MODELING THE ARMS RACE

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1 Assumptions

We define the following variables:

- $x(t) \geq 0$ is the amount of weaponry that country X has at time t,
- $y(t) \geq 0$ is the amount of weaponry that country Y has at time t,
- $a \geq 0$ is the coefficient of weapon production of country X,
- $b \geq 0$ is the coefficient of weapon production of country Y.

We restrict this model to a specific time interval in which a and b stay constant, even though in a real-world situation, the production rates might fluctuate.

2 Simple Model

3 A More Realistic Model

Now we consider the situation when both countries decide to update their inventory of weaponry due to new technological advances and old weapons being expired. Therefore, we define additional parameters in the differential equations:

$$\frac{dx}{dt} = ay - cx \tag{1}$$

$$\frac{dy}{dt} = bx - dy \tag{2}$$

where $c \geq 0$ and $d \geq 0$ are constant coefficients of the destruction of old weapons in countries X and Y, respectively.

We can solve this system of differential equations by constructing a coefficient matrix:

$$M = \begin{bmatrix} -c & a \\ b & d \end{bmatrix}.$$

Our characteristic polynomial is $\lambda^2 + (d + c)\lambda + cd - ab$. Computing th

4 Case Study ?

Due to limited time and resources, we define "weaponry" in this case to only include fighting planes and their associated weapons such as bombs and missiles.