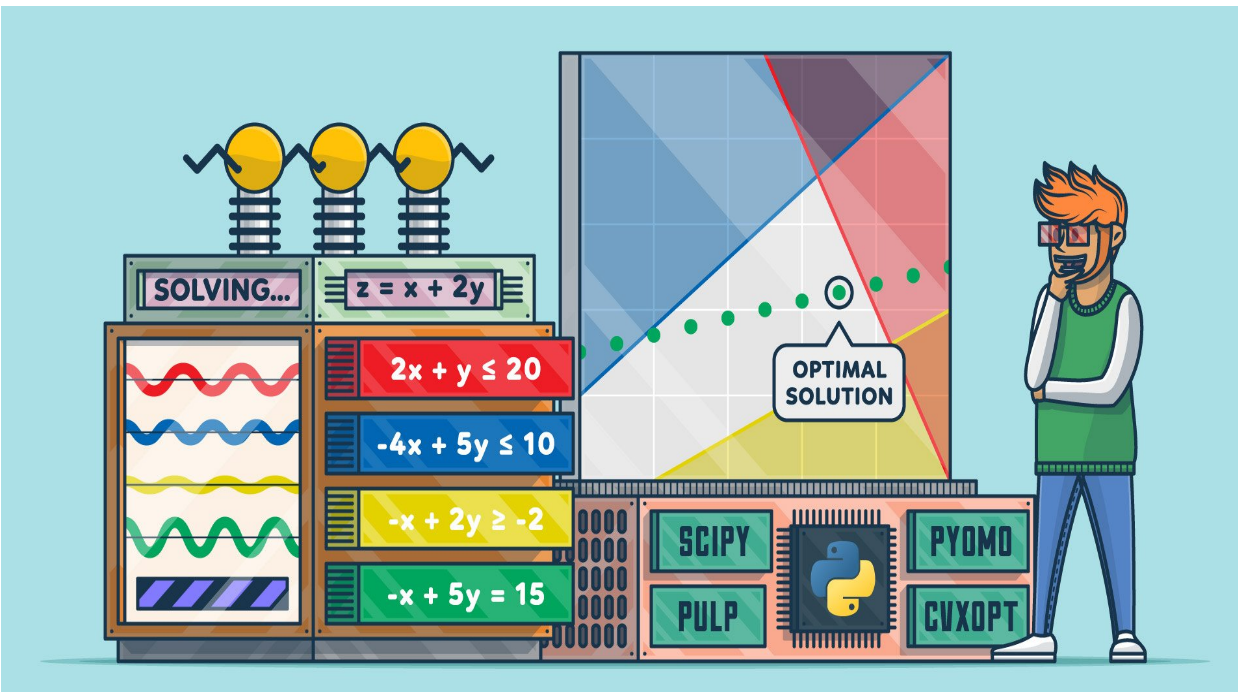
# Optimization Report

## Project 2 – Integer Programming



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# Project 2 – Integer Programming

## Executive Summary

The purpose of this project is to develop a data-driven strategy for allocating a $10 million marketing budget to maximize total Return on Investment (ROI). This analysis moves beyond "gut feel" decision-making by using mathematical optimization to model the complex, piecewise linear returns of each marketing platform.

We formulated two distinct models to evaluate ROI estimates from two independent consulting firms:

1. A Linear Program (LP) for the first firm's concave (diminishing return) data.
2. A Mixed-Integer Program (MIP) to correctly handle the second firm's more complex, non-concave data.

Our main takeaway is that the underlying data structure fundamentally changes the optimal strategy. The simple LP model fails when applied to the non-concave data, illustrating the necessity of the more advanced MIP. Our analysis quantifies the financial loss of using the wrong model and finds that the $3M platform cap is a critical and active constraint. We also extend this MIP to build a 12-month reinvestment plan and analyze its budget stability.

## Business Context

Marketing budgets represent a significant portion of company spending, but their effectiveness varies dramatically. The goal of this project is to move our allocation process from one based on "negotiation skills or individual managers" to a fact-based, profit-maximizing strategy.

We are tasked with recommending how to spread a $10M marketing budget across ten different platforms (e.g., TV, Print, Facebook, Email, etc.) . The core business problem is to find the optimal spending mix that honors all managerial constraints while generating the highest possible return.

## Technical Methodology

To solve this problem, we employed two different optimization formulations in Gurobi, as required by the two datasets.

1. **Linear Program (LP) for Concave Data** :
   For the first dataset (roi_company1.csv), the ROI was non-increasing, resulting in a concave return function. This allowed us to formulate a simple Linear Program.

   **Decision variables:** $(x_{i,j})$ - The amount (in millions) invested in platform i, tier j.

   **Objective :** Maximize total return - Max $\sum_{i,j} \quad x_{i,j} \, . \, ROI_{i,j}$

**Constraints:**

**Tier Capacity:** $x_{i,j} \leq width_{i,j}$ **Total Budget:** $\sum_{i,j} \quad x_{i,j}$ = 10

**Business Rules**:

(a) Print + TV ≤ Facebook + Email

(b) Social Media ≥ 2 × (SEO + AdWords)

(c) Max $3M per platform

2. **Mixed-Integer Program (MIP) for Non-Concave Data** :
   The second dataset (roi_company2.csv) was non-concave, meaning a simple LP would fail by skipping "valleys" to get to higher-ROI "peaks". To solve this, we formulated a Mixed-Integer Program (MIP) using the specified piecewise linear approximation method:

   **Binary variables** $z_{i,j} \epsilon$ {0,1} - Indicates whether tier j of platform i is active.

   **Continuous variables** $\lambda_{i,k} \epsilon$ [0,1] - Convex weights for breakpoints.
   **Objective function:** Maximise $\sum_i \quad \sum_k \quad \lambda_{i,k} . F_{i,k}$

   where $F_{i,k}$ is the cumulative return at breakpoint k for platform i.

   **Constraints :**

   | **Convex combination** | **One Active segment** | **SOS2 Adjacency** |
   |---|---|---|
   | $\sum_k \quad \lambda_{i,k}$=1 | $\sum_j \quad z_{i,j}$ = 1 | $\lambda_{i,k} \leq z_{i,k-1} + z_{i,k}$ |

## Part 3: Optimal Allocation with Concave Data (LP Model)

### Objective & Methodology

For the first scenario, we analyzed the data from roi_company1.csv. As specified in the project brief, this dataset is "concave," meaning it exhibits non-increasing, or diminishing, returns. This property is ideal for a **Linear Program (LP)**.

We formulated an LP model in Gurobi where the decision variables represented the continuous amount of money (in millions) invested in each individual tier of each platform. The model's objective was to maximize the total return, subject to the $10M total budget and the three business constraints.

```
 # Total Budget Constraint
m.addConstr(
gp.quicksum(total_invest_i[p] for p in platforms) <= TOTAL_BUDGET,
name="Total_Budget")
```

```
# Constraint (a): Print + TV <= Facebook + Email
   m.addConstr(
       gp.quicksum(total_invest_i[p] for p in PRINT_TV if p in platforms) <=
       gp.quicksum(total_invest_i[p] for p in FB_EMAIL if p in platforms),
       name="Constraint_A")
```

```
# Constraint (b): Social >= 2 * (SEO + AdWords)
   m.addConstr(
       gp.quicksum(total_invest_i[p] for p in SOCIAL_MEDIA if p in platforms) >=
       2 * gp.quicksum(total_invest_i[p] for p in SEO_ADWORDS if p in platforms),
       name="Constraint_B")
```

