

# CPSC 532W Assignment 6

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Here is a link to the repository:

[https://github.com/aliseyfi75/Probabilistic-Programming/tree/master/Assignment\\_6](https://github.com/aliseyfi75/Probabilistic-Programming/tree/master/Assignment_6)

## 1 Code

### 1.1 evaluator

```
1 def evaluate(exp, env=None):
2
3     if env is None:
4         env = standard_env()
5
6     if type(exp) is list:
7         op, *args = exp
8         if op == 'sample':
9             alpha = evaluate(args[0], env=env)
10            d = evaluate(args[1], env=env)
11            s = d.sample()
12            k = evaluate(args[2], env=env)
13            sigma = {'type' : 'sample', 'alpha' : alpha, 'sample' : s}
14            return k, [s], sigma
15        elif op == 'observe':
16            alpha = evaluate(args[0], env=env)
17            d = evaluate(args[1], env=env)
18            c = evaluate(args[2], env=env)
19            k = evaluate(args[3], env=env)
20            sigma = {'type' : 'observe', 'alpha' : alpha, 'log_prob' : d.log_prob(c)}
21            return k, [c], sigma
22        elif op == 'if':
23            cond,conseq,alt = args
24            if evaluate(cond, env=env):
25                return evaluate(conseq, env=env)
26            else:
27                return evaluate(alt, env=env)
28        elif op == 'fn':
29            params, body = args #fn is: ['fn', ['arg1','arg2','arg3'], body_exp]
30            return Procedure(params, body, env)
31        else: #func eval
32            proc = evaluate(op, env=env)
33            values = [evaluate(e, env=env) for e in args]
34            sigma = {'type' : 'proc'}
35            return proc, values, sigma
36    elif type(exp) is str:
37        if exp[0] == "\": # strings have double, double quotes
38            return exp[1:-1]
39        if exp[0:4] == 'addr':
40            return exp[4:]
41        lowest_env = env.find(exp)
42        return lowest_env[exp]
43    elif type(exp) is float or type(exp) is int or type(exp) is bool:
44        return torch.tensor(exp)
45    else:
46        raise ValueError('Expression type unkown')
```

Listing 1: evaluator.py - evaluate

## 1.2 SMC

```
1 particles = []
2 weights = []
3 logZs = []
4 output = lambda x: x
5
6 for i in range(n_particles):
7     res = evaluate(exp, env=None)('addr_start', output)
8     logW = 0.
9     particles.append(res)
10    weights.append(logW)
11
12 done = False
13 smc_cnter = 0
14 while not done:
15     new_address = ''
16     print('In SMC step {}, Zs: {}'.format(smc_cnter, logZs))
17     for i in range(n_particles):
18         res = run_until_observe_or_end(particles[i])
19         if 'done' in res[2]:
20             particles[i] = res[0]
21             if i == 0:
22                 done = True
23                 address = ''
24             else:
25                 if not done:
26                     raise RuntimeError('Failed SMC, finished one calculation before the
other')
27             else:
28                 if i == 0:
29                     new_address = res[2]['alpha']
30                 else:
31                     address = res[2]['alpha']
32                     if address != new_address:
33                         raise RuntimeError('Failed SMC, address changed')
34
35             log_prob = res[2]['log_prob']
36             weights[i] = weights[i] + log_prob
37             particles[i] = res
38
39     if not done:
40         logZn, particles = resample_particles(particles, weights)
41         logZs.append(logZn)
42         weights = [0.] * n_particles
43     smc_cnter += 1
44 logZ = sum(logZs)
45 return logZ, particles
```

Listing 2: smc.py - SMC

```
1 def resample_particles(particles, log_weights):
2     paricles_length = len(particles)
3     weights = torch.exp(torch.FloatTensor(log_weights)) # convert to weights
4     normalized_weights = weights + 1e-10 # add a small number to avoid zero weights
5     normalized_weights = normalized_weights / normalized_weights.sum() # normalize weights
6
7     logZ = torch.log(torch.mean(weights)) # calculate logZ
8
9     indices = torch.multinomial(normalized_weights, paricles_length, replacement=True)
10    new_particles = [particles[i] for i in indices]
11
12    return logZ, new_particles
```

