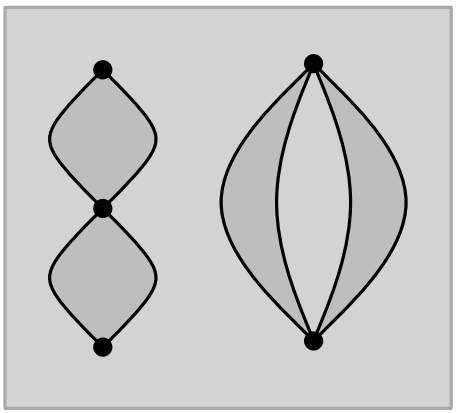
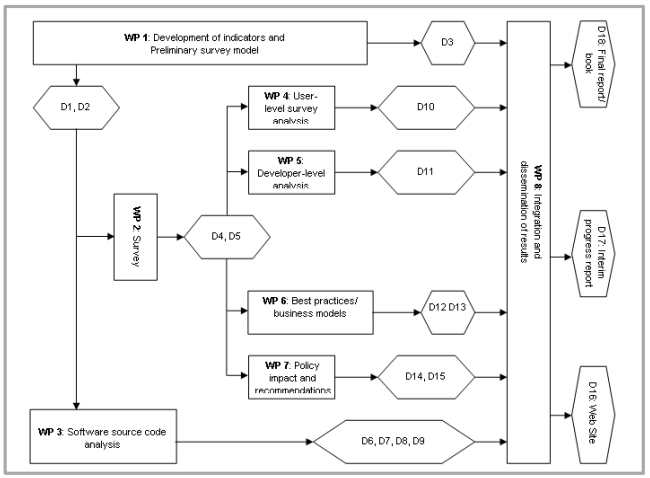
Trees and Series-Parallel Graphs

Lecture Graph Drawing Algorithms · 192.053

Martin Nöllenburg

10.04.2018





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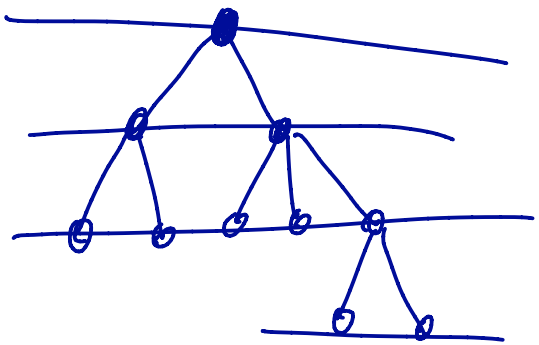
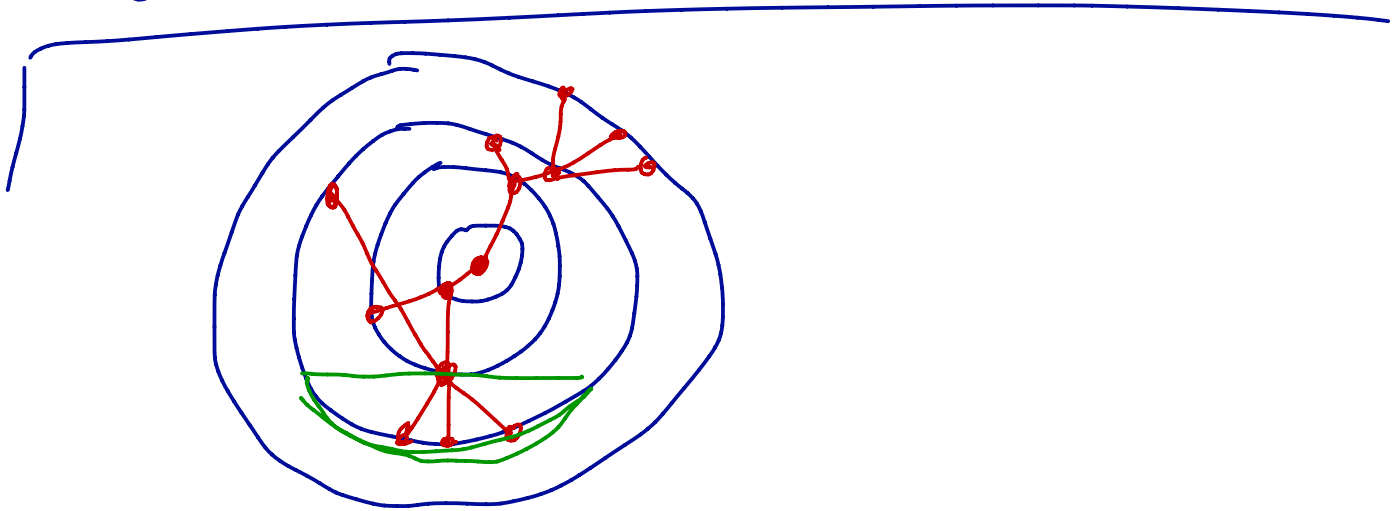
ALGORITHMS AND COMPLEXITY GROUP

Recall

What did we learn about tree drawings last time?



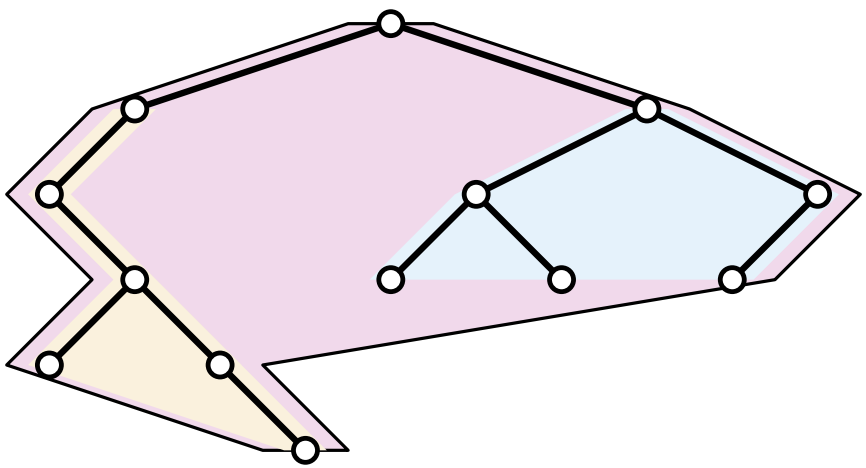
- symmetric placement
- small width

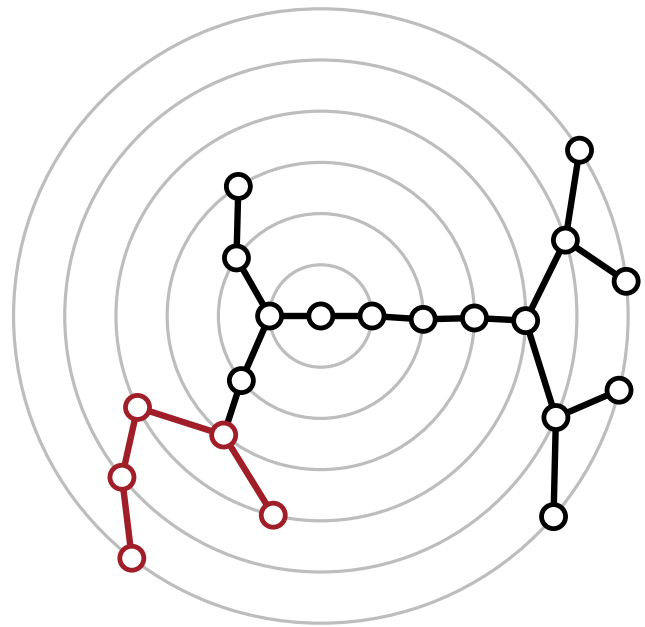


Recall

What did we learn about tree drawings last time?

- layered drawings: simple DFS-based algorithm
- improved contour-based 2-phase algorithm
- width minimization is generally NP-hard
- radial drawings (careful with crossings!)





Overview

Layered drawings of rooted trees

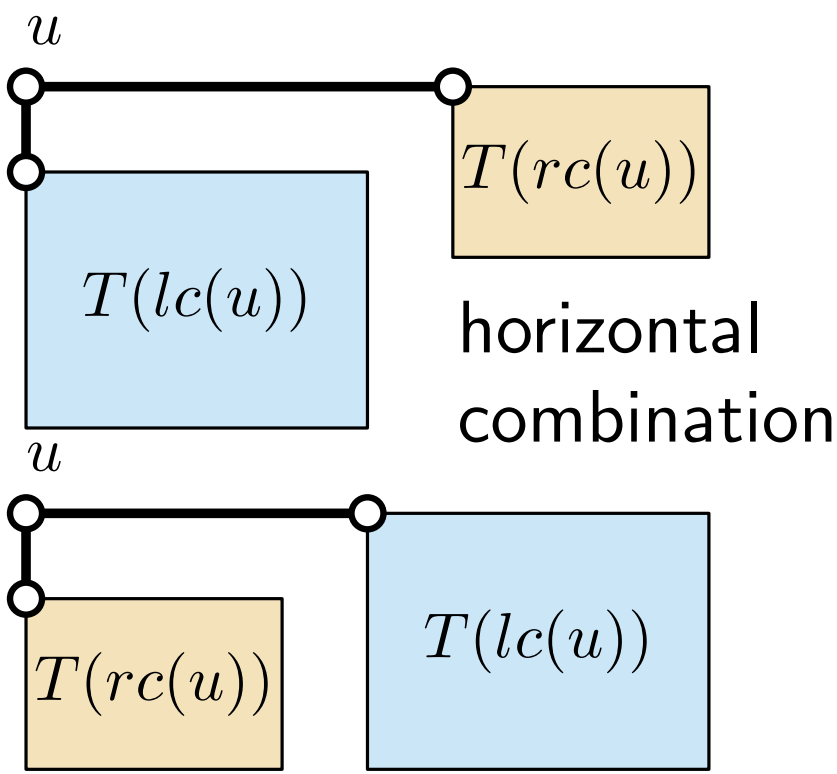
Radial tree drawings

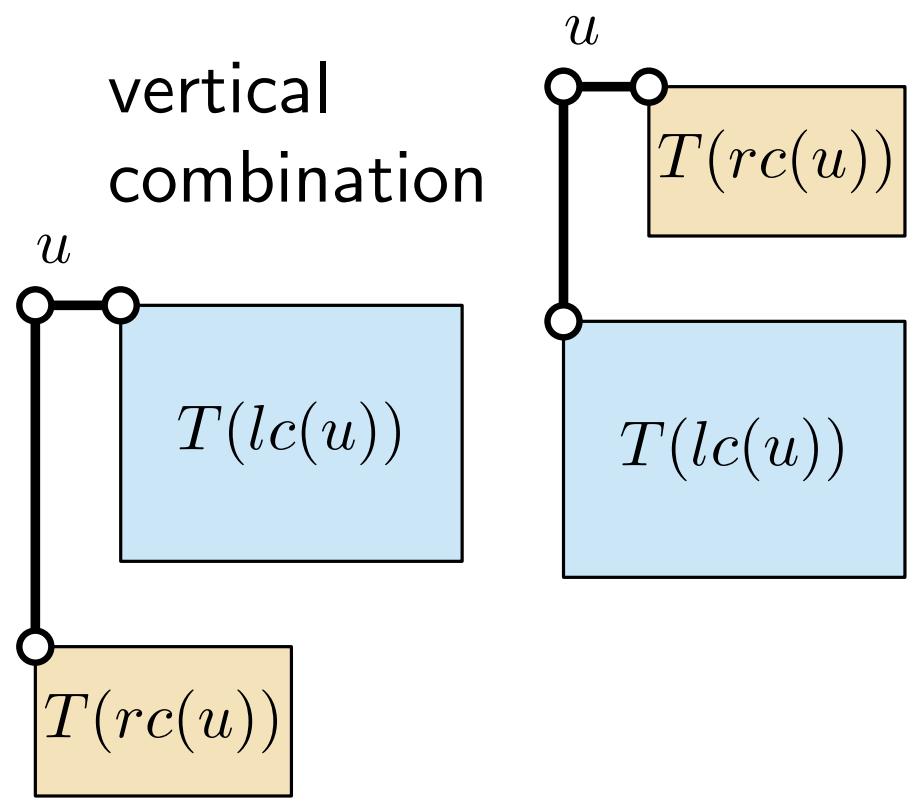
Compact orthogonal drawings of rooted trees

Compact Orthogonal Tree Drawings: HV Layout

Def: HV layout is a planar, straight-line grid drawing of a rooted binary tree with

- children either vertically aligned and below the parent or horizontally aligned and to the right
- recursively defined by placing bounding boxes of subtrees without intersections





Algorithm RightHeavyHVTreeDraw

- Construct drawings of left and right subtrees recursively (postorder)

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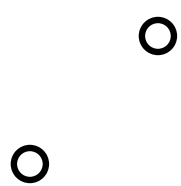

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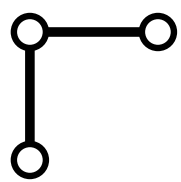

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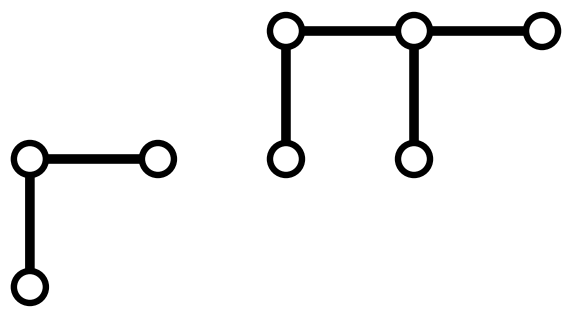

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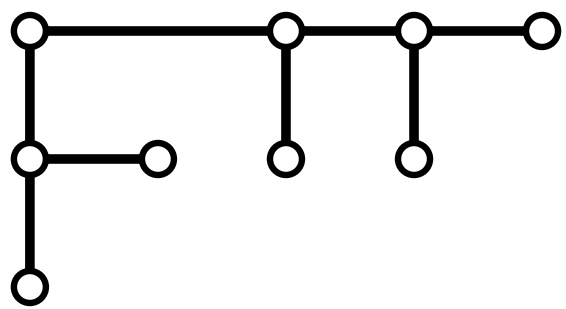

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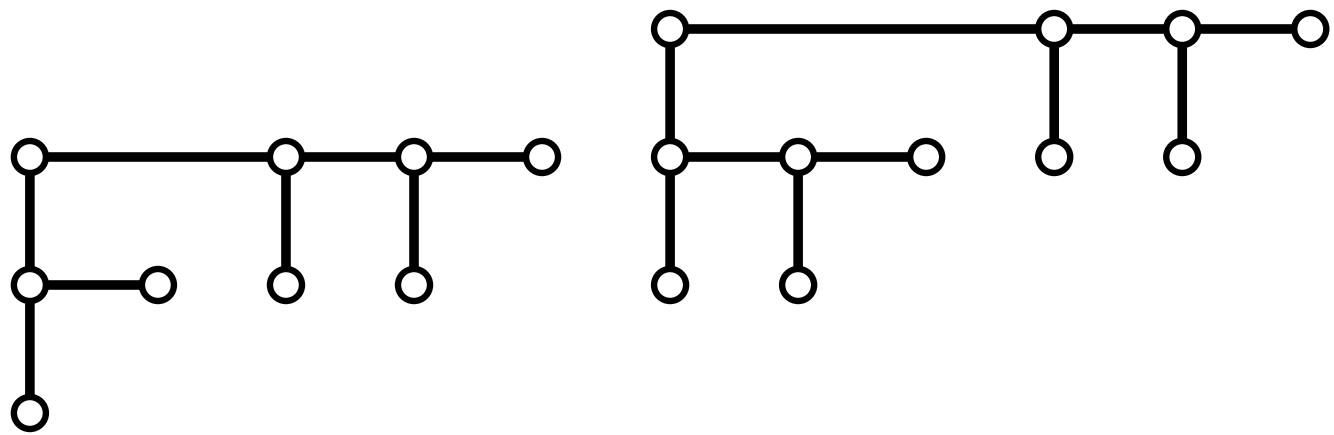

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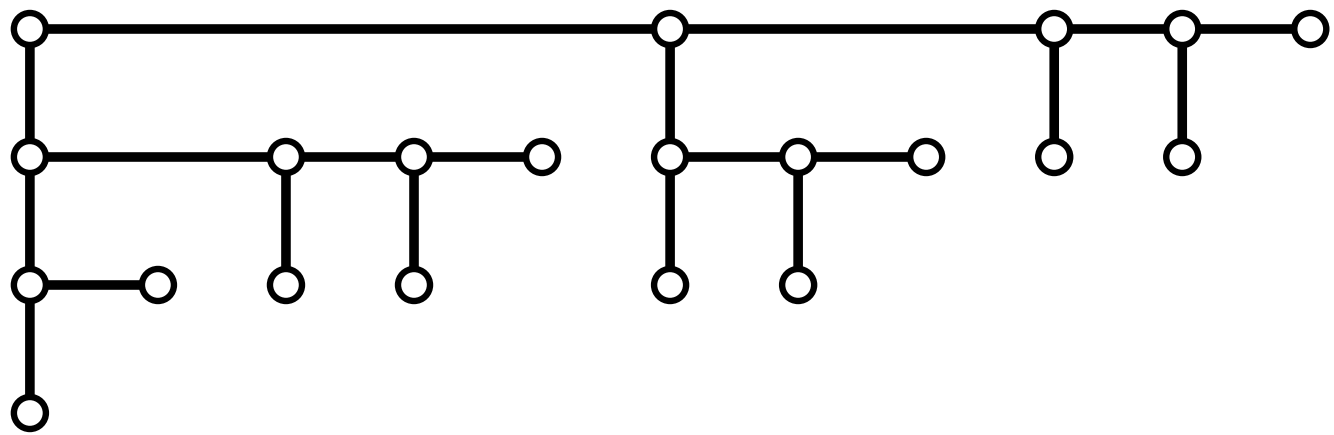

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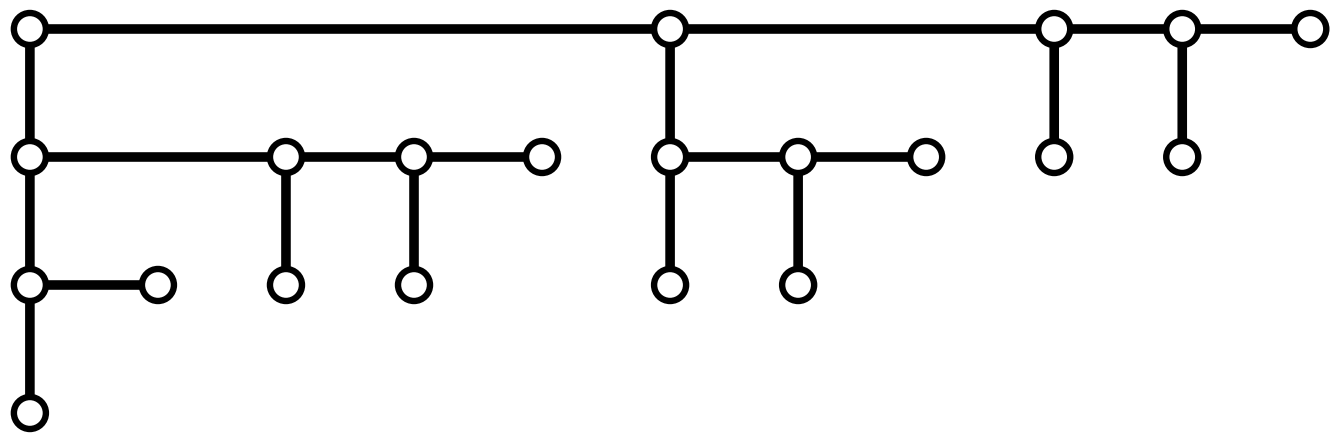

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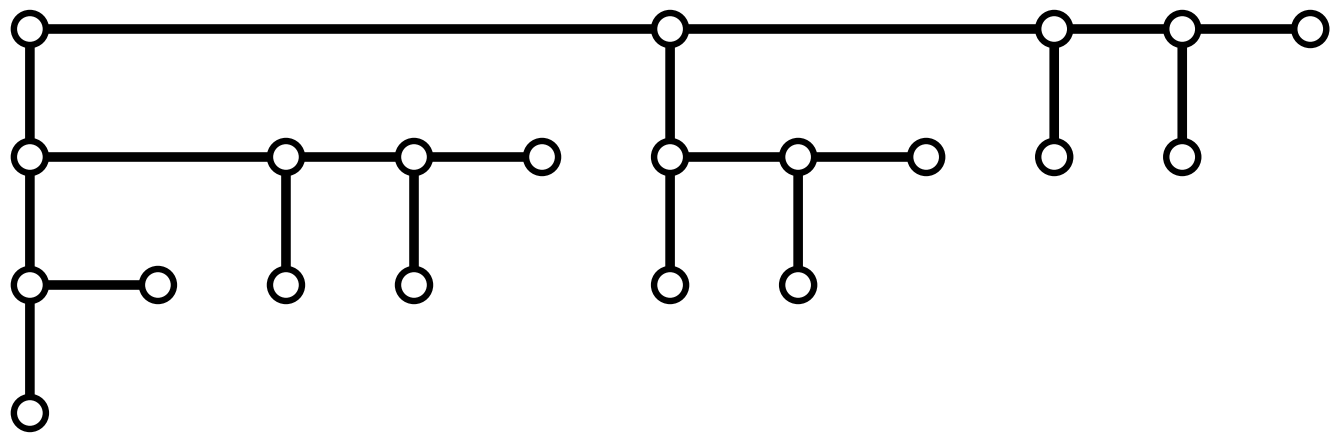

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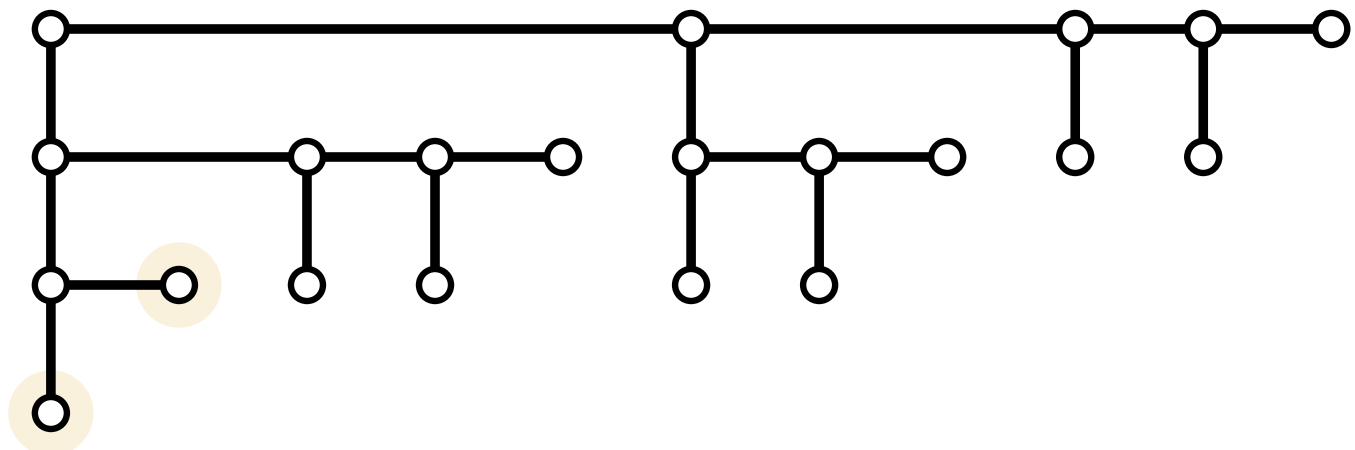

