
# COMP90049 Knowledge Technologies

Data Mining
Association Analysis (Lecture **Set 8)**
**2017**

Ramamohanarao (Rao) Kotagiri
Department of Computing and Information Systems
The Melbourne School of Engineering

Some of slides are derived from Prof Vipin Kumar and modified, http://www-users.cs.umn.edu/~kumar/

# Association Rule Mining

**Given a set of ( several millions of) transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction**

## Market-Basket transactions

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

## Example of Association Rules

{Diaper} → {Beer},
{ Bread} → {Milk},

{ Bread, Milk} → {Diaper}

Implication means co-occurrence, not causality!

# Definition: Frequent Itemset

**Itemset**

- A collection of one or more items
  Example: {Milk, Bread, Diaper}
- k-itemset
  An itemset that contains k items

**Support count ($\sigma$)**

- Frequency of occurrence of an itemset
- E.g. $\sigma$({Bread, Milk, Diaper}) = 2

**Support**

- Fraction of transactions that contain an itemset
- E.g. s({Milk, Bread, Diaper}) = 2/5

**Frequent Itemset**

- An itemset whose support is greater than or equal to a *minsup* threshold

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

# Definition: Association Rule

## Association Rule

- An implication expression of the form X → Y, where X and Y are itemsets
- Example:
  {Milk, Diaper} → {Beer}

| TID | Items |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Milk, Diaper, Beer, Bread |
| 5 | Milk, Diaper, Bread, Coke |

## Rule Evaluation Metrics

- Support (s)
  Fraction of transactions that contain both X and Y
- Confidence (c)
  Measures how often items in Y appear in transactions that contain X

Example:

$$\{\text{Milk, Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk, Diaper, Beer})}{|T|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk, Diaper, Beer})}{\sigma(\text{Milk, Diaper})} = \frac{2}{3} = 0.67$$

## Association Rule Mining Task

Given a set of transactions T, the goal of association rule mining is to find all rules having

- support ≥ *minsup* threshold
- confidence ≥ *minconf* threshold

Brute-force approach:

- List all possible association rules
- Compute the support and confidence for each rule
- Prune rules that fail the *minsup* and *minconf* thresholds

⇒ Computationally prohibitive!

# Mining Association Rules

| TID | Items |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

## Example of Rules:

{Milk,Diaper} → {Beer} (s=2/5, c=2/3)
{Milk,Beer} → {Diaper} (s=2/5, c=2/2)
{Diaper,Beer} → {Milk} (s=2/5, c=2/3)
{Beer} → {Milk,Diaper} (s=2/5, c=2/3)
{Diaper} → {Milk,Beer} (s=2/5, c=2/4)
{Milk} → {Diaper,Beer} (s=2/5, c=2/4)

## Observations:

- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

# Mining Association Rules

**Two-step approach:**

1. **Frequent Itemset Generation**

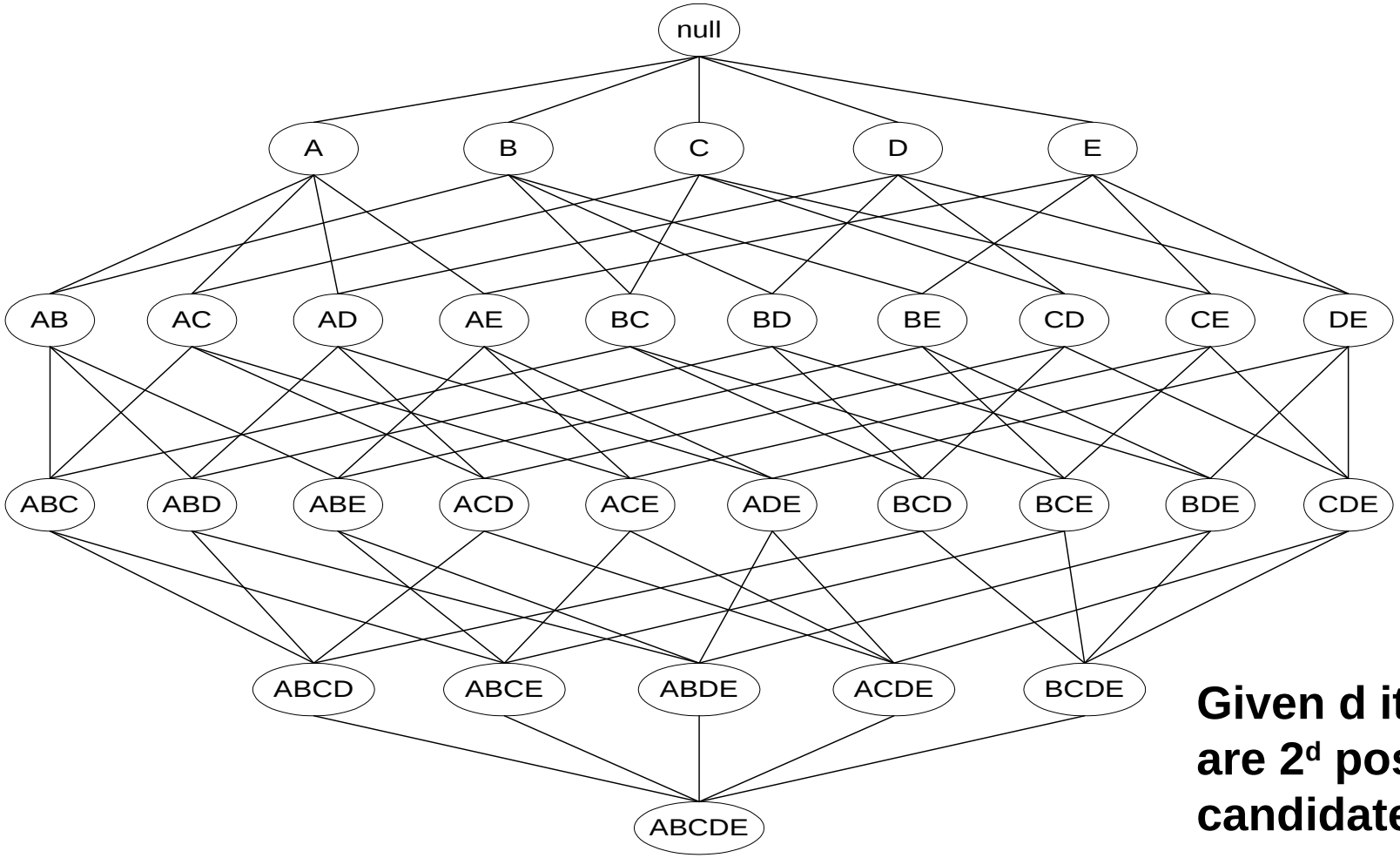
   **Generate all itemsets whose support $\geq$ minsup**

2. **Rule Generation**

   **Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset**

**Frequent itemset generation is still computationally expensive**

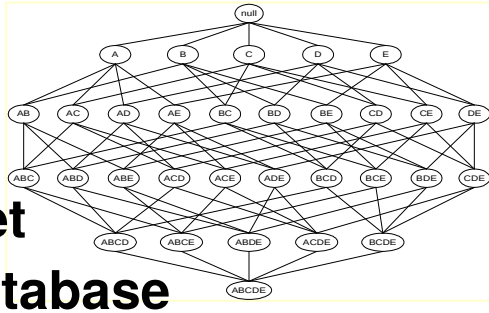
# Frequent Itemset Generation




**Given d items, there are $2^d$ possible candidate itemsets**

# Frequent Itemset Generation

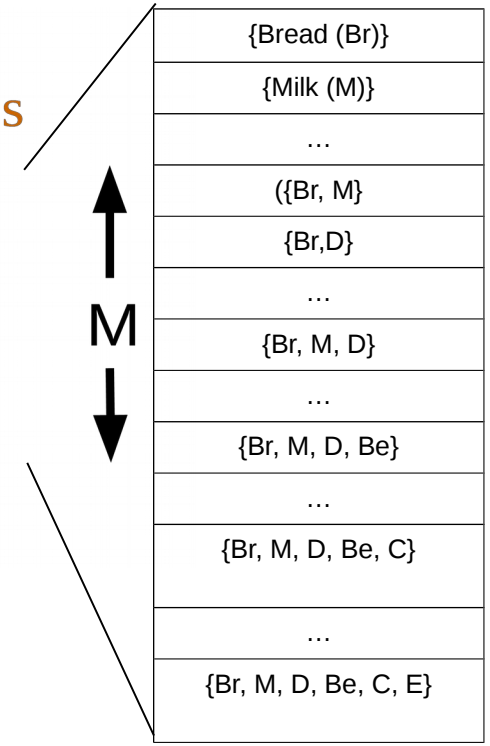
Lattice




**Brute-force approach:**

- **Each itemset in the lattice is a candidate frequent itemset**
- **Count the support of each candidate by scanning the database**

List of Candidates

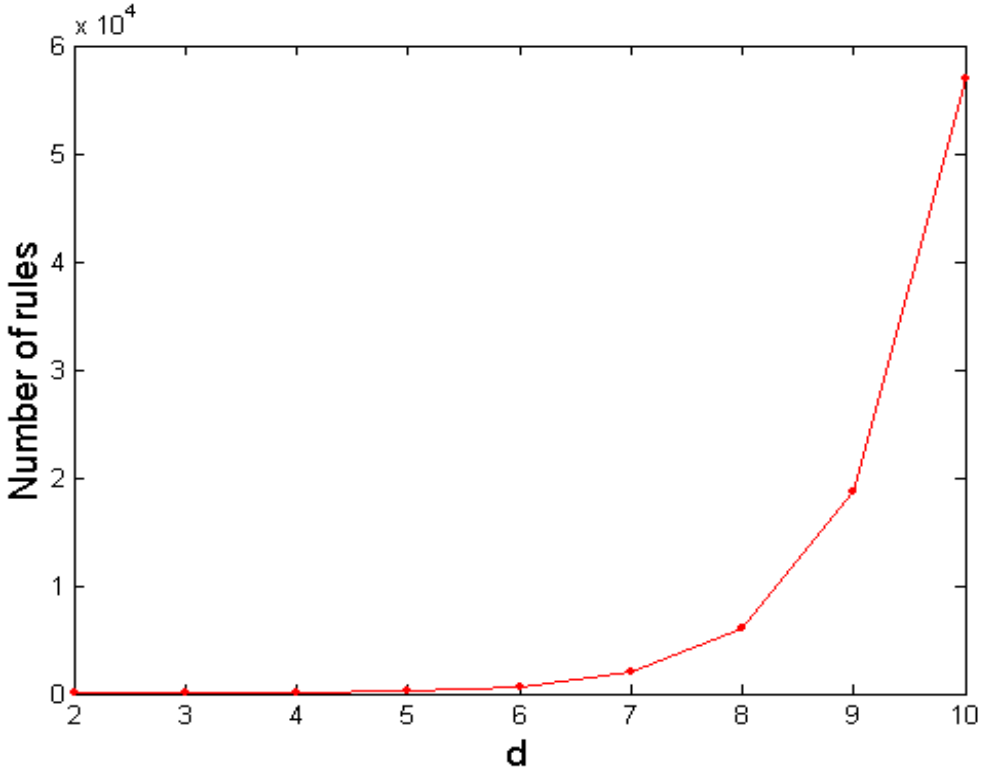




- Match each transaction against every candidate
- Complexity ~ O(NMw) => Expensive since $M = 2^d$ !!!

# Computational Complexity

## Given d unique items:

- Total number of itemsets = $2^d$
- Total number of possible association rules:




#ways left side items can be chosen out of d items

#ways right side items can be chosen using the remaining d-k items

$$R = \sum_{k=1}^{d-1} \binom{d}{k} \times \sum_{j=1}^{d-k} \binom{d-k}{j}$$

$$= 3^d - 2^{d+1} + 1$$

**If d=6, R = 602 rules**

An example d= 3 and item set = {abc}

{a} → {b} {a} → {c} {a} → {bc}
{b} → {a} {b} → {c} {b} → {ac}
{c} → {a} {c} → {b} {c} → {ab}
{ab} → {c} {ac} → {b} {bc} → {a}

# Frequent Itemset Generation Strategies

## Reduce the number of candidates (M)

- Complete search: $M=2^d$
- Use pruning techniques to reduce M

## Reduce the number of transactions (N)

- Reduce size of N as the size of itemset increases
- Used by DHP (Direct Hashing and Pruning) and vertical-based mining algorithms

## Reduce the number of comparisons (NM)

- Use efficient data structures to store the candidates or transactions
- No need to match every candidate against every transaction

# Reducing Number of Candidates

**Apriori principle:**

- If an itemset is frequent, then all of its subsets must also be frequent

**Apriori principle holds due to the following property of the support measure:**

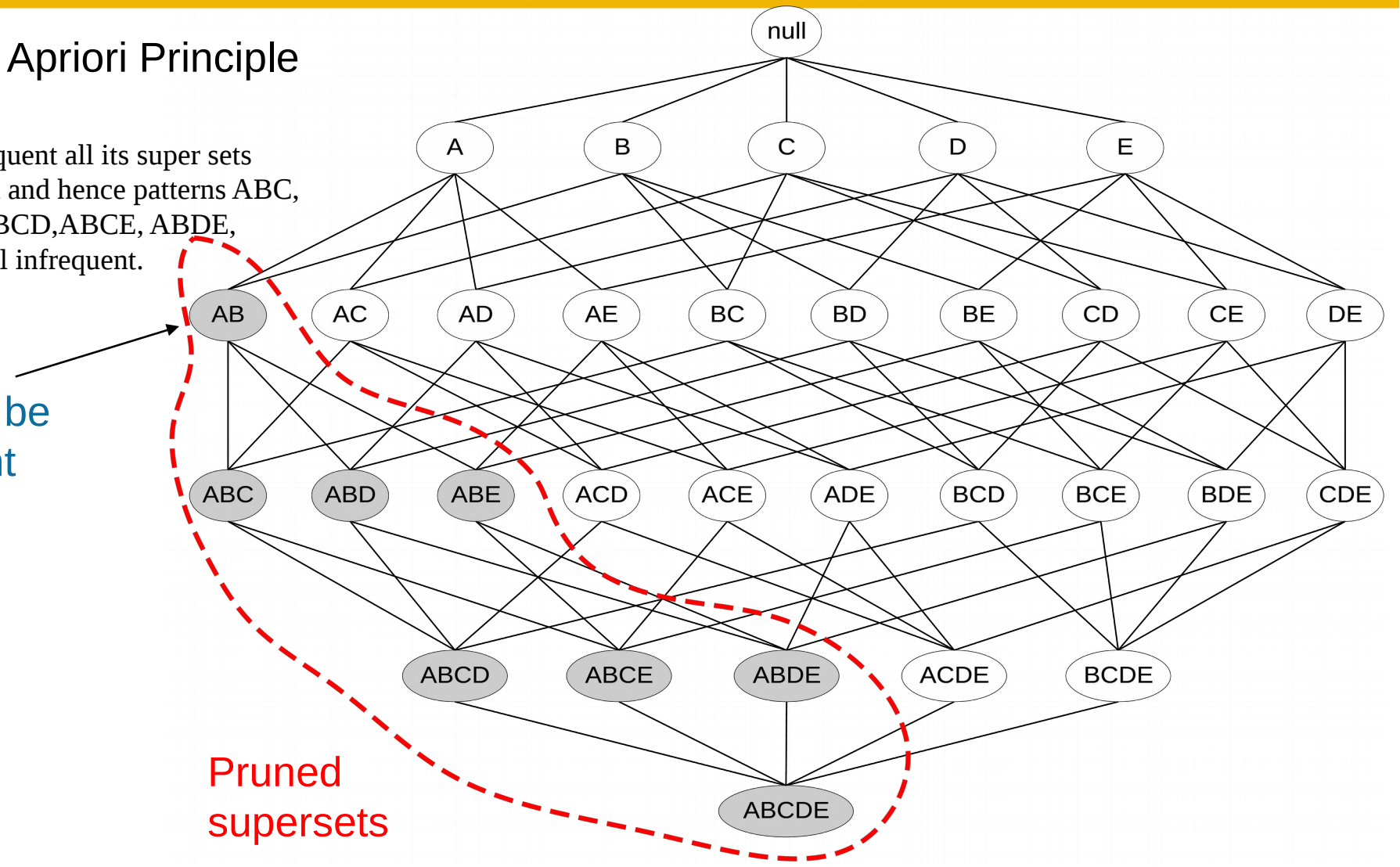
$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

$$Where\ s(X)\ \text{support of}\ X$$

- Support of an itemset never exceeds the support of its subsets
- This is known as the anti-monotone property of support

# Illustrating Apriori Principle

If AB is infrequent all its super sets are in frequent and hence patterns ABC, ABD, ABE,ABCD,ABCE, ABDE, ABCDE are all infrequent.



# Illustrating Apriori Principle

| TID | Items |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

## Items (1-itemsets)

| Item | Count |
|---|---|
| Bread | 4 |
| Coke | 2 |
| Milk | 4 |
| Beer | 3 |
| Diaper | 4 |
| Eggs | 1 |

Minimum Support = 3

Pairs (2-itemsets)
(No need to generate candidates involving Coke or Eggs as min support = 3)

| Itemset | Count |
|---|---|
| {Bread,Milk} | 3 |
| {Bread,Beer} | 2 |
| {Bread,Diaper} | 3 |
| {Milk,Beer} | 2 |
| {Milk,Diaper} | 3 |
| {Beer,Diaper} | 3 |

Triplets (3-itemsets)

| Itemset | Count |
|---|---|
| {Bread,Milk,Diaper} | 2 |

If every subset up to 3 itemsets are considered,
Number of subsets $= {}^6C_1$(itemset size of 1)+ ${}^6C_2$ (itemset size of 2)+ ${}^6C_3$ (itemset size of 3)= 41

With support-based pruning (see tables above),
6 + 6 + 1 = 13

# Apriori Algorithm

## Method:

- Let k=1
- Generate frequent itemsets of length k
- Repeat until no new frequent itemsets are identified
  - Generate length (k+1) candidate itemsets from length k frequent itemsets
  - Prune candidate itemsets containing subsets of length k+1 that are infrequent
  - Count the support of each candidate by scanning the DB
  - Eliminate candidates that are infrequent, leaving only those that are frequent

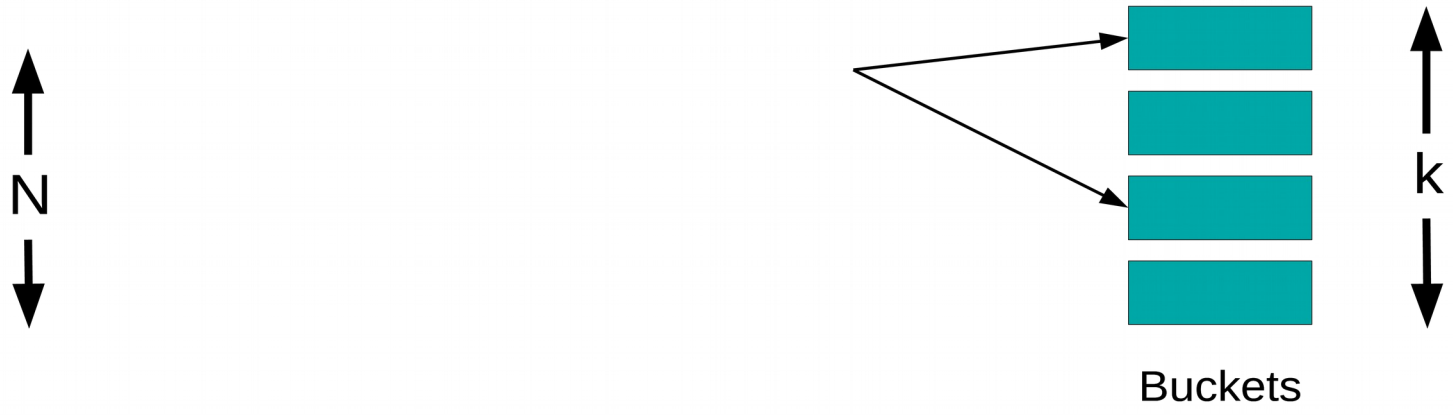
# Reducing Number of Comparisons

## Candidate counting:

- Scan the database of transactions to determine the support of each candidate itemset
- To reduce the number of comparisons, store the candidates in a hash structure

Instead of matching each transaction against every candidate, match it against candidates contained in the hashed buckets