Listing 3: smc.py - resample\_particles

## 2 Results

### 2.1 Task 1

#### 2.1.1 Number of particles: 1

##### 2.1.1.1 Posterior expectation

Posterior expected value of until success is: **22**

##### 2.1.1.2 Histogram

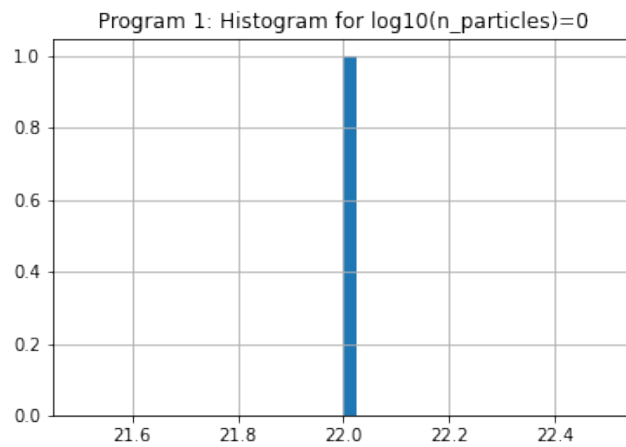


Figure 1: Histogram of posterior distribution of until success

##### 2.1.1.3 marginal evidence estimate

marginal evidence estimate of until success is: **NA**

### 2.1.2 Number of particles: 10

#### 2.1.2.1 Posterior expectation

Posterior expected value of until success is: **73.7**

#### 2.1.2.2 Histogram

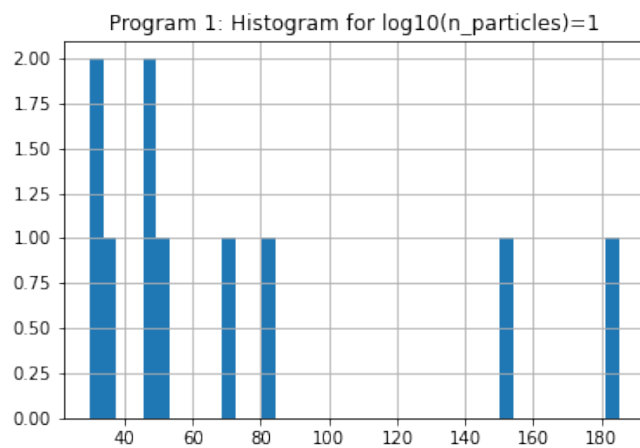


Figure 2: Histogram of posterior distribution of until success

#### 2.1.2.3 marginal evidence estimate

marginal evidence estimate of until success is: **NA**

### 2.1.3 Number of particles: $10^2$

#### 2.1.3.1 Posterior expectation

Posterior expected value of until success is: **97.35**

#### 2.1.3.2 Histogram

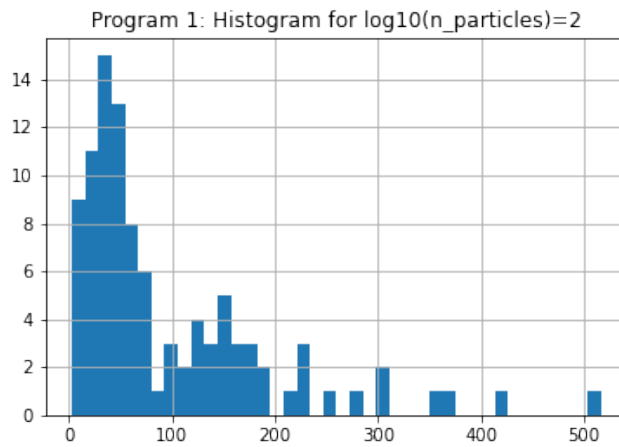


Figure 3: Histogram of posterior distribution of until success

#### 2.1.3.3 marginal evidence estimate

marginal evidence estimate of until success is: **NA**

#### 2.1.4 Number of particles: $10^3$

##### 2.1.4.1 Posterior expectation

Posterior expected value of until success is: **97.474**

##### 2.1.4.2 Histogram

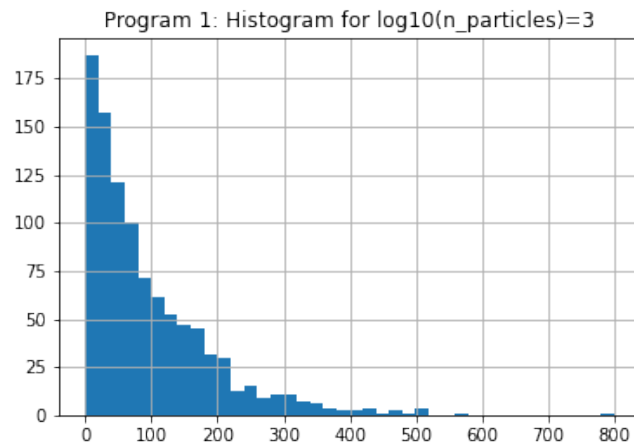


Figure 4: Histogram of posterior distribution of until success

##### 2.1.4.3 marginal evidence estimate

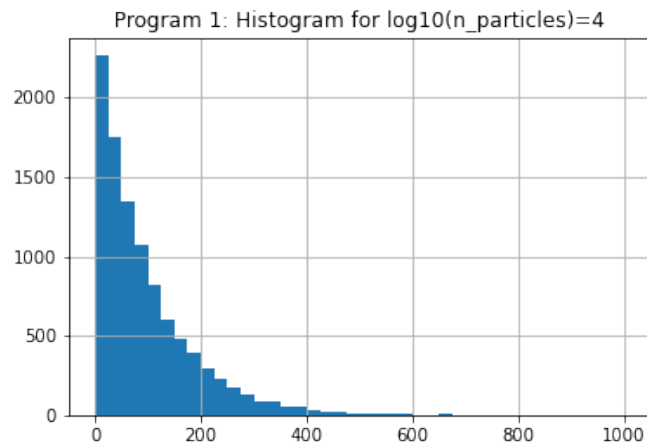
marginal evidence estimate of until success is: **NA**

### 2.1.5 Number of particles: $10^4$

#### 2.1.5.1 Posterior expectation

Posterior expected value of until success is: **96.4183**

#### 2.1.5.2 Histogram



## 2.1.6 Number of particles: $10^5$

### 2.1.6.1 Posterior expectation

Posterior expected value of until success is: **98.9192**

### 2.1.6.2 Histogram

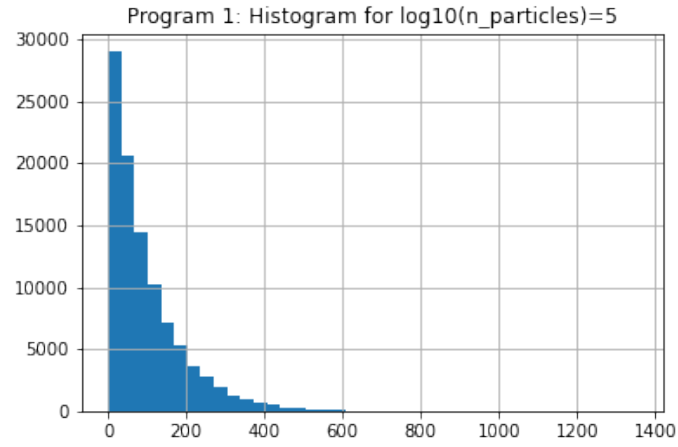


Figure 6: Histogram of posterior distribution of until success

### 2.1.6.3 marginal evidence estimate

marginal evidence estimate of until success is: **NA**

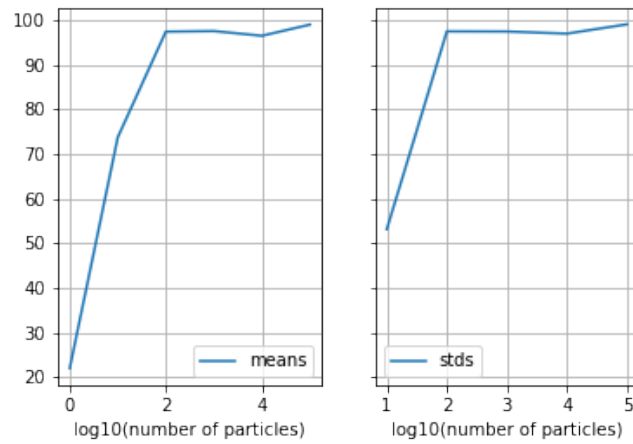


Figure 7: Marginal mean and std of until success for different number of particles



