SHARIF UNIVERSITY OF TECHNOLOGY, ELECTRICAL ENGINEERING DEPARTMENT

Project 2: Model Estimation from Process Reaction Curves

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1 MODEL ESTIMATION

1.1 Compute a model for the system using each of the techniques listed. For each case you need to clearly state the following:

(a). For each case, you need to calculate the model using at least two different inputs on two different time intervals. Clearly discuss why you have chosen each of the two inputs. Pay attention to the issues you need to follow for each technique.

(b). If the model you obtain for each input is different, explain why the difference exists. Select what you believe to be the better model of the two and explain why you made the choice?

In the first part, we should pay attention to these tips:

- ☐ We compute the model for two step inputs with amplitude 0.5 and 1. But for not to repeat too much diagrams and equations, we only draw the response for step input with amplitude 1 (all of the diagrams belong to step with amplitude 1). Results for step input with amplitude 0.5 are available in Table 1.3. and Table 1.4.
- ☐ Results for step input with amplitude 1 are available in Table 1.1. and Table 1.5.
- ☐ Step input with amplitude 1, has 900 seconds for process and 100 seconds for initial delay. Also for step input with amplitude 0.5, it has 1000 seconds for process and 120 seconds for initial delay.
- ☐ All the processing are done with this student number: 94105569
- ☐ The original model output (versus time) is shown in Figure 1.1.
- ☐ The Random input is shown in Figure 1.2.
- ☐ Our step input has $30s$ delay by default.
- ☐ Our Random input has $100s$ delay by default.
- ☐ It is better to omit the noise from the input. We can use a low pass filter.
- ☐ Using last 200 data for calculating k in Area method.
- ☐ It's a better idea to use first half of data for Calculating IAE and ISE.
- ☐ For calculating $A0$ and $A1$ we used Tustin method.

- ☐ In all figures, blue diagrams are the system's response and red ones are model's response.
- ☐ The process output measurement is corrupted with a normally (Gaussian) distributed random noise.
- ☐ Results and measures of step with amplitude 1 is better than amplitude 0.5 because the signal to noise ratio (SNR) is larger.

Before describing the methods, noticing some points are important. First, the simulation time should be large enough so that the response of the system reaches its steady state. If the simulation time be small, the system's gain will not be estimated well. Second, by increasing the amplitude of the input signal, the signal to noise ratio is increased and thus a good system parameters will be estimated. But this is not allowed in and industrial system, because of the physical system does not allow us to increase the input signal too much. The other important point is that because of the fact that the noise is Gaussian and static, the integral over the entire time is zero; Therefore, for the integral-based approach, it is better to increase the simulation time.

Another important point is that the output signal is summed up with noise, and it should be omitted. To filter out the noise, we should use a low-pass filter with broadband. We can do this noise reduction using the command "filtfilt".

In method of moments, in order to calculate the moment, we need to integrate in a limited time. The input signal must be in the form of a pulse. Also as the integrals depend in the time t, we should not let the simulation takes a very long time, As well as the input. On the other hand, we can not decrease the time too much, because in small time scales, the effect of noise on the signal is significant.

The advantage of Area method is that it gives us the opportunity to calculate our needed parameters not only with some points' data, but using the area and integral for calculation, so our errors will be minimized.

The advantage of method of moments is that they decrease the usage of saving data. In the other words, in this method we need less data to calculate model's parameters. Because we only use the fact that differentiate of system and model responses have to be equal in each orders in $S = 0$.

The differences between the models comes from their accuracy in determining the parameters. We will discuss differences between the models in part 2 in detail. Some models have good curves in tending to system but they have not a good ISE and IAE. The models results will be more accurate, when order of the models increases. According to the results and curves in part one, we choose 4-Parameter 2nd order model using method of moments. And reasons and explains will be discussed in the following.