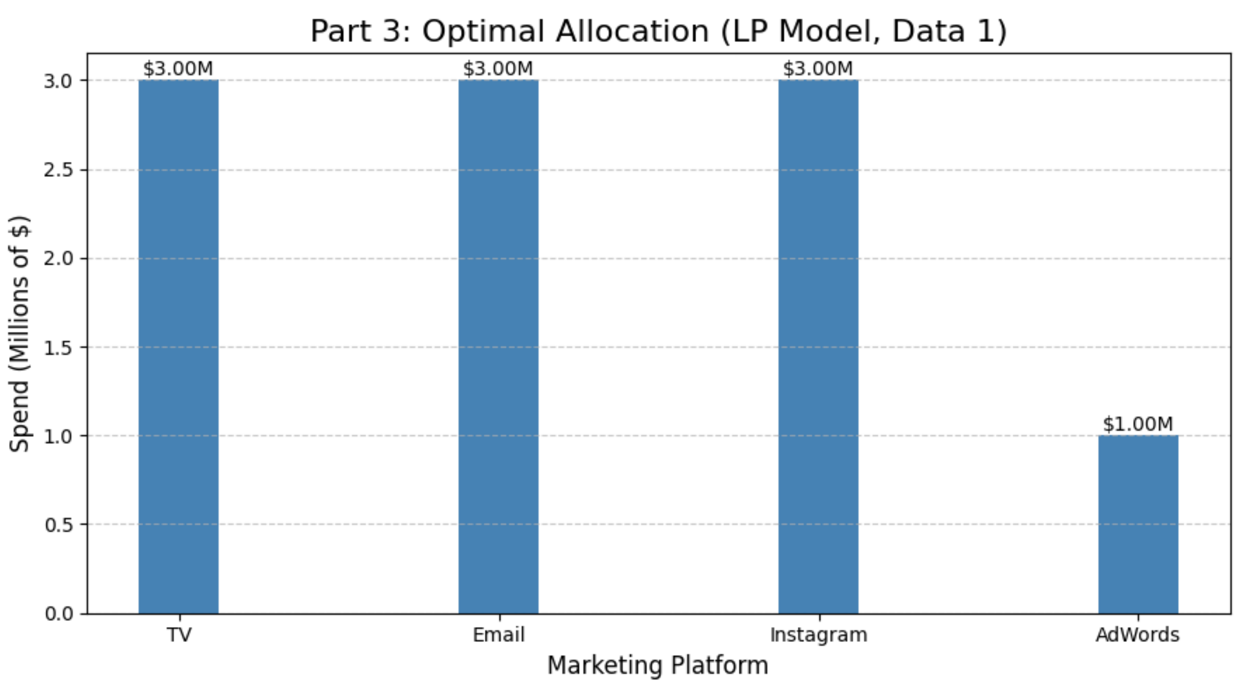
```
# Constraint (c): Max $3M per platform
   m.addConstrs(
       (total_invest_i[p] <= PLATFORM_CAP for p in platforms),name="Constraint_C")
```

**Optimal Allocation**

The model ran successfully and used the entire $10M budget. The maximum possible return for this scenario is **$0.5436 million**.The optimal strategy is not to diversify widely, but to concentrate the budget into the four highest-performing platforms. The recommended allocation is as follows:

| **Platform** | **Spend (M$)** | **Return (M$)** | **Blended ROI (%)** |
|---|---|---|---|
| TV | $3.0 | $0.1824 | 6.1% |
| Instagram | $3.0 | $0.1714 | 5.7% |
| Email | $3.0 | $0.1479 | 4.9% |
| AdWords | $1.0 | $0.0419 | 4.2% |
| **Total** | **$10.0** | **$0.5436** | |

This allocation is also visualized in the chart below, showing the heavy investment in TV, Instagram, and Email, all of which hit the $3M cap.




**Analysis of Constraints**

An essential part of the analysis is to see which "rules" actively influenced the final plan. Our constraint check provided clear answers:

- **Constraint (c) - $3M Platform Cap (Active):** This was the most influential constraint. Three platforms (TV, Instagram, and Email) all hit the **$3.0M** spending limit. This tells us that, if allowed, the model would have invested *even more* in these platforms. The boss's rule was critical in forcing diversification.
- **Constraint (a) - Print/TV vs. FB/Email (Active):** This constraint was also active. The $3.0M spent on TV (and $0 on Print) was *exactly equal* to the $3.0M spent on Email (and $0 on Facebook). This rule directly shaped the allocation, preventing further investment in TV, which had the highest blended ROI.
- **Constraint (b) - Social vs. SEO/AdWords (Not Active):** This rule was not a limiting factor. The $3.0M spent on Social Media (Instagram) was already well above the required $2.0M (2 * $1.0M for AdWords).

## Part 5: Optimal Allocation with Non-Concave ROI Data (MIP Model)

### Objective & Methodology

The data from roi_company2.csv was analyzed for the second case. The non-concave curves exhibited stepwise returns, indicating that the marginal ROI may fluctuate between tiers, either increasing or decreasing. These factors render them unsuitable for a straightforward Linear Program (LP).

To accurately model this behavior, a Mixed-Integer Programming (MIP) model was developed using Gurobi. This model utilized continuous variables ($\lambda$) to represent the fractional investment within a segment, while binary variables ($z$) indicated the active ROI tier for each platform. The objective remained to maximize the return on investment; however, it was necessary to adhere to the $10 million budget and the established three business rules. The MIP structure ensured that the allocation adhered to the actual non-concave shape of each platform's ROI curve.

```
 # Total Budget Constraint

if df_roi['ROI'].min() >= 0:

    m.addConstr(

        gp.quicksum(total_invest_i[p] for p in platforms) == TOTAL_BUDGET,

        name="Total_Budget"
```

```
    )
  else:
    m.addConstr(
        gp.quicksum(total_invest_i[p] for p in platforms) <= TOTAL_BUDGET,
        name="Total_Budget"
    )
```

**Optimal Allocation (MIP Model)**

The MIP model for non-concave ROI data performed effectively within the $10 million budget limit. The model had the potential to utilize less than the entire budget if specific tiers provided lower returns. However, in this instance, the model effectively utilized the full $10 million budget, resulting in a total return of $0.4528 million.

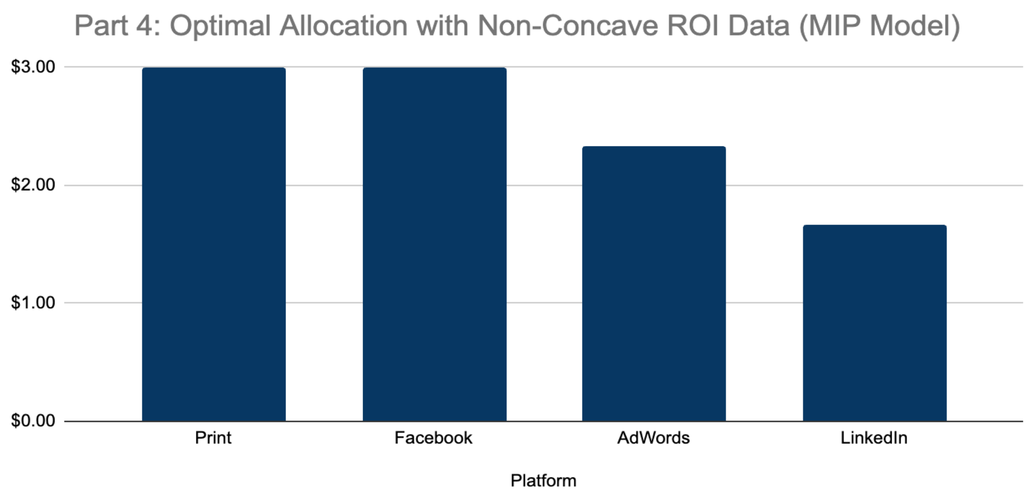
The findings indicate that MIP allocation strategy demonstrates a greater level of balance compared to the concave (LP) scenario. The MIP optimizer diversified its investments rather than limiting all funds into high-performing platforms. Each platform received funding according to its most profitable return on investment tier. The analysis demonstrates that the non-concave structure of the ROI curves enables the solver to identify local optima by selecting platform tiers that provide significant incremental gains within their respective ranges.

| Platform | Spend (M$) | Return (M$) | Blended ROI (%) |
|---|---|---|---|
| Print | $3.000 | $0.1426 | 4.8% |
| Facebook | $3.000 | $0.1299 | 4.3% |
| AdWords | $2.333 | $0.1065 | 4.6% |
| LinkedIn | $1.667 | $0.0738 | 4.4% |
| Total | $10.000 | $0.4528 | 4.5% |

This allocation satisfies all business constraints:

- The total spend on Print and TV does not exceed that of Facebook and Email.
- Social media investments remain at least twice the amount spent on SEO and AdWords.
- No single platform exceeds the $3M cap.

The MIP model shows superior performance in managing nonlinear ROI relationships compared to the concave LP case. The model's binary activation variables (z) ensured that investments only occurred at levels that would yield a favorable return on investment. This prevented discontinuities from merging in a manner that was illogical. The results indicate that the MIP model successfully illustrates how non-linear ROI operates in real-world situations. This results in an optimal and well-balanced combination of marketing strategies.



