LiCS ASSIGNMENT

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1. Introduction

We are attempting question 1 in which we play the classical tic-tac-toe game with one added rule.

Rule :- Both the players can swap their marks with other player's marks in their respective turns(considered as a move) only once in a game i.e. markX can change position with markO and vice-versa. This swap will be considered as a move.

2. General Rules

2.1 Board

We define the Board.

1	2	3
4	5	6
7	8	9

As [1,2,3,4,5,6,7,8,9]. A position filled by x is represented by X, position filled by o is represented by O and an empty position represented by e.

An empty board is represented by [e,e,e,e,e,e,e,e,e].

2.2 Winning Condition

A player wins if it completes 3 marks in a row or a column or a diagonal.

Same as a classical tic-tac-toe game.

2.3 Some Definitions

(a) mark(player, position) - marks player mark at the position.

(b) winInOne(player) – player wins in next turn by placing in its mark in winning position.

(c) winInOneSwap(player) – player wins in next turn by swapping opponent's mark to its own.

(d) stopThreat(player) – player stops the opponent player from winning in next move which is same as placing the player's mark in winInOne(opponentPlayer).

(e) swapPosition(position1, position2) – swaps position 1 and 2 in which one has X and other has O.

(f) player – it represents current player's mark(X or O).

(g) anyMove(player) - fill any one blank space with player's mark or swap with opponent's mark.

(h) makeLine(player) – it tries to achieve condition P or Q in next move.

(i) centreMove(player) – it places mark in position 5(centre).

2.4 Statements

We define two types of board positions P and Q which will always imply win for X such that:-

<u>P :</u>

Case(1) [X,O,X,e,e,e,e,e,e], [e,e,X,e,e,O,e,e,X], [e,e,e,e,e,e,X,O,X], [X,e,e,O,e,e,X,e,e]. These 4 are symmetric. For proof, we will solve for first one.

Case(2) [e,X,e,e,O,e,e,X,e], [e,e,e,X,O,X,e,e,e]. Both are symmetric.

For proof, we will solve for first one.

Case(3) [X,e,e,e,O,e,e,e,X], [e,e,X,e,O,e,X,e,e]. Both are symmetric. For proof, we will solve for first one.

Proof of P => win(X):-

(1) This has 2 subcases-

(1.1)$\forall c \in \{4,6,7,8,9\}$
$[mark(O,c) \wedge mark(X,5) \wedge anyMove(O) \Rightarrow (winInOne(X) \vee winInOneSwap(X))]$

(1.2) [mark(O,5)∧stopThreat(X)∧anyMove(O)=>winInOneSwap(X)]

(2) This has 2 subcases -

(2.1)∀c ∈{4,6}
[mark(O,c)∧stopThreat(X)∧anyMove(O)=>winInOneSwap(X)]

(2.2)∀c ∈{1,3,7,9}
[mark(O,c)∧stopThreat(X)∧anyMove(O)=>(winInOne(X) ∨ winInOneSwap(X))]

(3) This has 2 subcases-

(3.1)∀c ∈{3,7}
[mark(O,c)∧stopThreat(X)∧anyMove(O)=>winInOne(X)]

(3.2)∀c ∈{2,4,6,8}
[mark(O,c)∧stopThreat(X)∧anyMove(O)=>(winInOne(X) ∨ winInOneSwap(X))]

(1.1 ∧ 1.2 ∧ 2.1 ∧ 2.2 ∧ 3.1 ∧ 3.2)=>win(X)

Thus, P=>win(X)

Q:

Case(1) [X,e,e,X,e,e,O,e,e], [O,e,e,X,e,e,X,e,e], [X,X,O,e,e,e,e,e,e], [O,X,X,e,e,e,e,e,e], [e,e,X,e,e,X,e,e,O], [e,e,O,e,e,X,e,e,X], [e,e,e,e,e,e,X,X,O], [e,e,e,e,e,e,O,X,X]. These 8 are symmetric. For proof, we will solve for first one.

Case(2) [e,X,e,e,X,e,e,O,e], [e,e,e,X,X,O,e,e,e], [e,O,e,e,X,e,e,X,e], [e,e,e,O,X,X,e,e,e]. These 4 are symmetric. For proof, we will solve for first one.

Case(3) [X,e,e,e,X,e,e,e,O], [e,e,X,e,X,e,O,e,e], [O,e,e,e,X,e,e,e,X], [e,e,O,e,X,e,X,e,e]. These 4 are symmetric. For proof, we will solve for first one.