Hash Structure



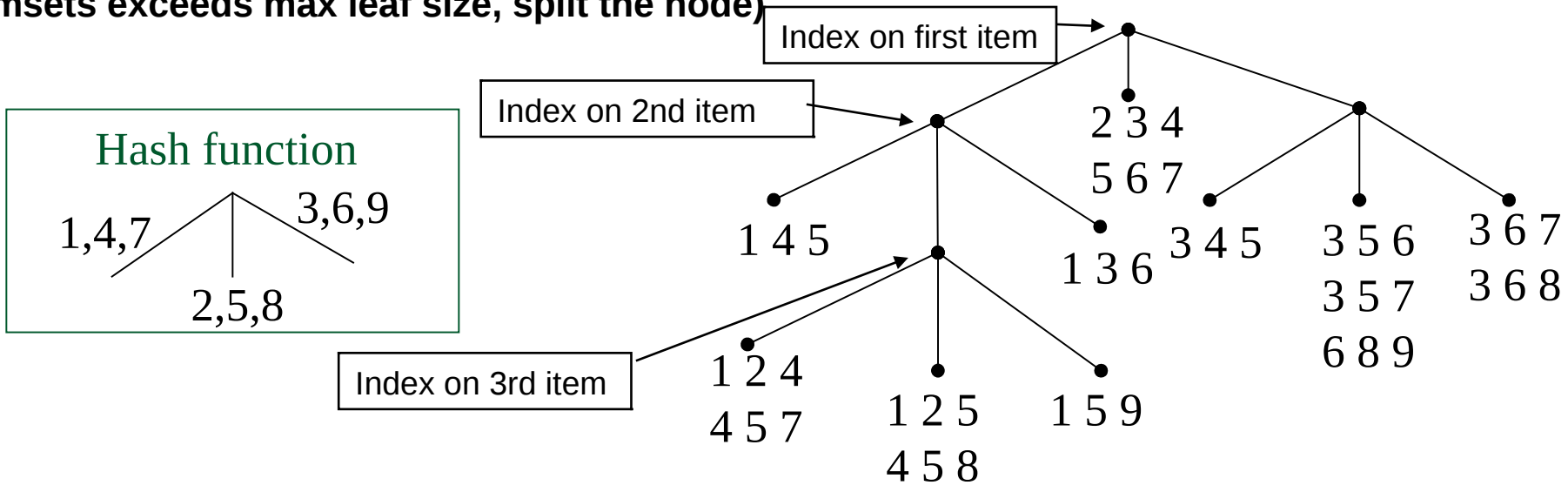
# Generate Hash Tree

**Suppose you have 15 candidate itemsets of length 3:**

**{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}**

**We need:**

- **Hash function**
- **Max leaf size: max number of itemsets stored in a leaf node (if number of candidate itemsets exceeds max leaf size, split the node)**

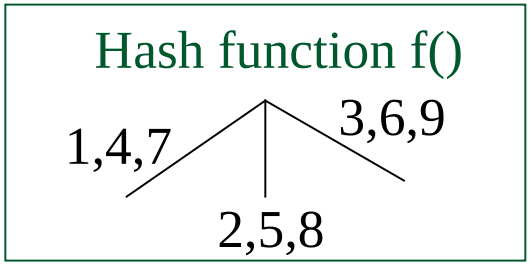


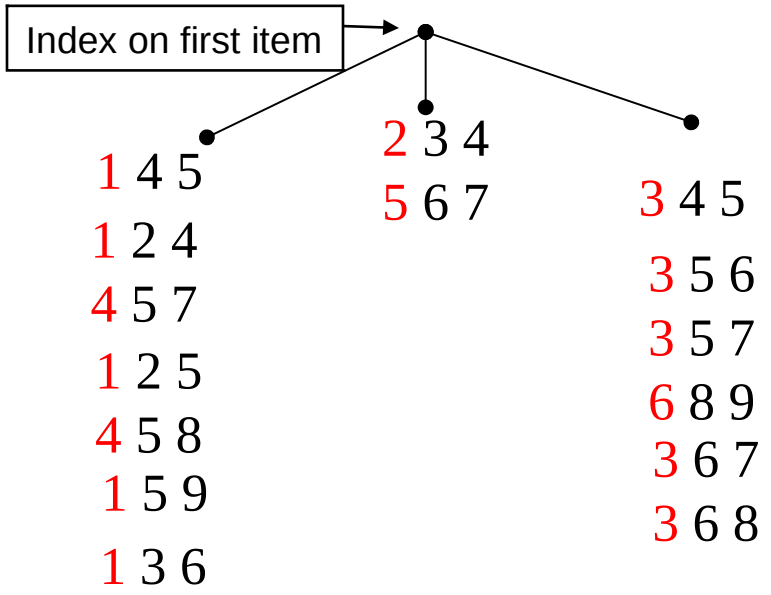
# Generate Hash Tree

**Suppose you have 15 candidate itemsets of length 3:**

**{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}**

f(1 or 4 or 7) = left branch
f(2 or 5 or 8) = middle branch
f(3 or 6 or 9) = right branch

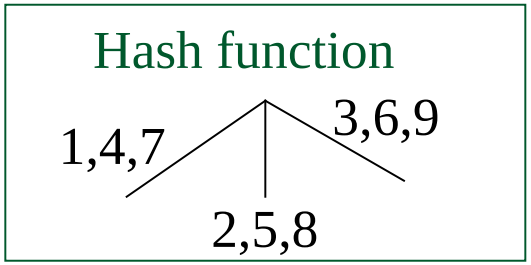


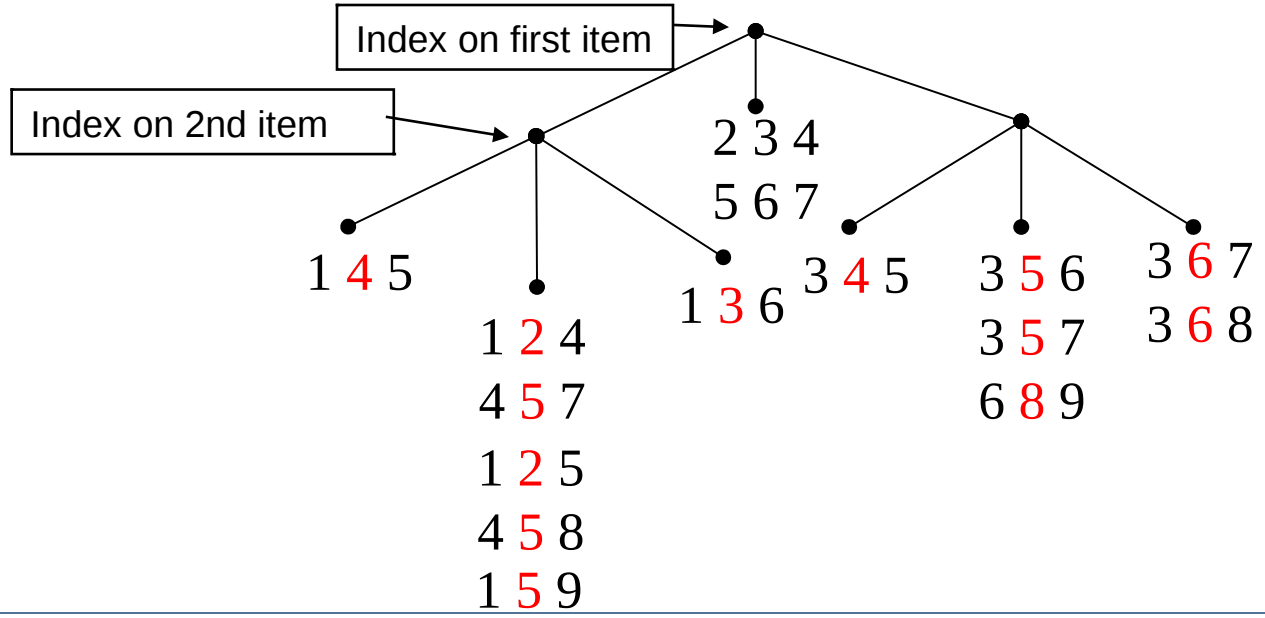



# Generate Hash Tree

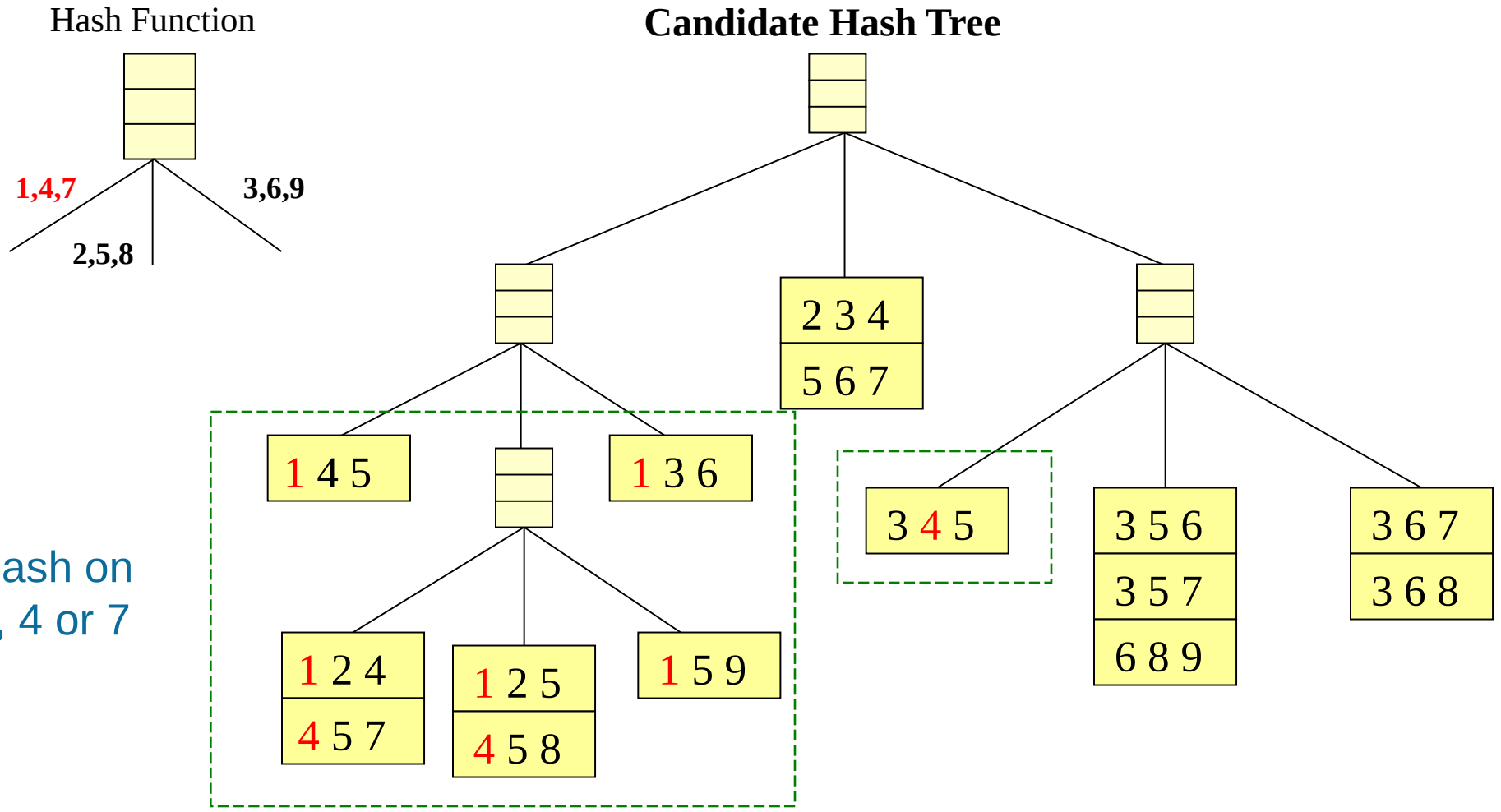
**Suppose you have 15 candidate itemsets of length 3:**