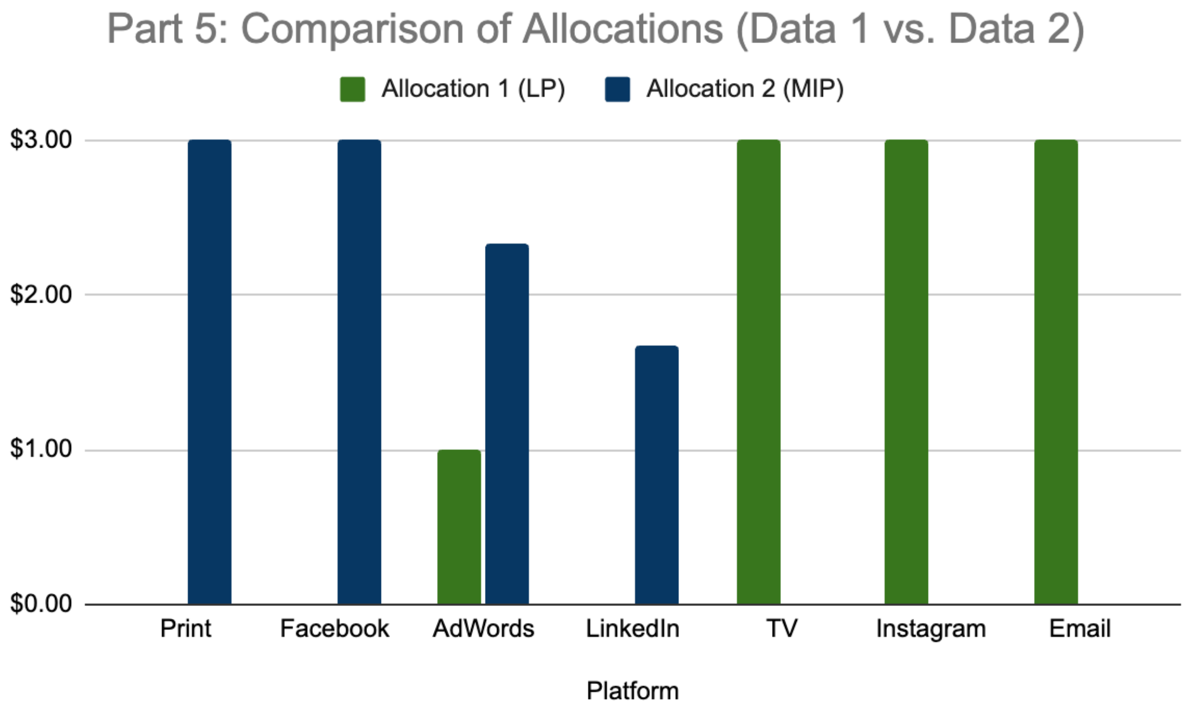

## Part 5: Comparative and Robustness Analysis

### Cross-Scenario Comparison

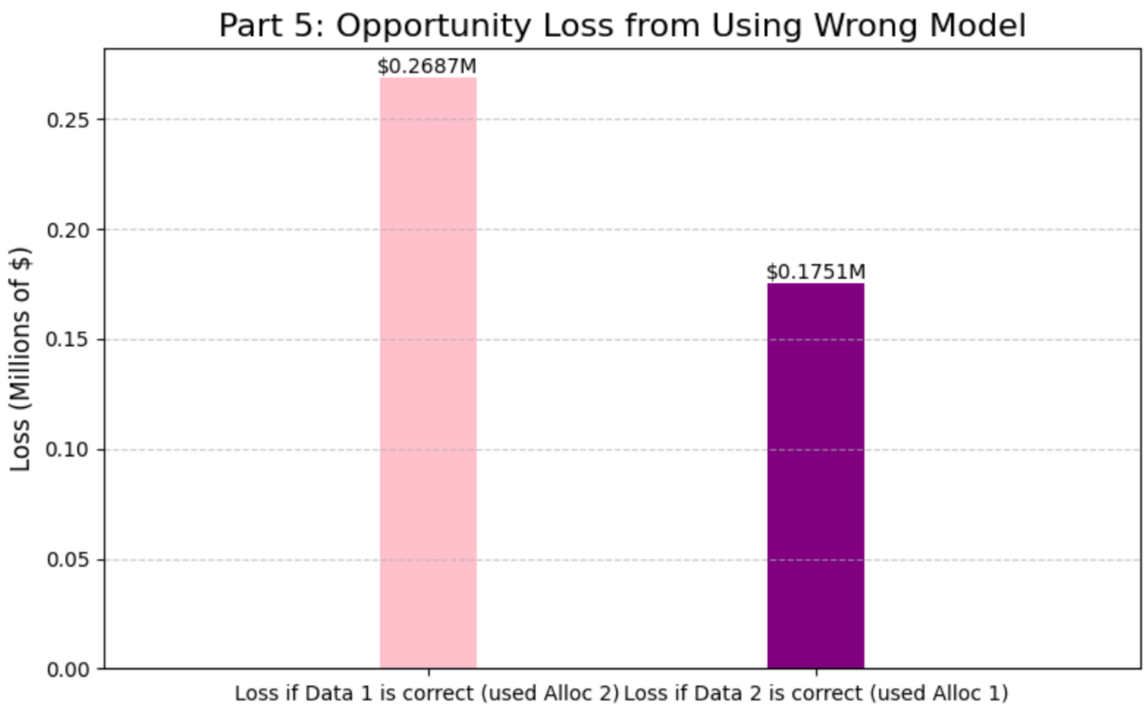
To test the sensitivity of both allocation models, we evaluated each model's recommended allocation under the other dataset's ROI assumptions.

- **Scenario 1 – Concave ROI data (Data 1) assumed correct:**
  The initial assessment indicated that Data 1 for the concave ROI was considered accurate. The allocation through Linear Programming (LP) yielded the highest return, amounting to $0.5436 million. The application of Mixed-Integer Programming (MIP) allocation, which is most effective with non-concave data, on this concave dataset resulted in a total return of $0.2749 million. This indicated an objective loss of $0.2687 million. The use of a non-concave allocation in a concave setting is likely to be ineffective.
- **Scenario 2 – Non-concave ROI data (Data 2) assumed correct:**
  In scenario 2, we assumed that the non-concave ROI data, referred to as Data 2, was accurate. The MIP model generated the highest revenue, amounting to $0.4528 million. The testing of the LP allocation, which had been optimized for concave data, on a non-

concave dataset resulted in a total return of $0.2777 million. This indicates an objective loss of $0.1751 million. This result demonstrates that each allocation is most effective when the ROI assumptions upon which it was based are accurate.




The results suggest that allocations specific to certain models are not applicable in both concave and non-concave environments. The stepwise ROI patterns in the non-concave data suggest that the continuous allocation logic of the LP does not sufficiently address the discrete changes in marginal returns. In contrast, the MIP model is more appropriate for handling non-linear structures.




**Constraint (c) Activity – $3 Million Platform Cap**

Constraint (c), which limits investment in any single platform to **$3 million**, was found to be **active in both models**.

- In the LP model, the constraint bound was reached for **TV, Instagram, and Email**.
- In the MIP model, it was bound for **Print and Facebook**.

This confirms that the cap meaningfully influences the optimization by preventing the solver from allocating the entire budget to one or two very high-ROI platforms. Its activation across multiple platforms shows that the constraint is **operationally relevant**, not redundant.

**Robustness Check – With Cap vs Without Cap**

To evaluate robustness, both models were re-solved without the per-platform cap.

| Case | True Data | Compared Allocation | Loss WITH Cap (M$) | Loss WITHOUT Cap (M$) | Δ (With – Without) | Interpretation |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| **A** | Data 1 (Concave) | MIP allocation | 0.2687 | 0.2824 | −0.0137 | Cap reduced loss → improved robustness |
| **B** | Data 2 (Non-Concave) | LP allocation | 0.1751 | 0.2247 | −0.0496 | Cap reduced loss → improved robustness |

In both cases, adding the $3M cap trimmed the losses. Put simply, the cap stops the model from pushing too much money into any one bet, which helps when ROI estimates shift or the data is a bit noisy. It also nudges the plan toward healthier diversification instead of overloading a single channel. Taken together, that makes the $3M limit a sensible, real-world guardrail: it's practical risk management that makes performance more consistent across different ROI scenarios.

