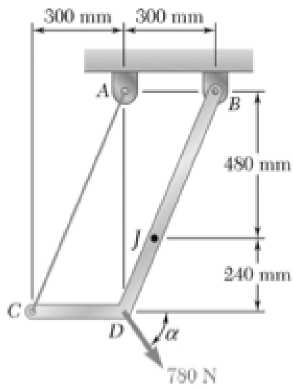
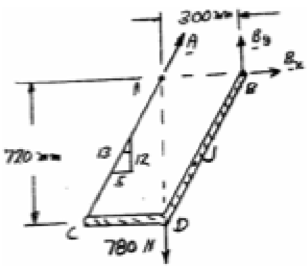
PROBLEM 7.3



Determine the internal forces at Point J when $\alpha = 90°$.

SOLUTION

<u>Reactions</u> $(\alpha = 90°)$



$$\Sigma M_A = 0: \quad \mathbf{B}_y = 0$$

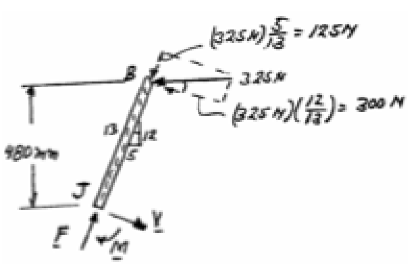
$$+\uparrow \Sigma F_y = 0: \quad A\left(\frac{12}{13}\right) - 780 \text{ N} = 0$$

$$A = 845 \text{ N} \qquad \mathbf{A} = 845 \text{ N} \nearrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad (845 \text{ N})\frac{5}{13} + B_x = 0$$

$$B_x = -325 \text{ N} \qquad \mathbf{B}_x = 325 \text{ N} \longleftarrow$$

FBD BJ:

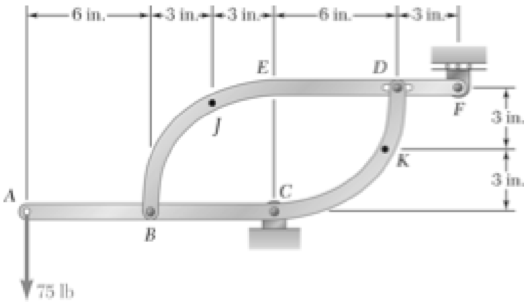


$$+\nearrow \Sigma F = 0: \quad 125 \text{ N} - F = 0 \qquad \mathbf{F} = 125.0 \text{ N} \measuredangle 67.4° \blacktriangleleft$$

$$+\searrow \Sigma F = 0: \quad V - 300 \text{ N} = 0 \qquad \mathbf{V} = 300 \text{ N} \nwarrow 22.6° \blacktriangleleft$$

$$+\circlearrowright \Sigma M = 0: \quad (325 \text{ N})(0.480 \text{ m}) - M = 0$$

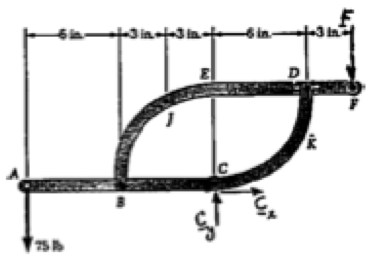
$$M = +156 \text{ N}\cdot\text{m} \qquad \mathbf{M} = 156.0 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



PROBLEM 7.8

Two members, each consisting of a straight and a quarter-circular portion of rod, are connected as shown and support a 75-lb load at A. Determine the internal forces at Point K.

SOLUTION



Free body: Entire frame

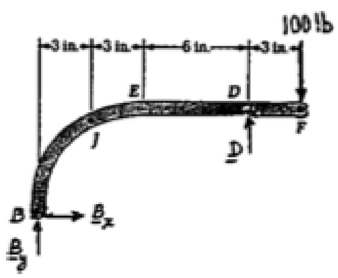
$$+\circlearrowleft \Sigma M_C = 0: \quad (75\text{ lb})(12\text{ in.}) - F(9\text{ in.}) = 0$$

$$\mathbf{F} = 100\text{ lb}\downarrow \quad \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 75\text{ lb} - 100\text{ lb} = 0$$

$$C_y = +175\text{ lb} \qquad \mathbf{C} = 175\text{ lb}\uparrow \quad \triangleleft$$



Free body: Member *BEDF*

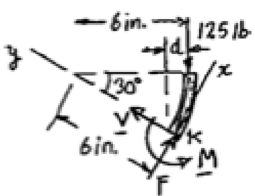
$$+\circlearrowleft \Sigma M_B = 0: \quad D(12\text{ in.}) - (100\text{ lb})(15\text{ in.}) = 0$$

$$\mathbf{D} = 125\text{ lb}\uparrow \quad \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 125\text{ lb} - 100\text{ lb} = 0$$

$$B_y = -25\text{ lb} \qquad \mathbf{B} = 25\text{ lb}\downarrow \quad \triangleleft$$



