Scaling and Transforming Data

Apples to Oranges: Comparing Test Scores

In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

#Let's say we're trying to compare how students perform on college entrance exams at tw
#schools in the US. One is in California, where a large majority of their students tend
#the other is in Illinois, where students favor taking the ACT. Is there anyway we see
#stack up to one another?

#Sunnydale High vs. Shermer High
SAT_scores = [690, 330, 600, 350, 540, 440, 650, 480, 570, 420, 360, 620, 580, 600,
 390, 420, 510, 640, 350, 470, 570, 430, 410, 420, 380, 420, 510, 620,
 470, 700, 520, 560, 480, 540, 450, 550, 520, 460, 410, 550, 400, 350,
 780, 590, 510, 410, 520, 340, 430, 370, 560, 560, 500, 560, 490, 550,
 430, 520, 710, 520, 460, 390, 550, 410, 480, 450, 520, 610, 380, 620,
 530, 460, 460, 660, 520, 580, 490, 560, 520, 380, 440, 610, 530, 350,
 630, 440, 450, 590, 430, 640, 500, 290, 560, 390, 320, 470, 700, 540,
 440, 550]
ACT_scores = [24, 18, 32, 23, 22, 26, 18, 23, 17, 28, 15, 20, 20, 17, 19, 24, 17, 29,
 21, 31, 22, 13, 17, 17, 26, 16, 25, 30, 26, 14, 14, 22, 14, 29, 26, 27,
 25, 20, 19, 17, 31, 20, 20, 25, 19, 24, 23, 24, 24, 23, 17, 18, 21, 26,
 21, 21, 28, 22, 22, 21, 18, 10, 16, 25, 31, 23, 24, 18, 28, 18, 20, 23,
 22, 17, 16, 17, 29, 25, 18, 19, 20, 22, 29, 18, 17, 24, 15, 33, 30, 17,
 11, 25, 24, 20, 21, 21, 29, 25, 22, 18]

columns = ["SAT", "ACT"]
score_df = pd.DataFrame(np.array([SAT_scores, ACT_scores]).T, columns=columns)
print(score_df.head())
```

```
   SAT  ACT
0  690   24
1  330   18
2  600   32
3  350   23
4  540   22
```

What is a transformation?

To do this, we're going to need to transform the data in some way. Specifically, when I talk about a transformation, all that means is that we're going to apply some function f(x) to each input, and get our new outputs. Which means, something as simple as x + 0 counts as a transformation, as does the much more complicated expression below.

$$\hat{f}(k) = \int_{-\infty}^{\infty} f(x)e^{-2\pi ikx}dx$$

Of course, adding zero isn't a particularly *useful* transformation. As for the second one, it's certainly useful, but not something you'll have to worry about in this course. I'll be sure to point out all the essential transformations you're going to run across when reading other peoples' analyses, and provide you with the tools necessary to get started on your own. Now, relating transformations back to our original question regarding test scores...

Scaling Techniques

Max-Min Normalization

$$x' = \frac{x - \min(x)}{\max(x) - \min(x)}$$

However, in our particular case, it might be more helpful to look at Standardization, or in other words, compute z-scores. Recall that this just captures the difference between your data point and the mean, relative to the spread of the overall distribution.

Standardization (z-score)

$$x' = \frac{x - \mu}{\sigma}$$

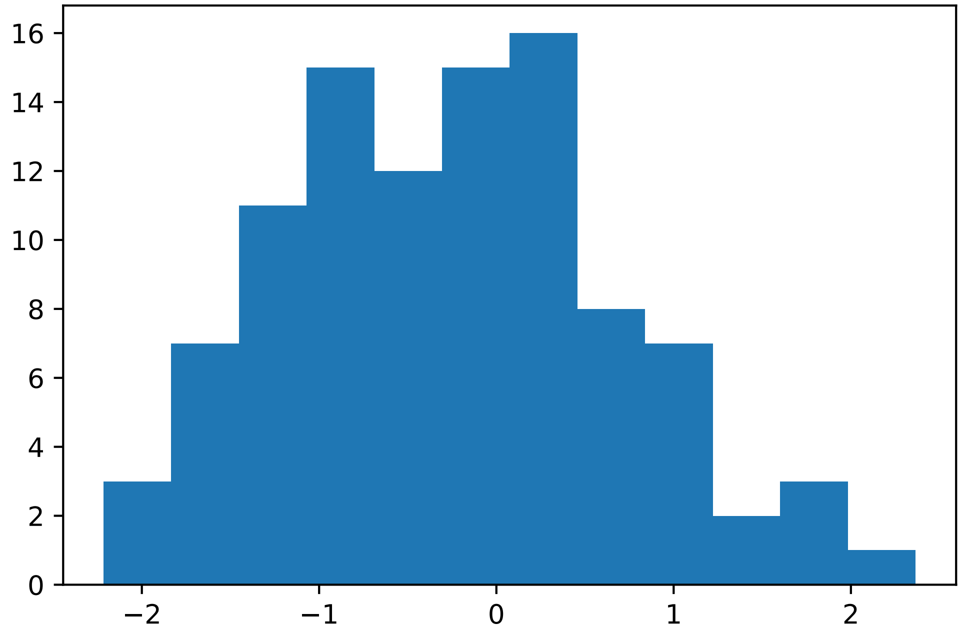
Instead of using the raw scores, we've used the z-scores to compute mu and sigma, and then looked up the national averages and SD for both exams so that we can benchmark each school relative to how the rest of the country performed.

In [2]:

