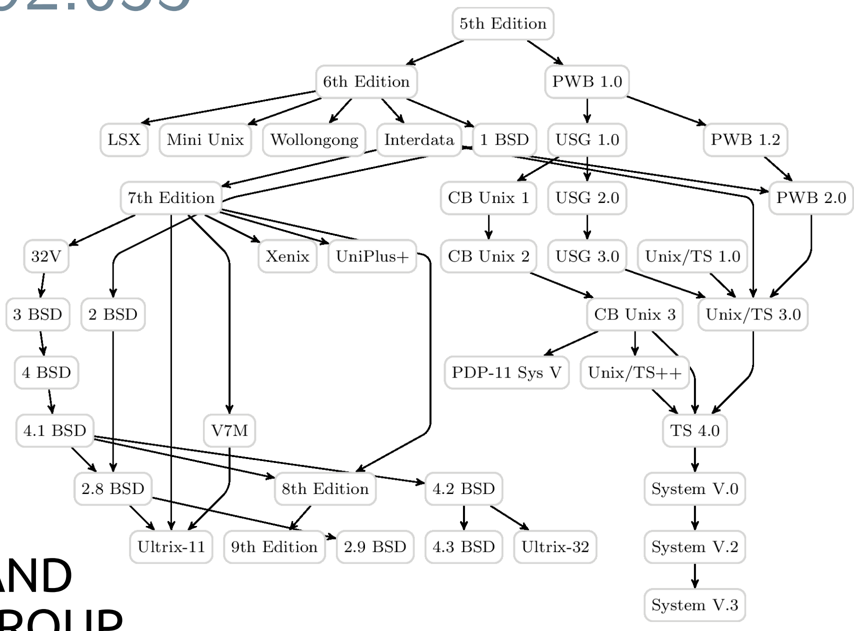
Layered Graph Drawing – Part 2

Lecture Graph Drawing Algorithms · 192.053

Martin Nöllenburg

29.05.2018

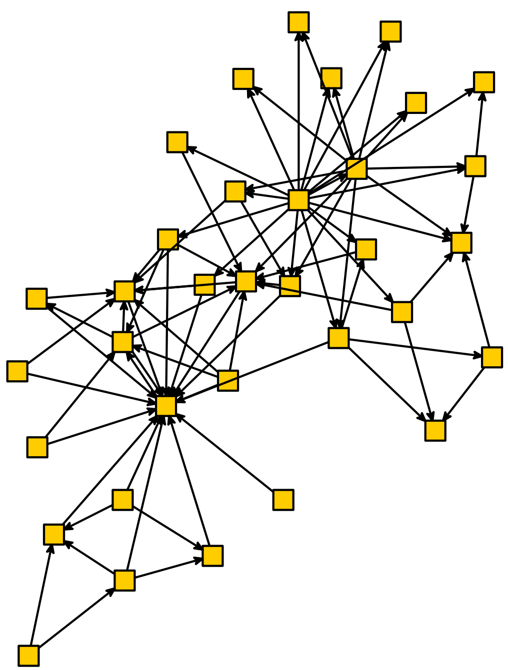


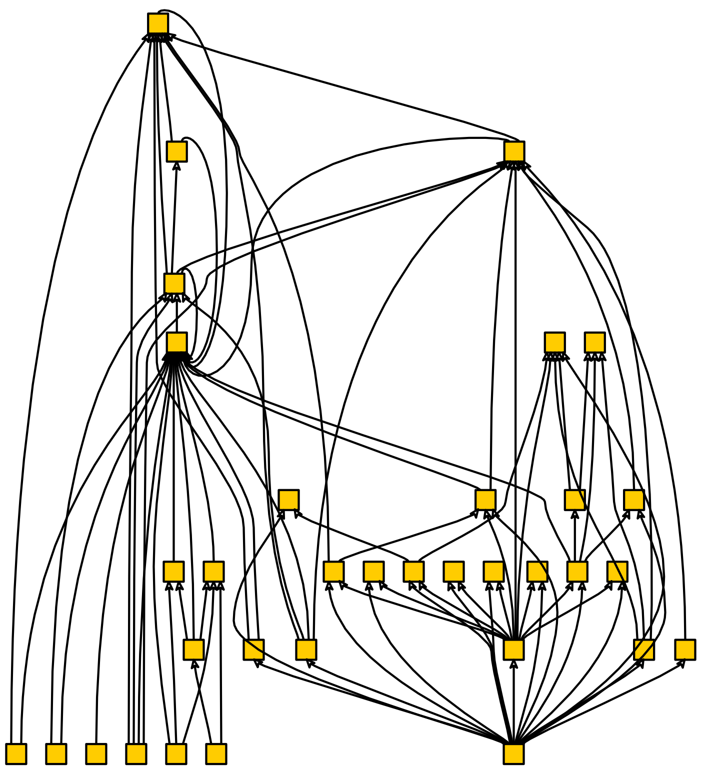
ALGORITHMS AND COMPLEXITY GROUP

Layered Graph Layout

Input: directed graph $D = (V, A)$

Output: drawing of D that emphasizes its hierarchical structure





Layered Graph Layout

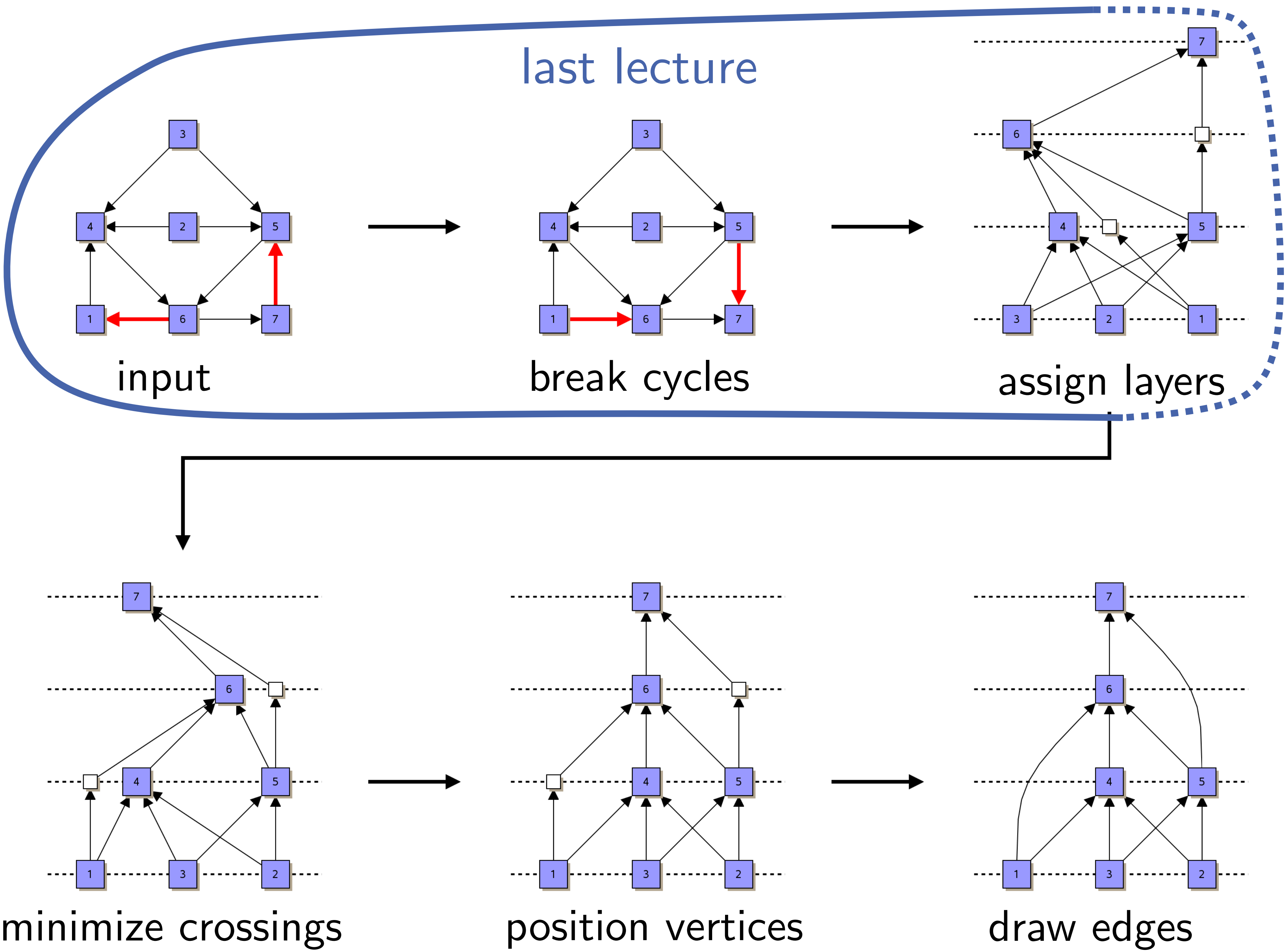
Input: directed graph $D = (V, A)$

Output: drawing of D that emphasizes its hierarchical structure

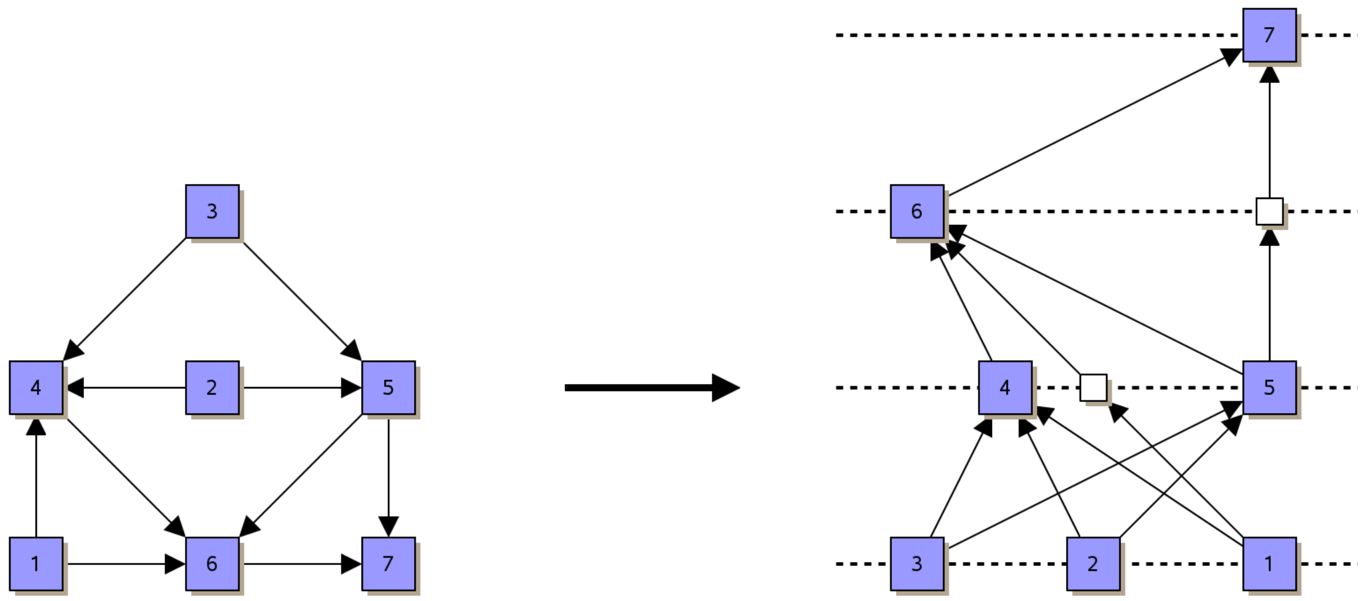
Criteria:

- many edges pointing upward (or some other given direction)
- ideally short, straight and vertical edges
- vertices placed on (few) horizontal layers
- few edge crossings
- evenly distributed vertices

Overview Sugiyama Framework



Step 2: Assign Layers



Layer Assignment

Input: directed acyclic graph $D = (V, A)$

Output: partition of V into disjoint subsets (**layers**) $L_1, \dots, L_h$
s.t. $(u, v) \in A$, $u \in L_i$, $v \in L_j \Rightarrow i < j$

Define: y-Coordinate $y(u) = i \Leftrightarrow u \in L_i$

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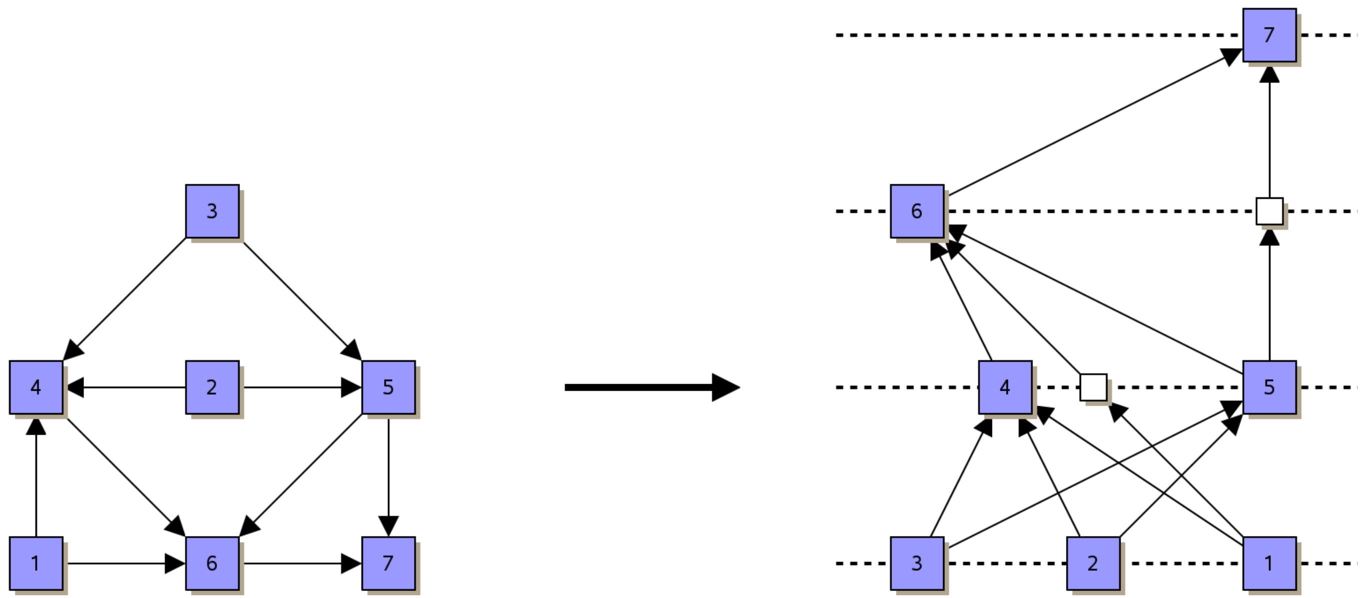
Some optimization criteria:

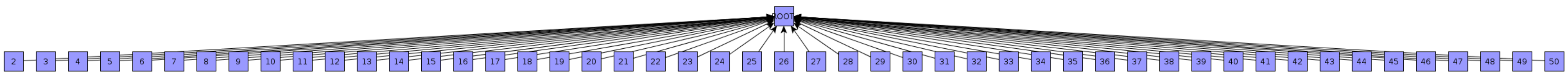
- minimize the number h of layers ($=$ height of the layouts)
- minimize the width, i.e., $\max\{|L_i| \mid 1 \leq i \leq h\}$
- minimize the longest edge, i.e., $\max\{j - i \mid (u, v) \in A,\, u \in L_i,\, v \in L_j\}$
- minimize the total edge length ($\approx$ number of dummy vertices)

Last Lecture

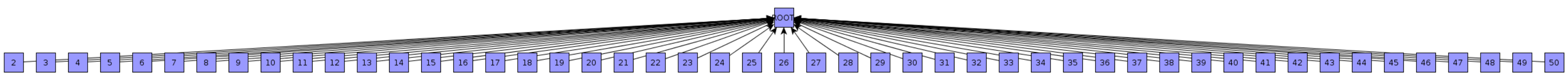


- Height minimization using topological sorting (linear time)
  $\rightarrow$ puts each vertex on lowest possible layer
- Minimization of total edge length using integer linear programming (ILP)
  $\rightarrow$ polynomial time via LP relaxation as constraint matrix is totally unimodular

Height/Edge Length Minimization is not all



Height/Edge Length Minimization is not all

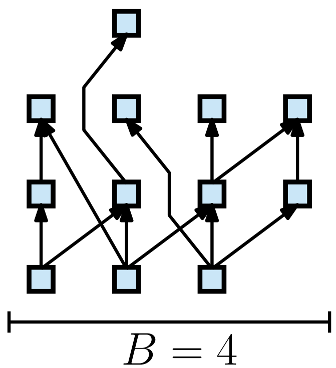


$\rightarrow$ bound the width!