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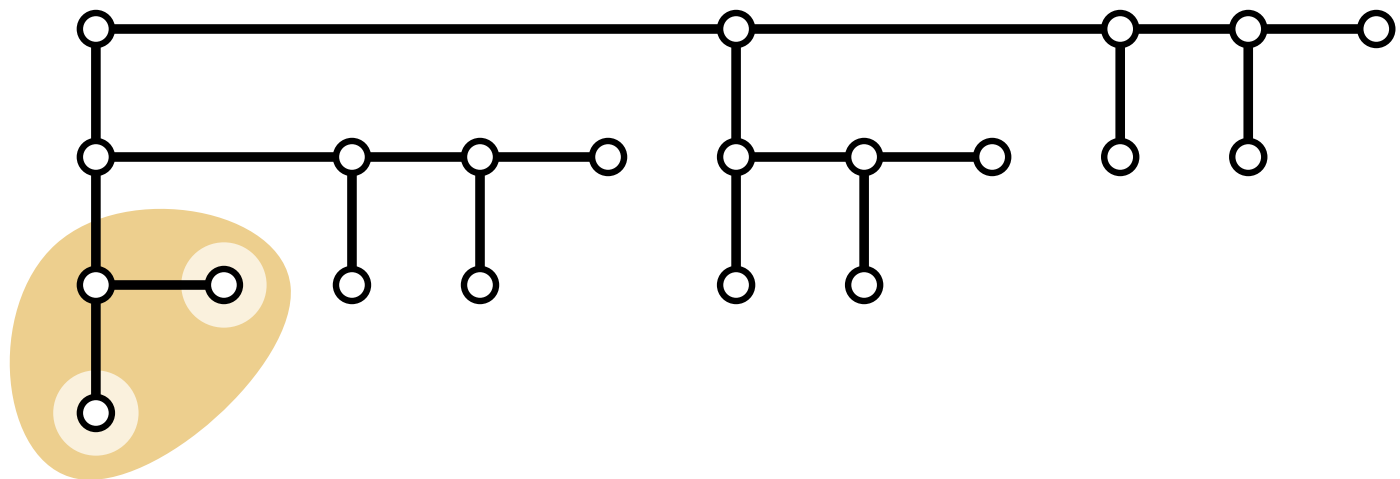

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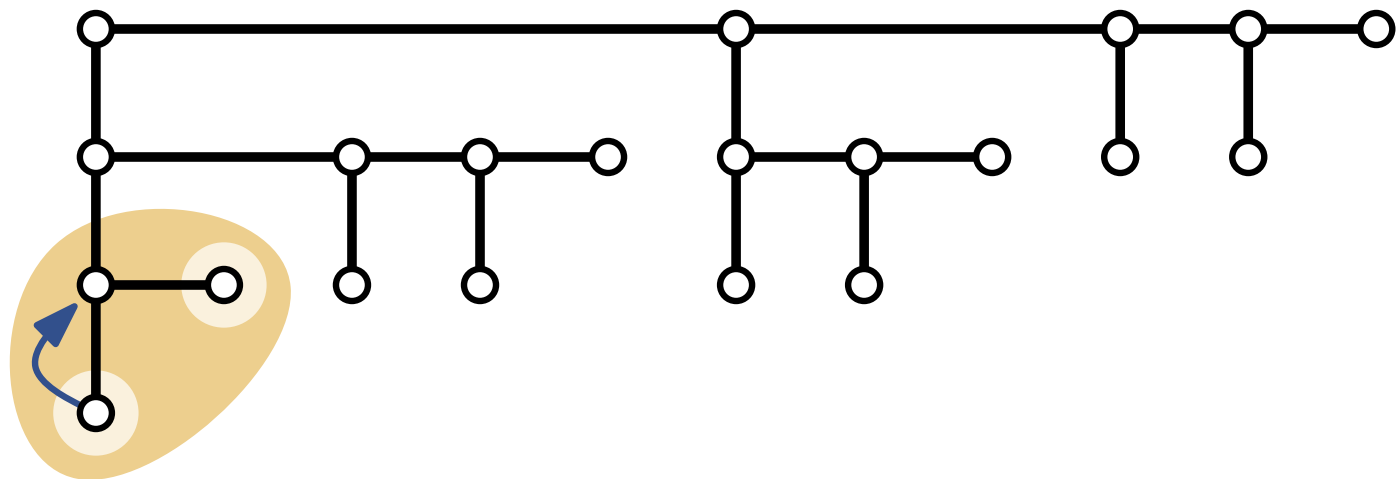

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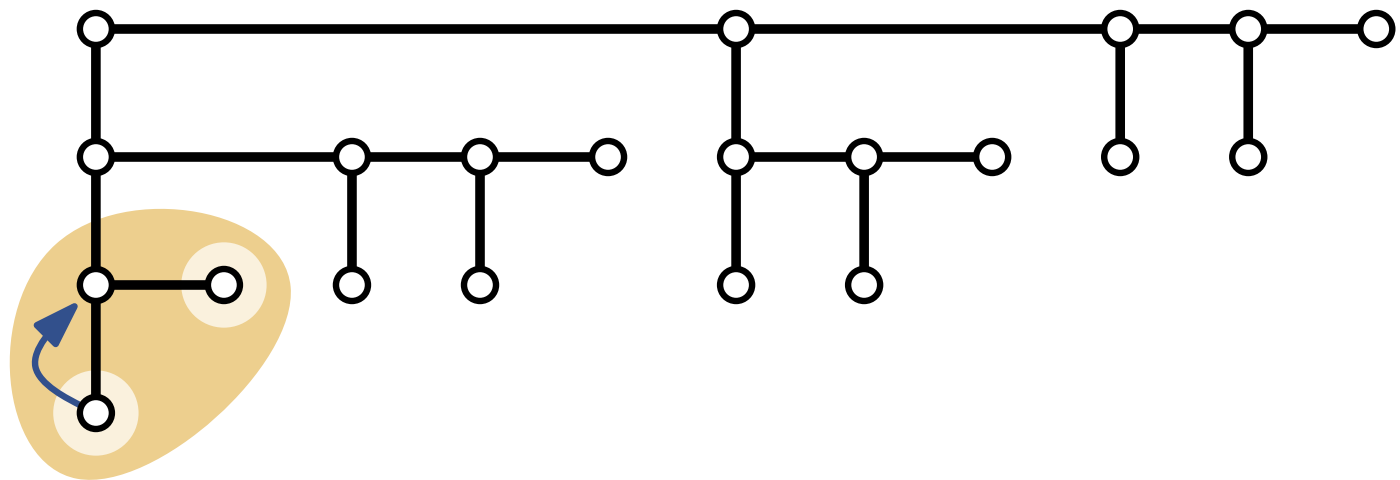

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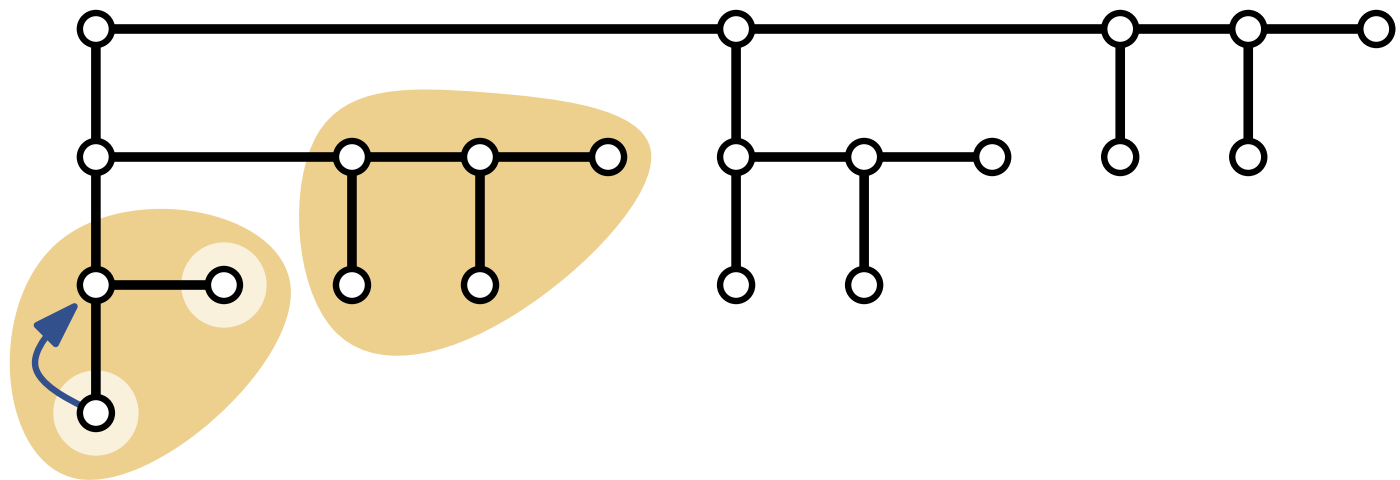

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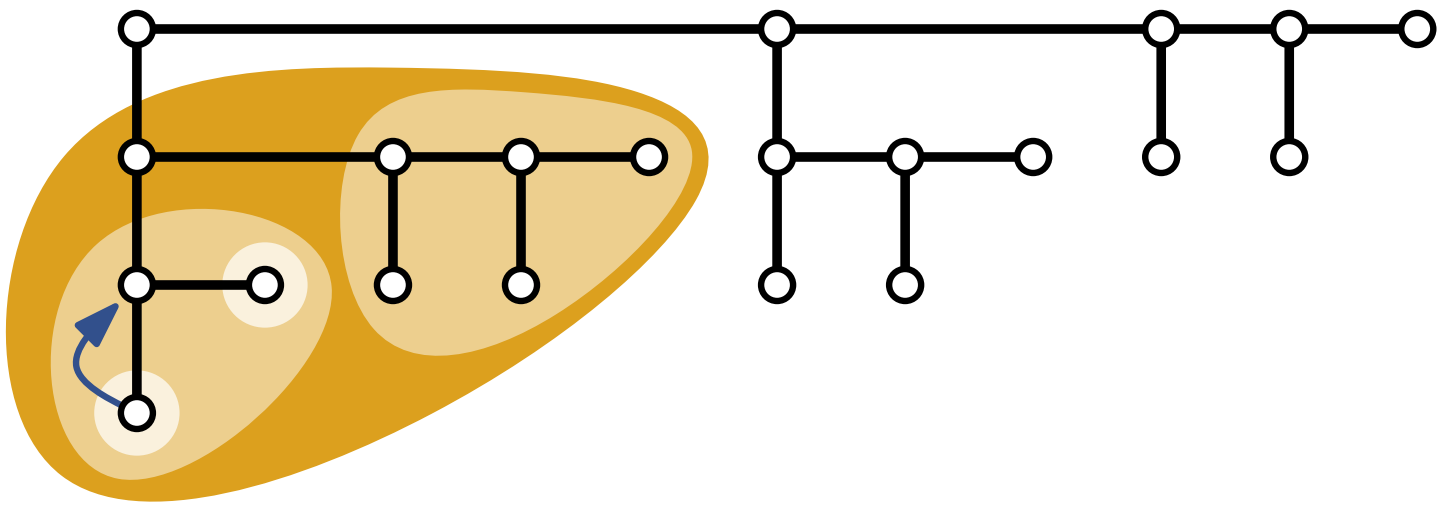

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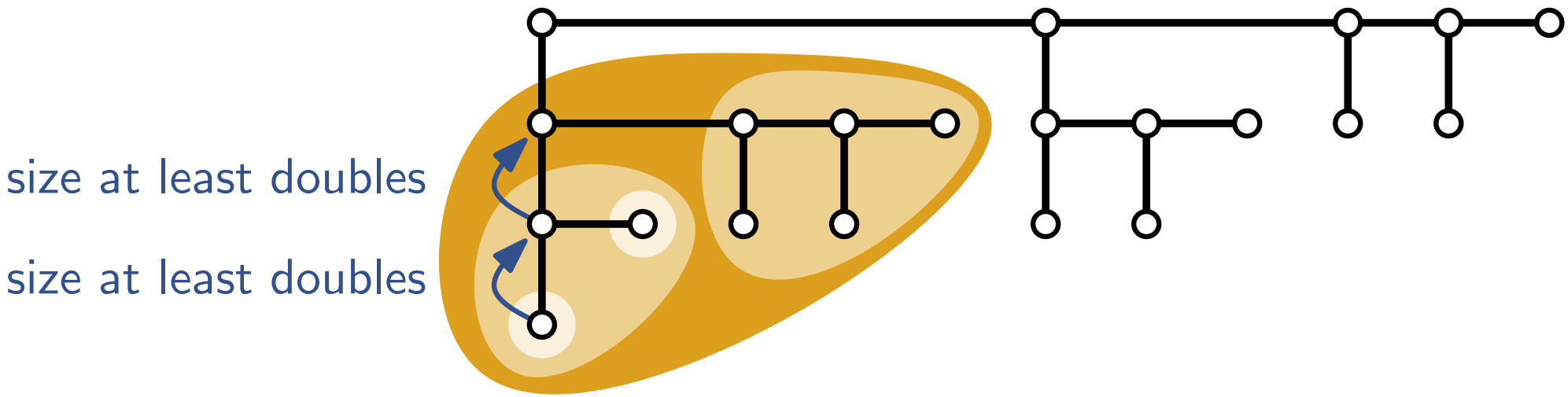

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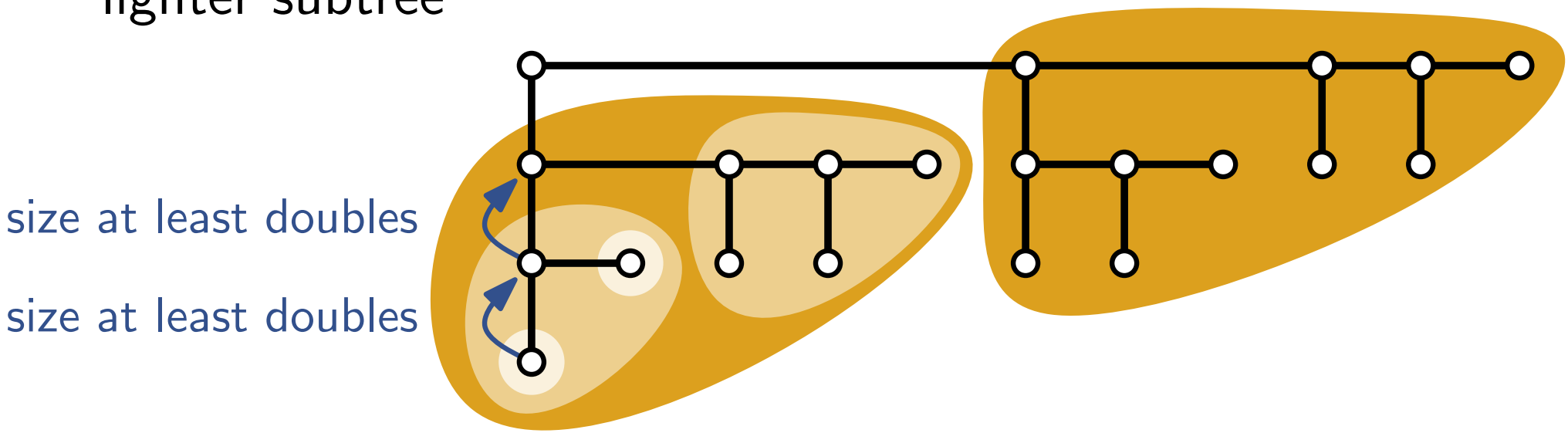

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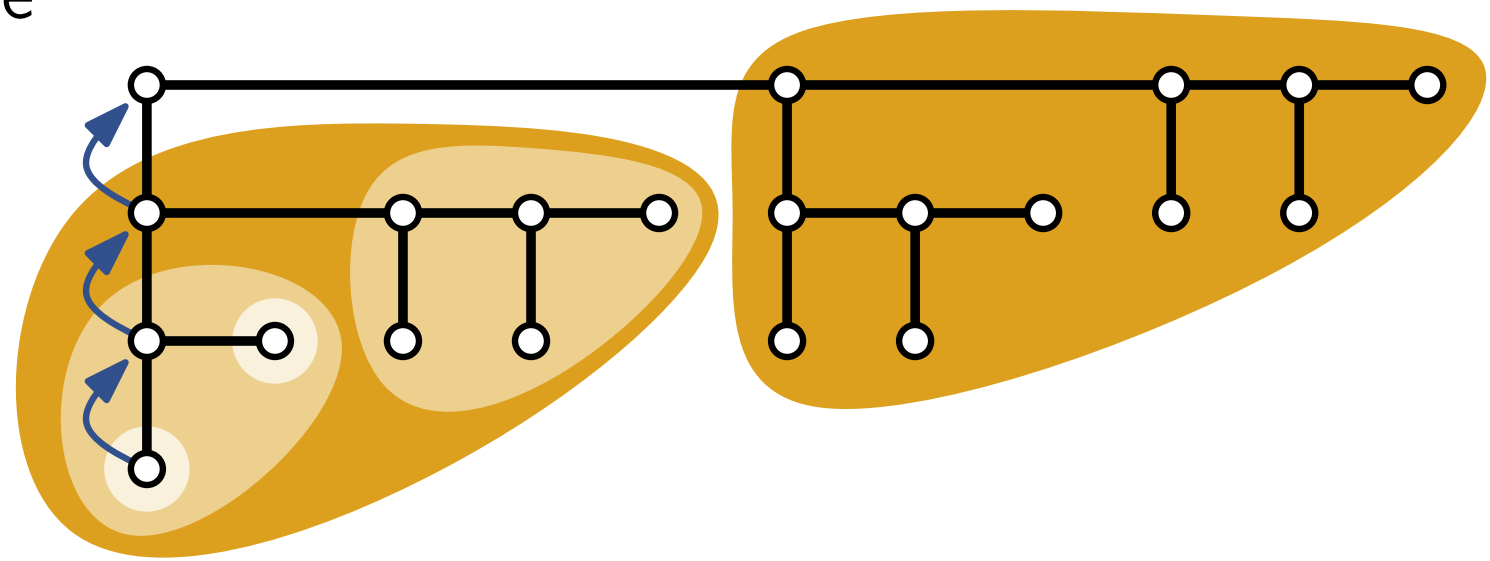

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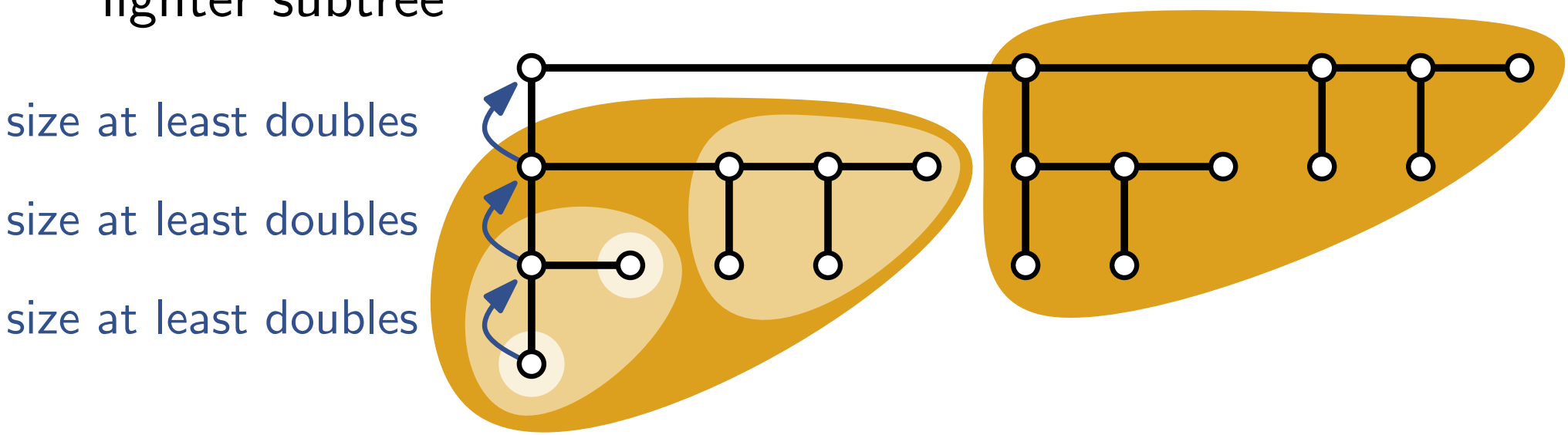

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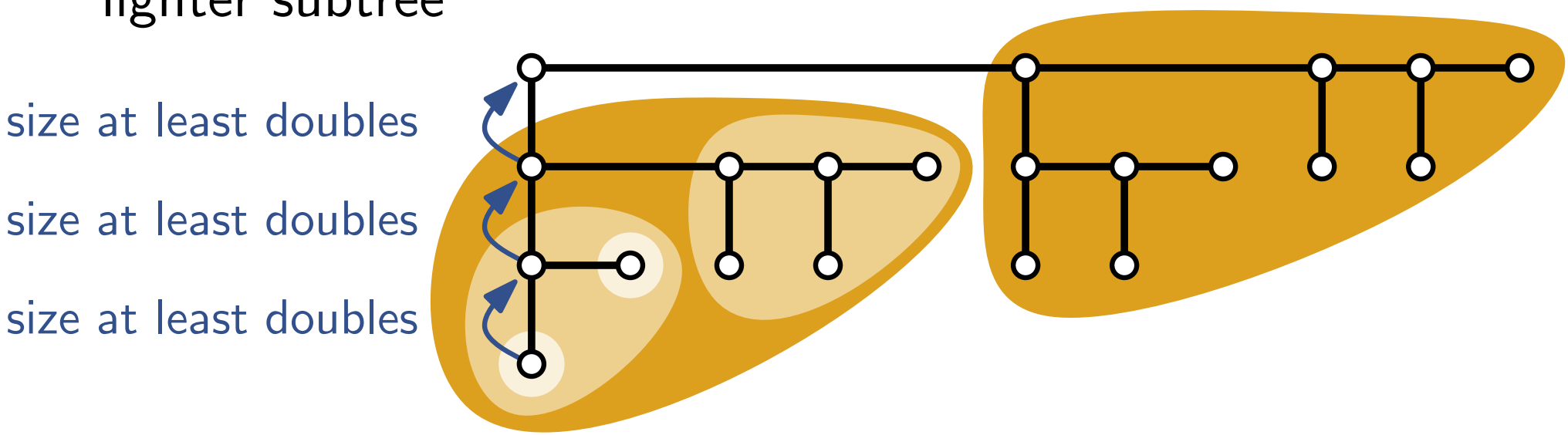