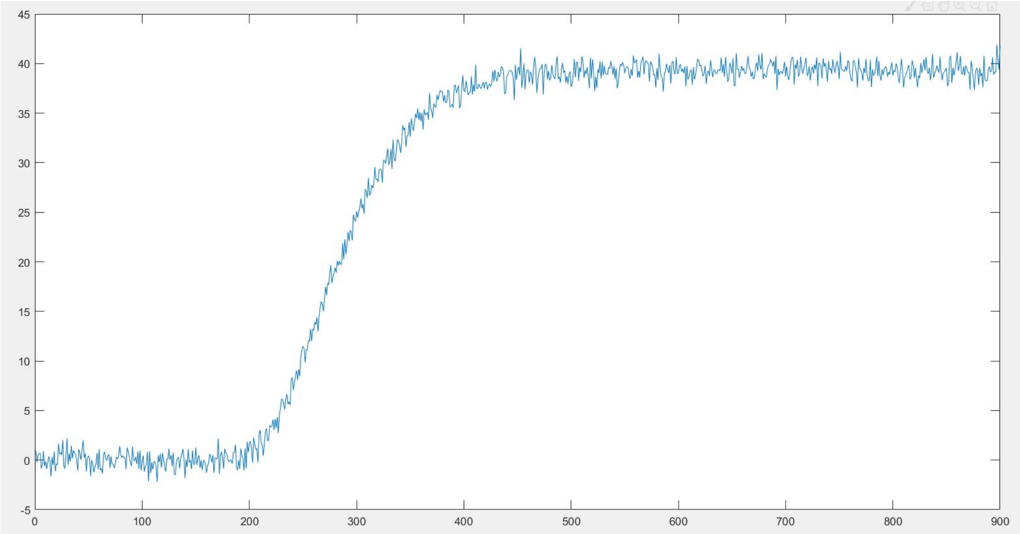

Figure 1.1: System output with 30 s delay - step input

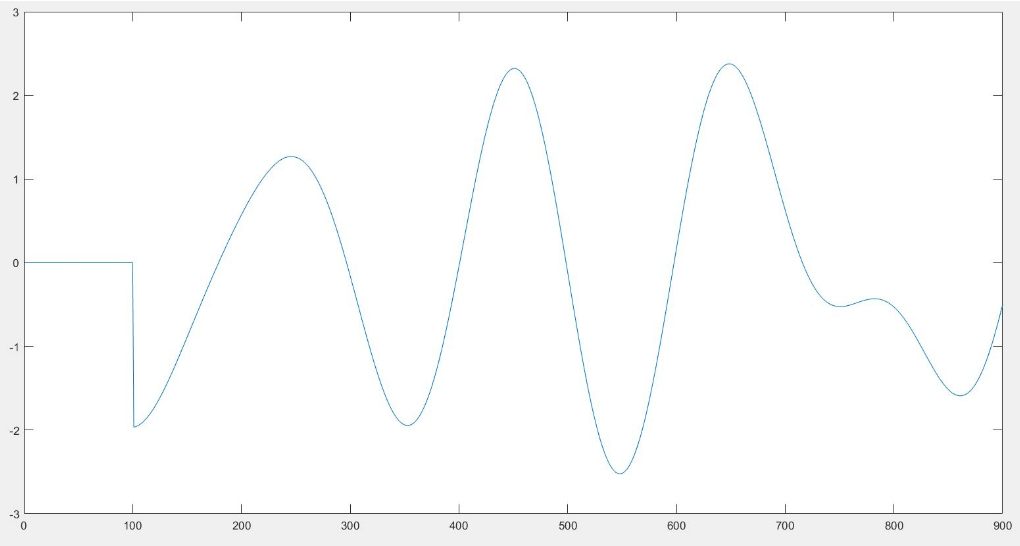

Figure 1.2: Random input with 100 s delay

3-Parameter 1st order model using area methods - Step input:

$$G(s) = \frac{k}{1+Ts}e^{-Ls} \tag{1.1}$$

We obtain the parameters with calculating $A0$ and $A1$ through Figure1.3.

$$k = 39.42 \tag{1.2}$$

$$A0 = 11724 \tag{1.3}$$

$$T_{ar} = \frac{A0}{k} = 297.45 \tag{1.4}$$

$$A1 = 921.5 \tag{1.5}$$

$$T = \frac{e \times A1}{k} = 63.55 \tag{1.6}$$

```
integral_a0 = sum(k - signal) * 0.3;
t_ar = integral_a0 / k;
integral_a1 = sum(signal(1:ceil(t_ar/0.3))) * 0.3;
T = exp(1) * integral_a1 / k;
L = t_ar - T;
```

Figure 1.3: Code for calculating parameters of 3-Parameter 1st order model - area method

$$L = t_{ar} - T = 233.90 \tag{1.7}$$

Finally, we can see our first model as follow. It is shown by Figure.1.4.

$$G(s) = \frac{39.42}{1 + 63.55 \times s} e^{-233.90 \times s} \tag{1.8}$$

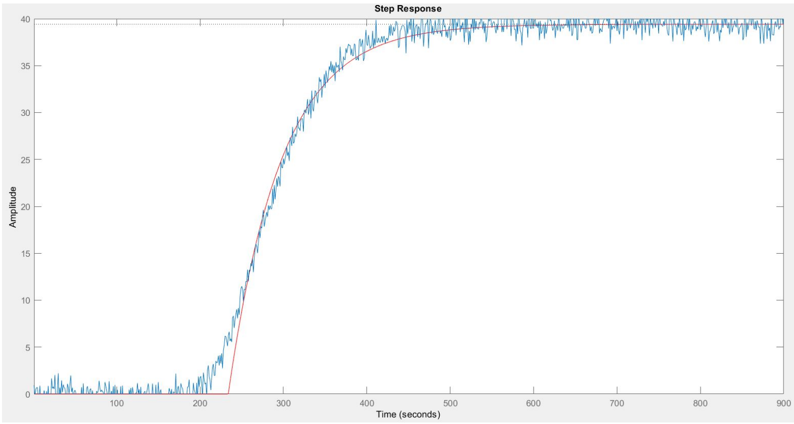

Figure 1.4: 3-Parameter 1st order model using area methods - Step input

3-Parameter 1st order model using area methods - Random input:
We can see our system and also our model with our Random input (Figure1.2.) which is presented by Figure1.5.

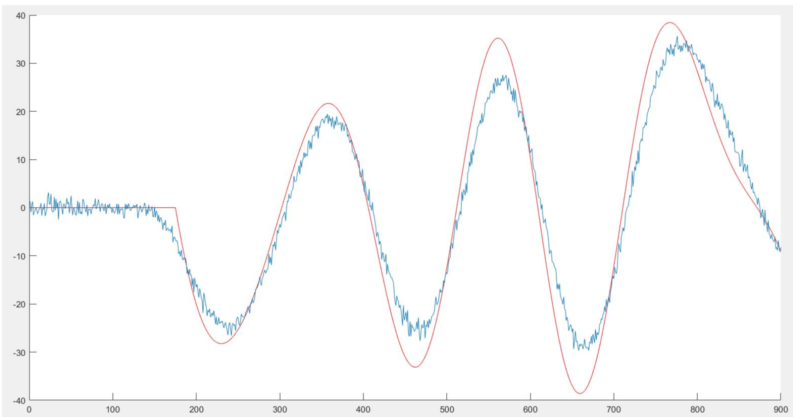

Figure 1.5: 3-Parameter 1st order model using area methods - Random input

3-Parameter 2nd order model using area methods - Step input:

$$G(s) = \frac{k}{(1+Ts)^2} e^{-Ls} \tag{1.9}$$

Same as previous model:

$$k = 39.42 \tag{1.10}$$

$$A0 = 11724 \tag{1.11}$$

$$T_{ar} = \frac{A0}{k} = 297.45 \tag{1.12}$$

$$A1 = 921.5 \tag{1.13}$$

Then we calculate other parameters:

$$T = \frac{e^2 \times A1}{4 \times k} = 78.61 \tag{1.14}$$

$$L = t_{ar} - 2 \times T = 211.08 \tag{1.15}$$

Finally, our second model is (result in Figure 1.6.):

$$G(s) = \frac{39.42}{(1 + 78.61 \times s)^2} e^{-211.08 \times s} \tag{1.16}$$

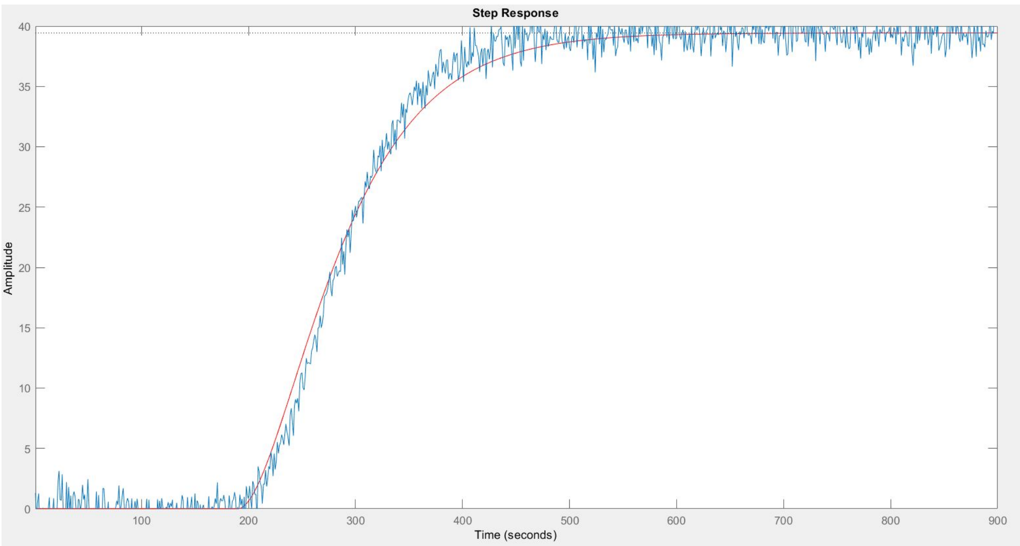

Figure 1.6: 3-Parameter 2nd order model using area methods - Step input

3-Parameter 2nd order model using area methods - Random input:
We can see our system and also our model with our Random input (Figure1.2.) which is presented by Figure1.7.

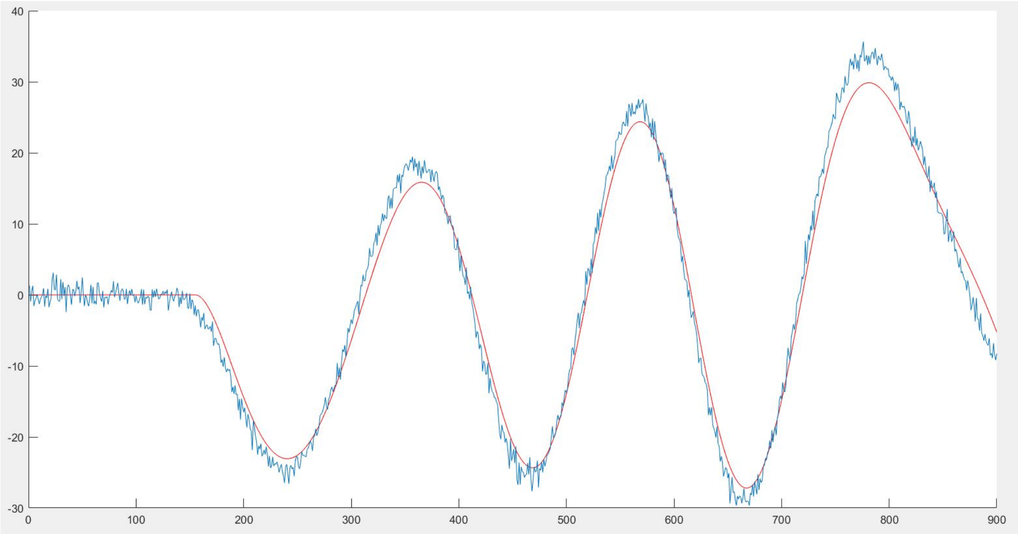

Figure 1.7: 3-Parameter 2nd order model using area methods - Random input

3-Parameter 1st order model using method of moments - Step input:
We generate a pulse to obtain the impulse response of our system. We can see our pulse in Figure 1.8. Then we determine our model's parameters from the impulse response of the system with method of moment.

