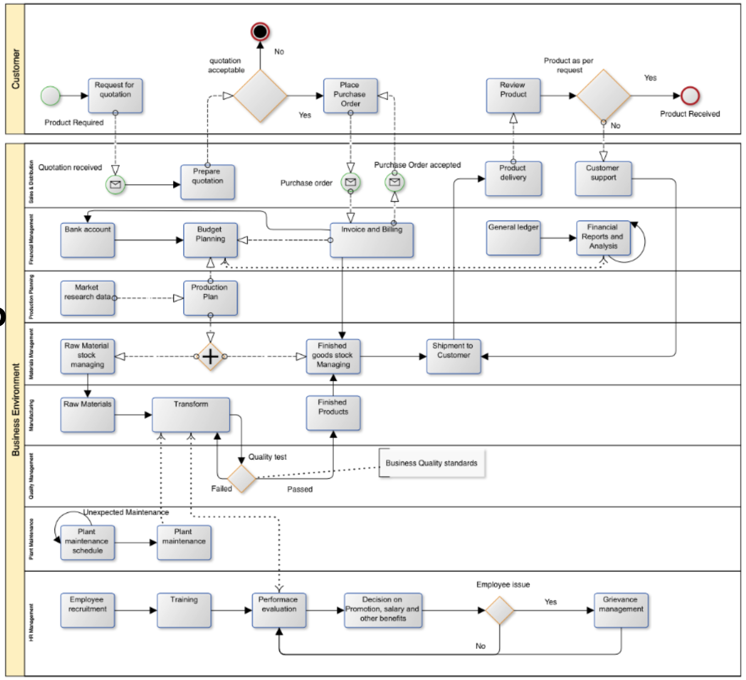
Orthogonal Graph Drawing

Lecture Graph Drawing Algorithms · 192.053

Martin Nöllenburg

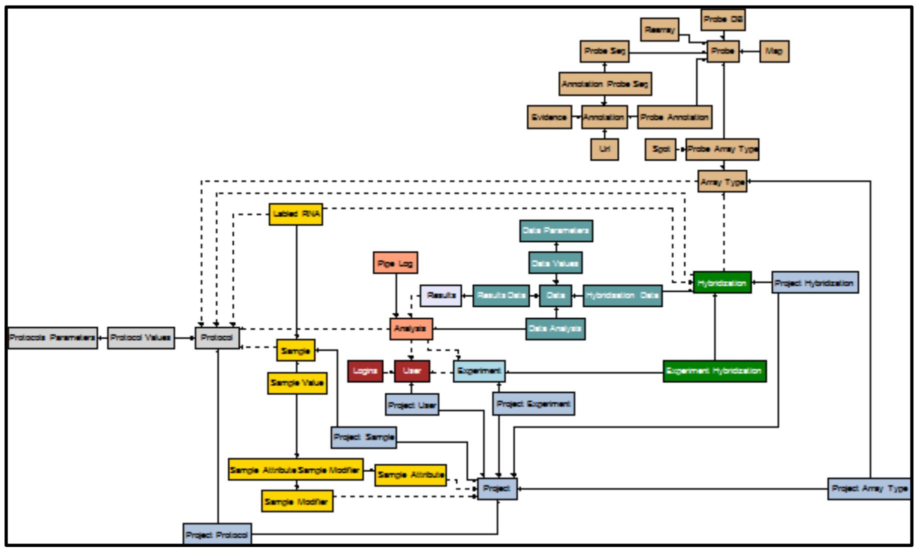
05.06.2018

ALGORITHMS AND COMPLEXITY GROUP

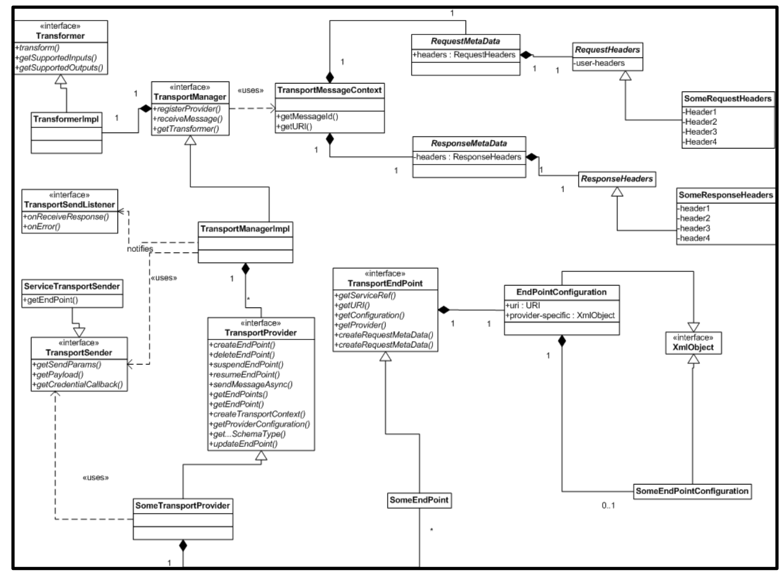


Orthogonal Drawings

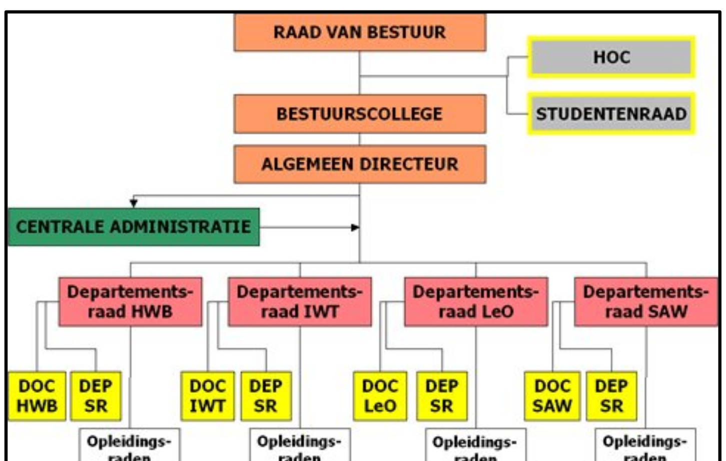
- Edges consist of vertical and horizontal segments
- Many applications due to clean and schematic appearance



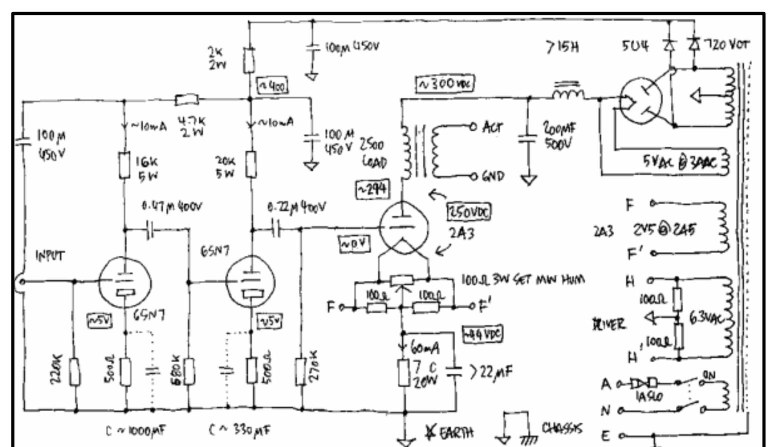
ER diagram in OGDF



UML diagram by Oracle



Organigram of HS Limburg



Circuit diagram by Jeff Atwood

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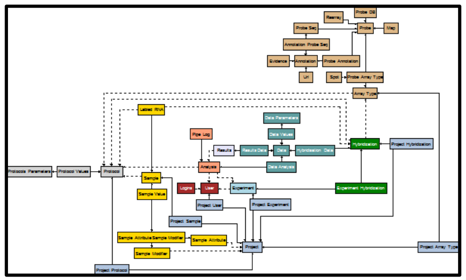

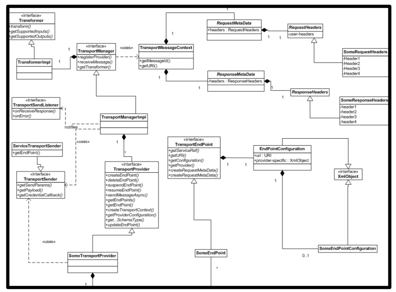

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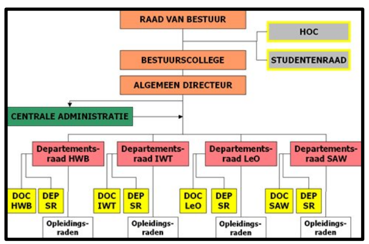
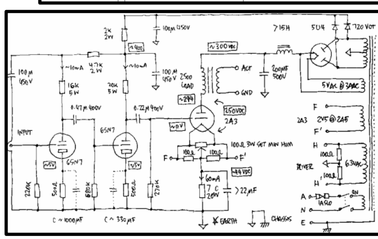
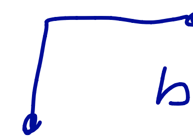




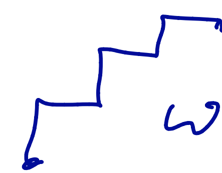
Circuit diagram by Jeff Atwood

What makes a good orthogonal drawing?

- bend minimization
- small edge length
- compact area



better



worse

(Planar) Orthogonal Drawings

Three-step approach: *Topology – Shape – Metrics*

[Tamassia SIAM J. Comput. 1987]

$V = \{v_1, v_2, v_3, v_4\}$
$E = \{v_1v_2, v_1v_3, v_1v_4, v_2v_3, v_2v_4\}$

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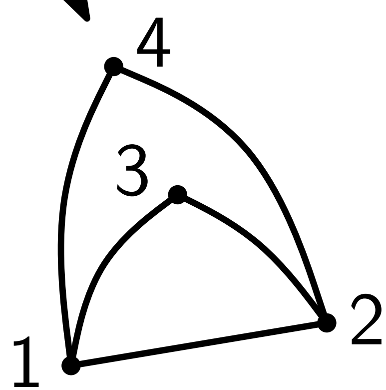
Reduce crossings

combinatorial embedding/ planarization



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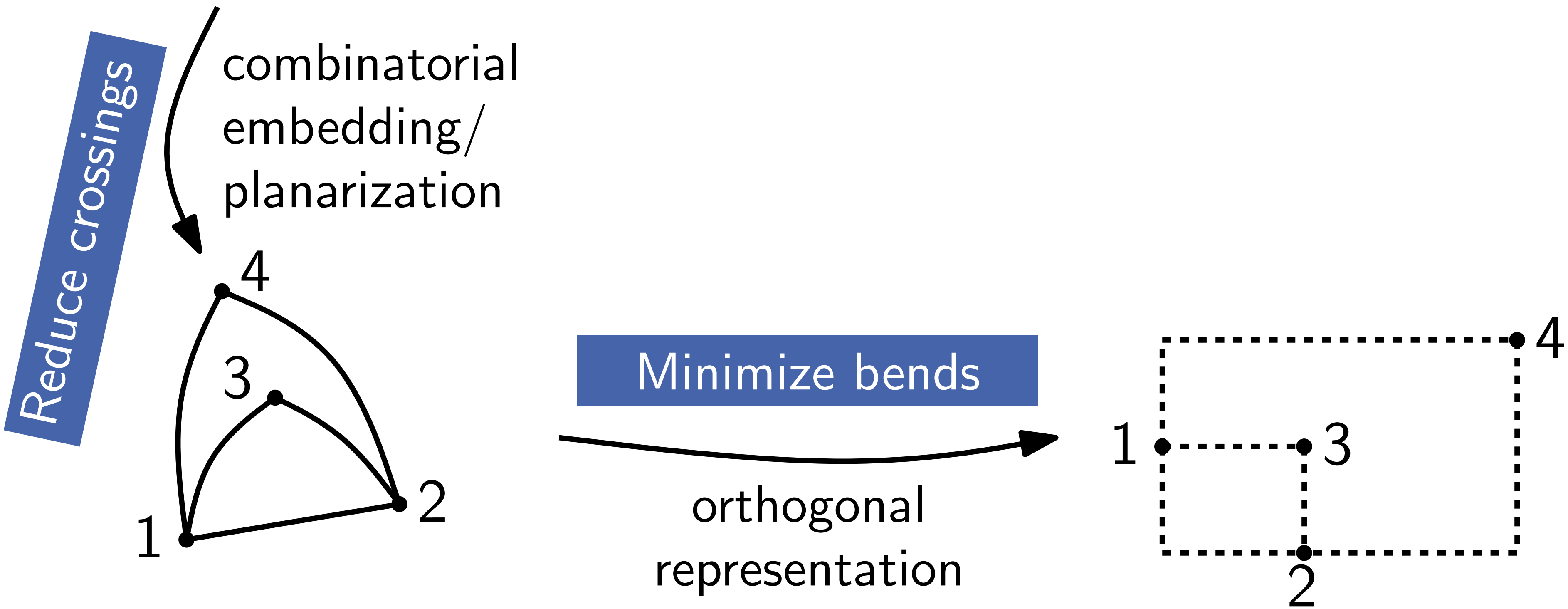