Fixed-Width Layer Assignment

Input: directed acyclic graph $D = (V, A)$, width B

Output: layer assignment $\mathcal{L}$ of minumum height with at most B nodes per layer



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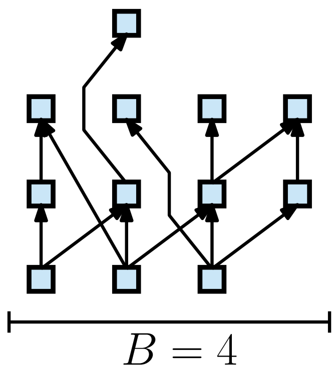

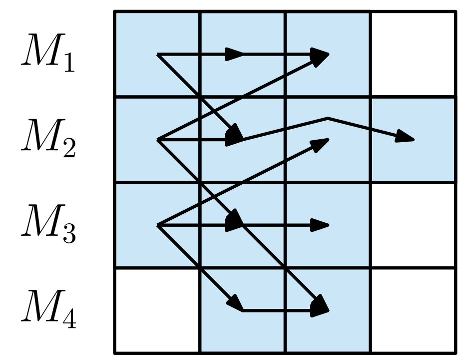

$\rightarrow$ this is equivalent to the following job scheduling problem:

Minimum Precedence Constrained Scheduling (MPCS)

Input: n jobs $J_1, \ldots, J_n$ with identical unit processing time, precedence constraints $J_i < J_k$, and B identical machines

Output: Schedule of minimum length that satisfies all the precedence constraints

Complexity

Theorem: For n jobs $J_1, \ldots, J_n$ of equal length, a precedence relation $<$, a number B of identical machines, and an integer T it is NP-complete to decide if a schedule of length at most T exists, even for $T = 3$.

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Proof:

- reduce NP-complete problem CLIQUE to MPCS with $T = 3$

CLIQUE: Given graph $G = (V, E)$ and $k \in \mathbb{N}$, is there a complete subgraph on $\geq k$ vertices in G?

Define: $k' = |V| - k$ $\quad L = \frac{k \cdot (k-1)}{2}$ (= # edges of Clique) $L' = |E| - L$

$B = \max\{k, L + k', L'\} + 1$

Jobs: J_v for every $v \in V$, J_e for every $e \in E$

for each edge $e = (u, v) \in E$ $\quad J_u < J_e, \; J_v < J_e$

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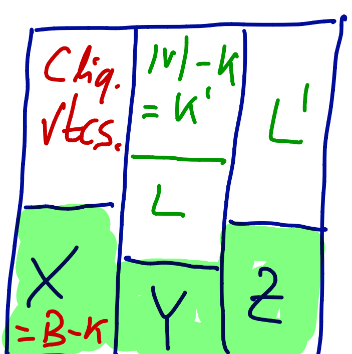
Dummy jobs X_j, $j = 1, \dots, B-K$ Z_j, $j = 1, \dots, B-L'$

Y_j, $j = 1, \dots, B-L-K'$

$X_j < Y_i < Z_\ell$

$\#\text{jobs}: \cancel{|V|} + \cancel{|E|} + (B-\cancel{K}) + (B-\cancel{L}-\underbrace{K'}_{\cancel{|V|-K}}) + (B-\underbrace{L'}_{\cancel{|E|-L}}) = 3B$

claim: G has a k-Clique $\Leftrightarrow$ Schedule of length $T=3$ exists



$\square$

Complexity

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Corollary: If $\mathcal{P} \neq \mathcal{NP}$ there is no polynomial-time approximation algorithm for MPCS with approximation ratio $< 4/3$.

Approximation

Theorem: MPCS has a polynomial-time approximation algorithm with approximation ratio $\leq 2 - \frac{1}{B}$.

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List scheduling algorithm:

$n = \#$ jobs

- order jobs arbitrarily as a list $\mathcal{L}$
- if a machine is free, assign to it the first feasible job in $\mathcal{L}$; if no feasible job exists machine remains idle

observations $\quad OPT \geq \frac{n}{B}$, $\quad OPT \geq \ell$ = longest "path" in the order $<$

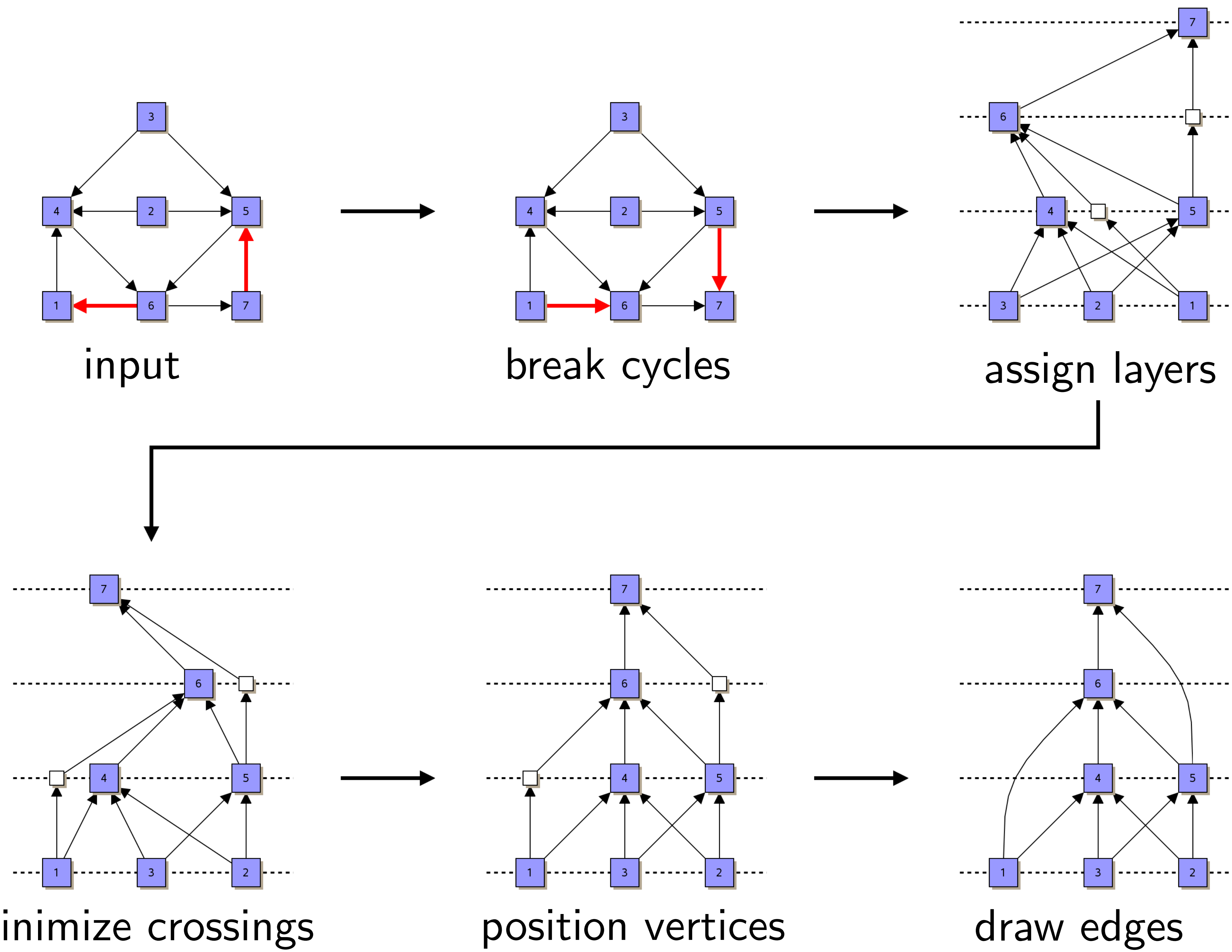
define: m_i = # of idle steps of machine i, observe: $m_i \leq \ell$

w.l.o.g. $m_1 = 0$

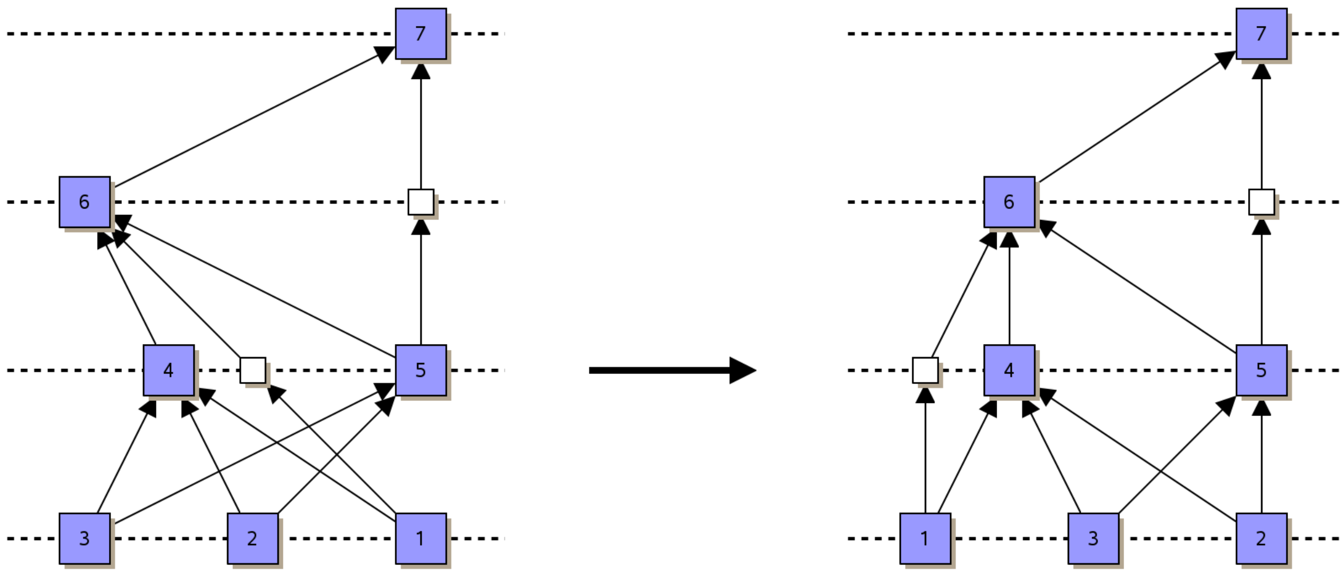
$$T = \frac{1}{B} \cdot \left(n + \sum_{i=2}^{B} m_i\right) \leq \frac{1}{B}\left(n + (B-1)\ell\right) = \frac{n}{B} + \left(1 - \frac{1}{B}\right)\ell$$

$$\leq OPT + \left(1 - \frac{1}{B}\right) OPT$$

Overview


input
break cycles
assign layers
minimize crossings
position vertices
draw edges

Step 3: Crossing Minimization



What would you do?

Problem Statement

Given: DAG $D = (V, A)$, layer assignment of all vertices

Find: Permutation of the vertices on each layer, such that the number of crossing is minimized

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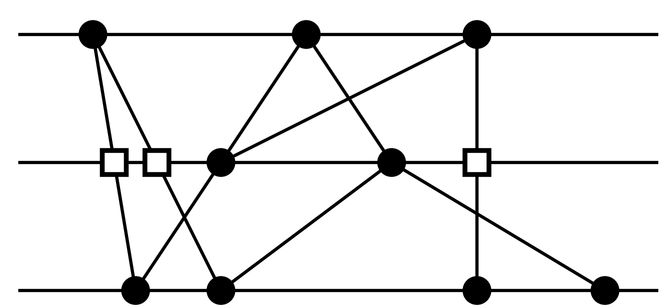
Properties

- problem is NP-hard even for two layers ($\textsc{Bipartite Crossing Number}$ [Garey, Johnson '83])
- no approaches optimizing several layers simultaneously
- usually iterative optimization for two adjacent layers
  $\rightarrow$ insert dummy vertices in each layer crossed by long edges



One-sided Crossing Minimization (OSCM)

Given: 2-layer graph $G = (L_1, L_2, E)$ and bijective vertex ordering $x_1 \colon L_1 \to \{1, 2, \ldots, |L_1|\}$

Find: vertex ordering $x_2 \colon L_2 \to \{1, 2, \ldots, |L_2|\}$, such that the number of crossing among E is minumum

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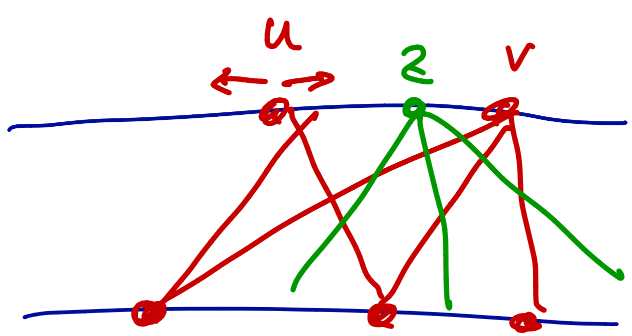
Observation:

- number of crossing in 2-layer drawing of G depends only on vertex orderings, not on the exact positions
- for $u, v \in L_2$ the number of crossing among incident edges depends only on $x_2(u) < x_2(v)$ or $x_2(v) < x_2(u)$



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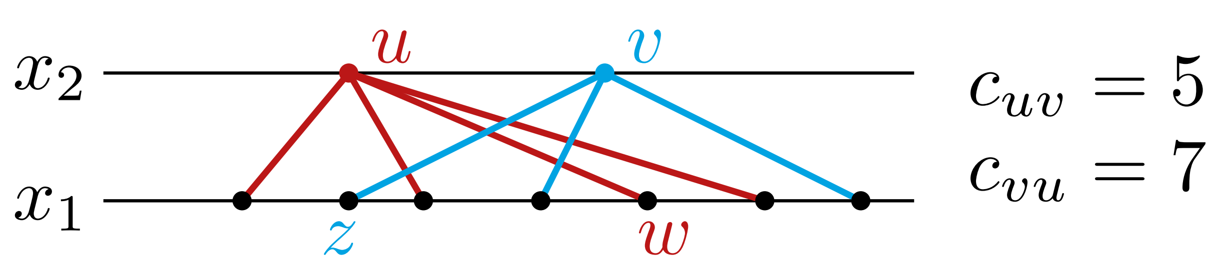
Def: crossing value

$$c_{uv} := |\{(uw, vz) : w \in N(u), z \in N(v), x_1(z) < x_1(w)\}|$$

for $x_2(u) < x_2(v)$



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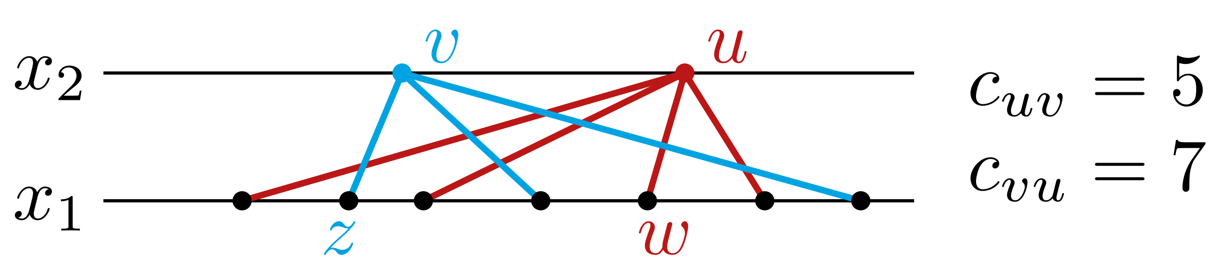