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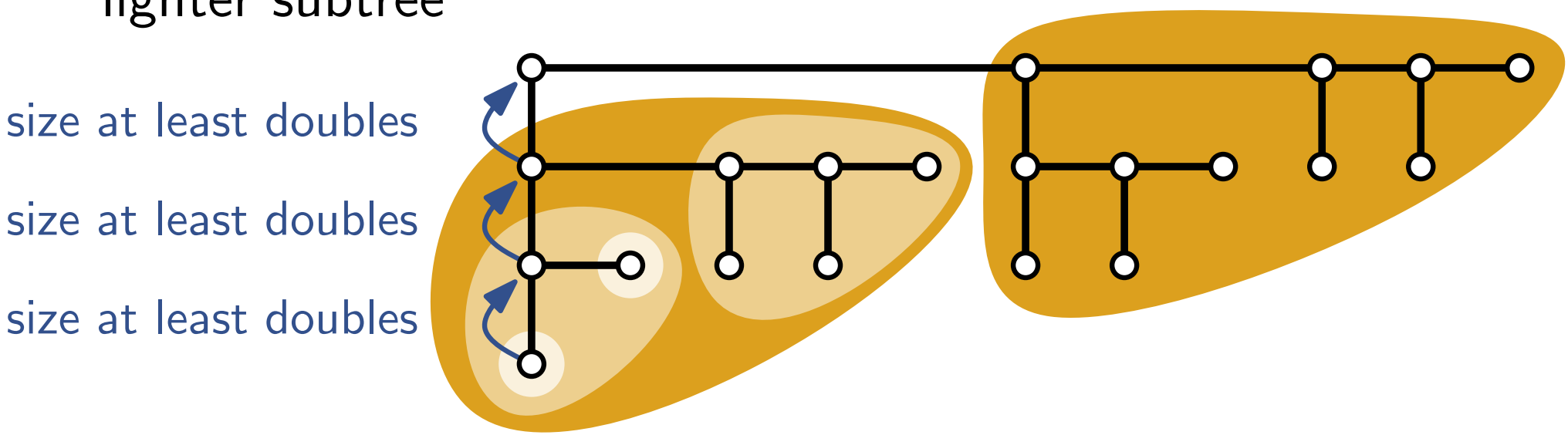

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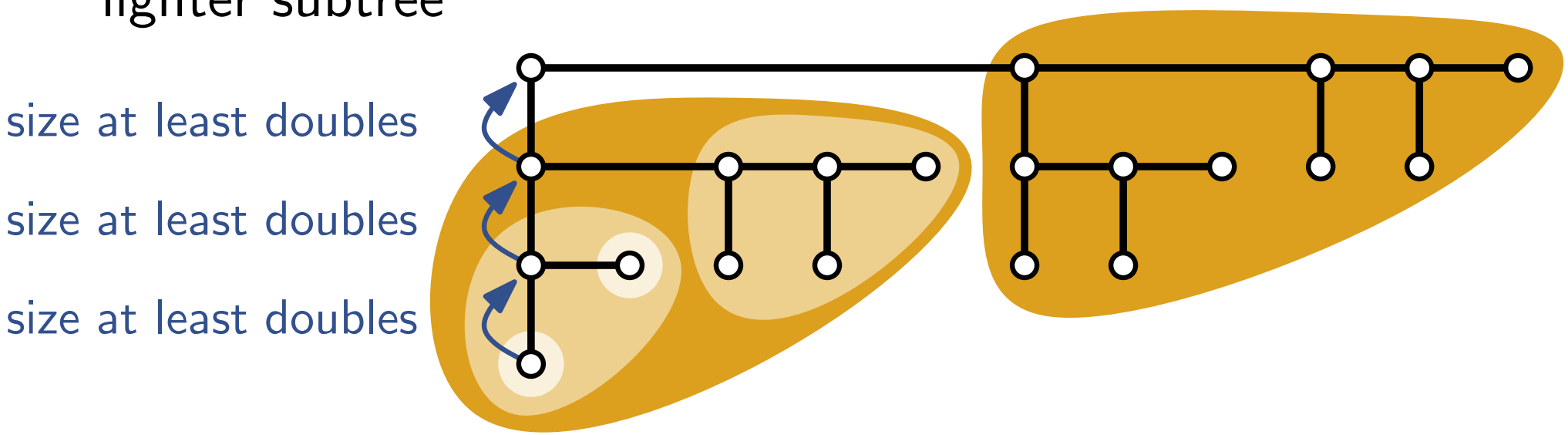

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But: we might have changed child order

Recall: Layout Problem

Graph visualization problem

given: graph $G = (V, E)$
find: drawing Γ of G that

- complies with the given drawing conventions
- optimizes the given aesthetics
- satisfies the partial/local constraints

$\rightarrow$ often lead to NP-hard optimization problems!
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we have seen:

- (binary/ordered) trees
- planar, straight-line (grid) drawings
- optional: centered parent, layered, upward, ordered, isomorphic subtrees
- optimization: area
- NP-hard: width minimization
- competing: width vs. isomorphic subtrees, area vs. ordered

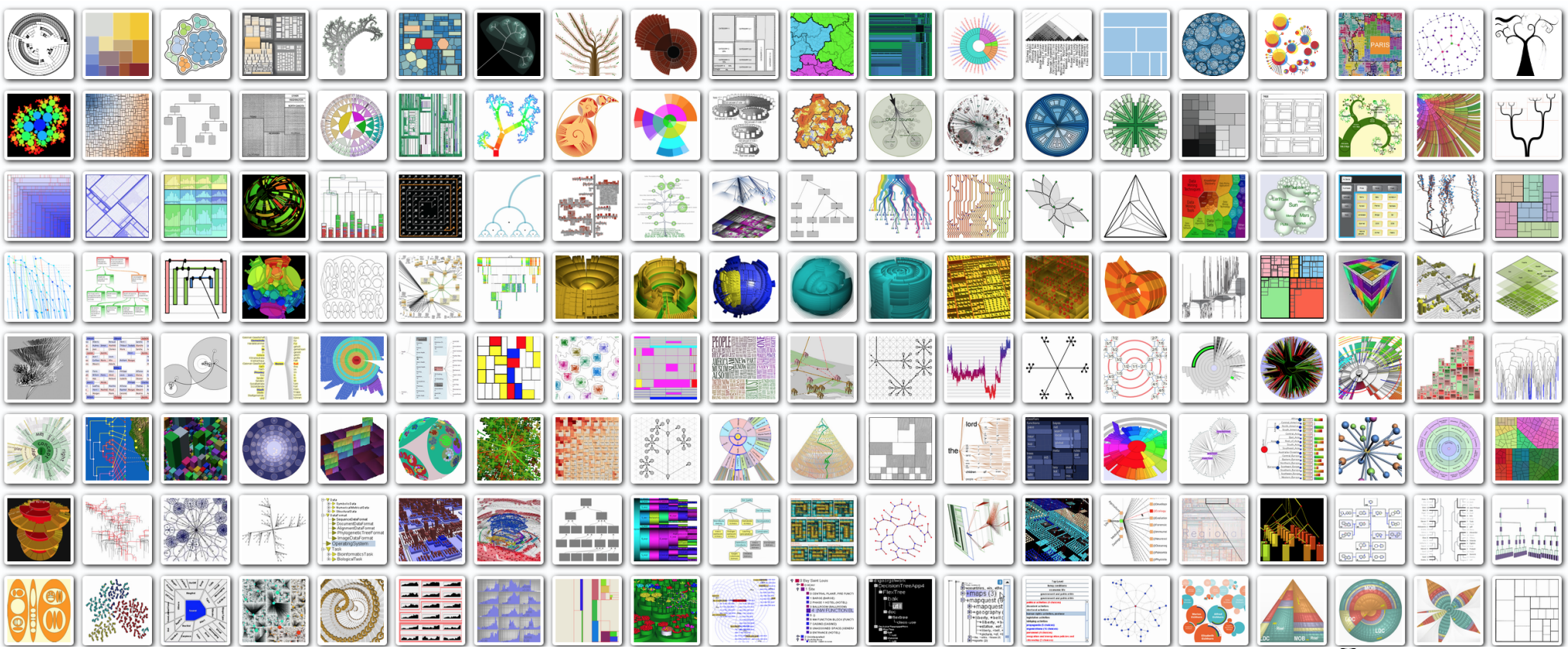
Summary and Outlook

- layered tree drawings
- radial tree drawings
- HV tree drawings

Summary and Outlook

- layered tree drawings
- radial tree drawings
- HV tree drawings

Trees are practically important graph class! Many layout algorithms exist and new ones are developed.



Further Reading: Area Minimization for Trees

Area minimization and area bounds of tree drawings in many flavors is still an active (theoretical) research topic.

Some examples:

- unordered, upward, binary: $O(n \log \log n)$ (1996)
- ordered, upward, binary: $O(n \log n)$ (2003)
- ordered, non-upward, orthogonal, binary: $O(nc^{\sqrt{\log n}})$ (2018)
- unordered, non-upward, binary: $\Theta(n)$ (2003)
- unordered, upward, general: $O(n\sqrt{\log n}(\log \log n)^k)$ (2018)
- ordered, upward, general: $O(nc^{\sqrt{\log n}})$ (1999)
- ...

see [Chan, 2018] for more

$\rightarrow$ unknown, whether most bounds are tight

Overview

Series-Parallel Graphs

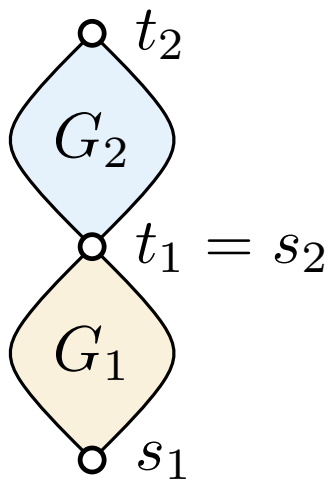
Series-Parallel Graphs

A simple (directed) graph G is **series-parallel**, if

- it consists of a single edge (s, t) with source s and sink t
- it is composed of two series-parallel graphs G_1, G_2 with sources s_1, s_2 and sinks t_1, t_2, which are combined using one of the following composition rules:

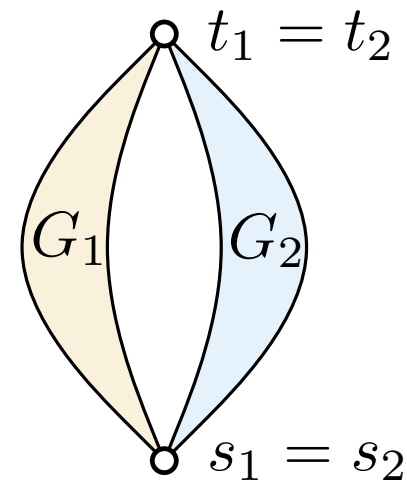
Series composition:

- identify t_1 and s_2,
- set s_1 as the source of G, t_2 as the sink of G



Parallel composition:

- identify s_1 and s_2 as new source
- identify t_1 and t_2 as new sink



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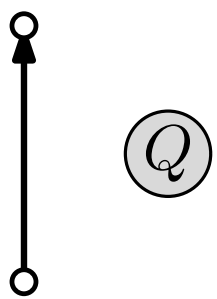

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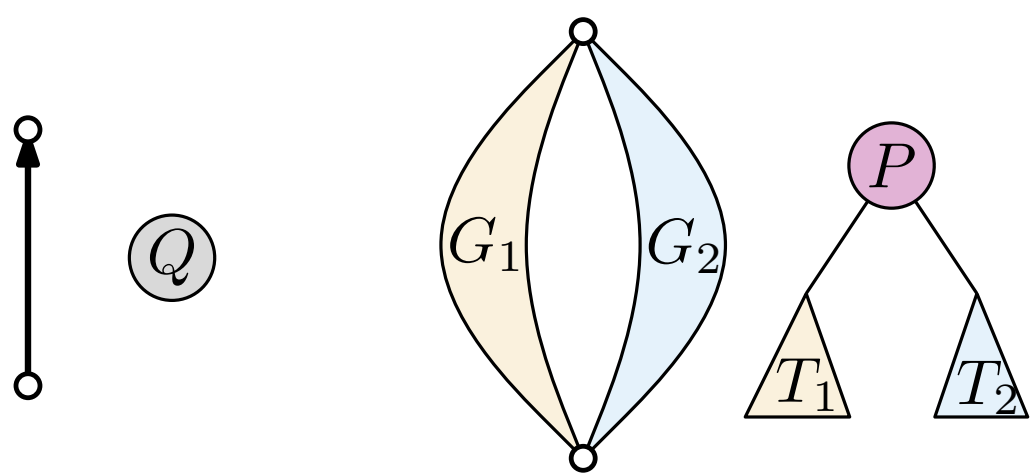

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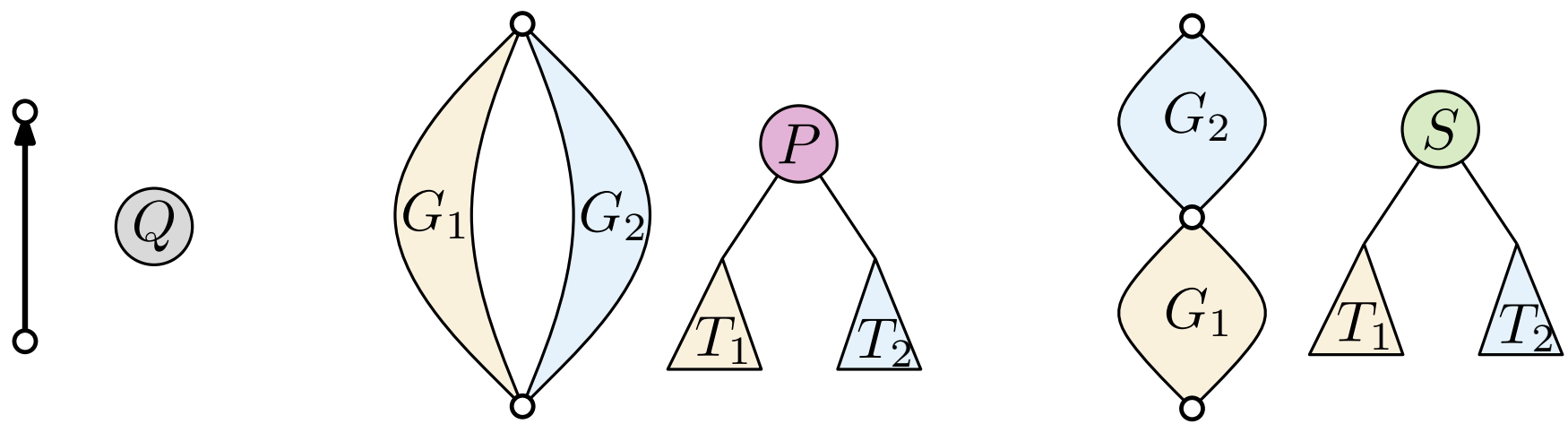
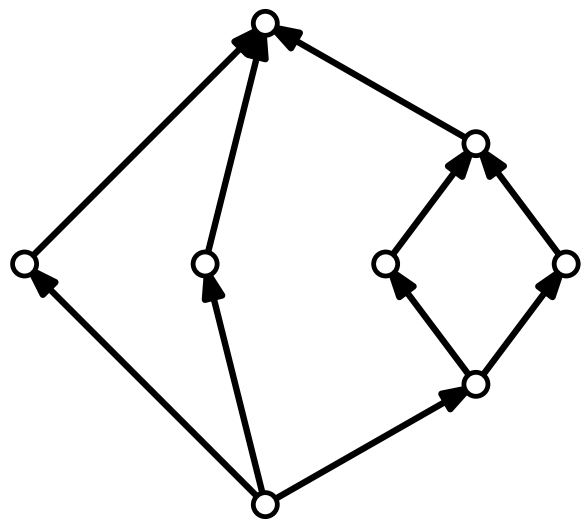
- If G is a **series composition** of G_1 (with tree T_1) and G_2 (with tree T_2), then the root of T is an **S-node** with left and right subtrees T_1 and T_2

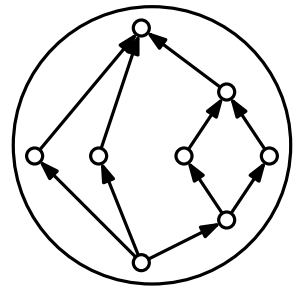


Decomposition Tree Example



Is this a series-parallel graph?

Decomposition Tree Example



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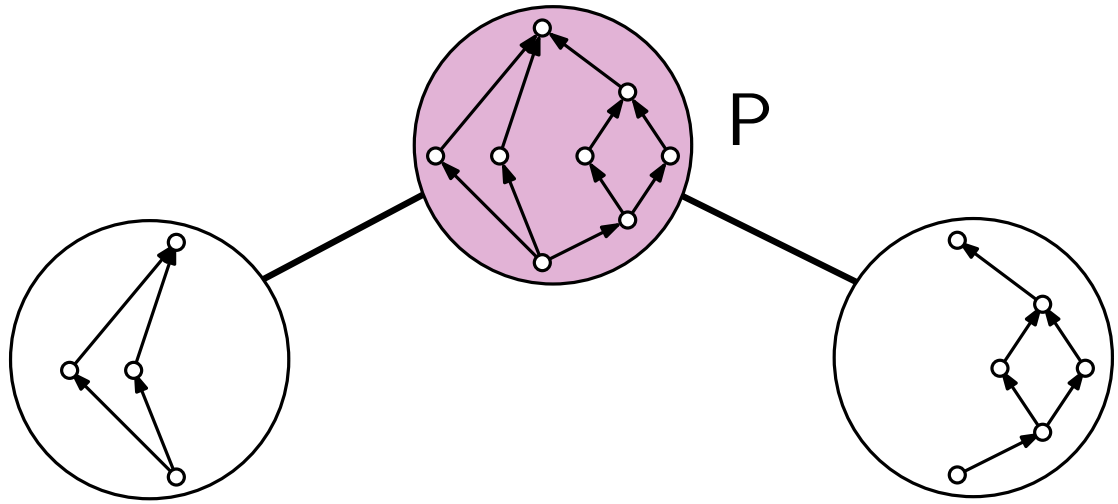

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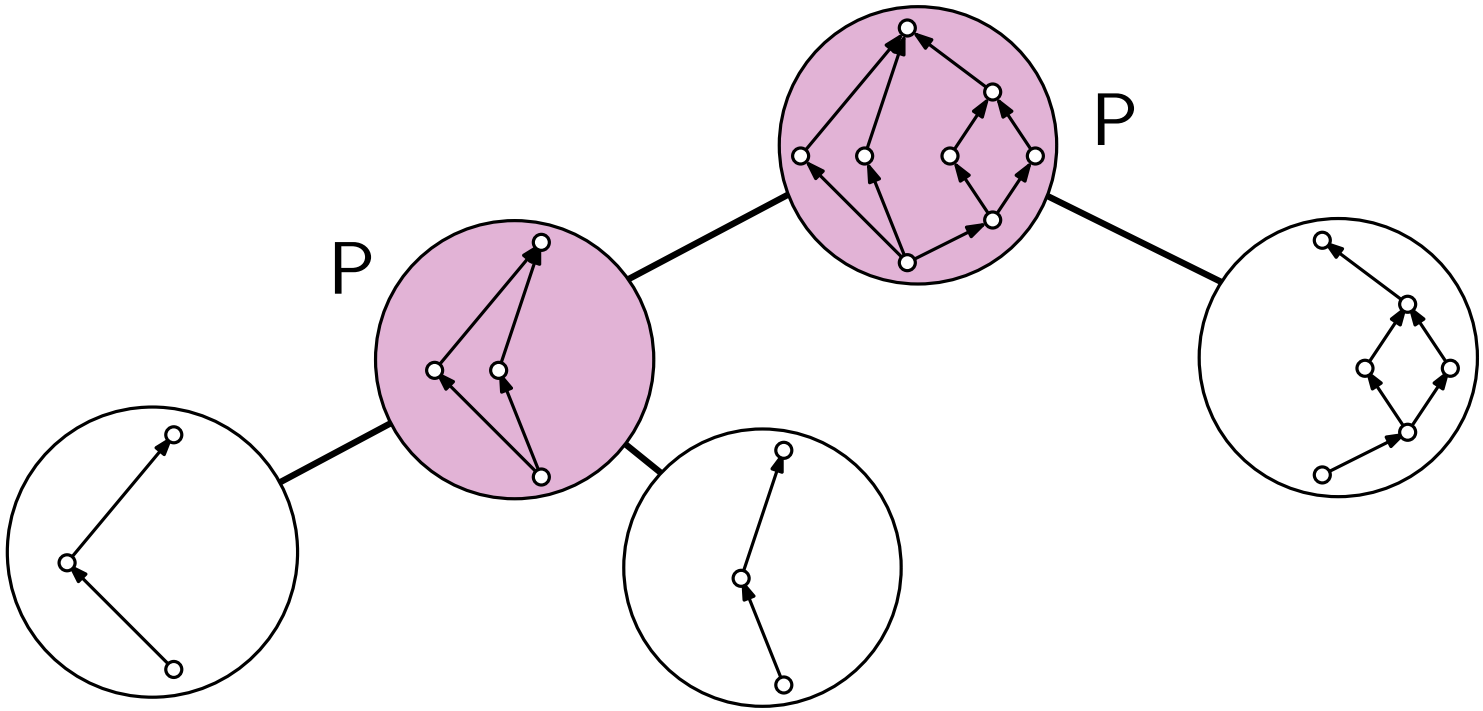

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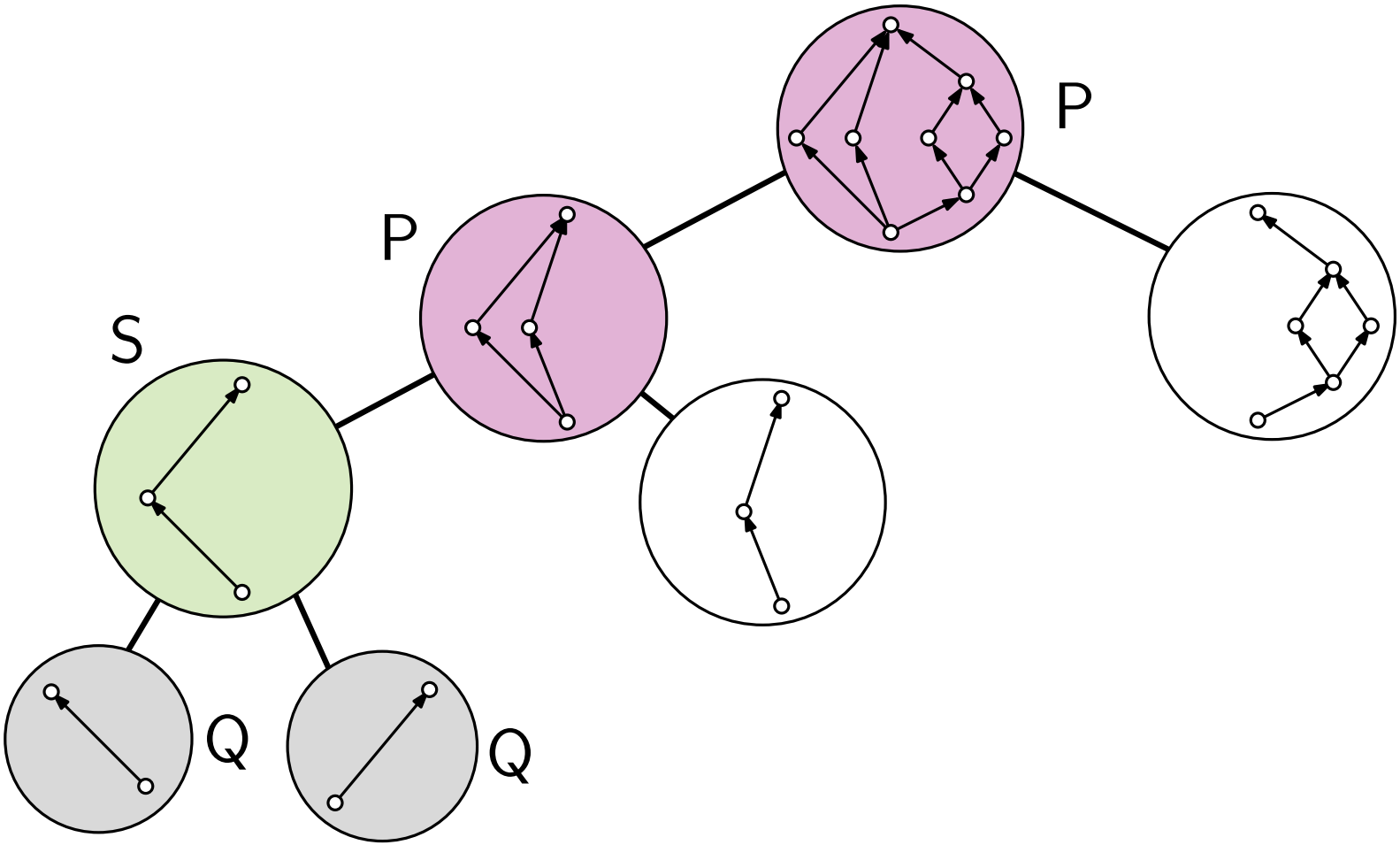