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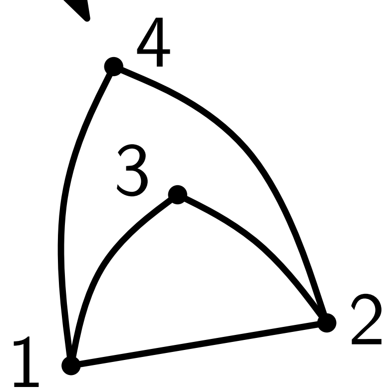
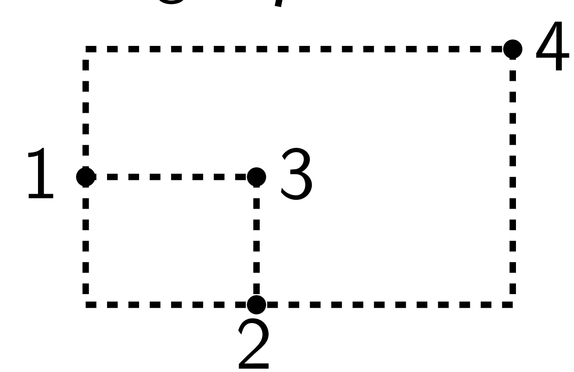


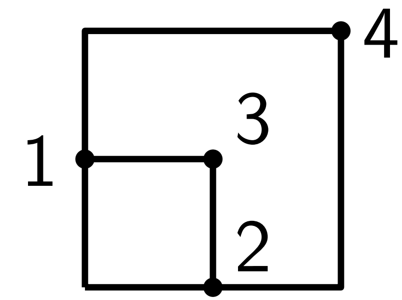
Minimize bends

orthogonal representation



planar orthogonal drawing

Minimize area



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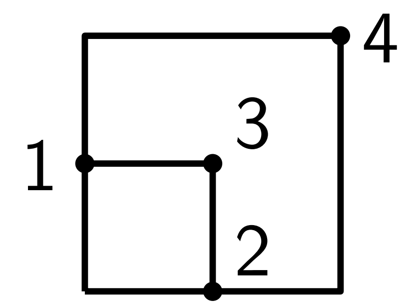
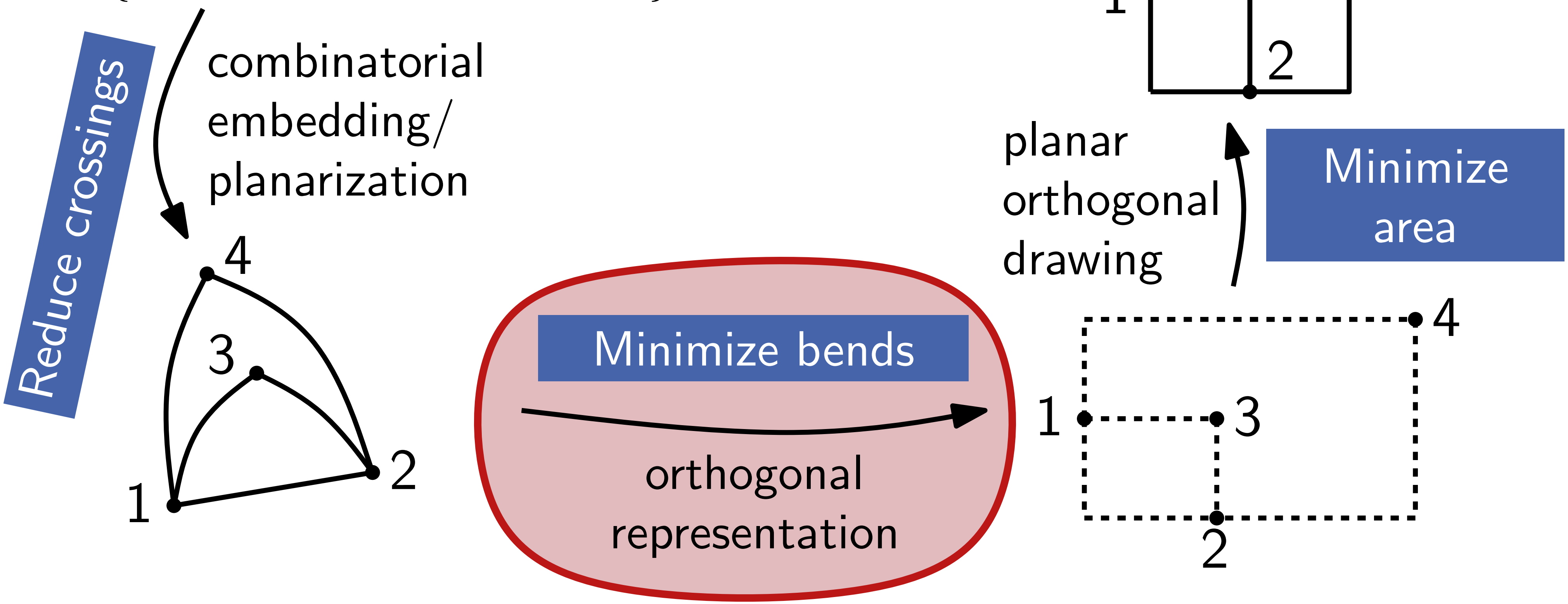
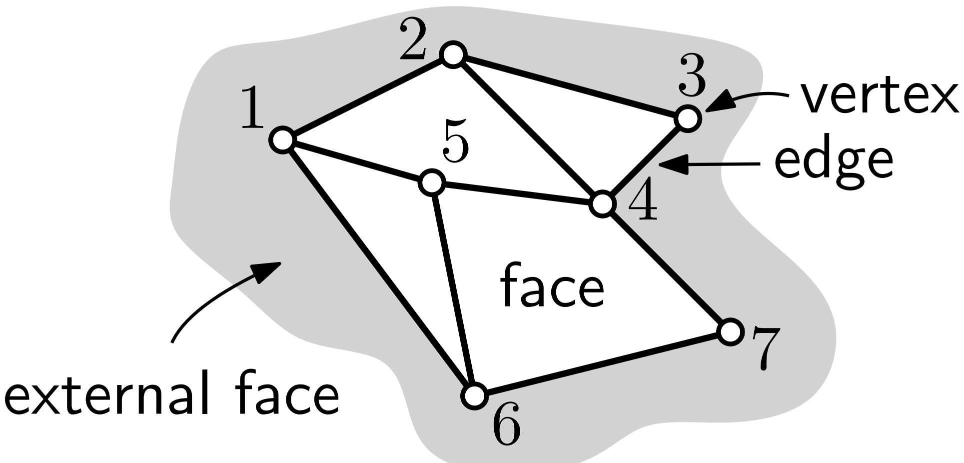




Recall: Planar Graphs

Planar graphs are an important graph class in graph drawing and graph theory.

Def: A **planar graph** G is a simple graph that can be drawn/ embedded in the plane $\mathbb{R}^2$ without edge crossings.

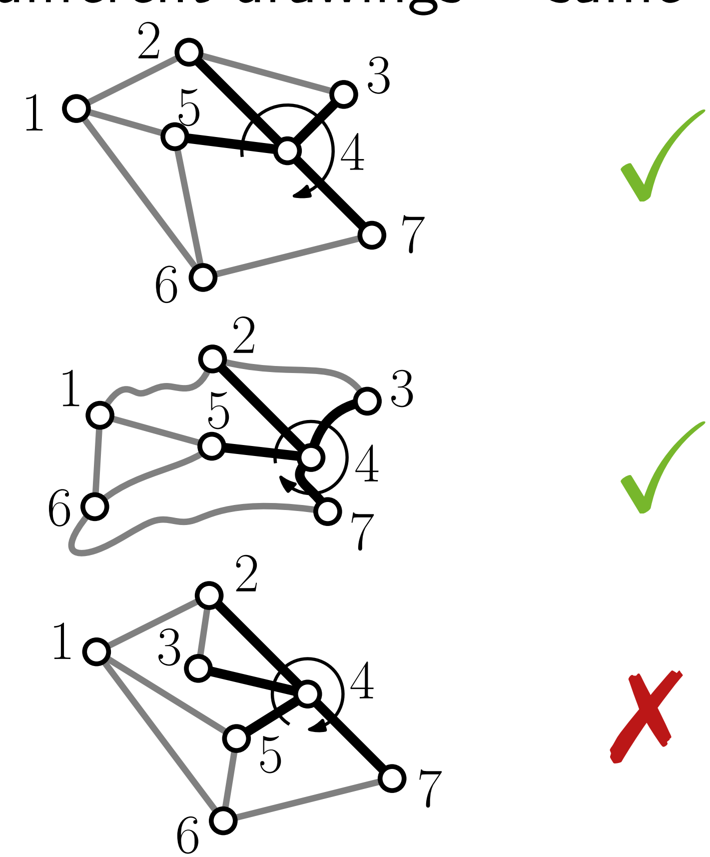


Def: The **rotation scheme** of a planar drawing is the circular ordering of the edges incident to each vertex.

Two planar drawings have the same embedding if they have the same rotation scheme and the same external face.

different drawings

same embedding?