Further Properties

Def: crossing number of G with orders x_1 and x_2 for L_1 and L_2 is denoted by $\mathsf{cr}(G, x_1, x_2)$;
for fixed x_1 let $\mathrm{opt}(G, x_1) = \min_{x_2} \mathsf{cr}(G, x_1, x_2)$

Lemma: The following properties hold:

- $\mathsf{cr}(G, x_1, x_2) = \sum_{x_2(u) < x_2(v)} c_{uv}$
- $\mathrm{opt}(G, x_1) \geq \sum_{u,v \in L_2} \min\{c_{uv}, c_{vu}\}$

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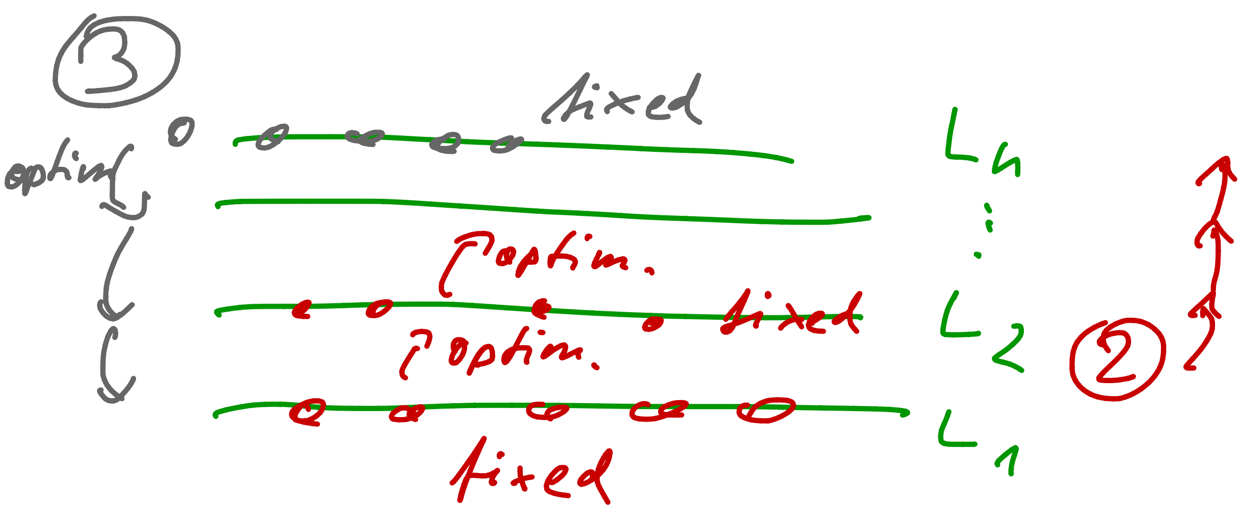
The value of $\text{cr}(G, x_1, x_2)$ can be computed in $O(n \log n)$ time in a divide-and-conquer algorithm similar to merge sort

Iterative Crossing Minimization

Let $G = (V, E)$ be a DAG with layers $L_1, \dots, L_h$.

1) compute a random ordering x_1 for layer L_1
2) for $i = 1, \dots, h-1$ consider layers L_i and L_{i+1} and minimize $\mathrm{cr}(G, x_i, x_{i+1})$ with fixed x_i ($\rightarrow$ **OSCM**)
3) for $i = h-1, \dots, 1$ consider layers L_{i+1} and L_i and minimize $\mathrm{cr}(G, x_i, x_{i+1})$ with fixed x_{i+1} ($\rightarrow$ **OSCM**)
4) repeat (2) and (3) until no further improvement happens
5) possibly repeat steps (1)–(4) with another x_1
6) return the best found solution



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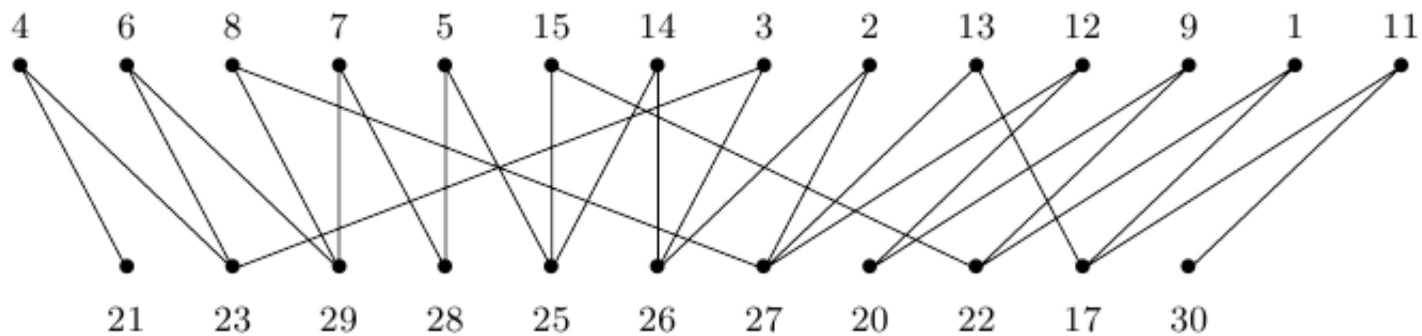
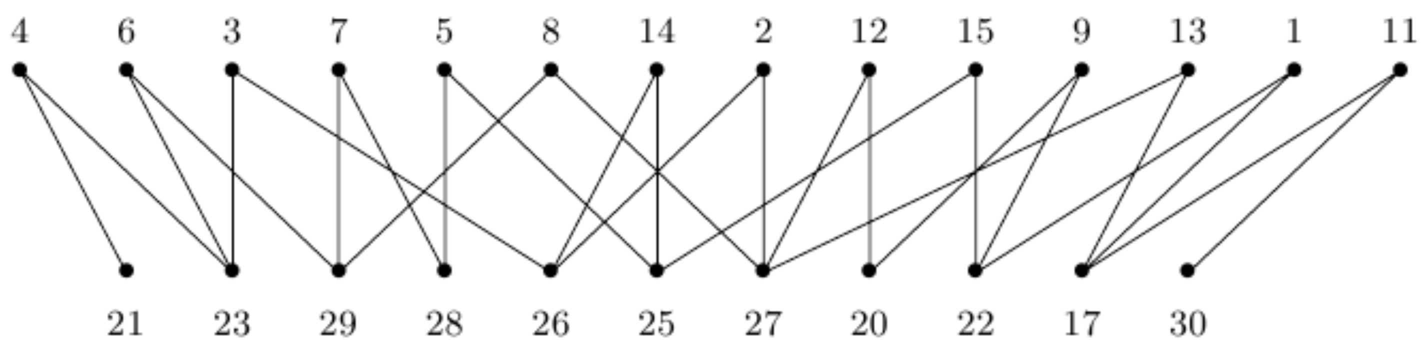
Theorem: The One-Sided Crossing Minimization (OSCM) problem is NP-hard (Eades, Wormald 1994).

Algorithms for OSCM

Heuristics:
- barycenter
- median

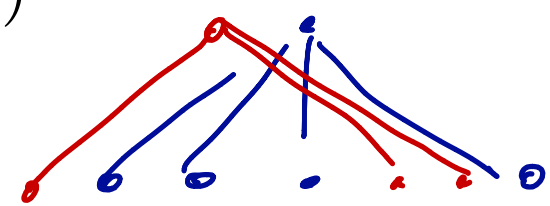
Exact:
- ILP model





Barycenter Heuristic (Sugiyama, Tagawa, Toda 1981)

Idea: few crossings when vertices are close to their neighbors

- set
$$x_2(u) = \frac{1}{\deg(u)} \sum_{v \in N(u)} x_1(v)$$


- in case of equality introduce small gap

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Properties:

- easy to implement
- fast
- usually very good results
- finds optimum if $\mathrm{opt}(G, x_1) = 0$
- may perform $\Theta(\sqrt{n})$ times worse than optimal for some graphs

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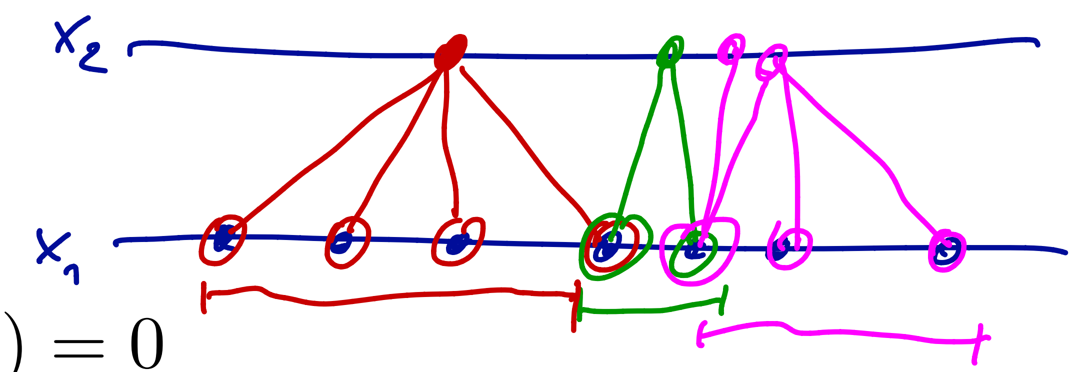

Why do we find a crossing-free solution if it exists?

Median Heuristic (Eades, Wormald 1994)

Idea: set position to median of the neighbors

- for vertex $v \in L_2$ with neighbors $v_1, \dots, v_k$ set $x_2(v) = \mathrm{med}(v) = x_1(v_{\lceil k/2 \rceil})$ or $x_2(v) = 0$ if $N(v) = \emptyset$
- if $x_2(u) = x_2(v)$ and u, v have different degree parity, place the odd degree vertex to the left
- if $x_2(u) = x_2(v)$ and u, v have the same degree parity, place an arbitrary one to the left
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Properties:

- easy to implement
- fast
- mostly good performance
- finds optimum when $\mathrm{opt}(G, x_1) = 0$
- **factor-3 approximation**

Median Heuristic: Approximation Factor

Theorem: Let $G = (L_1, L_2, E)$ be a 2-layer graph and x_1 an arbitrary ordering of L_1. Then it holds that

$$\mathrm{med}(G, x_1) \leq 3\,\mathrm{opt}(G, x_1).$$

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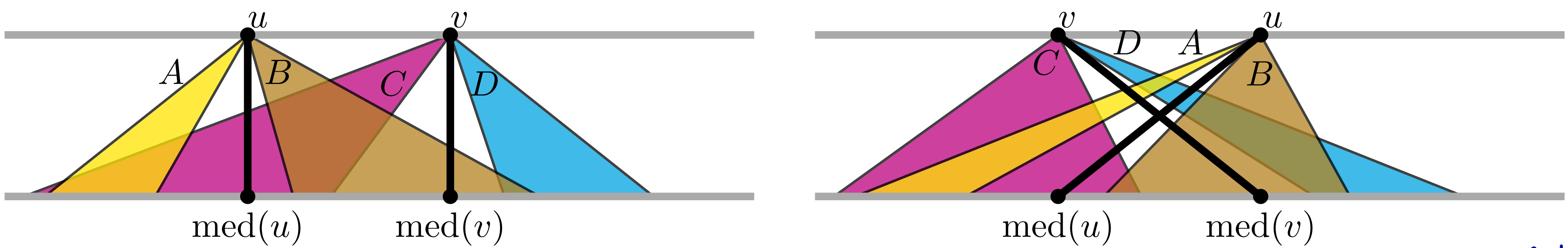
Proof: Let $u, v \in L_2$ with $x_2(u) < x_2(v)$



Let $|A| = a$, $|B| = b$, $|C| = c$, $|D| = d$

$$\varepsilon = \begin{cases} 0, & \text{if } \mathrm{med}(u) = \mathrm{med}(v) \\ 1, & \text{otherwise} \end{cases}$$

(1) $c_{vu} \geq \varepsilon + ad + a + d$

(2) $c_{uv} \leq ac + db + bc + c + b$

deg(u) odd: $a = b$ | even: $a + 1 = b$

deg(v) odd: $c = d$ | even: $c + 1 = d$

$\Rightarrow b \leq a + 1$, $c \leq d$

insert into (2)

(3) $c_{uv} \leq ad + ad + d + ad + d + d + a + 1 = 3ad + 3d + a + 1$

Median Heuristic: Approximation Factor

Proof: (cont'd)

1) $c_{vu} \geq ad + a + d + \varepsilon$
2) $c_{uv} \leq ac + bc + bd + c + b$
3) $c_{uv} \leq 3ad + 3d + a + 1$

Show: $c_{uv} \leq 3 c_{vu}$

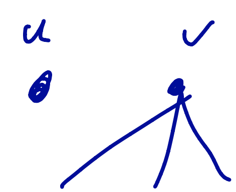
assume $c_{uv} > 3 c_{vu} \Leftrightarrow c_{uv} - 3 c_{vu} > 0$

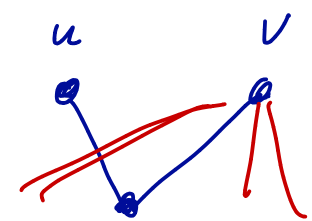
(1),(3) $\Rightarrow \cancel{3ad} + \cancel{3d} + a + 1 - \cancel{3 \cdot (ad} + a + \cancel{d} + \varepsilon) > 0$

$\Leftrightarrow 2a - 1 + 3\varepsilon < 0 \Rightarrow a = \varepsilon = 0 \Rightarrow med(u) = med(v)$

$a = 0 \Rightarrow deg(u) \leq 2$

- if $deg(u) \leq 1$ $\quad c_{uv} - 3 c_{vu} \leq 0$ ✓





$med(u) = med(v)$

- $deg(u) = 2$, $b = 1$ because u is even and left $\Rightarrow deg(v)$ even.

$c + 1 = d$

(1) $a = 0, \varepsilon = 0$
$\Rightarrow c_{vu} \geq d$

(2) $a = 0, b = 1$
$\Rightarrow c_{uv} \leq 2c + d + 1$

$$\left.\begin{array}{l} \\ \end{array}\right\} c_{uv} - 3 c_{vu} \leq 3d - 1 - 3d = -1 \leq 0 \; ↯$$

contradiction

Median Heuristic: Approximation Factor

Proof: (cont'd)

> **Lemma:** The following properties hold:
> - $\mathrm{cr}(G, x_1, x_2) = \sum_{x_2(u)<x_2(v)} c_{uv}$
> - $\mathrm{opt}(G, x_1) \geq \sum_{u,v \in L_2} \min\{c_{uv}, c_{vu}\}$

$$\mathrm{med}(G, x_1) = \mathrm{cr}(G, x_1, x_2) = \sum_{x_2(u)<x_2(v)} c_{uv} \leq 3 \cdot \sum_{x_2(u)<x_2(v)} \min\{c_{uv}, c_{vu}\} \leq 3 \cdot \mathrm{opt}(G, x_1)$$

x_2: computed order

□

Integer Linear Programming

Properties:

- branch-and-cut technique für DAGs of bounded size
- finds optimal solution
- no guarantee to find solution in polynomial time
- suitable for small to medium-size graphs

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minimize $\sum_{u<v} (c_{uv} - c_{vu}) x_{uv}$ s.t.