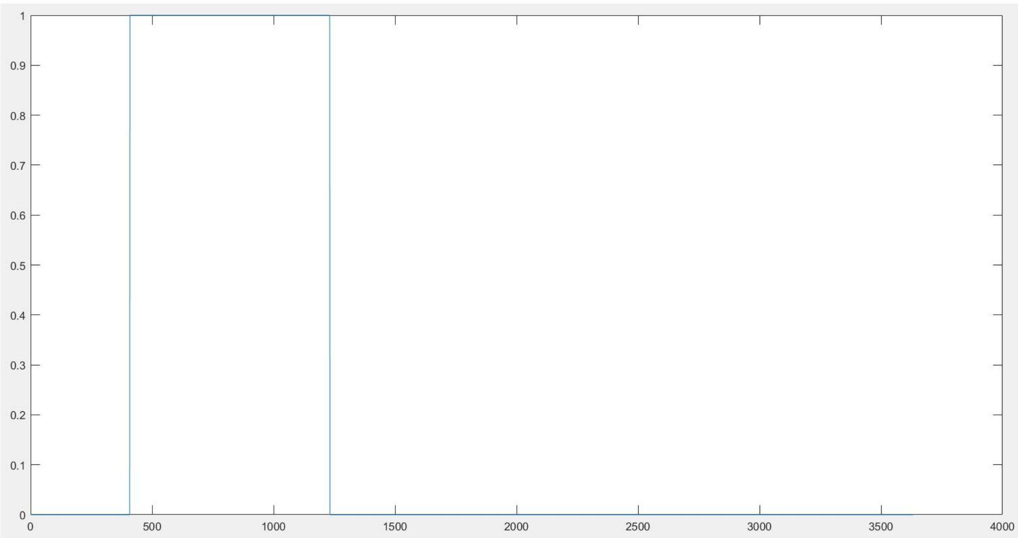

Figure 1.8: Pulse input for moment method

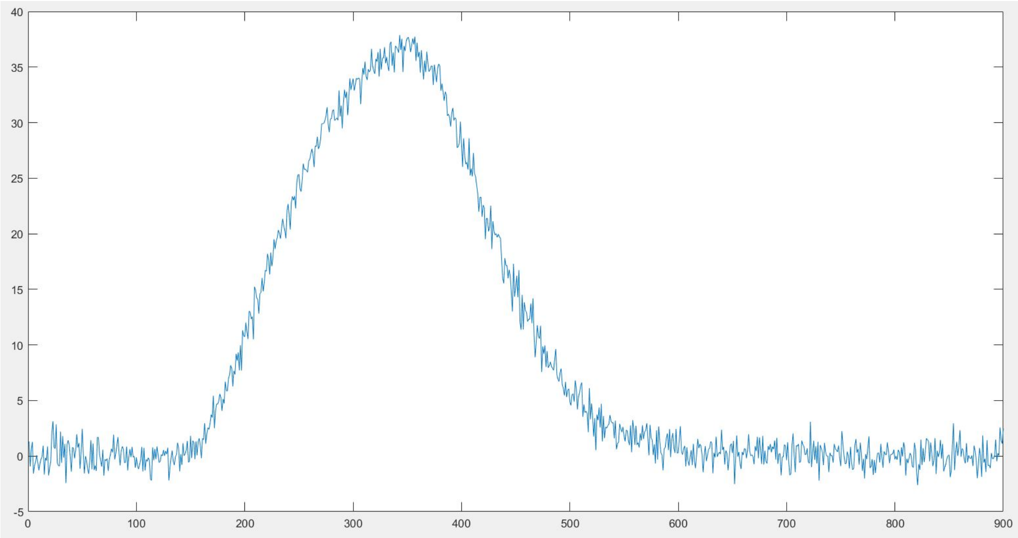

Figure 1.9: Impulse response of the system with delay time 100 s

$$G(s) = \frac{k}{1+Ts}e^{-Ls} \tag{1.17}$$

$$k = 40.10 \tag{1.18}$$

$$T_{ar} = 342.3 \tag{1.19}$$

$$T = 85.87 \tag{1.20}$$

$$L = 256.5 \tag{1.21}$$

We can see the code of calculation in figure 1.10. So, the final model is below. And also we can see the response of our model and system in Figure 1.11.

$$G(s) = \frac{40.10}{1+85.87\times s}e^{-256.5\times s} \tag{1.22}$$

```
time = simout7.time;
t_integral = sum(system_output.* time) * 0.3 / 200;
t_ar_moment = t_integral / k_moment;

t2_integral = sum(system_output.* (time.^2)) * 0.3 / 200;
T_moment = (t2_integral / k_moment - t_ar_moment^2)^(1/2);
L_moment = t_ar_moment - T_moment;
```

Figure 1.10: Code for calculating parameters of 3-Parameter 1st order model - Moments method