**{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}**

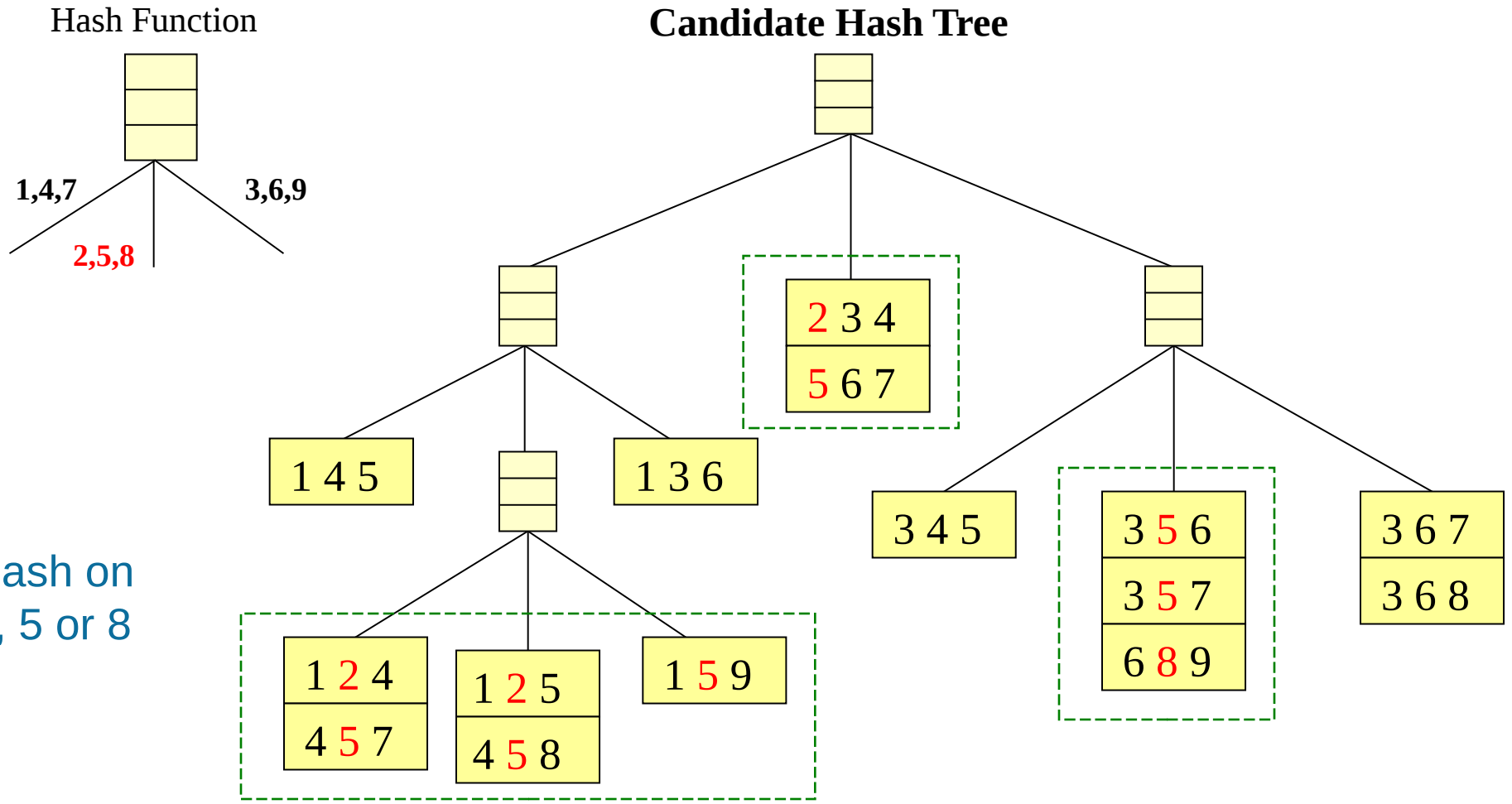


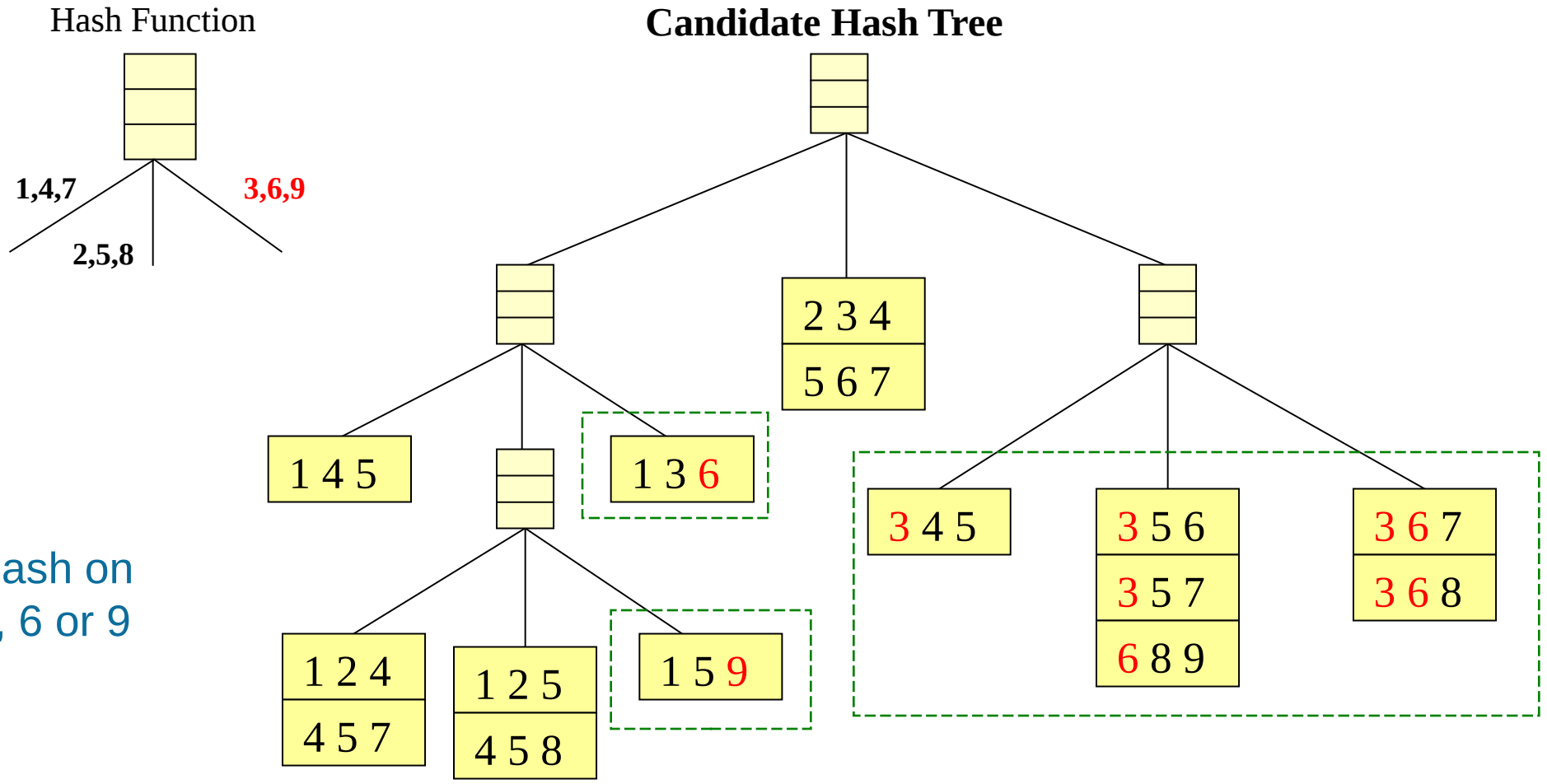
# Association Rule Discovery: Hash tree



# Association Rule Discovery: Hash tree

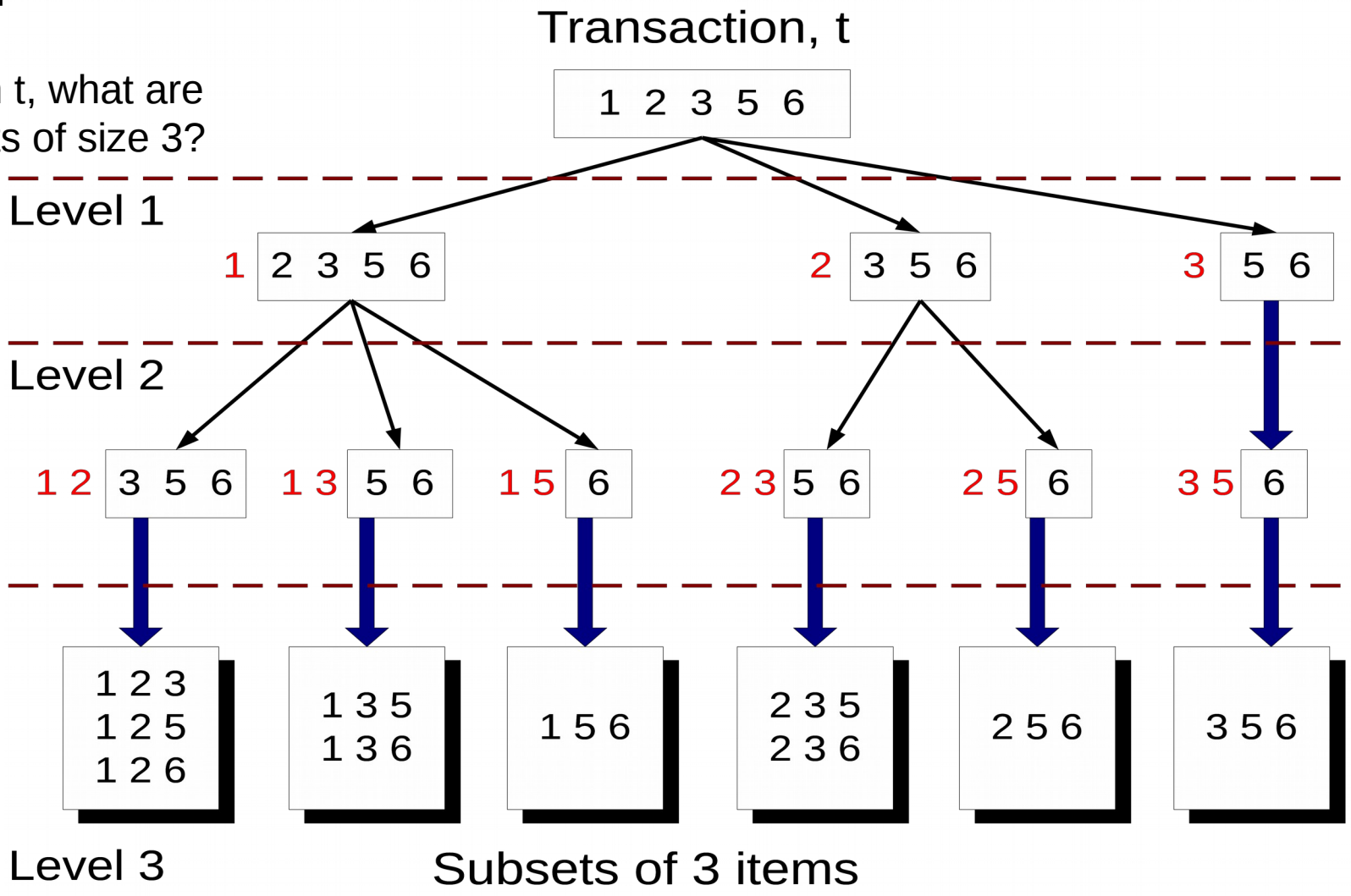


# Association Rule Discovery: Hash tree

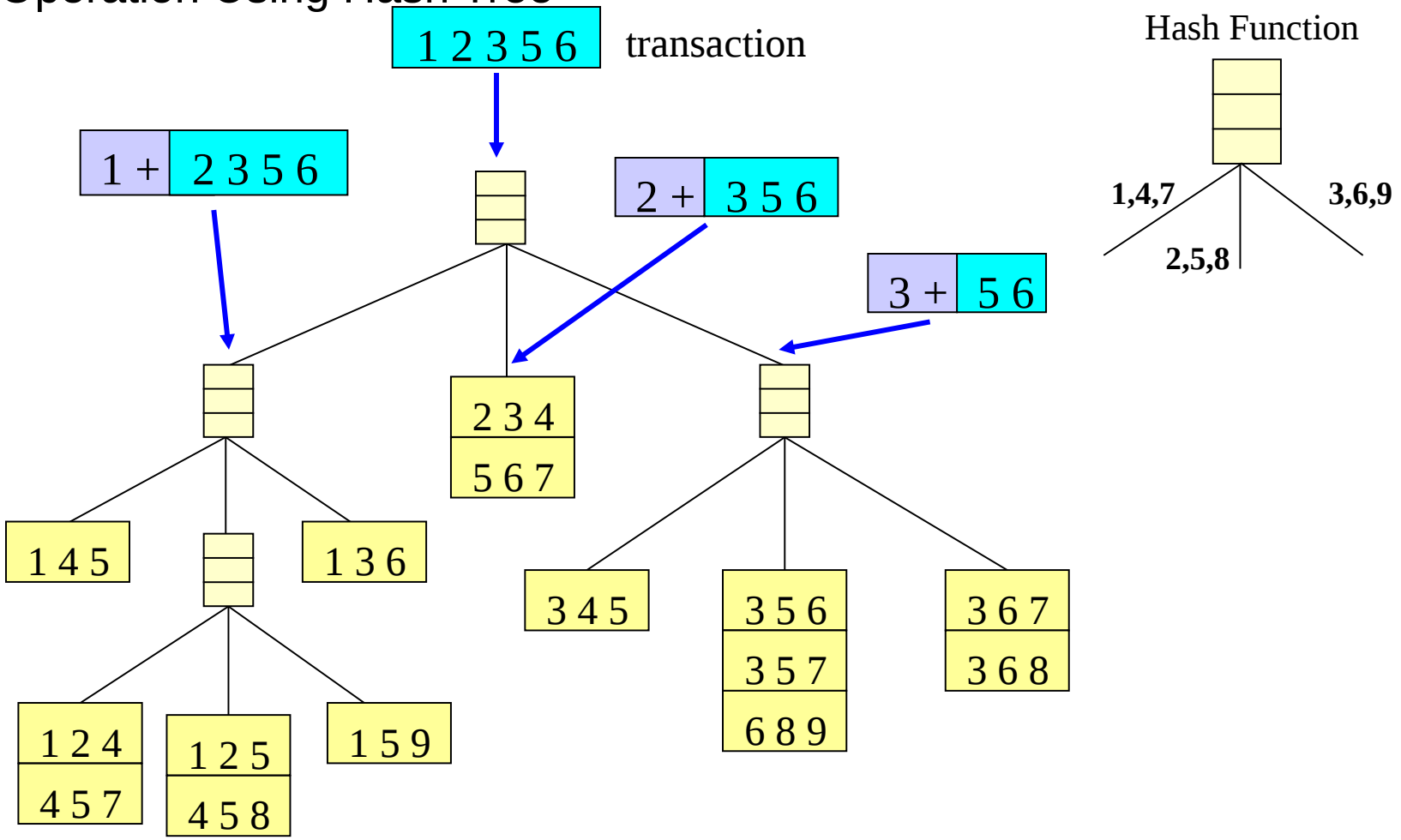


# Subset Operation

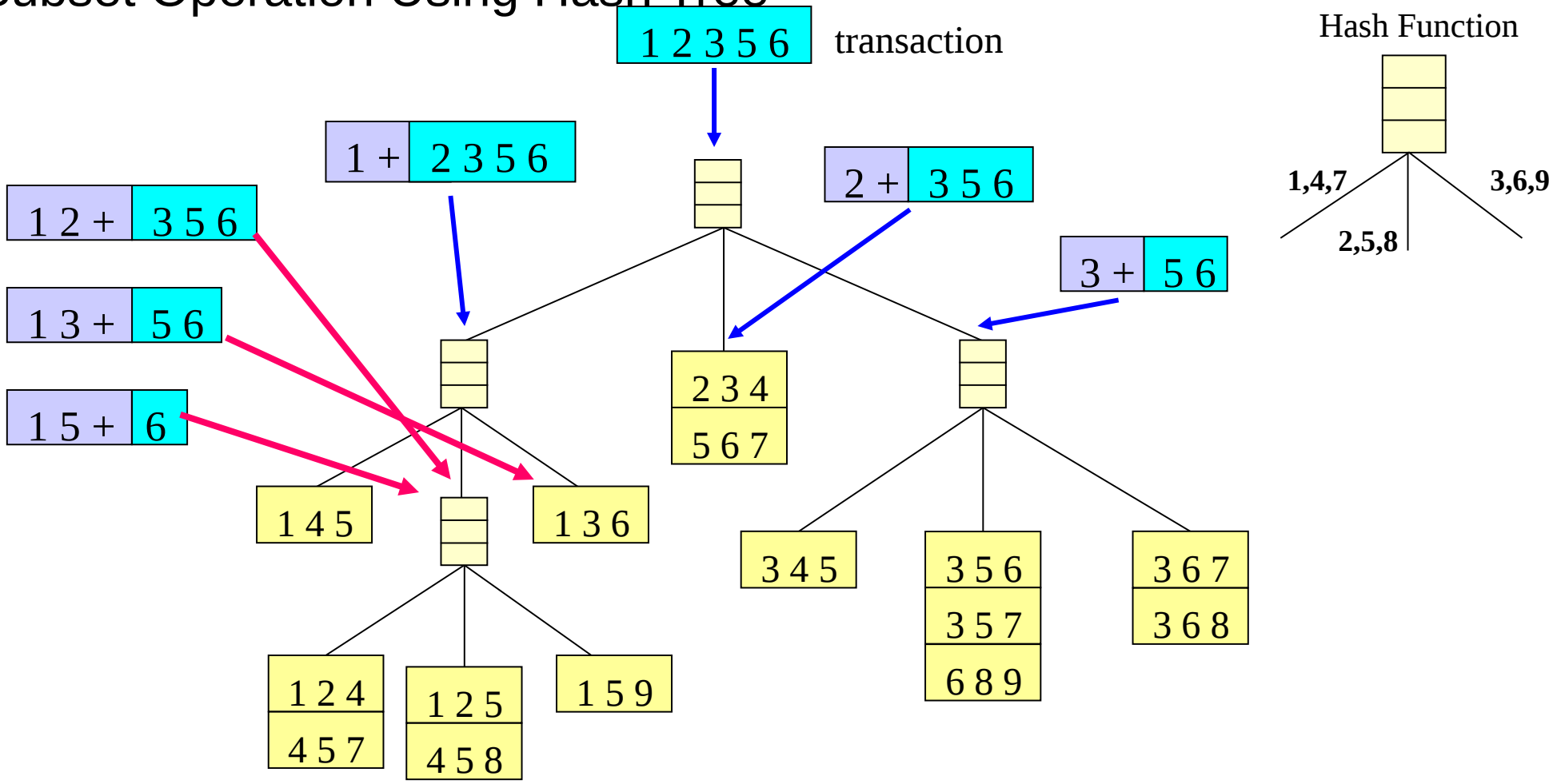
Given a transaction t, what are the possible subsets of size 3?



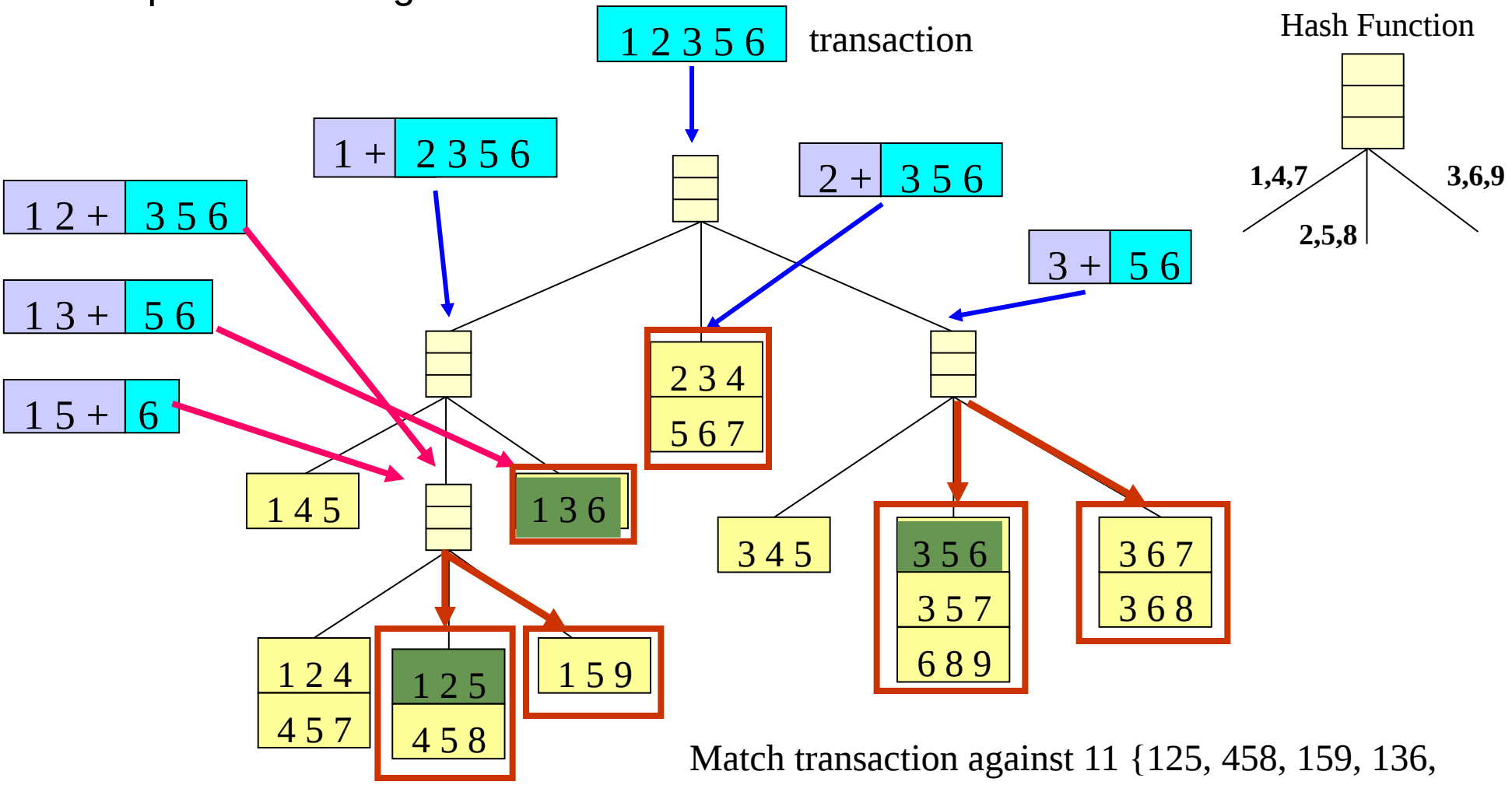
# Subset Operation Using Hash Tree



# Subset Operation Using Hash Tree



# Subset Operation Using Hash Tree




Match transaction against 11 {125, 458, 159, 136, 234, 567, 356, 357, 689, 367, 368} out of 15 candidates.

# Rule Generation

**Given a frequent itemset L, find all non-empty subsets F ⊂ L such that F → L – F satisfies the minimum confidence requirement**

- If {A,B,C,D} is a frequent itemset, candidate rules:

A →BCD, B →ACD, C →ABD, D →ABC

AB →CD, AC → BD, AD → BC, BC →AD,
BD →AC, CD →AB,

ABC →D, ABD →C, ACD →B, BCD →A,

**If |L| = k, then there are $2^k$ – 2 candidate association rules (ignoring L → ⊘ and ⊘ → L)**

# Rule Generation

$$Support_D(X) = \frac{|\{x \mid X \subseteq x, x \in D\}|}{|D|} = p(X) \leq 1$$ Support of X in D is the proportion of records in D that have itemset X

$$\text{Confidence}_D(X \rightarrow Y) = p(Y \mid X) = \frac{Support_D(X \cup Y)}{Support_D(X)} \leq 1$$

**How to efficiently generate rules from frequent itemsets?**

- In general, confidence does not have an anti-monotone property

  c(ABC →D) can be larger or smaller than c(AB →D)

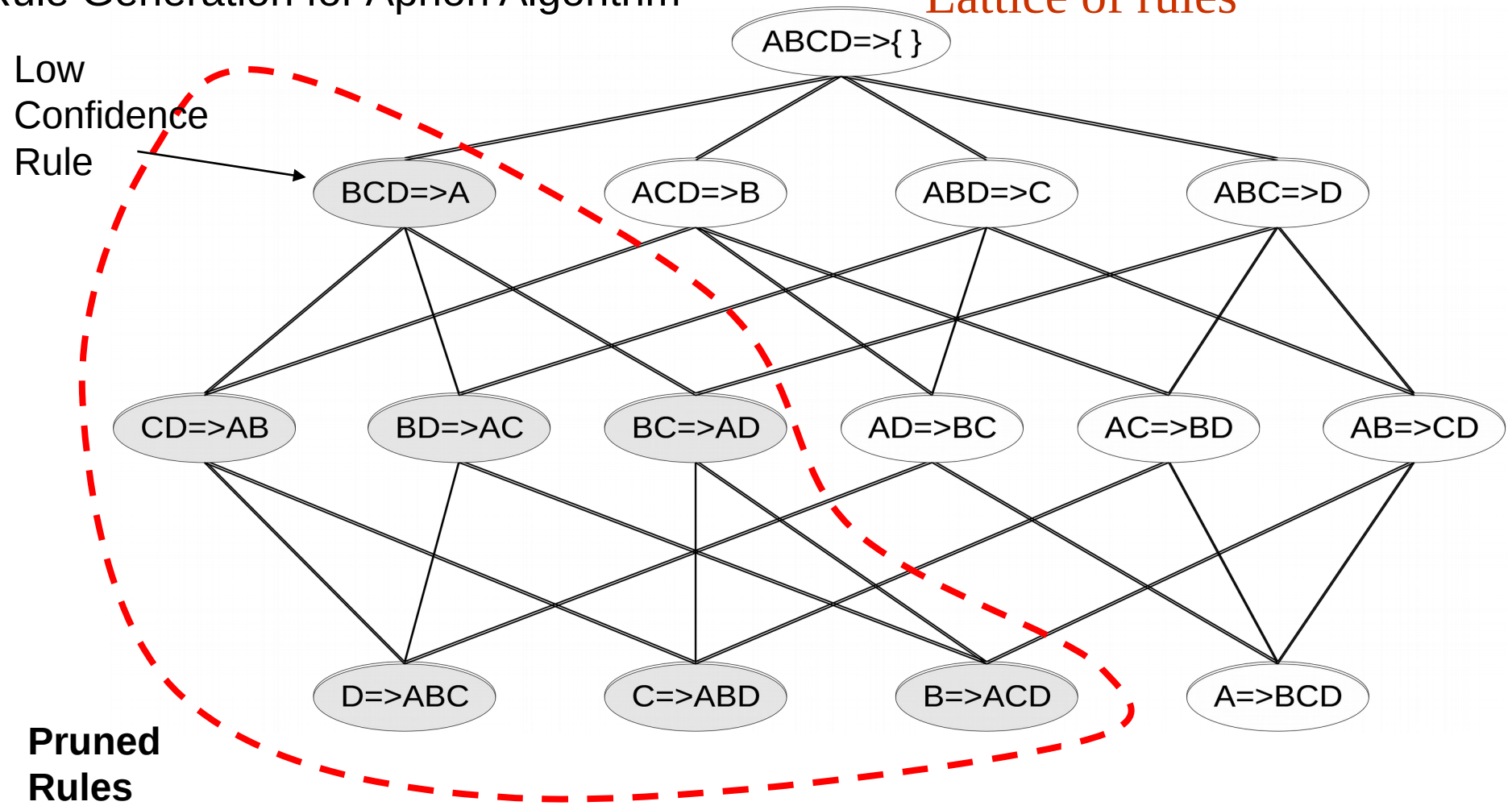
- But confidence of rules generated from the same itemset has an anti-monotone property
- e.g., L = {A,B,C,D}:

c(ABC → D) = sup(ABCD)/sup(ABC) ≥ c(AB → CD) = sup(ABCD)/sup(AB) ≥ c(A → BCD) = sup(ABCD)/sup(A)

sup(ABC) <= sup(AB) <= sup(A)

Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

# Rule Generation for Apriori Algorithm

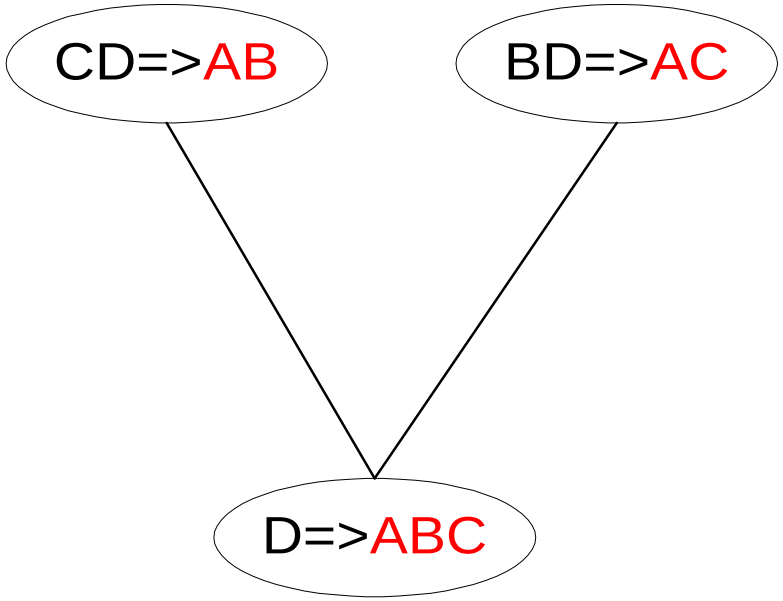


# Rule Generation for Apriori Algorithm

**Candidate rule is generated by merging two rules that share the same prefix
in the rule consequent**

**join(CD=>AB,BD=>AC)
would produce the candidate
rule D => ABC**

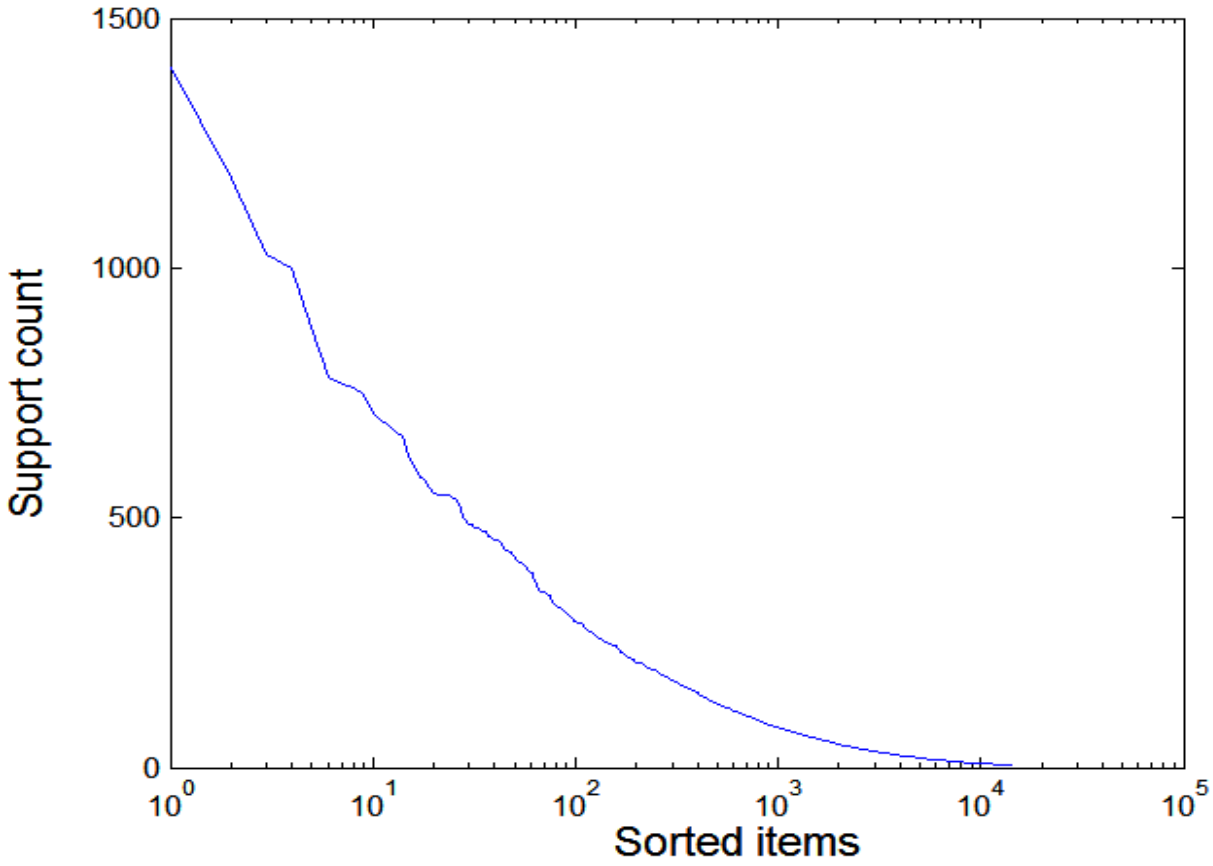
**Prune rule D=>ABC if its
subset AD=>BC does not have
high confidence**



# Effect of Support Distribution

## Many real data sets have skewed support distribution

**Support distribution of a retail data set**



The rest of slides would be of great interest for you to read but due to lack of time the rest of the slides are not covered in the lectures and are also not examinable. If you have time and interest and I recommend you to study these slides!