Proof of Q => win(X):-

(1)

(1.1) ∀c ∈{3,5,8,9}[(mark(O,c)∧stopThreat(X)∧anyMove(O)) =>(winInOne(X) ∨ winInOneSwap(X))]

(1.2) ∀c ∈{2,6}[mark(O,c)∧mark(X,5)∧anyMove(O)=>(winInOne(X) ∨ winInOneSwap(X))]

(2)

(2.1) ∀c ∈{1,3,4,6}, ∃ d ∈{2,4} [mark(O,c)∧mark(X,d)∧anyMove(O)=>(winInOne(X) ∨ winInOneSwap(X))]

(2.2) ∀c ∈{7,9}[(mark(O,c)∧stopThreat(X)∧anyMove(O)) =>(winInOne(X) ∨ winInOneSwap(X))]

(3)

(3.1) $\forall c \in \{3,6,7,8\}[(mark(O,c) \wedge stopThreat(X) \wedge anyMove(O)) \Rightarrow (winInOne(X) \vee winInOneSwap(X))]$

(3.2) $\forall c \in \{2,4\}, \exists d \in \{2,4\}$
$[mark(O,c) \wedge mark(X,d) \wedge anyMove(O) \Rightarrow winInOne(X)]$

3. First Player's Strategy

In every move, it places (in the following priority order):

(1) winInOne(player)

(2) winInOneSwap(player)

(3) stopThreat(player)

(4) makeLine(player)

(5) centreMove(player)

(6) anyMove(player).

4. FOL Proof

We assume first player has mark X and opponent has mark O, without loss of generity. We have three cases for first move.

Case(1): mark(X, 5) => [e,e,e,e,X,e,e,e,e]

Case(2): mark(X, 1) =>[X,e,e,e,e,e,e,e,e],

mark(X, 3) => [e,e,X,e,e,e,e,e,e],

mark(X, 7) => [e,e,e,e,e,e,X,e,e],

mark(X, 9) => [e,e,e,e,e,e,e,e,X]

These 4 board positions are same due to symmetry. We will show proof only for mark(X,1).

Case(3): mark(X, 2) =>[e,X,e,e,e,e,e,e,e],

mark(X, 4) => [e,e,e,X,e,e,e,e,e],

mark(X, 6) => [e,e,e,e,e,X,e,e,e],

mark(X, 8) => [e,e,e,e,e,e,e,X,e]

These 4 board positions are same due to symmetry. We will show proof only for mark(X, 2).

Now taking case(1) which has 2 subcases-

Case(1.1): [O,e,e,e,X,e,e,e,e], [e,e,O,e,X,e,e,e,e], [e,e,e,e,X,e,O,e,e], [e,e,e,e,X,e,e,e,O]. All 4 are symmetric. For proof, we will solve for first one.

$\exists$ c $\notin$\{1, 5\}[(mark(X, 5) $\wedge$ mark(O, 1) $\wedge$ mark(X, c) => Q) => win(X)]

Case(1.2): [e,O,e,e,X,e,e,e,e], [e,e,e,O,X,e,e,e,e], [e,e,e,e,X,O,e,e,e], [e,e,e,e,X,e,e,O,e]. All 4 are symmetric. For proof, we will solve for first one.

$\exists$ c $\notin$\{2, 5\}[(mark(X, 5) $\wedge$ mark(O, 2) $\wedge$ mark(X, c) => Q) => win(X)]

Now taking case(2) which has 5 subcases-

Case(2.1): [X,O,e,e,e,e,e,e,e], [X,e,e,O,e,e,e,e,e] .Both are symmetric. For proof, we will solve for first one.

$\exists$ c $\notin$\{1, 2\}[(mark(X, 1) ∧ mark(O, 2) ∧ mark(X, c) => P) => win(X)]

Case(2.2): [X,e,O,e,e,e,e,e,e], [X,e,e,e,e,e,O,e,e] .Both are symmetric. For proof, we will solve for first one.

$\exists$ c $\notin$\{1, 3\}[(mark(X, 1) ∧ mark(O, 3) ∧ mark(X, c) => Q) => win(X)]

Case(2.3): [X,e,e,e,e,e,e,e,O]

$\exists$ c $\notin$\{1, 9\}[(mark(X, 1) ∧ mark(O, 9) ∧ mark(X, c) => Q) => win(X)]

Case(2.4): [X,e,e,e,O,e,e,e,e]

$\exists$ c $\notin$\{1, 5\}[(mark(X, 1) ∧ mark(O, 5) ∧ mark(X, c) => P) => win(X)]

Case(2.5): [X,e,e,e,e,O,e,e,e], [X,e,e,e,e,e,e,O,e]. Both are symmetric. For proof, we will solve for first one.

(mark(X, 1) ∧ mark(O, 6) ∧ mark(X, 5))

Now taking case(2.5) which has 4 subcases-

Case(2.5.1): [X,O,e,e,X,O,e,e,e], [X,e,O,e,X,O,e,e,e], [X,e,e,O,X,O,e,e,e], [X,e,e,e,X,O,O,e,e], [X,e,e,e,X,O,e,O,e].

$\exists$ c $\notin$ {1, 5, 6, 9}[mark(X, 1) ∧ mark(O, 6) ∧ mark(X, 5) ∧ mark(O, c) => winInOne(X)]

Case(2.5.2): [X,e,e,e,X,O,e,e,O]

$\forall$c $\notin$ {1, 5, 6, 9}[mark(X, 1) ∧ mark(O, 6) ∧ mark(X, 5) ∧ mark(O, 9) ∧ stopThreat(X) ∧ mark(O, c) => (winInOne(X) ∨ winInOneSwap(X))].