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- transitivity

How to model transitivity in ILP?

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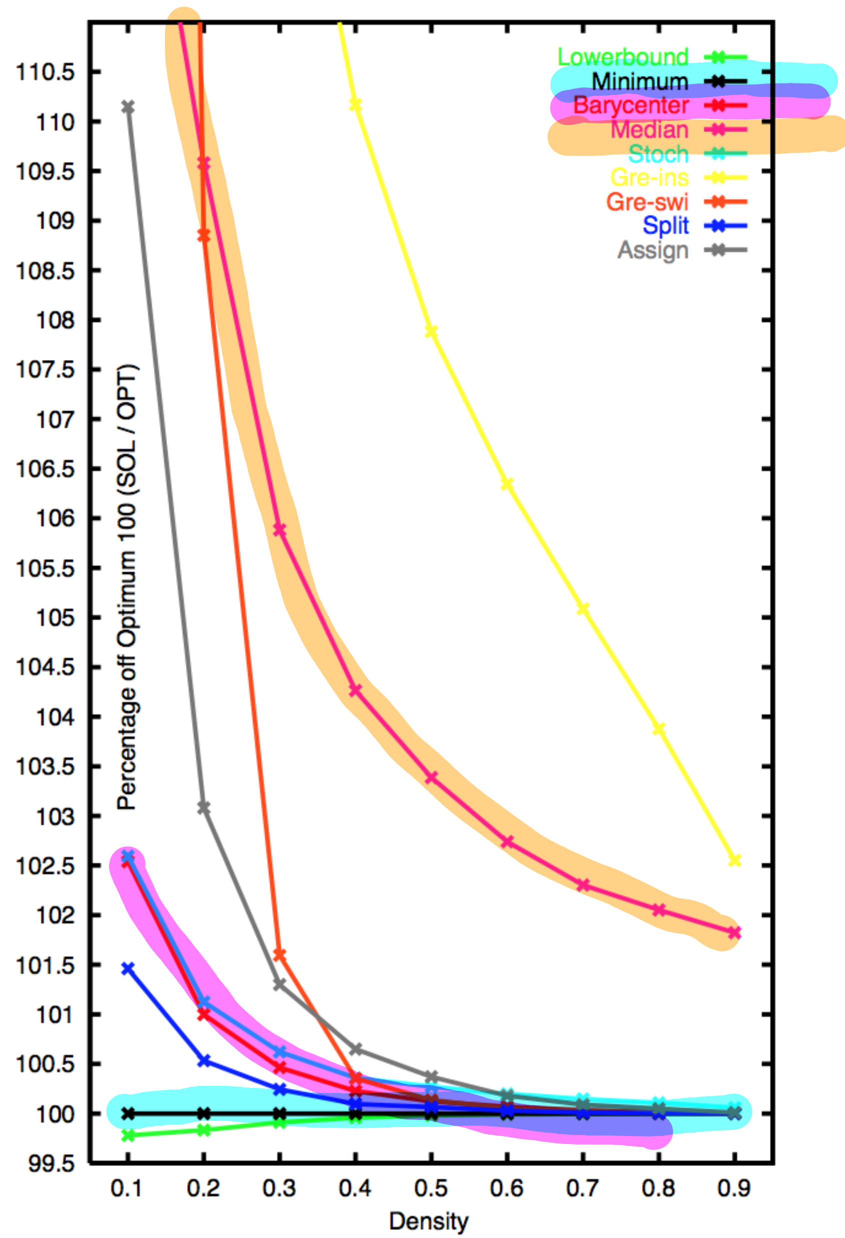
- $0 \le x_{uv} + x_{vw} - x_{uw} \le 1$ for all $u < v < w \in L_2$

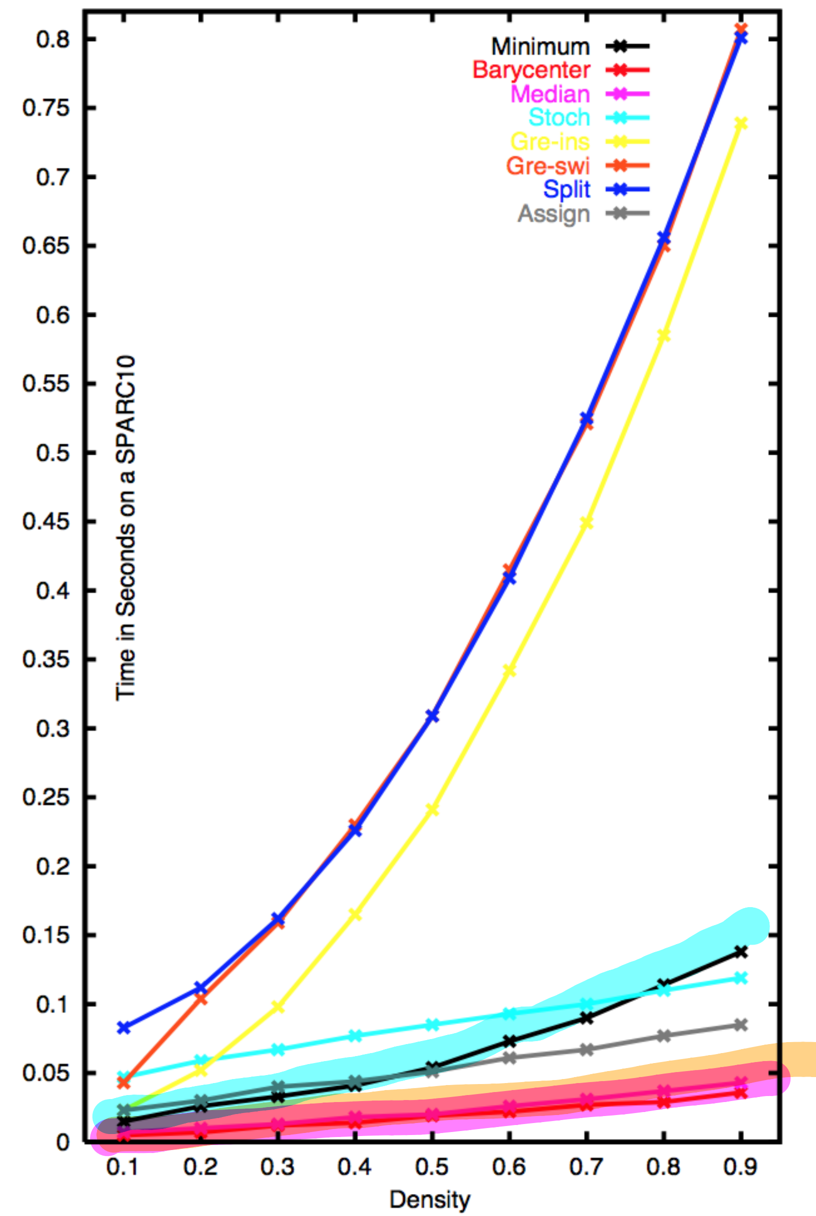
$u \quad v \quad w$

$u < v$ und $v < w$

$\Rightarrow u < w$

Experimental Evaluation (Jünger, Mutzel 1997)

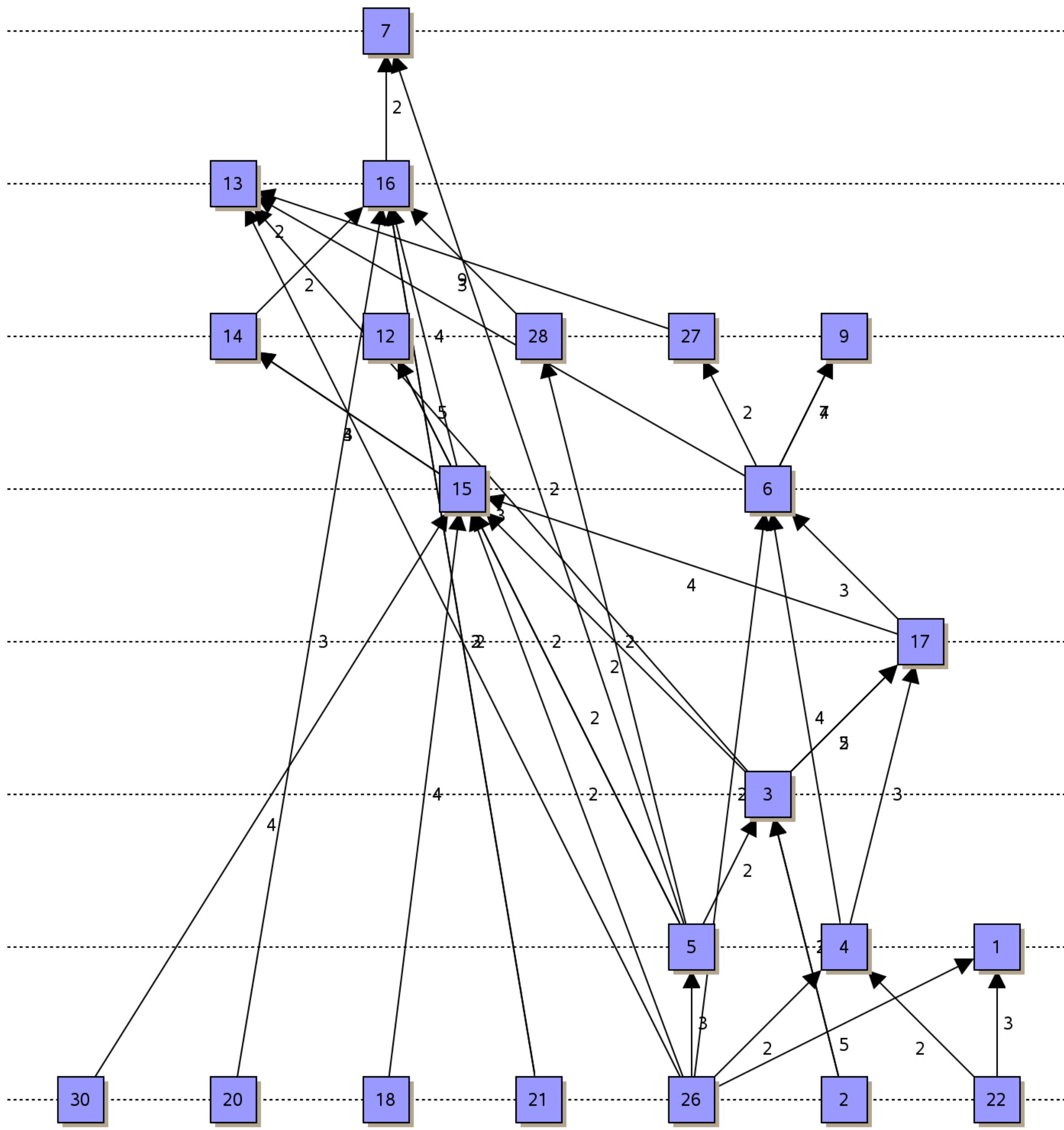




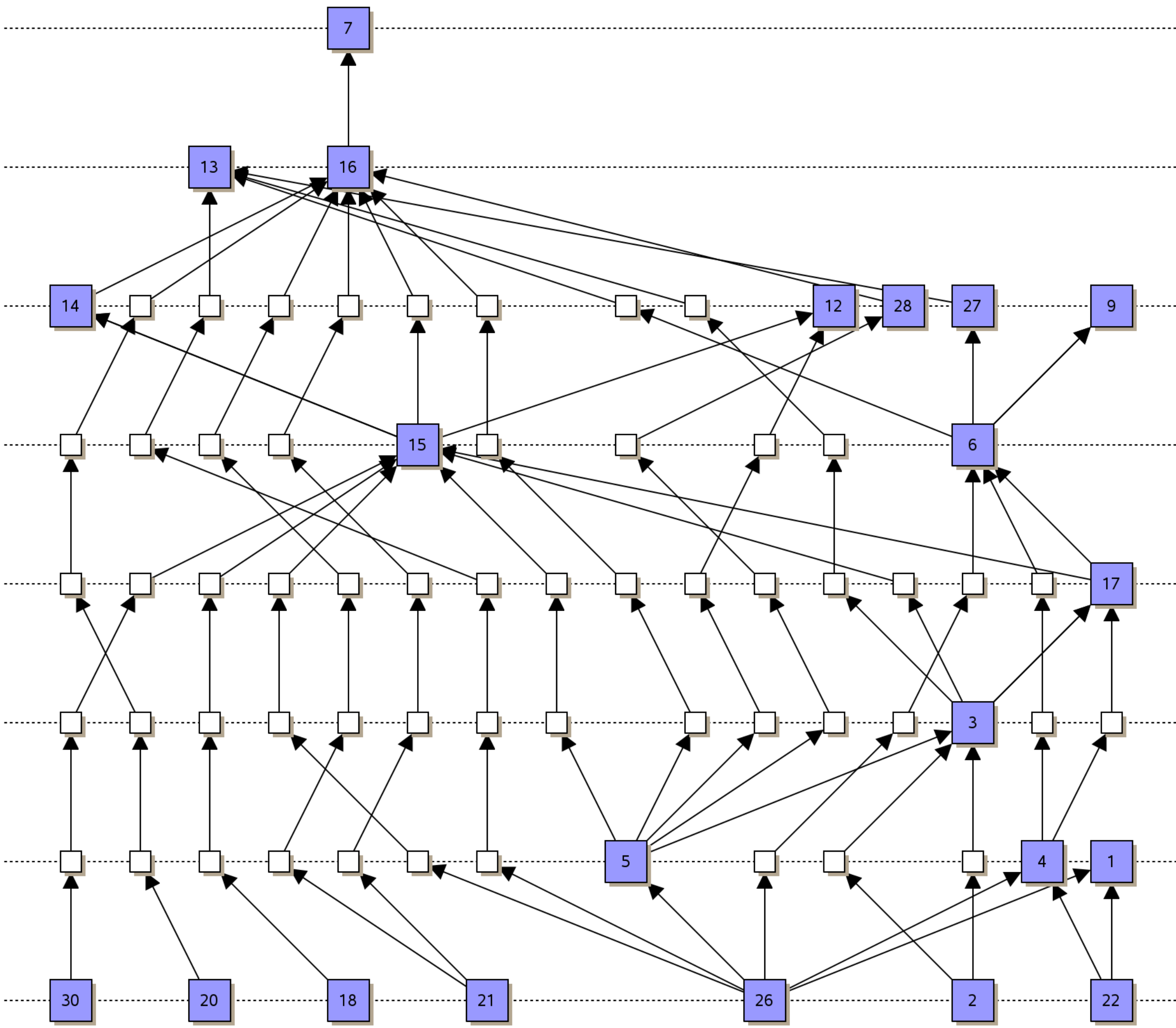
Quality averaged over 100 instances on 20 + 20 vertices bipartite graphs with increasing density