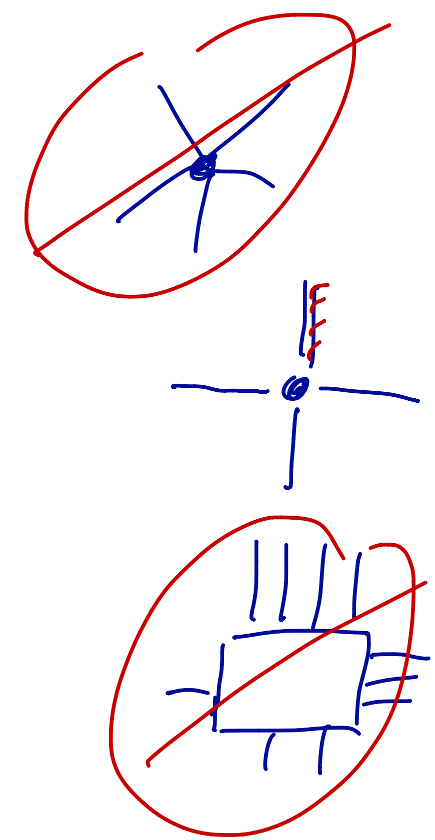


Bend Minimization with Fixed Embedding

Problem: Geometric Bend Minimization

given:
- planar graph $G = (V, E)$ with maximum degree 4
- (combinatorial) embedding $\mathcal{F}$ with external face f_0

find: orthogonal grid drawing with same embedding and minimum number of bends



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first consider the following

Problem: Combinatorial Bend Minimization

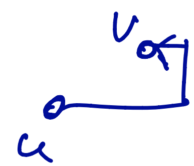
given:
- planar graph $G = (V, E)$ with maximum degree 4
- (combinatorial) embedding $\mathcal{F}$ with external face f_0

find: **orthogonal representation** $H(G)$ with same embedding and minimum number of bends

Orthogonal Representation

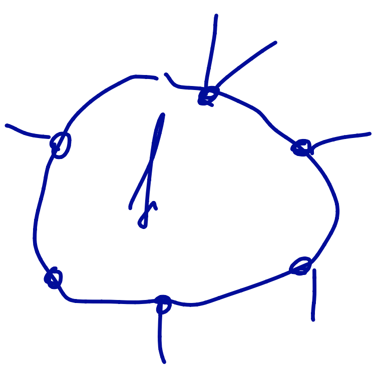
Given: planar Graph $G = (V, E)$, set of faces $\mathcal{F}$, external face f_0
Find: orthogonal representation $H(G) = \{H(f) \mid f \in \mathcal{F}\}$

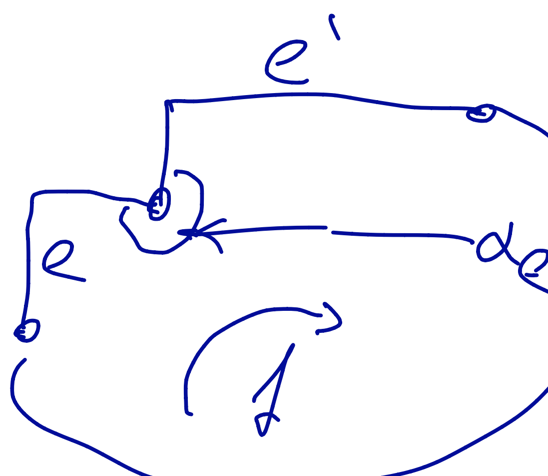
Face representation $H(f)$**:** clockwise ordered sequence of edge descriptions (e, δ, α) with



$\hat{=} \quad \delta = (1,1)$

- e edge of f
- δ is sequence in $\{0,1\}^*$ (0 = right bend, 1 = left bend)
- α is angle $\in \{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$ between e and next edge e'



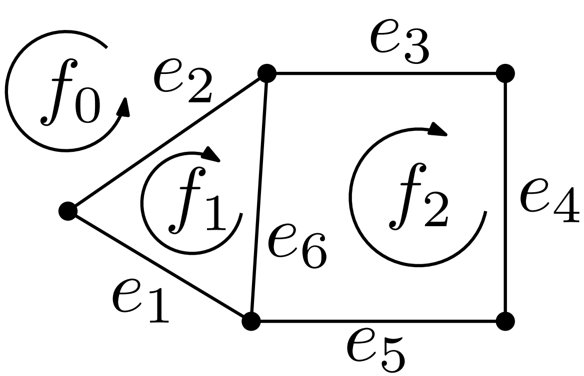


Orthogonal Representation: Example

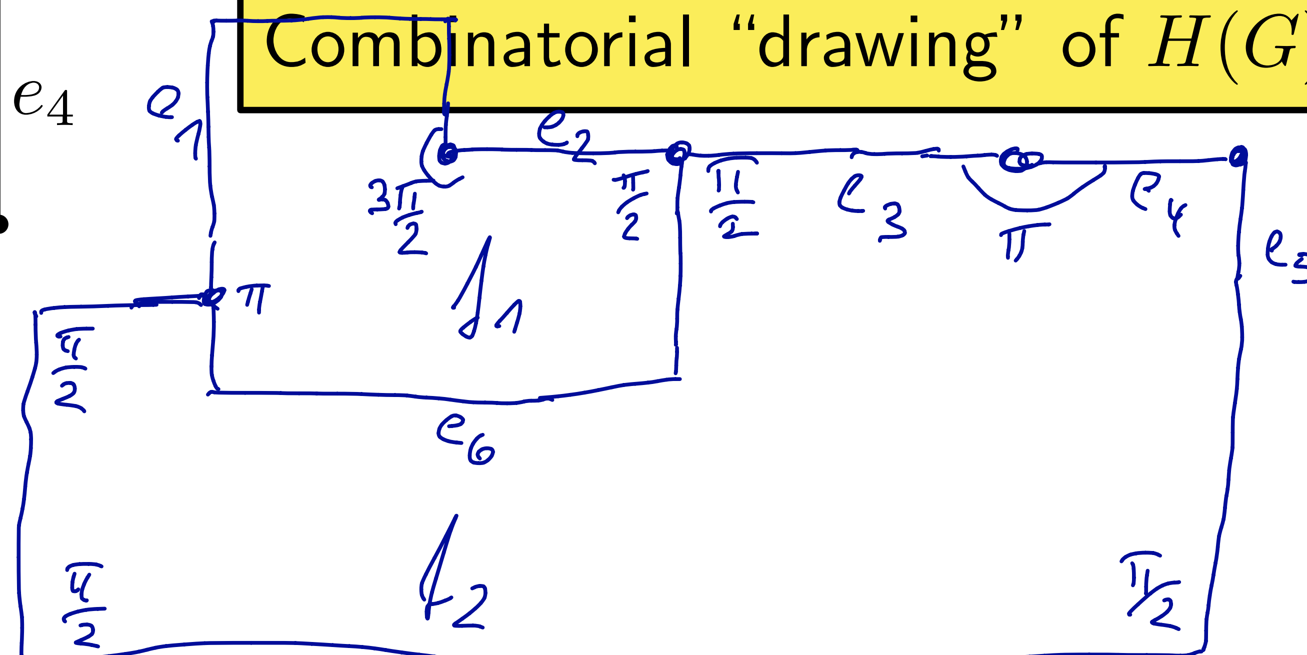
$$H(f_0) = ((e_1, 11, \tfrac{\pi}{2}), (e_5, 111, \tfrac{3\pi}{2}), (e_4, \emptyset, \pi), (e_3, \emptyset, \pi), (e_2, \emptyset, \tfrac{\pi}{2}))$$

$$H(f_1) = ((e_1, 00, \tfrac{3\pi}{2}), (e_2, \emptyset, \tfrac{\pi}{2}), (e_6, 00, \pi))$$

$$H(f_2) = ((e_5, 000, \tfrac{\pi}{2}), (e_6, 11, \tfrac{\pi}{2}), (e_3, \emptyset, \pi), (e_4, \emptyset, \tfrac{\pi}{2}))$$



Combinatorial “drawing” of $H(G)$?



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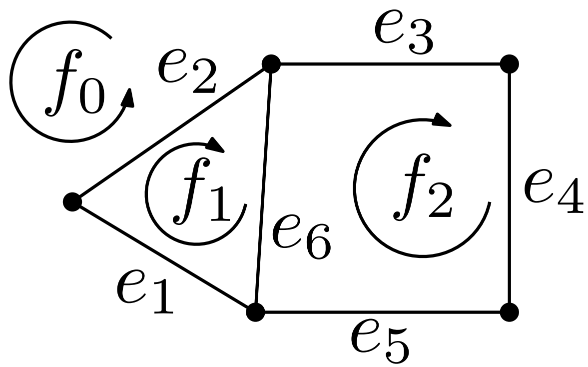

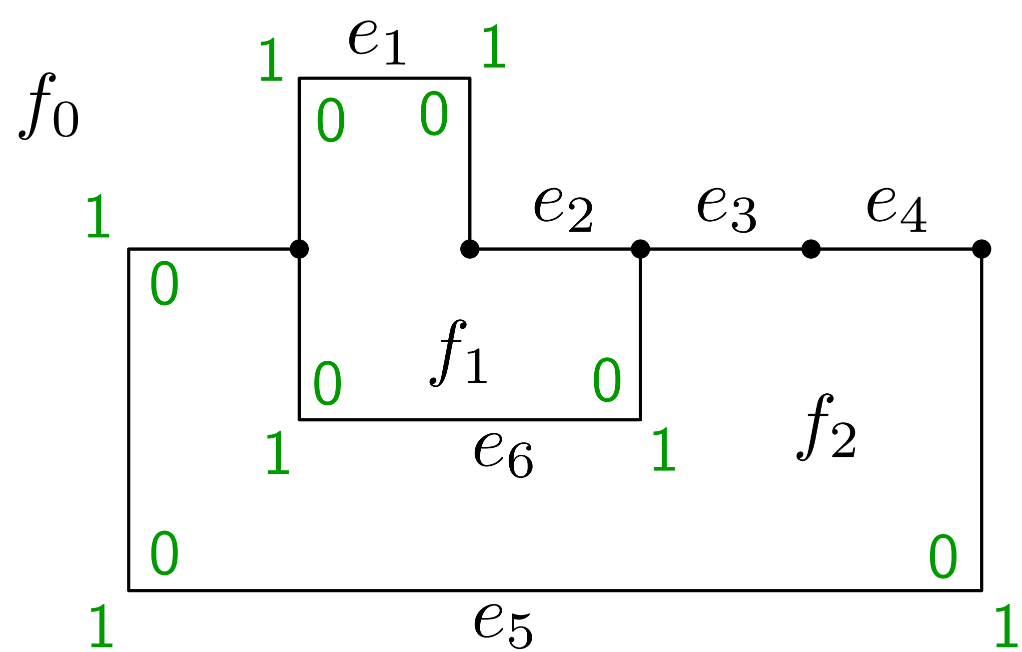

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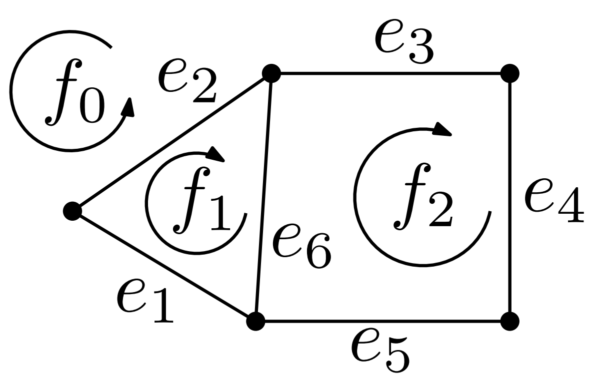
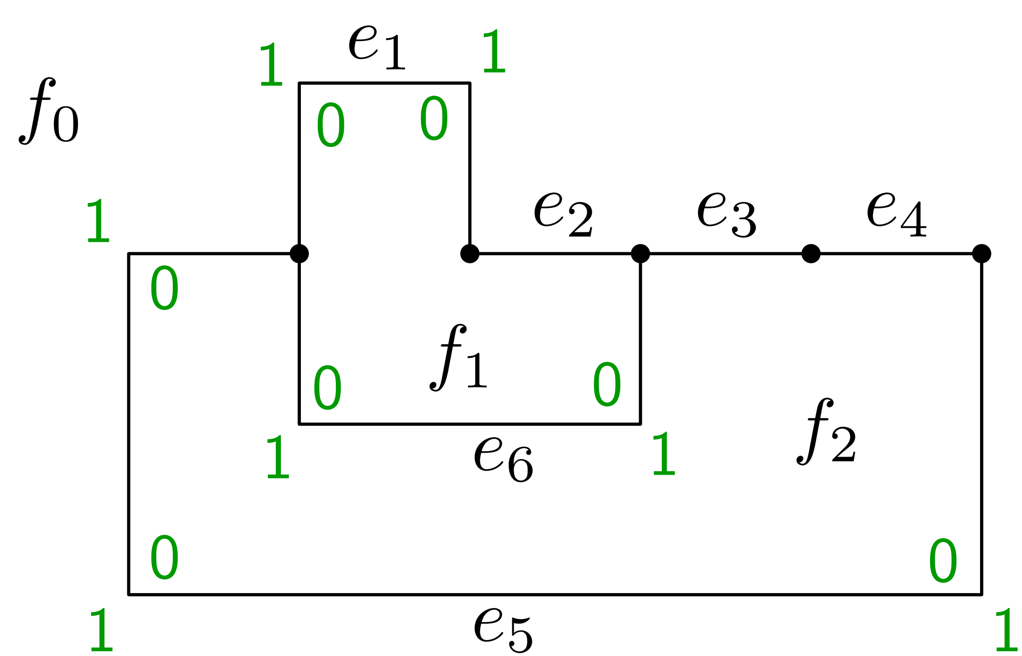
is f_0 in correct order?