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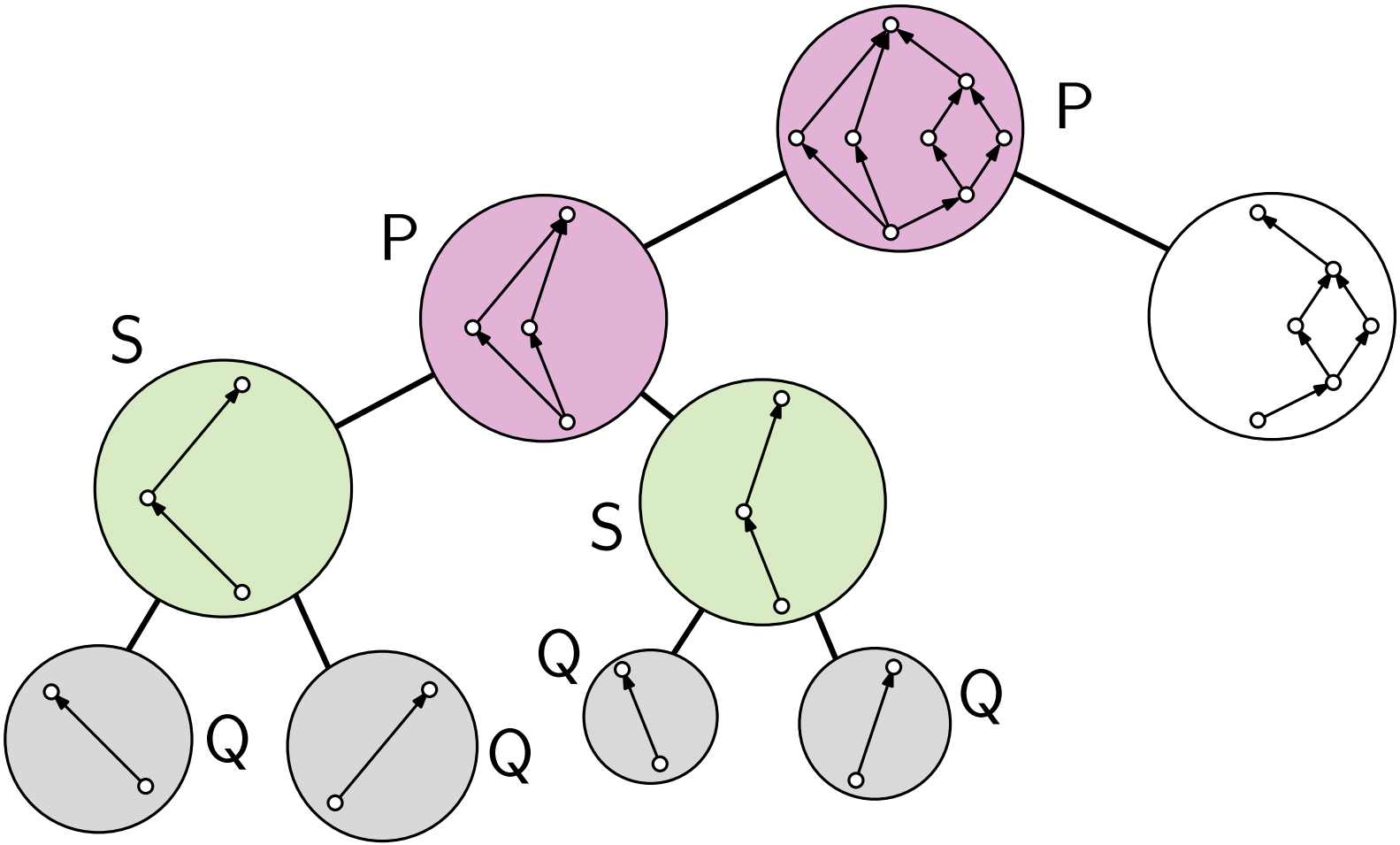

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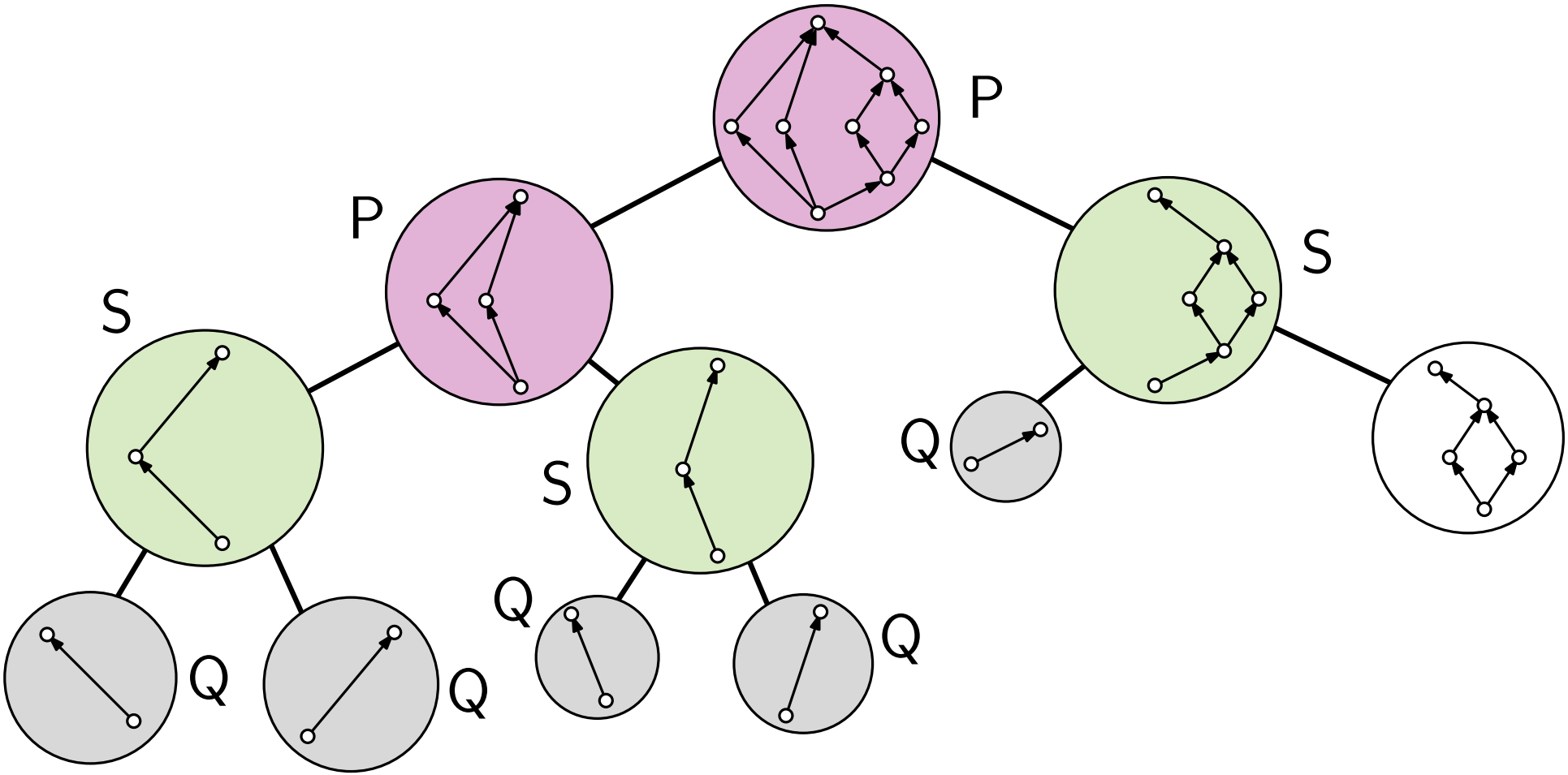

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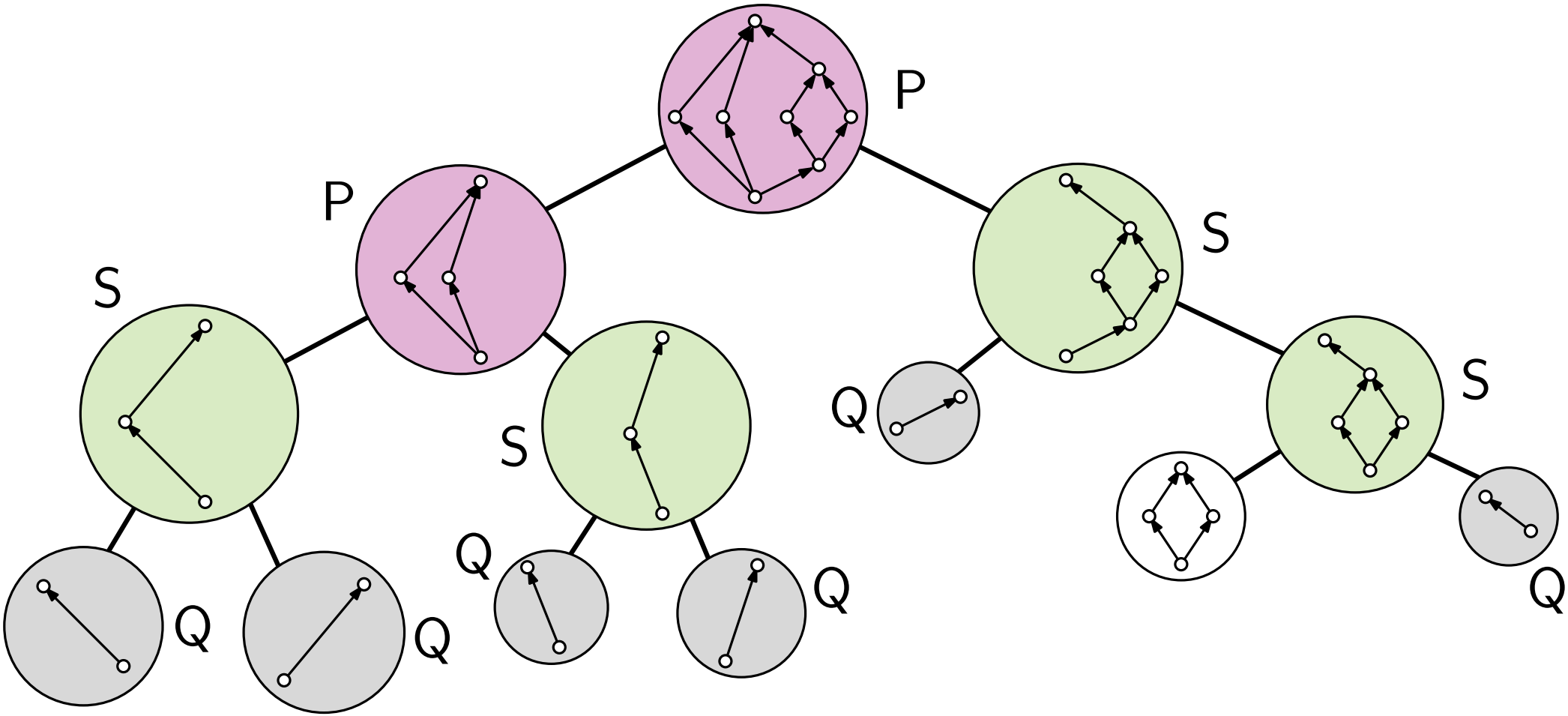

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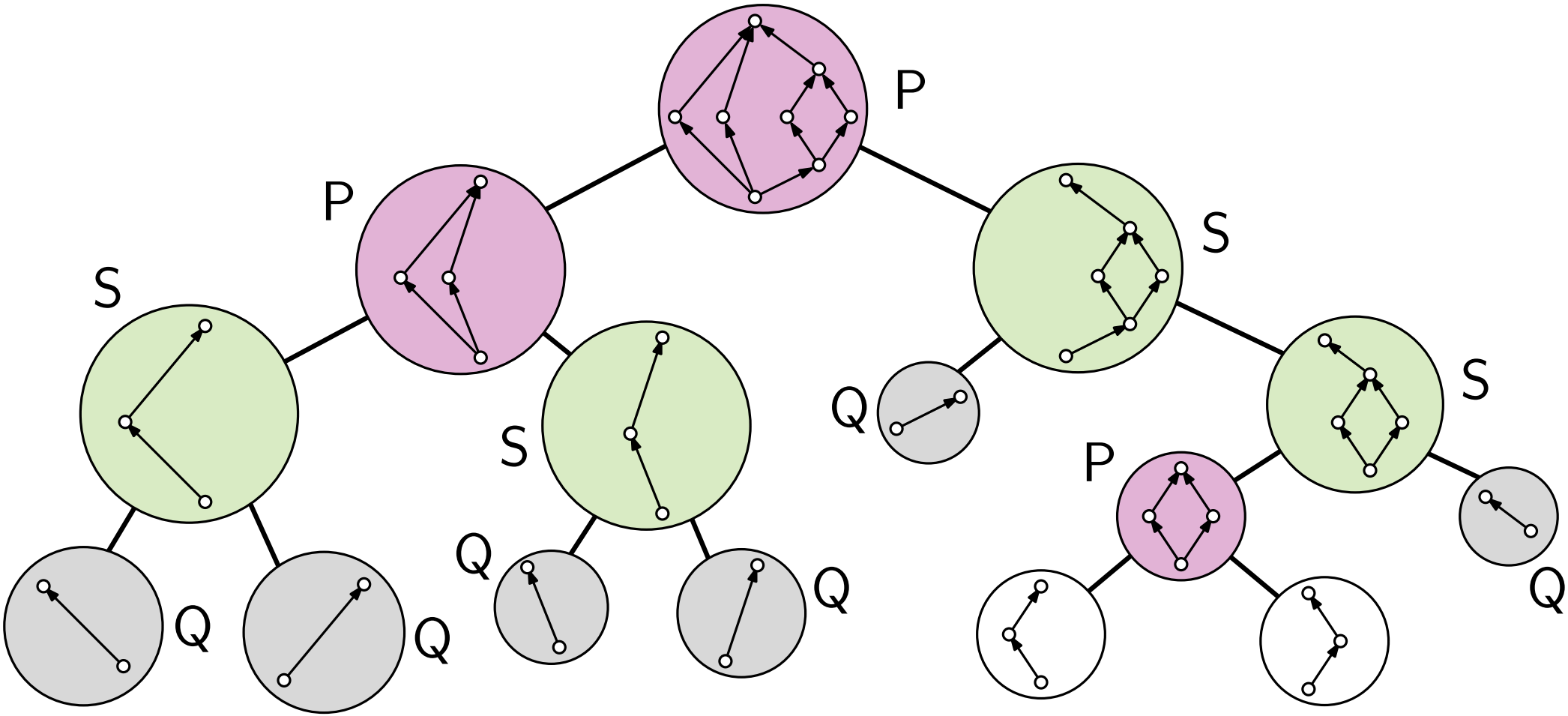

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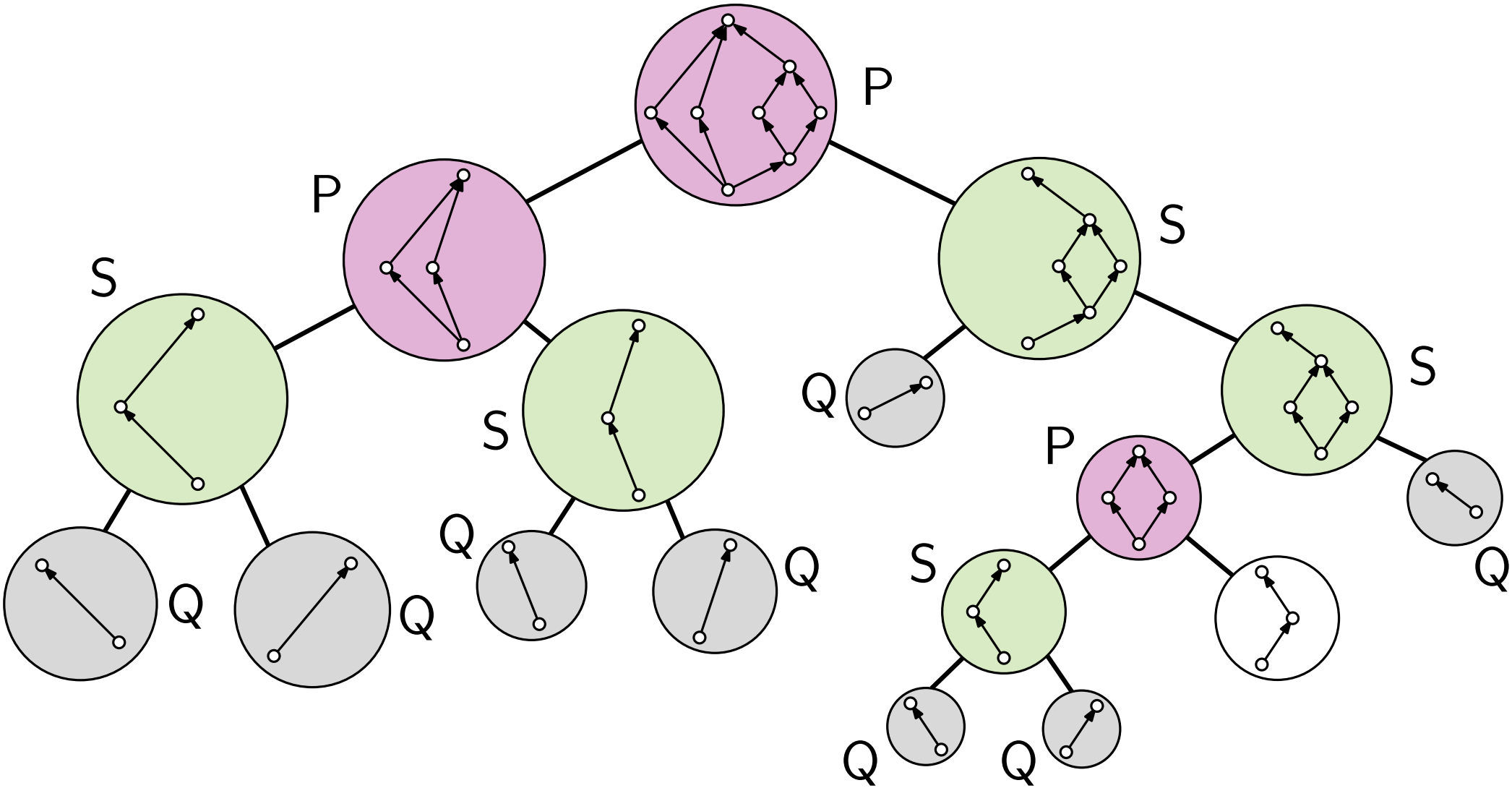

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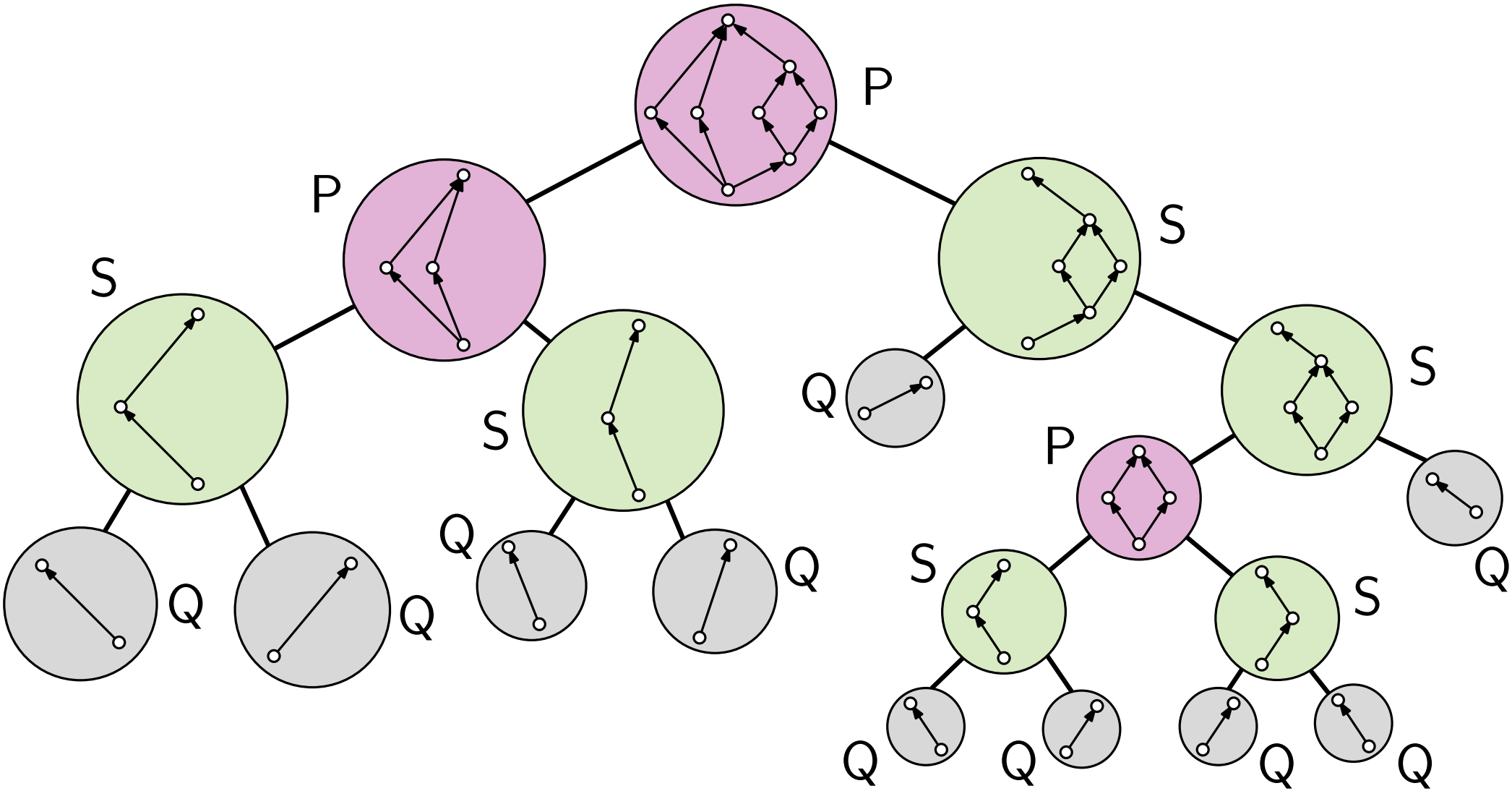