Please watch the following video with regard to AI and Data Mining on 19 May 2017 by Google CEO: Sundar Pichai
https://www.recode.net/2017/5/19/15666704/google-lens-key-example-ai-first-computer-vision

- I will be away on 6 June 2017
- I am available generally from next week: Monday morning; Tuesday afternoon; Wednesday morning; Friday morning for consultation. Please come in groups as it will be useful for many.

# Effect of Support Distribution

**How to set the appropriate *minsup* threshold?**

- If *minsup* is set too high, we could miss itemsets involving interesting rare items (e.g., expensive products)

- If *minsup* is set too low, it is computationally expensive and the number of itemsets is very large

**Using a single minimum support threshold may not be effective**

# Factors Affecting Complexity

**Choice of minimum support threshold**

- lowering support threshold results in more frequent itemsets
- this may increase number of candidates and max length of frequent itemsets

**Dimensionality (number of items) of the data set**

- more space is needed to store support count of each item
- if number of frequent items also increases, both computation and I/O costs may also increase

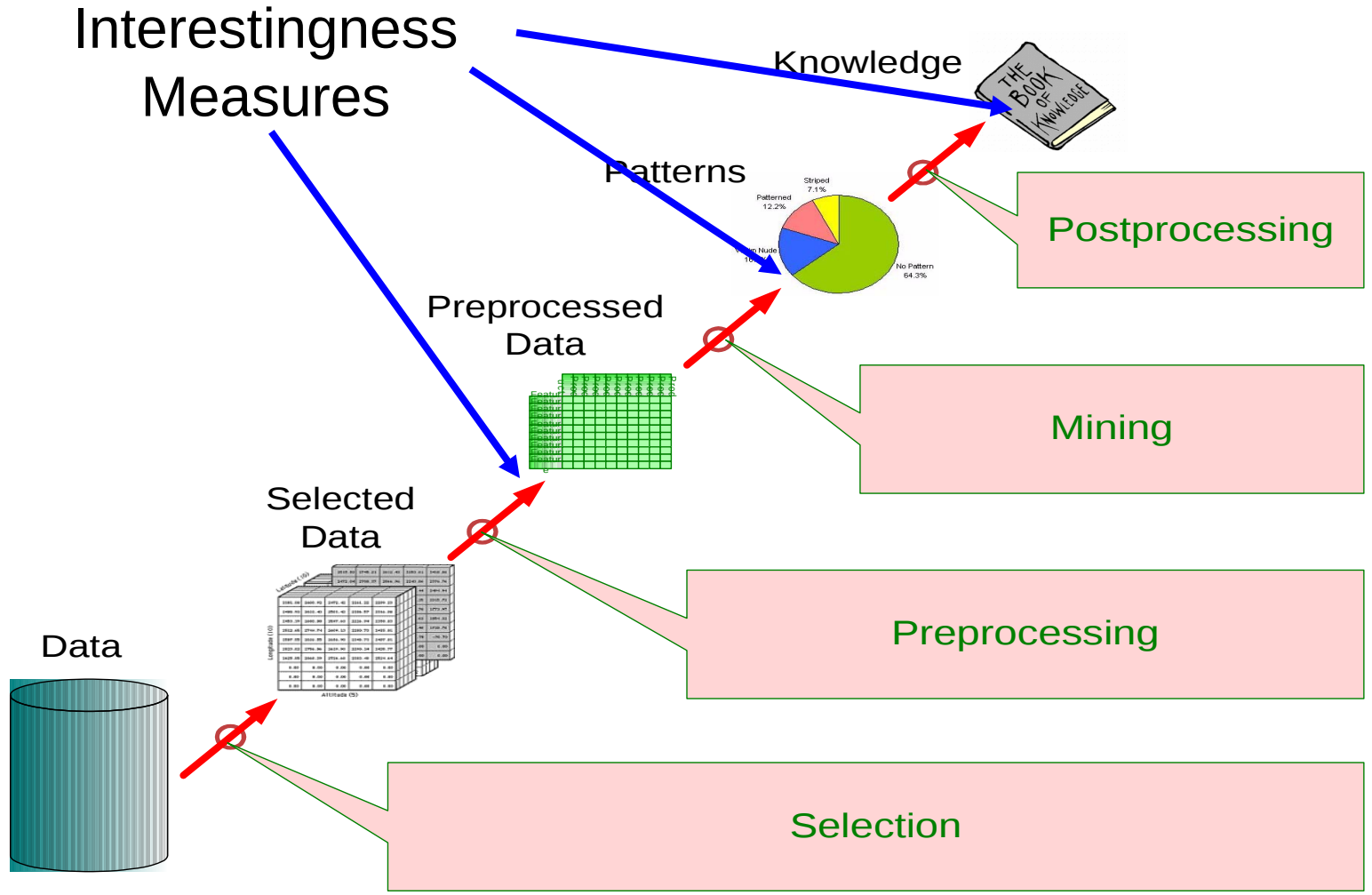
**Size of database**

- since Apriori makes multiple passes, run time of algorithm may increase with number of transactions

**Average transaction width**

- transaction width increases with denser data sets
- This may increase max length of frequent itemsets and traversals of hash tree (number of subsets in a transaction increases with its width)

# Application of Interestingness Measure



# Pattern Evaluation

**Association rule algorithms tend to produce too many rules**

- many of them are uninteresting or redundant
- Redundant if {A,B,C} → {D} and {A,B} → {D}
  have same support & confidence

**Interestingness measures can be used to prune/rank the derived patterns**

**In the original formulation of association rules, support & confidence are the only measures used**

# Computing Interestingness Measure

**Given a rule X → Y, information needed to compute rule interestingness can be obtained from a contingency table**

Contingency table for X → Y

| | Y | $\overline{Y}$ | |
|---|---|---|---|
| X | $f_{11}$ | $f_{10}$ | $f_{1+}$ |
| $\overline{X}$ | $f_{01}$ | $f_{00}$ | $f_{o+}$ |
| | $f_{+1}$ | $f_{+0}$ | \|T\| |

$f_{11}$: support of X and $\overline{Y}$
$f_{10}$: support of $\overline{X}$ and Y
$f_{01}$: support of $\overline{X}$ and $\overline{Y}$
$f_{00}$: support of X and Y

Used to define various measures

- support, confidence, lift, Gini, J-measure, etc.

# Drawback of Confidence

$$Support_D(X) = \frac{|\{x \mid X \subseteq x, x \in D\}|}{|D|} = p(X) \leq 1$$ Support of X in D is the proportion of records in D that have itemset X

$$\text{Confidence}_D(X \rightarrow Y) = p(Y \mid X) = \frac{Support_D(X \cup Y)}{Support_D(X)} \leq 1$$

| | Coffee | $\overline{\text{Coffee}}$ | |
|---|---|---|---|
| Tea | 15 | 5 | 20 |
| $\overline{\text{Tea}}$ | 75 | 5 | 80 |
| | 90 | 10 | 100 |

## Association Rule: Tea → Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

⇒ Although confidence is high, rule is misleading

⇒ P(Coffee|$\overline{\text{Tea}}$) = 0.9375

# Statistical Independence

## Population of 1000 students

- 600 students know how to swim (S)
- 700 students know how to bike (B)
- 420 students know how to swim and bike (S,B)

- $P(S \wedge B) = 420/1000 = 0.42$
- $P(S) \times P(B) = 0.6 \times 0.7 = 0.42$

- $P(S \wedge B) = P(S) \times P(B)$ => Statistical independence
- $P(S \wedge B) > P(S) \times P(B)$ => Positively correlated
- $P(S \wedge B) < P(S) \times P(B)$ => Negatively correlated

# Statistical-based Measures

**Measures that take into account statistical dependence**

$$Lift \text{ also called } Interest = \frac{P(Y \mid X)}{P(Y)} = \frac{P(X,Y)}{P(X)P(Y)}$$

$$PS = P(X,Y) - P(X)P(Y)$$

$$\phi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1 - P(X)]P(Y)[1 - P(Y)]}}$$

# Compact Representation of Frequent Itemsets

**Some itemsets are redundant because they have identical support as their supersets**

| TID | A1 | A2 | A3 | A4 | A5 | A6 | A7 | A8 | A9 | A10 | B1 | B2 | B3 | B4 | B5 | B6 | B7 | B8 | B9 | B10 | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | C9 | C10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

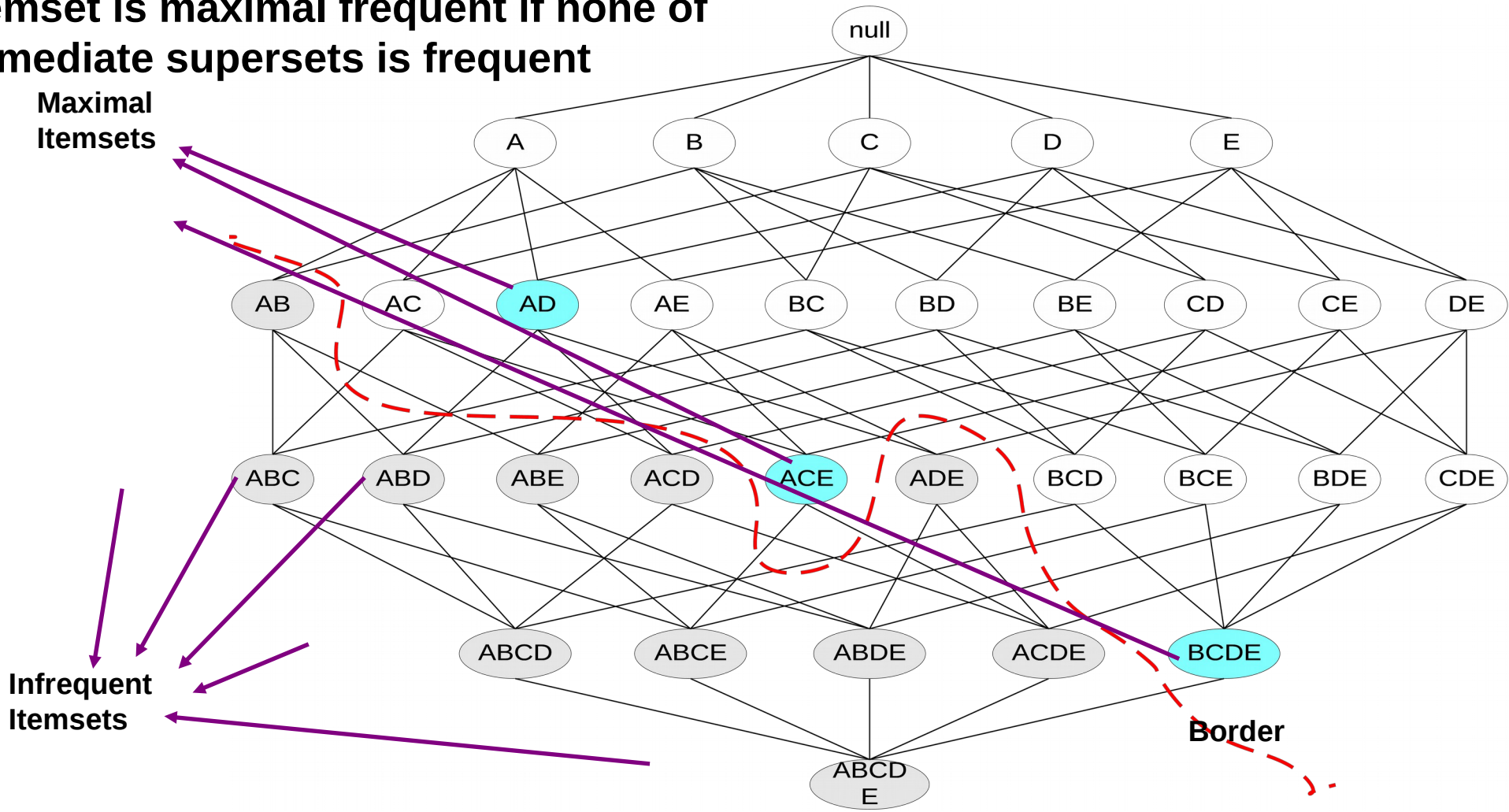
**Number of frequent itemsets**

$$= 3 \times \sum_{k=1}^{10} \binom{10}{k}$$

**Need a compact representation**

# Maximal Frequent Itemset

**An itemset is maximal frequent if none of its immediate supersets is frequent**



# Closed Itemset

**An itemset is closed if none of its immediate supersets has the same support as the itemset**

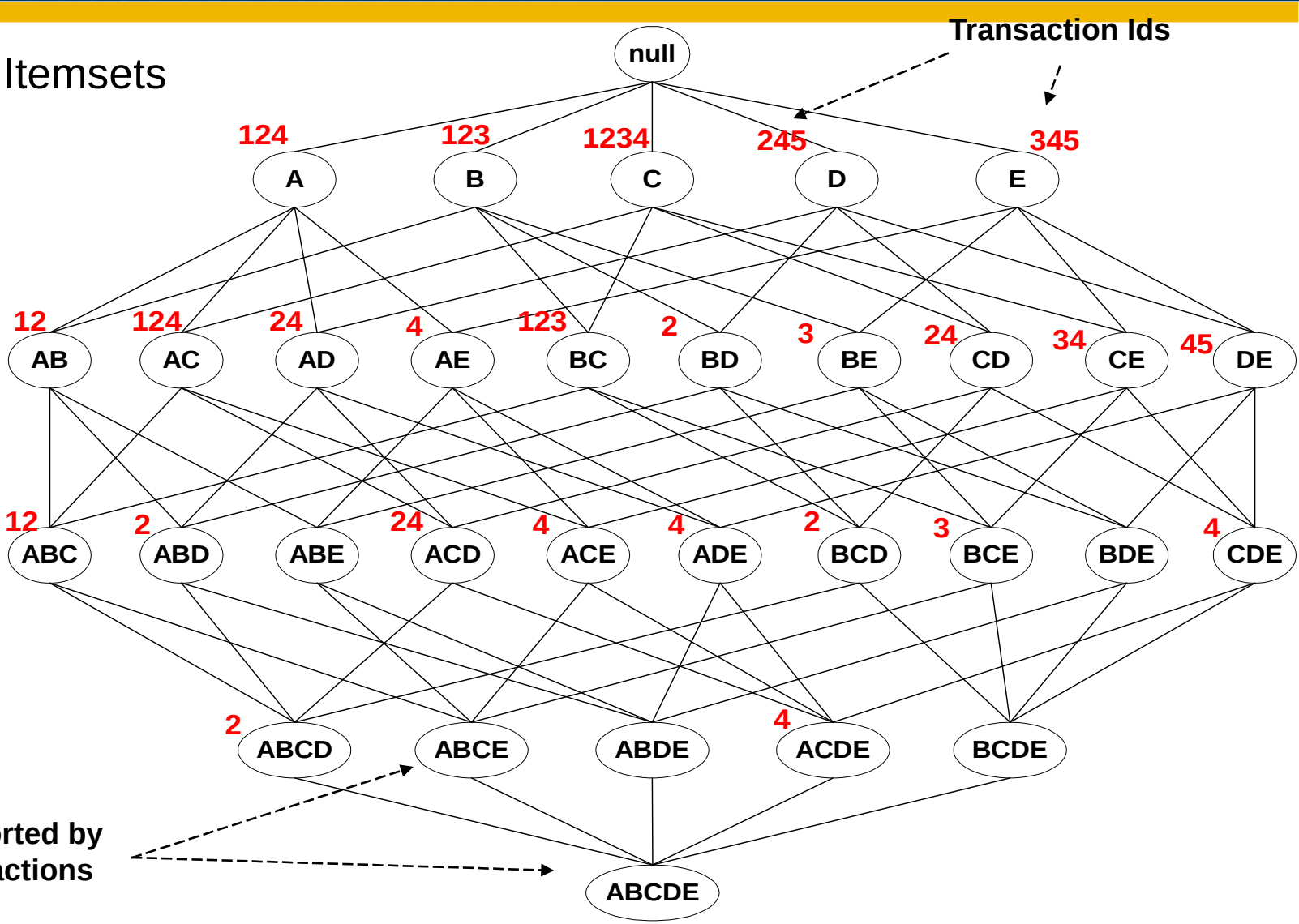
| TID | Items |
|---|---|
| 1 | {A,B} |
| 2 | {B,C,D} |
| 3 | {A,B,C,D} |
| 4 | {A,B,D} |
| 5 | {A,B,C,D} |

| Itemset | Support |
|---|---|
| {A} | 4 |
| {B} | 5 |
| {C} | 3 |
| {D} | 4 |
| {A,B} | 4 |
| {A,C} | 2 |
| {A,D} | 3 |
| {B,C} | 3 |
| {B,D} | 4 |
| {C,D} | 3 |

| Itemset | Support |
|---|---|
| {A,B,C} | 2 |
| {A,B,D} | 3 |
| {A,C,D} | 2 |
| {B,C,D} | 3 |
| {A,B,C,D} | 2 |

