CPSC 532W Assignment 6

Ali Seyfi - 97446637

Here is a link to the repository:
`https://github.com/aliseyfi75/Probabilistic-Programming/tree/master/Assignment_6`

1 Code

1.1 evaluator

```
def evaluate(exp, env=None):

    if env is None:
        env = standard_env()

    if type(exp) is list:
        op, *args = exp
        if op == 'sample':
            alpha = evaluate(args[0], env=env)
            d = evaluate(args[1], env=env)
            s = d.sample()
            k = evaluate(args[2], env=env)
            sigma = {'type' : 'sample', 'alpha' : alpha, 'sample' : s}
            return k, [s], sigma
        elif op == 'observe':
            alpha = evaluate(args[0], env=env)
            d = evaluate(args[1], env=env)
            c = evaluate(args[2], env=env)
            k = evaluate(args[3], env=env)
            sigma = {'type' : 'observe', 'alpha' : alpha, 'log_prob' : d.log_prob(c)}
            return k, [c], sigma
        elif op == 'if':
            cond,conseq,alt = args
            if evaluate(cond, env=env):
                return evaluate(conseq, env=env)
            else:
                return evaluate(alt, env=env)
        elif op == 'fn':
            params, body = args #fn is:  ['fn', ['arg1','arg2','arg3'], body_exp]
            return Procedure(params, body, env)
        else: #func eval
            proc = evaluate(op, env=env)
            values = [evaluate(e, env=env) for e in args]
            sigma = {'type' : 'proc'}
            return proc, values, sigma
    elif type(exp) is str:
        if exp[0] == "\"":  # strings have double, double quotes
            return exp[1:-1]
        if exp[0:4] == 'addr':
            return exp[4:]
        lowest_env = env.find(exp)
        return lowest_env[exp]
    elif type(exp) is float or type(exp) is int or type(exp) is bool:
        return torch.tensor(exp)
    else:
        raise ValueError('Expression type unkown')
```

Listing 1: evaluator.py - evaluate

1.2 SMC

```
    particles = []
    weights = []
    logZs = []
    output = lambda x: x

    for i in range(n_particles):
        res = evaluate(exp, env=None)('addr_start', output)
        logW = 0.
        particles.append(res)
        weights.append(logW)

    done = False
    smc_cnter = 0
    while not done:
        new_address = ''
        print('In SMC step {}, Zs: '.format(smc_cnter), logZs)
        for i in range(n_particles):
            res = run_until_observe_or_end(particles[i])
            if 'done' in res[2]:
                particles[i] = res[0]
                if i == 0:
                    done = True
                    address = ''
                else:
                    if not done:
                        raise RuntimeError('Failed SMC, finished one calculation before the
    other')
            else:
                if i == 0:
                    new_address = res[2]['alpha']
                else:
                    address = res[2]['alpha']
                    if address != new_address:
                        raise RuntimeError('Failed SMC, address changed')

                log_prob = res[2]['log_prob']
                weights[i] = weights[i] + log_prob
                particles[i] = res

        if not done:
            logZn, particles = resample_particles(particles, weights)
            logZs.append(logZn)
            weights = [0.] * n_particles
        smc_cnter += 1
    logZ = sum(logZs)
    return logZ, particles
```

Listing 2: smc.py - SMC

```
def resample_particles(particles, log_weights):
    paricles_length = len(particles)
    weights = torch.exp(torch.FloatTensor(log_weights)) # convert to weights
    normalized_weights = weights + 1e-10 # add a small number to avoid zero weights
    normalized_weights = normalized_weights / normalized_weights.sum() # normalize weights

    logZ = torch.log(torch.mean(weights)) # calculate logZ

    indices = torch.multinomial(normalized_weights, paricles_length, replacement=True)
    new_particles = [particles[i] for i in indices]

    return logZ, new_particles
```