Running times on the same instances

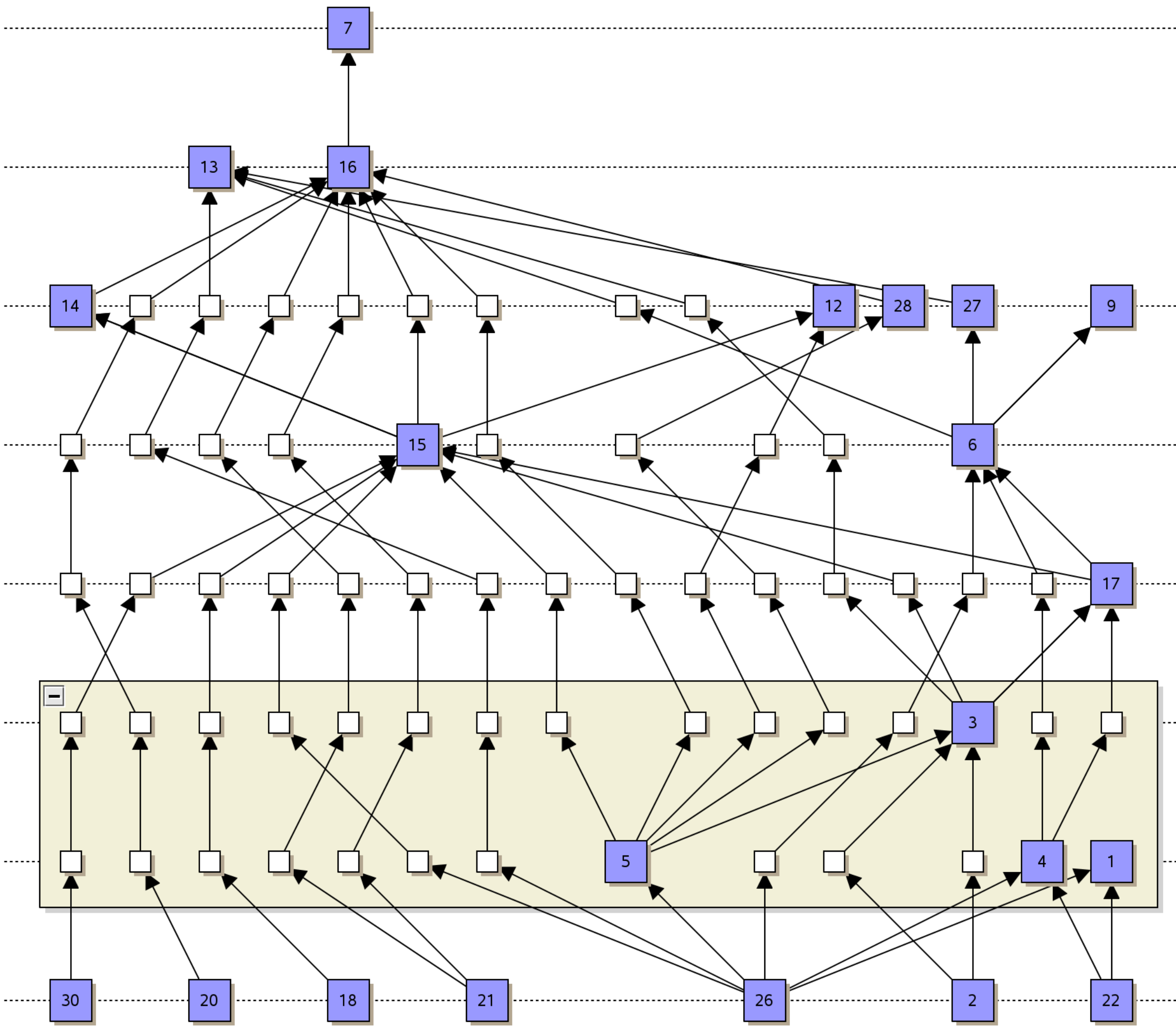
Example



Example



Example



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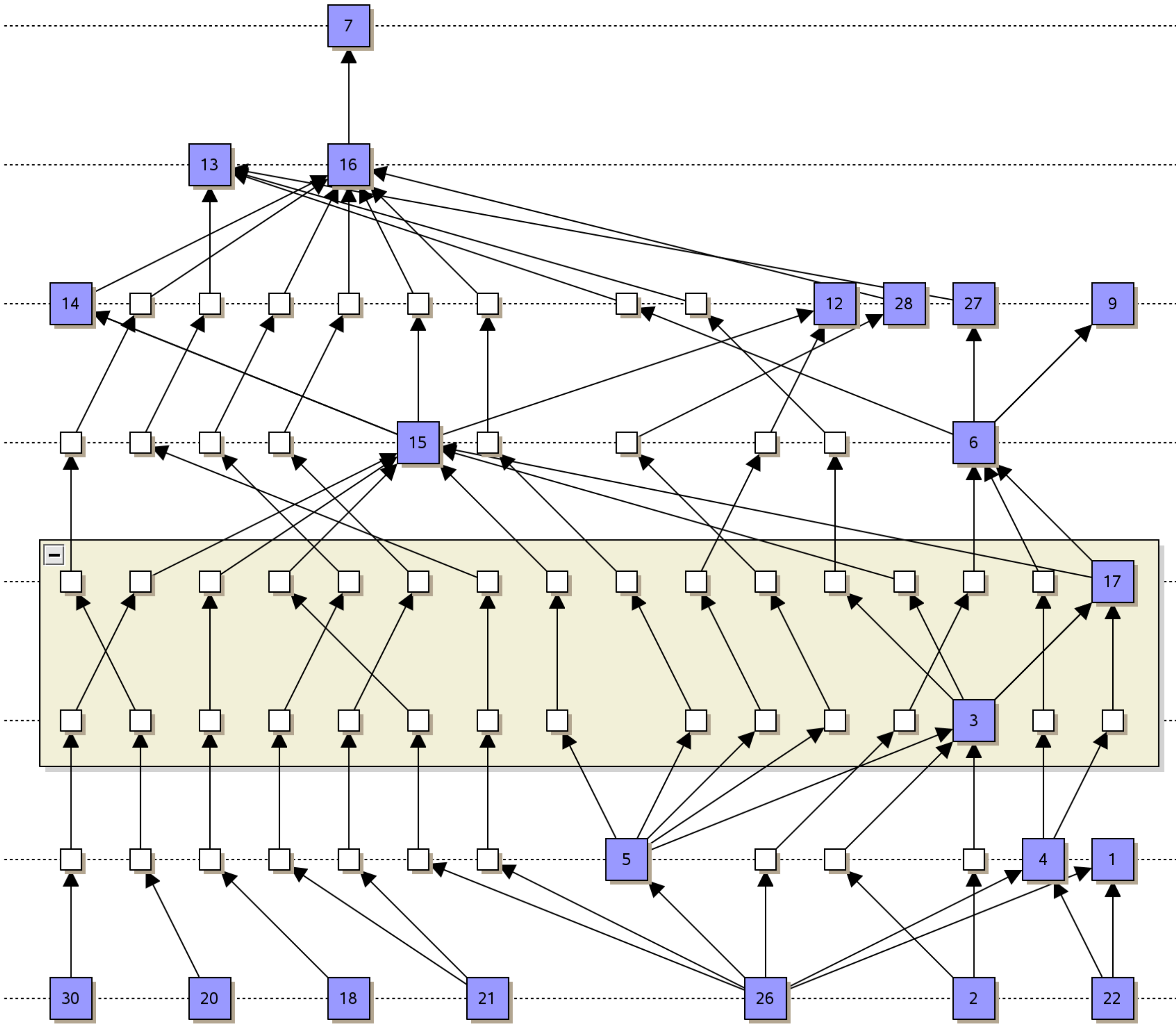

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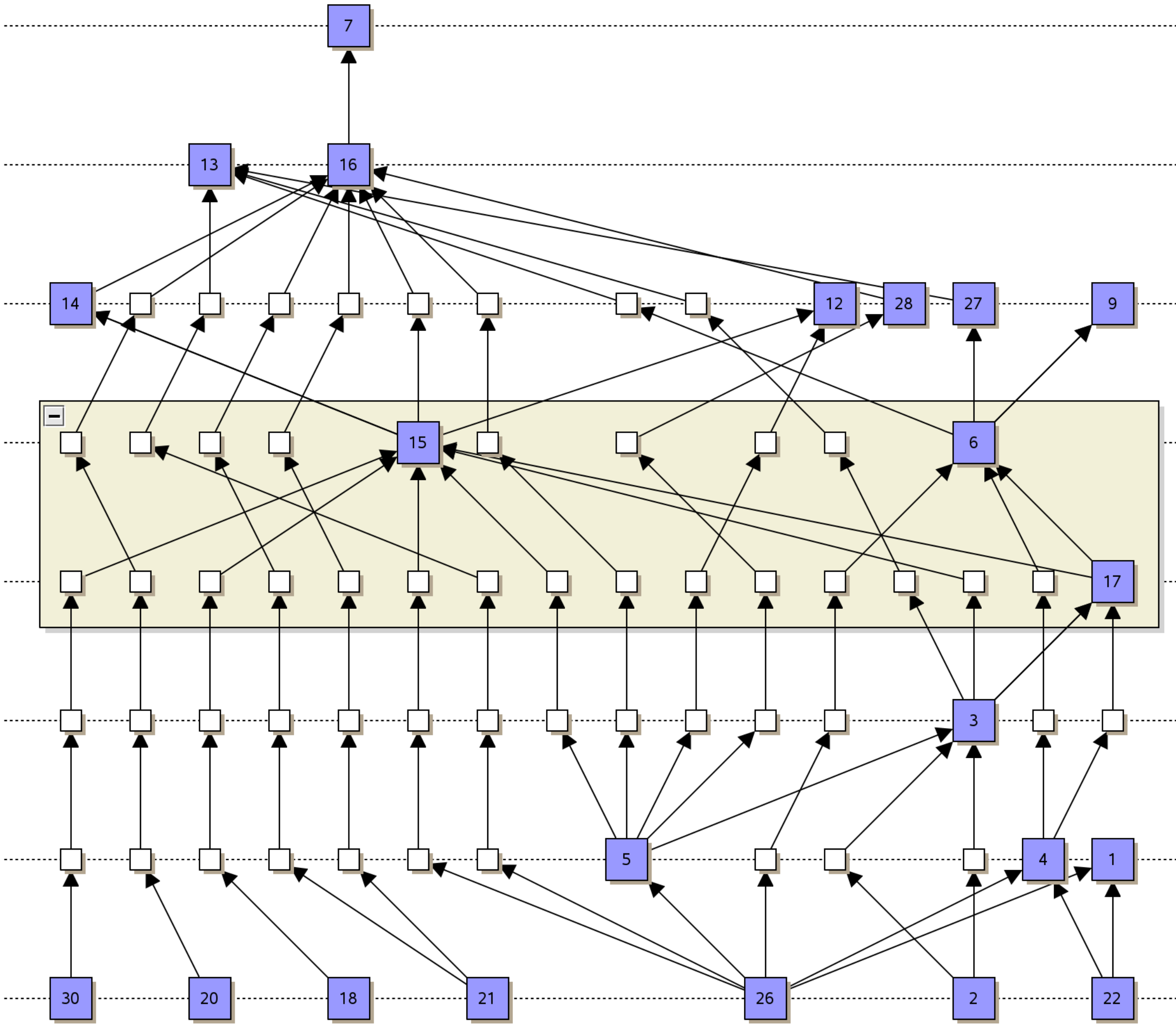

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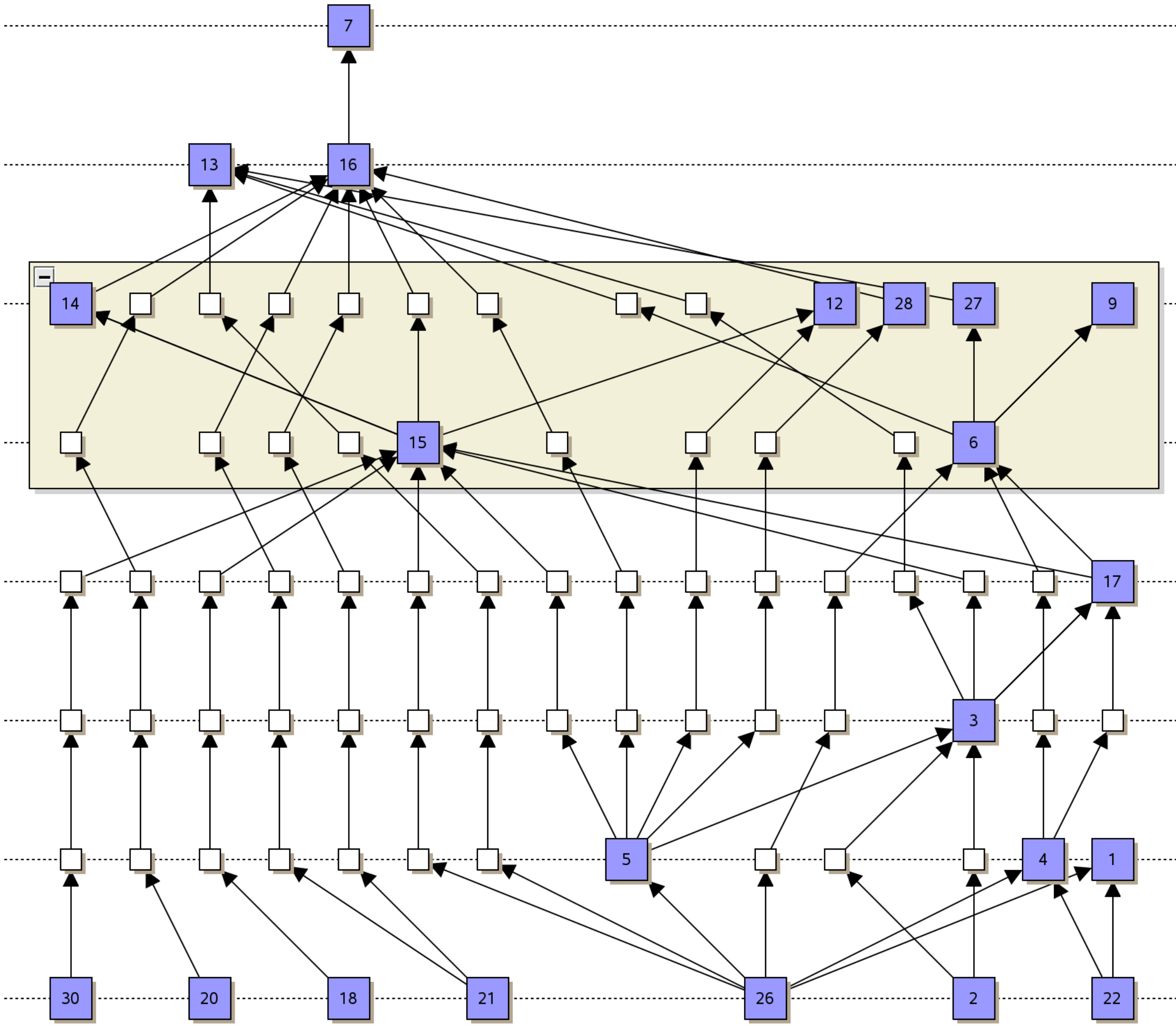