```
#2017 data obtained from https://nces.ed.gov/programs/digest/current_tables.asp (Tables
SAT_mean = 527
SAT_sd = 107

ACT_mean = 20.7
ACT_sd = 5.5

SAT_norm = (score_df['SAT'] - SAT_mean) / SAT_sd
ACT_norm = (score_df['ACT'] - ACT_mean) / ACT_sd
normalized_df = pd.DataFrame({'SAT': SAT_norm, 'ACT': ACT_norm})

plt.hist(normalized_df['SAT'], bins=12)

#Even though we haven't discussed histograms yet, we can still get a vague sense of wha
#Specifically, note that it somewhat resembles a normal distribution:
#with a hump--somewhat left of center--that tails off on both ends.

#Now let's go ahead and pause for a quick refresher. Try your hand at calculating z-sco
```

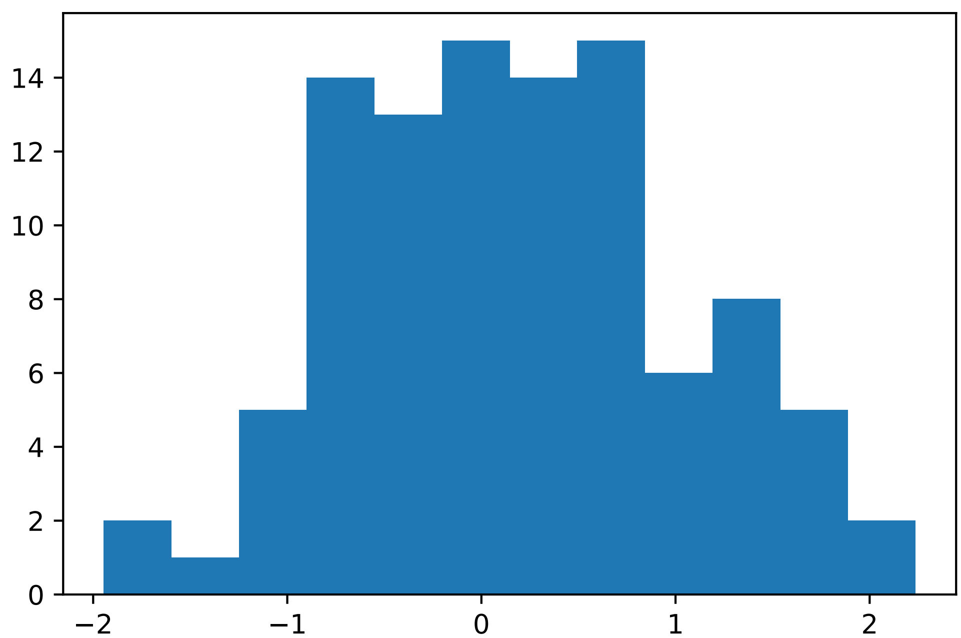
Out[2]:

```
(array([ 3.,  7., 11., 15., 12., 15., 16.,  8.,  7.,  2.,  3.,  1.]),
 array([-2.21495327, -1.83333333, -1.4517134 , -1.07009346, -0.68847352,
        -0.30685358,  0.07476636,  0.45638629,  0.83800623,  1.21962617,
         1.60124611,  1.98286604,  2.36448598]),
 <BarContainer object of 12 artists>)
```



In [3]:
```
plt.hist(normalized_df['ACT'], bins=12)
```

Out[3]:
```
(array([ 2.,  1.,  5., 14., 13., 15., 14., 15.,  6.,  8.,  5.,  2.]),
 array([-1.94545455, -1.5969697 , -1.24848485, -0.9       , -0.55151515,
        -0.2030303 ,  0.14545455,  0.49393939,  0.84242424,  1.19090909,
         1.53939394,  1.88787879,  2.23636364]),
 <BarContainer object of 12 artists>)
```



In [4]:
```
#As we can see, the distribution from Shermer High seems to be shifted to the right eve
#compared to Sunnydale from before. However, just to make sure, let's print out some ce

print("Sunnydale Mean:", normalized_df['SAT'].mean())
print("Sunnydale Median:", normalized_df['SAT'].median())

print("Shermer Mean:", normalized_df['ACT'].mean())
print("Shermer Median:", normalized_df['ACT'].median())