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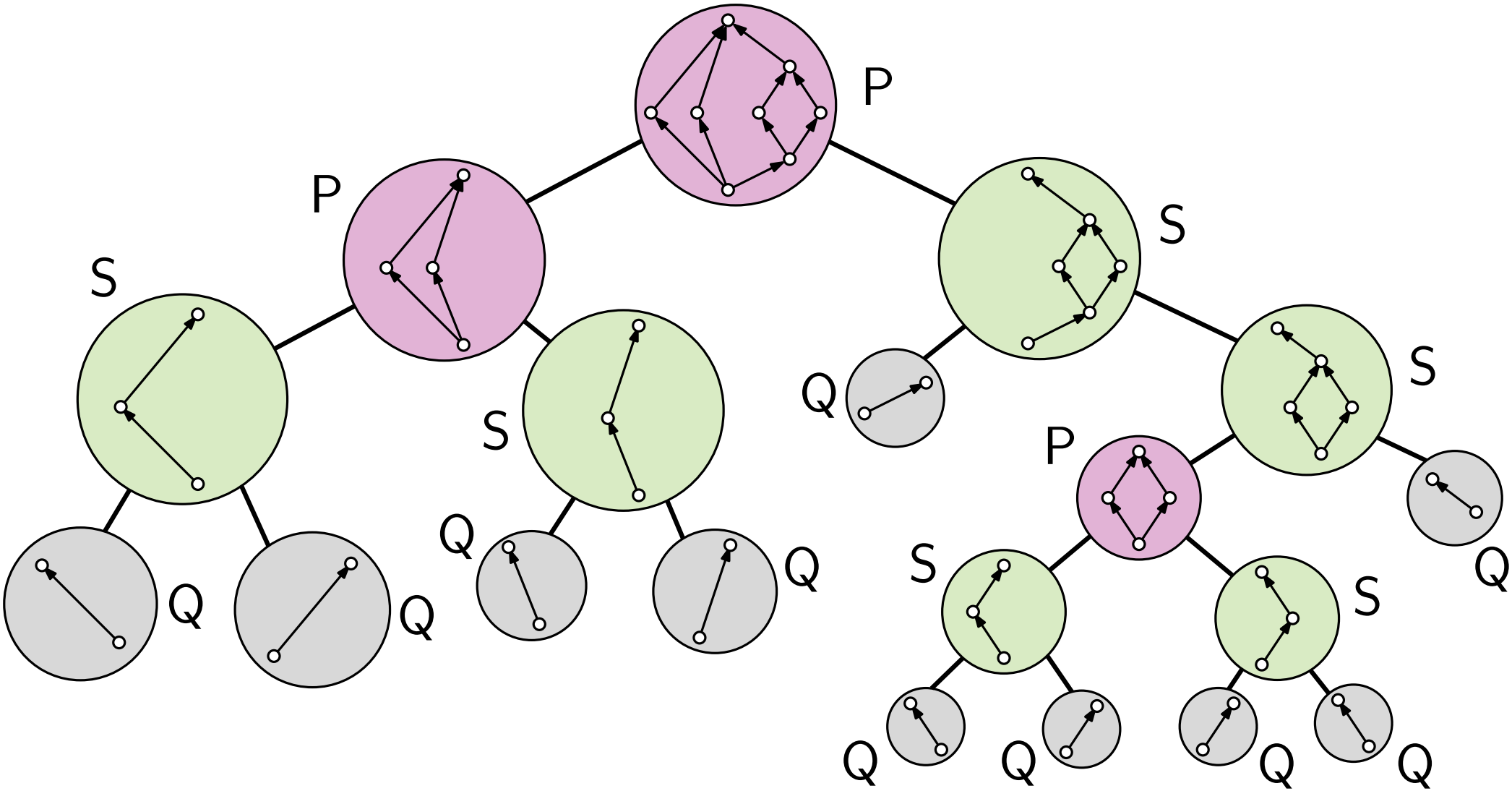


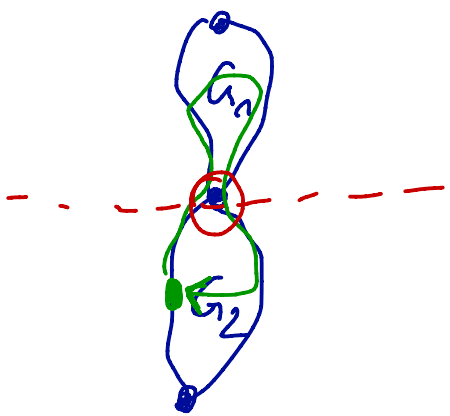
$\rightarrow$ A similar recursive decomposition tree, the *SPQR-Tree* exists for biconnected planar graphs and is frequently used as a tool in graph drawing.

Some Properties

Lemma: Series-parallel graphs are planar and acyclic.

How could you prove this?

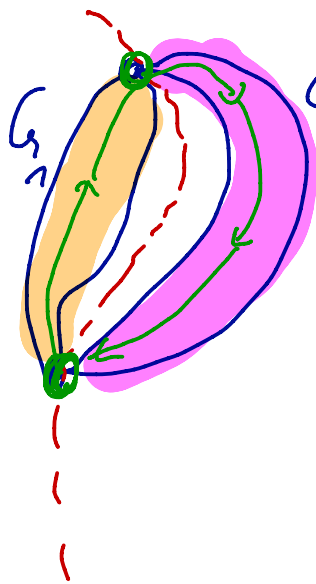
• S-composition:



planarity by design ✓

acyclic ✓

• P-composition



planar ✓

acyclic:
we identify two sources and two sinks
→ new sink has no outgoing edges

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Lemma: Series-parallel graphs are planar and acyclic.

- Given a graph G with n vertices, it can be tested in $O(n)$ time whether G is series-parallel. If so, the decomposition tree can be constructed in $O(n)$ time. [Valdes et al. '82]

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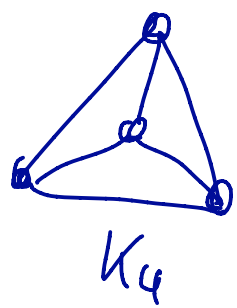

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- Series-parallel graphs have treewidth ≤ 2 and are K_4-minor-free.
- Many NP-hard problems can be solved in linear time on series-parallel graphs, e.g., [Takamizawa et al. '82]
  - maximum independent set
  - maximum matching
  - minimum dominating set

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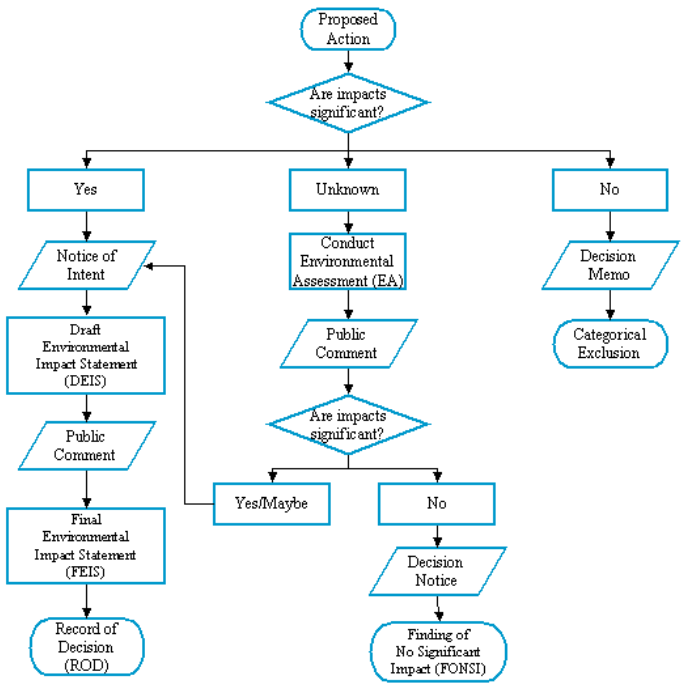
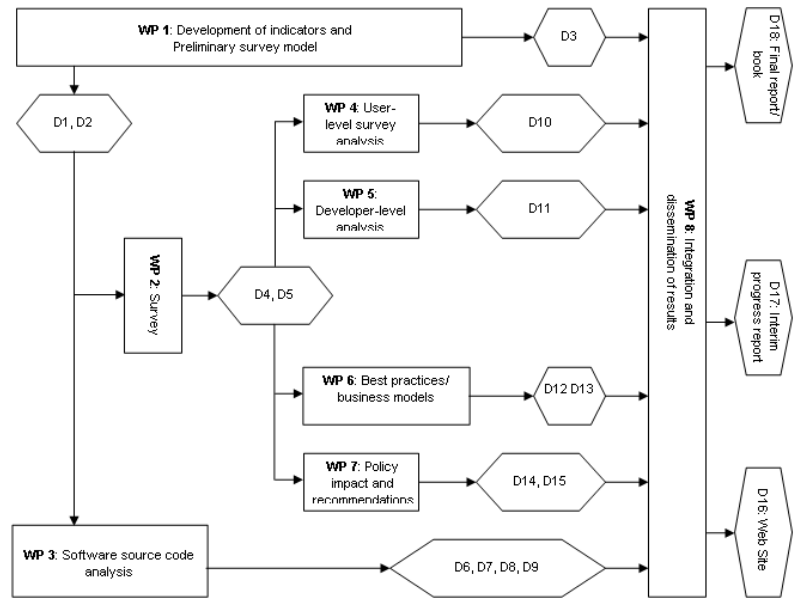


flow chart



PERT diagram

(Program Evaluation and Review Technique)

- Series-parallel graphs are used to model electrical circuits, flow charts, PERT diagrams etc.

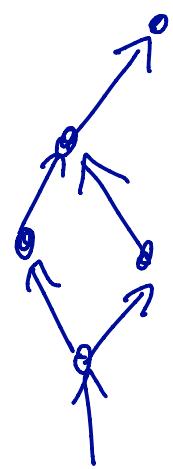
Drawing Series-Parallel Graphs

What are desirable properties of drawings of series-parallel graphs?

- what are reasonable drawing conventions?
- which aesthetics shall be optimized?

Let's collect some properties!

- straight edges
- upward drawings
- no crossings



optimize?

- area
- minimize longest edge
- similar edge lengths
- isomorphic structures drawn the same
- symmetries

Bad News: Exponential Area Lower Bound

Theorem: There exists a family of series-parallel graphs G_n ($n \in \mathbb{N}$) such that any order-preserving upward planar straight-line drawing of G_n requires area $\Omega(4^n)$. [Bertolazzi et al. '94]

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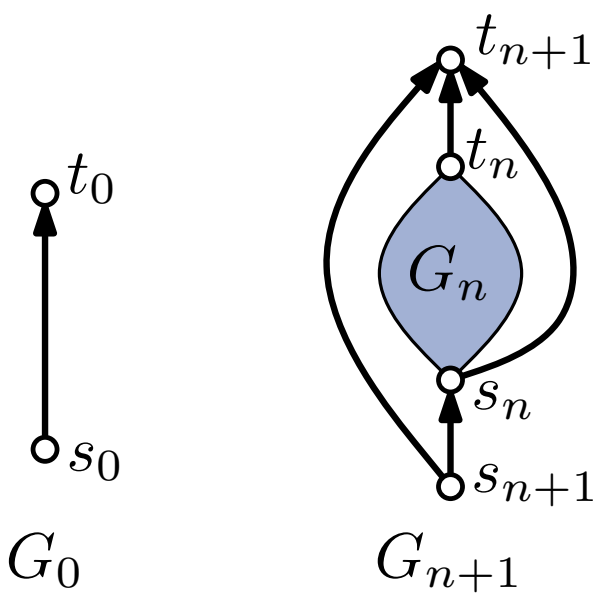

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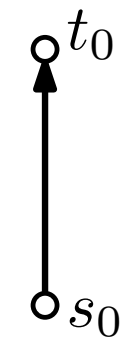
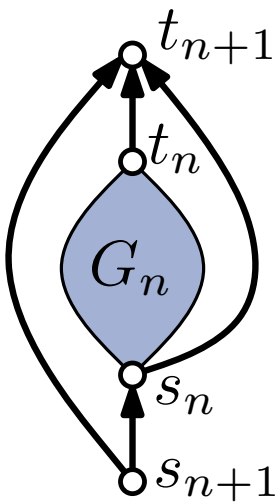
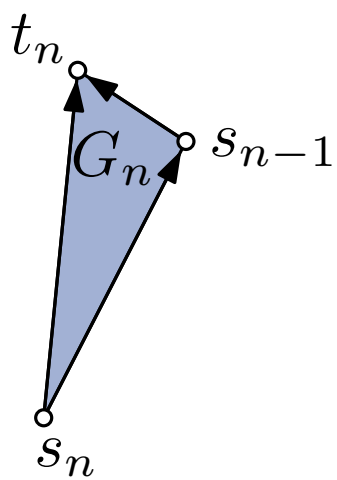


G_0



G_{n+1}



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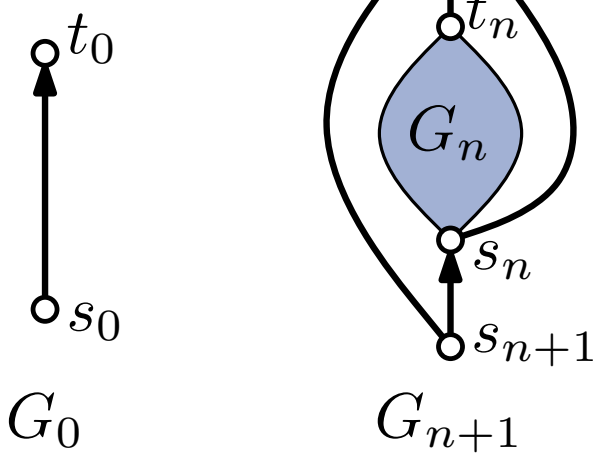

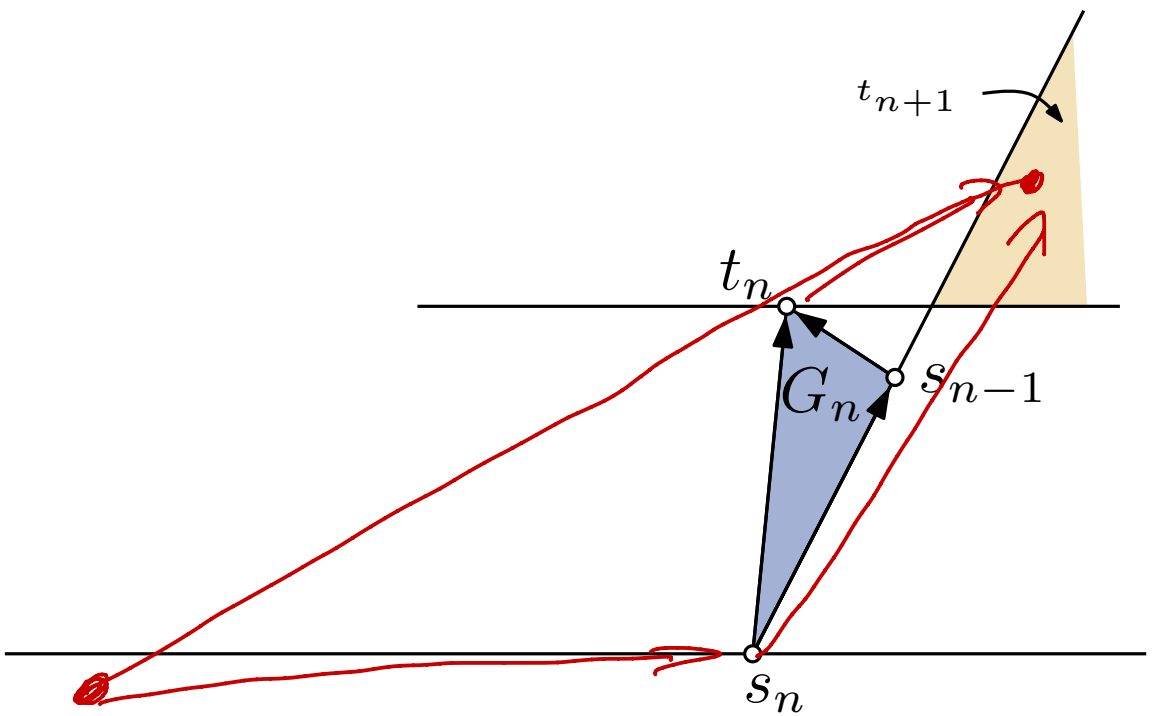

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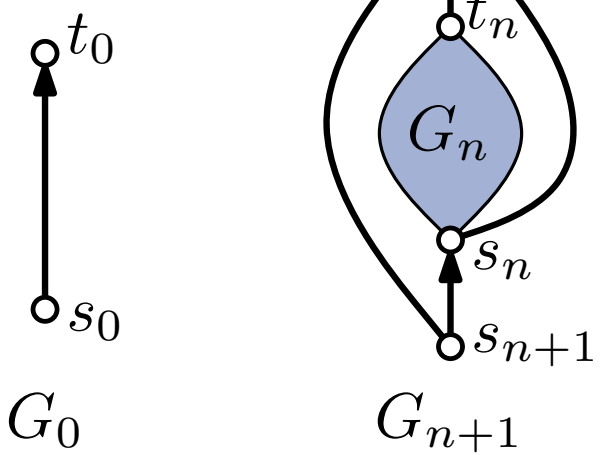

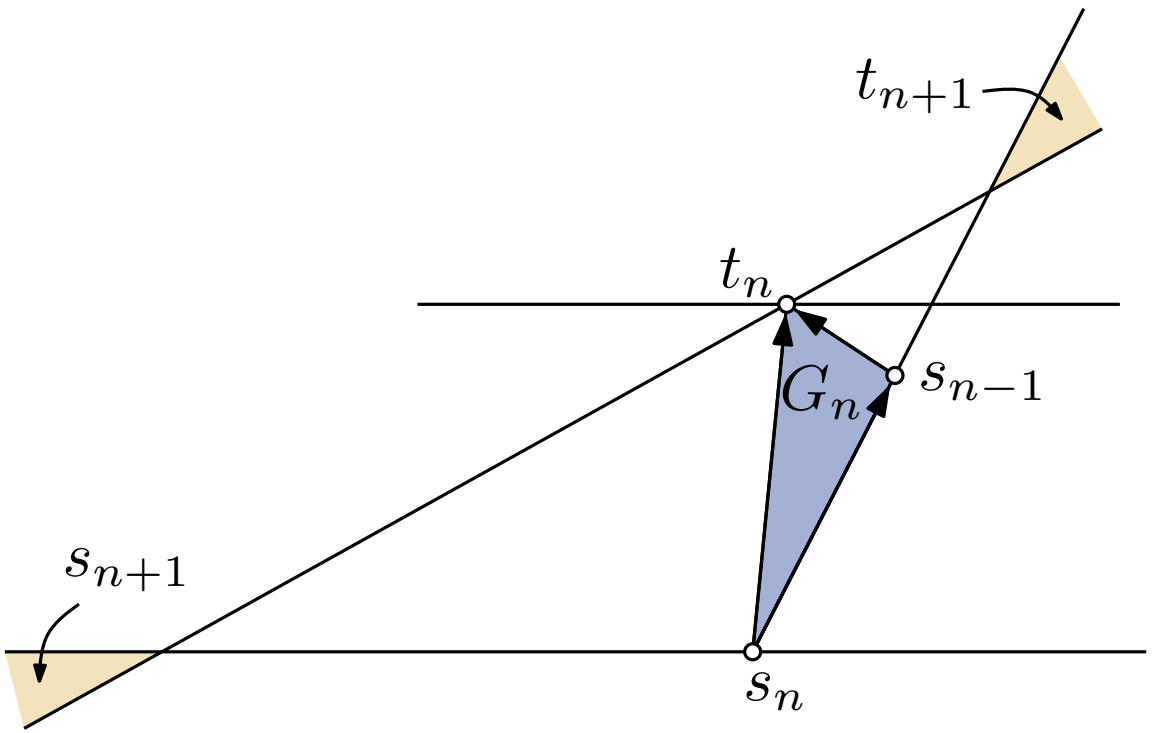

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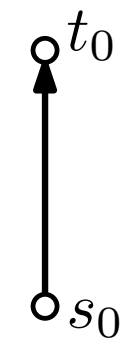

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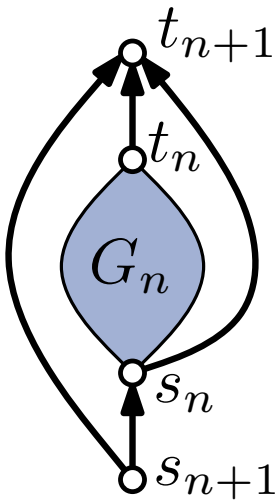

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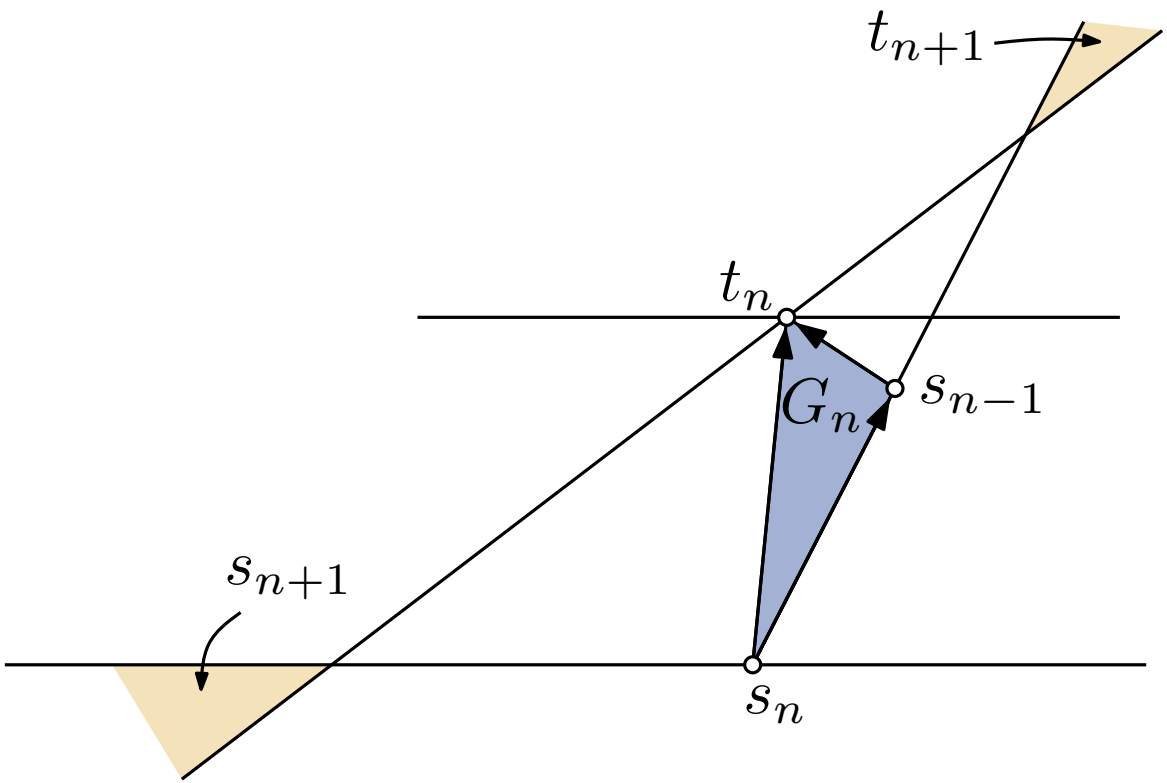

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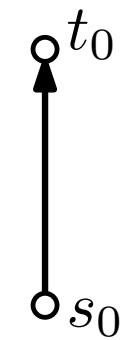

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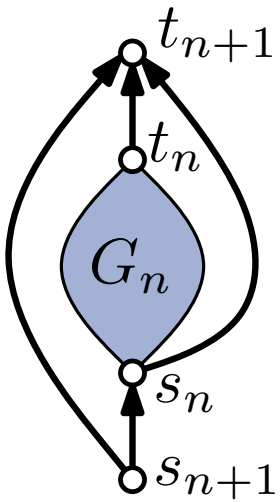

