به نام خدا

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گزارش پروژه 1

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Q1).

The differential equation for the rotation θ of the principal axis is (Figure 1.3 a):

$$M\ddot{\theta} = P - R - F(\dot{\theta} - V_1) - G\psi$$

The equation of motion of the friction ring is (Figure 1.3 b):

$$B\ddot{\psi} = F(\dot{\theta} - V_1) - Y\dot{\psi} - W$$

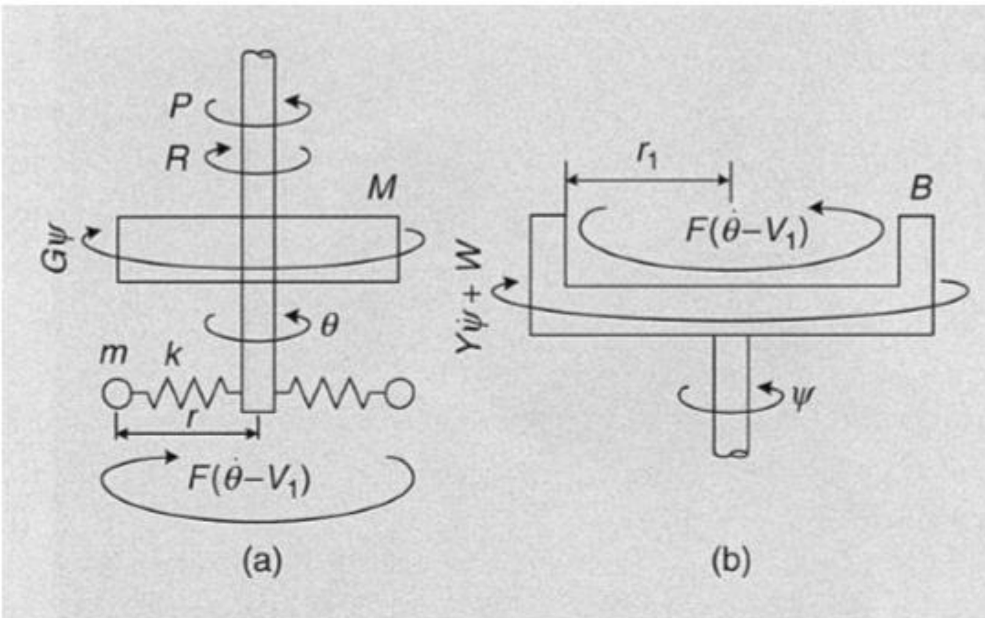


Figure 1.3 *A free body diagram of Jenkin's Governor*

Combining two equations to a linear differential equation that is third order in the velocity $\omega = \dot{\theta}$:

$$MB\ddot{\omega} + (MY + FB)\ddot{\omega} + FY\dot{\omega} + FG\omega = u(t)$$

Q2).

$$MB\ddot{\omega} + (MY + FB)\ddot{\omega} + FY\dot{\omega} + FG\omega = u(t)$$

the velocity $\dot{\theta}$ varies within very narrow limits around the value V_1:

$$\dot{\theta} \triangleq V_1 + \Delta\dot{\theta}, \qquad \Delta\theta^2 \cong 0$$

Make equation for $u(t)$:

$$u(t) = B(\ddot{P} - \ddot{R}) + Y(\dot{P} - \dot{R}) + GFV_1 + GW$$

If:

$$\dot{P} = constant, \qquad \dot{R} = constant$$

We solve $\omega(t)$ as follow:

$$\omega(t) = \dot{\theta}(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_3 e^{s_3 t} + V$$

V is nominal velocity:

$$V = V_1 + W/F$$

Steady state of problem:

$$GFV = GFV_1 + GW$$

Q3).

S_1, S_2, S_3 are the roots of the cubic characteristic equation:

$$MBs^3 + (MY + FB)s^2 + FYs + FG = 0$$

The real roots and the real parts of the complex conjugate roots of the characteristic equation above must all be negative.

