COMP90049 Knowledge Technologies

Introduction to basic probability
(Lecture Set2) 2017

Rao Kotagiri
Department of Computing and Information Systems

The Melbourne School of Engineering

Some of slides are derived from Prof Vipin Kumar and modified, http://www-users.cs.umn.edu/~kumar/

Probability

Discrete Random Variables

• A is a Boolean-valued random variable if A denotes an event, and there is some degree of uncertainty as to whether A occurs. This uncertinity is stated in terms of probability

Examples:

A = The next toss of coin is Head

A = The next toss of a coin is Tail

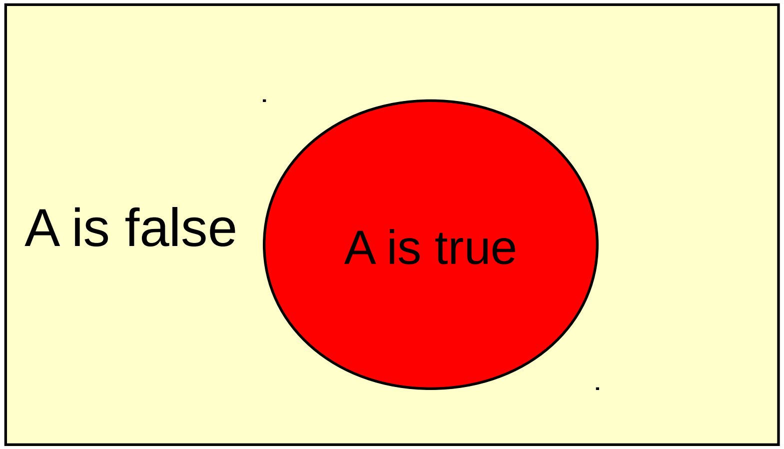
A = The flights will resume next day

Probability

Discrete Random Variables

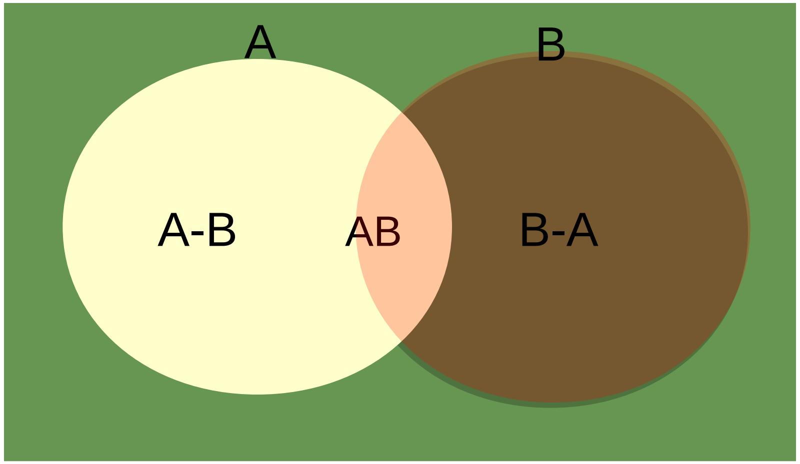
• P(A) = “the fraction of worlds in which A is true” or the fraction of times the event is true in independent trails

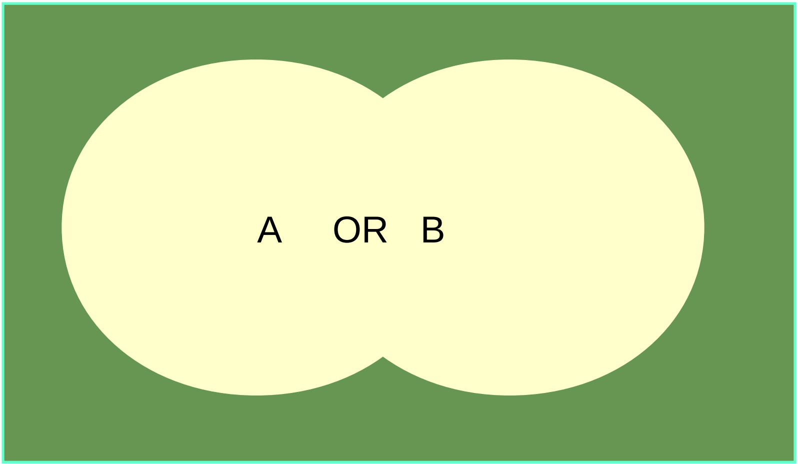
P(A) = Proportion of area of reddish oval.