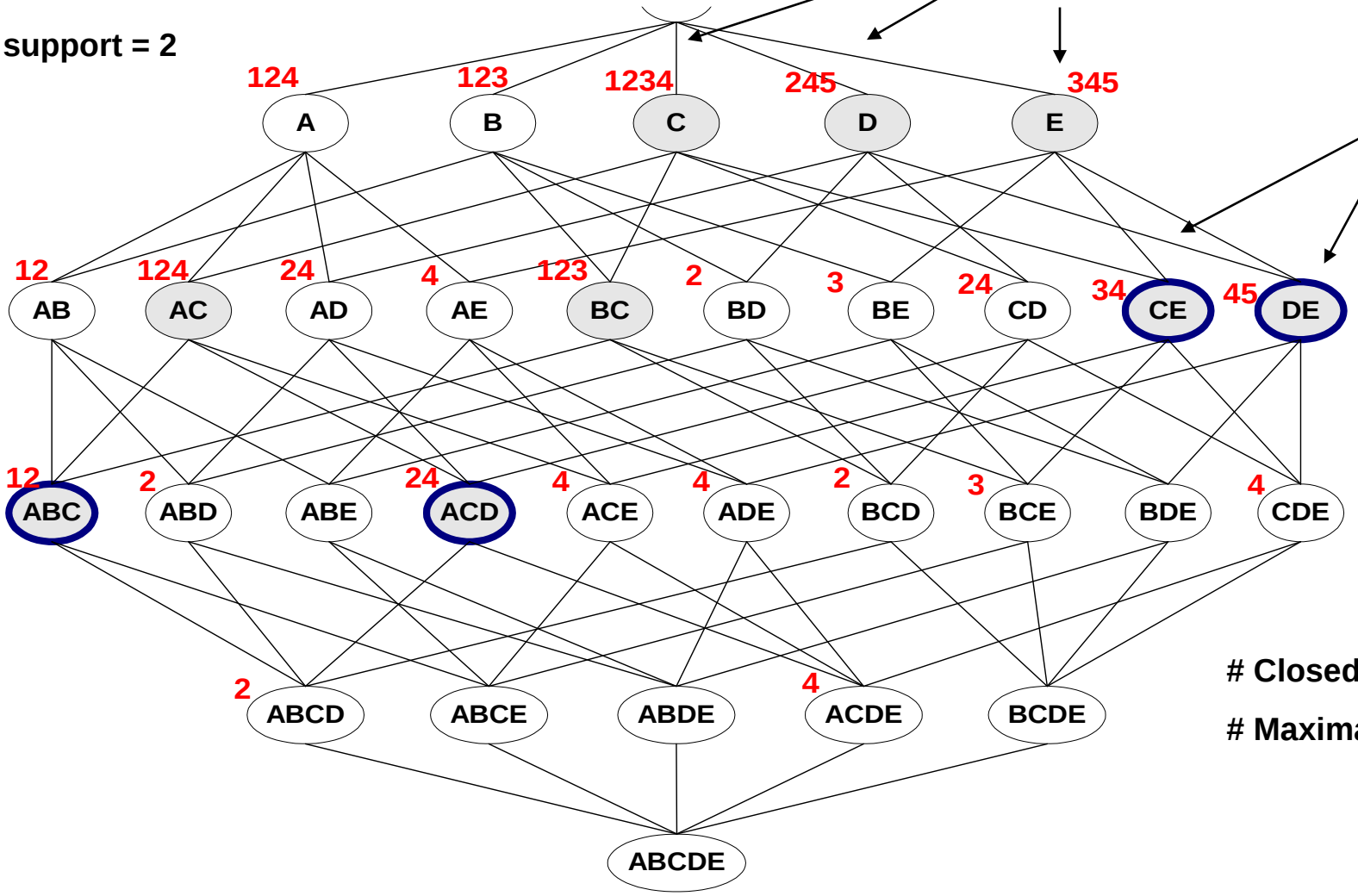
# Maximal vs Closed Itemsets

| TID | Items |
|---|---|
| 1 | ABC |
| 2 | ABCD |
| 3 | BCE |
| 4 | ACDE |
| 5 | DE |



# Maximal vs Closed Frequent Itemsets

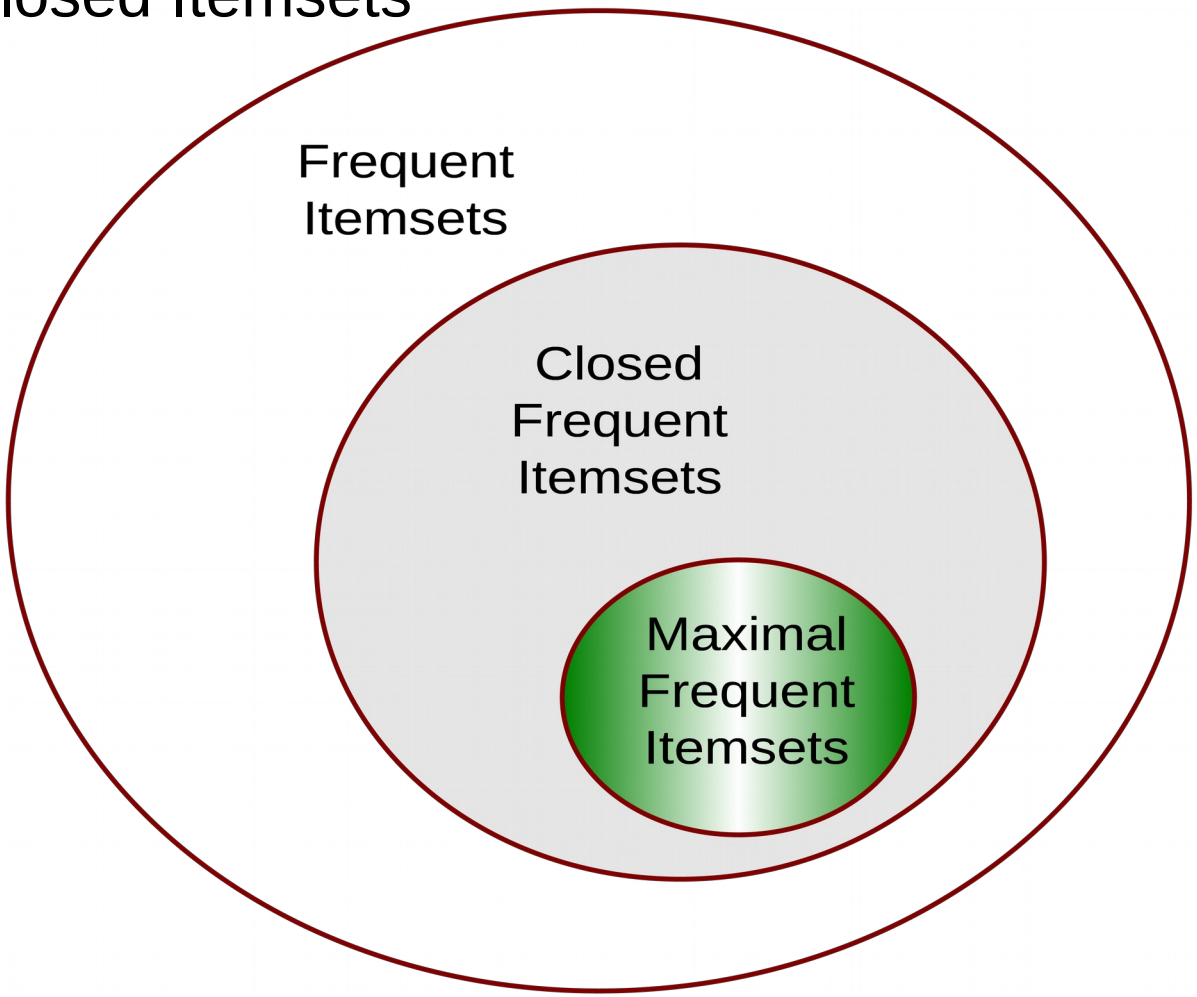
**Minimum support = 2**




**# Closed = 9**

**# Maximal = 4**

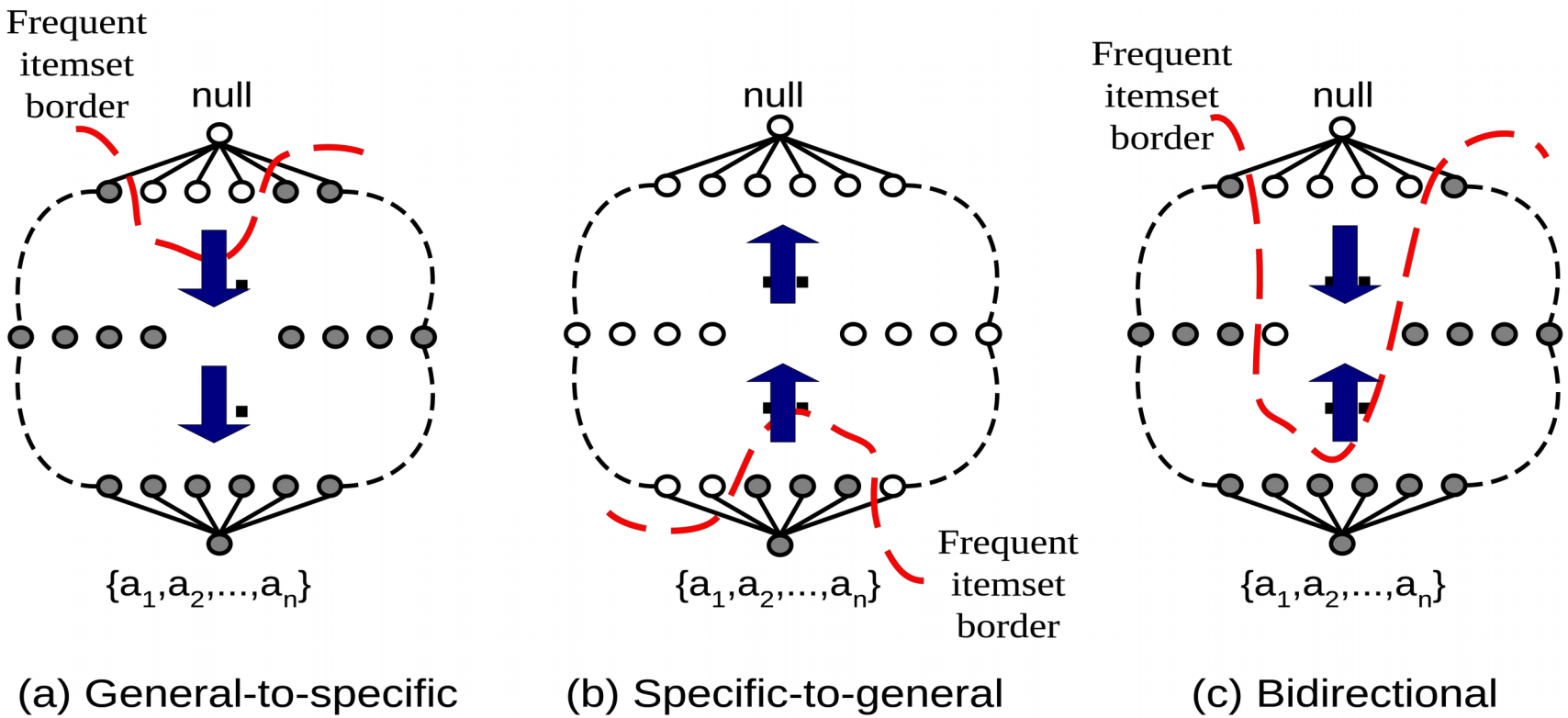
# Maximal vs Closed Itemsets



Alternative Methods for Frequent Itemset Generation

## Traversal of Itemset Lattice

- General-to-specific vs Specific-to-general

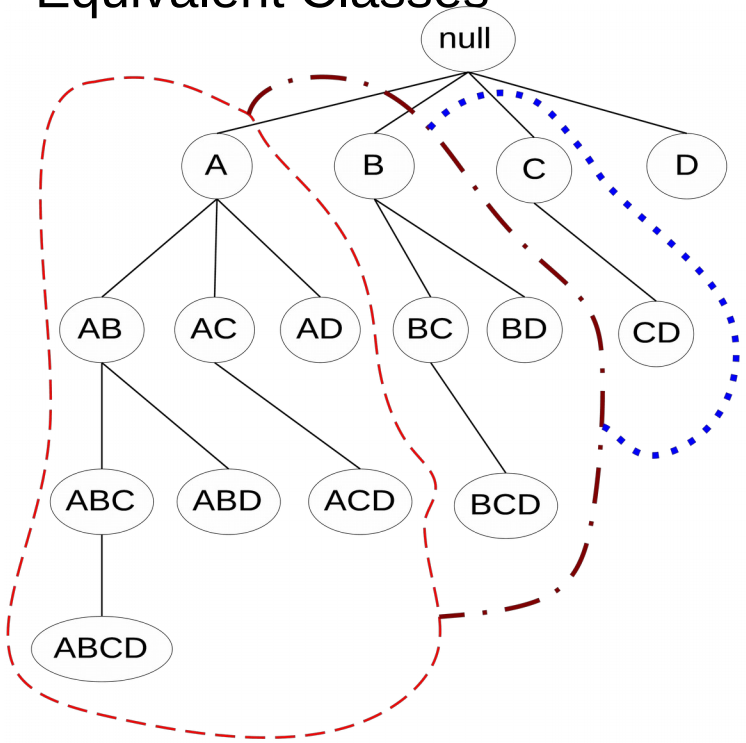



(a) General-to-specific (b) Specific-to-general (c) Bidirectional

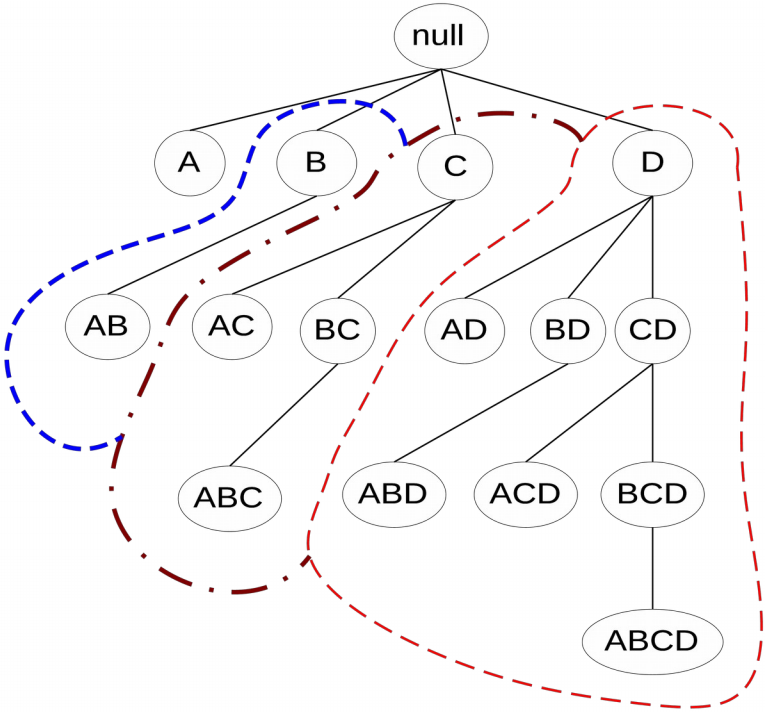
Alternative Methods for Frequent Itemset Generation

## Traversal of Itemset Lattice

- Equivalent Classes




(a) Prefix tree

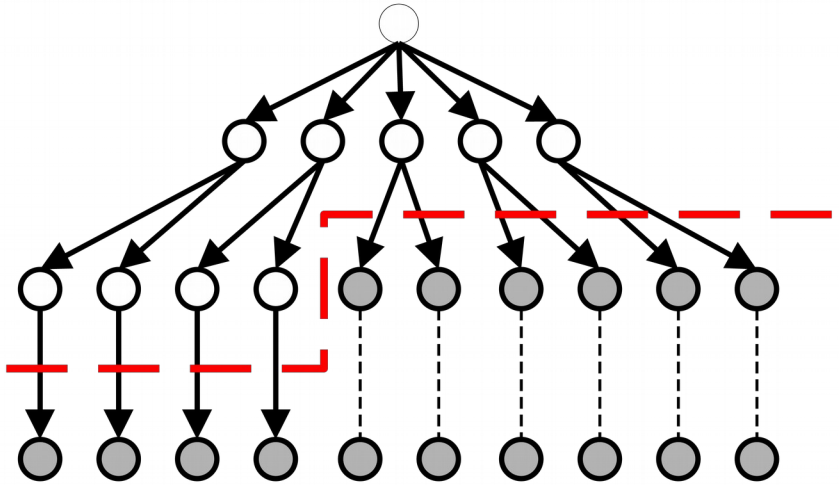



(b) Suffix tree

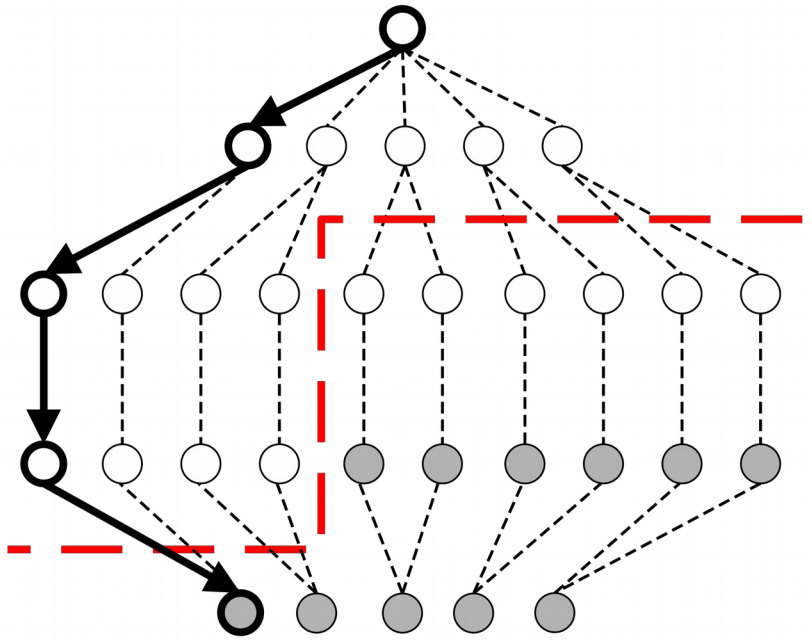
Alternative Methods for Frequent Itemset Generation

## Traversal of Itemset Lattice

- Breadth-first vs Depth-first



(a) Breadth first



(b) Depth first

Alternative Methods for Frequent Itemset Generation

## Representation of Database

- horizontal vs vertical data layout

Horizontal Data Layout

Vertical Data Layout

# FP-growth Algorithm

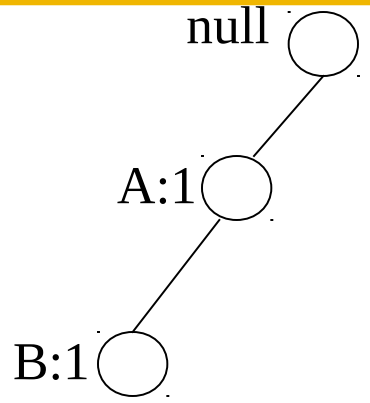
**Use a compressed representation of the database using an FP-tree**

**Once an FP-tree has been constructed, it uses a recursive divide-and-conquer approach to mine the frequent itemsets**

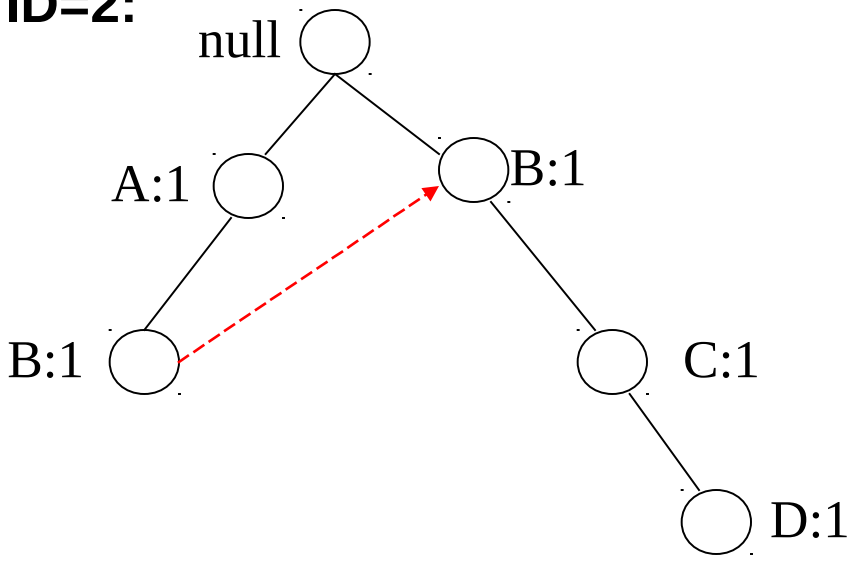
# FP-tree construction

| TID | Items |
|---|---|
| 1 | {A,B} |
| 2 | {B,C,D} |
| 3 | {A,C,D,E} |
| 4 | {A,D,E} |
| 5 | {A,B,C} |
| 6 | {A,B,C,D} |
| 7 | {B,C} |
| 8 | {A,B,C} |
| 9 | {A,B,D} |
| 10 | {B,C,E} |