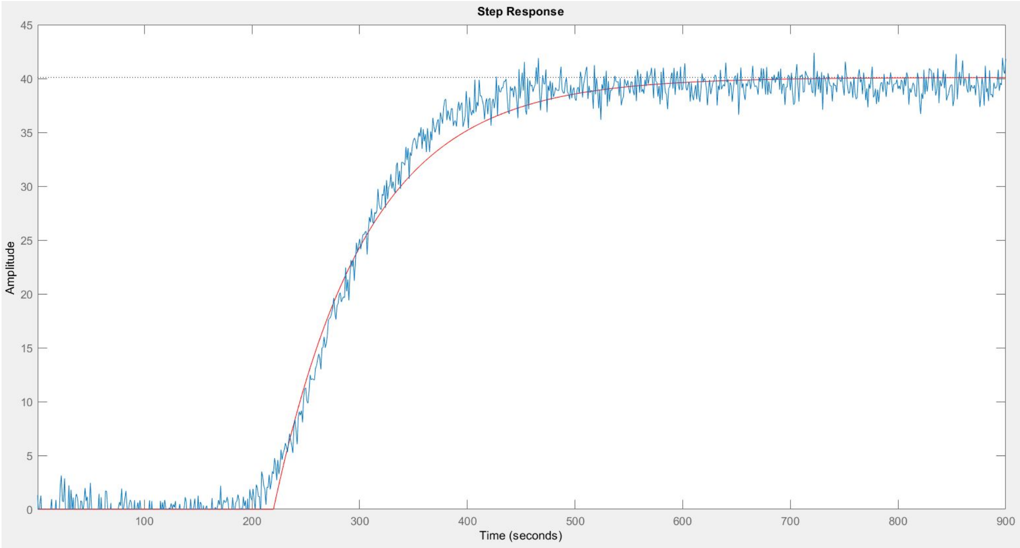

Figure 1.11: 3-Parameter 1st order model using method of moments - Step input

3-Parameter 1st order model using method of moments - Random input:
We can see our system and also our model with our Random input (Figure1.2.) which is presented by Figure1.12.

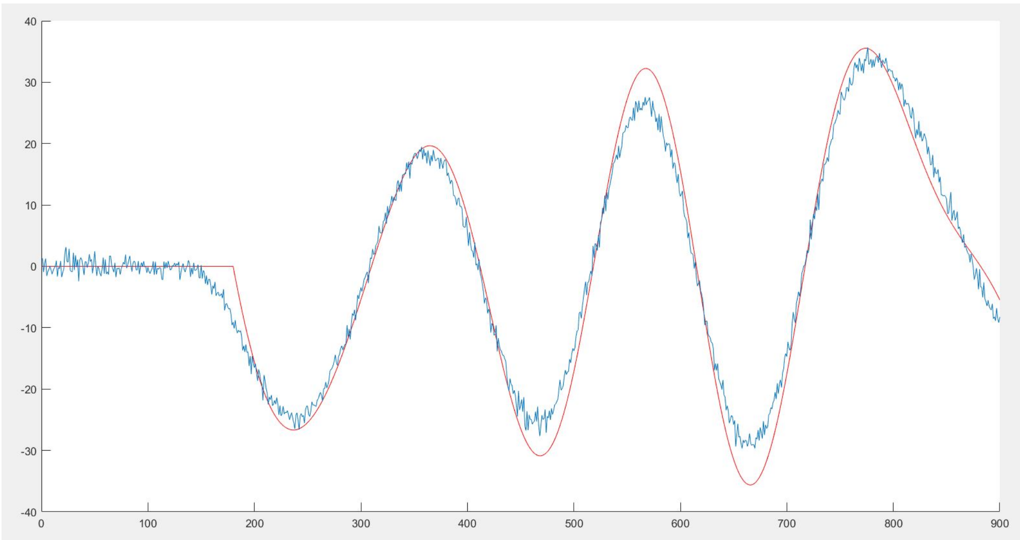

Figure 1.12: 3-Parameter 1st order model using method of moments - Random input

3-Parameter 2nd order model using method of moments - Step input:

$$G(s) = \frac{k}{(1+Ts)^2} e^{-Ls} \tag{1.23}$$

Same as previous model:

$$k = 40.10 \tag{1.24}$$

$$T_{ar} = 342.3 \tag{1.25}$$

And for 2nd order we have (code in Figure 1.13.):

$$T = 60.6 \tag{1.26}$$

$$L = 221.0 \tag{1.27}$$

So, the final model is below. And also we can see the response of our model and system in

```
T_moment_2 = (t2_integral / (2*k_moment) - ((1/2)*t_ar_moment^2))^(1/2);
L_moment_2 = t_ar_moment - 2 * T_moment_2;
```

Figure 1.13: Code for calculating parameters of 3-Parameter 2nd order model - Moments method

Figure 1.14.

$$G(s) = \frac{40.10}{(1 + 60.6 \times s)^2} e^{-221.0 \times s} \tag{1.28}$$

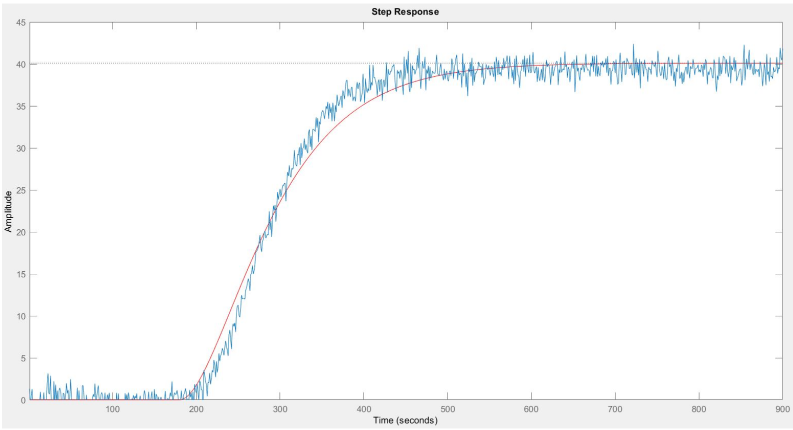

Figure 1.14: 3-Parameter 2nd order model using method of moments - Step input

3-Parameter 2nd order model using method of moments - Random input:
We can see our system and also our model with our Random input (Figure1.2.) which is presented by Figure1.15.

