MODELING THE ARMS RACE

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1 Assumptions

We define the following variables:

- $x(t) \geq 0$ is the amount of weaponry that country X has at time t,
- $y(t) \geq 0$ is the amount of weaponry that country Y has at time t,
- $a \geq 0$ is the coefficient of weapon production of country X,
- $b \geq 0$ is the coefficient of weapon production of country Y.

We restrict this model to a specific time interval in which a and b stay constant, even though in a real-world situation, the production rates might fluctuate.

2 Simple Model

3 A More Realistic Model

Now we consider the situation when both countries decide to update their inventory of weaponry due to new technological advances and old weapons being expired. Therefore, we define additional parameters in the differential equations:

$$\frac{dx}{dt} = ay - cx \tag{1}$$

$$\frac{dy}{dt} = bx - dy \tag{2}$$

where $c \geq 0$ and $d \geq 0$ are constant coefficients of the destruction of old weapons in countries X and Y, respectively.

We can solve this system of differential equations by constructing a coefficient matrix:

$$M = \begin{bmatrix} -c & a \\ b & d \end{bmatrix}.$$

Our characteristic polynomial is $\lambda^2 + (d+c)\lambda + cd - ab$. Computing the eigenvalues and eigenvectors yield

$$\lambda_1 = -\frac{1}{2}d - \frac{1}{2}c + \frac{1}{2}\sqrt{4ab - 2cd + c^2 + d^2}; \quad \vec{v}_1 = \begin{bmatrix} \frac{a}{\lambda_1} \\ 1 \end{bmatrix}$$

$$\lambda_2 = -\frac{1}{2}d - \frac{1}{2}c - \frac{1}{2}\sqrt{4ab - 2cd + c^2 + d^2}; \quad \vec{v}_2 = \begin{bmatrix} \frac{a}{\lambda_2} \\ 1 \end{bmatrix}$$

4 Case Study: Cold War

We consider the arm race between the United States and the USSR during the Cold War. Due to limited time and resources, we define "weaponry" in this case to only include warheads. We also exclude other external factors such as internal political conflicts in each country, current wars at the time, and treaties.

5 Reference