Listing 3: smc.py - resample_particles

2 Results

2.1 Task 1

2.1.1 Number of particles: 1

2.1.1.1 Posterior expectation

Posterior expected value of until success is: **22**

2.1.1.2 Histogram

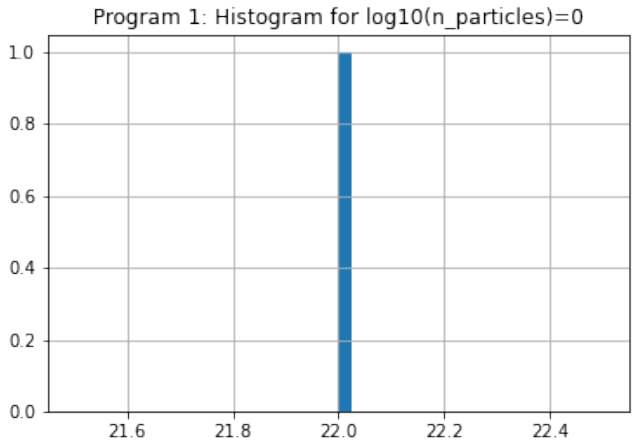

Figure 1: Histogram of posterior distribution of until success

2.1.1.3 marginal evidence estimate

marginal evidence estimateof until success is: **NA**

2.1.2 Number of particles: 10

2.1.2.1 Posterior expectation

Posterior expected value of until success is: **73.7**

2.1.2.2 Histogram

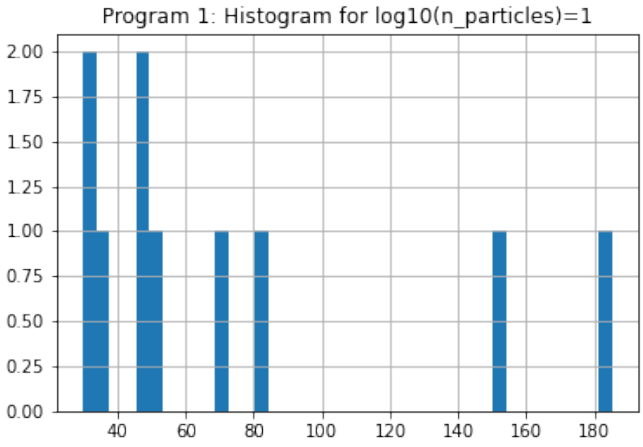

Figure 2: Histogram of posterior distribution of until success

2.1.2.3 marginal evidence estimate

marginal evidence estimateof until success is: **NA**

2.1.3 Number of particles: 10^2

2.1.3.1 Posterior expectation

Posterior expected value of until success is: **97.35**

2.1.3.2 Histogram

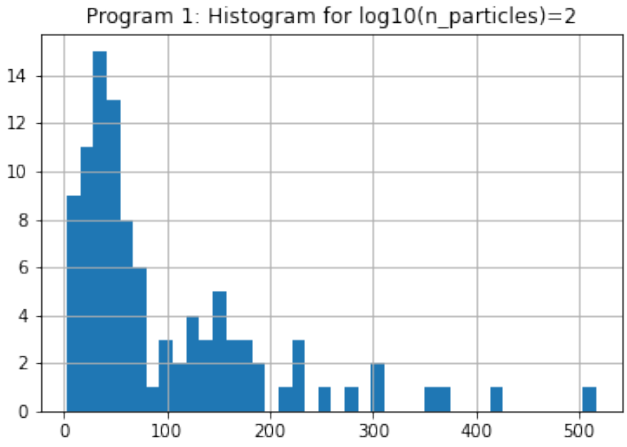

Figure 3: Histogram of posterior distribution of until success

2.1.3.3 marginal evidence estimate

marginal evidence estimateof until success is: **NA**

2.1.4 Number of particles: 10^3

2.1.4.1 Posterior expectation

Posterior expected value of until success is: **97.474**

2.1.4.2 Histogram

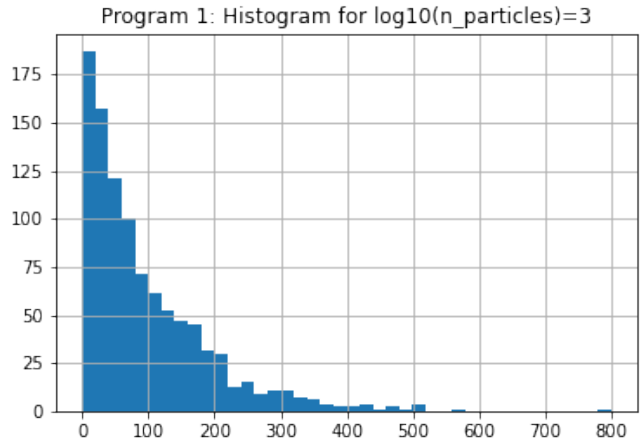

Figure 4: Histogram of posterior distribution of until success

2.1.4.3 marginal evidence estimate

marginal evidence estimateof until success is: **NA**

2.1.5 Number of particles: 10^4

2.1.5.1 Posterior expectation

Posterior expected value of until success is: **96.4183**

2.1.5.2 Histogram

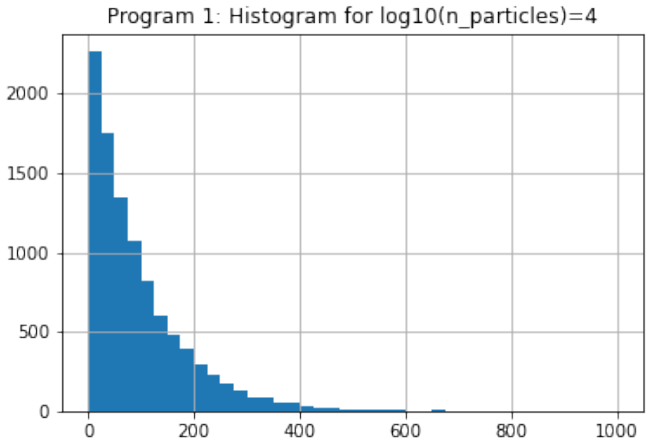

Figure 5: Histogram of posterior distribution of until success

2.1.5.3 marginal evidence estimate

marginal evidence estimateof until success is: **NA**

2.1.6 Number of particles: 10^5

2.1.6.1 Posterior expectation

Posterior expected value of until success is: **98.9192**

2.1.6.2 Histogram

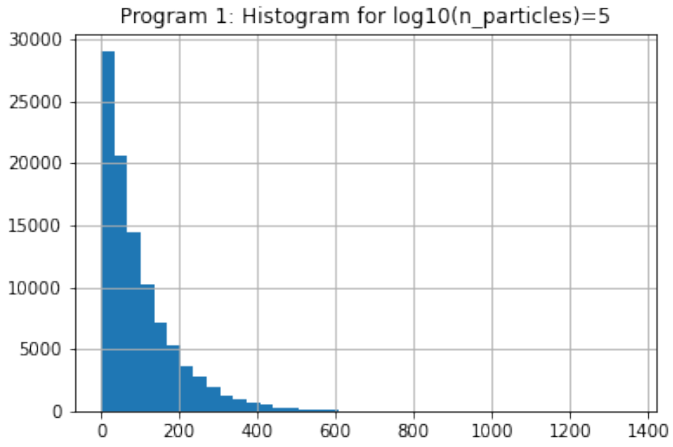

Figure 6: Histogram of posterior distribution of until success

2.1.6.3 marginal evidence estimate

marginal evidence estimateof until success is: **NA**

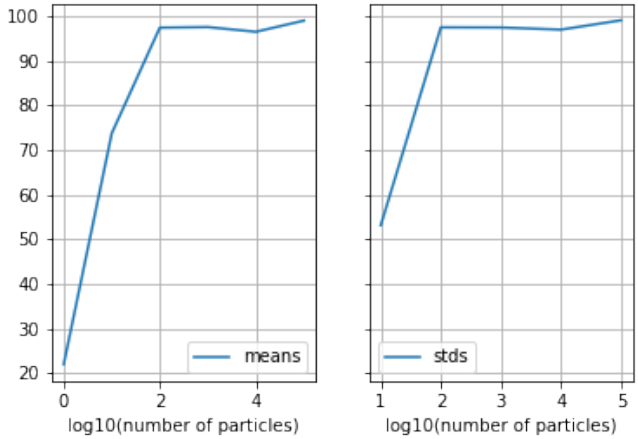

Figure 7: Marginal mean and std of until success for different number of particles