The Axioms of Probability

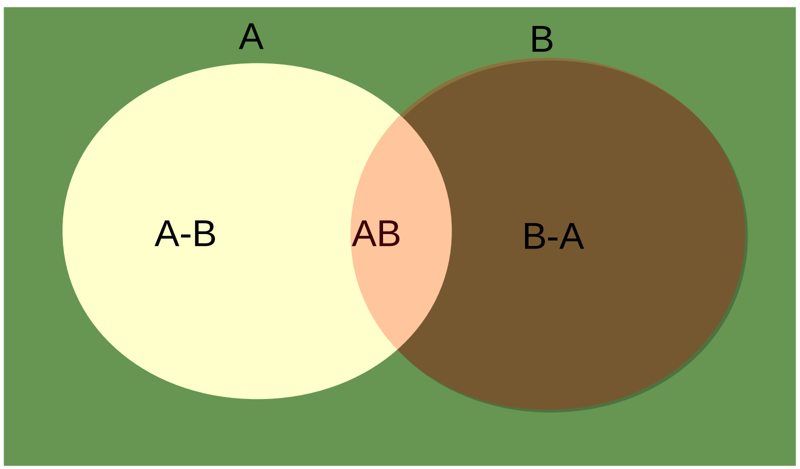
0 <= P(A) <= 1, Head and Tail are mutually exclusive.
P(True) = 1 e.g., p(A = Head or A = Tail) = 1 (we also write P(Head or Tail))
P(False) = 0 e.g. P(A = Head and A = Tail) = 0 (we also write P(Head and Tail))
If A and B are not mutually exclusive
P(A or B) = P(A) + P(B) - P(A and B)





The Axioms of Probability

P(A) = P(A and B) + P(A and Not B)

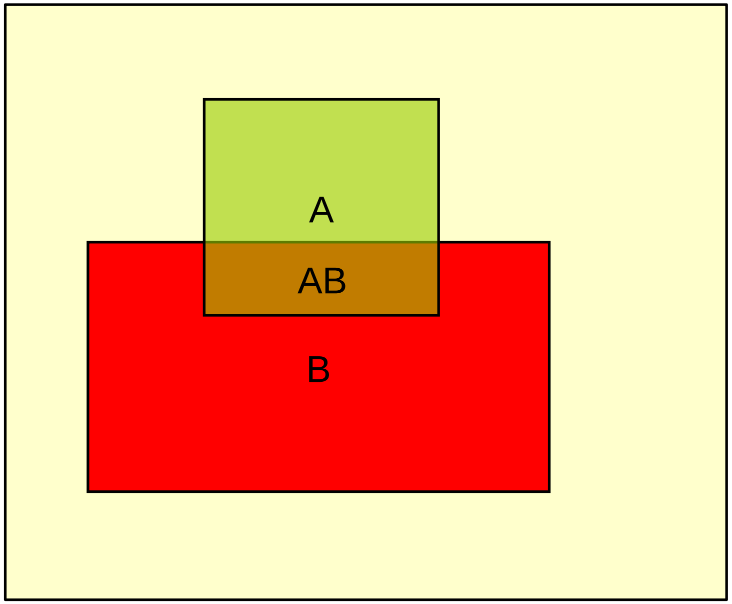


Conditional Probability

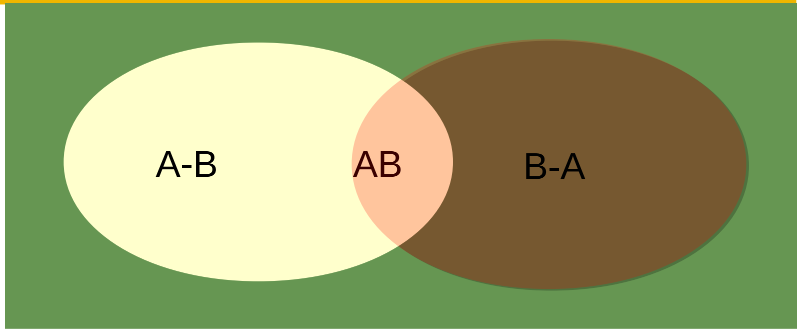
P(A|B) = Fraction of time A is true knowing B is true
= P(A and B)/P(B)