## Part 6: Optimal Allocation with Non-Concave Data (Mixed IP Model) and Minimum Amount Constraint

For the sixth part, we used the data from roi_company2.csv and the minimum investment thresholds from min_amount.csv.
While the approach to modelling is similar to that of Part 4, this scenario required implementing an **"activation rule"** for each marketing medium — **meaning that an investment below a certain threshold would not generate meaningful impact.**

To model this, we formulated a **Mixed Integer Program (MIP)** in **Gurobi**.
The decision variables represented:

- The **continuous investment amount** (in millions of dollars) allocated to each tier of each platform

```
# Continuous decision variables for investment in each tier

y[p, idx] = m.addVar(lb=0.0, ub=r["TierCap"], vtype=GRB.CONTINUOUS,
name=f"y[{p},{idx}]")
```

Where, `y[p, idx]` is the amount invested in tier `idx` of platform `p`.
The variable is **continuous**, representing partial or full investment up to the tier capacity.

- Each platform's **total spend** was also captured through an aggregated continuous variable:

```
x[p] = m.addVar(lb=0.0, ub=PLATFORM_CAP, vtype=GRB.CONTINUOUS,
name=f"x[{p}]")
```

where `x[p]` = total spend on platform *p*, the total spend on platform p is bounded by the maximum cap (3 million USD).

- **Binary activation indicators** for whether a platform or tier was utilized.

  Binary variables were introduced to model **whether a platform or a tier is activated (invested in)**.
  These are defined as:

```
# Binary variable for whether the platform is used

w[p] = m.addVar(vtype=GRB.BINARY, name=f"w[{p}]")
```

```
# Binary variable for whether a particular tier is active

z[p, idx] = m.addVar(vtype=GRB.BINARY, name=f"z[{p},{idx}]")
```

Where, `w[p] = 1` denotes platform *p* is selected and receives at least its minimum investment, and `z[p, idx] = 1` → tier *idx* of platform *p* is active (investment allowed in that tier).

- These binary indicators implement the **all-or-nothing activation rule**, ensuring investments only occur when they meet or exceed the minimum effectiveness threshold.

- The **objective** of the model was to **maximize the total return** from all platforms, calculated as the sum of (ROI × investment) across all active tiers.

```
m.setObjective(quicksum(tiers_by_p[p].loc[idx, "ROI"] * y[p, idx]

for p in platforms for idx in range(len(tiers_by_p[p]))), GRB.MAXIMIZE)
```

- This objective function calculates the sum of each tier's **Return on Investment (ROI)** multiplied by the corresponding **amount invested (y)**.

  By maximizing this expression, the model identifies the optimal allocation of the $10 million marketing budget that yields the highest overall return.

This was subject to the following constraints:

- The total investment in traditional media (*Print* and *TV*) must not exceed the total investment in *Facebook* and *Email*. - **Constraint A/ Boss Constraint A**

```
m.addConstr(x["Print"] + x["TV"] <= x["Facebook"] + x["Email"], name="boss_a")
```

- The total spending across all *social media platforms* must be at least **twice** the combined spending on *SEO* and *AdWords*. - **Constraint B/ Boss Constraint B**

```
social = ["Facebook", "LinkedIn", "Instagram", "Snapchat", "Twitter"]

m.addConstr(quicksum(x[p] for p in social) >= 2.0 * (x["SEO"] + x["AdWords"]),
name="boss_b")
```

- Each platform’s total spend cannot exceed **$3 million** - **Constraint C**

```
for p in platforms:

 m.addConstr(x[p] <= PLATFORM_CAP, name=f"cap[{p}]")
```

- Then the **minimum amount constraint for the marketing platforms**

```
m.addConstr(x[p] <= PLATFORM_CAP * w[p], name=f"cap_link[{p}]")

m.addConstr(x[p] >= min_amt[p] * w[p], name=f"min_link[{p}]")
```

- For each platform *p*, a binary variable `w[p]` indicates whether the platform is activated (invested in).

  If the platform is not selected (`w[p] = 0`), its investment `x[p]` must be **0**.

  If it is selected (`w[p] = 1`), the investment must be **at least the minimum required spend** (`min_amt[p]`) and no more than the cap (Platform Cap is $3 Million PLATFORM_CAP=`3.0`).

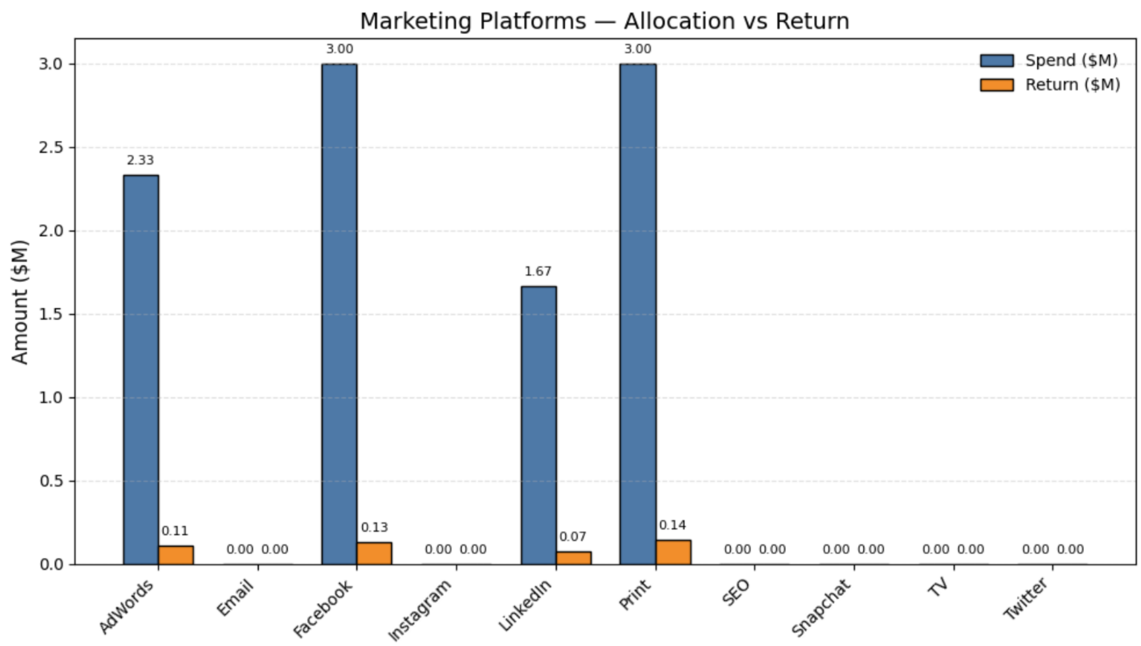
**Optimal Allocation**

The model ran successfully and used the entire $10M budget. The maximum possible return for this scenario is **$0.4528 million** (or $452,800).

The allocation obtained is as follows:

| Platform | Spend($M) | Return($M) | ROI (Return/Spend) |
|---|---|---|---|
| Adwords | 2.333 | 0.106 | 0.046 |
| Facebook | 3.000 | 0.130 | 0.043 |
| LinkedIn | 1.667 | 0.074 | 0.044 |
| Print | 3.000 | 0.143 | 0.048 |

This allocation is also depicted visually using a side-by-side bar chart that depicts the Marketing Spend/ Marketing Allocation and the Return respectively from the marketing platforms.




- From the plot, it can be noted that the $10M marketing budget was allocated as $2.33 M, $3M, $1.67M and $3M to Adwords, Facebook, LinkedIn and Print respectively.
- The corresponding returns from these platforms were $0.11M, $0.13M, $0.07M and $0.14M respectively.

**Analysis of Constraints**

- **Constraint (a) — Print/TV vs. Facebook/Email (Active)**

  This constraint remained active in the solution.
  The spending on Print ($3.0M) was balanced by the corresponding digital investment of $3.0M on Facebook and $0 on Email, satisfying the equality condition of the constraint.
  This constraint effectively prevented the model from pushing more funds into Print, which had strong ROI performance, maintaining parity between traditional and digital media spending as per management policy.

- **Constraint (b) — Social Media vs. SEO/AdWords (Not Active)**

  This constraint was **non-active**.

  Although AdWords received a sizable investment of $2.33M, there were no active investments in other social platforms such as Instagram, Snapchat, or Twitter.