$H(f_2) = ((e_5, 000, \frac{\pi}{2}), (e_6, 11, \frac{\pi}{2}), (e_3, \emptyset, \pi), (e_4, \emptyset, \frac{\pi}{2}))$





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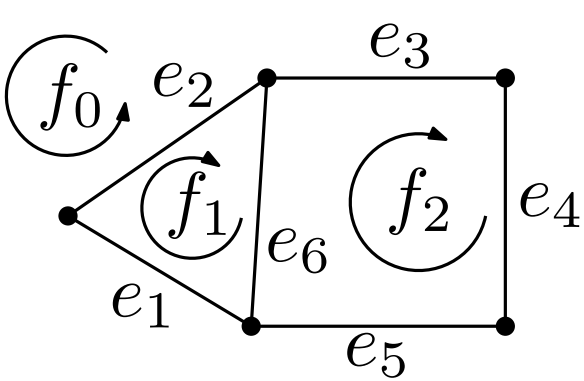

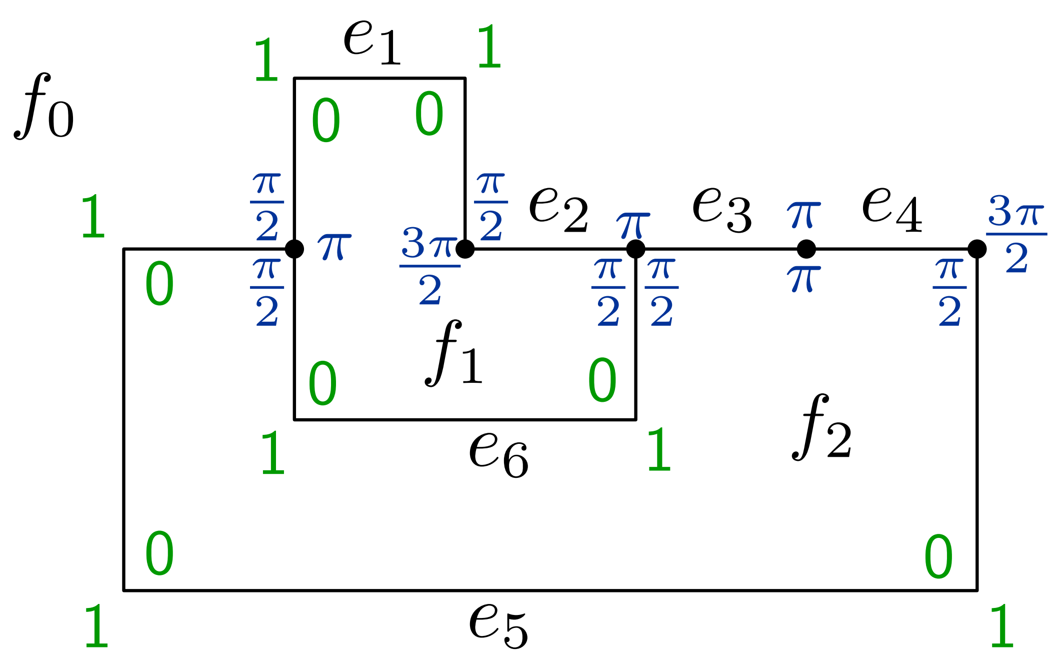

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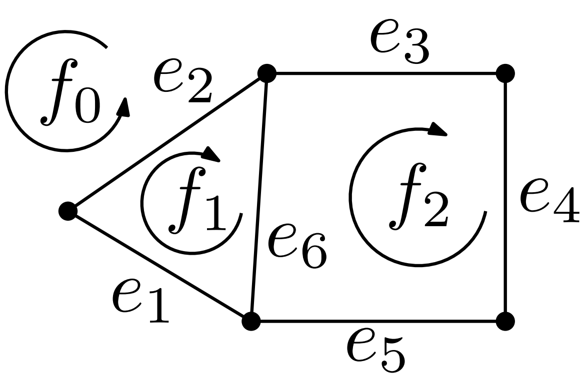

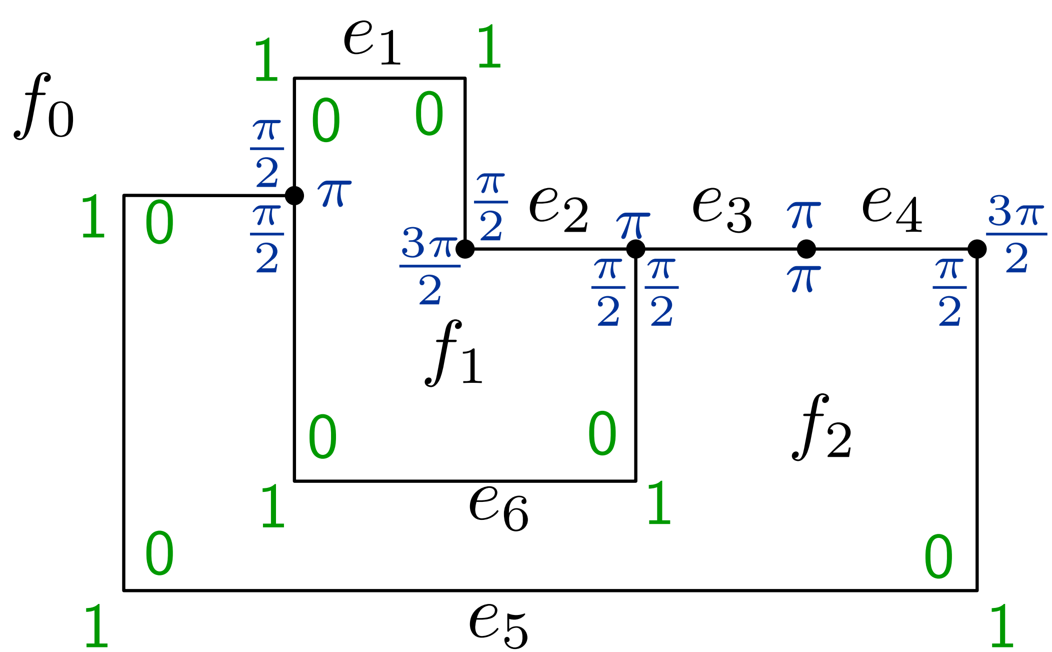

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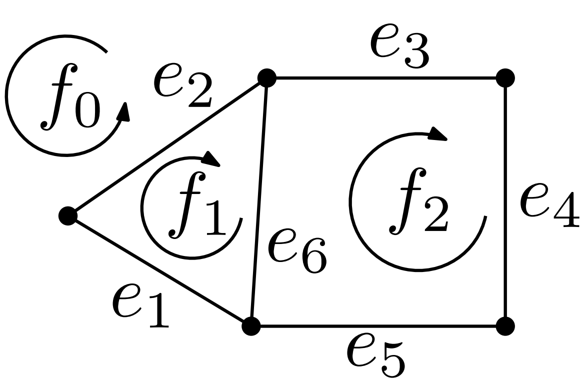

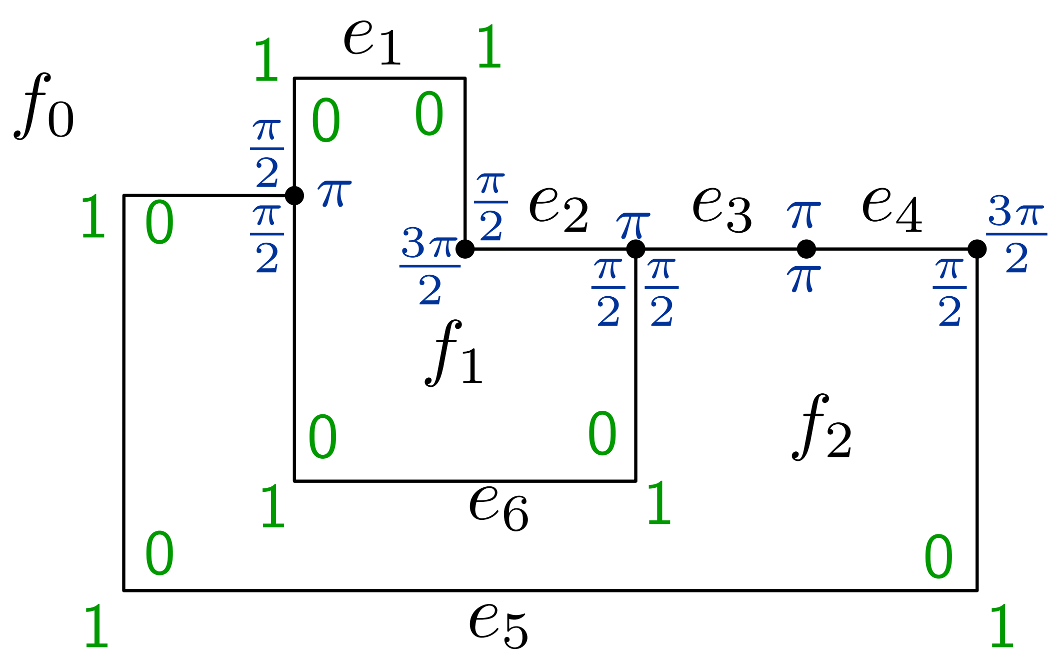

concrete coordinates are not fixed yet!

Correctness of an Orthogonal Representation

(H1) $H(G)$ corresponds to $\mathcal{F}, f_0$

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(H2) for an edge (u, v) shared by faces f and g with $((u, v), \delta_1, \alpha_1) \in H(f)$ and $((v, u), \delta_2, \alpha_2) \in H(g)$ sequence δ_1 is reversed and inverted δ_2

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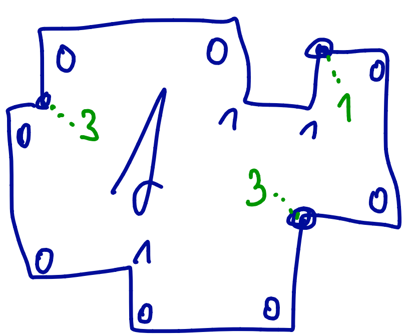
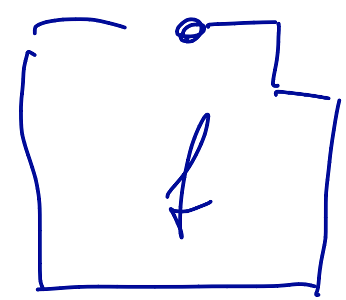
(H3) Let $|\delta|_0$ (resp. $|\delta|_1$) be the number of zeros (resp. ones) in δ and $r = (e, \delta, \alpha)$. For $C(r) := |\delta|_0 - |\delta|_1 + 2 - 2\alpha/\pi$ it holds: $\sum_{r \in H(f)} C(r) = 4$ for $f \neq f_0$ and $\sum_{r \in H(f_0)} C(r) = -4$

closing a face by a net turn of $4 \cdot \frac{\pi}{2} = 360°$!