## 2.2 Task 2

### 2.2.1 Number of particles: 1

#### 2.2.1.1 Posterior expectation

Posterior expected value of mu is: **0.8076**

#### 2.2.1.2 Histogram

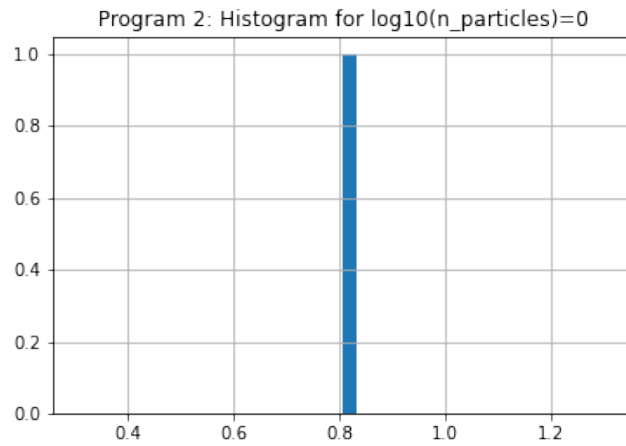


Figure 8: Histogram of posterior distribution of mu

#### 2.2.1.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-35.7038**

marginal evidence estimate of mu is:  $3.1191 * 10^{-16}$

## 2.2.2 Number of particles: 10

### 2.2.2.1 Posterior expectation

Posterior expected value of mu is: **4.9418**

### 2.2.2.2 Histogram

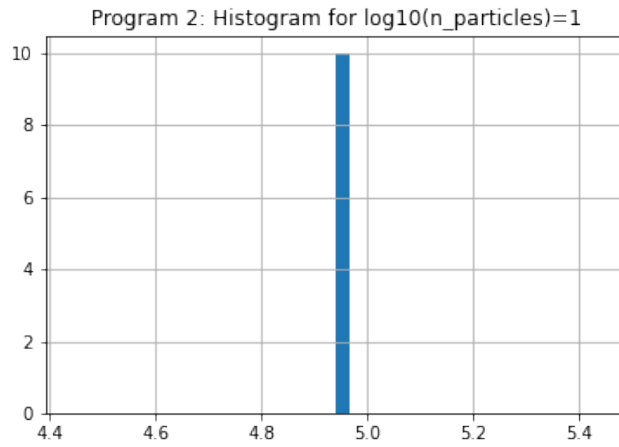


Figure 9: Histogram of posterior distribution of mu

### 2.2.2.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-15.9816**

marginal evidence estimate of mu is:  $1.1462 * 10^{-7}$

### 2.2.3 Number of particles: $10^2$

#### 2.2.3.1 Posterior expectation

Posterior expected value of mu is: **4.3897**

#### 2.2.3.2 Histogram

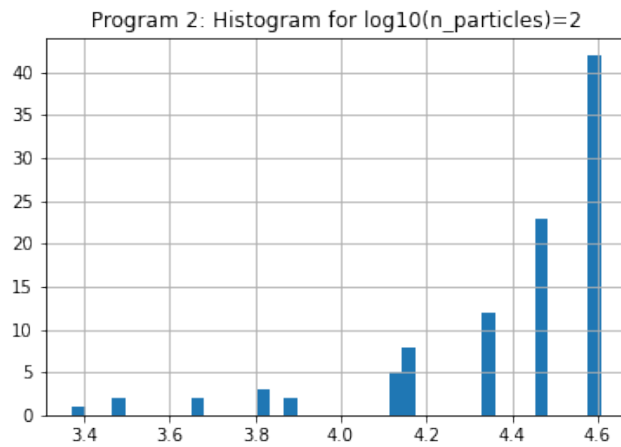


Figure 10: Histogram of posterior distribution of mu

#### 2.2.3.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-9.0384**

marginal evidence estimate of mu is: **0.0001187**

## 2.2.4 Number of particles: $10^3$

### 2.2.4.1 Posterior expectation

Posterior expected value of mu is: **7.5386**

### 2.2.4.2 Histogram

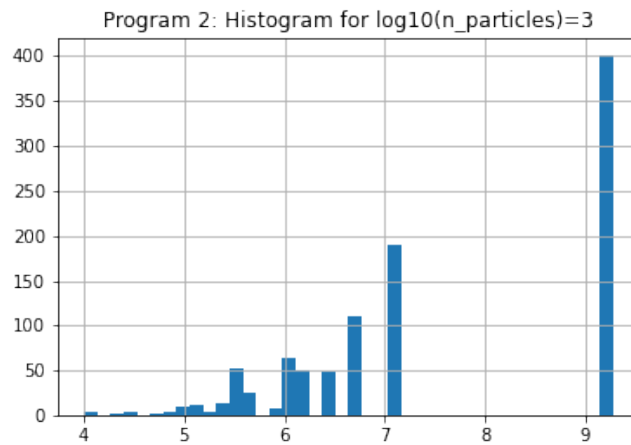


Figure 11: Histogram of posterior distribution of mu

### 2.2.4.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-7.9729**

marginal evidence estimate of mu is: **0.000344**

## 2.2.5 Number of particles: $10^4$

### 2.2.5.1 Posterior expectation

Posterior expected value of mu is: **7.2384**