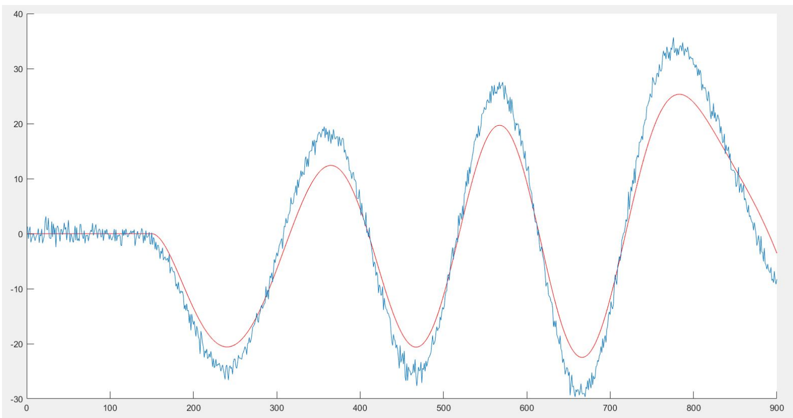

Figure 1.15: 3-Parameter 2nd order model using method of moments - Random input

4-Parameter 2nd order model using method of moments - step input:
Our calculation is presented in appendix B.

$$G(s) = \frac{k}{(1+T_1 s)\times(1+T_2 s)} e^{-Ls} \tag{1.29}$$

$$k = 40.10 \tag{1.30}$$

$$T_1 = 31.41 \tag{1.31}$$

$$T_2 = 69.56 \tag{1.32}$$

$$L = 198.70 \tag{1.33}$$

So our model is presented as below. We can see the result of the 4-Parameter 2nd order model using method of moments to step input in figure 1.16.

$$G(s) = \frac{40.10}{(1+31.41\times s)\times(1+69.56\times s)} e^{-198.70\times s} \tag{1.34}$$

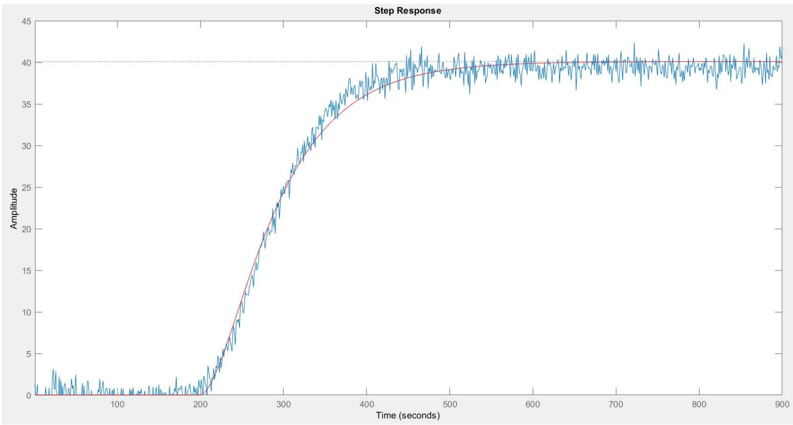

Figure 1.16: 4-Parameter 2nd order model using method of moments - step input

4-Parameter 2nd order model using method of moments - random input:
We can see our system and also our model with our Random input (Figure1.2.) which is presented by Figure1.17.

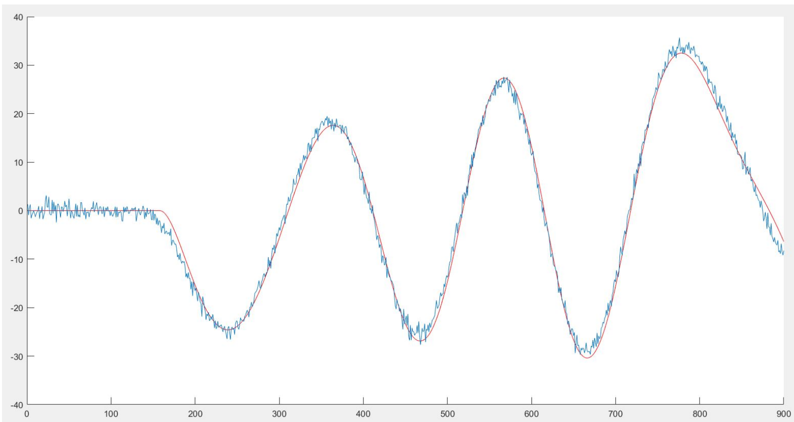

Figure 1.17: 4-Parameter 2nd order model using method of moments - random input

1.2 Complete Tables 1.1 to 1.2. The modeling error indices in Tables 1.1 to 1.2 need to be computed over the same time interval for all models. In each table, the same input should be used to calculate all columns. In Table 1.1 you should use a suitable step input. For Table 1.2 use random signal as the input. In your answer to this part you need to discuss the following points:

(a). How are the ISE and the IAE calculated and what is the difference between the two?

(b). What is a suitable time interval to calculate the errors for and why?

(a). The formula for calculating the ISE and IAE is in the following. The difference between these two is that the ISE method gives more weight to large errors due to the power of 2. For computing both of them, as the integral goes to infinity, we need the time of simulation to be large.

IAE is calculated with this equation:

$$IAE = \int_0^\infty |e(t)|dt \tag{1.35}$$

ISE is calculated with this equation:

$$ISE = \int_0^\infty e^2(t)dt \tag{1.36}$$

(b). Simulation interval should be selected in order that the response of the system and models reach their steady state. On the other hand, because of the infinite time integral of the error indicators discussed earlier, a large simulation time is better. On the contrary, because of the noise, the larger time interval give us the bigger errors. This increase is misleading of the indicators of error (error of the model should compute from the beginning up to the steady time not after that). Also for the random input, the time frame should be in a way such that a reasonable amount of random signal is determined.