**After reading TID=1:**



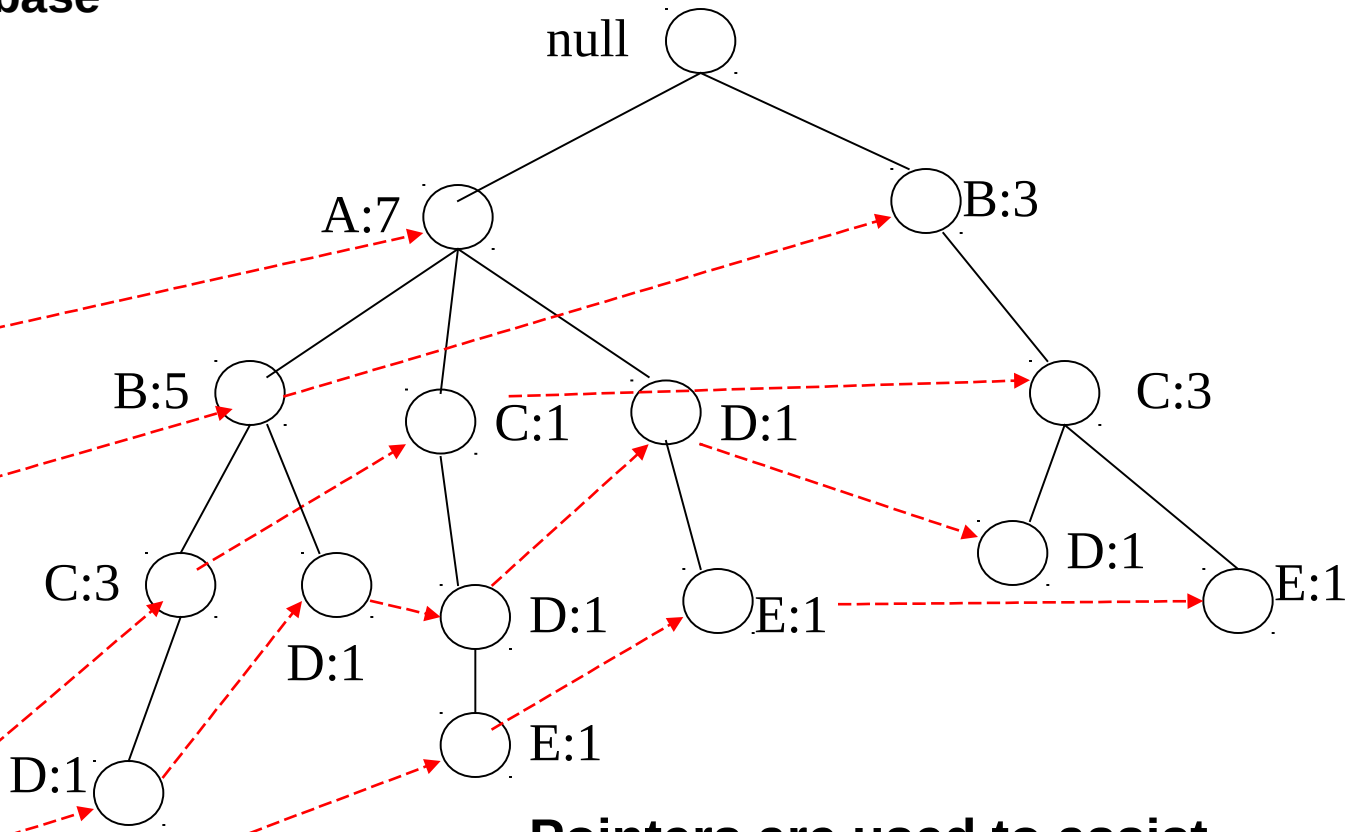

**After reading TID=2:**



| TID | Items |
|---|---|
| 1 | {A,B} |
| 2 | {B,C,D} |
| 3 | {A,C,D,E} |
| 4 | {A,D,E} |
| 5 | {A,B,C} |
| 6 | {A,B,C,D} |
| 7 | {B,C} |
| 8 | {A,B,C} |
| 9 | {A,B,D} |
| 10 | {B,C,E} |

**Transaction Database**

# FP-Tree Construction

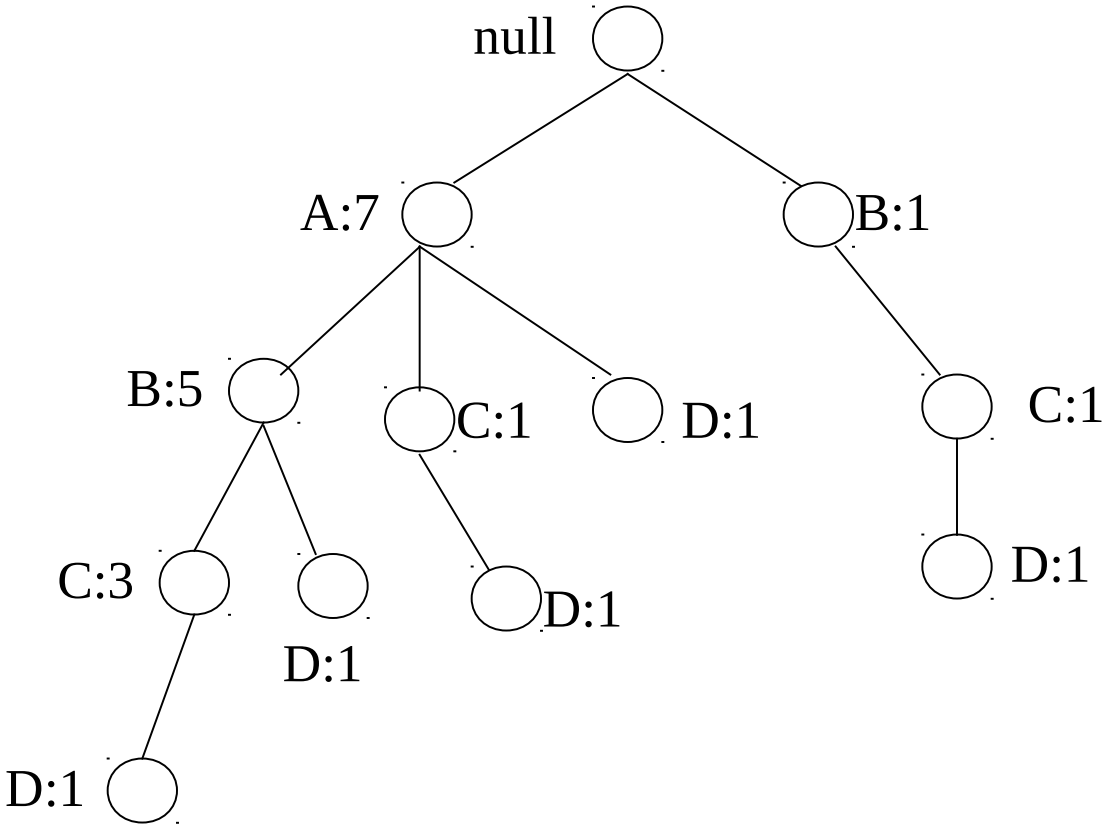



**Header table**

| Item | Pointer |
|---|---|
| A | |
| B | |
| C | |
| D | |
| E | |

**Pointers are used to assist frequent itemset generation**

# FP-growth




**Conditional Pattern base for D:**

**P = {(A:1,B:1,C:1),**
**(A:1,B:1),**
**(A:1,C:1),**
**(A:1),**
**(B:1,C:1)}**

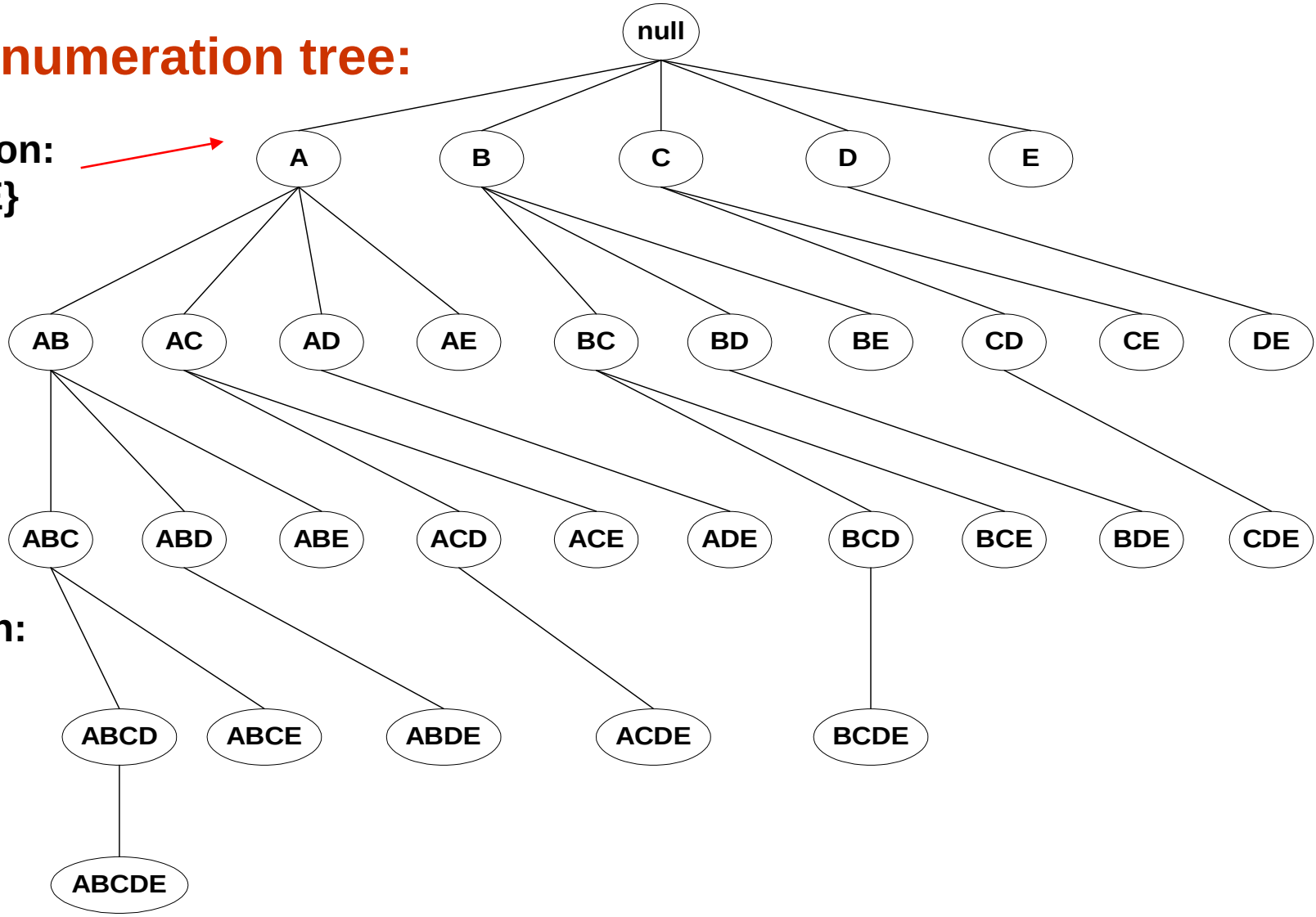
**Recursively apply FP-growth on P**

**Frequent Itemsets found (with sup > 1):**
**AD(4), BD(3), CD(3), ACD(2), BCD(2), ABD(2)**

Tree Projection

## Set enumeration tree:

**Possible Extension:**
**E(A) = {B,C,D,E}**




**Possible Extension:**
**E(ABC) = {D,E}**

# Tree Projection

**Items are listed in lexicographic order**

**Each node P stores the following information:**

- Itemset for node P
- List of possible lexicographic extensions of P: E(P)
- Pointer to projected database of its ancestor node
- Bitvector containing information about which transactions in the projected database contain the itemset

# Projected Database

**Original Database:**

| TID | Items |
|---|---|
| 1 | {A,B} |
| 2 | {B,C,D} |
| 3 | {A,C,D,E} |
| 4 | {A,D,E} |
| 5 | {A,B,C} |
| 6 | {A,B,C,D} |
| 7 | {B,C} |
| 8 | {A,B,C} |
| 9 | {A,B,D} |
| 10 | {B,C,E} |

**Projected Database for node A:**

| TID | Items |
|---|---|
| 1 | {B} |
| 2 | {} |
| 3 | {C,D,E} |
| 4 | {D,E} |
| 5 | {B,C} |
| 6 | {B,C,D} |
| 7 | {} |
| 8 | {B,C} |
| 9 | {B,D} |
| 10 | {} |

**For each transaction T, projected transaction at node A is** $T - A$ If $A \in T$; $\{\}$ Otherwise

# ECLAT

**For each item, store a list of transaction ids (tids)**

## Horizontal Data Layout

| TID | Items |
|---|---|
| 1 | A,B,E |
| 2 | B,C,D |
| 3 | C,E |
| 4 | A,C,D |
| 5 | A,B,C,D |
| 6 | A,E |
| 7 | A,B |
| 8 | A,B,C |
| 9 | A,C,D |
| 10 | B |

## Vertical Data Layout

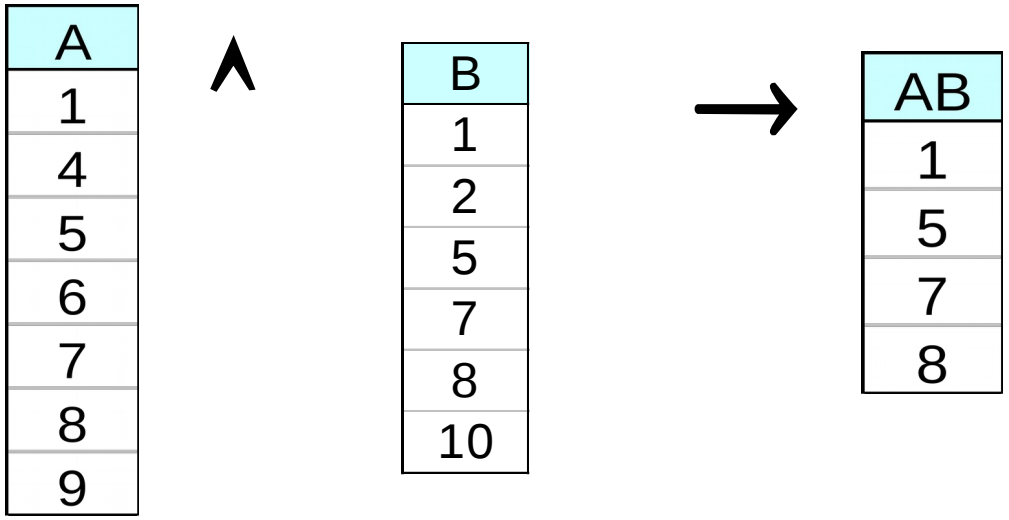
| A | B | C | D | E |
|---|---|---|---|---|
| 1 | 1 | 2 | 2 | 1 |
| 4 | 2 | 3 | 4 | 3 |
| 5 | 5 | 4 | 5 | 6 |
| 6 | 7 | 8 | 9 | |
| 7 | 8 | 9 | | |
| 8 | 10 | | | |
| 9 | | | | |

↓

**TID-list**

# ECLAT

**Determine support of any k-itemset by intersecting tid-lists of two of its (k-1) subsets.**




**3 traversal approaches:**

- top-down, bottom-up and hybrid

**Advantage: very fast support counting**

**Disadvantage: intermediate tid-lists may become too large for memory**

# Multiple Minimum Support

**How to apply multiple minimum supports?**

- MS(i): minimum support for item i
- e.g.: MS(Milk)=5%, MS(Coke) = 3%,
  MS(Broccoli)=0.1%, MS(Salmon)=0.5%
- MS({Milk, Broccoli}) = min (MS(Milk), MS(Broccoli))
  = 0.1%

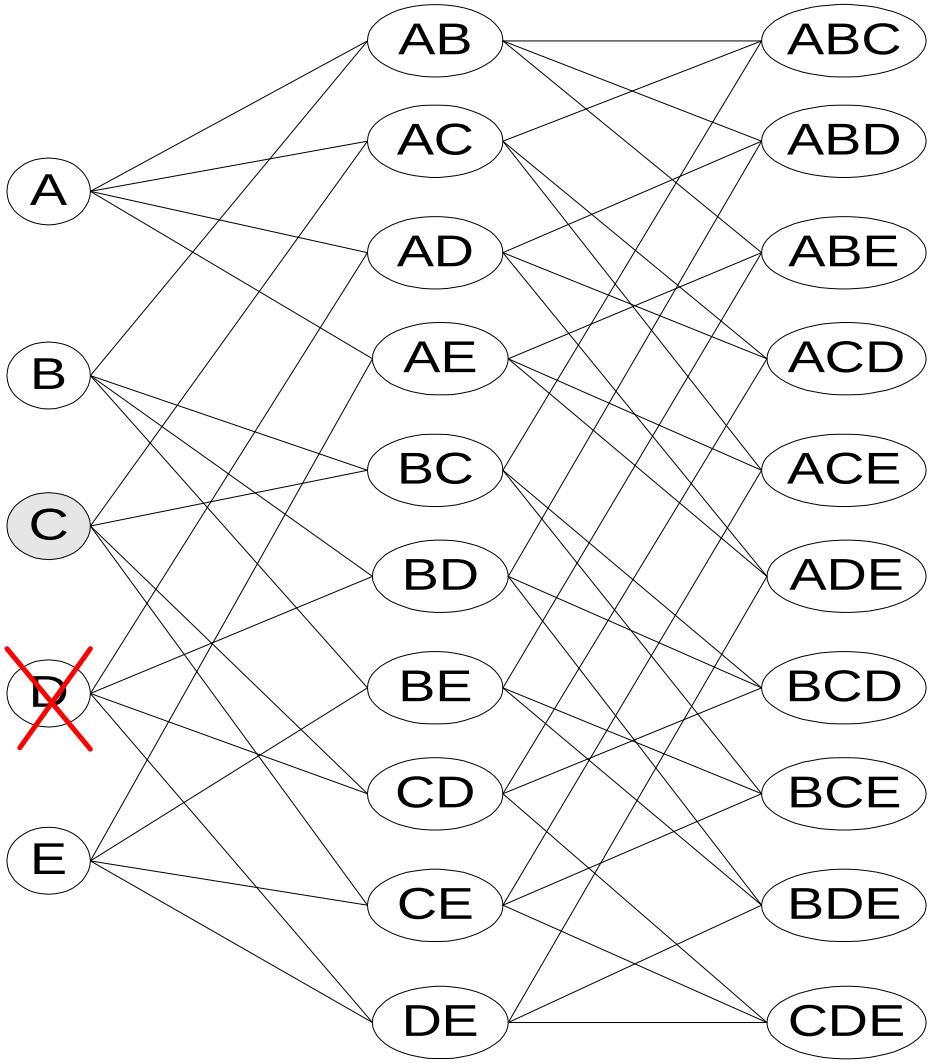
- Challenge: Support is no longer anti-monotone

  Suppose: Support(Milk, Coke) = 1.5% and
  Support(Milk, Coke, Broccoli) = 0.5%

  {Milk,Coke} is infrequent but {Milk,Coke,Broccoli} is frequent due to different minimum support requirements!

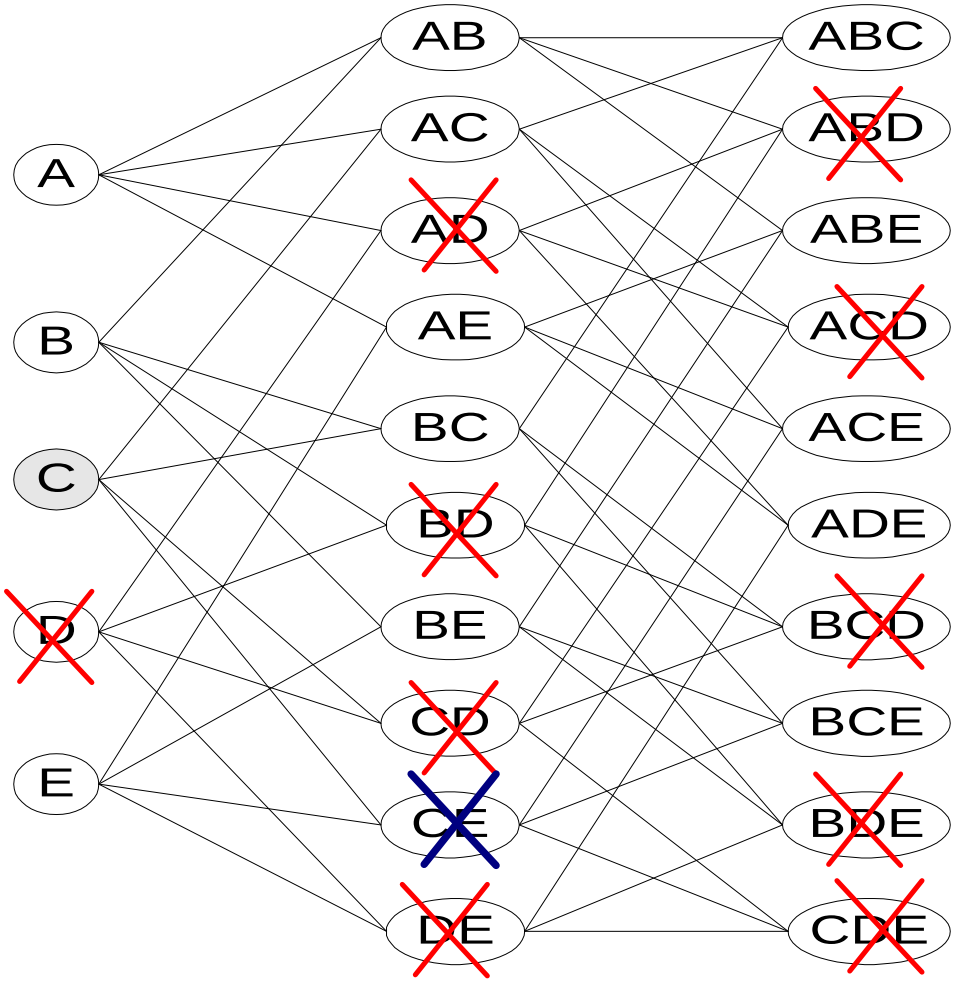
# Multiple Minimum Support

| Item | MS(I) | Sup(I) |
|---|---|---|
| A | 0.10% | 0.25% |
| B | 0.20% | 0.26% |
| C | 0.30% | 0.29% |
| D | 0.50% | 0.05% |
| E | 3% | 4.20% |



# Multiple Minimum Support

| Item | MS(I) | Sup(I) |
|---|---|---|
| A | 0.10% | 0.25% |
| B | 0.20% | 0.26% |
| C | 0.30% | 0.29% |
| D | 0.50% | 0.05% |
| E | 3% | 4.20% |



# Multiple Minimum Support (Liu 1999)

## Order the items according to their minimum support (in ascending order)

- e.g.: MS(Milk)=5%, MS(Coke) = 3%,
  MS(Broccoli)=0.1%, MS(Salmon)=0.5%
- Ordering: Broccoli, Salmon, Coke, Milk

## Need to modify Apriori such that:

- $L_1$ : set of frequent items
- $F_1$ : set of items whose support is $\geq$ MS(1)
  where MS(1) is $\min_i$( MS(i) )
- $C_2$ : candidate itemsets of size 2 is generated from $F_1$
  instead of $L_1$

# Multiple Minimum Support (Liu 1999)

## Modifications to Apriori:

- In traditional Apriori,

  A candidate (k+1)-itemset is generated by merging two frequent itemsets of size k

  The candidate is pruned if it contains any infrequent subsets of size k

- Pruning step has to be modified:

  Prune only if subset contains the first item

  e.g.: Candidate={Broccoli, Coke, Milk} (ordered according to minimum support)

  {Broccoli, Coke} and {Broccoli, Milk} are frequent but {Coke, Milk} is infrequent

  Candidate is not pruned because {Coke,Milk} does not contain the first item, i.e., Broccoli.