### 2.2.5.2 Histogram

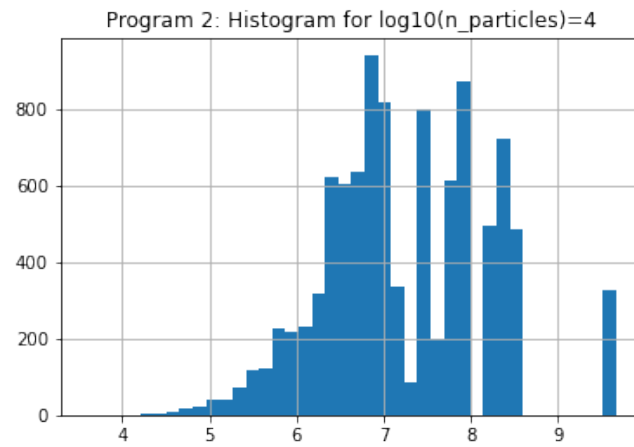


Figure 12: Histogram of posterior distribution of mu

### 2.2.5.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-8.23083**

marginal evidence estimate of mu is: **0.00027**

## 2.2.6 Number of particles: $10^5$

### 2.2.6.1 Posterior expectation

Posterior expected value of mu is: **7.2103**

### 2.2.6.2 Histogram

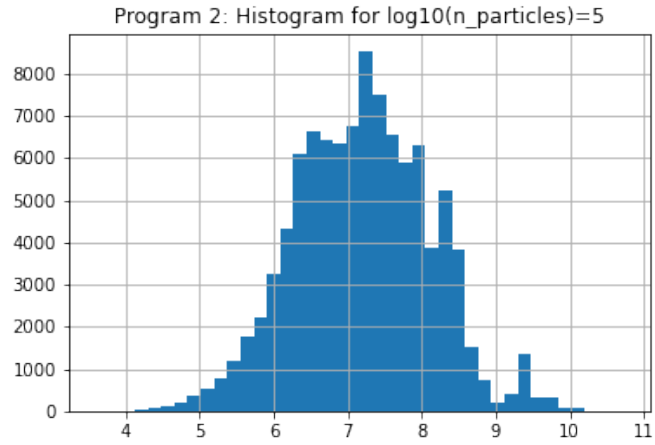


Figure 13: Histogram of posterior distribution of mu

### 2.2.6.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-8.23384**

marginal evidence estimate of mu is: **0.000266**

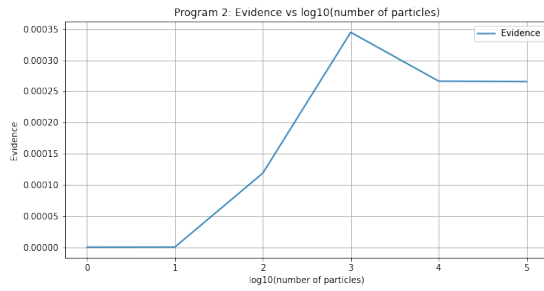


Figure 14: Marginal evidence estimate of mu for different number of particles

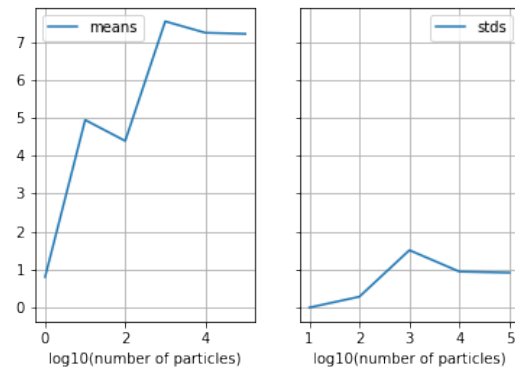


Figure 15: Marginal mean and std of mu for different number of particles

## 2.3 Task 3

### 2.3.1 Number of particles: 1

#### 2.3.1.1 Posterior expectation

Posterior expected value of states in each step is: **1,0,0,2,2,0,1,0,2,2,2,2,2,2,2,1**

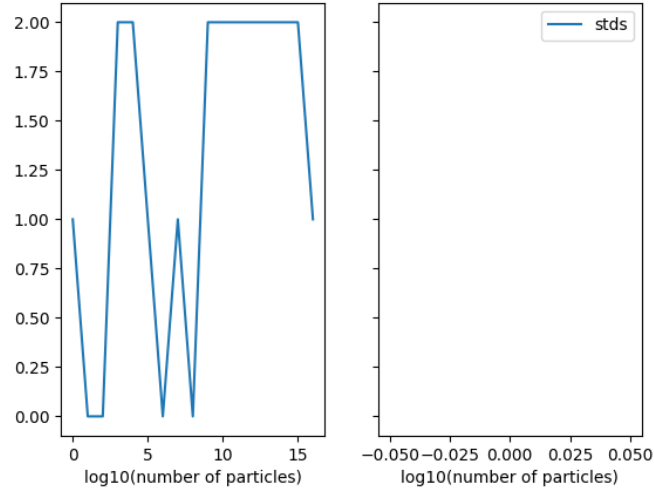


Figure 16: Marginal mean and std of states in each step

#### 2.3.1.2 Histogram

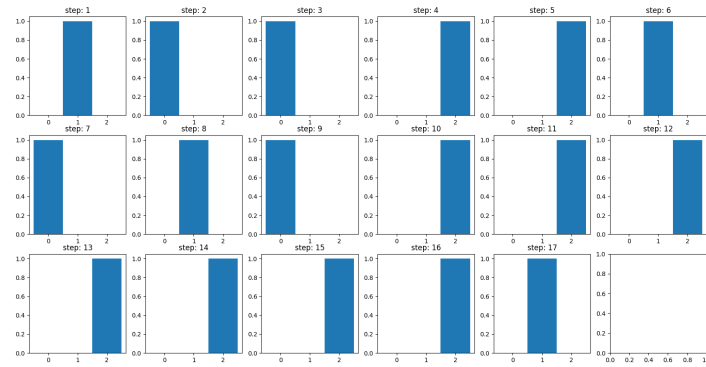


Figure 17: Histogram of posterior distribution of states in each step

#### 2.3.1.3 marginal evidence estimate

marginal log evidence estimate of states in each step is: **-52.73303**

marginal evidence estimate of states in each step is:  $1.254118 * 10^{-23}$

### 2.3.2 Number of particles: 10

#### 2.3.2.1 Posterior expectation

Posterior expected value of states in each step is: []