In Table 1.1. and Table 1.3. we choose 900 and 1000 seconds for process, 100 and 120 seconds for first delay. In Table 1.2. we choose 900 seconds for process and 30 seconds for first delay.

Model	IAE	ISE	Maximum Error
$3-P1^{st}O($ Area $)$	10760	17.22×10^4	26.351
$3-P2^{nd}O($ Area $)$	6614	5.503×10^4	15.661
$3-P1^{st}O($ Moment $)$	10390	13.70×10^4	22.431
$3-P2^{nd}O($ Moment $)$	5080	3.276×10^4	13.206
$4-P2^{nd}O($ Moment $)$	4326	4.175×10^4	11.452

Table 1.1: Table of comparison of modeling errors - Step input 'amplitude 1'

1.3 Compare Tables 1.1 to 1.2. Discuss the result and explain if they match with your expectations. Based on these date select determine what you believe to be the best model. State the reasons for your choice.

Due to our expectation, by increasing the order of the models or increasing the degree of freedom of our model and let the model place its poles differently wherever it wants, the accuracy of our model should be increased. As we can see in results figures and also in the table of errors,

Model	IAE	ISE	Maximum Error
$3-P1^{st}O$(Area)	1094	4474	13.171
$3-P2^{nd}O$(Area)	714	1839	6.164
$3-P1^{st}O$(Moment)	728	2214	9.562
$3-P2^{nd}O$(Moment)	1122	4476	10.619
$4-P2^{nd}O$(Moment)	916	2703	9.976

Table 1.2: Table of comparison of modeling errors - Random input

Model	IAE	ISE	Maximum Error
$3-P1^{st}O$(Area)	11461	18.02×10^4	27.954
$3-P2^{nd}O$(Area)	7215	6.141×10^4	16.545
$3-P1^{st}O$(Moment)	11305	14.52×10^4	23.431
$3-P2^{nd}O$(Moment)	6250	4.447×10^4	14.760
$4-P2^{nd}O$(Moment)	5663	3.290×10^4	12.119

Table 1.3: Table of comparison of modeling errors - Step input 'amplitude 0.5'

Model	K	L	T	T_1	T_2
$3-P1^{st}O$(Area)	39.87	235.5	67.4	-	-
$3-P2^{nd}O$(Area)	39.87	213.1	75.3	-	-
$3-P1^{st}O$(Moment)	40.21	258.9	81.8	-	-
$3-P2^{nd}O$(Moment)	40.21	227.4	62.7	-	-
$4-P2^{nd}O$(Moment)	40.21	202.6	-	33.8	72.0

Table 1.4: Table of models' parameters - Step input 'amplitude 0.5'

Model	K	L	T	T_1	T_2
$3-P1^{st}O$(Area)	39.42	233.9	63.5	-	-
$3-P2^{nd}O$(Area)	39.42	211.1	78.6	-	-
$3-P1^{st}O$(Moment)	40.10	256.5	85.9	-	-
$3-P2^{nd}O$(Moment)	40.10	221.0	60.6	-	-
$4-P2^{nd}O$(Moment)	40.10	198.7	-	31.4	69.6

Table 1.5: Table of models' parameters - Step input 'amplitude 1'

the results are as we expect and the 4-parameter 2nd order model have the less errors and it fits best to our system response both in random and step inputs.

1.4 Compare the Nichols diagram of the system and the model you have selected as your model of choice. What does this tell you? Comment on the results and whether it matches the results predicted by tables 1.1 and 1.2.

In step input, 4-Parameter 2nd order model using method of moments is the best one. Here is its Nichols diagram in Figure 1.18.

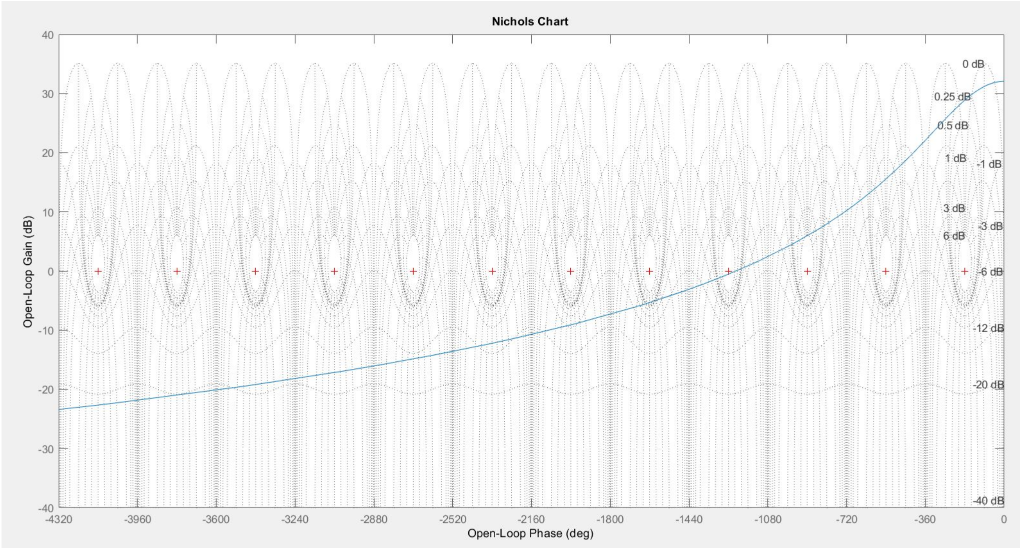

Figure 1.18: 4-Parameter 2nd order model using method of moments - Nichols diagram

In random input, 4-Parameter 2nd order model using area methods is the best one. Here is its Nichols diagram in Figure 1.19.