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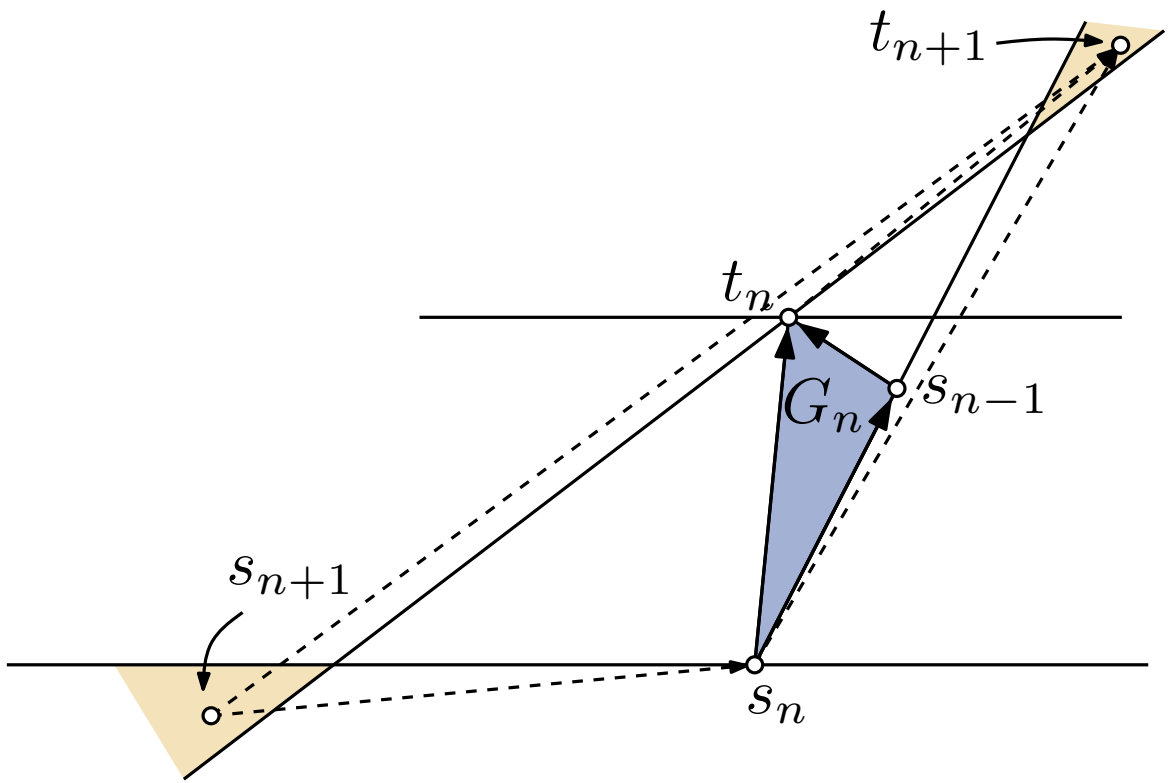

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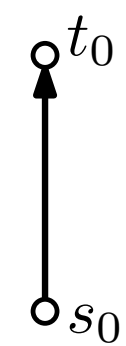

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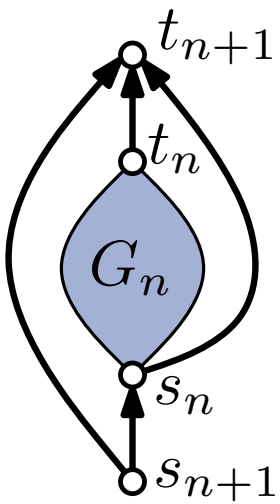

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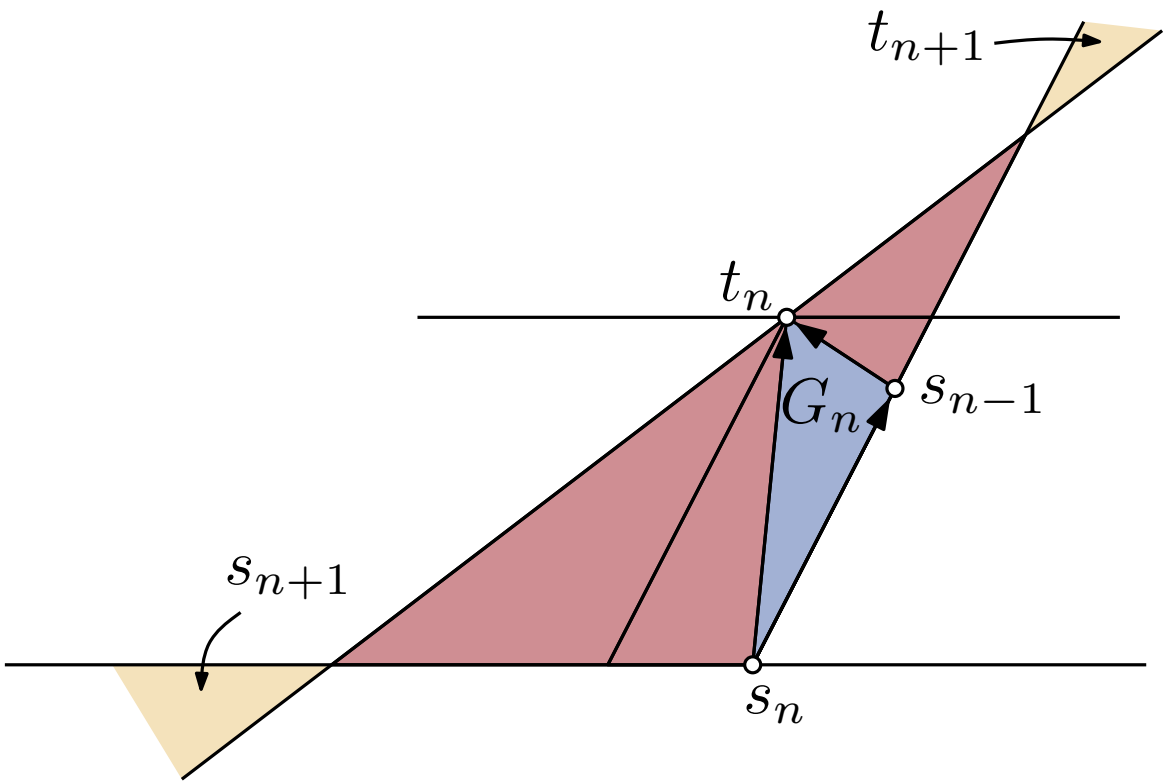

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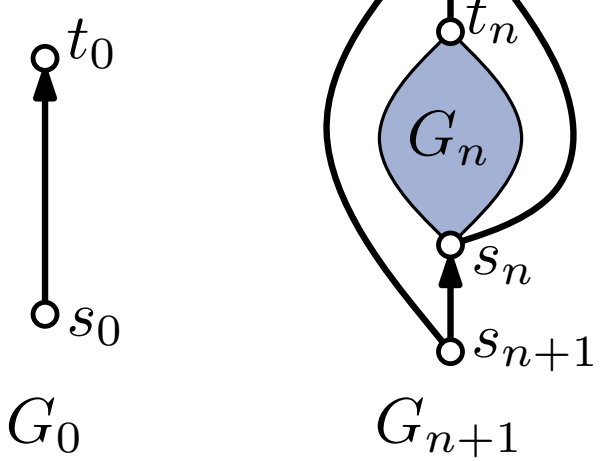

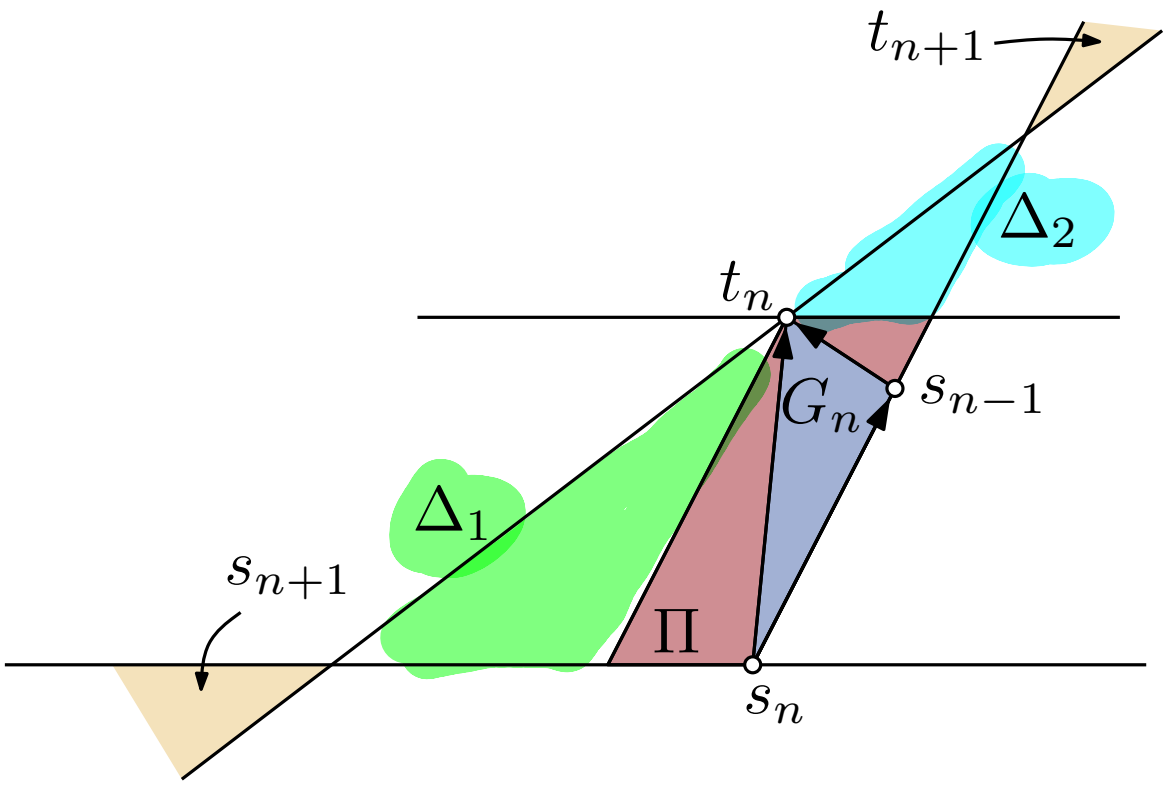

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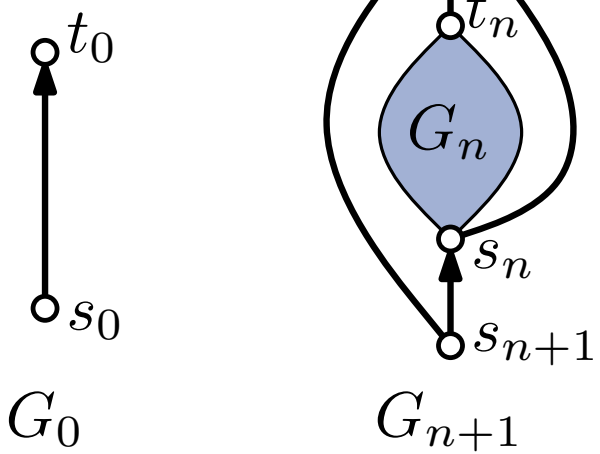

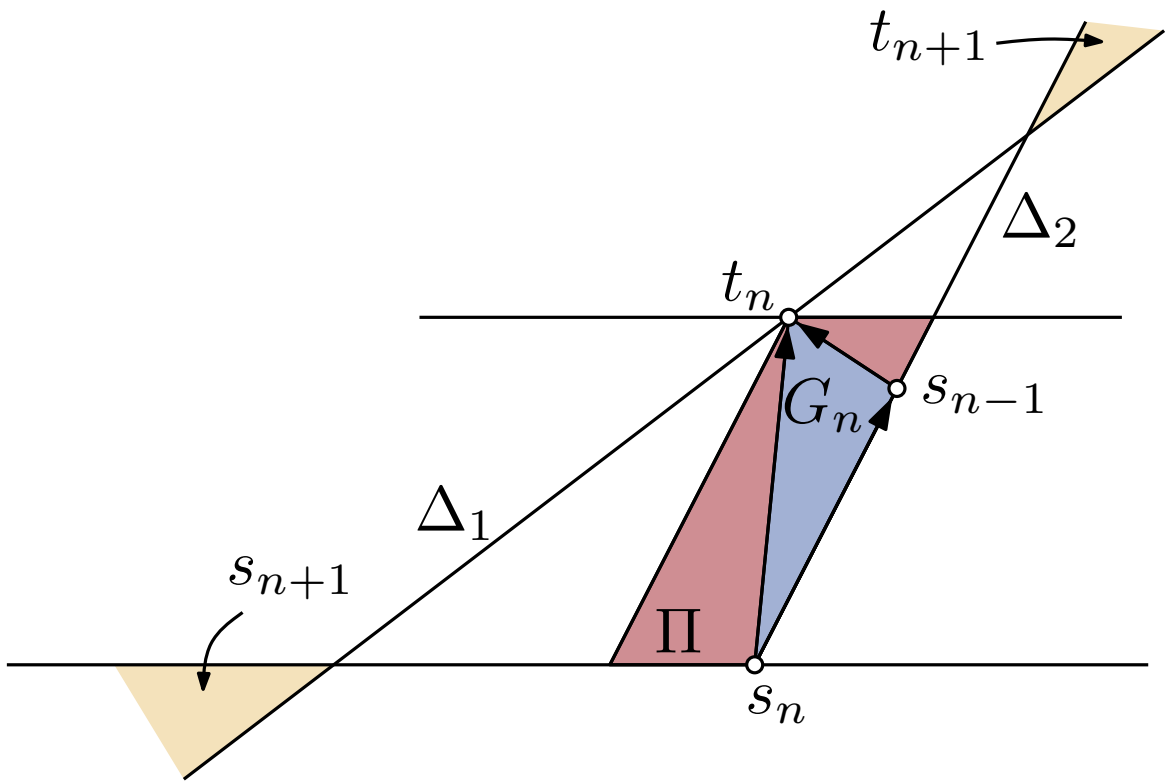

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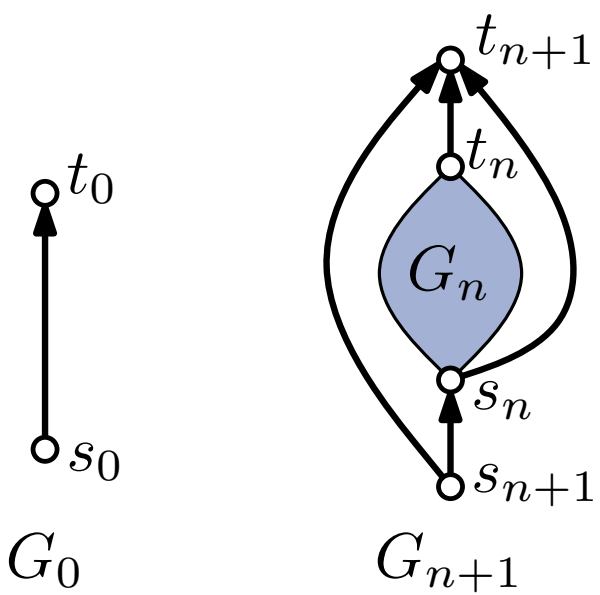

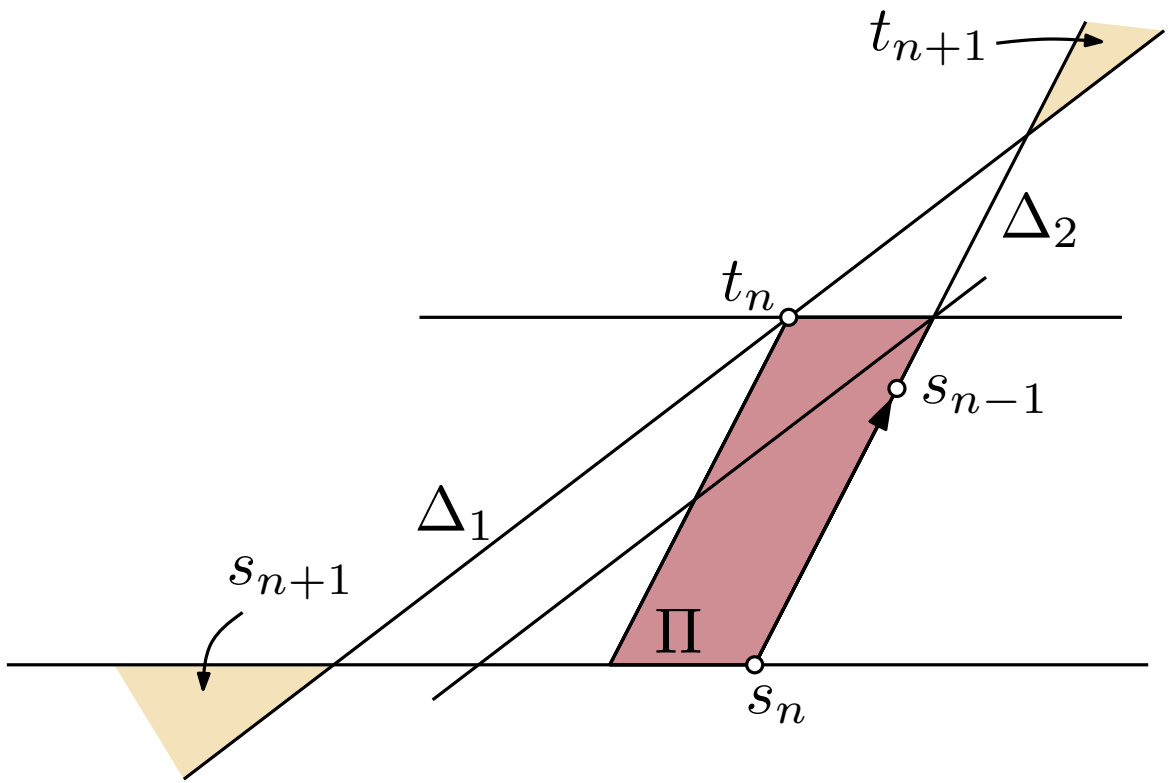

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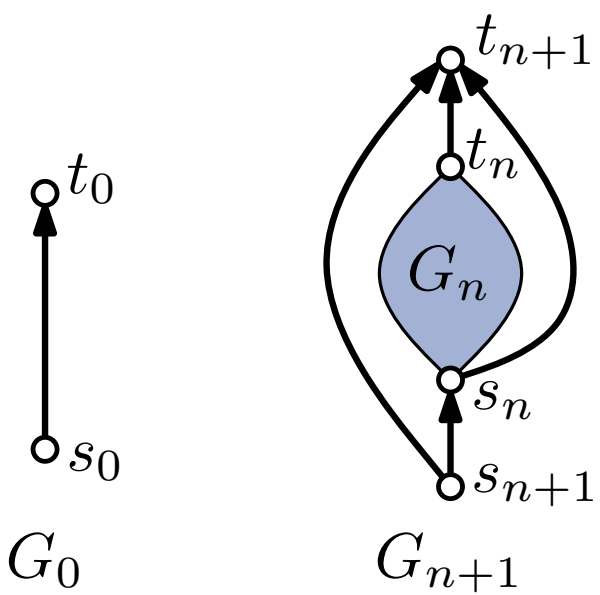

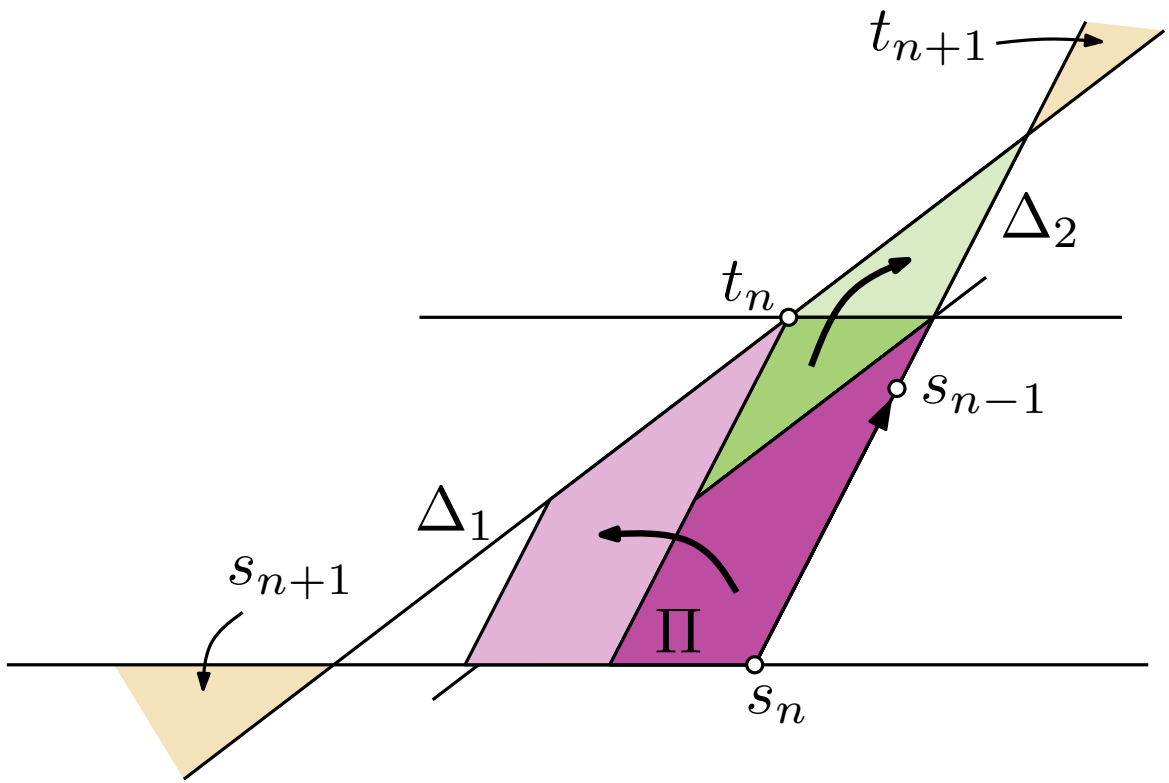

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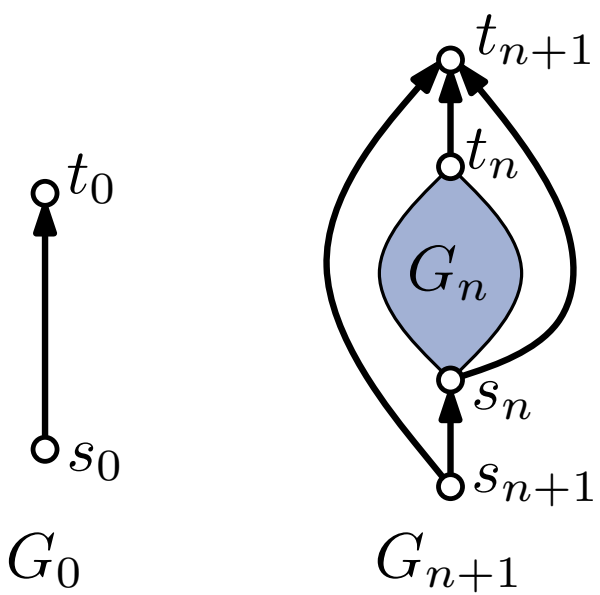
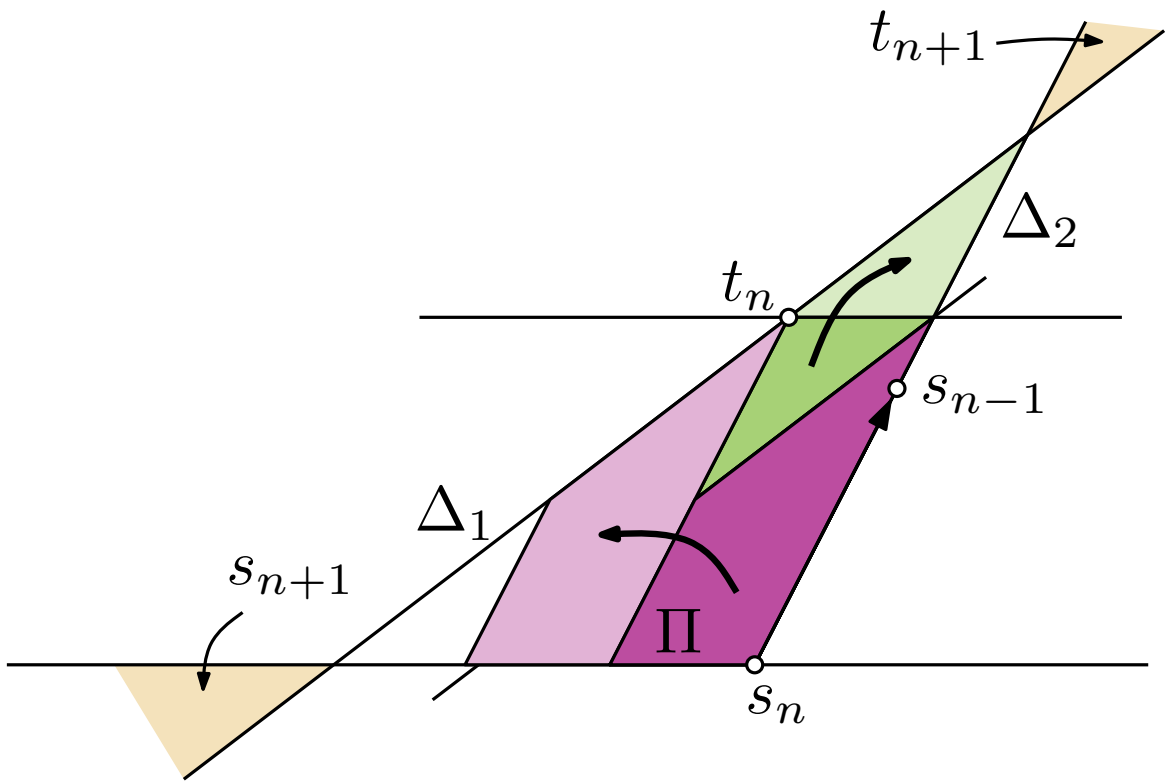