2.2 Task 2

2.2.1 Number of particles: 1

2.2.1.1 Posterior expectation

Posterior expected value of mu is: **0.8076**

2.2.1.2 Histogram

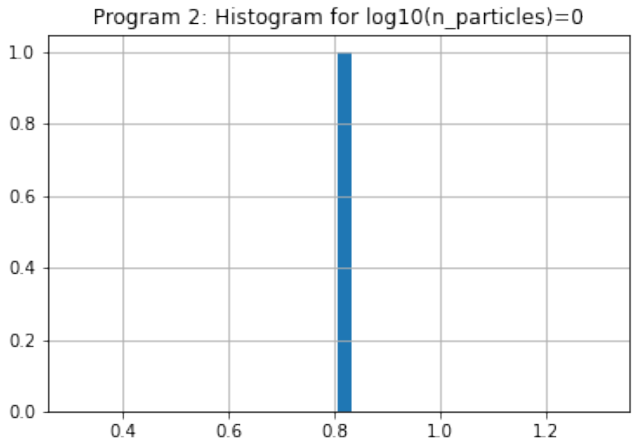

Figure 8: Histogram of posterior distribution of mu

2.2.1.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-35.7038**
marginal evidence estimate of mu is: $3.1191 * 10^{-16}$

2.2.2 Number of particles: 10

2.2.2.1 Posterior expectation

Posterior expected value of mu is: **4.9418**

2.2.2.2 Histogram

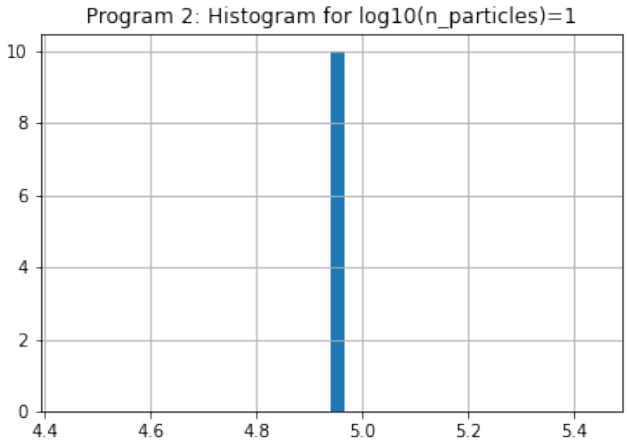

Figure 9: Histogram of posterior distribution of mu

2.2.2.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-15.9816**
marginal evidence estimate of mu is: $1.1462 * 10^{-7}$

2.2.3 Number of particles: 10^2

2.2.3.1 Posterior expectation

Posterior expected value of mu is: **4.3897**

2.2.3.2 Histogram

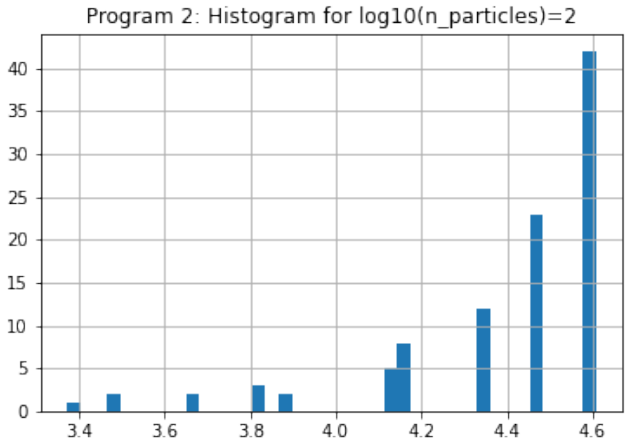

Figure 10: Histogram of posterior distribution of mu

2.2.3.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-9.0384**
marginal evidence estimate of mu is: **0.0001187**

2.2.4 Number of particles: 10^3

2.2.4.1 Posterior expectation

Posterior expected value of mu is: **7.5386**

2.2.4.2 Histogram

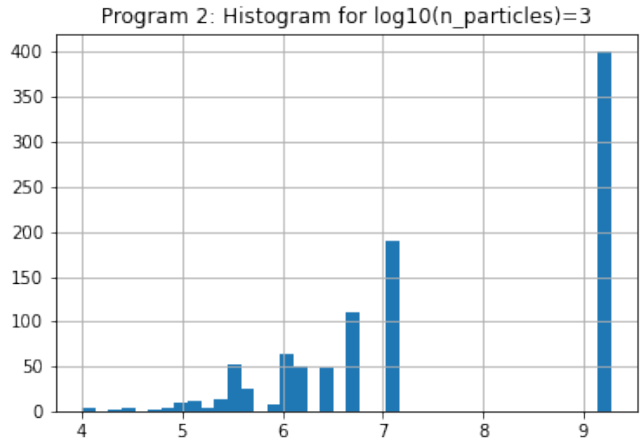

Figure 11: Histogram of posterior distribution of mu

2.2.4.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-7.9729**
marginal evidence estimate of mu is: **0.000344**

