

Q1 Single equivalent couple acting on the system,

$$M = M_1 + M_2 \quad (1)$$

Position vector of ED, $ED = (192 \text{ mm})\mathbf{k}$

$$\text{Force vector } F_1 = -(12.5 \text{ N})\mathbf{i}$$

Couple made by the force F_1 ,

$$\begin{aligned} M_1 &= ED \times F_1 \\ &= (192 \text{ mm})\mathbf{k} \times [-(12.5)\mathbf{i}] \\ &= -(2400 \text{ N}\cdot\text{mm})\mathbf{j} \end{aligned}$$

Position vector of GF, $GF = -(160 \text{ mm})\mathbf{i}$

Position vector of BF,

$$\begin{aligned} BF &= -(160 \text{ mm})\mathbf{i} - \left(\frac{144}{2} \text{ mm}\right)\mathbf{j} + \left(\frac{192}{2} \text{ mm}\right)\mathbf{k} \\ &= -(160 \text{ mm})\mathbf{i} - (72 \text{ mm})\mathbf{j} + (96 \text{ mm})\mathbf{k} \end{aligned}$$

Magnitude of BF,

$$\begin{aligned} BF &= \sqrt{160^2 + 72^2 + 96^2} \\ &= 200 \text{ mm} \end{aligned}$$

Force vector of BF,

$$\begin{aligned} F_2 &= F_2 \left(\frac{BF}{BF} \right) = (50 \text{ N}) \times \left(\frac{-(160 \text{ mm})\mathbf{i} - (72 \text{ mm})\mathbf{j} + (96 \text{ mm})\mathbf{k}}{200 \text{ mm}} \right) \\ &= (-40 \text{ N})\mathbf{i} - (18 \text{ N})\mathbf{j} + (24 \text{ N})\mathbf{k} \end{aligned}$$

Couple made by the force F_2

$$\begin{aligned} M_2 &= GF \times F_2 \\ &= -(160 \text{ mm})\mathbf{i} \times [(-40 \text{ N})\mathbf{i} - (18 \text{ N})\mathbf{j} + (24 \text{ N})\mathbf{k}] \\ &= (2880 \text{ N}\cdot\text{mm})\mathbf{k} + (3840 \text{ N}\cdot\text{mm})\mathbf{j} \end{aligned}$$

From equation (1), we get equivalent couple,

$$\begin{aligned} M &= M_1 + M_2 \\ &= -(2400 \text{ N}\cdot\text{mm})\mathbf{j} + (2880 \text{ N}\cdot\text{mm})\mathbf{k} + (3840 \text{ N}\cdot\text{mm})\mathbf{j} \\ &= (1440 \text{ N}\cdot\text{mm})\mathbf{j} + (2880 \text{ N}\cdot\text{mm})\mathbf{k} \end{aligned}$$

Magnitude of the equivalent couple,

$$\begin{aligned} M &= \sqrt{(1440)^2 + (2880)^2} \\ &= 3219.937 \text{ N}\cdot\text{mm} \\ &= \boxed{3.219 \text{ N}\cdot\text{m}} \end{aligned}$$

From the equivalent couple, we get Moment in x-direction,

$$M_x = 0 \text{ N}\cdot\text{m}$$

Moment in y-direction

$$M_y = 1440 \text{ N}\cdot\text{m} \\ = 1.44 \text{ N}\cdot\text{m}$$

Moment in z-direction

$$M_z = 2880 \text{ N}\cdot\text{mm} \\ = 2.88 \text{ N}\cdot\text{m}$$

The angle of equivalent moment with respect to x-axis,

$$\theta_x = \cos^{-1}\left(\frac{M_x}{M}\right) \\ = \cos^{-1}\left(\frac{0}{3.219}\right) = \underline{\underline{90^\circ}}$$