# Example: Lift/Interest

| | Coffee | $\overline{\text{Coffee}}$ | |
|---|---|---|---|
| Tea | 15 | 5 | 20 |
| $\overline{\text{Tea}}$ | 75 | 5 | 80 |
| | 90 | 10 | 100 |

## Association Rule: Tea → Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

⇒ Lift = 0.75/0.9= 0.8333 (< 1, therefore is negatively associated)

# Drawback of Lift & Interest

| | Y | $\overline{Y}$ | |
|---|---|---|---|
| X | 10 | 0 | 10 |
| $\overline{X}$ | 0 | 90 | 90 |
| | 10 | 90 | 100 |

$$Lift = \frac{1.0}{0.1} = 10$$

| | Y | $\overline{Y}$ | |
|---|---|---|---|
| X | 90 | 0 | 90 |
| $\overline{X}$ | 0 | 10 | 10 |
| | 90 | 10 | 100 |

$$Lift = \frac{1.0}{0.9} = 1.11$$

$$Lift \text{ also called } Interest = \frac{P(Y \mid X)}{P(Y)} = \frac{P(X,Y)}{P(X)P(Y)}$$

$$PS = P(X,Y) - P(X)P(Y)$$

$$\phi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1 - P(X)]P(Y)[1 - P(Y)]}}$$

**Statistical independence:**

**If P(X,Y)=P(X)P(Y) => Lift = 1**

**There are lots of measures proposed in the literature**

**Some measures are good for certain applications, but not for others**

**What criteria should we use to determine whether a measure is good or bad?**

**What about Apriori-style support based pruning? How does it affect these measures?**

| # | Measure | Formula |
|---|---|---|
| 1 | $\phi$-coefficient | $\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$ |
| 2 | Goodman-Kruskal's ($\lambda$) | $\frac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$ |
| 3 | Odds ratio ($\alpha$) | $\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$ |
| 4 | Yule's $Q$ | $\frac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB})+P(A,\overline{B})P(\overline{A},B)} = \frac{\alpha-1}{\alpha+1}$ |
| 5 | Yule's $Y$ | $\frac{\sqrt{P(A,B)P(\overline{AB})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})}+\sqrt{P(A,\overline{B})P(\overline{A},B)} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$ |
| 6 | Kappa ($\kappa$) | $\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$ |
| 7 | Mutual Information ($M$) | $\frac{\sum_i \sum_j P(A_i,B_j)\log \frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i)\log P(A_i), -\sum_j P(B_j)\log P(B_j))}$ |
| 8 | J-Measure ($J$) | $\max\left(P(A,B)\log(\frac{P(B\|A)}{P(B)})+P(A\overline{B})\log(\frac{P(\overline{B}\|A)}{P(\overline{B})}),\right.$ $\left.P(A,B)\log(\frac{P(A\|B)}{P(A)})+P(\overline{A}B)\log(\frac{P(\overline{A}\|B)}{P(\overline{A})})\right)$ |
| 9 | Gini index ($G$) | $\max\Big(P(A)[P(B\|A)^2+P(\overline{B}\|A)^2]+P(\overline{A})[P(B\|\overline{A})^2+P(\overline{B}\|\overline{A})^2]$ $-P(B)^2-P(\overline{B})^2,$ $P(B)[P(A\|B)^2+P(\overline{A}\|B)^2]+P(\overline{B})[P(A\|\overline{B})^2+P(\overline{A}\|\overline{B})^2]$ $-P(A)^2-P(\overline{A})^2\Big)$ |
| 10 | Support ($s$) | $P(A,B)$ |
| 11 | Confidence ($c$) | $\max(P(B\|A),P(A\|B))$ |
| 12 | Laplace ($L$) | $\max\left(\frac{NP(A,B)+1}{NP(A)+2},\frac{NP(A,B)+1}{NP(B)+2}\right)$ |
| 13 | Conviction ($V$) | $\max\left(\frac{P(A)P(\overline{B})}{P(A\overline{B})},\frac{P(B)P(\overline{A})}{P(B\overline{A})}\right)$ |
| 14 | Interest ($I$) | $\frac{P(A,B)}{P(A)P(B)}$ |
| 15 | cosine ($IS$) | $\frac{P(A,B)}{\sqrt{P(A)P(B)}}$ |
| 16 | Piatetsky-Shapiro's ($PS$) | $P(A,B)-P(A)P(B)$ |
| 17 | Certainty factor ($F$) | $\max\left(\frac{P(B\|A)-P(B)}{1-P(B)},\frac{P(A\|B)-P(A)}{1-P(A)}\right)$ |
| 18 | Added Value ($AV$) | $\max(P(B\|A)-P(B),P(A\|B)-P(A))$ |
| 19 | Collective strength ($S$) | $\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})}\times\frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$ |
| 20 | Jaccard ($\zeta$) | $\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$ |
| 21 | Klosgen ($K$) | $\sqrt{P(A,B)}\max(P(B\|A)-P(B),P(A\|B)-P(A))$ |

# Properties of A Good Measure

**Piatetsky-Shapiro:**
**3 properties a good measure M must satisfy:**

- M(A,B) = 0 if A and B are statistically independent
- M(A,B) increase monotonically with P(A,B) when P(A) and P(B) remain unchanged
- M(A,B) decreases monotonically with P(A) [or P(B)] when P(A,B) and P(B) [or P(A)] remain unchanged

# Comparing Different Measures

10 examples of contingency tables:

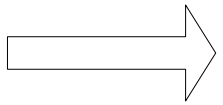
| Example | $f_{11}$ | $f_{10}$ | $f_{01}$ | $f_{00}$ |
|---|---|---|---|---|
| E1 | 8123 | 83 | 424 | 1370 |
| E2 | 8330 | 2 | 622 | 1046 |
| E3 | 9481 | 94 | 127 | 298 |
| E4 | 3954 | 3080 | 5 | 2961 |
| E5 | 2886 | 1363 | 1320 | 4431 |
| E6 | 1500 | 2000 | 500 | 6000 |
| E7 | 4000 | 2000 | 1000 | 3000 |
| E8 | 4000 | 2000 | 2000 | 2000 |
| E9 | 1720 | 7121 | 5 | 1154 |
| E10 | 61 | 2483 | 4 | 7452 |

Rankings of contingency tables using various measures:

| # | $\phi$ | $\lambda$ | $\alpha$ | $Q$ | $Y$ | $\kappa$ | $M$ | $J$ | $G$ | $s$ | $c$ | $L$ | $V$ | $I$ | $IS$ | $PS$ | $F$ | $AV$ | $S$ | $\zeta$ | $K$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | 1 | 1 | 3 | 3 | 3 | 1 | 2 | 2 | 1 | 3 | 5 | 5 | 4 | 6 | 2 | 2 | 4 | 6 | 1 | 2 | 5 |
| E2 | 2 | 2 | 1 | 1 | 1 | 2 | 1 | 3 | 2 | 2 | 1 | 1 | 1 | 8 | 3 | 5 | 1 | 8 | 2 | 3 | 6 |
| E3 | 3 | 3 | 4 | 4 | 4 | 3 | 3 | 8 | 7 | 1 | 4 | 4 | 6 | 10 | 1 | 8 | 6 | 10 | 3 | 1 | 10 |
| E4 | 4 | 7 | 2 | 2 | 2 | 5 | 4 | 1 | 3 | 6 | 2 | 2 | 2 | 4 | 4 | 1 | 2 | 3 | 4 | 5 | 1 |
| E5 | 5 | 4 | 8 | 8 | 8 | 4 | 7 | 5 | 4 | 7 | 9 | 9 | 9 | 3 | 6 | 3 | 9 | 4 | 5 | 6 | 3 |
| E6 | 6 | 6 | 7 | 7 | 7 | 7 | 6 | 4 | 6 | 9 | 8 | 8 | 7 | 2 | 8 | 6 | 7 | 2 | 7 | 8 | 2 |
| E7 | 7 | 5 | 9 | 9 | 9 | 6 | 8 | 6 | 5 | 4 | 7 | 7 | 8 | 5 | 5 | 4 | 8 | 5 | 6 | 4 | 4 |
| E8 | 8 | 9 | 10 | 10 | 10 | 8 | 10 | 10 | 8 | 4 | 10 | 10 | 10 | 9 | 7 | 7 | 10 | 9 | 8 | 7 | 9 |
| E9 | 9 | 9 | 5 | 5 | 5 | 9 | 9 | 7 | 9 | 8 | 3 | 3 | 3 | 7 | 9 | 9 | 3 | 7 | 9 | 9 | 8 |
| E10 | 10 | 8 | 6 | 6 | 6 | 10 | 5 | 9 | 10 | 10 | 6 | 6 | 5 | 1 | 10 | 10 | 5 | 1 | 10 | 10 | 7 |

# Property under Variable Permutation

| | B | $\overline{B}$ |
|---|---|---|
| A | p | q |
| $\overline{A}$ | r | s |



| | A | $\overline{A}$ |
|---|---|---|
| B | p | r |
| $\overline{B}$ | q | s |

Does M(A,B) = M(B,A)?

Symmetric measures:

- support, lift, collective strength, cosine, Jaccard, etc

Asymmetric measures:

- confidence, conviction, Laplace, J-measure, etc

# Property under Row/Column Scaling

## Grade-Gender Example (Mosteller, 1968):

| | Male | Female | |
|---|---|---|---|
| High | 2 | 3 | 5 |
| Low | 1 | 4 | 5 |
| | 3 | 7 | 10 |

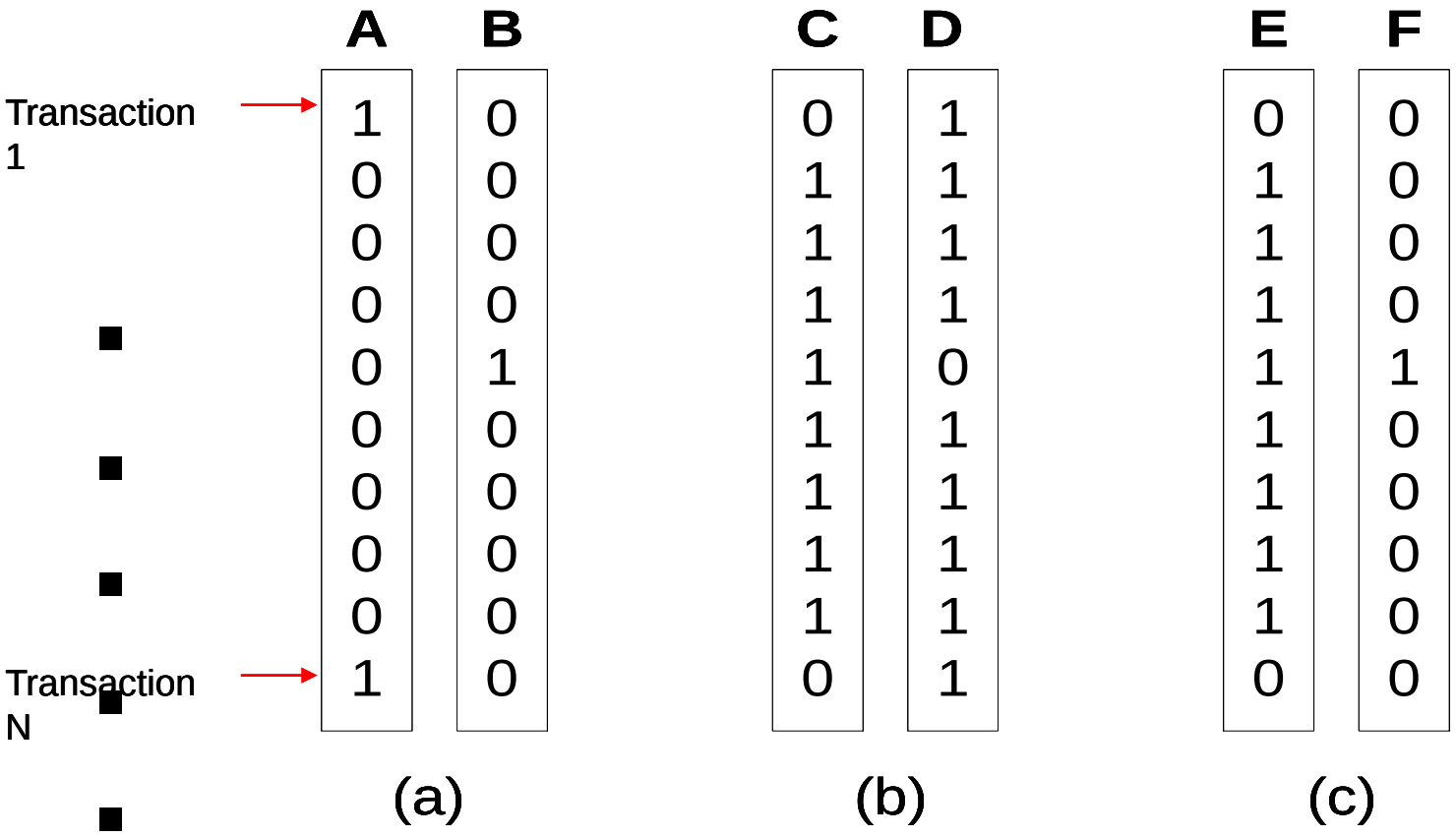
| | Male | Female | |
|---|---|---|---|
| High | 4 | 30 | 34 |
| Low | 2 | 40 | 42 |
| | 6 | 70 | 76 |

2x 10x

Mosteller:

Underlying association should be independent of the relative number of male and female students in the samples

# Property under Inversion Operation



Example: $\phi$-Coefficient

$$\phi\text{-}\ coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1 - P(X)]P(Y)[1 - P(Y)]}}$$

**$\phi$-coefficient is analogous to correlation coefficient for continuous variables**

| | Y | $\overline{Y}$ | |
|---|---|---|---|
| $\overline{X}$ | 60 | 10 | 70 |
| X | 10 | 20 | 30 |
| | 70 | 30 | 100 |

| | Y | $\overline{Y}$ | |
|---|---|---|---|
| X | 20 | 10 | 30 |
| $\overline{X}$ | 10 | 60 | 70 |
| | 30 | 70 | 100 |

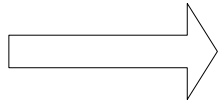
$$\phi = \frac{0.6 - 0.7 \times 0.7}{\sqrt{0.7 \times 0.3 \times 0.7 \times 0.3}} = 0.5238$$

$$\phi = \frac{0.2 - 0.3 \times 0.3}{\sqrt{0.7 \times 0.3 \times 0.7 \times 0.3}} = 0.5238$$

**$\phi$ Coefficient is the same for both tables**

Property under Null Addition

| | B | $\overline{B}$ |
|---|---|---|
| A | p | q |
| $\overline{A}$ | r | s |



| | B | $\overline{B}$ |
|---|---|---|
| A | p | q |
| $\overline{A}$ | r | s + k |

Invariant measures:

- support, cosine, Jaccard, etc

Non-invariant measures:

- correlation, Gini, mutual information, odds ratio, etc

# Different Measures have Different Properties

| Symbol | Measure | Range | P1 | P2 | P3 | O1 | O2 | O3 | O3' | O4 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\Phi$ | Correlation | -1 … 0 … 1 | Yes | Yes | Yes | Yes | No | Yes | Yes | No |
| $\lambda$ | Lambda | 0 … 1 | Yes | No | No | Yes | No | No* | Yes | No |
| $\alpha$ | Odds ratio | 0 … 1 … $\infty$ | Yes* | Yes | Yes | Yes | Yes | Yes* | Yes | No |
| Q | Yule's Q | -1 … 0 … 1 | Yes | Yes | Yes | Yes | Yes | Yes | Yes | No |
| Y | Yule's Y | -1 … 0 … 1 | Yes | Yes | Yes | Yes | Yes | Yes | Yes | No |
| $\kappa$ | Cohen's | -1 … 0 … 1 | Yes | Yes | Yes | Yes | No | No | Yes | No |
| M | Mutual Information | 0 … 1 | Yes | Yes | Yes | Yes | No | No* | Yes | No |
| J | J-Measure | 0 … 1 | Yes | No | No | No | No | No | No | No |
| G | Gini Index | 0 … 1 | Yes | No | No | No | No | No* | Yes | No |
| s | Support | 0 … 1 | No | Yes | No | Yes | No | No | No | No |
| c | Confidence | 0 … 1 | No | Yes | No | Yes | No | No | No | Yes |
| L | Laplace | 0 … 1 | No | Yes | No | Yes | No | No | No | No |
| V | Conviction | 0.5 … 1 … $\infty$ | No | Yes | No | Yes** | No | No | Yes | No |
| I | Interest | 0 … 1 … $\infty$ | Yes* | Yes | Yes | Yes | No | No | No | No |
| IS | IS (cosine) | 0 .. 1 | No | Yes | Yes | Yes | No | No | No | Yes |
| PS | Piatetsky-Shapiro's | -0.25 … 0 … 0.25 | Yes | Yes | Yes | Yes | No | Yes | Yes | No |
| F | Certainty factor | -1 … 0 … 1 | Yes | Yes | Yes | No | No | No | Yes | No |
| AV | Added value | 0.5 … 1 … 1 | Yes | Yes | Yes | No | No | No | No | No |
| S | Collective strength | 0 … 1 … $\infty$ | No | Yes | Yes | Yes | No | Yes* | Yes | No |
| $\zeta$ | Jaccard | 0 .. 1 | No | Yes | Yes | Yes | No | No | No | Yes |
| K | Klosgen's | $\max\left(\sqrt{\frac{2}{\sqrt{3}}-1}\left(2-\sqrt{3}-\frac{1}{\sqrt{3}}\right),0\right) \ldots \frac{2}{3\sqrt{3}}$ | Yes | Yes | Yes | No | No | No | No | No |