  Since LinkedIn and Facebook together already satisfied the minimum ratio required (i.e., total social spend ≥ 2 × search spend), the condition did not restrict the allocation.

The rule therefore had minimal influence on the final outcome.

- **Constraint (c) — $3M Platform Cap (Active)**

  This constraint was again **highly influential**.
  The platforms Facebook and Print both reached their upper limit of $3.0M, while no other platform exceeded this level.
  This indicates that these channels had the highest marginal returns within their tiers, and the model would have allocated even more to them if the cap allowed.
  The $3M ceiling therefore directly limited potential further investment in these top-performing media and ensured diversification across other platforms like LinkedIn and AdWords.

- **Minimum Investment Threshold (Active for Select Platforms)**

  The **minimum amount constraint** played a crucial role in filtering out low-performing platforms.
  Channels such as SEO, TV, and Email received no allocation, as their expected returns did not justify even the minimum required spend.
  On the other hand, platforms like AdWords and LinkedIn crossed their respective thresholds, confirming their inclusion as economically viable investments under this rule.

## Part 7: Optimal Monthly Allocation with Non-Concave Data (Mixed IP Model) and Minimum Amount Constraint along with Monthly ROI data

### Methodology

For the seventh part, we extended the model to incorporate **dynamic budgeting through reinvestment**. The monthly ROI data provided in roi_monthly.csv was used to optimize marketing allocations across all platforms for each month of the year.

Unlike Part 6, where the total budget was fixed, this scenario introduced a **reinvestment mechanism**, where **half of the return from the previous month was added to the base $10M budget** for the following month. This allowed the marketing spend to evolve dynamically based on performance, linking each month’s optimization to the success of the previous period.

The optimization for each month was formulated as a **Mixed Integer Program (MIP)** in Gurobi, similar to the previous model. The **objective function** maximized the total return from all active platforms, computed as the sum of *(ROI × investment)* across all tiers.

### Objective Function

The goal is to maximize the total return from all platforms for **each month**, calculated as the sum of the product of each tier's ROI and its allocated spend.

```
m.setObjective(quicksum(tiers_by_p[p].loc[idx, "ROI"] * y[p, idx] for p in platforms for idx in range(len(tiers_by_p[p]))), GRB.MAXIMIZE)
```

Where, `y[p, idx]` = amount invested (in $M) in tier `idx` of platform `p`.

`ROI[p, idx]` = return rate for that tier (from `roi_monthly.csv`).

The objective maximizes total monetary return per month given that month's budget and constraints.

### Dynamic Budgeting for each month

```
BASE_BUDGET = 10.0

REINVEST_FRACTION = 0.5

prev_return = 0.0



for i, mname in enumerate(months_in_file):

    if i == 0:

        budget = BASE_BUDGET
```

```
else:
    budget = BASE_BUDGET + REINVEST_FRACTION * prev_return
```

Each month starts with a $10 M base amount.

Half of the previous month's total return is **reinvested** — allowing better-performing months to increase the next month's available budget.

These are the main changes from part 6, the other parts such as the three boss constraints and the minimum amount constraint for each marketing platform remains the same as for part 6.

### Output

| **Month** | **Budget** |
|---|---|
| January | $10.0000 M |
| February | $10.2697 M |
| March | $10.2104 M |
| April | $10.2674 M |
| May | $10.2510 M |
| June | $10.3004 M |
| July | $10.2426 M |
| August | $10.2737 M |
| September | $10.2760 M |

| | |
|---|---|
| October | $10.2845 M |
| November | $10.3233 M |
| December | $10.2758 M |

The above table gives the budget allocations for each month from January to December to maximize the total return for each month

| Month | AdWords | Email | Facebook | Instagram | LinkedIn | Print | SEO | Snapchat | TV | Twitter | Total Return (M$) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Jan | 0.0000 | 0.0000 | 3.0000 | 0.0000 | 3.0000 | 3.0000 | 0.0000 | 0.0000 | 0.0 | 1.0000 | 0.5394 |
| Feb | 2.5500 | 0.0000 | 3.0000 | 0.0000 | 2.1000 | 2.6197 | 0.0000 | 0.0000 | 0.0 | 0.0000 | 0.4207 |
| Mar | 2.6421 | 0.0000 | 2.2841 | 0.0000 | 3.0000 | 2.2841 | 0.0000 | 0.0000 | 0.0 | 0.0000 | 0.5349 |
| Apr | 1.2674 | 0.0000 | 3.0000 | 0.0000 | 3.0000 | 3.0000 | 0.0000 | 0.0000 | 0.0 | 0.0000 | 0.5020 |
| May | 0.0000 | 3.0000 | 0.0000 | 1.2510 | 3.0000 | 3.0000 | 0.0000 | 0.0000 | 0.0 | 0.0000 | 0.6008 |
| Jun | 0.0000 | 0.0000 | 3.0000 | 0.0000 | 3.0000 | 3.0000 | 0.0000 | 0.0000 | 0.0 | 1.3004 | 0.4853 |
| Jul | 0.0000 | 0.0000 | 2.2971 | 0.0000 | 3.0000 | 2.2971 | 2.6485 | 0.0000 | 0.0 | 0.0000 | 0.5473 |
| Aug | 3.0000 | 0.0000 | 0.6368 | 0.0000 | 3.0000 | 0.6368 | 0.0000 | 0.0000 | 0.0 | 3.0000 | 0.5520 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Sep | 1.4760 | 2.1000 | 0.9000 | 0.0000 | 2.8000 | 3.0000 | 0.0000 | 0.0000 | 0.0 | 0.0000 | 0.5691 |
| Oct | 0.0000 | 0.0000 | 3.0000 | 0.0000 | 3.0000 | 3.0000 | 0.0000 | 0.0000 | 0.0 | 1.2846 | 0.6466 |
| Nov | 0.0000 | 0.0000 | 3.0000 | 0.0000 | 0.0000 | 3.0000 | 0.0000 | 1.3233 | 0.0 | 3.0000 | 0.5517 |
| Dec | 0.0000 | 3.0000 | 0.0000 | 0.0000 | 3.0000 | 3.0000 | 0.0000 | 0.0000 | 0.0 | 1.2758 | 0.5084 |

This pivoted table above depicts the total return along with marketing budget allocation across the marketing platforms from January to December.

**Analysis Of Constraints**

- **Constraint (a)/ First Boss Constraint**

  was **intermittently active** during the optimization.
  It often became binding in months where traditional media (Print and TV) had high ROI values, especially when the budget was still near the $10 M base level.
  As reinvestment increased the available funds later in the year, the digital side (Facebook + Email) naturally grew faster, reducing the tightness of this constraint.
  Overall, it acted as a stabilizer, maintaining the intended balance between offline and digital investments.

- **Constraint (b)/ Second Boss Constraint**

  This constraint was **rarely active**.
  Because social platforms such as Instagram, Snapchat, and LinkedIn typically offered strong ROIs, the model naturally invested heavily in them.
  As a result, this proportionality requirement was automatically satisfied in nearly all months, serving mainly as a policy safeguard rather than an active limiter.

- **Constraint (c)/ Third Boss Constraint**

  This constraint remained the **most frequently active rule**. Platforms with the highest monthly ROIs, particularly **TV**, **Instagram**, and **Email**, consistently hit this upper limit in several months. The per-platform cap limited further concentration of spending on these high-return channels and ensured a **diversified allocation** across media categories.

Without this restriction, these same platforms would have absorbed larger shares of the monthly budgets, especially during high-return periods.

- **Minimum Threshold Amount Constraint for each Marketing Platform**

  This constraint remained the **most frequently active rule**. Platforms with the highest remained important in shaping feasible solutions.
  It prevented very small or fragmented investments by enforcing either:

  **Full activation** — a platform must receive at least its threshold spend to be impactful, or
  **Deactivation** — no investment if the minimum can't be justified.

  This rule was most active for **medium-ROI channels** such as **LinkedIn**, **SEO**, and **Print**, where the required minimum spend occasionally exceeded what the model would allocate under pure ROI optimization.
  Consequently, it improved **efficiency** by pruning marginally effective investments.