2.2.5 Number of particles: 10^4

2.2.5.1 Posterior expectation

Posterior expected value of mu is: **7.2384**

2.2.5.2 Histogram

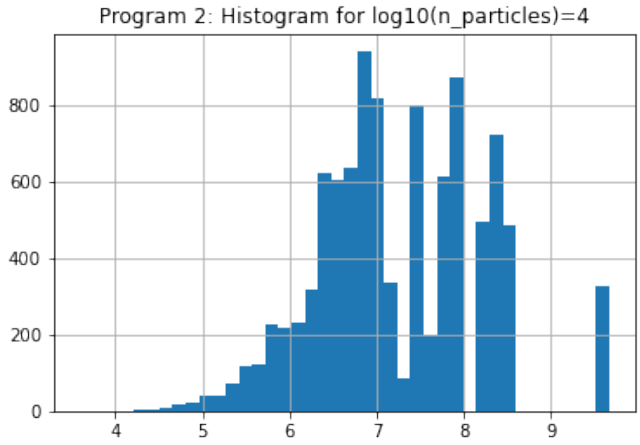

Figure 12: Histogram of posterior distribution of mu

2.2.5.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-8.23083**
marginal evidence estimate of mu is: **0.00027**

2.2.6 Number of particles: 10^5

2.2.6.1 Posterior expectation

Posterior expected value of mu is: **7.2103**

2.2.6.2 Histogram

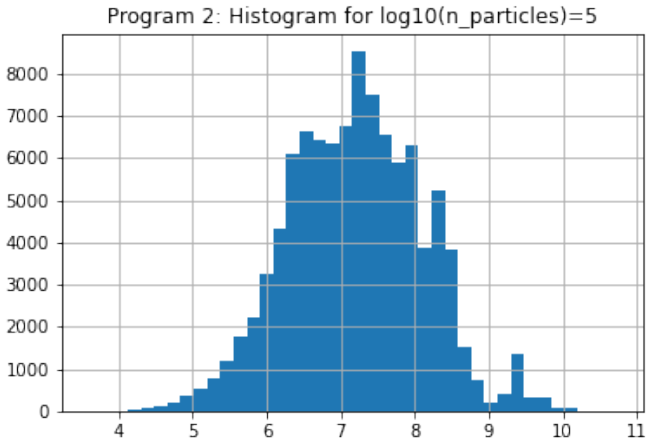

Figure 13: Histogram of posterior distribution of mu

2.2.6.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-8.23384**
marginal evidence estimate of mu is: **0.000266**

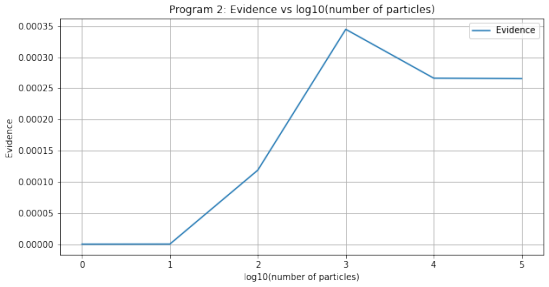

Figure 14: Marginal evidence estimate of mu for different number of particles

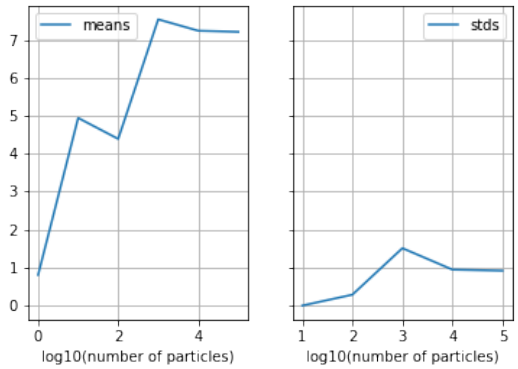

Figure 15: Marginal mean and std of mu for different number of particles

2.3 Task 3

2.3.1 Number of particles: 1

2.3.1.1 Posterior expectation

Posterior expected value of states in each step is: **[2., 1., 0., 1., 0., 2., 1., 0., 2., 2., 2., 2., 2., 1., 2., 1., 0.]**

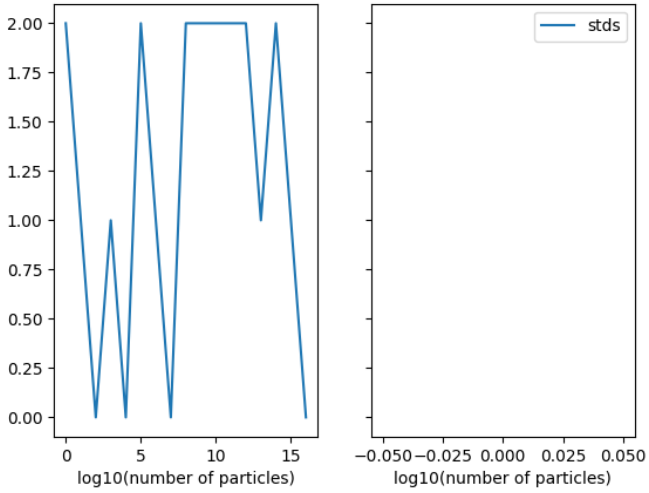

Figure 16: Marginal mean and std of states in each step

2.3.1.2 Histogram

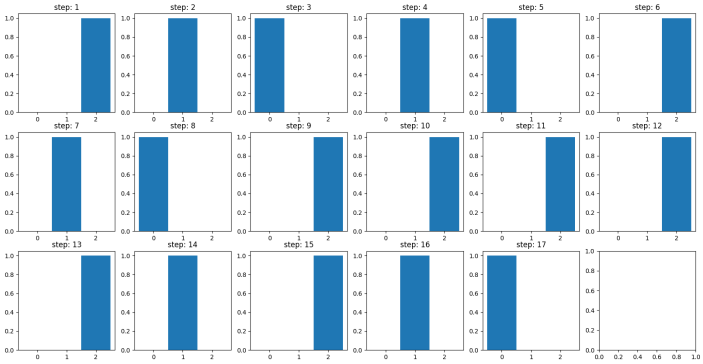

Figure 17: Histogram of posterior distribution of states in each step

2.3.1.3 marginal evidence estimate

marginal log evidence estimate of states in each step is: **-58.723**
marginal evidence estimate of states in each step is: $3.1398 * 10^{-26}$