Pair with your neighbor and discuss:

What does (H3) mean intuitively?



here: $8_0 - 3_1 - 7 + 6 = 4$

Correctness of an Orthogonal Representation

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(H4) For each vertex v the sum of incident angles is 2π

Combinatorial Bend Minimization

given:
- planar graph $G = (V, E)$ with maximum degree 4
- (combinatorial) embedding $\mathcal{F}$ with external face f_0

find: orthogonal representation $H(G)$ with same embedding and minimum number of bends

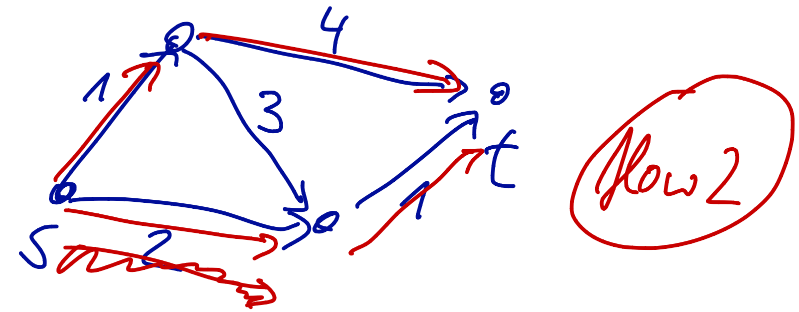
Idea: use a network flow model

- one unit of flow represents an angle $\pi/2$
- flow from vertices to faces represents angles at vertices
- flow between adjacent faces represents edge bends

Reminder: s-t Flow Network

Flow network $(D = (V, A); s, t; c)$ with

- directed graph $D = (V, A)$
- edge capacities $c : A \to \mathbb{R}_0^+$
- source $s \in V$, sink $t \in V$



A function $X : A \to \mathbb{R}_0^+$ is called s-t-**flow**, if X satisfies:

$$0 \leq X(u, v) \leq c(u, v) \qquad \forall (u, v) \in A \tag{1}$$

$$\sum_{(u,v) \in A} X(u, v) - \sum_{(v,u) \in A} X(v, u) = 0 \qquad \forall u \in V \setminus \{s, t\} \tag{2}$$

Reminder: General Flow Network

Flow network $(D = (V, A); \ell; c; b)$ with

- directed graph $D = (V, A)$
- edge capacity lower bounds $\ell : A \to \mathbb{R}_0^+$
- edge capacities $c : A \to \mathbb{R}_0^+$
- vertex production/consumption $b : V \to \mathbb{R}$ with $\sum_{u \in V} b(u) = 0$

A function $X : A \to \mathbb{R}_0^+$ is called **valid flow**, if X satisfies:

$$\ell(u,v) \leq X(u,v) \leq c(u,v) \qquad \forall (u,v) \in A \tag{3}$$

$$\underbrace{\sum_{(u,v) \in A} X(u,v)}_{\text{incoming flow}} - \underbrace{\sum_{(v,u) \in A} X(v,u)}_{\text{outgoing flow}} = b(u) \qquad \forall u \in V \tag{4}$$

Problems for Flow Networks

(A) Valid Flow:

Find a valid flow $X : A \to \mathbb{R}_0^+$, such that.

- lower bounds and capacities $\ell(e)$, $c(e)$ are respected
- consumption/production $b(v)$ is satisfied

Problems for Flow Networks

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For **cost function** $\mathrm{cost} : A \to \mathbb{R}_0^+$ define the cost of X as
$\mathrm{cost}(X) := \sum_{(u,v) \in A} \mathrm{cost}(u,v) \cdot X(u,v)$

Problems for Flow Networks

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- lower bounds and capacities $\ell(e)$, $c(e)$ are respected
- consumption/production $b(v)$ is satisfied

For **cost function** $\mathrm{cost} : A \to \mathbb{R}_0^+$ define the cost of X as
$\mathrm{cost}(X) := \sum_{(u,v) \in A} \mathrm{cost}(u,v) \cdot X(u,v)$

(B) Miminum Cost Flow

Find a valid flow $X : A \to \mathbb{R}_0^+$, that minimizes $\mathrm{cost}(X)$ (over all valid flows)

Flow Network for Bend Minimization

Flow network $N(G) = ((V \cup \mathcal{F}, A); \ell; c; b; \mathrm{cost})$

- $A = \{(v, f)_{e,e'} \in V \times \mathcal{F} \mid v$ incident to f between $e, e'\} \cup$
  $\{(f, g)_e \in \mathcal{F} \times \mathcal{F} \mid f, g$ adjacent through edge $e\}$

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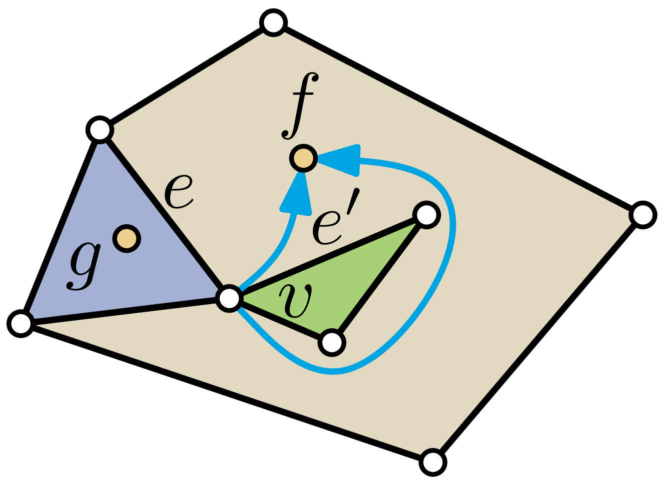

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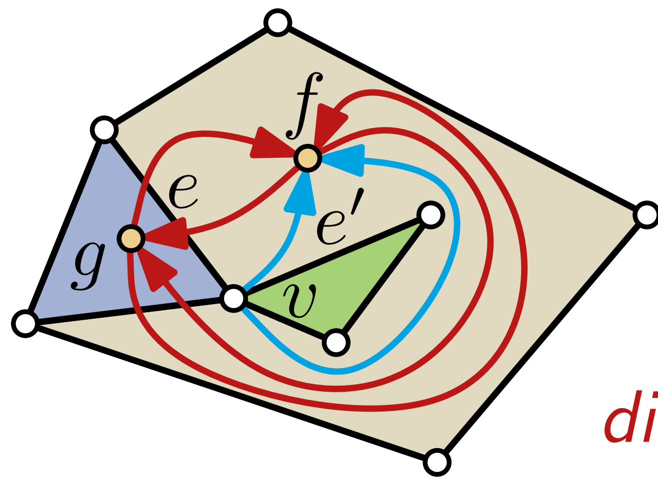

directed multigraph

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- $b(f) = -2(\underbrace{d_G(f)}_{\text{\# edges of } f} - 2) \quad \forall f \in \mathcal{F} \setminus \{f_0\}$
- $b(f_0) = -2(d_G(f_0) + 2)$

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$$\sum_{w \in (V \cup \mathcal{F})} b(w) = \sum_{v \in V} 4 - 2 \sum_{f \in \mathcal{F}} d_G(f) + \sum_{f \in \mathcal{F}} 4 - 8$$

$$= 4 \cdot |V| - 4|E| + 4|\mathcal{F}| - 8$$

$$= 4 \cdot (\underbrace{|V| - |E| + |\mathcal{F}| - 2}_{= 0 \text{ by Euler equation}}) = 0$$

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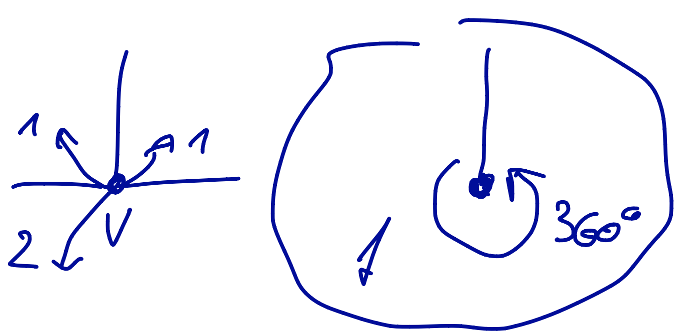
$\Rightarrow \sum_w b(w) = 0$ (Euler)

$\forall (f, g) \in A, f, g \in \mathcal{F}$ $\quad \ell(f, g) := 0 \leq X(f, g) \leq \infty =: c(f, g)$

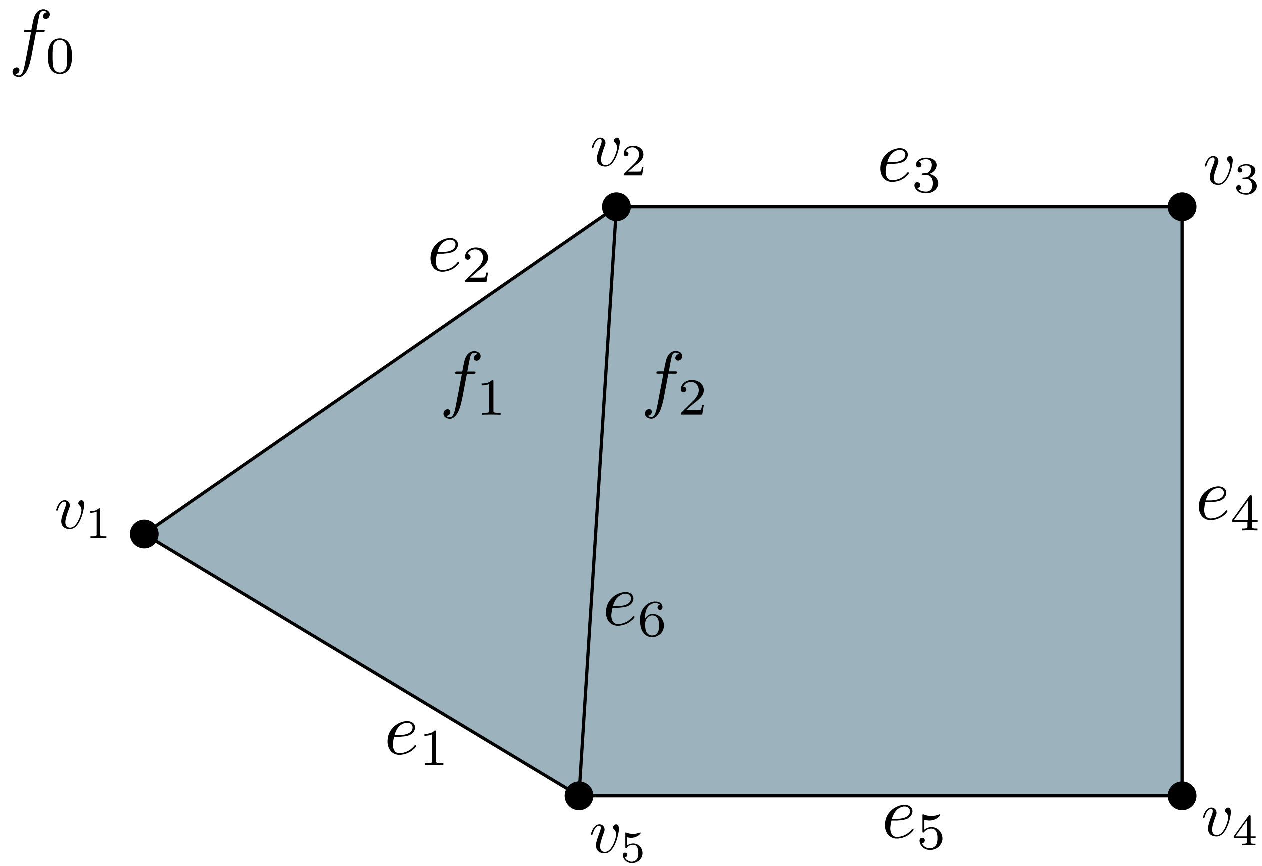
$\text{cost}(f, g) = 1$

$\forall (v, f) \in A, v \in V, f \in \mathcal{F}$ $\quad \ell(v, f) := 1 \leq X(v, f) \leq 4 =: c(v, f)$