$d = 6\text{ in.}(1-\cos 30^\circ)$
$d = 0.8038\text{ in.}$

Free body: *DK*

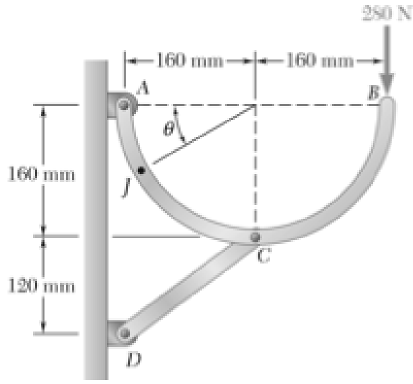
We found in Problem 7.11 that

$$\mathbf{D} = 125\text{ lb}\uparrow \text{ on } BEDF.$$

Thus $\qquad \mathbf{D} = 125\text{ lb}\downarrow \text{ on } DK. \quad \triangleleft$

$$+\nearrow \Sigma F_x = 0: \quad F - (125\text{ lb})\cos 30^\circ = 0$$

$$\mathbf{F} = 108.3\text{ lb} \measuredangle 60.0^\circ \quad \blacktriangleleft$$

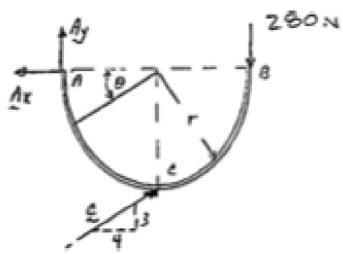


PROBLEM 7.11

A semicircular rod is loaded as shown. Determine the internal forces at Point J knowing that $\theta = 30°$.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: \quad r\left(\frac{4}{5}C\right) + r\left(\frac{3}{5}C\right) - 2r(280\text{ N}) = 0$$

$$\mathbf{C} = 400\text{ N} \nearrow$$

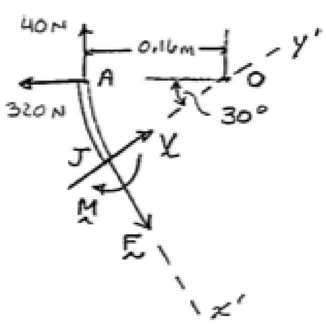
$$\xrightarrow{} \Sigma F_x = 0: \quad -A_x + \frac{4}{5}(400\text{ N}) = 0$$

$$\mathbf{A}_x = 320\text{ N} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y + \frac{3}{5}(400\text{ N}) - 280\text{ N} = 0$$

$$\mathbf{A}_y = 40.0\text{ N} \uparrow$$

FBD AJ:



$$\searrow \Sigma F_{x'} = 0: \quad F - (320\text{ N})\sin 30° - (40.0\text{ N})\cos 30° = 0$$

$$F = 194.641\text{ N}$$

$$\mathbf{F} = 194.6\text{ N} \nwarrow 60.0° \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: \quad V - (320\text{ N})\cos 30° + (40\text{ N})\sin 30° = 0$$

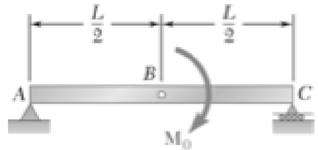
$$V = 257.13\text{ N}$$

$$\mathbf{V} = 257\text{ N} \angle 30.0° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_0 = 0: \quad (0.160\text{ m})(194.641\text{ N}) - (0.160\text{ m})(40.0\text{ N}) - M = 0$$

$$M = 24.743$$

$$\mathbf{M} = 24.7\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

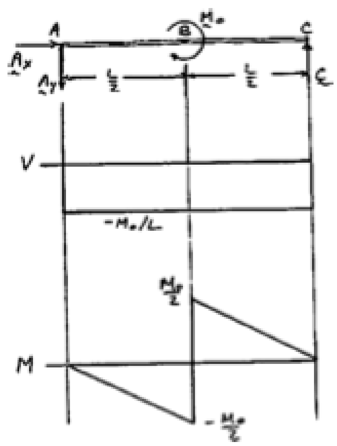


PROBLEM 7.33

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) **FBD Beam:**



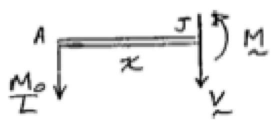
$$\circlearrowleft \Sigma M_C = 0: \quad LA_y - M_0 = 0$$

$$\mathbf{A}_y = \frac{M_0}{L} \downarrow$$

$$\uparrow \Sigma F_y = 0: \quad -A_y + C = 0$$

$$\mathbf{C} = \frac{M_0}{L} \uparrow$$

Along AB:



$$\uparrow \Sigma F_y = 0: \quad -\frac{M_0}{L} - V = 0$$

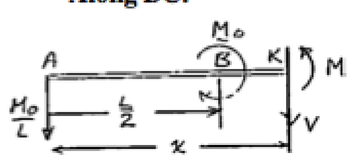
$$V = -\frac{M_0}{L}$$

$$\circlearrowleft \Sigma M_J = 0: \quad x\frac{M_0}{L} + M = 0$$

$$M = -\frac{M_0}{L}x$$

Straight with $\quad M = -\dfrac{M_0}{2}$ at B

Along BC:



$$\uparrow \Sigma F_y = 0: \quad -\frac{M_0}{L} - V = 0 \qquad V = -\frac{M_0}{L}$$