2.3.2 Number of particles: 10

2.3.2.1 Posterior expectation

Posterior expected value of states in each step is: **[1.4, 1.2, 2.0, 2.0, 1.0, 0.2, 1.0, 0.0, 2.0, 2.0, 0.0, 2.0, 2.0, 2.0, 1.0, 0.0, 1.0]**

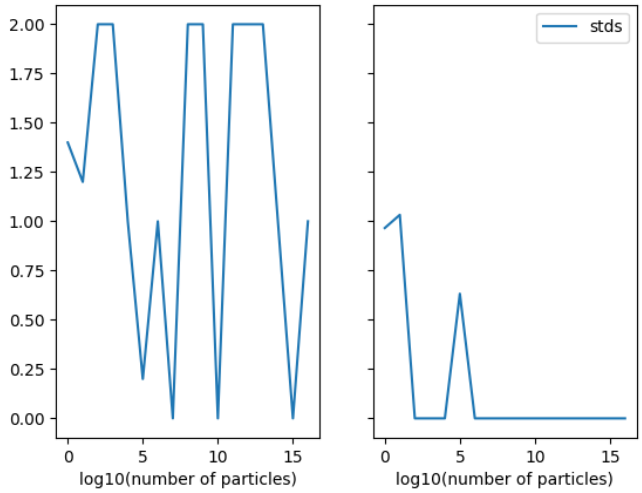

Figure 18: Marginal mean and std of states in each step

2.3.2.2 Histogram

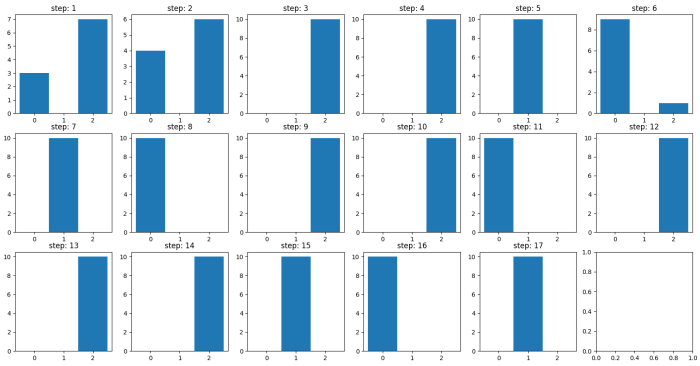

Figure 19: Histogram of posterior distribution of states in each step

2.3.2.3 marginal evidence estimate

marginal log evidence estimate of states in each step is: **-43.748**
marginal evidence estimate of states in each step is: 1.0011^{-19}

2.3.3 Number of particles: 10^2

2.3.3.1 Posterior expectation

Posterior expected value of states in each step is: **[1.35, 1.35, 1.66, 1.71, 1.00, 1.66, 1.85, 1.82, 1.17, 0.36, 0.00, 1.42, 1.76, 1.82, 1.28, 1.66, 0.90]**

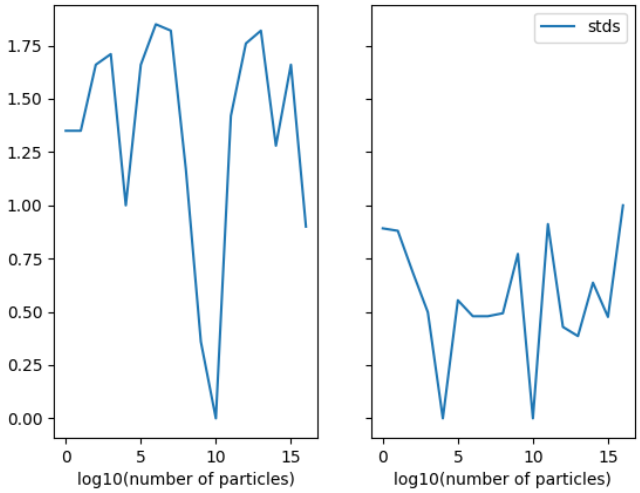

Figure 20: Marginal mean and std of states in each step

2.3.3.2 Histogram

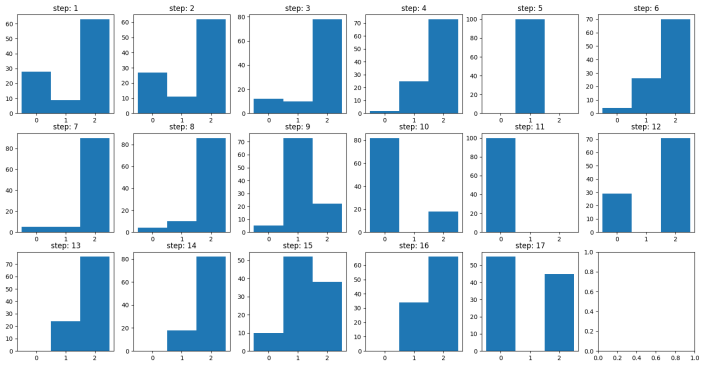

Figure 21: Histogram of posterior distribution of states in each step

2.3.3.3 marginal evidence estimate

marginal log evidence estimate of states in each step is: **-43.931**
marginal evidence estimate of states in each step is: $8.3384 * 10^{-20}$

