CENG 384 - Signals and Systems for Computer Engineers
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Written Assignment 4

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1. (a) $y[n](-3/4)y[n-1] + (1/8)y[n-2] = 2x[n]$

   (b) $H(e^{jw}) = Y(e^{jw})/X(e^{jw})$ applying the fourier transform to both sides of difference equation by using the linearity and time-shifting properties, we obtain the expression
   $H(e^{jw}) = Y(e^{jw})/X(e^{jw}) = 2/(1-(3/4)e^{-jw} + (1/8)e^{-2jw})$
   $H(e^{jw}) = 2/((1-(1/2)e^{-jw})(1-(1/4)e^{-jw})) = A/(1-(1/2)e^{-jw}) + B/(1-(1/4)e^{-jw})$ $A + B = 2$
   $(-A/4) - (B/2) = 0$
   A = 4
   B = -2
   $H(e^{jw}) = 4/(1-(1/2)e^{-jw}) - 2/(1-(1/4)e^{-jw})$

   (c) The impulse response of this system can be found from FT table by using frequency response
   $h[n] = (4(1/2)^n - 2(1/4)^n)u[n]$

   (d) $x[n] = (1/4)^n u[n]$
   $X[e^{jw}] = 1/(1-(1/4)e^{-jw})$
   $Y(e^{jw}) = H(e^{jw})X(e^{jw}) = 2/((1-(1/2)e^{-jw})(1-(1/4)e^{-jw}))1/(1-(1/4)e-jw)$
   $= 2/((1-(1/2)e^{-jw})(1-(1/4)e^{-jw})^2)$
   $Y(e^{jw}) = A/(1-(1/4)e^{-jw}) + B/((1-(1/4)e^{-jw})^2) + C/(1-(1/2)e^{-jw})$
   from this equation we find
   A = -4
   B = -2
   C = 8
   $Y(e^{jw}) = -4/(1-(1/4)e^{-jw}) - 2/((1-(1/4)e^{-jw})^2) + 8/(1-(1/2)e^{-jw})$
   and from FT table
   $y[n] = (-4(1/4)^n - 2(n+1)(1/4)^n + 8(1/2)^n)u[n]$

2. $h_1[n] = (1/3)^n u[n]$
   $H_1[e^{jw}] = 1/(1-(1/3)e^{-jw}) = 3/(3-e^{-jw})$
   H_2 can be found from H-H_1
   $H_2(e^{jw}) = (-12+5e^{-jw})/(12-7e^{jw}+e^{-2jw}) + 3/(e^{-jw}-3)$
   we can rewrite $(12-7e^{jw}+e^{-2jw})$ as $(e^{-jw}-4)(e^{-jw}-3)$
   and when we multiply 3 with $e^{-jw}-4$ we can make the summation

   $H_2(e^{jw}) = (-24+8e^{-jw})/(e^{-jw}-4)(e^{-jw}-3)$
   $= A/(e^{-jw}-4) + B/(e^{-jw}-3)$
   $A + B = 8$
   $-4B - 3A = -24$
   B = 0
   A = 8
   $H_2(e^{jw}) = 8/(e^{-jw}-4) = 2*(1/1-(1/4)e^{-jw})$
   and from FT table we get
   $h_2[n] = -2(1/4)^n u[n]$

3. (a) $f_1 = 2\pi/2\pi$ Hz
   $f_2 = 3\pi/2\pi$ Hz
   $f_1 = n_1 * f_0$

$f_2 = n_2 * f_0$
$f_1/n1 = f_0 = f_2/n_2$
$f_1/f_2 = n_1/n_2$
$1/(3/2) = 2\pi/3\pi$
$f_0 = 2/3$
$T_0 = 3/2$
$w = 2\pi/T_0$
$w = 4\pi/3$
$W > |w|$
From table 4.2 in textbook we know the fourier transform of x(t) is
$1 + \pi[\delta(w - 3\pi) - \delta(w + 3\pi)]$

(b) The Nyquist rate will be equal to twice the highest frequency in the signal so
$w_N = 2 * w_{max}$
$w_N = 2 * 3\pi$
$w_N = 6\pi$
and period for sampling is
$w_{max} = 2\pi/T$
$T = 2\pi/w_{max}$
$T = 2/3$

(c) $x_p(t) = \sum_{-\infty}^{\infty} x(kT)\delta(t - kT)$
From the multiplication property, we know that
$x_p(jw) = (1/2\pi)\int_{-\infty}^{\infty} X(j\theta)P(j(w - \theta))d\theta$
$P(jw) = (2\pi/T)\sum_{-\infty}^{\infty} \delta(w - kw_0)$
$X_p(jw) = (1/T)\sum_{-\infty}^{\infty} X(j(w - kw_s))$
$w_s = 2\pi/(2/3)$
$= (2/3)\sum_{-\infty}^{\infty} X(j(4\pi/3 - k3\pi))$

4. (a) $T = \frac{2\pi}{\omega_s} = 2$

$X_p(j\omega) = \frac{1}{T}\sum_{\forall k} X(j(\omega - k\omega_s))$

$X_d(e^{j\omega}) = X_p(j\frac{\omega}{T})$

$X_d(e^{j\omega}) = \begin{cases} \frac{2}{\pi}\omega & if\ |x| \leq \frac{\pi}{2} \\ 0 & otherwise \end{cases}$, $X_d(e^{j\omega}) = X_d(e^{j(\omega+T)})$

(b) From discrete time Fourier transform table, we know that

$e^{j\omega_0 n} \longleftrightarrow 2\pi\sum_{\forall k}\delta(\omega - \omega_0 - 2\pi k)$

$h[n] = cos\pi n = \frac{1}{2}(e^{j\pi n} + e^{-j\pi n})$

Then, $H(e^{j\omega}) = \frac{1}{2}(2\pi\sum_{\forall k}\delta(\omega - \pi - 2\pi k) + 2\pi\sum_{\forall k}\delta(\omega + \pi - 2\pi k))$

$= \pi(\sum_{\forall k}\delta(\omega - \pi - 2\pi k) + \delta(\omega + \pi - 2\pi k))$

(c) $Y_d(e^{jw}) = X_d(e^{jw}) * H_d(e^{jw})$
$H_d(e^{jw}) = j(w/T), \quad w < \pi$