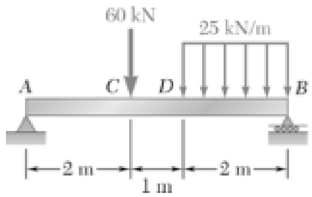
$$\circlearrowleft \Sigma M_K = 0: \quad M + x\frac{M_0}{L} - M_0 = 0 \qquad M = M_0\left(1 - \frac{x}{L}\right)$$

Straight with $\quad M = \dfrac{M_0}{2}$ at $B \qquad M = 0$ at C

(*b*) From diagrams:

$$|V|_{max} = M_0/L \blacktriangleleft$$

$$|M|_{max} = \frac{M_0}{2} \text{ at } B \blacktriangleleft$$

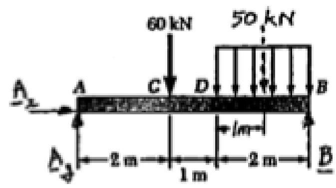


PROBLEM 7.39

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$+\circlearrowleft \Sigma M_A = 0: \quad B(5\text{ m}) - (60\text{ kN})(2\text{ m}) - (50\text{ kN})(4\text{ m}) = 0$

$B = +64.0\text{ kN}$ $\qquad \mathbf{B} = 64.0\text{ kN}\uparrow$ ◁

$\Sigma F_x = 0: \quad A_x = 0$

$+\uparrow \Sigma F_y = 0: \quad A_y + 64.0\text{ kN} - 6.0\text{ kN} - 50\text{ kN} = 0$

$A_y = +46.0\text{ kN}$ $\qquad \mathbf{A} = 46.0\text{ kN}\uparrow$ ◁

(*a*) Shear and bending-moment diagrams.

From *A* to *C*:

$+\uparrow \Sigma F_y = 0: \quad 46 - V = 0$ $\qquad V = +46\text{ kN}$ ◁

$+\circlearrowleft \Sigma M_y = 0: \quad M - 46x = 0$ $\qquad M = (46x)\text{kN}\cdot\text{m}$ ◁

From *C* to *D*:

$+\uparrow \Sigma F_y = 0: \quad 46 - 60 - V = 0$ $\qquad V = -14\text{ kN}$ ◁

$+\circlearrowleft \Sigma M_j = 0: \quad M - 46x + 60(x-2) = 0$

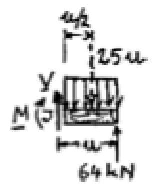
$M = (120 - 14x)\text{kN}\cdot\text{m}$

For $\quad x = 2\text{ m}: \quad M_C = +92.0\text{ kN}\cdot\text{m}$ ◁

For $\quad x = 3\text{ m}: \quad M_D = +78.0\text{ kN}\cdot\text{m}$ ◁

PROBLEM 7.39 (Continued)

From D to B:



$+\uparrow \Sigma F_y = 0$: $V + 64 - 25\mu = 0$

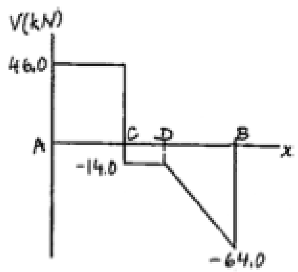
$$V = (25\mu - 64)\text{kN}$$

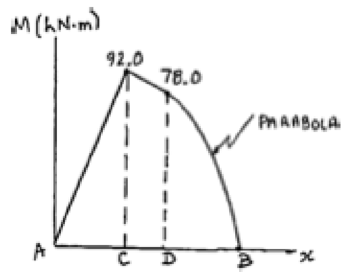
$+\circlearrowleft \Sigma M_j = 0$: $64\mu - (25\mu)\left(\frac{\mu}{2}\right) - M = 0$

$$M = (64\mu - 12.5\mu^2)\text{kN}\cdot\text{m}$$

For $\mu = 0$: $V_B = -64$ kN $\qquad M_B = 0$ ◁

(*b*) $|V|_{max} = 64.0$ kN ◀





$|M|_{max} = 92.0\ \text{kN}\cdot\text{m}$ ◀