$\text{cost}(v, f) = 0$



Example Flow Network



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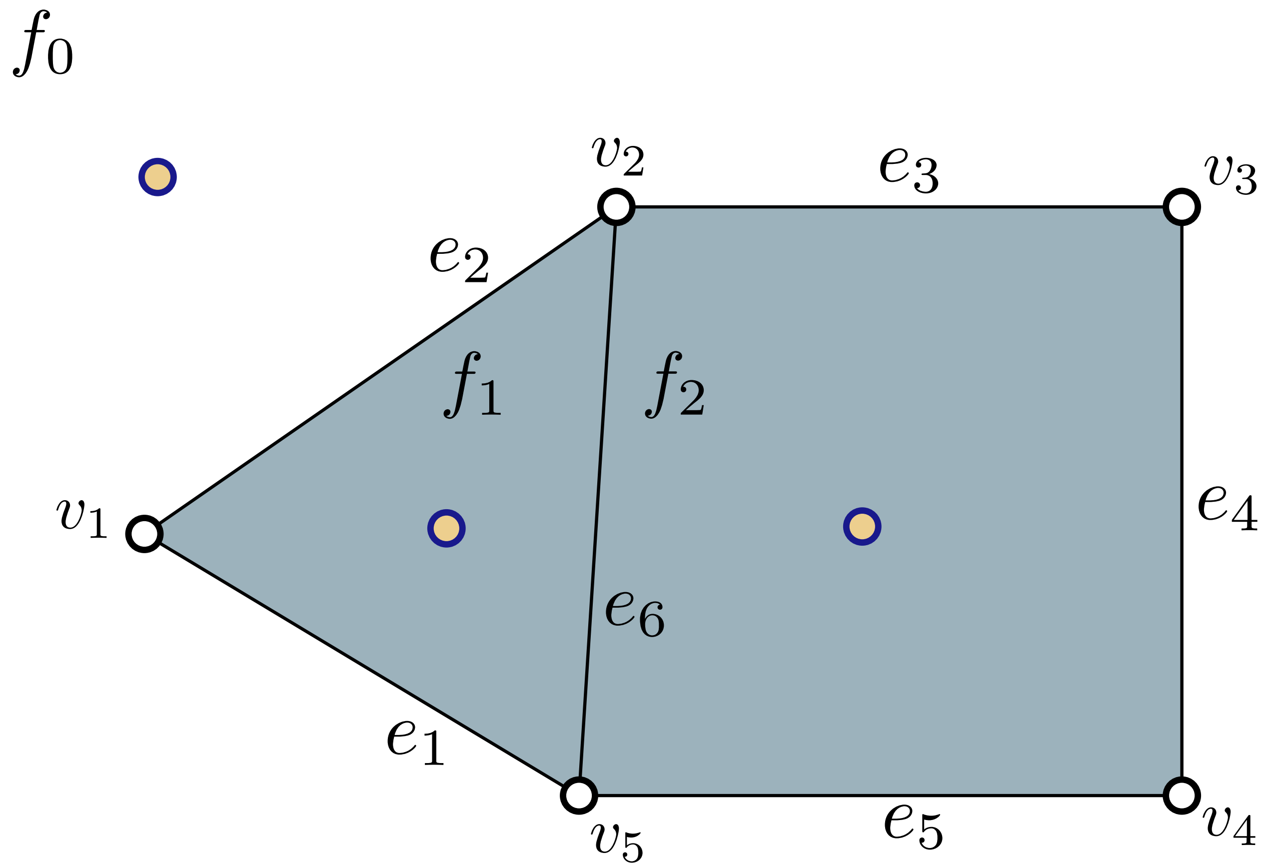

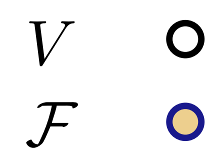

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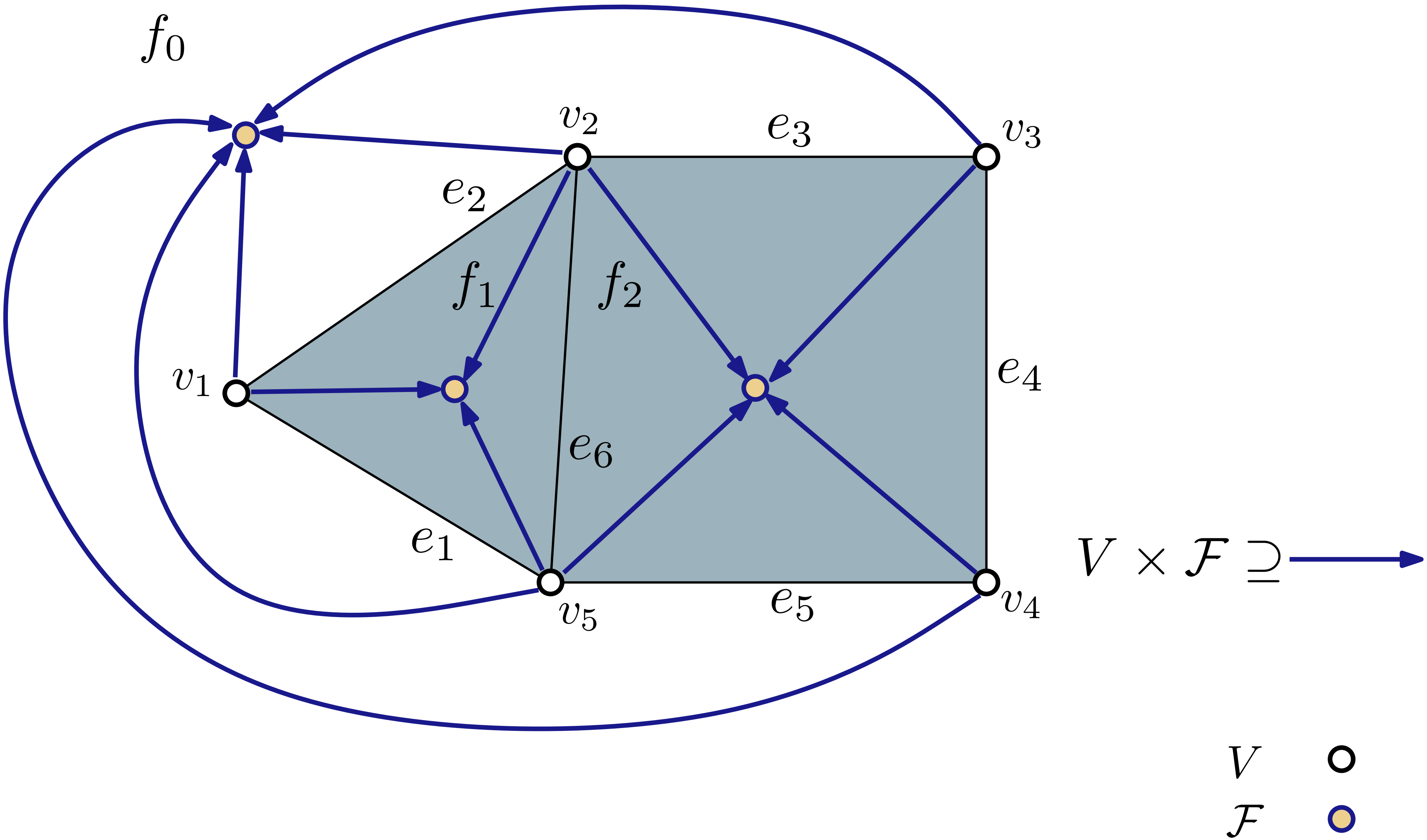

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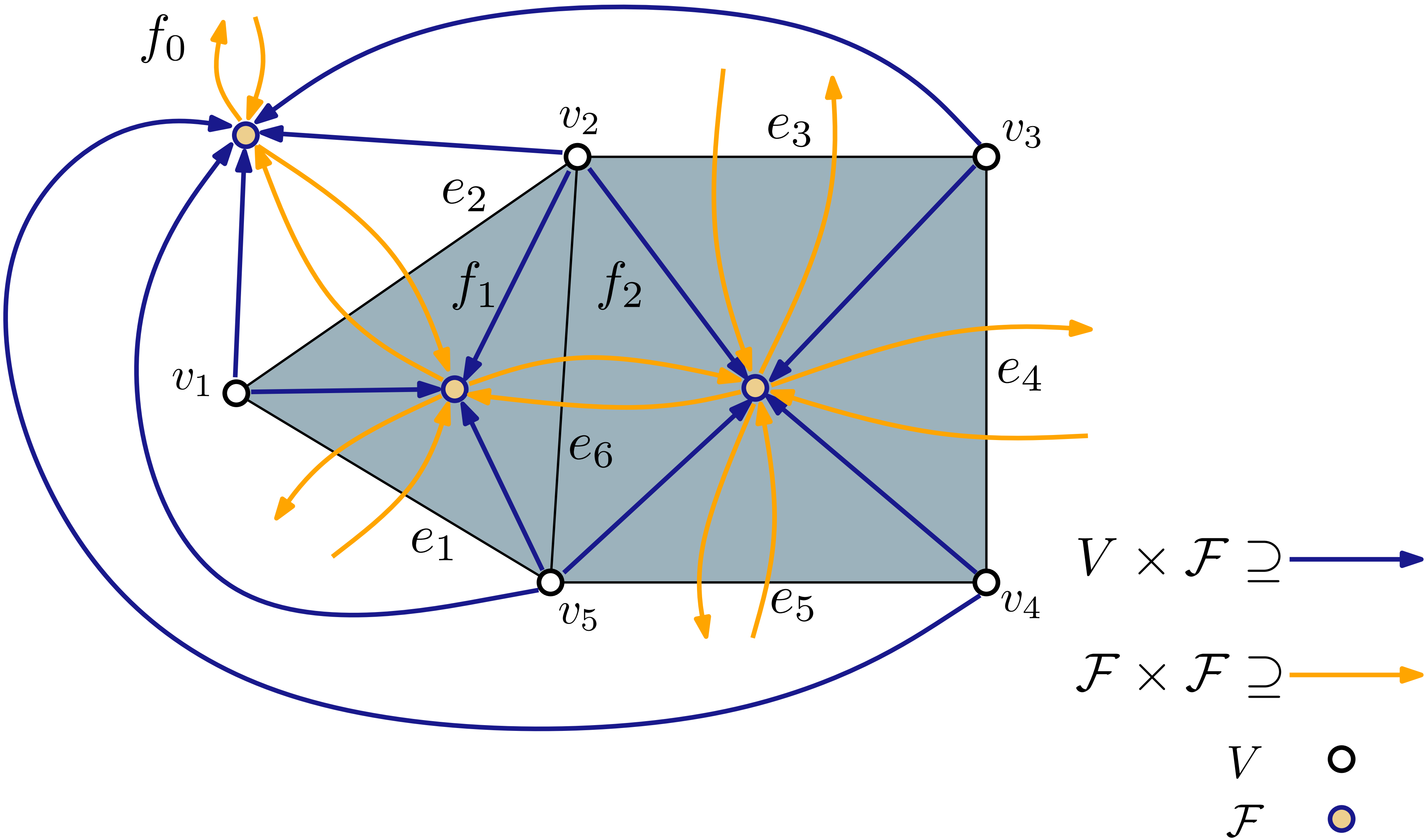