**Visualizations**

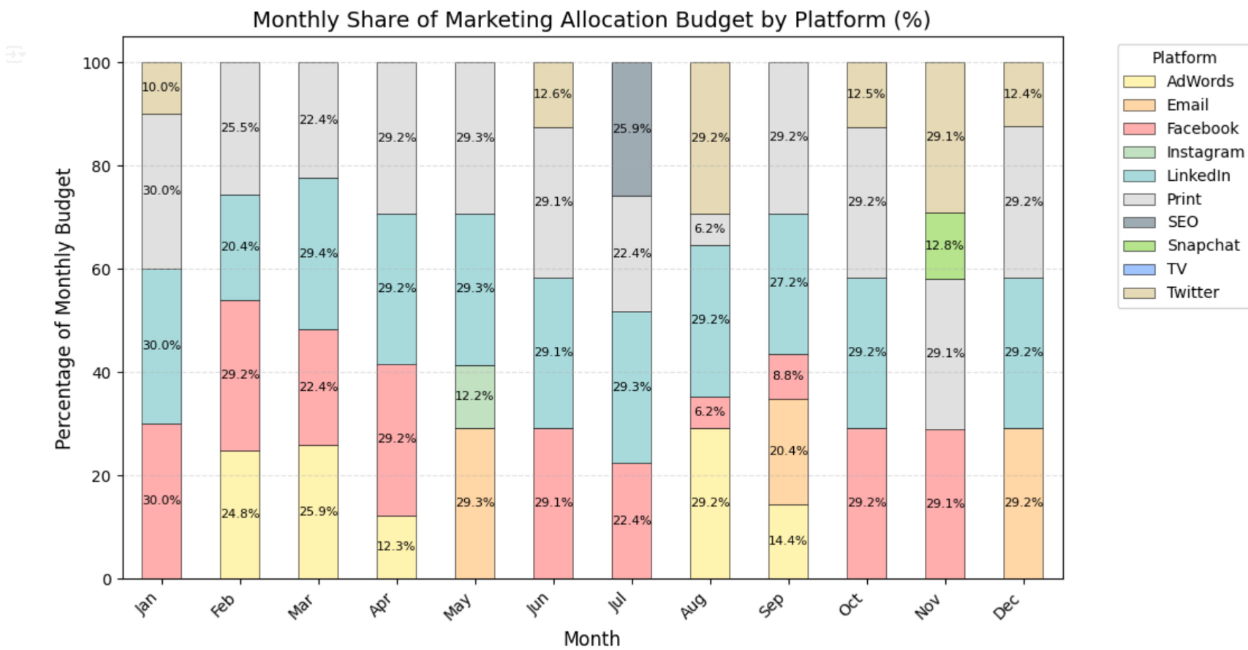
Visual- Monthly Marketing Allocation vs Total Return


This plot depicts the trend of the Total Allocation (blue line) and the Total Return (green line) from January to December.

From the first lineplot that shows the total allocation, it can be noted that there was a huge increase in marketing budget from January to February by 0.27 Million USD, whereas for the other months, the change in marketing budget was within the range of 0-0.6 Million USD

From the second lineplot that shows the Total Return, February had the lowest return of 0.42 Million USD whereas October had the highest return of 0.65 Million USD. Monthly Total Returns fluctuated between 0.42-0.65 Million USD

Visual - Monthly Composition of Marketing Spend across Platforms from January to December




This Stacked Bar Chart depicts the composition of marketing budget allocation across the 10 marketing platforms from January to December.

It should be noted that the **marketing budget isn't the same monthly** as we go from January to December. However, some patterns can be observed here.

- LinkedIn dominates the monthly allocation spends (except for February, September) across the months whenever it is considered for investment.
- There is a monthly allocation into the Print Platform for all the 12 months
- SEO is preferred only in July for investment
- August and September are the only months where the budget is spent across a maximum of 5 marketing platforms.

## Part 8: Assessing the Stability of the Monthly Allocation Budgeting Model

A stable budget is defined as a monthly allocation such that for each platform the monthly change in spend is no more than $1M.

However, the monthly allocation budgeting model obtained **wasn't stable** as the monthly change in allocation/spend for some of the marketing platforms was **greater than $1M**.

**Output**

--- Stability Check (Q8) ---

The allocation is NOT stable. Some platforms exceed $1M change between months.

Details (changes > $1M):

| Month | AdWords | Email | Facebook | Instagram | LinkedIn | Print | SEO | Snapchat | TV | Twitter |
|---|---|---|---|---|---|---|---|---|---|---|
| Feb | 2.550 | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN |
| Apr | 1.375 | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN |
| May | 1.267 | 3.0 | 3.00 | 1.251 | NaN | NaN | NaN | NaN | NaN | NaN |
| Jun | NaN | 3.0 | 3.00 | 1.251 | NaN | NaN | NaN | NaN | NaN | 1.300 |
| Jul | NaN | NaN | NaN | NaN | NaN | NaN | 2.649 | NaN | NaN | 1.300 |
| Aug | 3.000 | NaN | 1.66 | NaN | NaN | 1.660 | 2.649 | NaN | NaN | 3.000 |
| Sep | 1.524 | 2.1 | NaN | NaN | NaN | 2.363 | NaN | NaN | NaN | 3.000 |
| Oct | 1.476 | 2.1 | 2.10 | NaN | NaN | NaN | NaN | NaN | NaN | 1.285 |
| Nov | NaN | NaN | NaN | NaN | 3.0 | NaN | NaN | 1.323 | NaN | 1.715 |
| Dec | NaN | 3.0 | 3.00 | NaN | 3.0 | NaN | NaN | 1.323 | NaN | 1.724 |

From the above table, it can be noted that the allocation isn't stable as the change in monthly allocation for some platforms exceeded 1 Million USD from month to month. The stability criteria was violated multiple times. This output can also identify which months had the least and most number of stability violations.

- **In this output, the numerical values (change in marketing allocation from the previous month) are shown where the stability criteria (change in monthly allocation of marketing budget for each platform from previous month > 1 Million USD) has been violated. It is NaN otherwise.**
- The stability criteria has been violated the maximum number of times in the Months of August and December.
- February and April have one stability criteria violation each.
- March remains the only month, where the stability criteria hasn't been violated.

Detailed Tabular Output depicting stability violations across the marketing platforms

| Platform | Month From | Month To | Spend Change (M$) | Previous Spend (M$) | Current Spend (M$) |
|---|---|---|---|---|---|
| AdWords | Jan | Feb | 2.5500 | 0.0000 | 2.5500 |
| AdWords | Mar | Apr | 1.3746 | 2.6421 | 1.2674 |
| AdWords | Apr | May | 1.2674 | 1.2674 | 0.0000 |
| Email | Apr | May | 3.0000 | 0.0000 | 3.0000 |
| Facebook | Apr | May | 3.0000 | 3.0000 | 0.0000 |
| Instagram | Apr | May | 1.2510 | 0.0000 | 1.2510 |

| | | | | | |
|---|---|---|---|---|---|
| Email | May | Jun | 3.0000 | 3.0000 | 0.0000 |
| Facebook | May | Jun | 3.0000 | 0.0000 | 3.0000 |
| Instagram | May | Jun | 1.2510 | 1.2510 | 0.0000 |
| Twitter | May | Jun | 1.3004 | 0.0000 | 1.3004 |
| SEO | Jun | Jul | 2.6485 | 0.0000 | 2.6485 |
| Twitter | Jun | Jul | 1.3004 | 1.3004 | 0.0000 |
| AdWords | Jul | Aug | 3.0000 | 0.0000 | 3.0000 |
| Facebook | Jul | Aug | 1.6602 | 2.2971 | 0.6368 |
| Print | Jul | Aug | 1.6602 | 2.2971 | 0.6368 |
| SEO | Jul | Aug | 2.6485 | 2.6485 | 0.0000 |
| Twitter | Jul | Aug | 3.0000 | 0.0000 | 3.0000 |
| AdWords | Aug | Sep | 1.5240 | 3.0000 | 1.4760 |
| Email | Aug | Sep | 2.1000 | 0.0000 | 2.1000 |

| | | | | | |
|---|---|---|---|---|---|
| Print | Aug | Sep | 2.3632 | 0.6368 | 3.0000 |
| Twitter | Aug | Sep | 3.0000 | 3.0000 | 0.0000 |
| AdWords | Sep | Oct | 1.4760 | 1.4760 | 0.0000 |
| Email | Sep | Oct | 2.1000 | 2.1000 | 0.0000 |
| Facebook | Sep | Oct | 2.1000 | 0.9000 | 3.0000 |
| Twitter | Sep | Oct | 1.2846 | 0.0000 | 1.2846 |
| LinkedIn | Oct | Nov | 3.0000 | 3.0000 | 0.0000 |
| Snapchat | Oct | Nov | 1.3233 | 0.0000 | 1.3233 |
| Twitter | Oct | Nov | 1.7154 | 1.2846 | 3.0000 |
| Email | Nov | Dec | 3.0000 | 0.0000 | 3.0000 |
| Facebook | Nov | Dec | 3.0000 | 3.0000 | 0.0000 |
| LinkedIn | Nov | Dec | 3.0000 | 0.0000 | 3.0000 |
| Snapchat | Nov | Dec | 1.3233 | 1.3233 | 0.0000 |

| | | | | | |
|---|---|---|---|---|---|
| Twitter | Nov | Dec | 1.7242 | 3.0000 | 1.2758 |

This is another representation of the stability violations for each platform where the **Month To** column is where the stability violation has been recorded for each platform.
It can be noted that in total there were **33 stability violations** that occurred across these marketing platforms.