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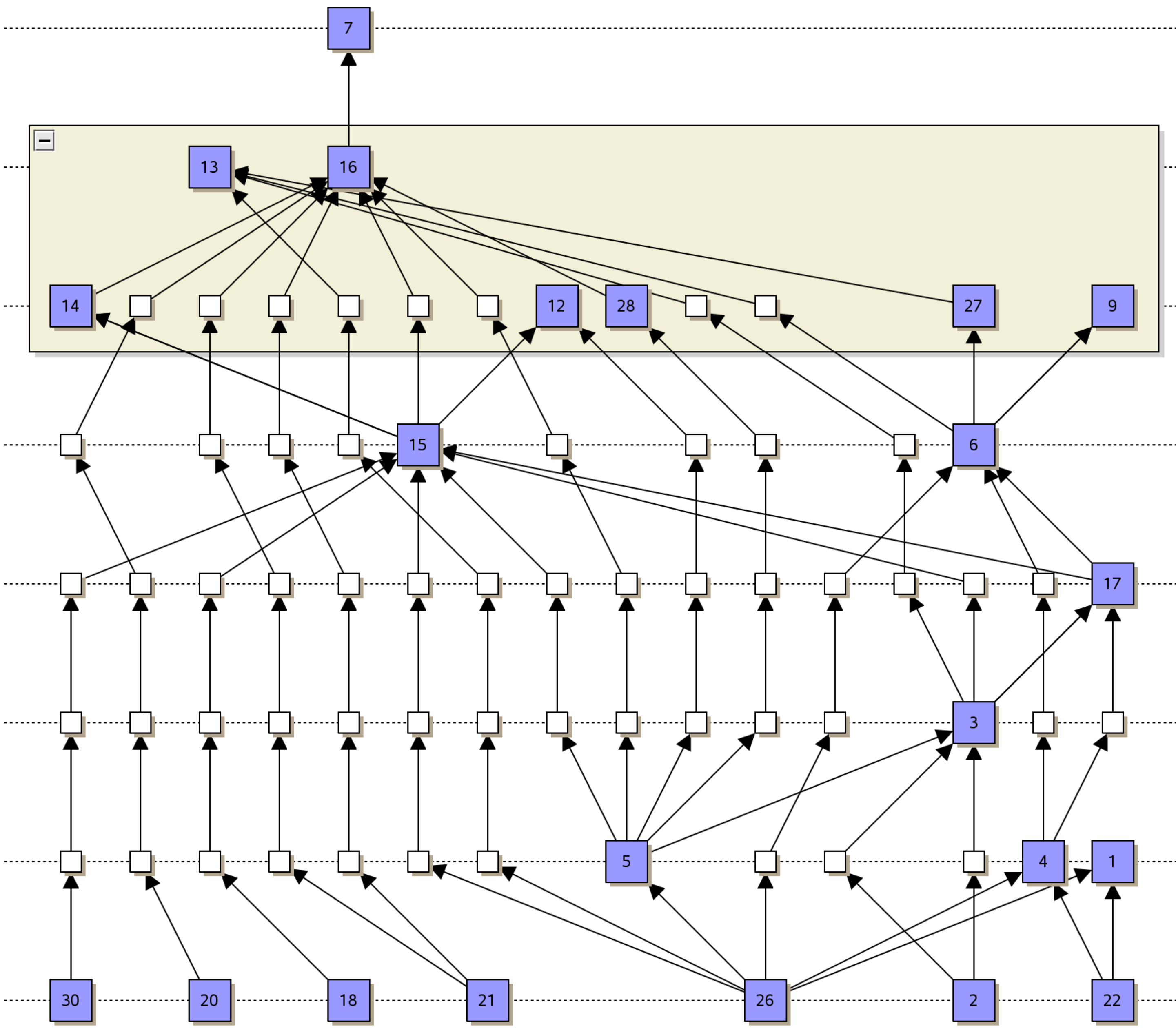

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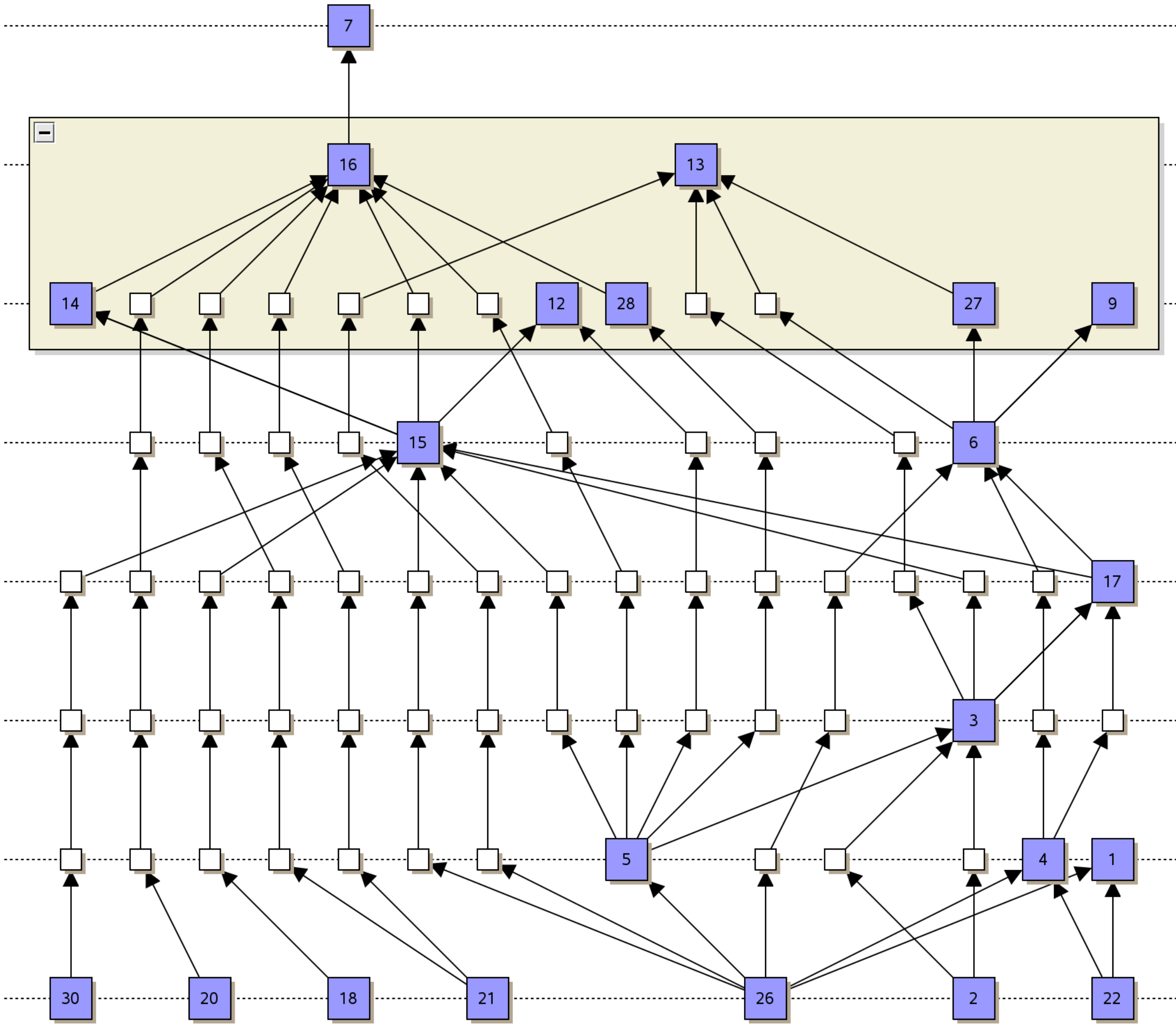

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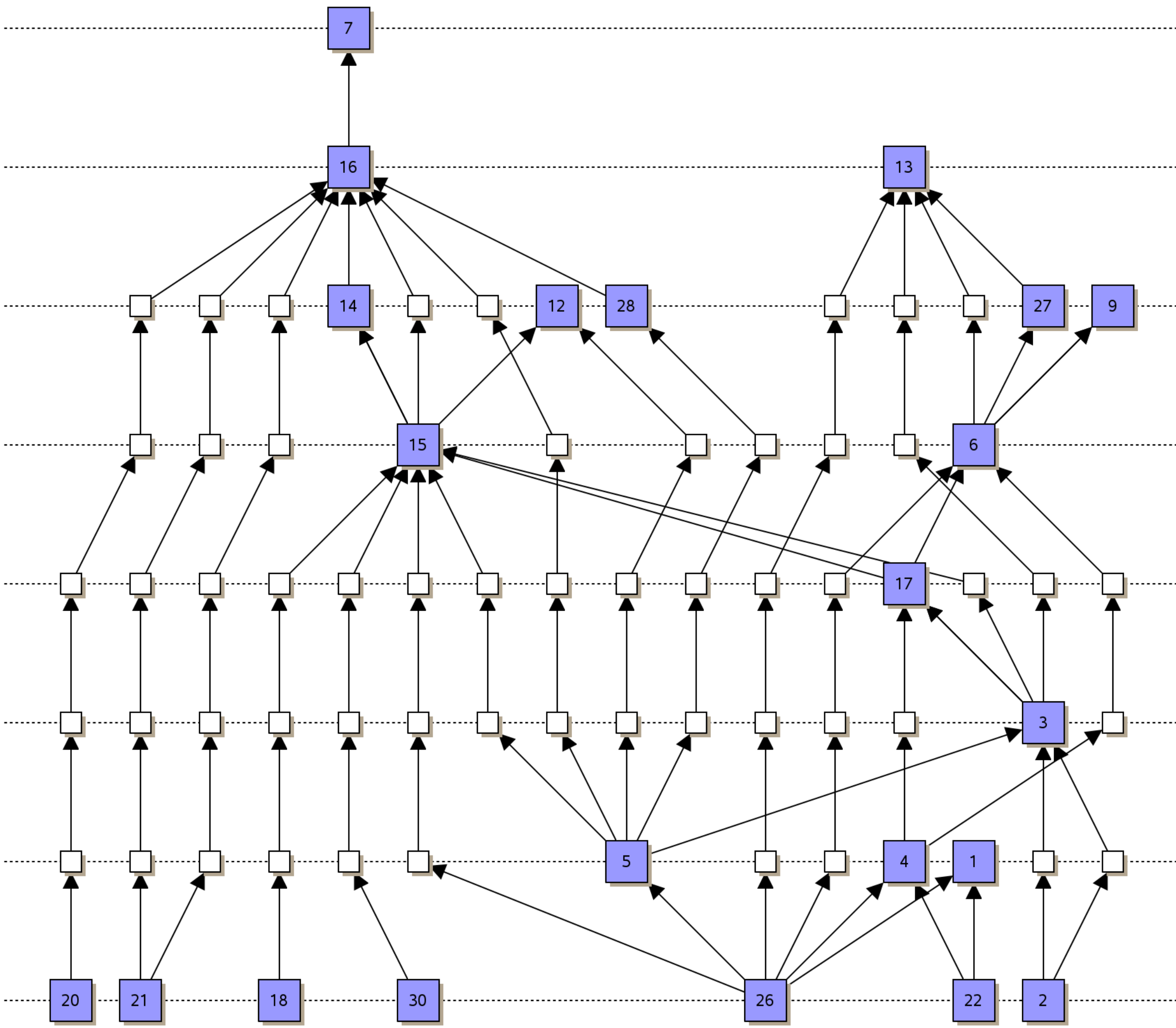
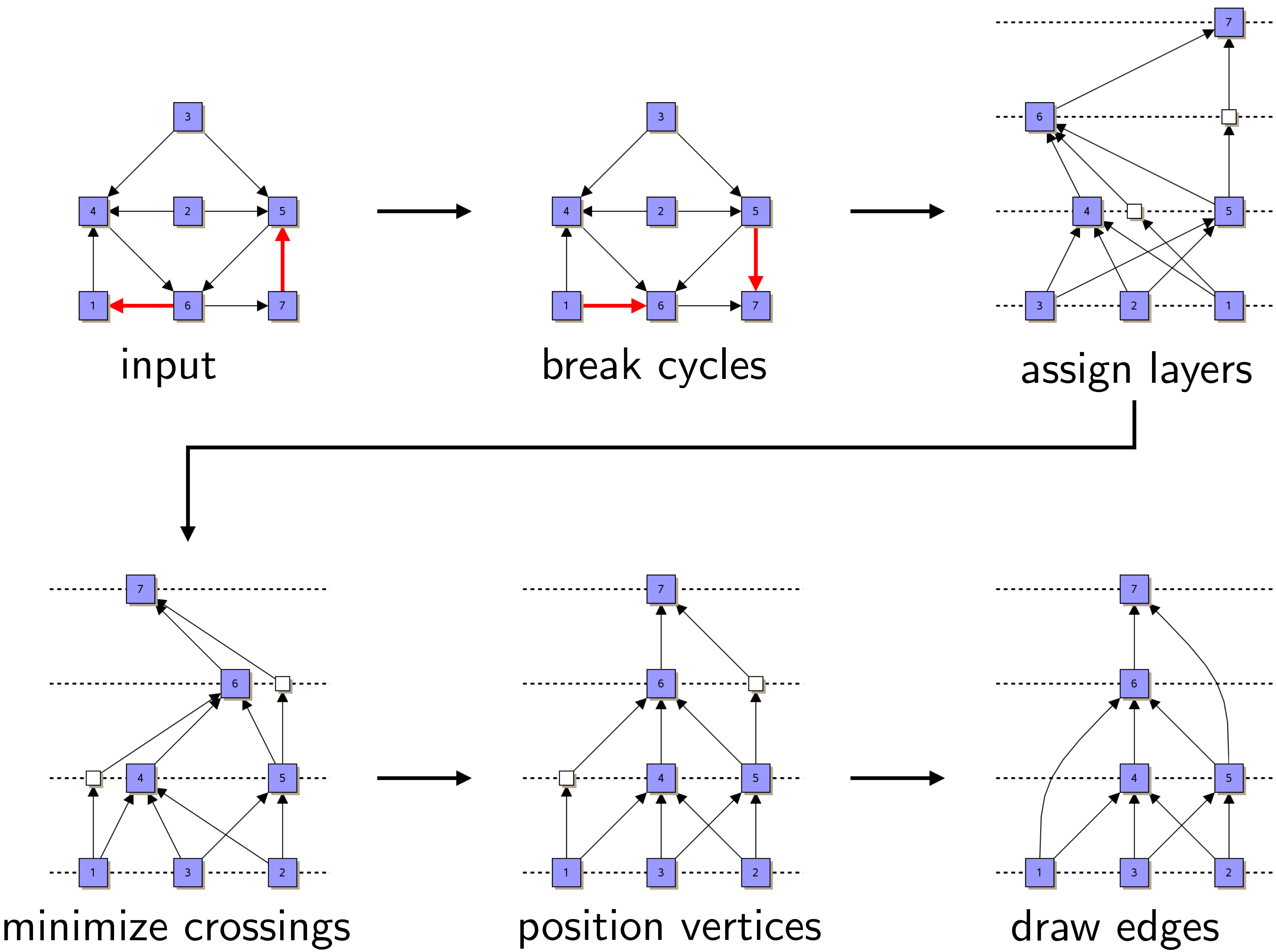
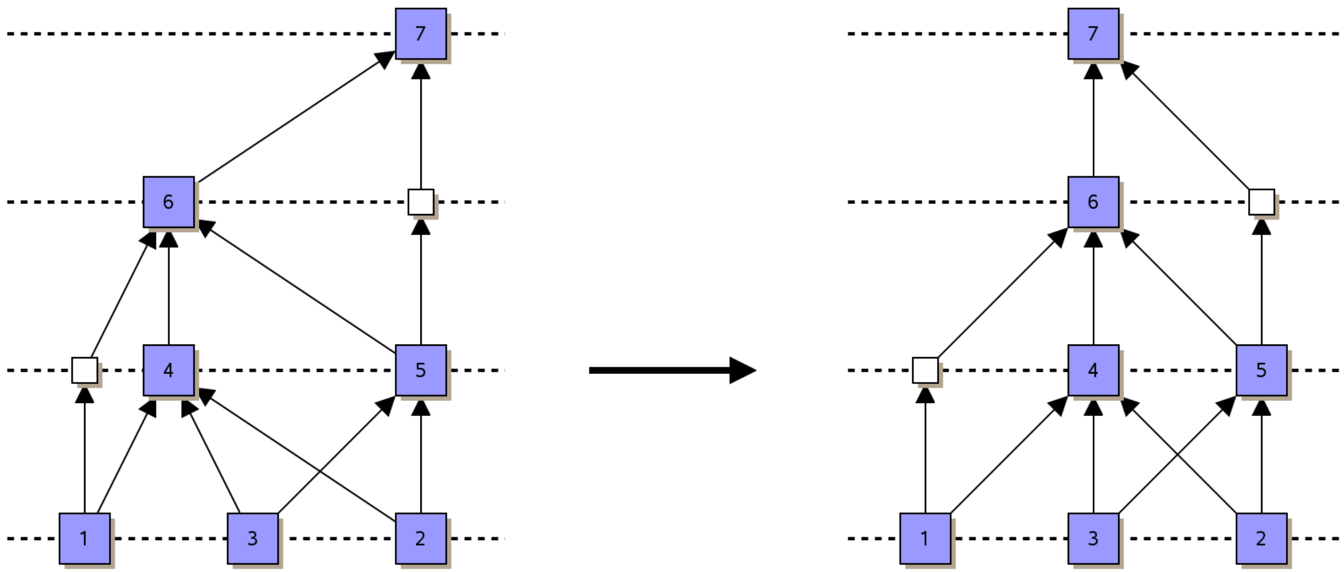


Overview



input
break cycles
assign layers
minimize crossings
position vertices
draw edges

Step 4: Coordinate Assignment



What are the goals?