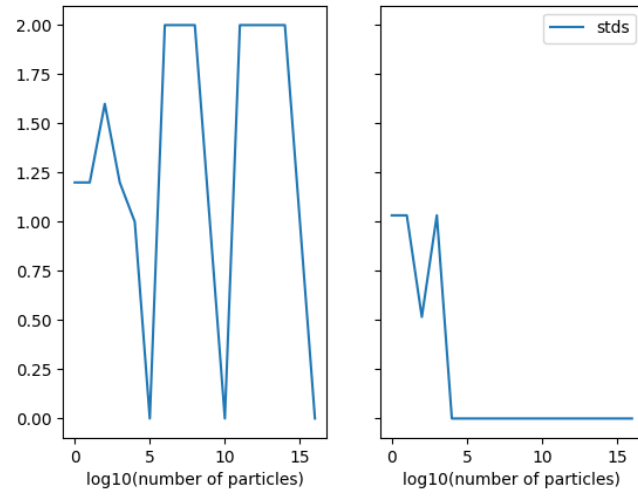


Figure 18: Marginal mean and std of states in each step



### 2.3.2.2 Histogram

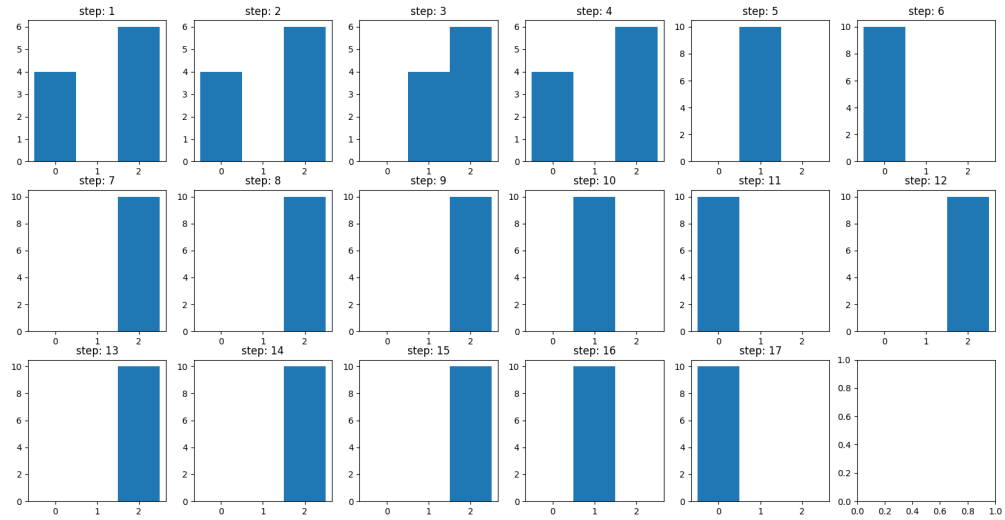


Figure 19: Histogram of posterior distribution of states in each step

### 2.3.2.3 marginal evidence estimate

marginal log evidence estimate of states in each step is: **-45.1086**

marginal evidence estimate of states in each step is:  $2.56789 * 10^{-20}$

### 2.3.3 Number of particles: $10^2$

#### 2.3.3.1 Posterior expectation

Posterior expected value of states in each step is: ?

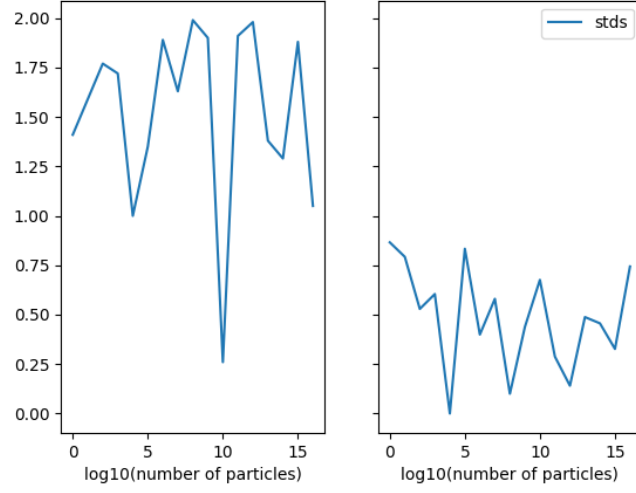


Figure 20: Marginal mean and std of states in each step

### 2.3.3.2 Histogram

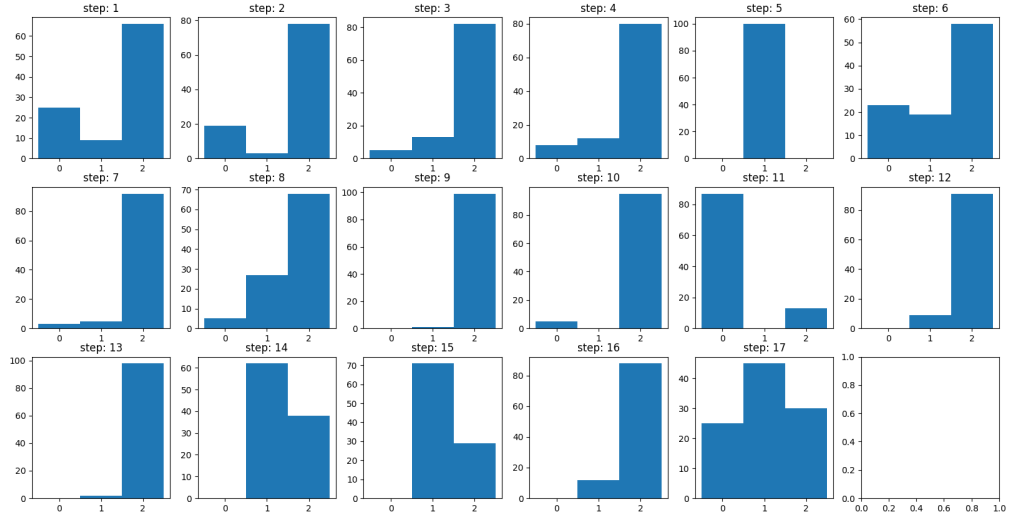


Figure 21: Histogram of posterior distribution of states in each step

### 2.3.3.3 marginal evidence estimate

marginal log evidence estimate of states in each step is: **-44.41735**  
marginal evidence estimate of states in each step is:  $5.12611 * 10^{-20}$

### 2.3.4 Number of particles: $10^3$

#### 2.3.4.1 Posterior expectation

Posterior expected value of states in each step is: ?