## Visualizations

Visual - Stability Violations across the Marketing Platforms

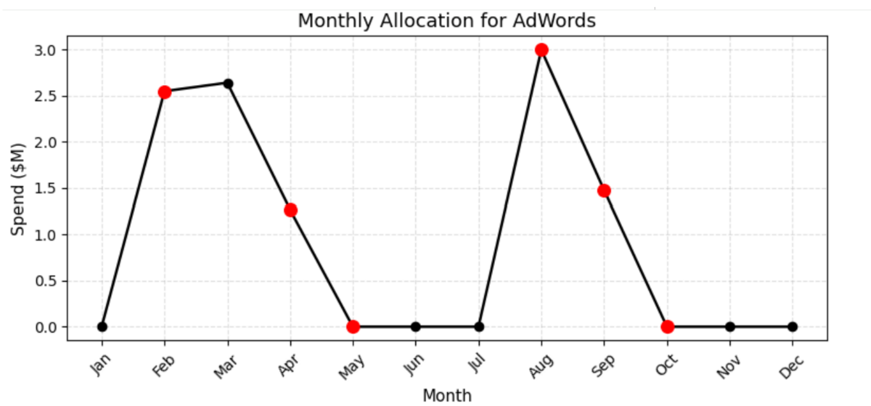

This visual helps answer which of the marketing platforms are the most stable and least stable by observing the number of violations for each of the marketing platforms.

- Twitter had the maximum number of violations at 7.
- Adwords had 6 violations,
- Email and Facebook had 5 each followed by Instagram, SEO, Print, LinkedIn, Snapchat at 2 each.
- TV was the most stable platform with zero violations.

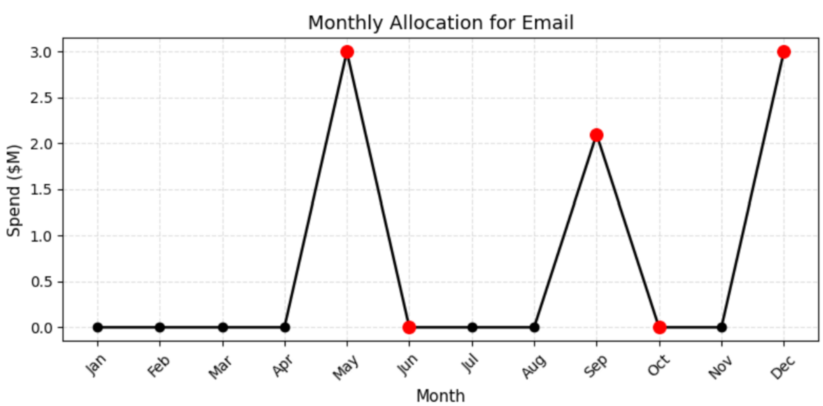
Visual -  Trend of Marketing Allocation for each Platforms from January to December

These line plots depict the trend of marketing budget allocations from January to December. **The red data points are the months where the stability criteria has been violated** (difference in marketing allocation is greater than $1M).

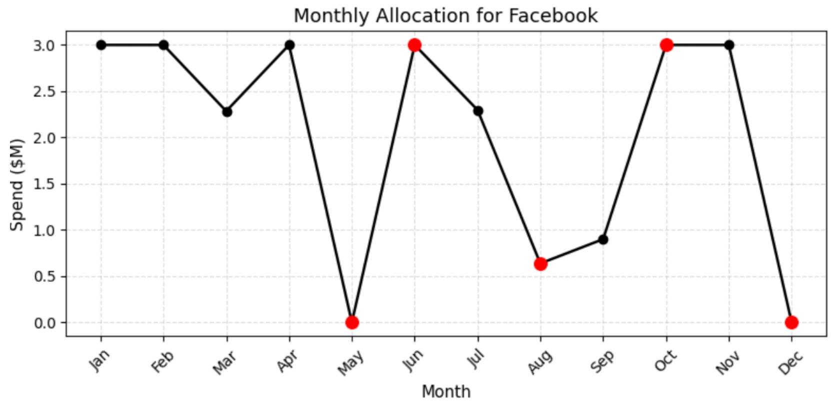
These plots help to identify which of the platforms were the **most and least stable and how frequently were the stability violations occurring for each of these platforms from January to December.**