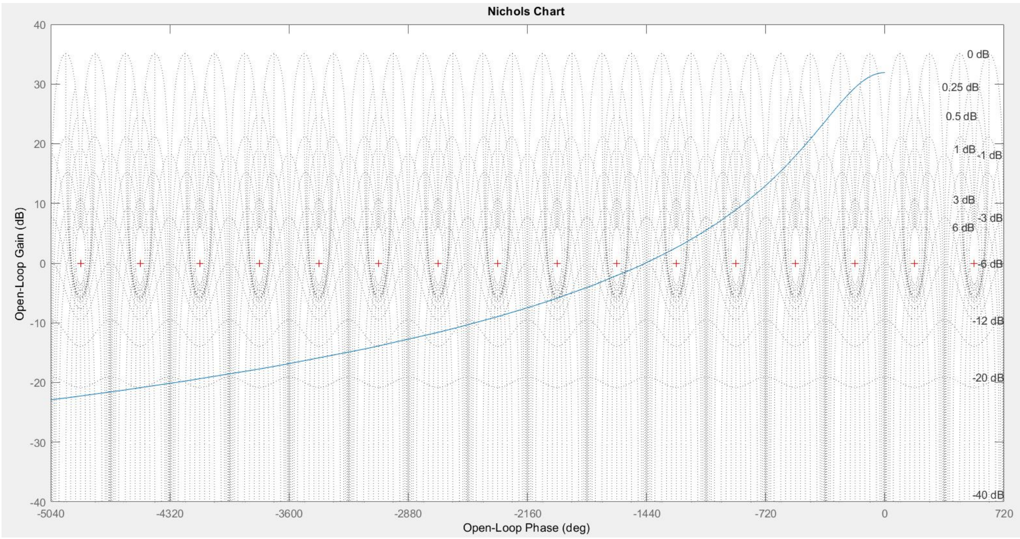

Figure 1.19: 4-Parameter 2nd order model using area methods - Nichols diagram

Unfortunately, as we don't have the system in the Matlab environment, there was no way to plot the Nichols diagram for the system itself. So we can not compare the Nichols diagram of our best model with the Nichols diagram of our system

2 APPENDIX A: METHOD OF AREA

For a system having the transfer function:

$$G(s) = \frac{k}{1+Ts}e^{-Ls} \tag{2.1}$$

We have:

$$A_1 = \int_0^{T_{ar}} s(t)dt = \int_0^{T} K\left(1-e^{-t/T}\right)dt = KTe^{-1} \tag{2.2}$$

The time constant is thus given by:

$$T = \frac{eA_1}{K} \tag{2.3}$$

The dead time is then given by:

$$L = T_{ar} - T = \frac{A_0}{K} - \frac{eA_1}{K} \tag{2.4}$$

The same idea can easily be applied to a system with the transfer function:

$$G(s) = \frac{k}{(1+Ts)^2}e^{-Ls} \tag{2.5}$$

In this case we have:

$$T_{ar} = L + 2T \tag{2.6}$$

For a system having transfer function above we have:

$$A_1 = \int_0^{T_{ar}} s(t)dt = \int_0^{2T} K\left(1-e^{-t/T} - \frac{t}{T}e^{-t/T}\right)dt = 4KTe^{-2} \tag{2.7}$$

The time constant is thus given by:

$$T = \frac{A_1 e^2}{4K} \tag{2.8}$$

And the dead time is:

$$L = T_{ar} - 2T = \frac{A_0}{K} - \frac{A_1 e^2}{2K} \tag{2.9}$$

3 APPENDIX B: METHOD OF MOMENTS

Consider the transfer function:

$$G(s) = \frac{k}{1+Ts}e^{-Ls} \tag{3.1}$$

It follows that:

$$K = G(0) = \int_0^\infty h(t)dt \tag{3.2}$$

Hence:

$$T^2 = \frac{G''(0)}{G(0)} - T_{ar}^2 = \frac{\int_0^\infty t^2 h(t)dt}{\int_0^\infty h(t)dt} - T_{ar}^2 \tag{3.3}$$

The dead time L can then be computed to:

$$L = T_{ar} - T \tag{3.4}$$

The method of moments will now be applied to determine the parameters of the transfer function:

$$G(s) = \frac{k}{(1+Ts)^2}e^{-Ls} \tag{3.5}$$

Hence:

$$K = G(0) = \int_0^\infty h(t)dt \tag{3.6}$$

And:

$$T^2 = \frac{G''(0)}{2G(0)} - \frac{1}{2}T_{ar}^2 = \frac{\int_0^\infty t^2 h(t)dt}{2\int_0^\infty h(t)dt} - \frac{1}{2}T_{ar}^2 \tag{3.7}$$

The dead time L can then be computed to:

$$T_{ar} = L + 2T \tag{3.8}$$

Also if we have this transfer function:

$$G(s) = \frac{k}{(1+T_1 s)\times(1+T_2 s)}e^{-Ls} \tag{3.9}$$

We could determine k, L, T_1 and T_2 if we calculate G'' and G'''.

References:

1. PID Controllers, Theory, Design and Tuning (2nd Edition) - Åström, Karl
2. Performance of FOPI with error filter based on controllers performance criterion (ISE, IAE and ITAE) - Mohd Hezri Marzaki