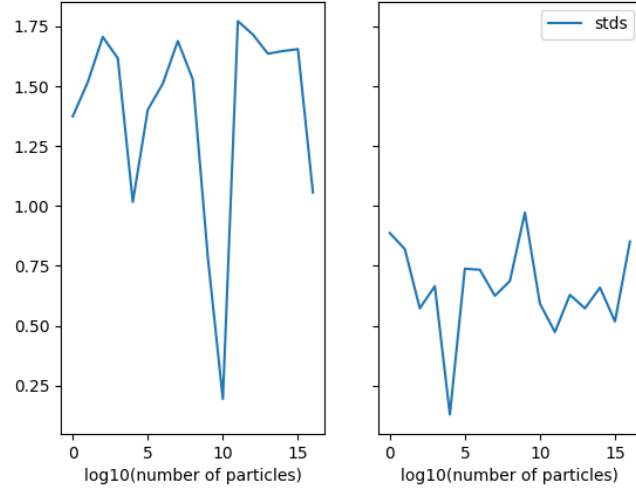


Figure 22: Marginal mean and std of states in each step

#### 2.3.4.2 Histogram

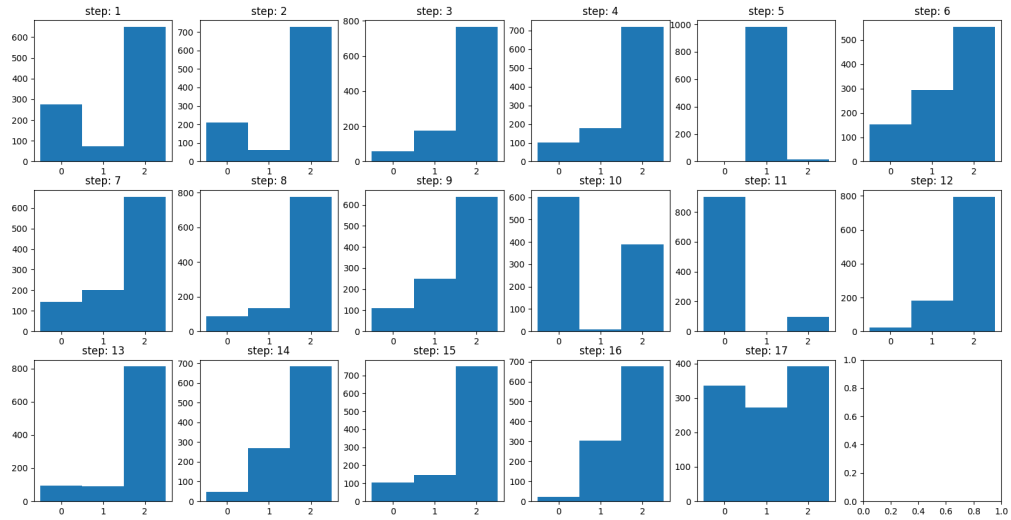


Figure 23: Histogram of posterior distribution of states in each step

### 2.3.4.3 marginal evidence estimate

marginal log evidence estimate of states in each step is: **-44.29677**  
marginal evidence estimate of states in each step is:  $5.783042 * 10^{-20}$

### 2.3.5 Number of particles: $10^4$

#### 2.3.5.1 Posterior expectation

Posterior expected value of states in each step is: ?

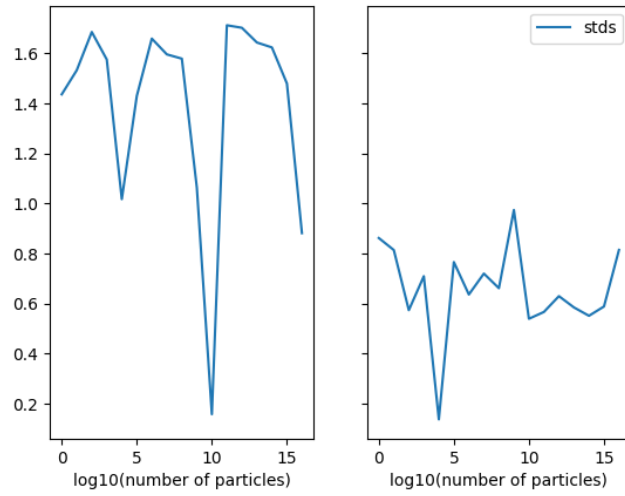


Figure 24: Marginal mean and std of states in each step

### 2.3.5.2 Histogram

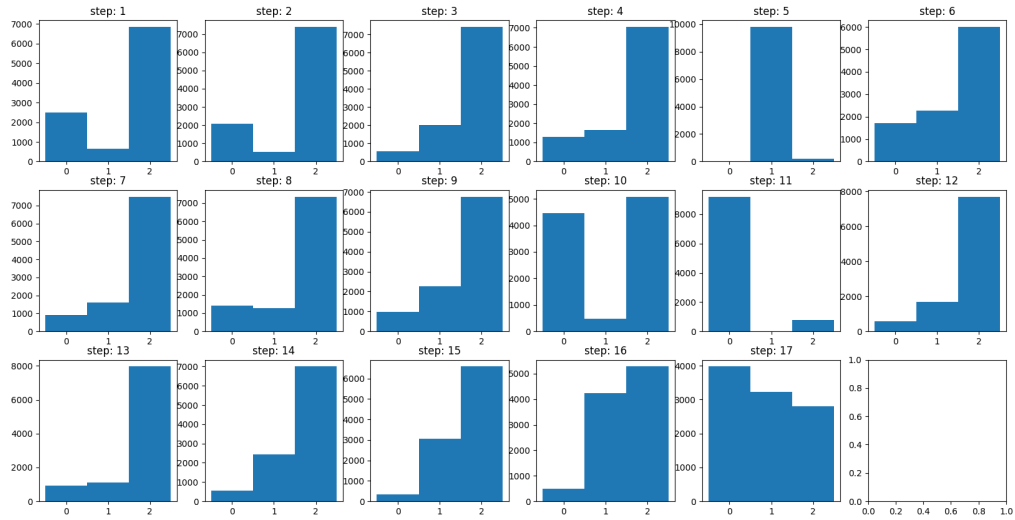


Figure 25: Histogram of posterior distribution of states in each step

### 2.3.5.3 marginal evidence estimate

marginal log evidence estimate of states in each step is: **-44.44874**

marginal evidence estimate of states in each step is:  $4.9677 * 10^{-20}$

### 2.3.6 Number of particles: $10^5$

#### 2.3.6.1 Posterior expectation

Posterior expected value of states in each step is: ?

