MODELING THE ARMS RACE

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Introduction

Every country is concerned about its national security. Maintaining an inventory of weaponry is one of the priorities in defense, but how a country does it depends on not only its own inventory, but also on other factors, such as technological advances, other countries's weaponry, and the tension among them. In this paper we seek to create a simple differential equations model to investigate the change in weaponry of two countries in relation to one another, and to improve our model by factoring another variable into our equations. Then we apply our model to the Cold War between the United States and the USSR where there was an arm race between the two countries. Due to limited time and resources, as well as techniques and skills in solving differential equations, our model has a lot of room for improvement, considering the number of variables not present in our equations.

1 Assumptions

We define the following variables:

- $x(t) \geq 0$ is the amount of weaponry that country X has at time t,
- $y(t) \geq 0$ is the amount of weaponry that country Y has at time t,
- $a \geq 0$ is the coefficient of weapon production of country X,
- $b \geq 0$ is the coefficient of weapon production of country Y.

We restrict this model to a specific time interval in which a and b stay constant, even though in a real-world situation, the production rates might fluctuate.

2 Simple Model

3 A More Realistic Model

Now we consider the situation when both countries decide to update their inventory of weaponry due to new technological advances and old weapons being expired. Therefore, we define additional parameters in the differential equations:

$$\frac{dx}{dt} = ay - cx \tag{1}$$
$$\frac{dy}{dt} = bx - dy \tag{2}$$

where $c \geq 0$ and $d \geq 0$ are constant coefficients of the destruction of old weapons in countries X and Y, respectively.

We can solve this system of differential equations by constructing a coefficient matrix:

$$M = \begin{bmatrix} -c & a \\ b & d \end{bmatrix}.$$

Our characteristic polynomial is $\lambda^2 + (d+c)\lambda + cd - ab$. Computing the eigenvalues and eigenvectors yield

$$\lambda_1 = -\frac{1}{2}d - \frac{1}{2}c + \frac{1}{2}\sqrt{4ab - 2cd + c^2 + d^2}; \quad \vec{v}_1 = \begin{bmatrix} \frac{a}{\lambda_1} \\ 1 \end{bmatrix}$$

$$\lambda_2 = -\frac{1}{2}d - \frac{1}{2}c - \frac{1}{2}\sqrt{4ab - 2cd + c^2 + d^2}; \quad \vec{v}_2 = \begin{bmatrix} \frac{a}{\lambda_2} \\ 1 \end{bmatrix}$$

4 Case Study: Cold War

We consider the arm race between the United States and the USSR during the Cold War. Due to limited time and resources, we define "weaponry" in this case to only include warheads. We also exclude other external factors such as internal political conflicts in each country, current wars at the time, and treaties.

5 Reference