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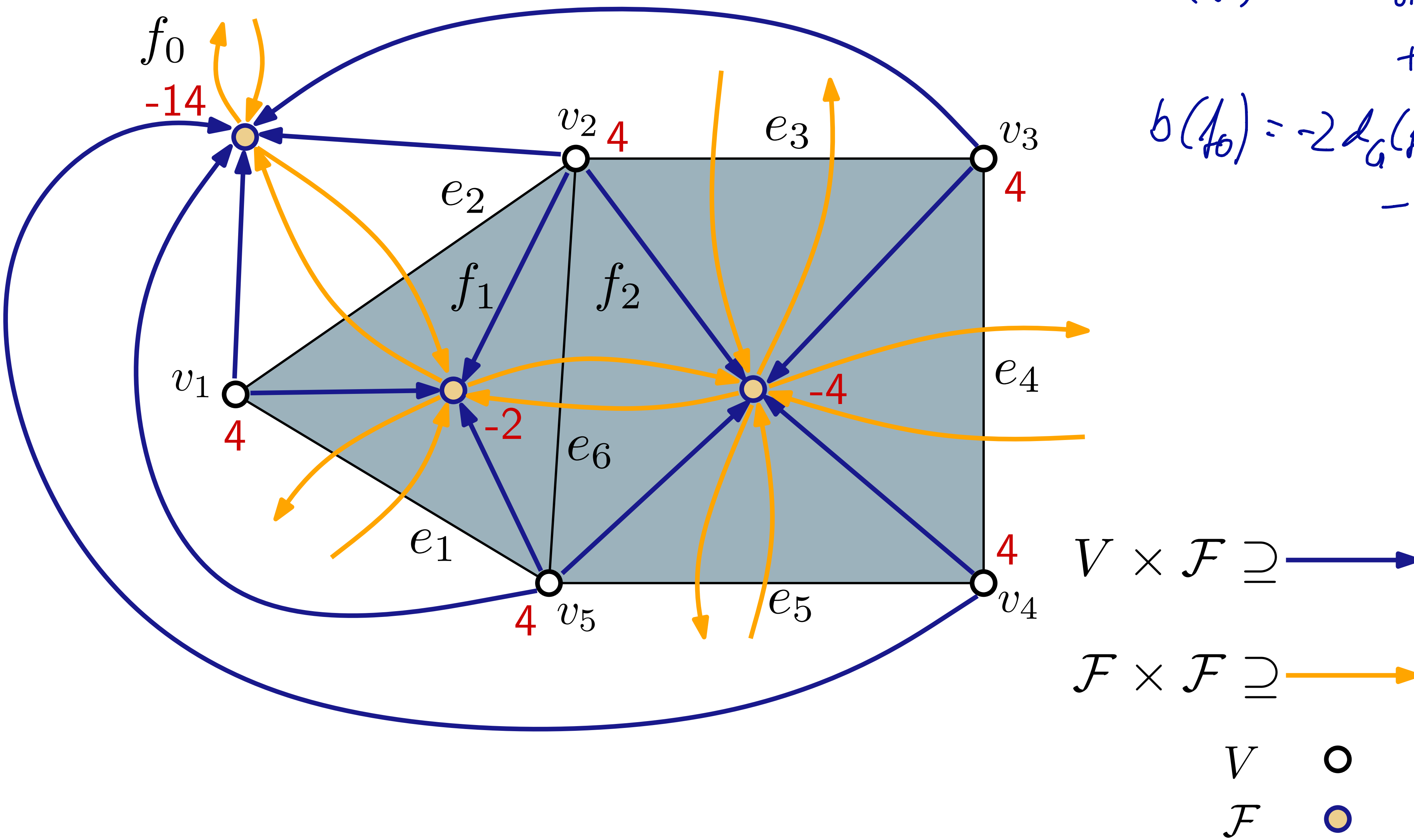

$b(f) = -2\, d_G(f) + 4$

$b(f_0) = -2\, d_G(f) - 4$

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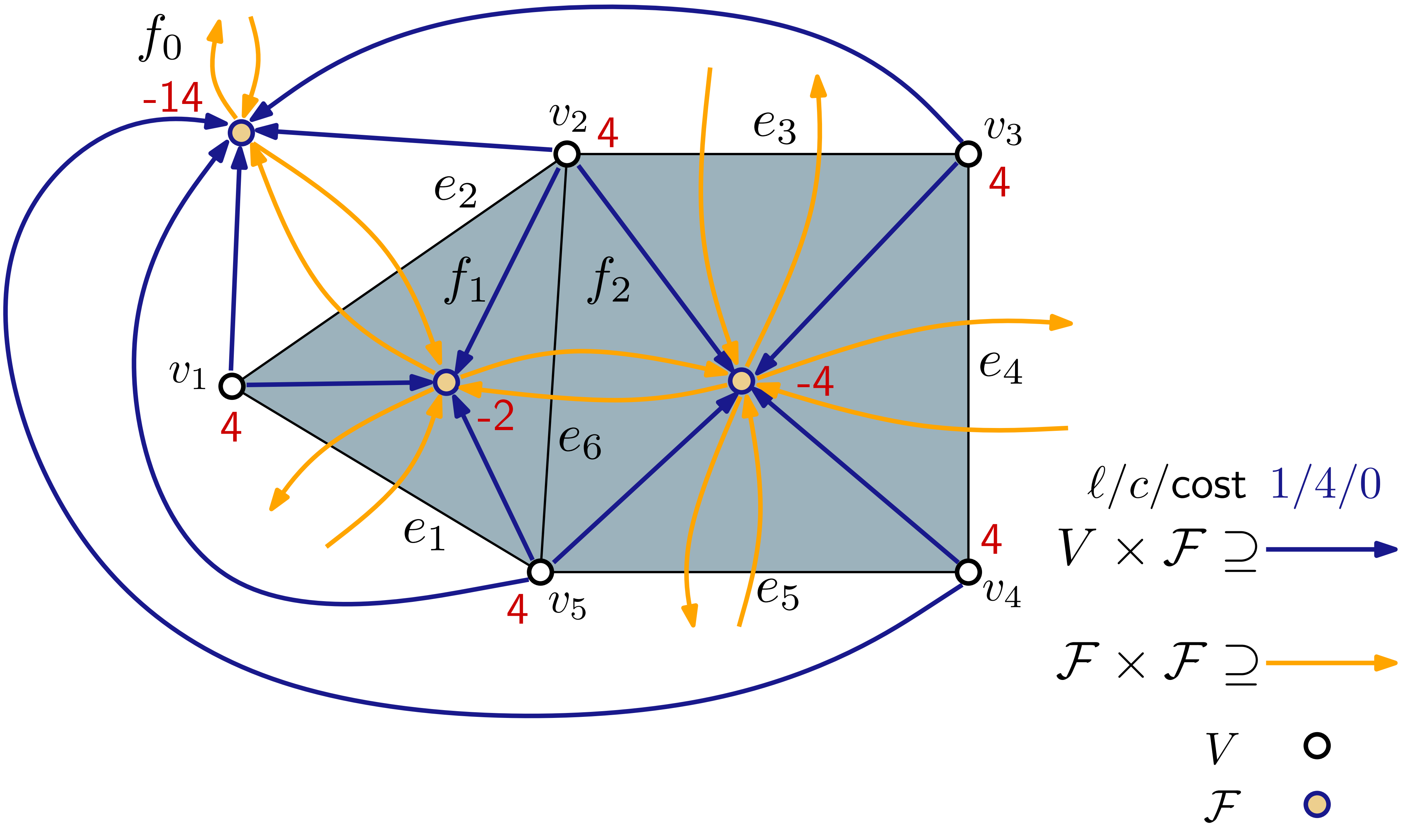

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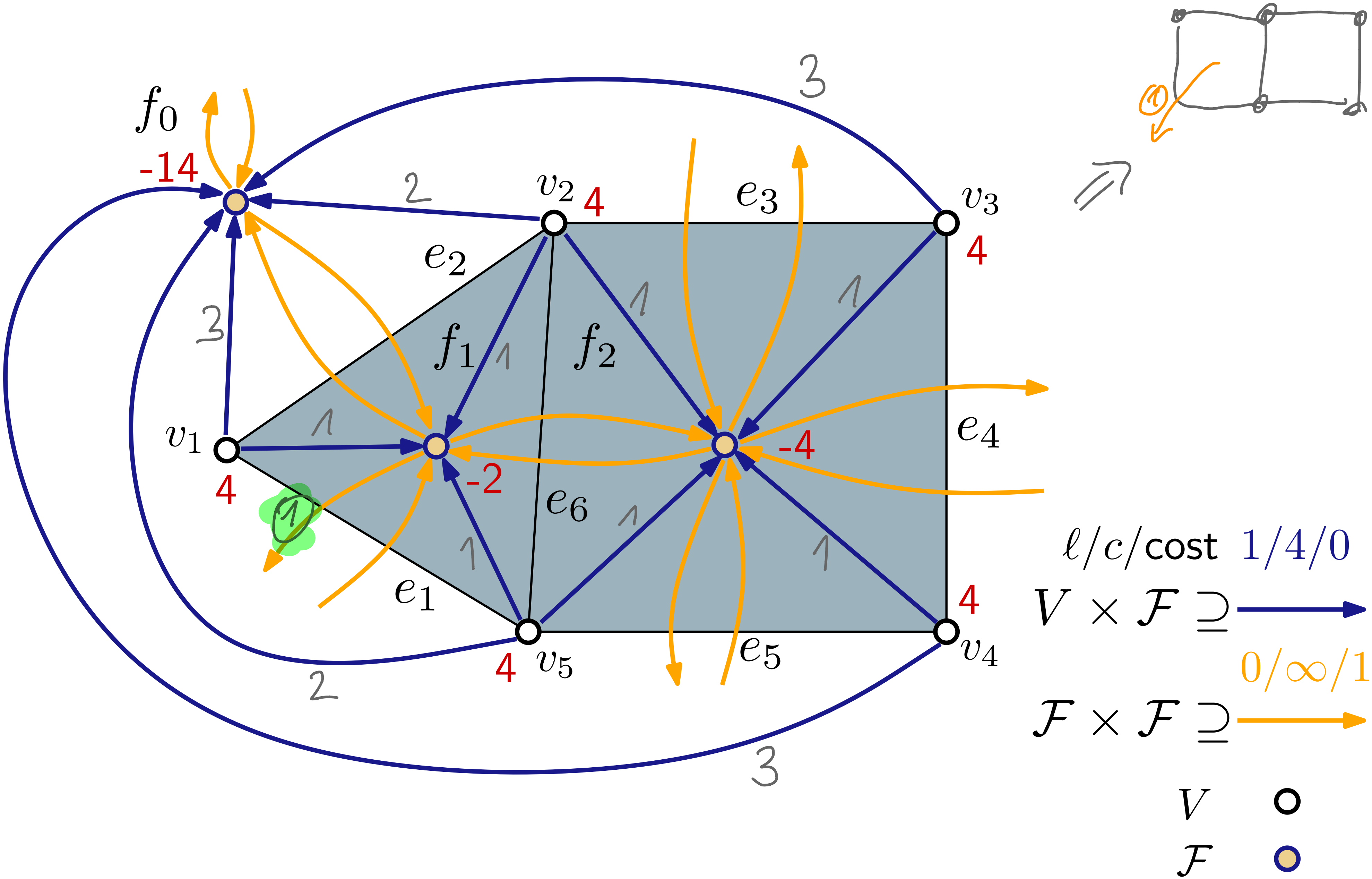