Edge Straightening

Goal: minimize deviation from straight line for edges with dummy vertices

Idea: quadratic programming

- let $p_{uv} = (u, d_1, \ldots, d_k, v)$ be a path with k dummy vertices between u and v
- let $a_i = x(u) + \frac{i}{k+1}(x(v) - x(u))$ be the x-coordinate of d_i assuming straight line
- minimize $\sum_{i=1}^{k}(x(d_i) - a_i)^2$ over all paths
- constraints: $x(w) - x(z) \geq \delta$ for all vertices on the same layer with w right of z (δ is a spacing parameter)

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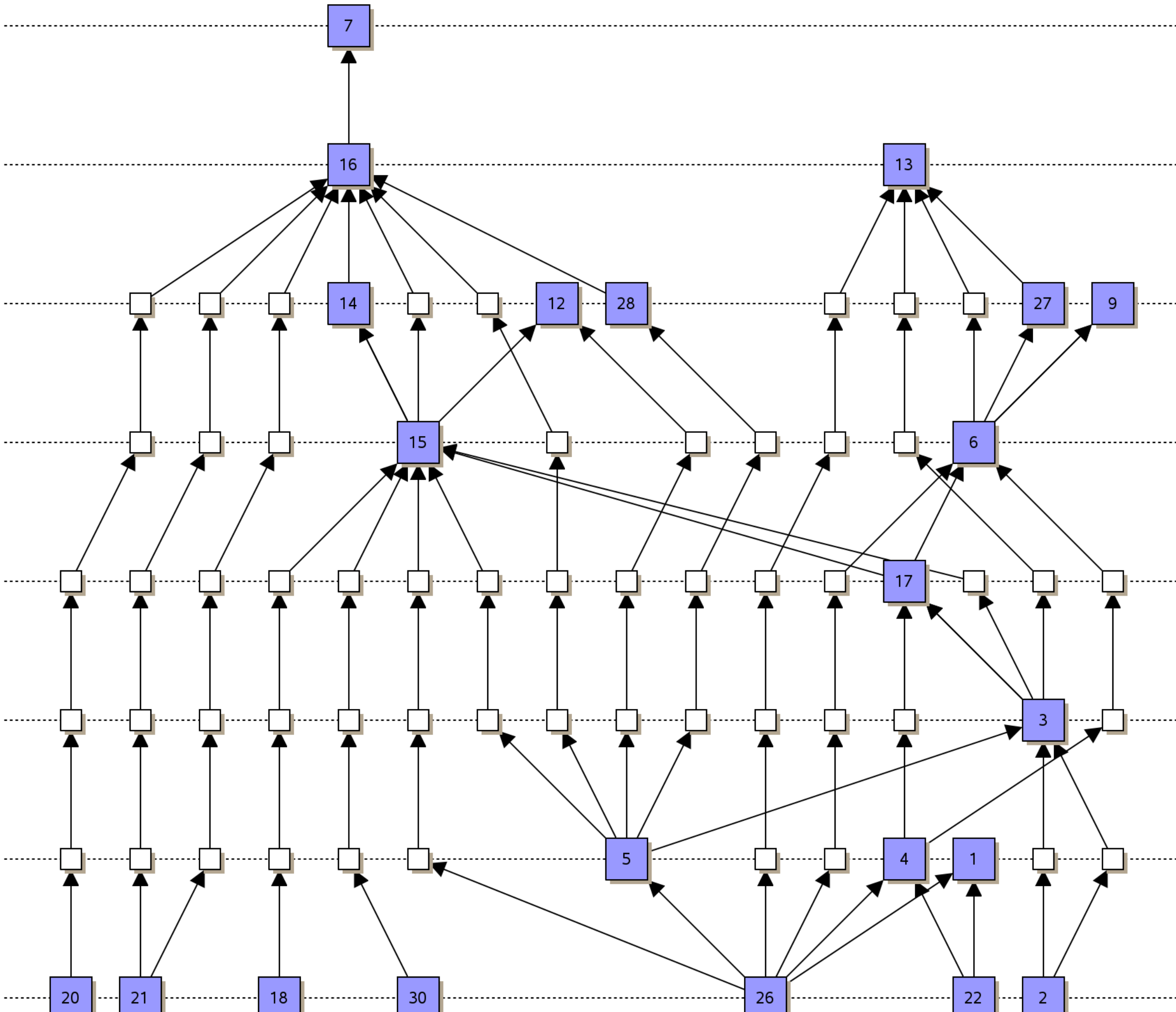
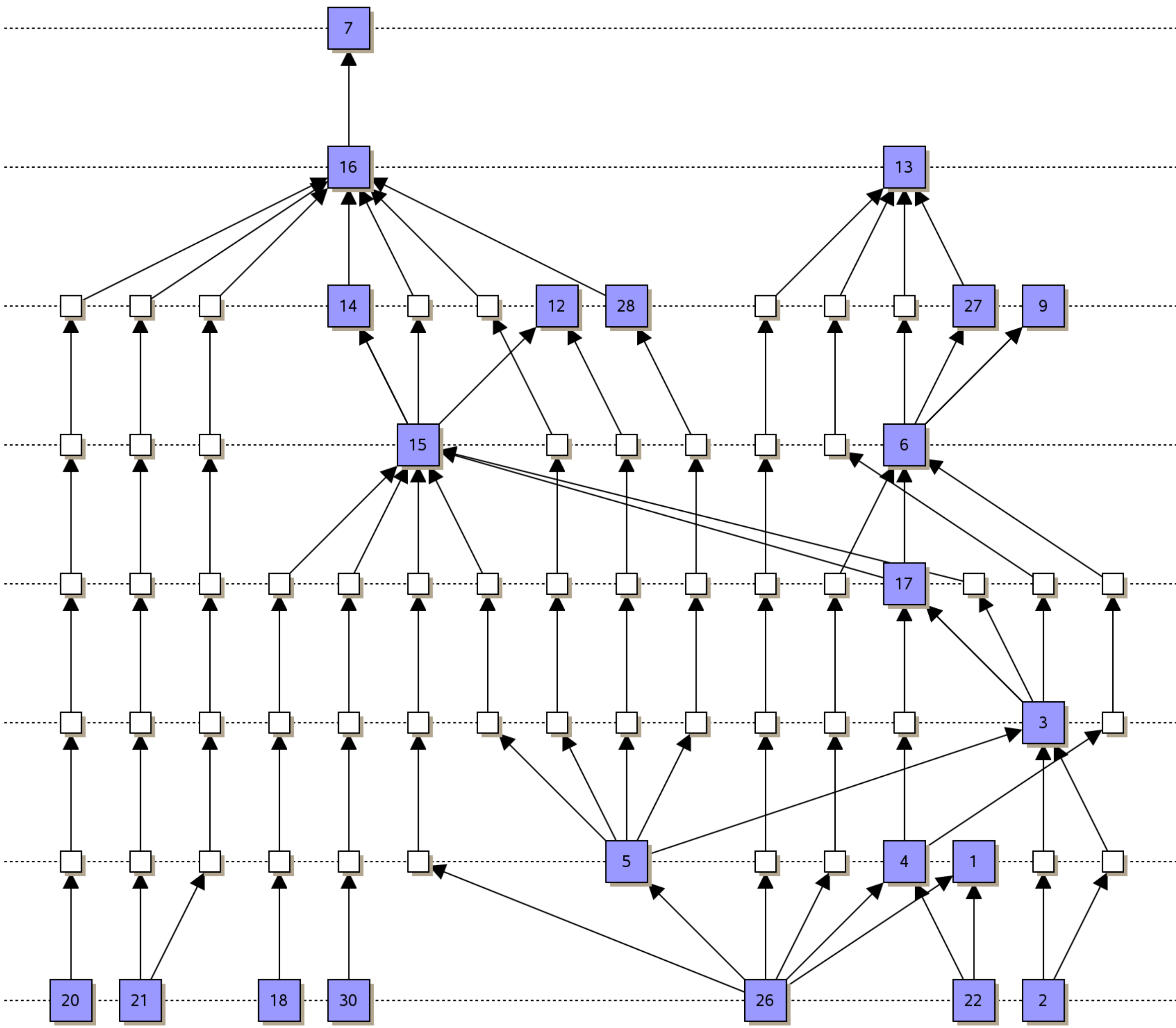
Properties:

- solving quadratic program often time expensive
- width can grow exponentially
- objective function can be modified for optimizing verticality