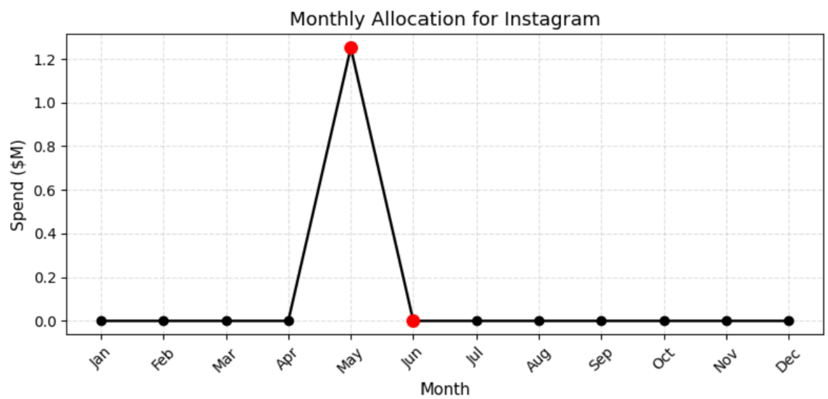

Adwords had 6 stability violations



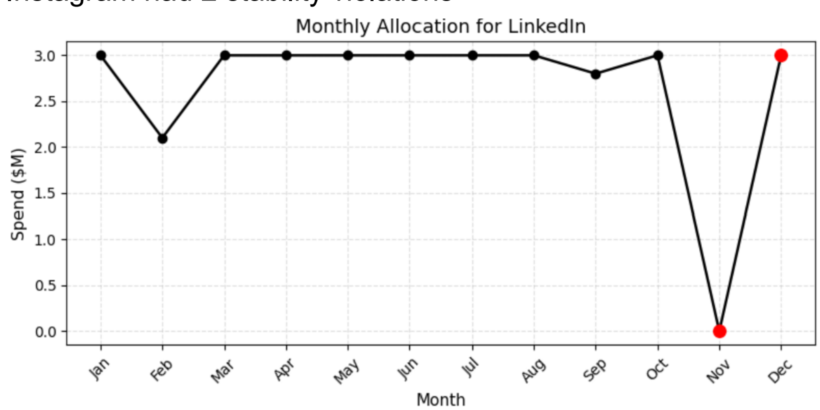

Email had 5 stability violations



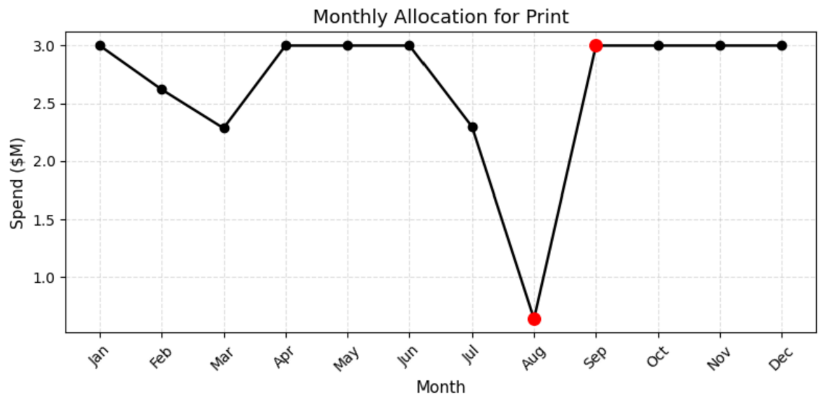

Facebook had 5 stability violations



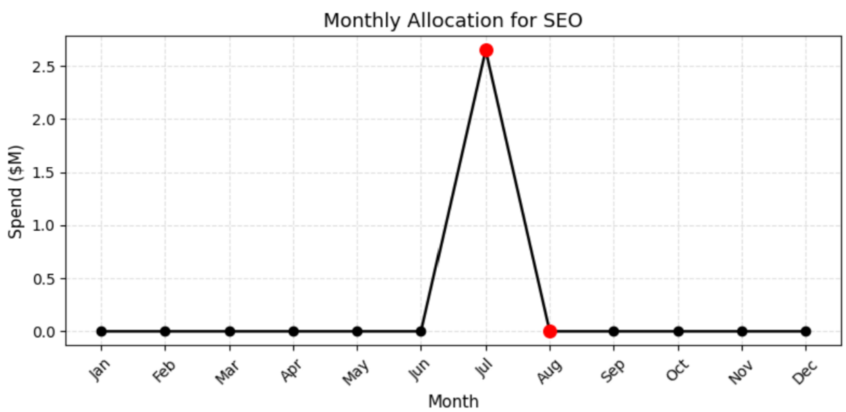

Instagram had 2 stability violations



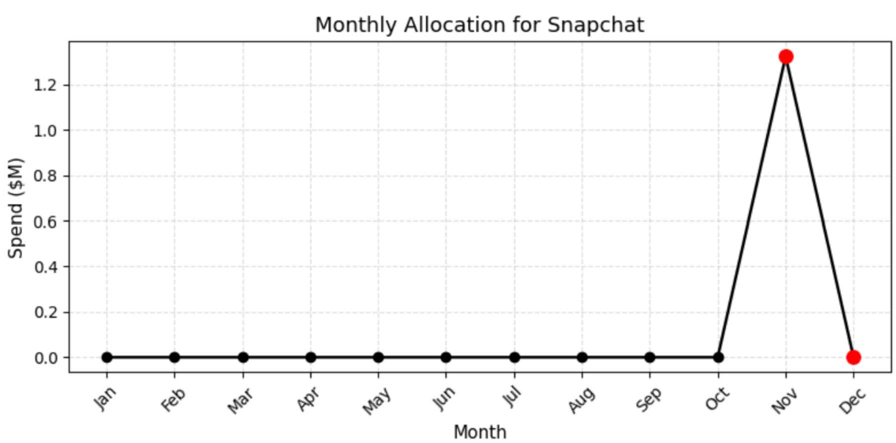

LinkedIn had 2 stability violations



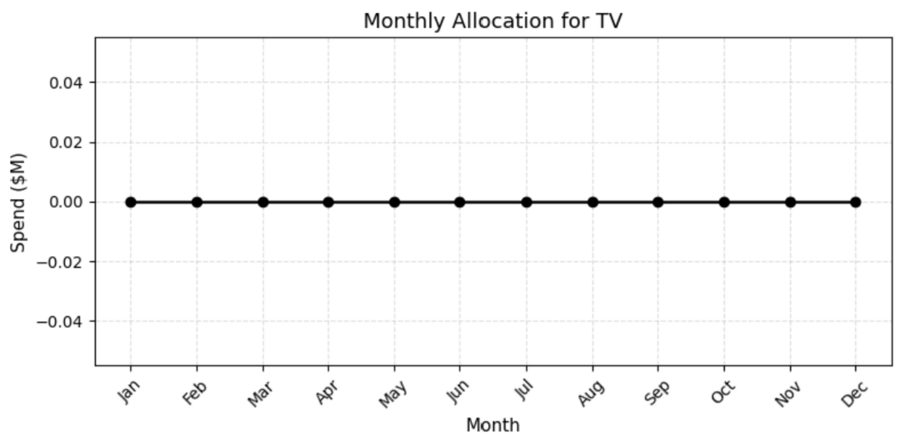

Print had 2 stability violations



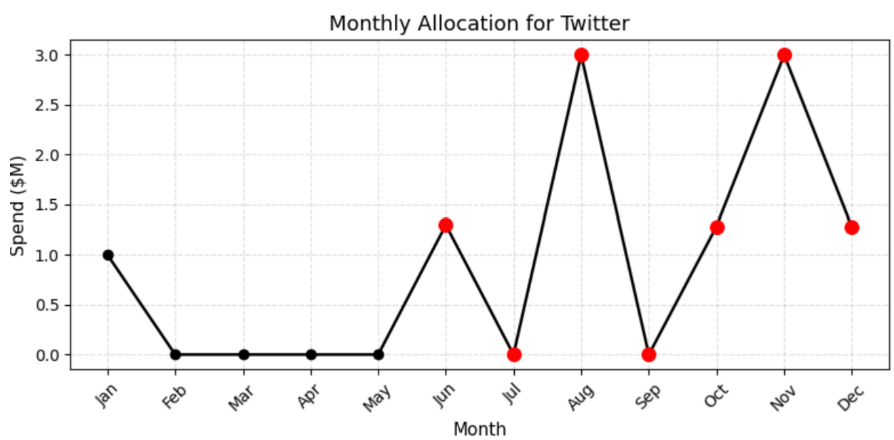

SEO had 2 stability violations




Snapchat had 2 stability violations




TV had no stability violations




Twitter had 7 stability violations.

From these plots, it can be observed that for Twitter was the most unstable platform to invest in, the stability criteria violation occurred continuously 7 times from May to December.

## Enforcing Stability to the Monthly Allocation Model

As the model obtained was **unstable**, with a total of **33 stability violations** across the marketing platforms from January to December, we can add **two constraints** to our monthly allocation model to enforce stability such that the change in monthly allocation across the marketing platforms is less than or equal to $1M.

The stability constraints can be written as:

$$x_{p,t} - x_{p,t-1} \leq 1$$
$$x_{p,t-1} - x_{p,t} \leq 1$$

where $x_{p,t}$ is the monthly spend (in $M) on platform $p$ in month $t$.

The above snippet shows the stability constraints that can be added to the model. These constraints restrict the change in marketing monthly spend on platform p for any 2 consecutive months to be within $1M.

## Conclusion

The purpose of this project was to develop a data-driven, optimization-based strategy for allocating a $10 million marketing budget to maximize total Return on Investment (ROI). Through a progressive series of modeling enhancements, we transitioned from a simple static optimization to a dynamic, multi-period decision framework capable of guiding realistic marketing strategies.

In the initial stages (Parts 3–5), the project established the foundation by modeling ROI data from two consulting firms.

- In Part 3, A Linear Programming (LP) model was first applied to concave ROI data, capturing diminishing returns and identifying optimal spend levels across marketing channels under key managerial constraints.
- Subsequently for Part 4, as the data was non-concave, the problem was reformulated as a Mixed Integer Program (MIP) to handle non-concave data and discrete platform activation, ensuring that investments were both strategically and operationally meaningful.
- Incorporating tiered ROI structures allowed the model to better represent diminishing returns within each platform, capturing real-world nonlinearity in marketing effectiveness.
- In Part 5, robustness analysis showed that the $3M platform cap was an influential constraint.

Building on this, the later phases (Parts 6–8) offered some key takeaways for realistic allocation strategies.

- Part 6 was built on top of Part 4 with an introduction of minimum investment thresholds to ensure that platforms only received funding when it was large enough to create meaningful impact. This constraint refined the model into a more selective and focused allocation strategy, avoiding inefficient micro-investments.
- Part 7 extended the model by introducing a dynamic reinvestment mechanism, where half of each month’s return was reinvested into the next month’s base budget. This created an adaptive budgeting process, allowing the marketing spend to evolve in response to prior performance.

- Finally, in Part 8, the stability of the monthly allocation model was assessed and it was observed that the allocation model isn't stable and stability constraints were modelled that can be incorporated into the model to limit abrupt month-to-month changes in spending across platforms, producing smoother, operationally practical budget recommendations while preserving high ROI performance.