P(A|B) = P(A and B)/P(B)

P(A and B) = P(A|B)*P(B) -- product rule



Probability



Product rule $$p(\mathbf{x},\mathbf{y}) = p(\mathbf{x}|\mathbf{y})p(\mathbf{y})$$

Sum rule
$$p(\mathbf{x}) = \sum_{\mathbf{y}} p(\mathbf{x},\mathbf{y})$$
$$p(\mathbf{x}) = \sum_{\mathbf{y}} p(\mathbf{x}|\mathbf{y})p(\mathbf{y})$$

From these we have Bayes theorem

$$p(y/x) = \frac{p(x,y)}{p(x)} \qquad p(\mathbf{y}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathbf{y})p(\mathbf{y})}{p(\mathbf{x})}$$

$$p(y|x) = \frac{p(x|y)p(y)}{\sum_y p(x|y)p(y)}$$

We use "~" to represent "not", E.g., Not B is represented by ~B

P(A) +P(~A) = 1; P(A) = P(A,B) + P(A, ~B)

P(x|y) +P(~x|y) = 1; But P(x|y) + P(x|~y) ~= 1;

Example:

B = restaurant is bad; S = menu is smudged;

~B = restaurant is good;

p(B) = ½ = 0.5 % Prior probability

p(S|B) = ¾; p(S| ~B) = 1/3 % Likelihood

We are interested to know p(B|S) % Posterior probability

p(B|S) = p(B,S)/p(S) = p(B, S)/[(p(S, B) + p(S,~B)]

= p(S|B)P(B)/[p(S|B)P(B) + p(S|~B)P(~B)] % Bayes theorem

= (3/4)*(1/2)/[(3/4)*(1/2) + (1/3)*(1/2)]

= 9/13 =0.69

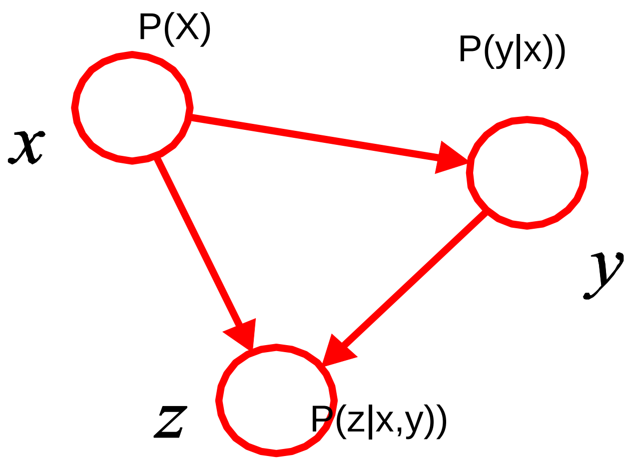
Probabilistic Graphical Models (PGM)

PGM provides new insights into existing models

Consider an arbitrary joint distribution

By successive application of the product rule $p(x, y, z)$

$$\begin{aligned} p(x, y, z) &= p(x)p(y, z|x) \\ &= p(x)p(y|x)p(z|x, y) \end{aligned}$$