2.3.4 Number of particles: 10^3

2.3.4.1 Posterior expectation

Posterior expected value of states in each step is: **[1.427, 1.617, 1.551, 1.614, 1.021, 1.426, 1.752, 1.749, 1.587, 1.151, 0.170, 1.774, 1.608, 1.723, 1.675, 1.505, 1.178]**

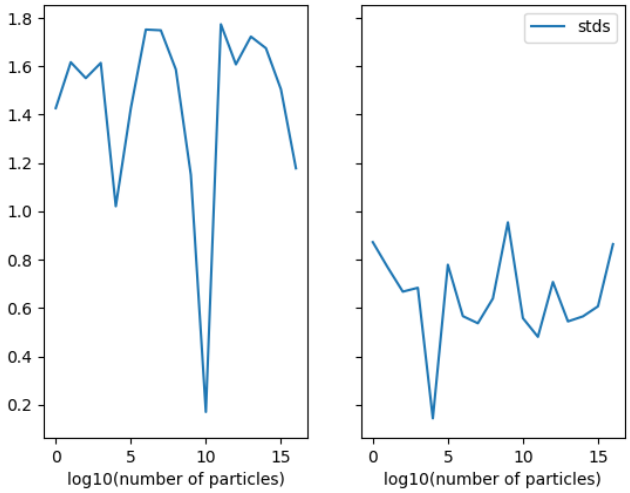

Figure 22: Marginal mean and std of states in each step

2.3.4.2 Histogram

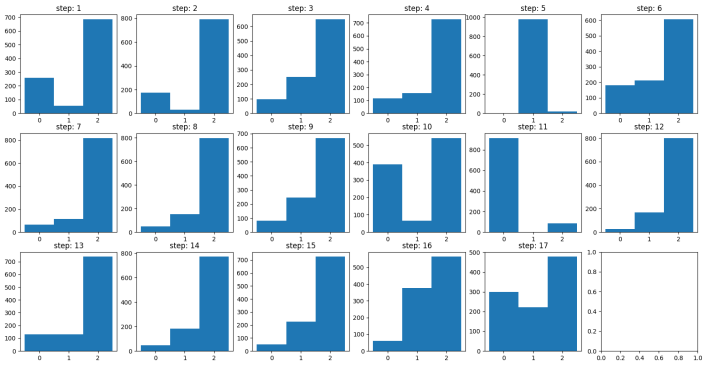

Figure 23: Histogram of posterior distribution of states in each step

2.3.4.3 marginal evidence estimate

marginal log evidence estimate of states in each step is: **-44.359**
marginal evidence estimate of states in each step is: $5.4340 * 10^{-20}$

2.3.5 Number of particles: 10^4

2.3.5.1 Posterior expectation

Posterior expected value of states in each step is: **[1.4412, 1.5189, 1.6974, 1.6059, 1.0146, 1.3829, 1.6333, 1.6718, 1.6207, 1.0641, 0.1592, 1.7524, 1.6479, 1.7258, 1.6390, 1.5436, 0.9160]**

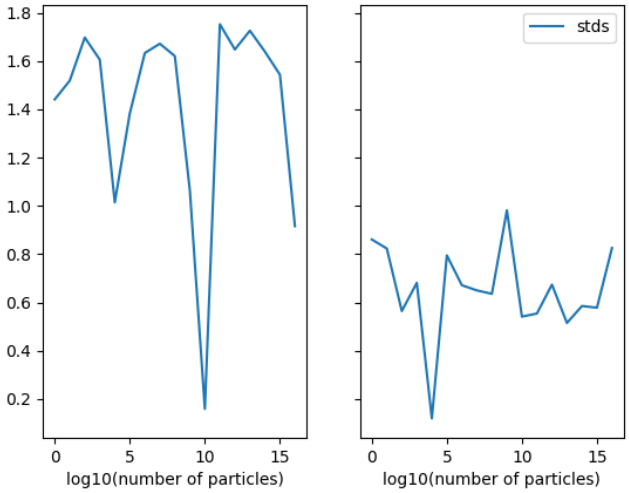

Figure 24: Marginal mean and std of states in each step

2.3.5.2 Histogram

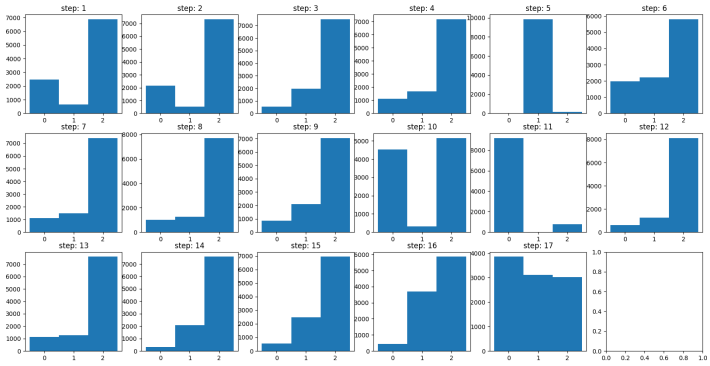

Figure 25: Histogram of posterior distribution of states in each step

2.3.5.3 marginal evidence estimate

marginal log evidence estimate of states in each step is: **-44.404**
marginal evidence estimate of states in each step is: $5.1935 * 10^{-20}$

2.3.6 Number of particles: 10^5

2.3.6.1 Posterior expectation

Posterior expected value of states in each step is: **[1.4314, 1.5516, 1.7008, 1.6037, 1.0174, 1.4171, 1.6541, 1.6618, 1.6076, 1.0369, 0.1375, 1.6632, 1.6574, 1.6778, 1.6381, 1.4908, 0.9158]**