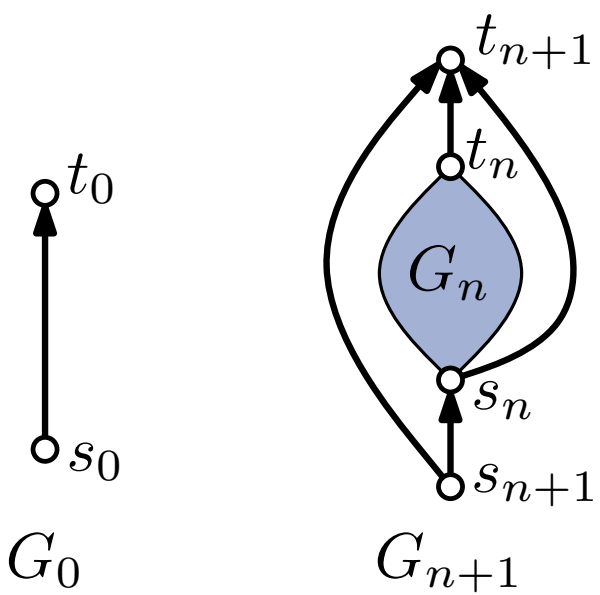
we have: $a(G_{n+1}) \geq a(\Pi) + a(\Delta_1) + a(\Delta_2)$ and $a(G_n) \leq 1/2 \cdot a(\Pi)$

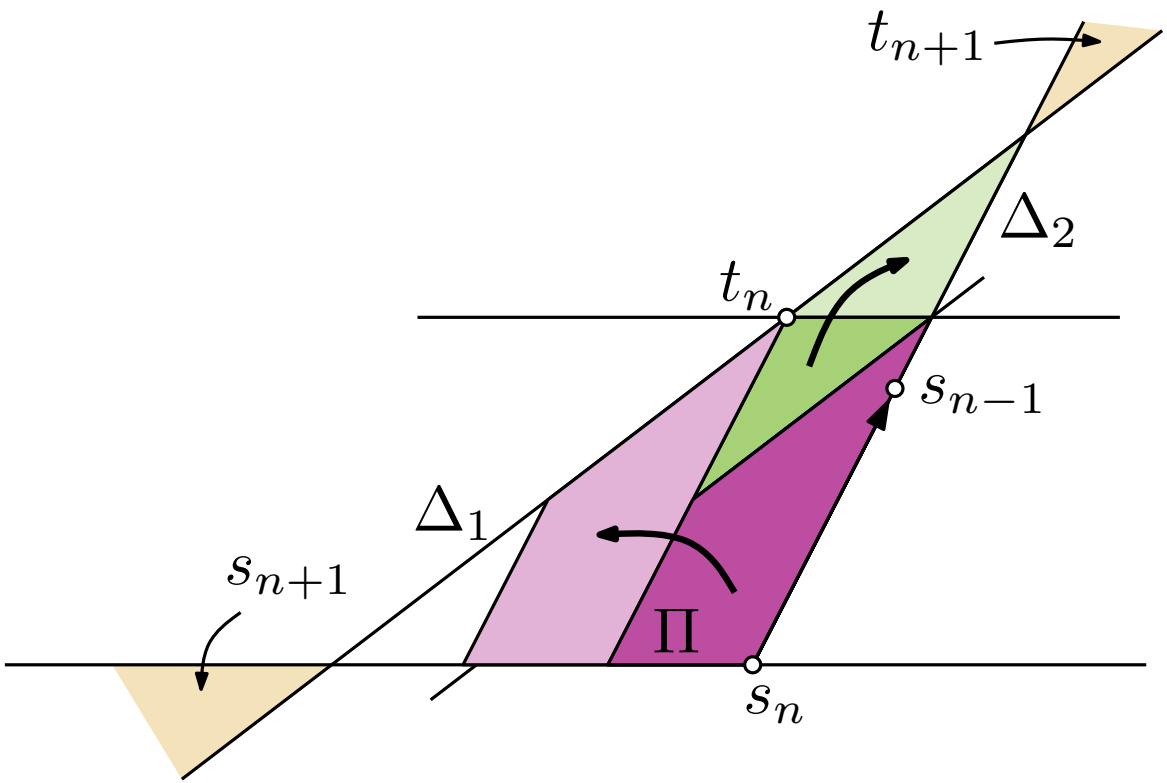
further: $a(\Delta_1) + a(\Delta_2) \geq a(\Pi) \Rightarrow a(G_{n+1}) \geq 2a(\Pi) \geq 4a(G_n)$

Bad News: Exponential Area Lower Bound

Theorem: There exists a family of series-parallel graphs G_n ($n \in \mathbb{N}$) such that any order-preserving upward planar straight-line drawing of G_n requires area $\Omega(4^n)$. [Bertolazzi et al. '94]

Proof (sketch):





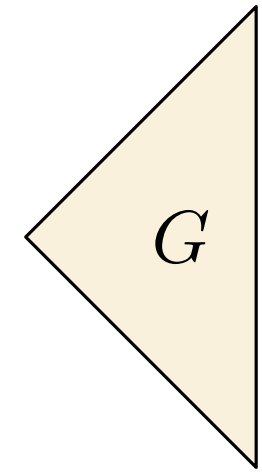
we have: $a(G_{n+1}) \geq a(\Pi) + a(\Delta_1) + a(\Delta_2)$ and $a(G_n) \leq 1/2 \cdot a(\Pi)$

further: $a(\Delta_1) + a(\Delta_2) \geq a(\Pi) \Rightarrow a(G_{n+1}) \geq 2a(\Pi) \geq 4a(G_n)$ □

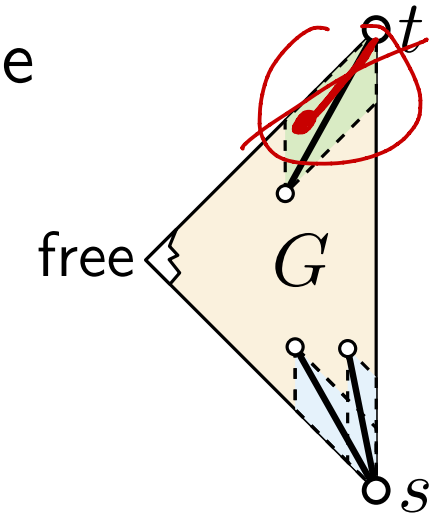
Good News: Polynomial-Area Drawings

Divide & conquer algorithm using decomposition tree

- draw G in an isosceles, right-angled triangle Δ



Good News: Polynomial-Area Drawings



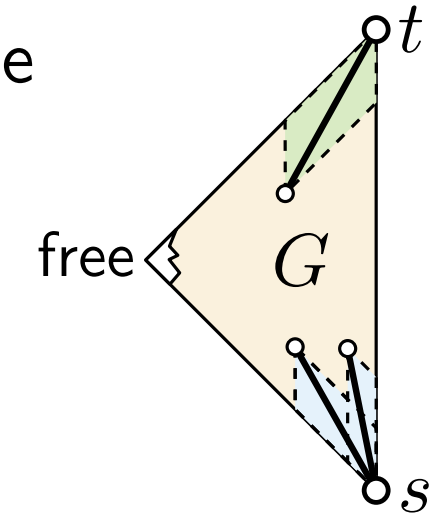
Divide & conquer algorithm using decomposition tree

- draw G in an isosceles, right-angled triangle Δ

Invariants:

- s and t in lower/upper corner of Δ
- left corner of Δ remains free (no vertex)
- for each neighbor v of s or t, the triangle-aligned parallelogram from v contains no vertex (except s or t)

Good News: Polynomial-Area Drawings



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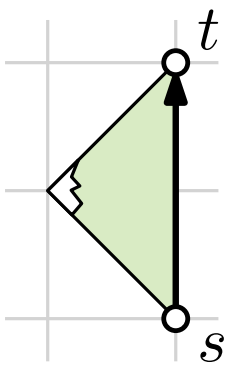
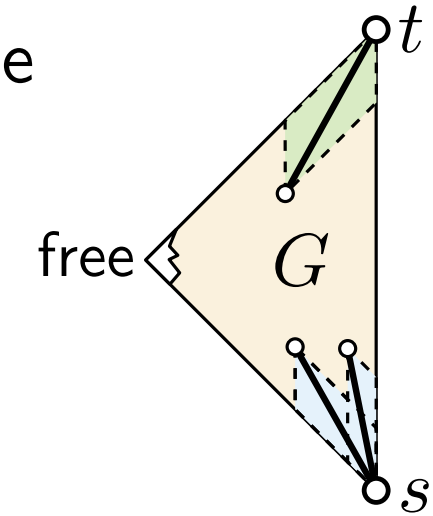


Q-node

Good News: Polynomial-Area Drawings

Divide & conquer algorithm using decomposition tree

- draw G in an isosceles, right-angled triangle Δ



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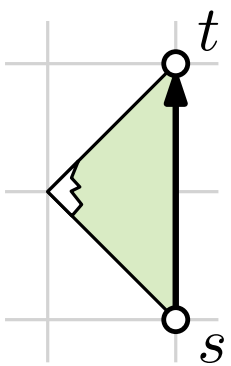
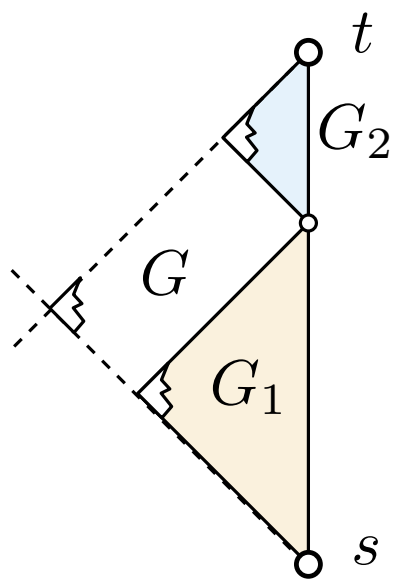


Q-node

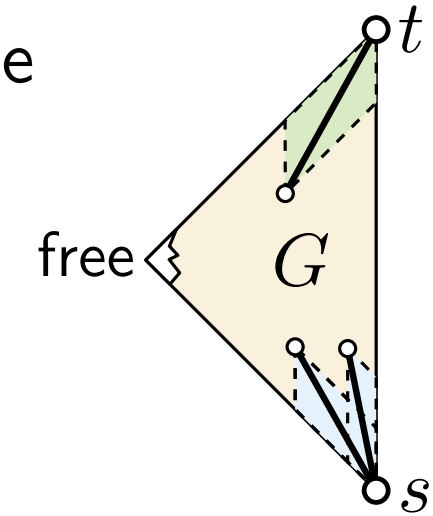


S-node

Good News: Polynomial-Area Drawings

Divide & conquer algorithm using decomposition tree

- draw G in an isosceles, right-angled triangle Δ



Invariants:

- s and t in lower/upper corner of Δ
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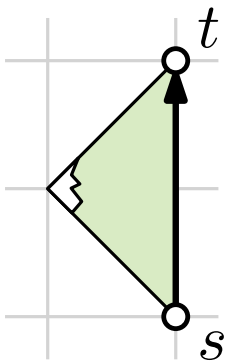
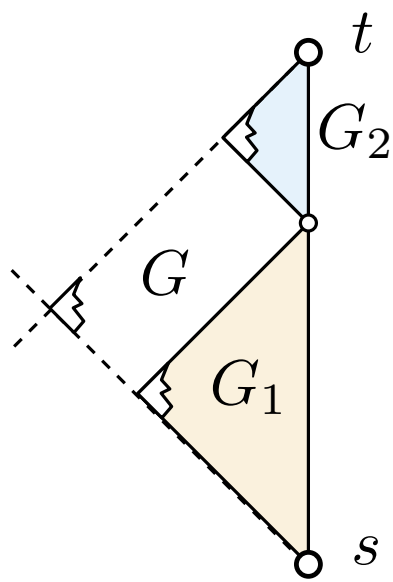
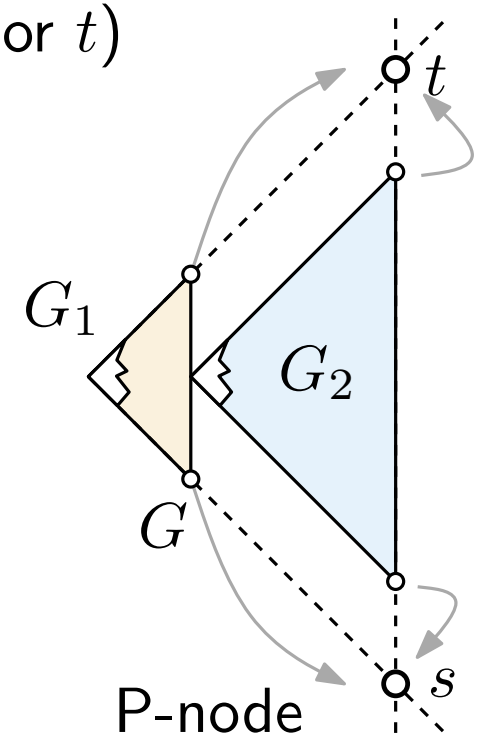


Q-node



S-node

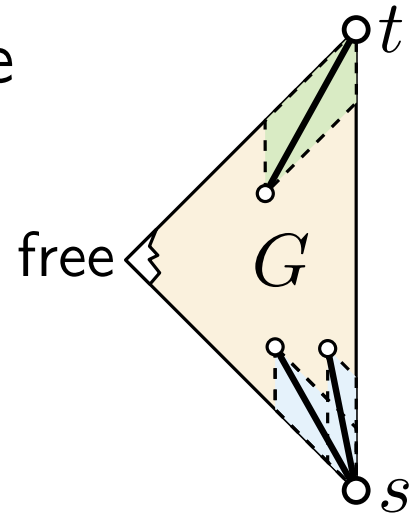


P-node

Good News: Polynomial-Area Drawings

Divide & conquer algorithm using decomposition tree

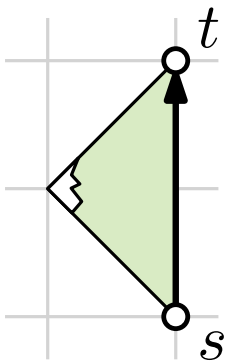
- draw G in an isosceles, right-angled triangle Δ



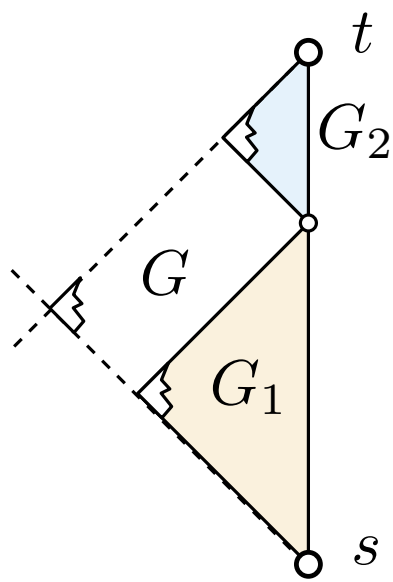
- simple graphs only (no multi-edges)

Invariants:

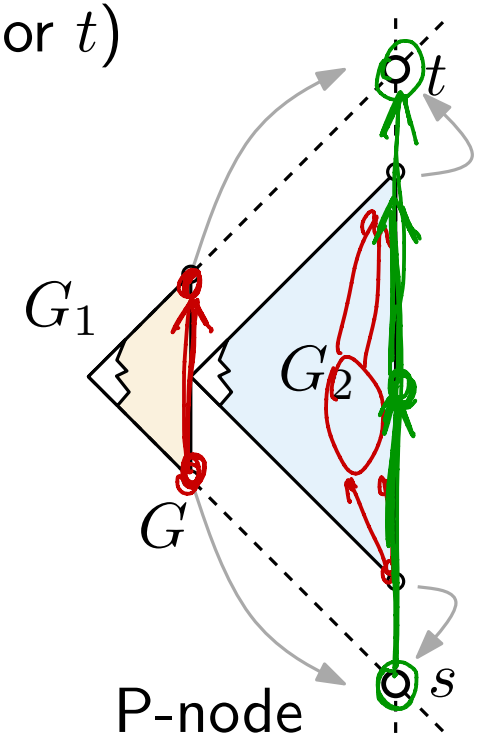
- s and t in lower/upper corner of Δ
- left corner of Δ remains free (no vertex)
- for each neighbor v of s or t, the triangle-aligned parallelogram from v contains no vertex (except s or t)



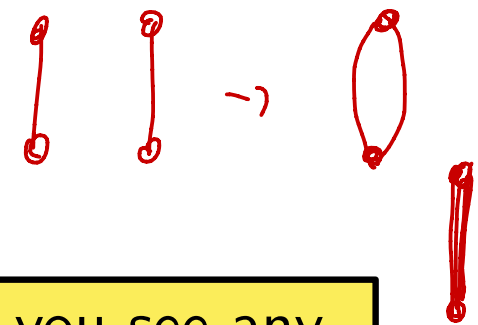
Q-node



S-node



P-node

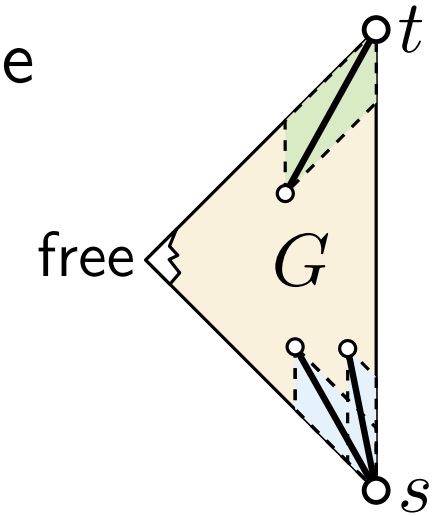


Do you see any problems?