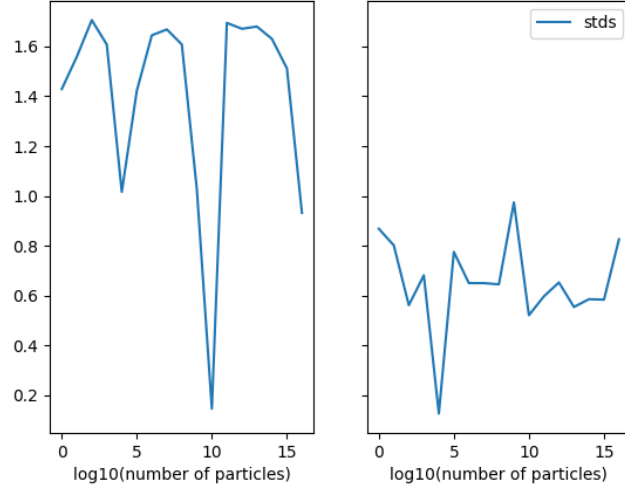


Figure 26: Marginal mean and std of states in each step

### 2.3.6.2 Histogram

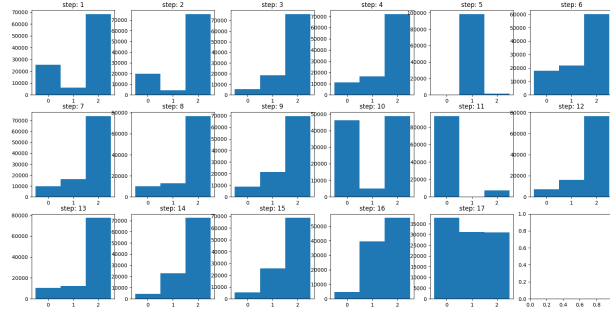


Figure 27: Histogram of posterior distribution of states in each step

### 2.3.6.3 marginal evidence estimate

marginal log evidence estimate of states in each step is: **-44.43314**  
marginal evidence estimate of states in each step is:  $5.04581 \times 10^{-20}$

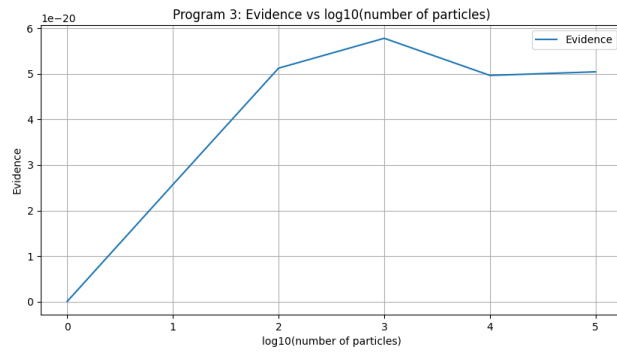


Figure 28: Marginal evidence estimate of  $\mu$  for different number of particles