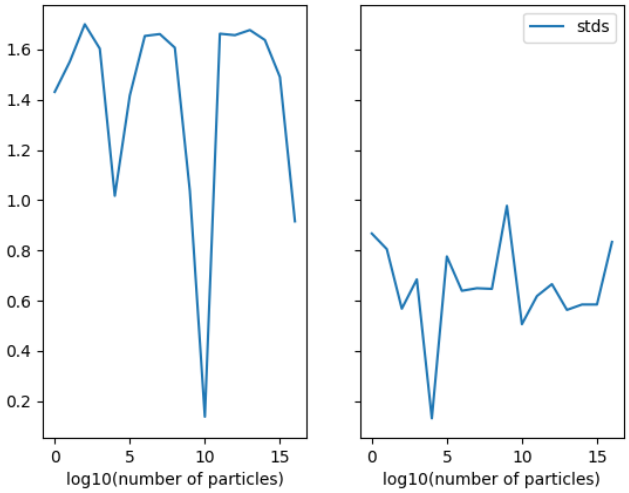

Figure 26: Marginal mean and std of states in each step

2.3.6.2 Histogram

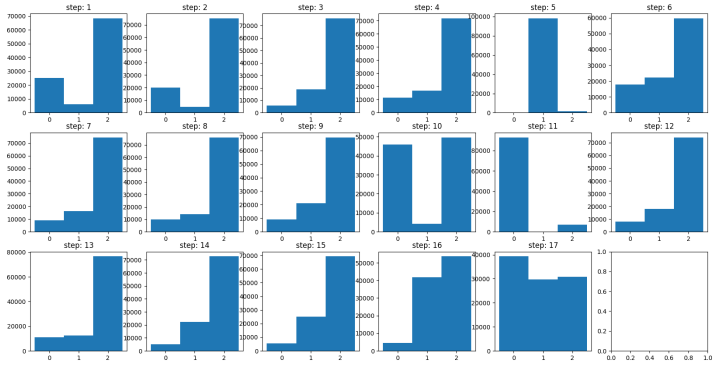

Figure 27: Histogram of posterior distribution of states in each step

2.3.6.3 marginal evidence estimate

marginal log evidence estimate of states in each step is: **-44.407**
marginal evidence estimate of states in each step is: $5.1775 * 10^{-20}$

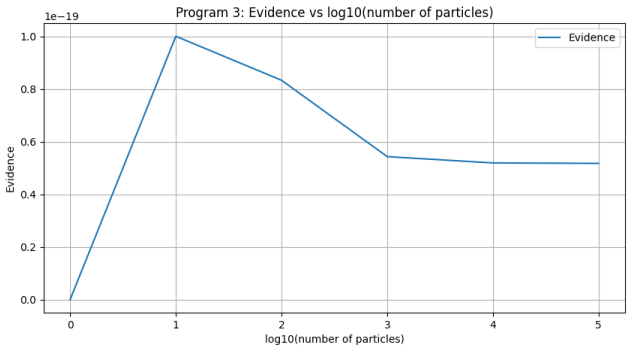

Figure 28: Marginal evidence estimate of mu for different number of particles

2.4 Task 4

2.4.1 Number of particles: 1

2.4.1.1 Posterior expectation

Posterior expected value of mu is: **2.8495**

2.4.1.2 Histogram

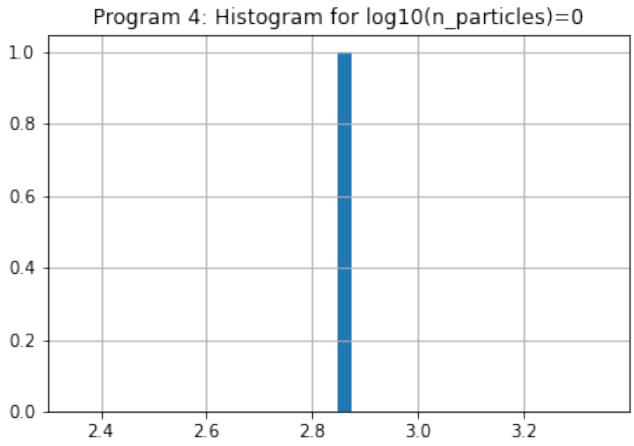

Figure 29: Histogram of posterior distribution of mu

2.4.1.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-35.165577**
marginal evidence estimate of mu is: $5.3429 * 10^{-16}$

2.4.2 Number of particles: 10

2.4.2.1 Posterior expectation

Posterior expected value of mu is: **3.8222**

2.4.2.2 Histogram

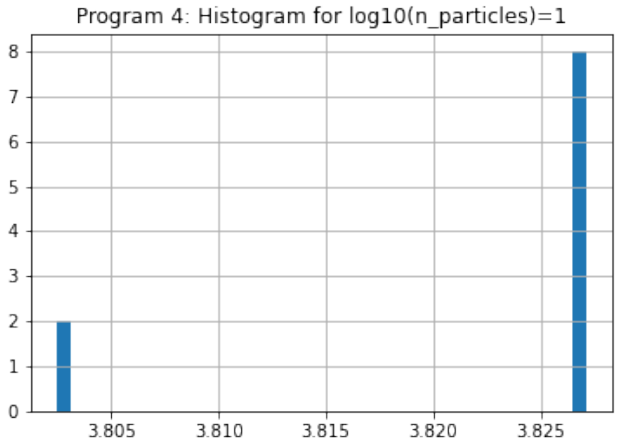

Figure 30: Histogram of posterior distribution of mu

2.4.2.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-15.06296**
marginal evidence estimate of mu is: $2.8724 * 10^{-7}$

