57^\circ}{\cos 45^\circ}$$

$$\boxed{= 4.25 \text{ kN}}$$

(4)



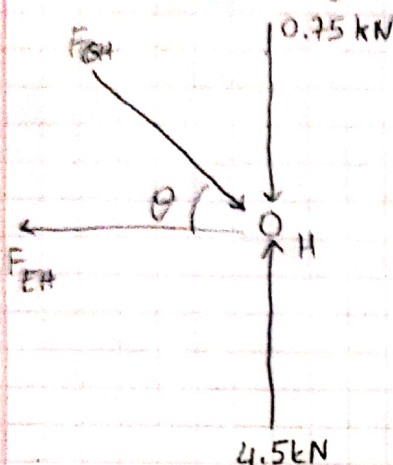
Balance the forces in the vertical direction.

$$F_{EF} + (1.75 \text{ kN}) = F_{DF} \sin \alpha + F_{GF} \sin \theta$$

$$F_{EF} + (1.75 \text{ kN}) = (3.36 \text{ kN}) \sin 26.57^\circ + (4.25 \text{ kN}) \sin 45^\circ$$

$$F_{EF} = 2.75 \text{ kN}$$

Free body diagram of H



Balance vertical forces

$$(0.75 \text{ kN}) + F_{GH} \sin \theta = (4.5) \text{ kN}$$

$$F_{GH} = \frac{3.75}{\sin 45^\circ}$$

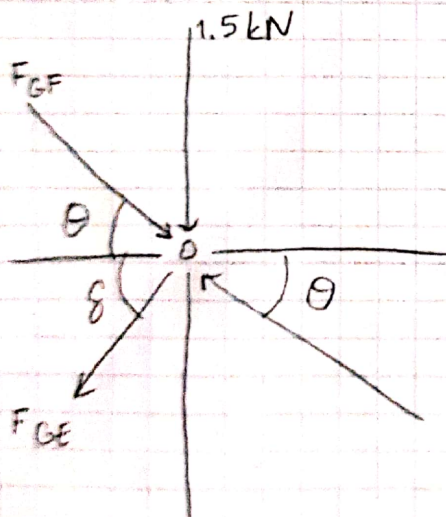
$$= 5.30 \text{ kN}$$

Balance the horizontal forces

$$F_{EH} = F_{GH} \cos \theta$$

$$F_{EH} = 5.30 \text{ kN} \cos 45^\circ = 3.75 \text{ kN}$$

Free body diagram of G



balance horizontal forces

$$F_{GE} \cos \delta + F_{GH} \cos \theta = F_{GF} \cos \theta$$

$$F_{GE} = \frac{(F_{GH} - F_{GF}) \cos \theta}{\cos \delta}$$

$$= \frac{(5.3 - 4.25) \cos 45^\circ}{\cos 45^\circ}$$

$$= 1.05 \text{ kN}$$



Q2 Apply moment equilibrium about joint A to find the reaction force at support F.

$$\sum M_A = 0$$

$$40(1.5) + 30(1.5 + 1.5) + 40(2) - F_y(2+2) = 0$$

$$F_y = 57.5 \text{ kN}$$

Apply force equilibrium along vertical direction

$$\sum F_y = 0$$

$$A_y + F_y - 40 - 30 = 0$$

$$A_y + 57.5 - 40 - 30 = 0$$

$$A_y = 32.5 \text{ kN}$$

$$\sum F_x = 0$$

$$A_x - 40 - 30 = 0$$

$$A_x = 70 \text{ kN}$$

Apply moment equilibrium about joint E.

$$\sum M_E = 0$$

$$-57.5(2) + F_{GH}(1.5) = 0$$

$$F_{GH} = 76.7 \text{ kN (T)}$$

for joint H.

$$\sum M_H = 0$$

$$-57.5(2+2) + 40(2) - F_{ED}(1.5) = 0$$

$$F_{ED} = -100 \text{ kN (C)}$$

$$\sum F_y = 0$$

$$57.5 - F_{EH} \left( \frac{3}{\sqrt{3^2 + 4^2}} \right) - 40 = 0$$

$$F_{EH} = 29.2 \text{ kN (T)}$$



(93)

In the first figure (left one)

A B is zero force member

There is no zero force member for the second figure