## 2.4 Task 4

### 2.4.1 Number of particles: 1

#### 2.4.1.1 Posterior expectation

Posterior expected value of mu is: **2.8495**

#### 2.4.1.2 Histogram

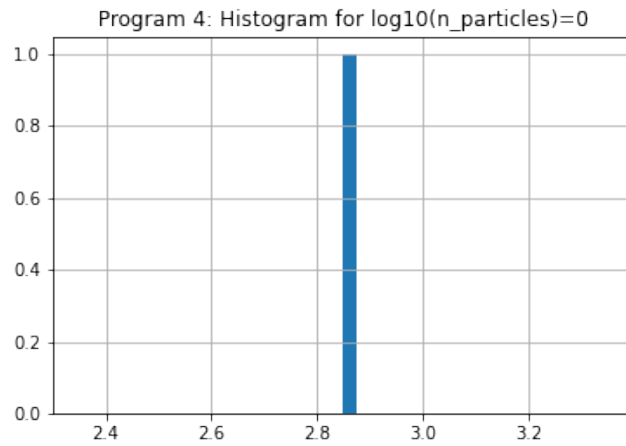


Figure 29: Histogram of posterior distribution of mu

#### 2.4.1.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-35.165577**

marginal evidence estimate of mu is:  $5.3429 * 10^{-16}$



## 2.4.2 Number of particles: 10

### 2.4.2.1 Posterior expectation

Posterior expected value of mu is: **3.8222**

### 2.4.2.2 Histogram

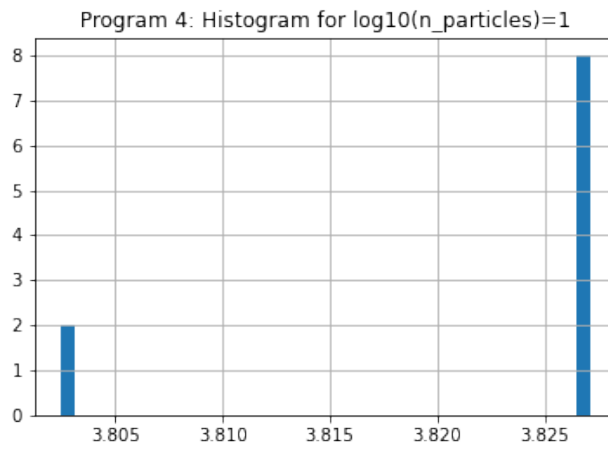


Figure 30: Histogram of posterior distribution of mu

### 2.4.2.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-15.06296**

marginal evidence estimate of mu is:  $2.8724 * 10^{-7}$

### 2.4.3 Number of particles: $10^2$

#### 2.4.3.1 Posterior expectation

Posterior expected value of mu is: **7.3450**

#### 2.4.3.2 Histogram

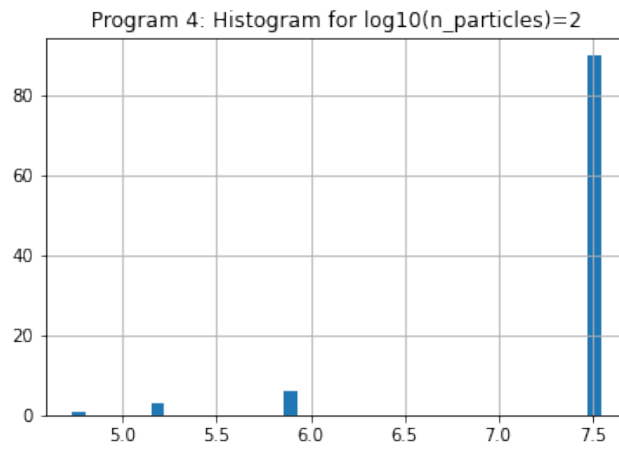


Figure 31: Histogram of posterior distribution of mu

#### 2.4.3.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-7.62482**

marginal evidence estimate of mu is: **0.000488**

#### 2.4.4 Number of particles: $10^3$

##### 2.4.4.1 Posterior expectation

Posterior expected value of mu is: **7.1552**

##### 2.4.4.2 Histogram

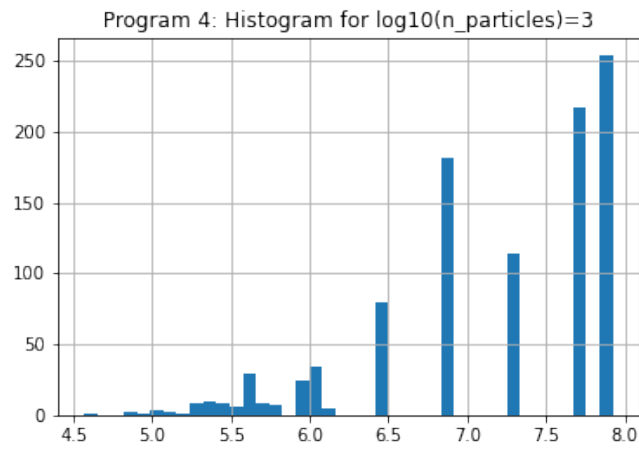


Figure 32: Histogram of posterior distribution of mu

##### 2.4.4.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-7.9910**

marginal evidence estimate of mu is: **0.00034**

### 2.4.5 Number of particles: $10^4$

#### 2.4.5.1 Posterior expectation

Posterior expected value of mu is: **7.1179**

#### 2.4.5.2 Histogram

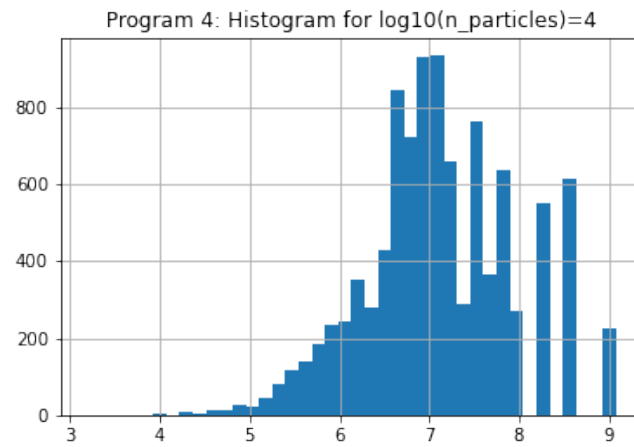


Figure 33: Histogram of posterior distribution of mu

#### 2.4.5.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-8.40138**

marginal evidence estimate of mu is: **0.00023**

## 2.4.6 Number of particles: $10^5$

### 2.4.6.1 Posterior expectation

Posterior expected value of mu is: **7.2363**

### 2.4.6.2 Histogram

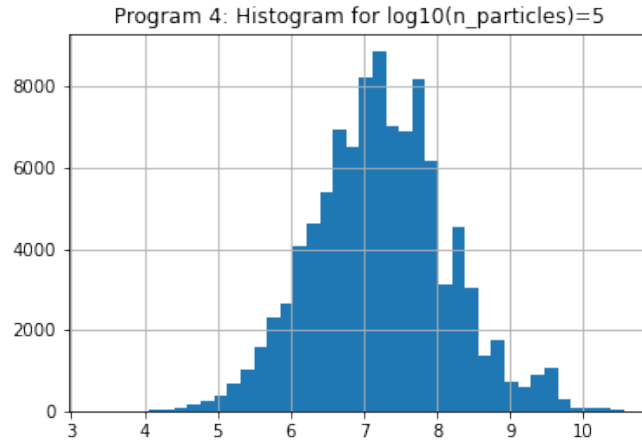


Figure 34: Histogram of posterior distribution of mu

### 2.4.6.3 marginal evidence estimate

marginal log evidence estimate of mu is: **-8.22765**

marginal evidence estimate of mu is: **0.00027**

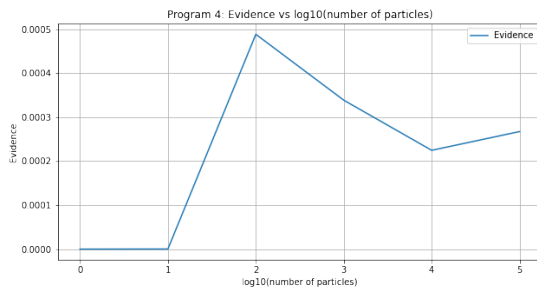


Figure 35: Marginal evidence estimate of mu for different number of particles

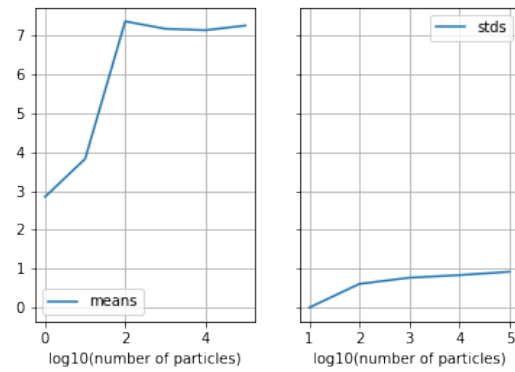


Figure 36: Marginal mean and std of mu for different number of particles