Naive Bayes Classifier

Learning Type: Supervised, Task: Classification, Algorithm: Naive Bayes Classifier

Bayes' theorem calculates the probability of the event using prior knowledge and likelihood based on new information. The Naive Bayes classifier is based on Bayes' theorem

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$

for k-classes classification:
A $\rightarrow$ y $\in y_1, y_2, ..., y_k$ and B $\rightarrow$ X = $x_1, x_2, ..., x_n$

$$P(y \mid X) = \frac{P(X \mid y) \times P(y)}{P(X)}$$

if features $X = x_1, x_2, ..., x_n$ are i.i.d, that is $P(x_1, x_2, x_3, \dots \mid y) = P(x_1 \mid y)P(x_2 \mid y)...P(x_n \mid y)$

$$P(y|x_1, x_2...x_n) = \frac{\prod_{i=1}^{n} P(x_i \mid y) \times P(y)}{\prod_{i=1}^{n} P(x_i)}$$

Since $\prod_{i=1}^{n} P(x_i)$ is constant therefore

$$score(y) = \operatorname{argmax}_y \prod_{i=1}^{n} P(x_i \mid y) \times P(y)$$

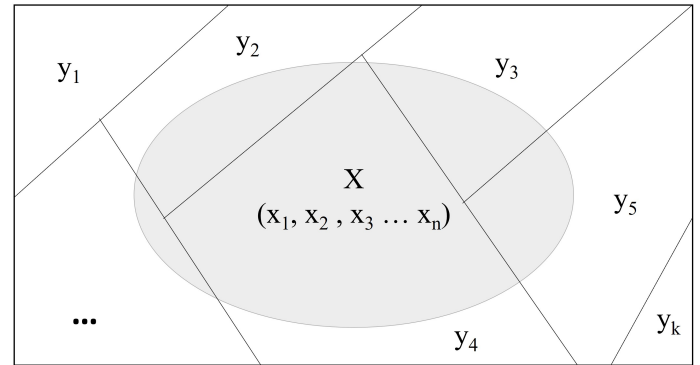$$Posterior\_probability = \frac{Likelihood \times Prior\_probability}{Evidence}$$

Illustration of Naive Bayes Classifier using Venn Diagram

Tennis Play Example

x_1	x_2	x_3	x_4	y
sunny	hot	high	weak	no
sunny	hot	high	strong	no
overcast	hot	high	weak	yes
rain	mild	high	weak	yes
rain	cool	normal	weak	yes
rain	cool	normal	strong	no
overcast	cool	normal	strong	yes
sunny	mild	high	weak	no
sunny	cool	normal	weak	yes
rain	mild	normal	weak	yes

x_1 = Outlook, x_2 = Temperature, x_3 = Humidity, x_4 = Wind, y = {yes, no}

Prior probability P(y)

$P(y = yes) = 6/10 = 0.6 \qquad P(y = no) = 4/10 = 0.4$

Likelihood P(X | y)

x_1	yes	no
sunny	1/6	3/4
overcast	2/6	0/4
rain	3/6	1/4

x_2	yes	no
hot	1/6	2/4
mild	2/6	1/4
cool	3/6	1/4

x_3	yes	no
high	2/6	3/4
normal	4/6	1/4

x_4	yes	no
weak	5/6	2/4
strong	1/6	2/4

New example: x_1 = sunny, x_2 = cool, x_3 = high and x_4 = strong

$$score(yes) = P(sunny \mid yes) P(cool \mid yes) P(high \mid yes) P(strong \mid yes) \times P(yes)$$
$$score(yes) = 0.167 \times 0.5 \times 0.333 \times 0.167 \times 0.6 = 0.002786$$
$$score(no) = P(sunny \mid no) P(cool \mid no) P(high \mid no) P(strong \mid no) \times P(no)$$
$$score(no) = 0.75 \times 0.25 \times 0.75 \times 0.5 \times 0.4 = 0.0281 \quad \text{(No Tennis)}$$