2.4.3 Number of particles: 10^2

2.4.3.1 Posterior expectation

Posterior expected value of mu is: **7.3450**

2.4.3.2 Histogram

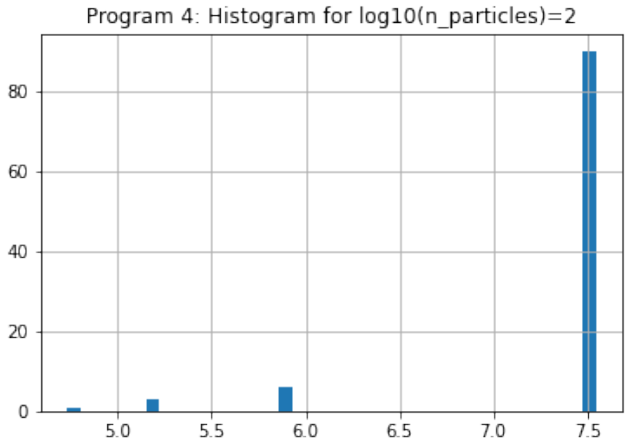

Figure 31: Histogram of posterior distribution of mu

2.4.3.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-7.62482**
marginal evidence estimate of mu is: **0.000488**

2.4.4 Number of particles: 10^3

2.4.4.1 Posterior expectation

Posterior expected value of mu is: **7.1552**

2.4.4.2 Histogram

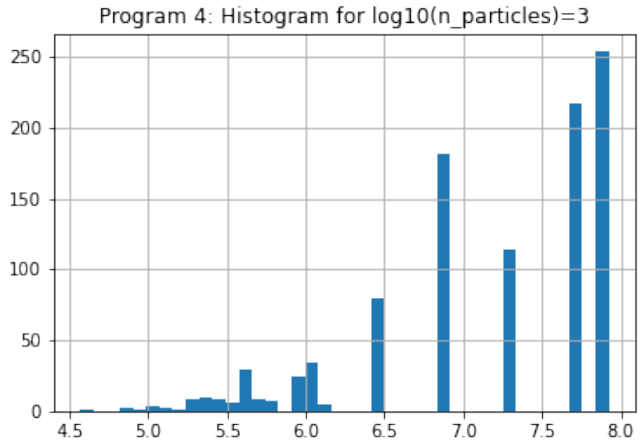

Figure 32: Histogram of posterior distribution of mu

2.4.4.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-7.9910**
marginal evidence estimate of mu is: **0.00034**

2.4.5 Number of particles: 10^4

2.4.5.1 Posterior expectation

Posterior expected value of mu is: **7.1179**

2.4.5.2 Histogram

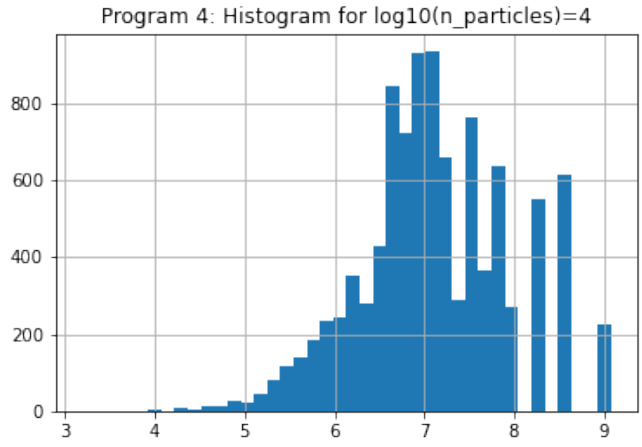

Figure 33: Histogram of posterior distribution of mu

2.4.5.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-8.40138**
marginal evidence estimate of mu is: **0.00023**

2.4.6 Number of particles: 10^5

2.4.6.1 Posterior expectation

Posterior expected value of mu is: **7.2363**

2.4.6.2 Histogram

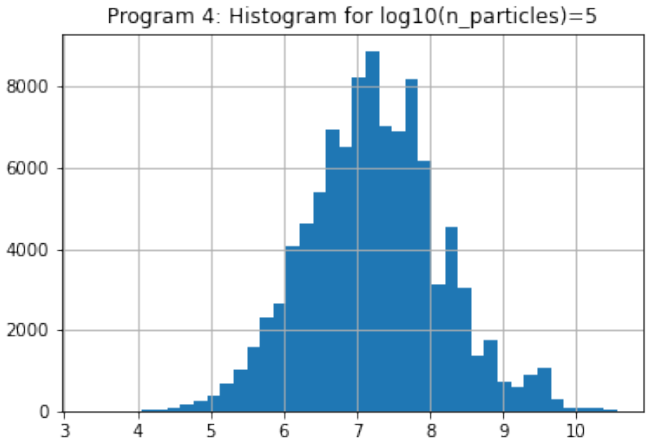

Figure 34: Histogram of posterior distribution of mu

2.4.6.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-8.22765**
marginal evidence estimate of mu is: **0.00027**

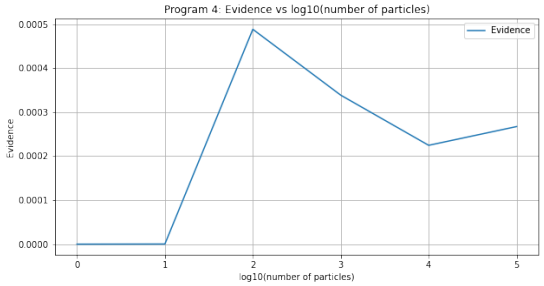

Figure 35: Marginal evidence estimate of mu for different number of particles

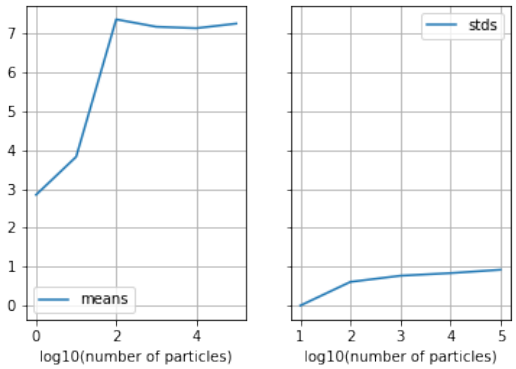

Figure 36: Marginal mean and std of mu for different number of particles