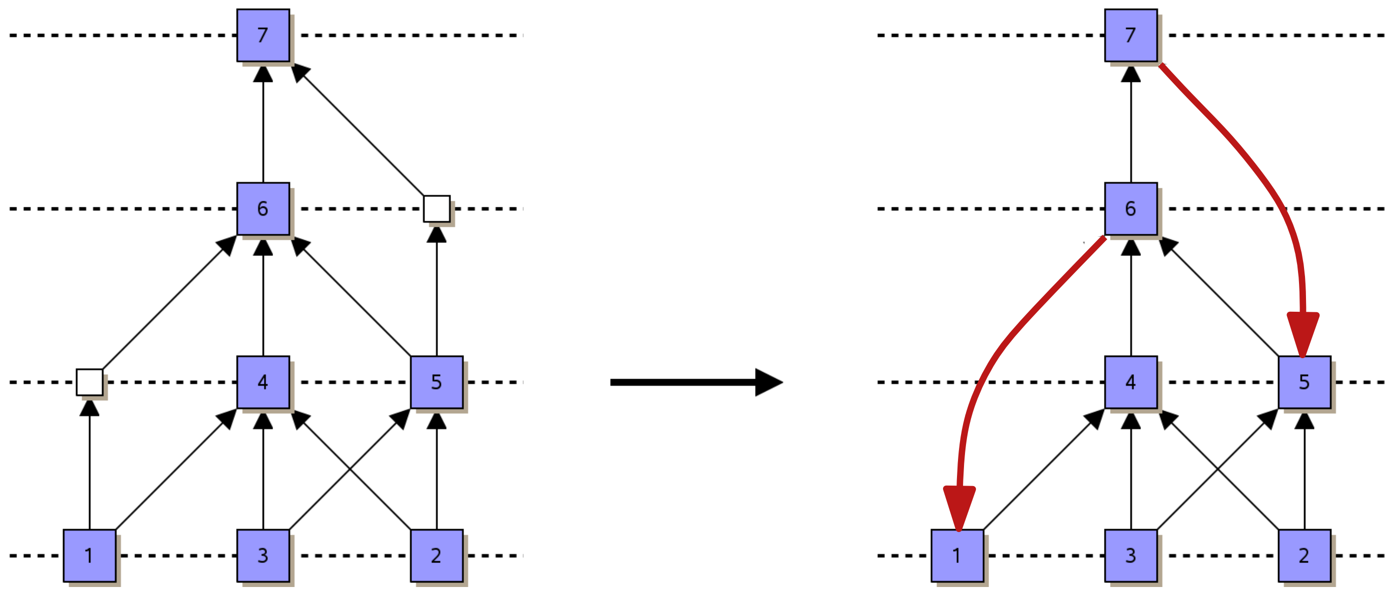
Example



Example

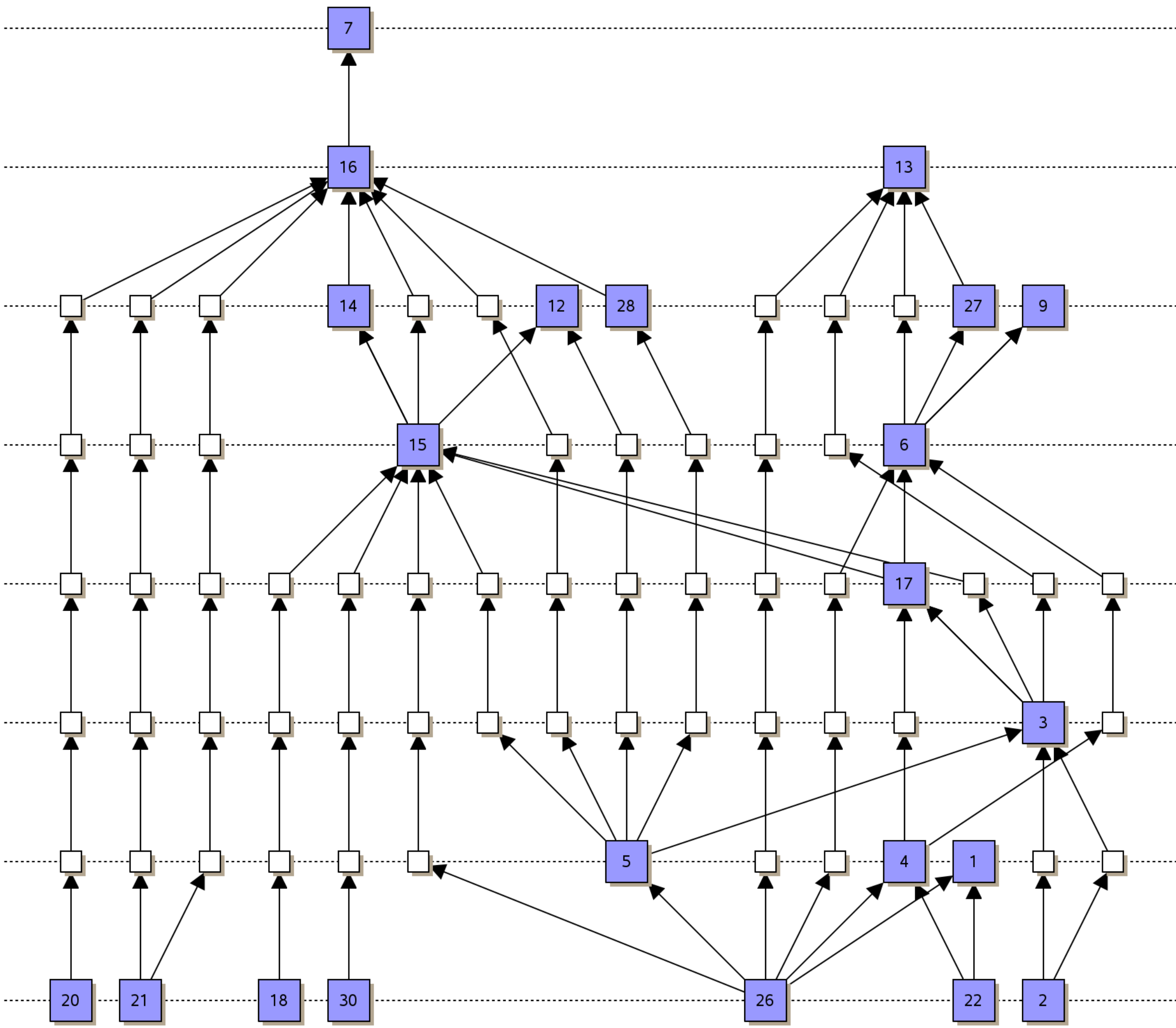


Step 5: Drawing edges

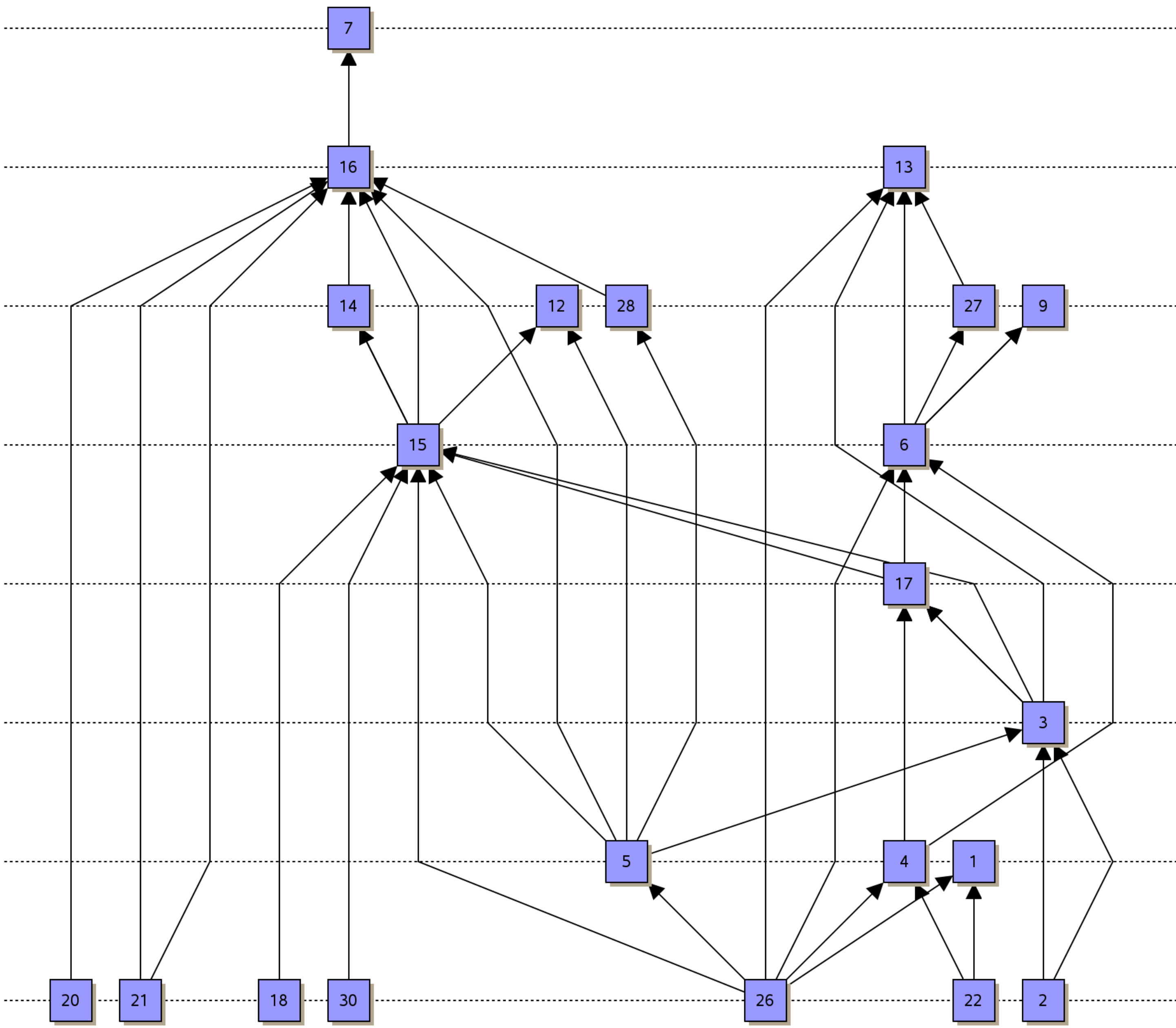


- postprocessing: optionally substitute polylines by Bézier curves
- draw all edges in original orientation

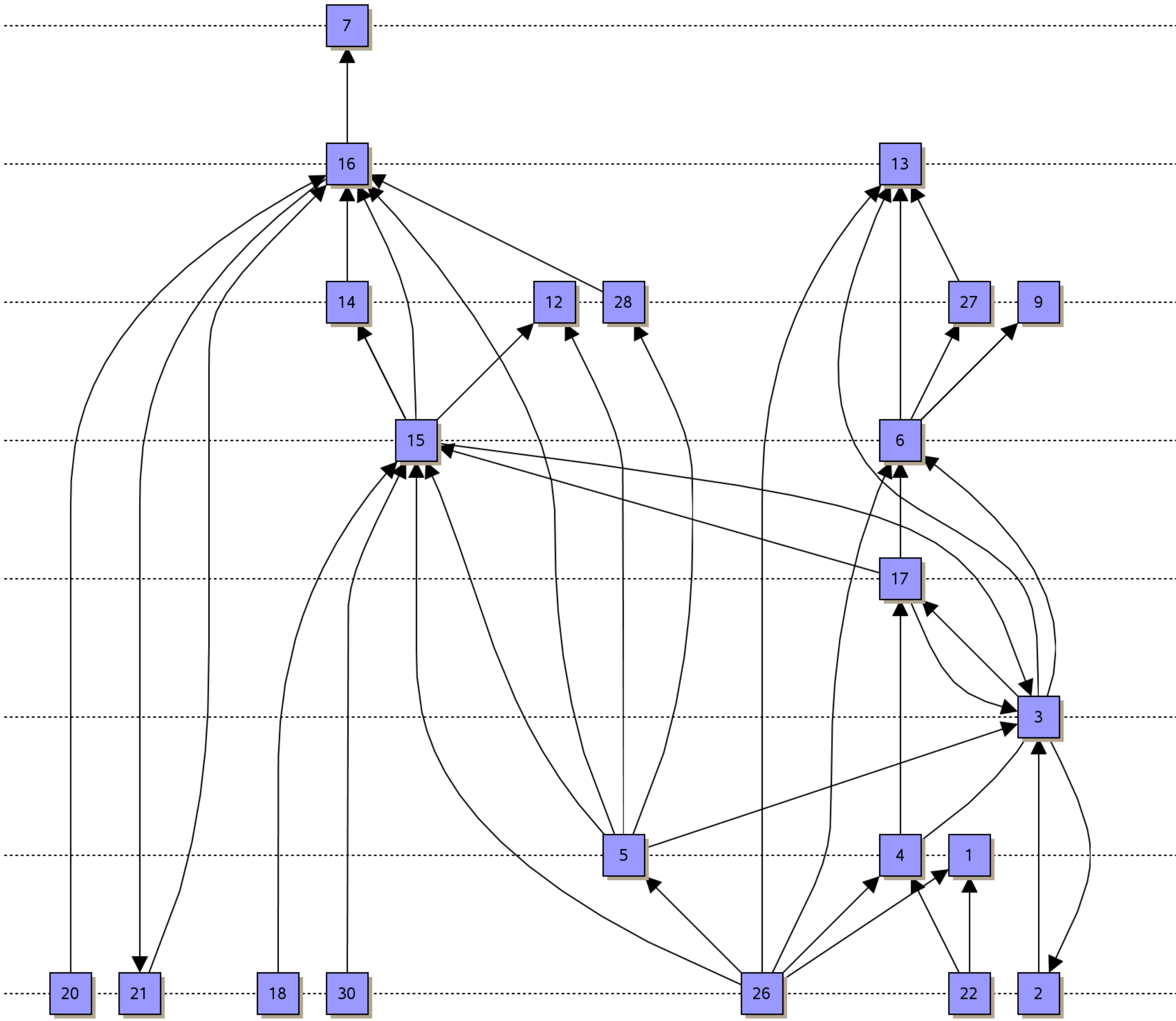
Example



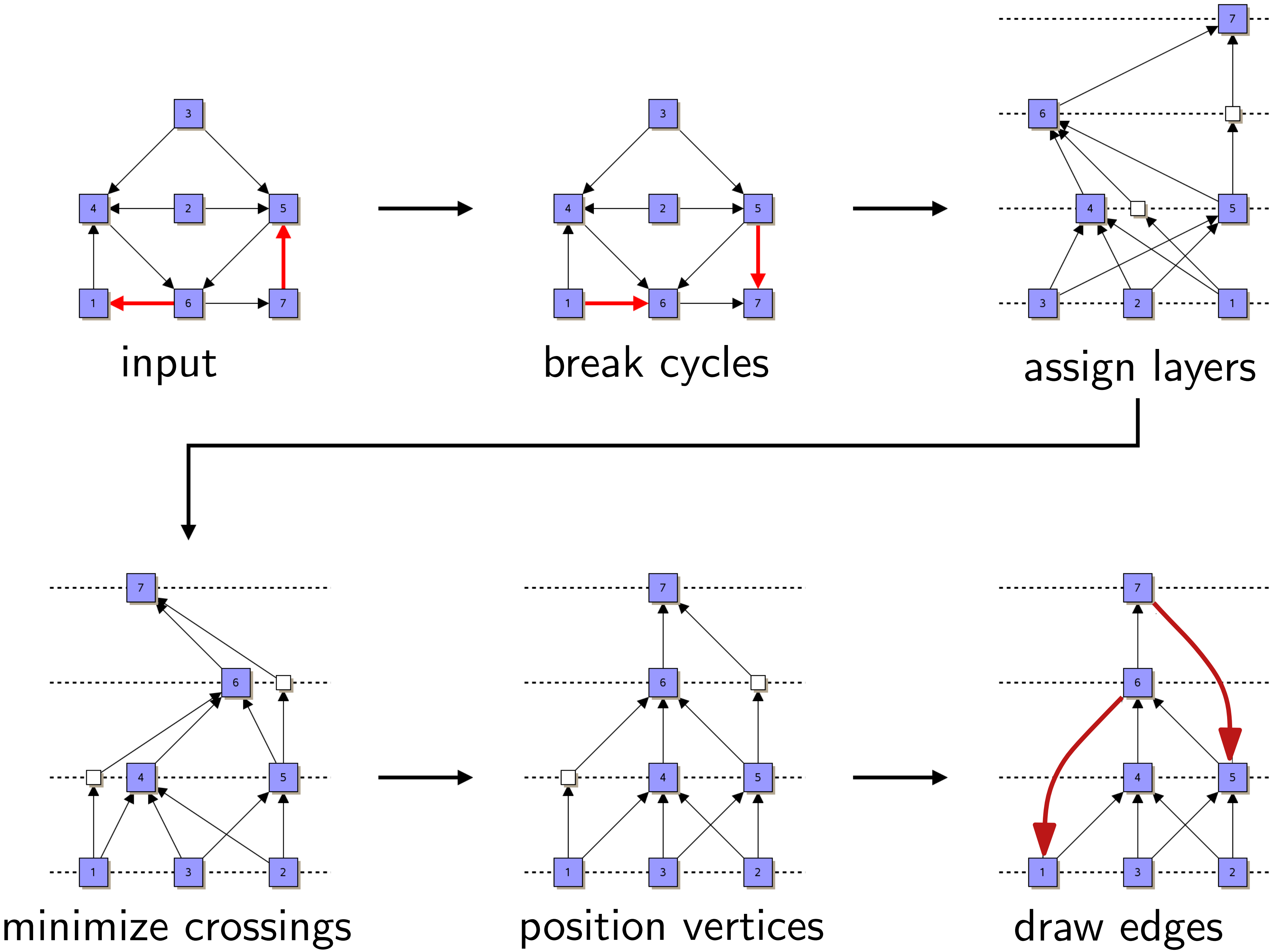
Example



Example



Summary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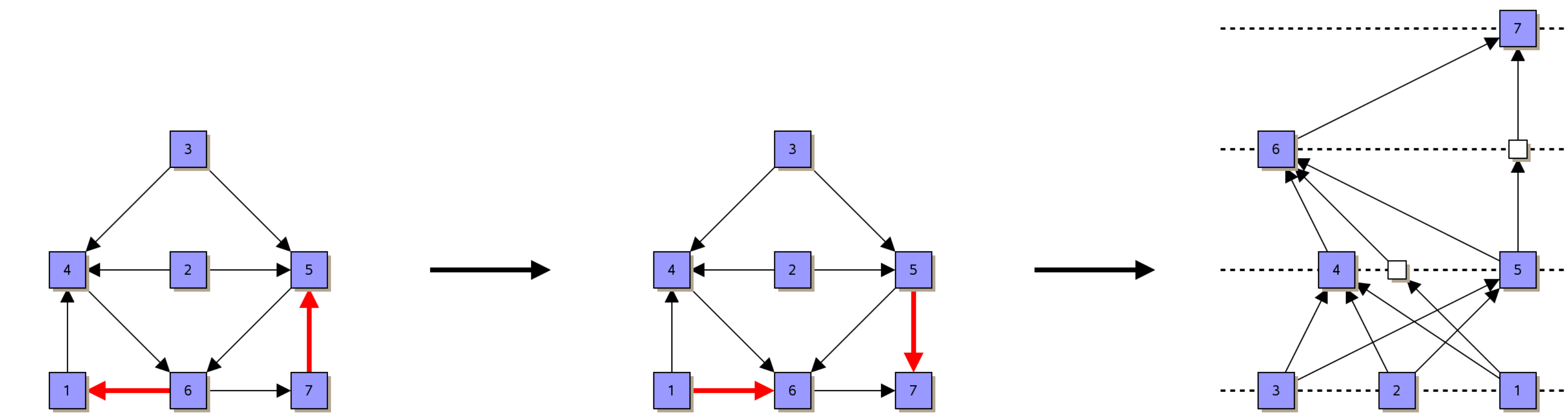
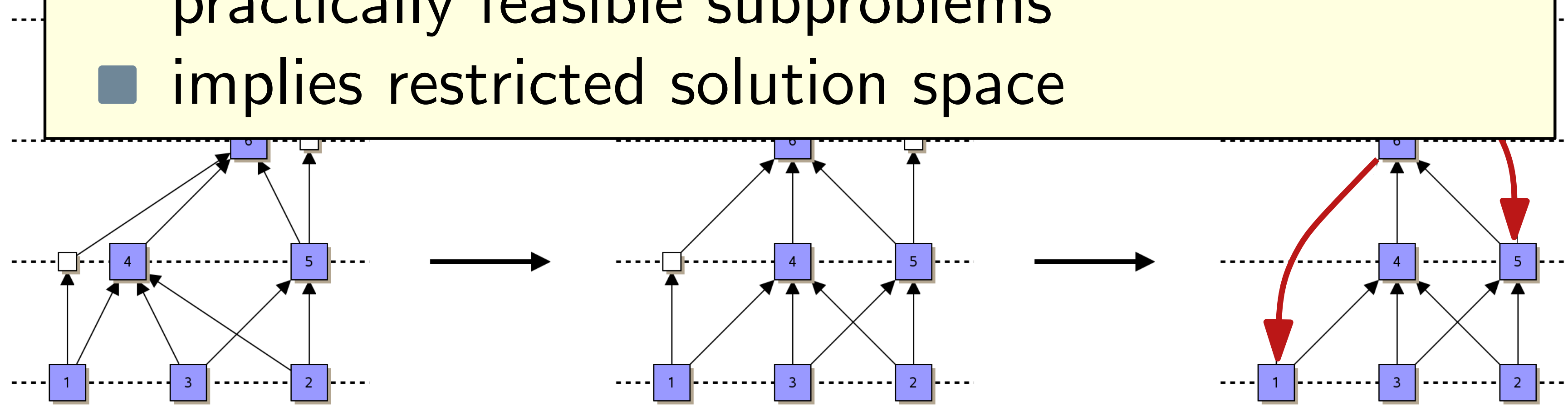


- flexible framework for drawing directed graphs
- sequential optimization of various criteria
- decomposition into often NP-hard but still practically feasible subproblems
- implies restricted solution space



minimize crossings position vertices draw edges

Announcements

- The class on June 12 is shifted to Monday, June 11 in the same time slot 9:00–11:00 in seminar room 186.
- Student presentations from exercise groups will take place on July 3, 10:00–12:00 and 13:00–15:00. Attendance required.
- Oral exam dates are July 10 and September 18. Registration in TISS will open June 1.