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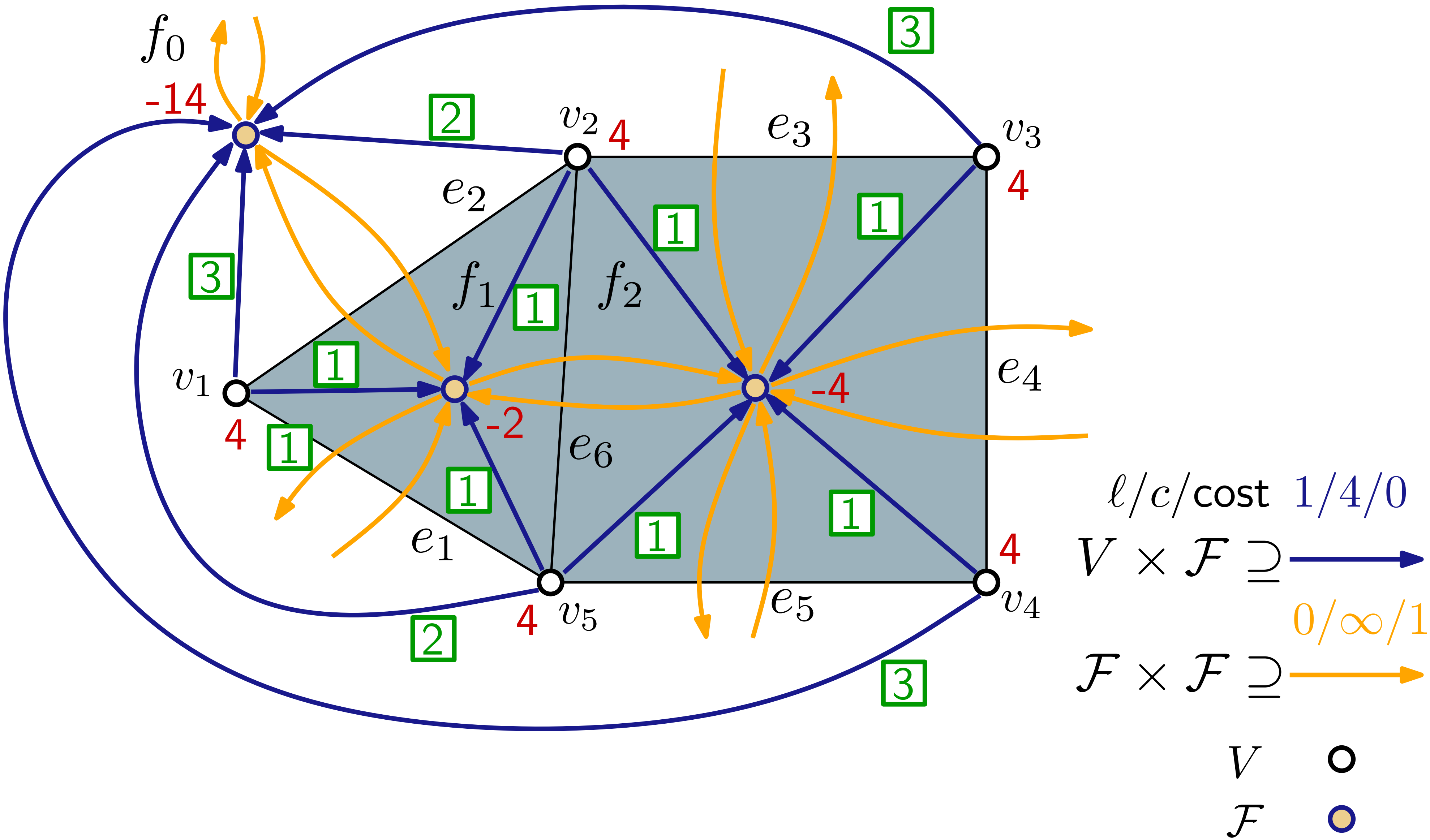

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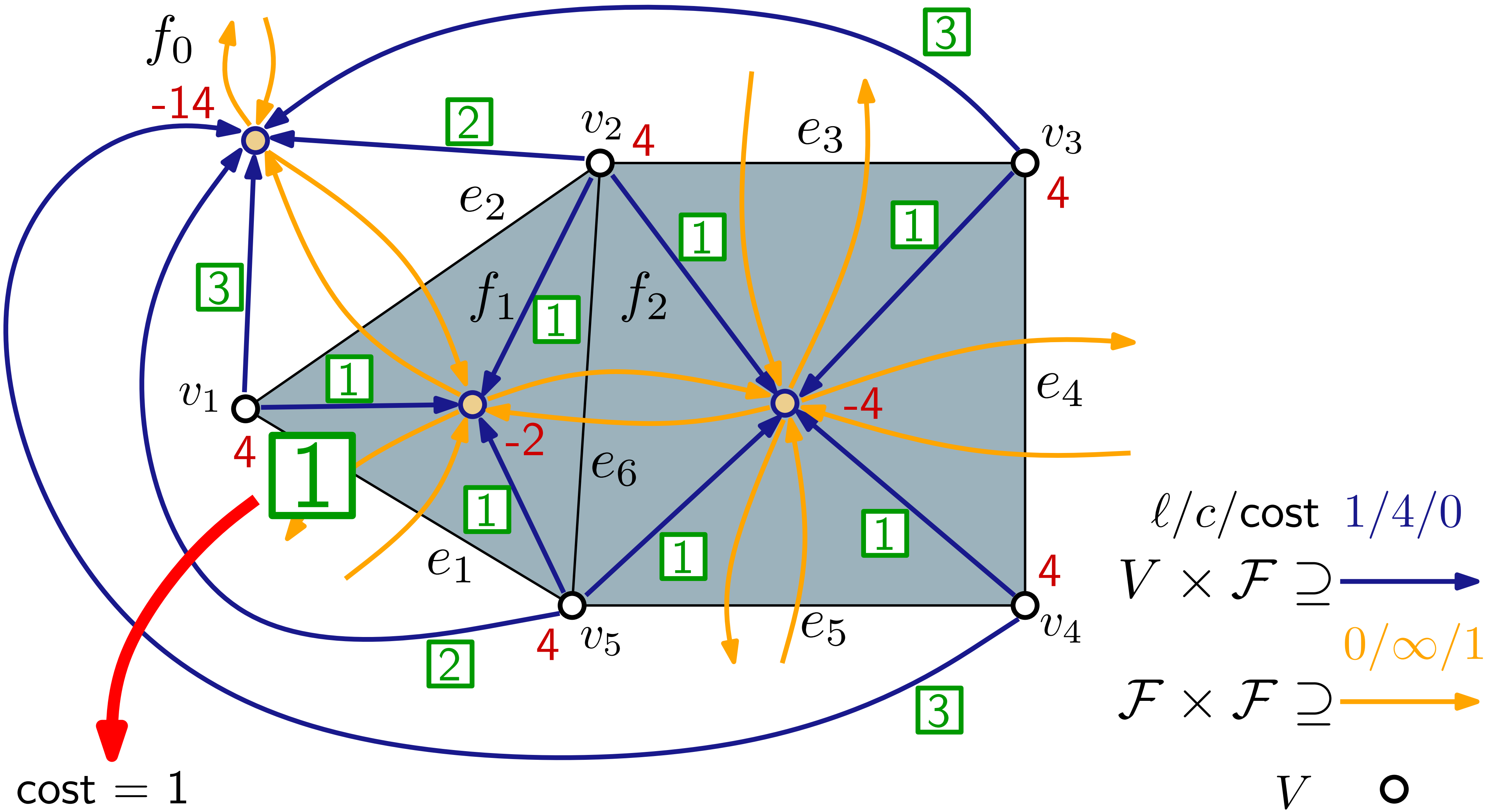

cost $= 1$
$\Rightarrow$ bend!
(to the outside)

Bend Minimization: Main Result

Theorem: A planar embedded graph $(G, \mathcal{F}, f_0)$ has a valid orthogonal description $H(G)$ with k bends iff the flow network $N(G)$ has a valid flow X with cost k.

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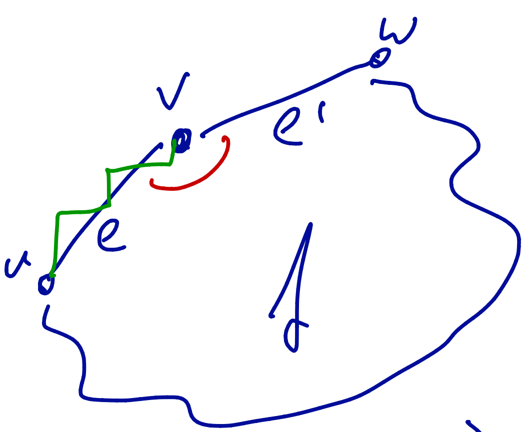
construct: orthogonal representation $H(G)$ with k bends

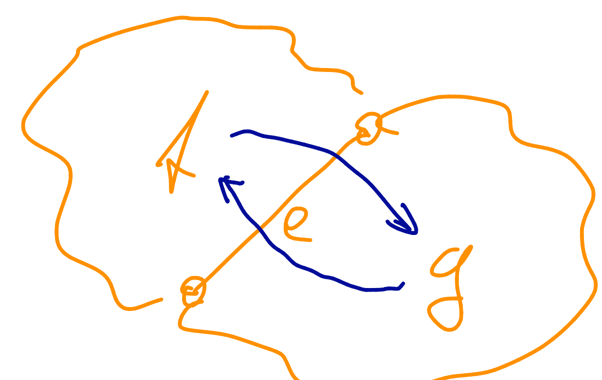
- transform flow into orthogonal representation
- show properties (H1)–(H4)

• for each face f, define $H(f)$, i.e. sequence of edge descriptors (e, δ_e, α_e)



$$\alpha_e = X(v,f) \cdot \frac{\pi}{2}$$

$$\delta_e = \underbrace{0 \ldots 0}_{X(f,g)} \underbrace{1 \ldots 1}_{X(g,f)}$$



$\Rightarrow H(G)$ has k bends

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$\Rightarrow$ given: orthogonal description $H(G)$ with k bends
construct: flow X in $N(G)$ with cost k

- define function $X\colon A \to \mathbb{R}_0^+$
- show that X is a valid flow and has cost k

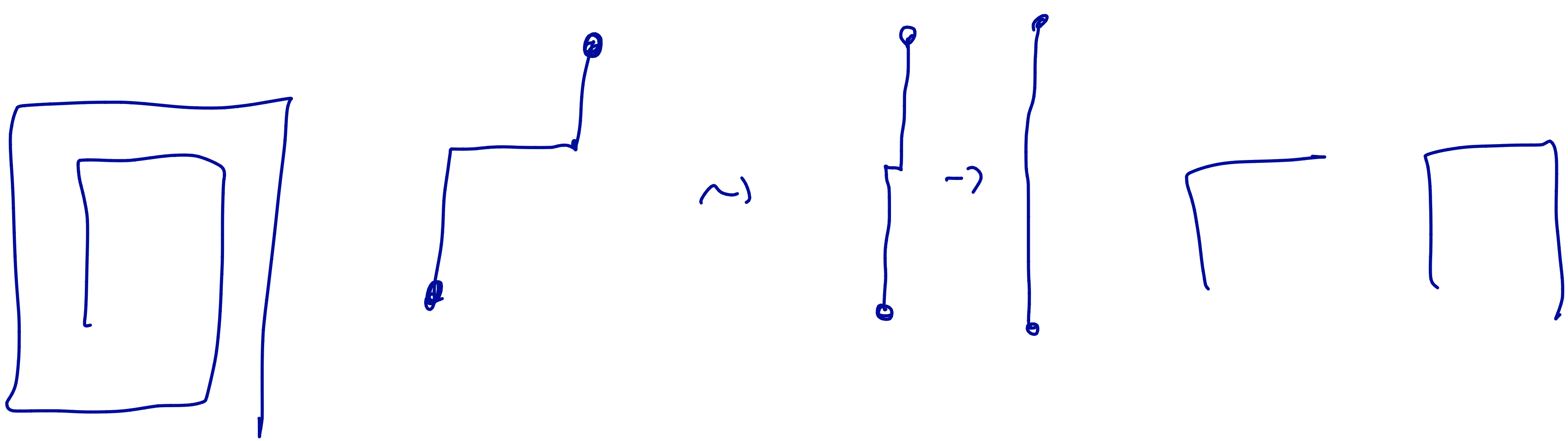
def: $X(v,f) = \begin{cases} 1 & \text{if } \alpha_{v,f} = \frac{\pi}{2} \\ 2 & \pi \\ 3 & 3\pi/2 \\ 4 & 2\pi \end{cases}$

$X(f,g) = |\delta_0|$

check: capacities ✓, flow conservation ✓, cost $= k$ ✓

Question

In a bend-minimal orthogonal representations of G can there be an edge with both left and right bends? No!



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- Bend minimization without a given combinatorial embedding is an NP-hard problem. [Garg, Tamassia 2001]
- Deciding whether a graph has an orthogonal drawing with at most $f(e)$ bends per edge e can be solved in polynomial time if $f(e) \geq 1$ for all $e \in E$. [Bläsius et al. 2014]

Announcements

- Next week the class of June 12 is shifted to Monday, June 11 in the same time slot 9:00–11:00 in seminar room 186.
- Student presentations from exercise groups will take place on July 3, 10:00–12:00 and 13:00–15:00. Attendance required.
- Oral exam dates are July 10 and September 18. Registration in TISS is open.