<u>Stability condition</u>:

$$(\frac{F}{M} + \frac{Y}{B})\frac{Y}{B} - \frac{G}{B} = positive\ value$$

So, we must have this condition on parameters of the problem:

$$(\frac{F}{M} + \frac{Y}{B})\frac{Y}{B} > \frac{G}{B}$$

Using Routh's array:

$$\begin{array}{ll} s^3: MB & FY \\ s^2: MY + FB & FG \\ s: \dfrac{(MY + FB)FY - (MB)(FG)}{MY + FB} & \\ 1: FG & \end{array}$$

For stability, all elements of the <u>first column of the Routh array must be positive</u>.

All <u>coefficients</u> in equation below must be positive:

$$MBs^3 + (MY + FB)s^2 + FYs + FG = 0$$

They are actually positive until parameters are physical values.

Q4).

a).

1. Sir William Thomson and Léon Foucault governor model.

2. Watt's Centrifugal Governor model.

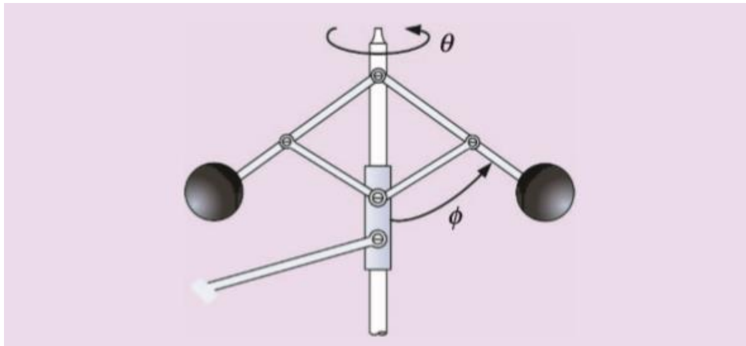
b).

Let’s solve for a famous example model: Sir William Thomson and Léon Foucault governor model

The equations of motion using the angular momentum $A\dot{\theta}$:

$$\frac{d}{dt}(A\dot{\theta}) = L$$

Where θ is the angle of revolution about the vertical axis, A is the moment of inertia of a revolving apparatus for θ motion, and L is the total torque acting on the axis. Let B be the moment of inertia of the flyballs in Figure below for ϕ motion.



Then, the sum of the kinetic and potential energies of Foucault's governor is:

$$E = \frac{1}{2}A\dot{\theta}^2 + \frac{1}{2}B\dot{\phi}^2 + P = \int L d\theta$$

where P is the potential energy of the apparatus, which is a function of the divergence angle ϕ of the centrifugal piece. Here, A and B are both functions of the angle ϕ.

Assume that $P_\phi = 0.5A_\phi V^2$, then we can rearrange the equation:

$$\frac{d}{dt}(B\dot{\phi}) = \frac{1}{2}A_\phi(\dot{\theta}^2 - V^2) + \frac{1}{2}B_\phi\dot{\phi}^2$$

By assuming:

$$\dot{\theta} \triangleq V + \omega, \qquad \phi \triangleq \phi_1 + \phi$$

Also, the linear differential equations are:

$$A\dot{\omega} + A_\phi V\dot{\phi} = L$$

$$B\ddot{\phi} - A_\phi V\omega = 0$$

To convert this apparatus into a governor, the equations become:

$$A\dot{\omega} + X\omega + K\dot{\phi} + G\phi = L$$

$$B\ddot{\phi} + Y\dot{\phi} - K\omega = 0$$

After model Linearization we have:

$$AB\dddot{\phi} + (AY + BX)\ddot{\phi} + (XY + K^2)\dot{\phi} + GK\phi = L$$

So, the stability condition of equation is (confirmed by the Routh stability criterion):

$$(\frac{X}{A} + \frac{Y}{B})(XY + K^2) > GK$$

model	condition
centrifugal piece at a constant distance	$(\frac{F}{M}+\frac{Y}{B})\frac{Y}{B}>\frac{G}{B}$
centrifugal piece with free movement	$(\frac{X}{A}+\frac{Y}{B})(XY+K^2)>GK$

Q5).