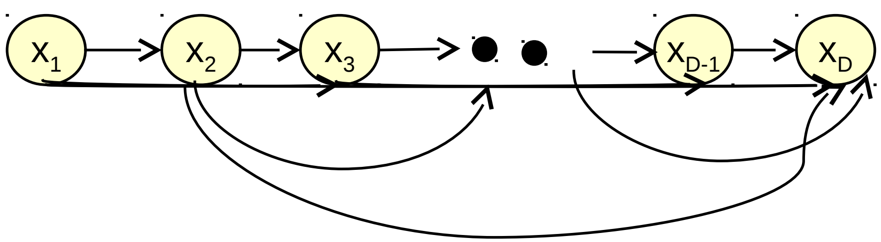


Directed Acyclic Graphs

Joint distribution
where pa$_i$ **denotes the parents of i**

$$p(x_1, \ldots, x_D) = \prod_{i=1}^{D} p(x_i|\mathrm{pa}_i)$$

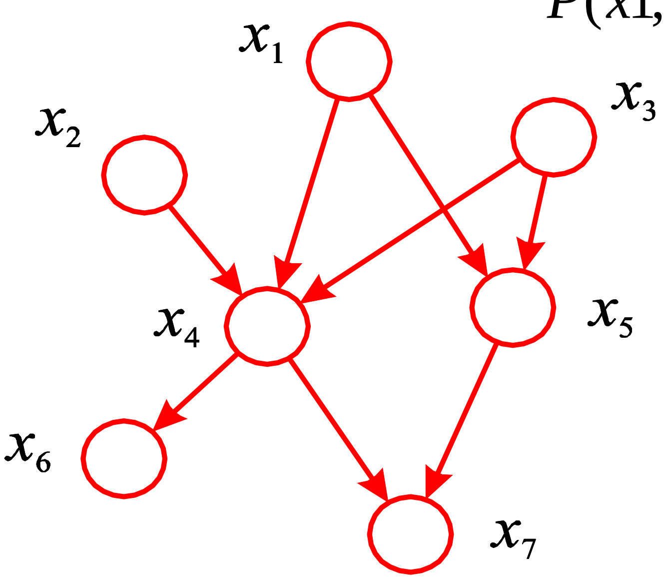
$$\begin{aligned} p(x_1, x_2, \ldots, x_D) &= p(x_1)\, p(x_2, x_3, \ldots x_D \mid x_1) \\ &= p(x_1)\, p(x_2 \mid x_1)\, p(x_3, x_4, \ldots x_D \mid x_1, x_2) \\ &= p(x_1)\, p(x_2 \mid x_1)\, p(x_3 \mid x_1, x_2)\, p(x_4, \ldots x_D \mid x_1, x_2, x_3) \\ &= p(x_1)\, p(x_2 \mid x_1)\, p(x_3 \mid x_1, x_2) \ldots p(x_D \mid x_1, x_2, \ldots, x_{D-1}) \end{aligned}$$



Directed Acyclic Graphs

Joint distribution
where pa$_i$ **denotes the parents of i**

$$p(x_1, \ldots, x_D) = \prod_{i=1}^{D} p(x_i|\mathrm{pa}_i)$$



$$P(x1, x2, x3, x4, x5, x6, x7) = \left\{ \begin{array}{l} P(x1) * P(x2) * p(x3) * \\ P(x4 \mid x1, x2, x3) * \\ P(x5 \mid x1, x3) * \\ P(x6 / x4) * \\ p(x7 \mid x4, x5) \end{array} \right\}$$

"Explaining Away"

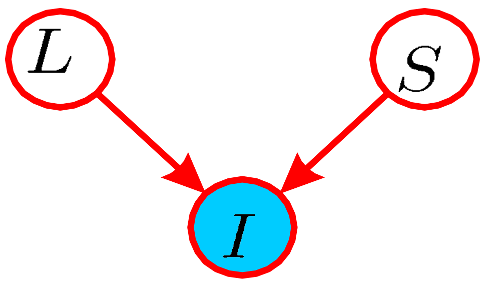
Conditional independence for directed graphs is similar, but with one subtlety

Illustration: pixel colour in an image

$$p(I,L,S) = p(L,S)p(I|L,S)$$

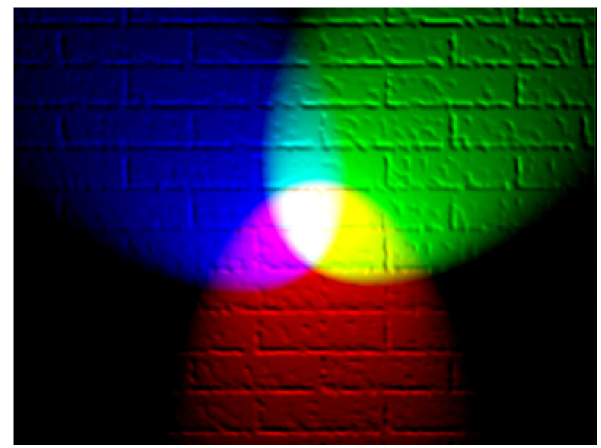
$$p(I,L,S) = p(L)p(S)p(I|L,S)$$

lighting colour of the room



surface colour of the painting

image colour



$$p(L,S) = p(L)p(S)$$

$$p(L,S|I) \neq p(L|I)p(S|I)$$

$$p(I,L,S) \neq p(I)*p(L|I)*p(S|I)$$