Case(2.5.3): [X,e,e,e,O,X,e,e,e]

$\exists$ c $\notin$ {1, 5, 6}[mark(X, 1) ∧ mark(O, 6) ∧ mark(X, 5) ∧ swap(5, 6) => winInOne(X)]

Case(2.5.4): [O,e,e,e,X,X,e,e,e]

$\exists$ c $\notin$ {1, 5, 6}[mark(X, 1) ∧ mark(O, 6) ∧ mark(X, 5) ∧ swap(1, 6) => winInOne(X)]

Now taking case(3) which has 5 subcases-

Case(3.1): [O,X,e,e,e,e,e,e,e], [e,X,O,e,e,e,e,e,e] .Both are symmetric. For proof, we will solve for first one.

$\exists$ c $\notin$ {1, 2}[(mark(X, 2) ∧ mark(O, 1) ∧ mark(X, c) => Q) => win(X)]

Case(3.2): [e,X,e,e,O,e,e,e,e].

$\exists$ c $\notin$ {2, 5}[(mark(X, 2) ∧ mark(O, 5) ∧ mark(X, c) => P) => win(X)]

Case(3.3): [e,X,e,e,e,e,e,O,e].

$\exists$ c $\notin$\{2, 8\}[(mark(X, 2) $\wedge$ mark(O, 8) $\wedge$ mark(X, c) => Q) => win(X)]

Case(3.4): [e,X,e,O,e,e,e,e,e], [e,X,e,e,e,O,e,e,e] .Both are symmetric. For proof, we will solve for first one.

(mark(X, 2) $\wedge$ mark(O, 4) $\wedge$ mark(X, 5))

Now taking case(3.4) which has 4 subcases-

Case(3.4.1): [O,X,e,O,X,e,e,e,e], [e,X,O,O,X,e,e,e,e], [e,X,e,O,X,O,e,e,e], [e,X,e,O,X,e,O,e,e], [e,X,e,O,X,e,e,e,O].

$\exists$ c $\notin$\{2,4,5,8\}[mark(X, 2) $\wedge$ mark(O, 4) $\wedge$ mark(X, 5) $\wedge$ mark(O, c) => winInOne(X)]

Case(3.4.2): [e,X,e,O,X,e,e,O,e].

$\forall$c $\notin$\{2,4,5,8\}[(mark(X, 2) $\wedge$ mark(O, 4) $\wedge$ mark(X, 5) $\wedge$ mark(O, c)=> Q) => win(X)]

Case(3.4.3): [e,X,e,X,O,e,e,e,e].

$\exists$ c $\notin$\{2,4,5\}[mark(X, 2) $\wedge$ mark(O, 4) $\wedge$ mark(X, 5) $\wedge$ swap(4, 5) => winInOne(X)]

Case(3.4.4): [e,O,e,X,X,e,e,e,e].

$\exists$ c $\notin$\{2,4,5\}[mark(X, 2) $\wedge$ mark(O, 4) $\wedge$ mark(X, 5) $\wedge$ swap(2, 4) => winInOne(X)]

Case(3.5): [e,X,e,O,e,e,e,e,e], [e,X,e,e,e,O,e,e,e] .Both are symmetric. For proof, we will solve for first one.

$\exists$ c $\notin$\{2, 7\}[(mark(X, 2) $\wedge$ mark(O, 7) $\wedge$ mark(X, c) => P) => win(X)]

(mark(X, 2) $\wedge$ mark(O, 7) $\wedge$ mark(X, 5))

Now taking case(3.5) which has 4 subcases-

Case(3.5.1): [O,X,e,e,X,e,O,e,e], [e,X,O,e,X,e,O,e,e], [e,X,e,O,X,e,O,e,e], [e,X,e,e,X,O,O,e,e], [e,X,e,e,X,e,O,e,O].

$\exists$ c $\notin$\{2,7,5,8\}[mark(X, 2) $\wedge$ mark(O, 7) $\wedge$ mark(X, 5) $\wedge$ mark(O, c) => winInOne(X)]

Case(3.5.2): [e,X,e,e,X,e,O,O,e].

$\forall$c $\notin$\{2,7,5,8,9\}[mark(X,2) $\wedge$ mark(O, 7) $\wedge$ mark(X, 5) $\wedge$ mark(O, 8) $\wedge$ stopThreat(X) $\wedge$ mark(O, c) => (winInOne(X)$\wedge$winInOneSwap(X))].

Case(3.5.3): [e,X,e,e,O,e,X,e,e].

$\exists$ c $\notin$\{2,7,5\}[mark(X, 2) $\wedge$ mark(O, 7) $\wedge$ mark(X, 5) $\wedge$ swap(5, 7) => winInOne(X)]

Case(3.5.4): [e,O,e,e,X,e,X,e,e].

$\exists c \notin \{2,7,5\}[mark(X, 2) \wedge mark(O, 7) \wedge mark(X, 5) \wedge swap(2, 7) => winInOne(X)]$

5. Conclusion

With this added rule we have proved that the first player will always win irrespective of the moves played by the second player.