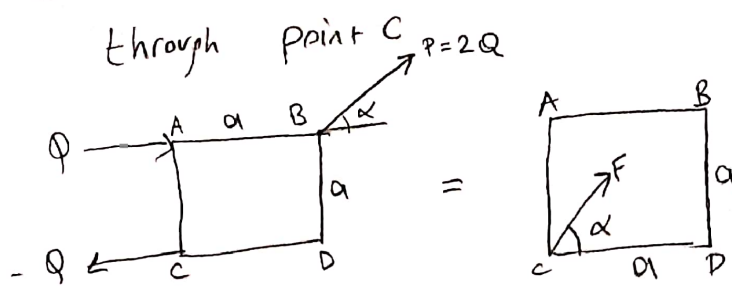
The angle of equivalent moment with respect to y-axis,

$$\theta_y = \cos^{-1}\left(\frac{M_y}{M}\right) \\ = \cos^{-1}\left(\frac{1.44}{3.219}\right) = \underline{\underline{63.426^\circ}}$$

The equivalent moment with respect to z-axis,

$$\theta_z = \cos^{-1}\left(\frac{M_z}{M}\right) \\ = \cos^{-1}\left(\frac{2.88}{3.219}\right) = \underline{\underline{26.531^\circ}}$$

Q2 If the line of action of the single equivalent force is to pass



Let F be the single equivalent force which replaces the given force and couple.
For similar loadings, the single equivalent force is $F = \sum \vec{F}$
 $= Q - Q + P = 2Q(\cos\alpha i + \sin\alpha j)$

For similar loadings, sum of moments about C should be the same in both the cases.

$$\sum M_C = M_C$$

$$-(Q)(a) - (P\cos\alpha)(a) + (P\sin\alpha)(a) = 0$$

$$-(Q)(a) - (P\cos\alpha)(a) + (P\sin\alpha)(a) = 0$$

$$2Q(\sin\alpha - \cos\alpha) = Q$$

$$\sin\alpha - \cos\alpha = 0.5$$

$$\sin\alpha = 0.5 + \cos\alpha$$

Square on both sides of the above equation:

$$\sin^2\alpha = (0.5 + \cos\alpha)^2$$

$$\sin^2\alpha = 0.25 + \cos^2\alpha + 2(0.5)\cos\alpha$$

$$\cos^2\alpha + \cos\alpha + 0.25 = 1 - \cos^2\alpha$$

$$2\cos^2\alpha + \cos\alpha - 0.75 = 0$$

$$\cos\alpha = \frac{-1 \pm \sqrt{1^2 - 4(2)(-0.75)}}{2(2)}$$

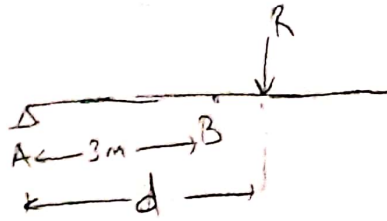
$$= \frac{-1 \pm 2.646}{4}$$

$$\cos\alpha = 0.4115$$

$$\alpha = \cos^{-1}(0.4115)$$

$$= 65.7^\circ$$

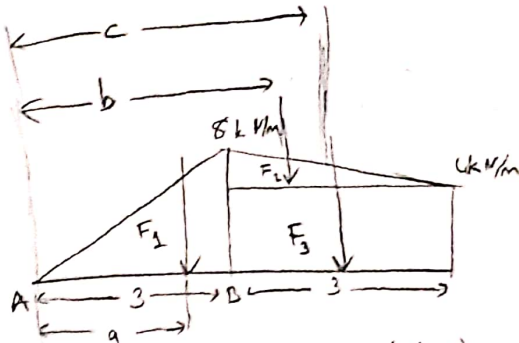
Q3



$$R = \frac{1}{2} \times 3 \times 8 + \frac{1}{2} \times 3 \times (8-4) + 4 \times 3$$

$$R = 12 + 6 + 12 = 28 \text{ kN}$$

resultant



$$a = 3 - \left(\frac{1}{3} \cdot 3\right) \quad b = 3 + \left(\frac{1}{3} \cdot 3\right)$$

$a = 2$ $b = 4$

$$c = 3 + \left(\frac{1}{2} \cdot 3\right)$$

$$c = \frac{9}{2} = 4.5$$

$$+ \circlearrowleft M_A = F_1 \cdot a + F_2 \cdot b + F_3 \cdot c$$

$$= 12 \times 2 + 6 \times 4 + 12 \times 4.5$$

$$= 102 \text{ kN/m}^2$$

$$M_A = R \times d$$

$$102 = 28 \times d \quad d = 3.64 \text{ m}$$