Good News: Polynomial-Area Drawings

Divide & conquer algorithm using decomposition tree

- draw G in an isosceles, right-angled triangle Δ



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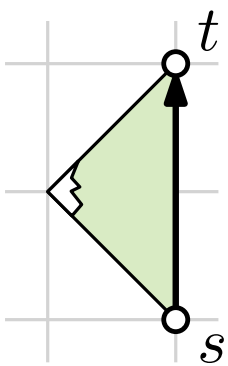
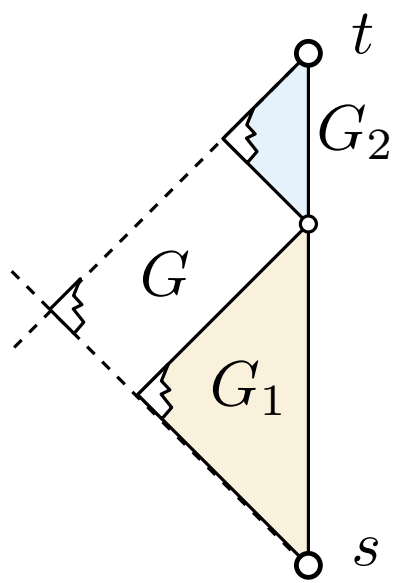
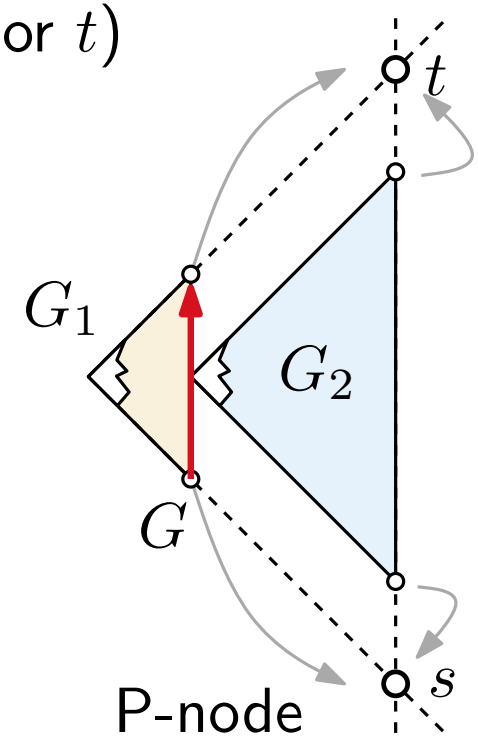


Q-node



S-node



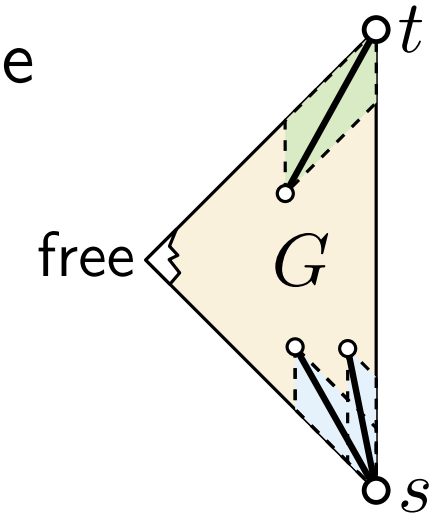
P-node

- in P-composition, G_1 cannot be a Q-node
- change subtree order (embedding) to have Q-nodes rightmost

Good News: Polynomial-Area Drawings

Divide & conquer algorithm using decomposition tree

- draw G in an isosceles, right-angled triangle Δ



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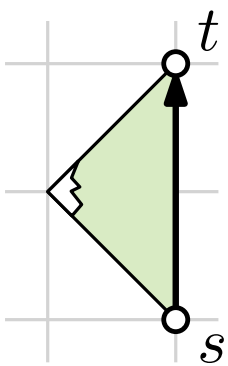
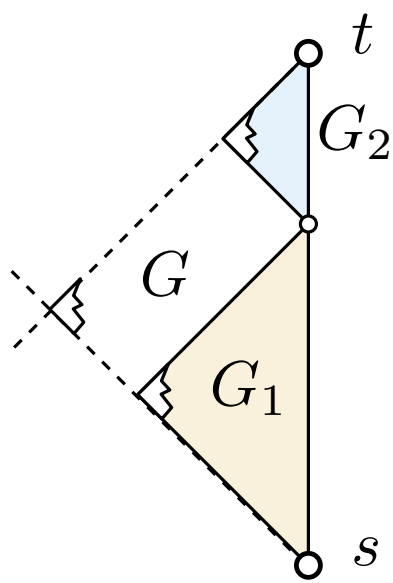
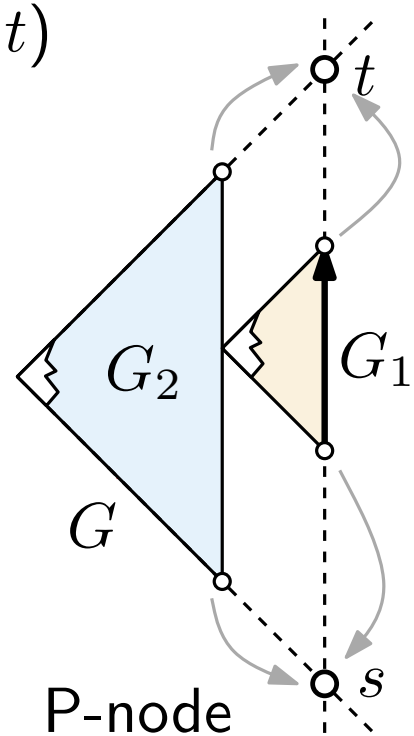


Q-node



S-node



P-node

- in P-composition, G_1 cannot be a Q-node
- change subtree order (embedding) to have Q-nodes rightmost

→ **right-pushed** drawing

Properties of Right-Pushed Drawings

Verify Invariants, planarity, and compute area:

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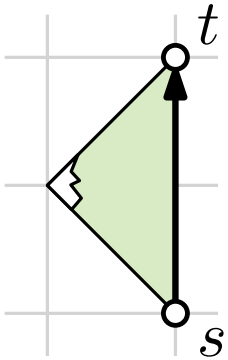
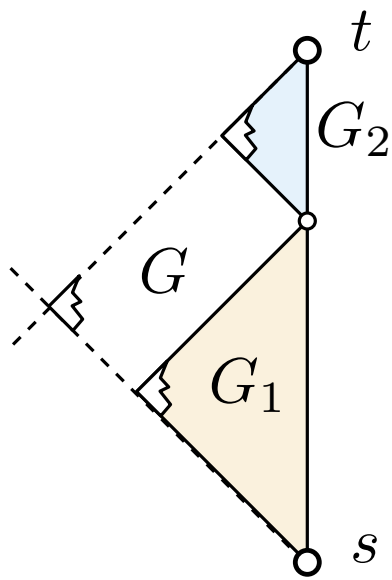
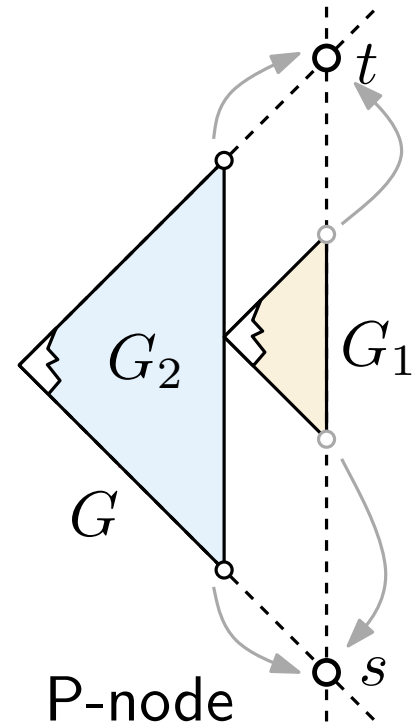


Q-node



S-node



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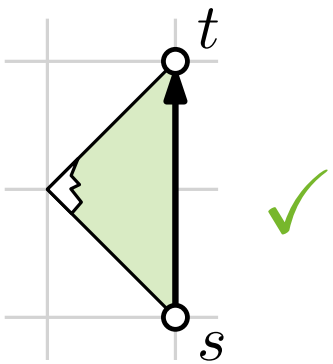
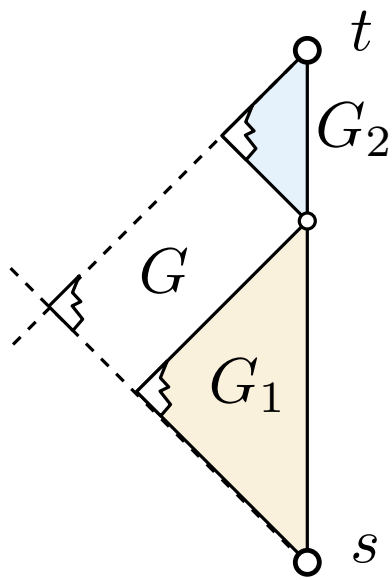
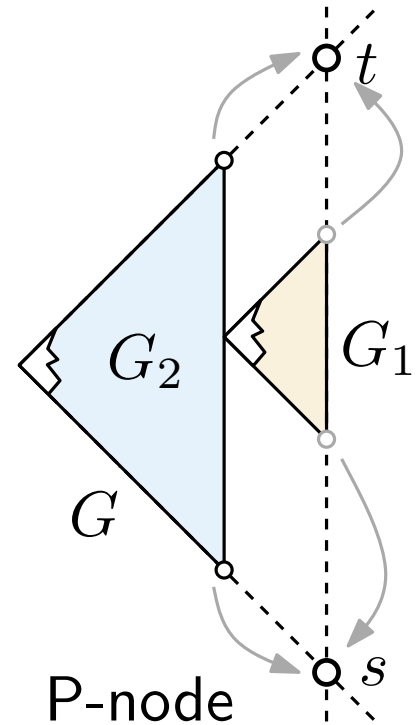


Q-node



S-node



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Properties of Right-Pushed Drawings

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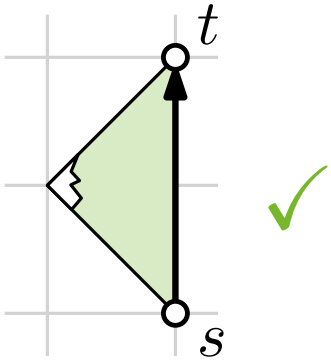
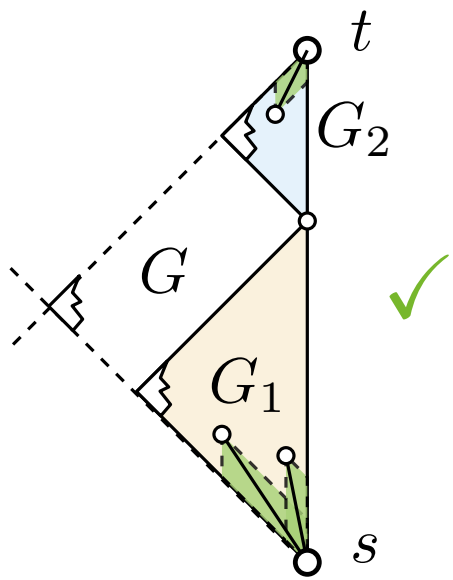
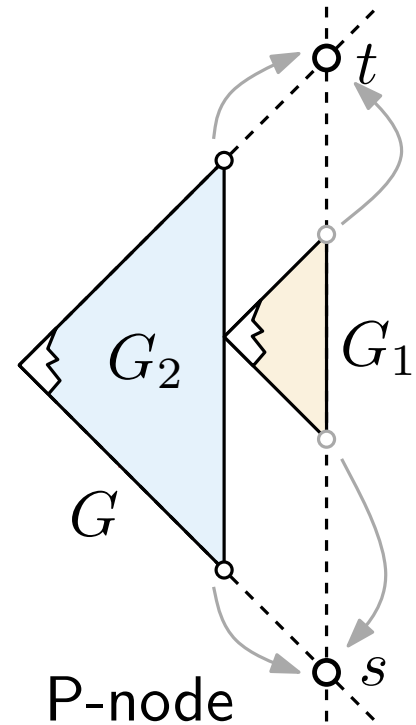


Q-node



S-node



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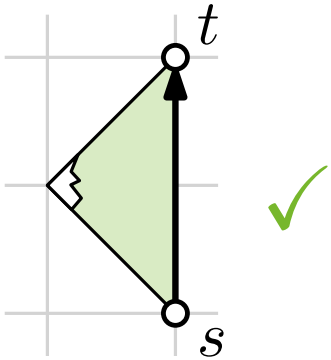
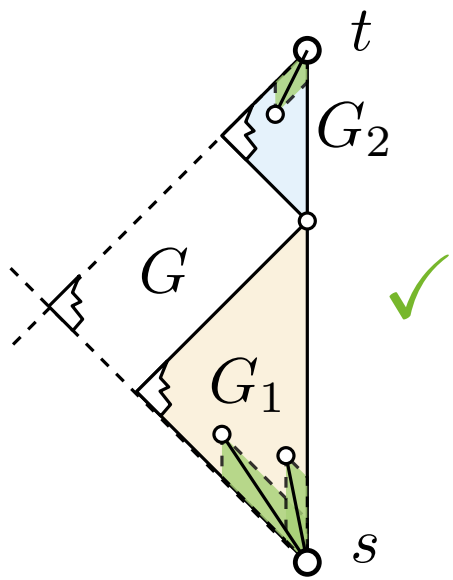
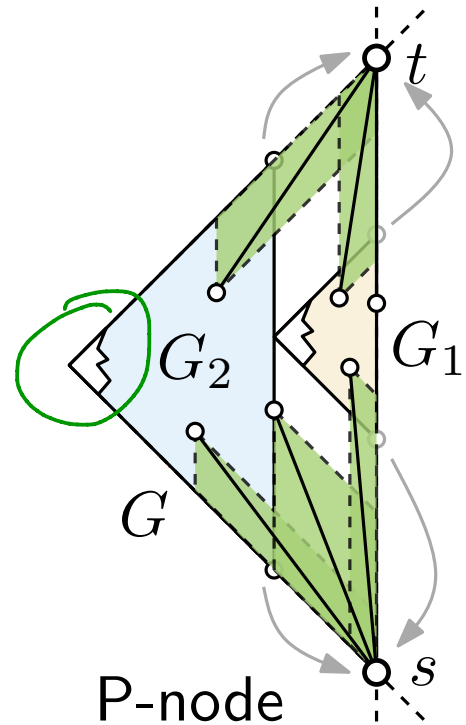


Q-node



S-node



P-node

Properties of Right-Pushed Drawings

Verify Invariants, planarity, and compute area:

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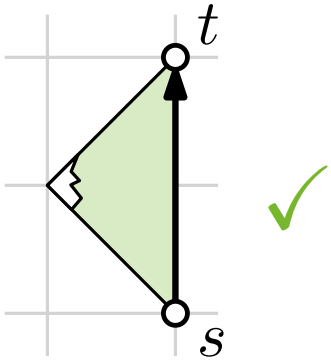
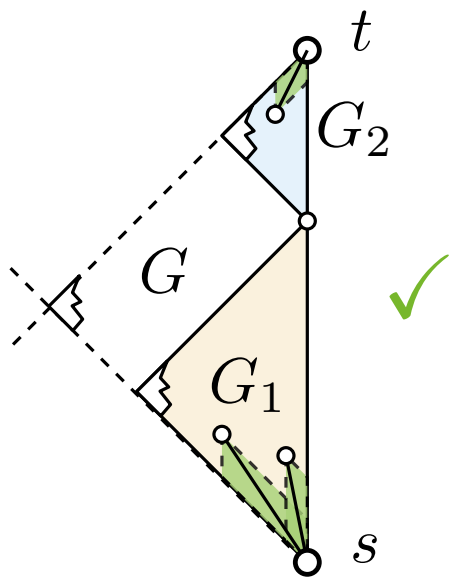
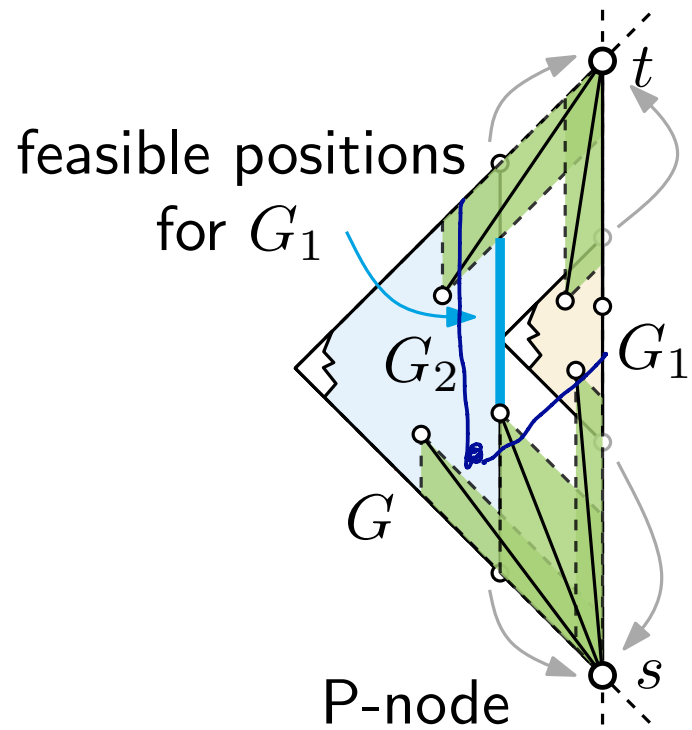


Q-node



S-node

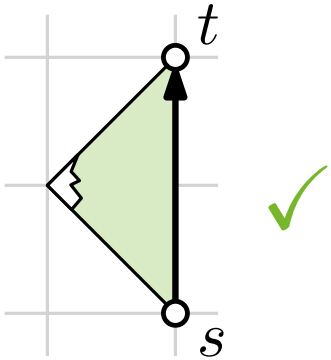


P-node

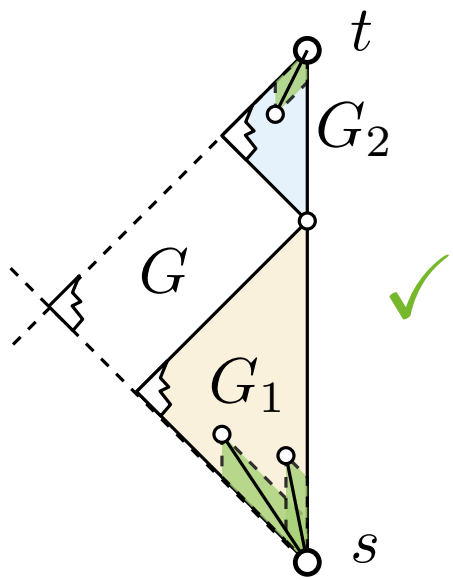
Properties of Right-Pushed Drawings

Verify Invariants, planarity, and compute area:

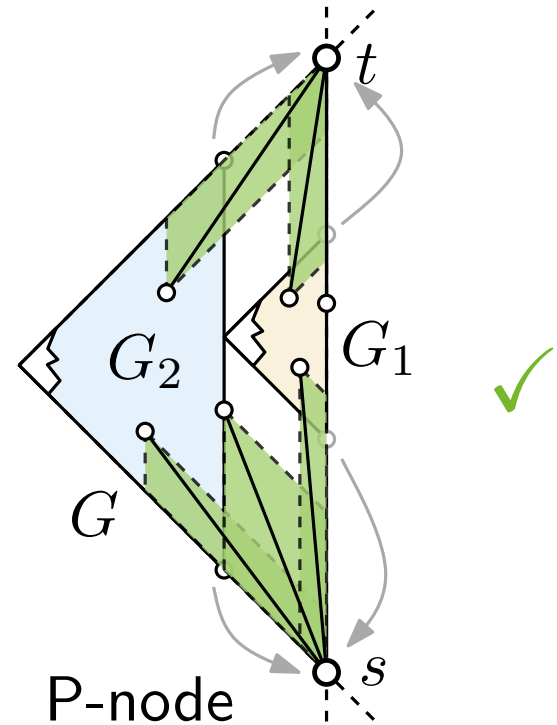
- s and t in lower/upper corner of Δ
- left corner of Δ remains free (no vertex)
- for each neighbor v of s or t, the triangle-aligned parallelogram from v contains no vertex (except s or t)



Q-node



S-node



P-node

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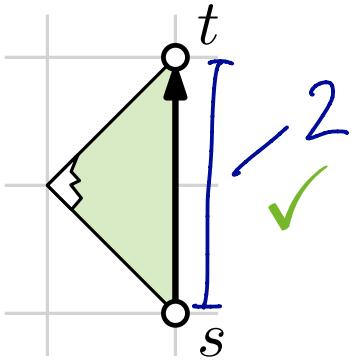
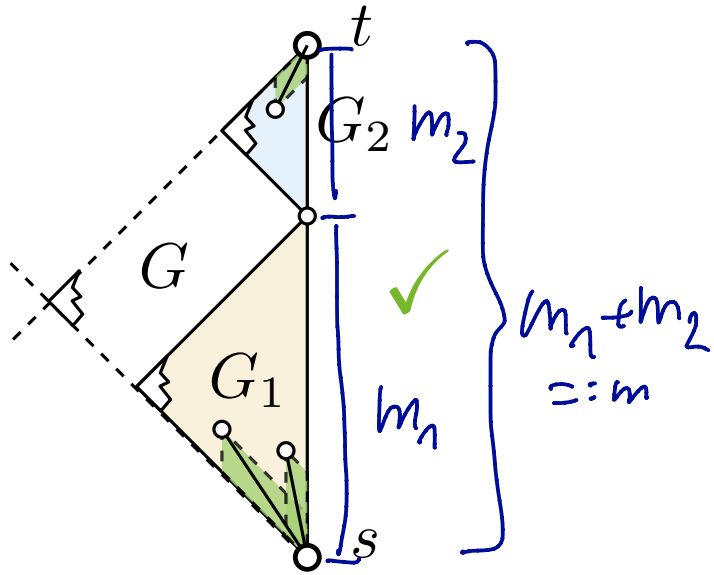


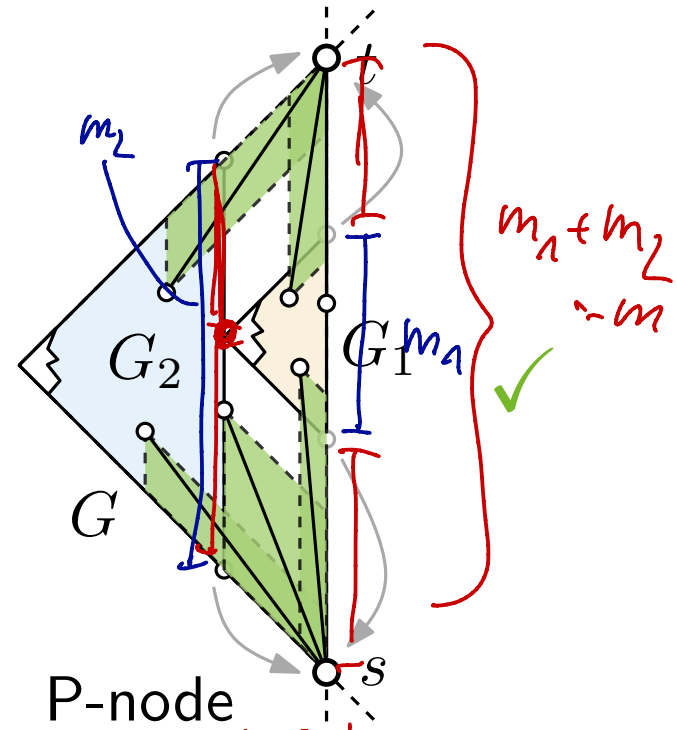
Q-node

area: 1



S-node

area: $O(m^2)$



P-node

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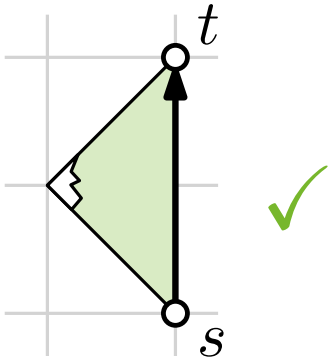
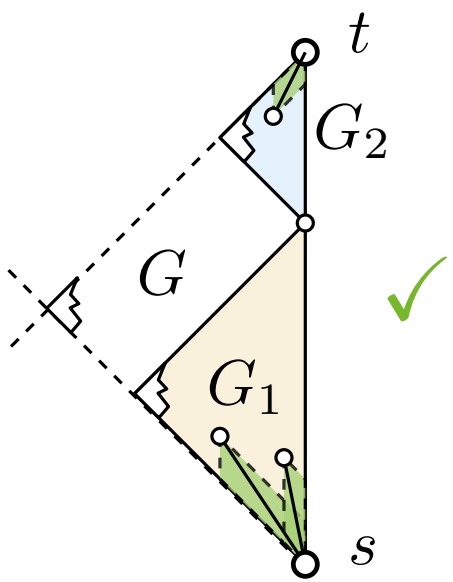
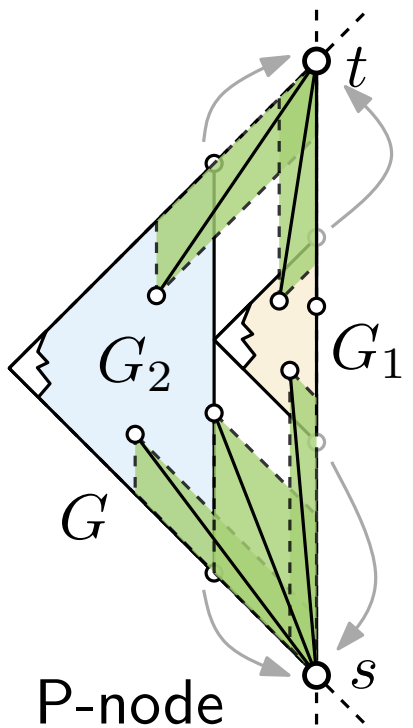


Q-node



S-node



P-node

Theorem: A simple, series-parallel graph G with m edges admits an (unordered) right-pushed upward planar drawing of area $O(m^2)$.

Summary

given: graph $G = (V, E)$
find: drawing Γ of G that

- complies with the given drawing conventions
- optimizes the given aesthetics
- satisfies the partial/local constraints

$\rightarrow$ often lead to NP-hard optimization problems!
$\rightarrow$ often several competing criteria

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today:

- series-parallel (directed) graphs
- planar, straight-line upward drawings
- optimization goal: compactness/small area
- exponential lower bound for ordered (embedded) graphs
- divide & conquer algorithm for unordered graphs: $O(m^2)$ area
- competing: small area vs. ordered/preserve embedding
- *further reading:* maximize displayed symmetries via upward planar automorphism groups [Hong et al. '00]