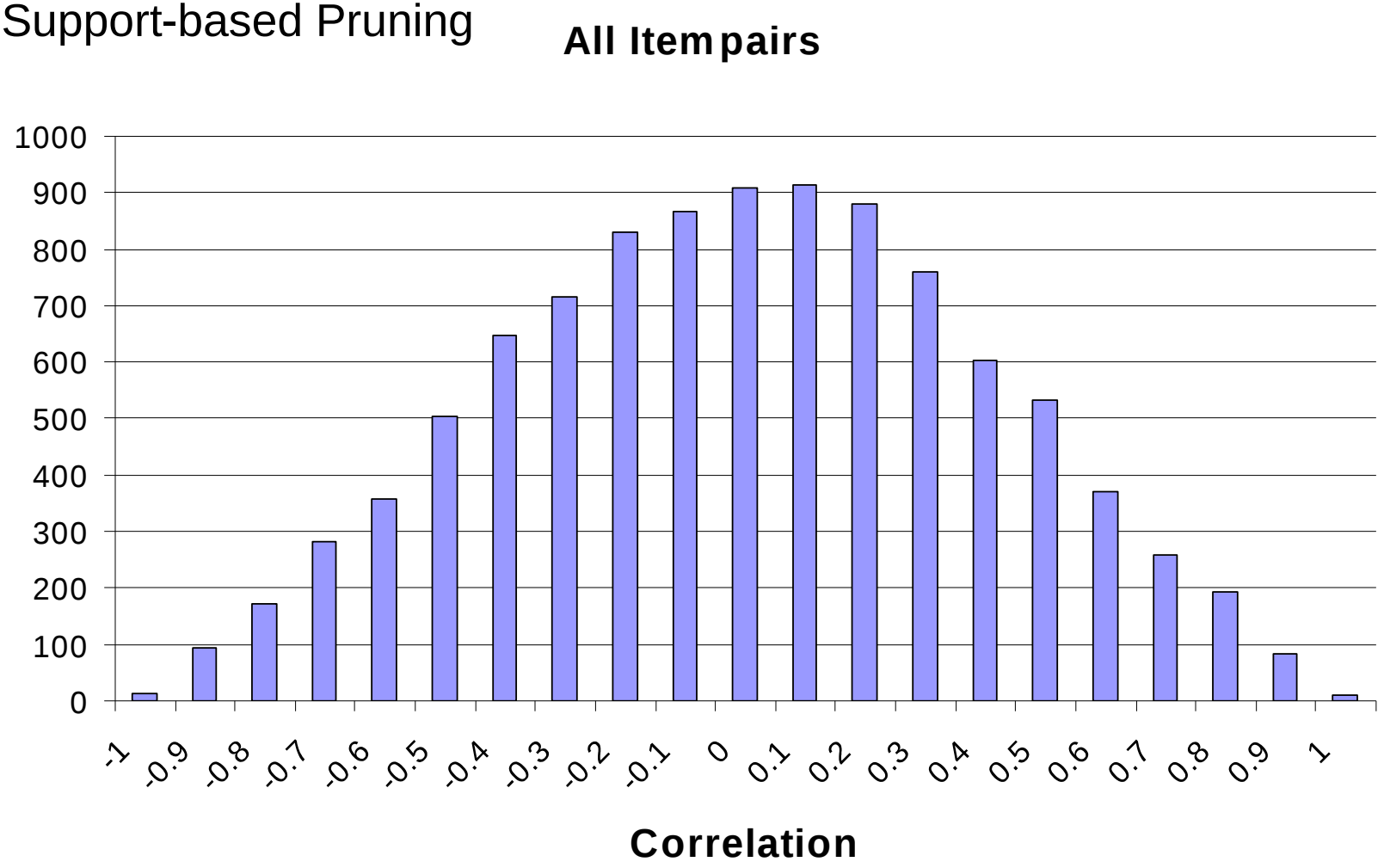
# Support-based Pruning

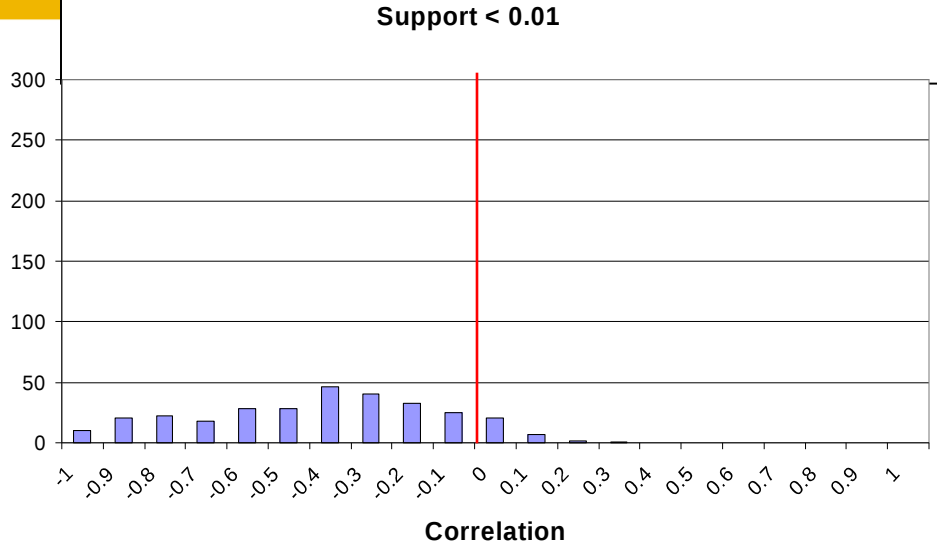
**Most of the association rule mining algorithms use support measure to prune rules and itemsets**

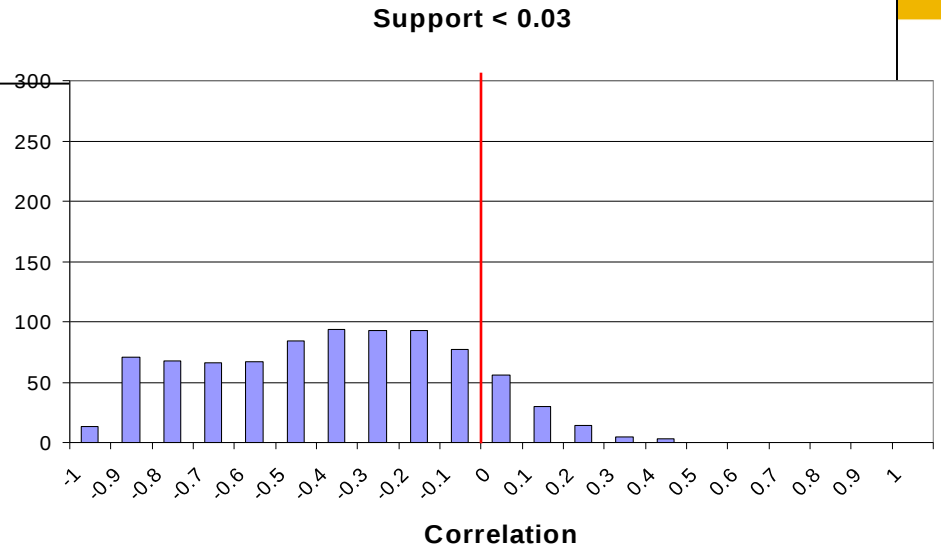
**Study effect of support pruning on correlation of itemsets**

- Generate 10000 random contingency tables
- Compute support and pairwise correlation for each table
- Apply support-based pruning and examine the tables that are removed

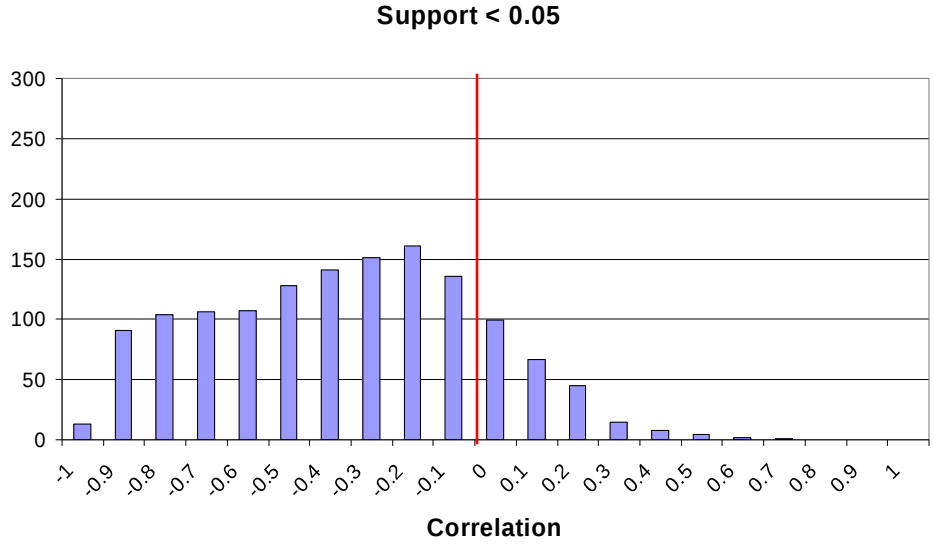
# Effect of Support-based Pruning






Support-based pruning eliminates mostly negatively correlated itemsets



# Effect of Support-based Pruning

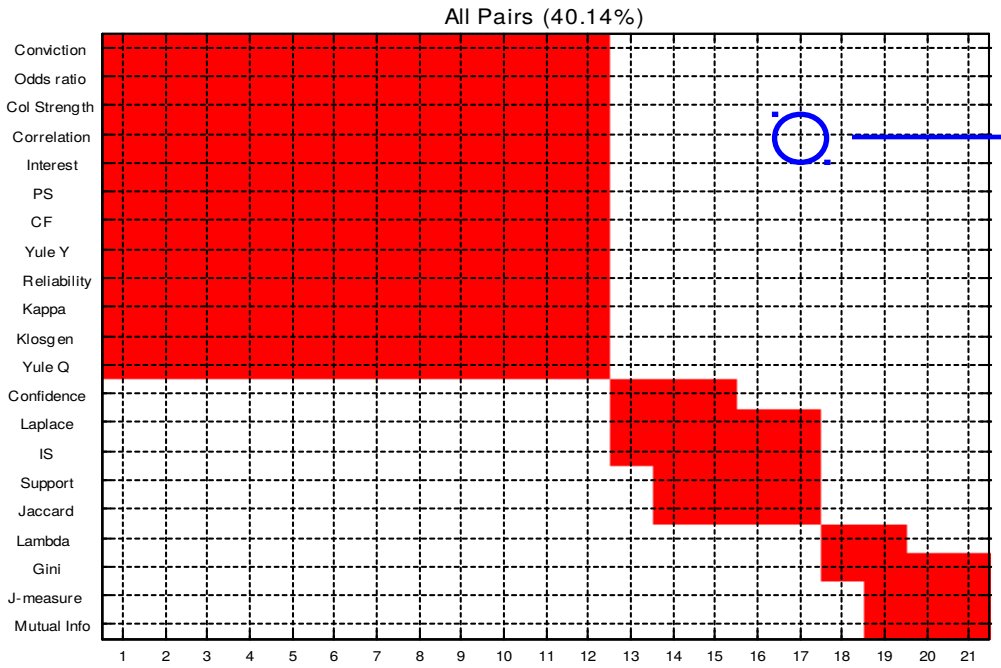
**Investigate how support-based pruning affects other measures**

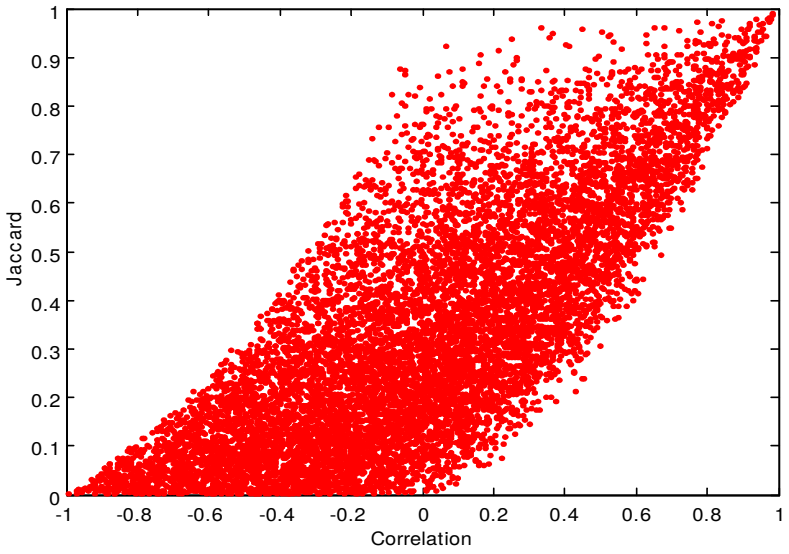
**Steps:**

- Generate 10000 contingency tables
- Rank each table according to the different measures
- Compute the pair-wise correlation between the measures

# Effect of Support-based Pruning

- Without Support Pruning (All Pairs)



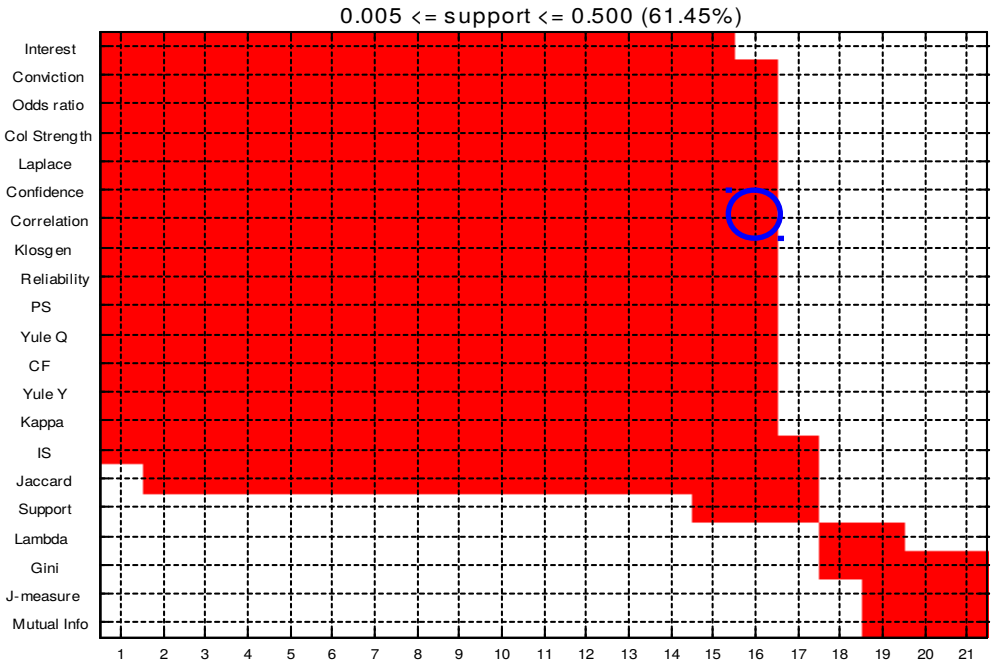



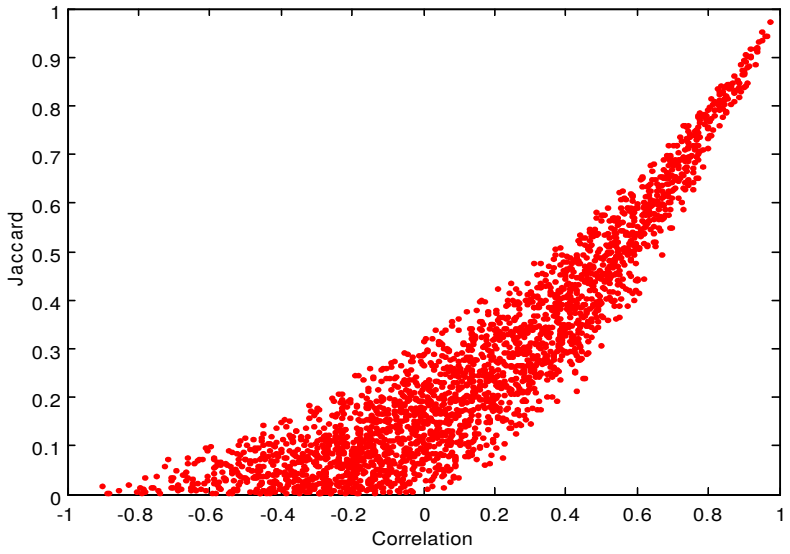

Scatter Plot between Correlation & Jaccard Measure

- Red cells indicate correlation between the pair of measures > 0.85
- 40.14% pairs have correlation > 0.85

# Effect of Support-based Pruning

- 0.5% ≤ support ≤ 50%



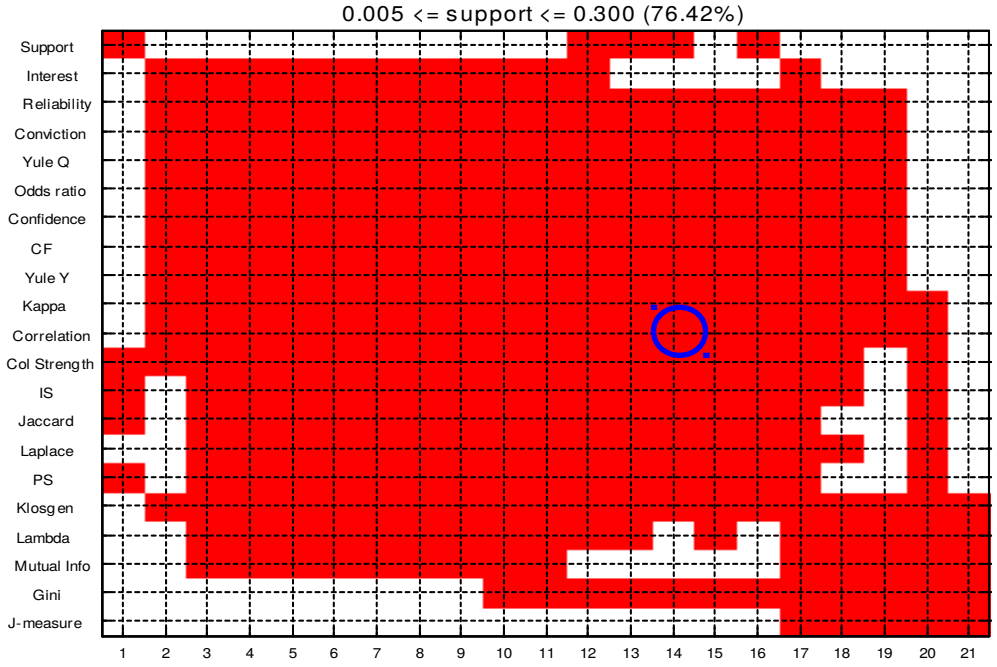



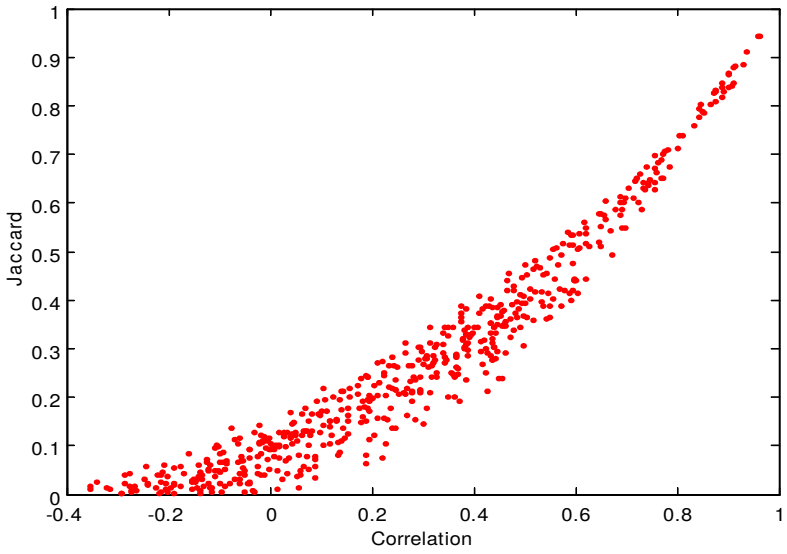

Scatter Plot between Correlation & Jaccard Measure:

- 61.45% pairs have correlation > 0.85

# Effect of Support-based Pruning

- 0.5% ≤ support ≤ 30%




Scatter Plot between Correlation & Jaccard Measure

- 76.42% pairs have correlation > 0.85

# Subjective Interestingness Measure

**Objective measure:**

- Rank patterns based on statistics computed from data
- e.g., 21 measures of association (support, confidence, Laplace, Gini, mutual information, Jaccard, etc).

**Subjective measure:**

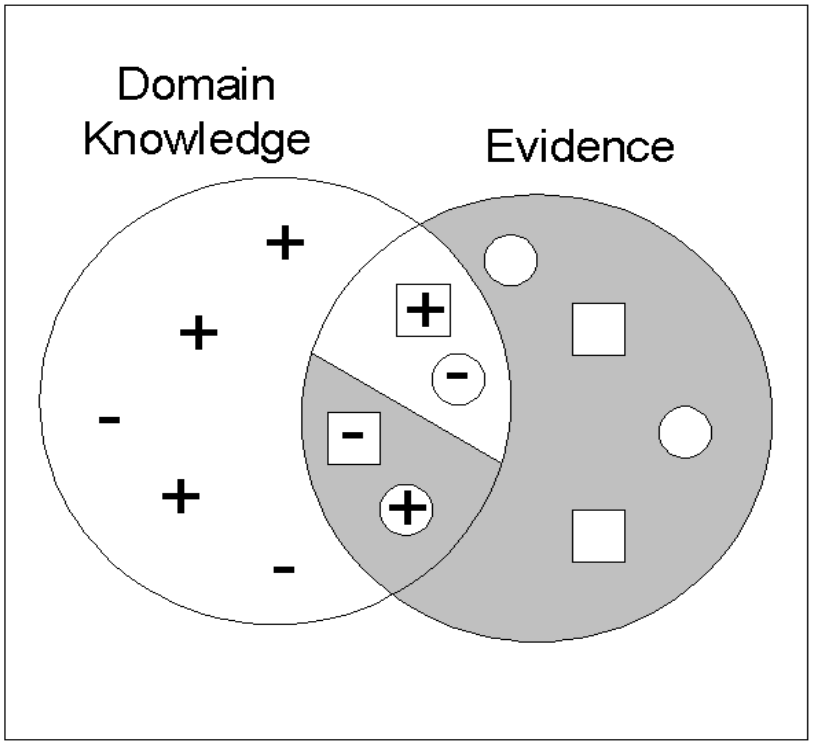
- Rank patterns according to user's interpretation

A pattern is subjectively interesting if it contradicts the expectation of a user (Silberschatz & Tuzhilin)

A pattern is subjectively interesting if it is actionable (Silberschatz & Tuzhilin)

# Interestingness via Unexpectedness

## Need to model expectation of users (domain knowledge)




+ Pattern expected to be frequent

- Pattern expected to be infrequent

Pattern found to be frequent

Pattern found to be infrequent

Expected Patterns

Unexpected Patterns

## Need to combine expectation of users with evidence from data (i.e., extracted patterns)

# Interestingness via Unexpectedness

## Web Data (Cooley et al 2001)

- Domain knowledge in the form of site structure
- Given an itemset F = {$X_1$, $X_2$, …, $X_k$} ($X_i$ : Web pages)

  L: number of links connecting the pages

  lfactor = L / (k × k-1)

  cfactor = 1 (if graph is connected), 0 (disconnected graph)

- Structure evidence = cfactor × lfactor
- Usage evidence

$$= \frac{P(X_1 \cap X_2 \cap \ldots \cap X_k)}{P(X_1 \cup X_2 \cup \ldots \cup X_k)}$$

- Use Dempster-Shafer theory to combine domain knowledge and evidence from data