#Indeed, both the mean and median for Shermer High are greater than those for Sunnydale
```

```
#intuition. Admittedly, this isn't very rigorous, but it does show how a simple transfo
#with some basic visual exploration is an effective way of getting some quick insights
#the end of this module, we'll take a look at some procedures to more confidently answe
#using grounded statistical techniques.
```

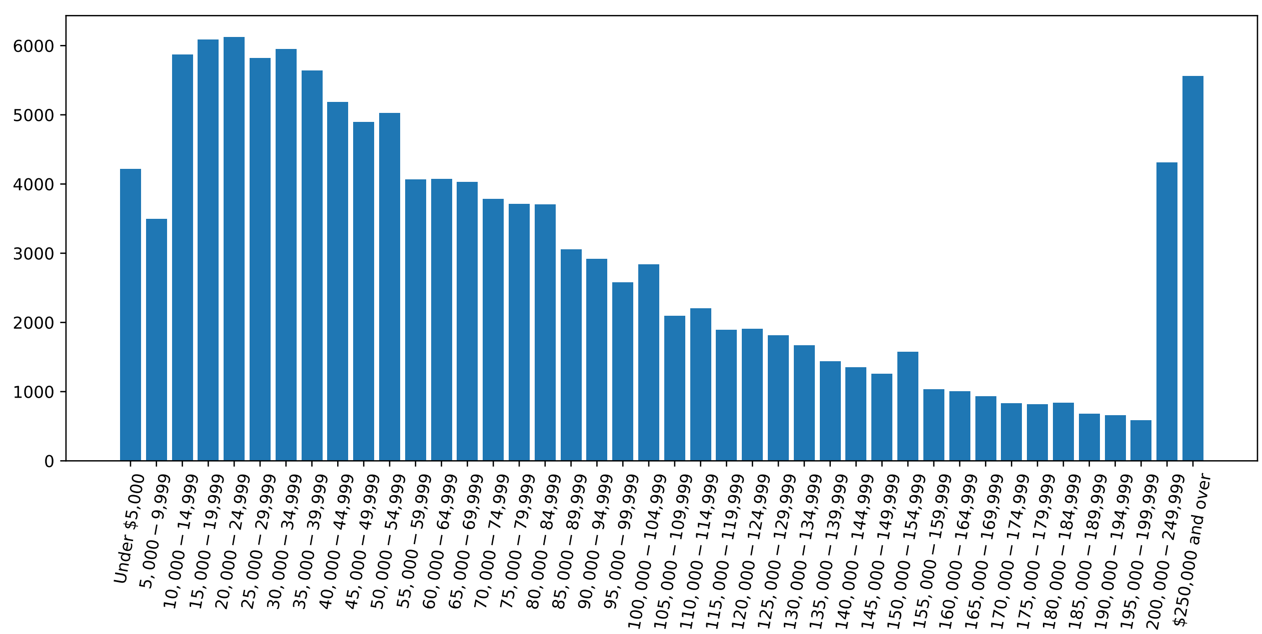
```
Sunnydale Mean: -0.25420560747663556
Sunnydale Median: -0.20560747663551399
Shermer Mean: 0.18363636363636374
Shermer Median: 0.1454545454545456
```

Other Transformations: Why do it?

In [5]:

```python
#Of course, while we can standardize data, this doesn't necessarily mean that result wi
#bell curve if the data aren't normal to begin with. This can pose a challenge when we
#statistical tools that assume some degree of normality such as z or t-tests. Fortunate
#can help us address this issue as well.
#To illustrate, we'll use income distribution data from the 2017 US census.

#2017 data obtained from US Census Bureau: Table A-1
#(https://www.census.gov/data/tables/2018/demo/income-poverty/p60-263.html)
N_households = 127586000
income_df = pd.read_csv('assets/income_dist_2017.csv')

axis = plt.figure(figsize=(13,5))
plt.xticks(rotation=80)
plt.bar(income_df['Range'], income_df['Households'])

#Now, there are few interesting features we can observe here. First of all, notice that
#positively, or right, skewed because it *tails* off to the right. With that said, the
#end seem a bit out of place. Why is this? Well, each range only spans $5,000, for inst
#$39,999. However, the second to last bin is from $200,000 to $249,999, while $250,000
#millionaires and billionaires. Though, if we did continue to segement out in $5,000 in
#expect to see a similar decreasing trend.
```

Out[5]: <BarContainer object of 42 artists>



In [6]:

```python
#For now though, just to keep things simple, let's go ahead and drop those last two bar
#we have integer values on the x-axis to run a transformation, let's just take the midp
```

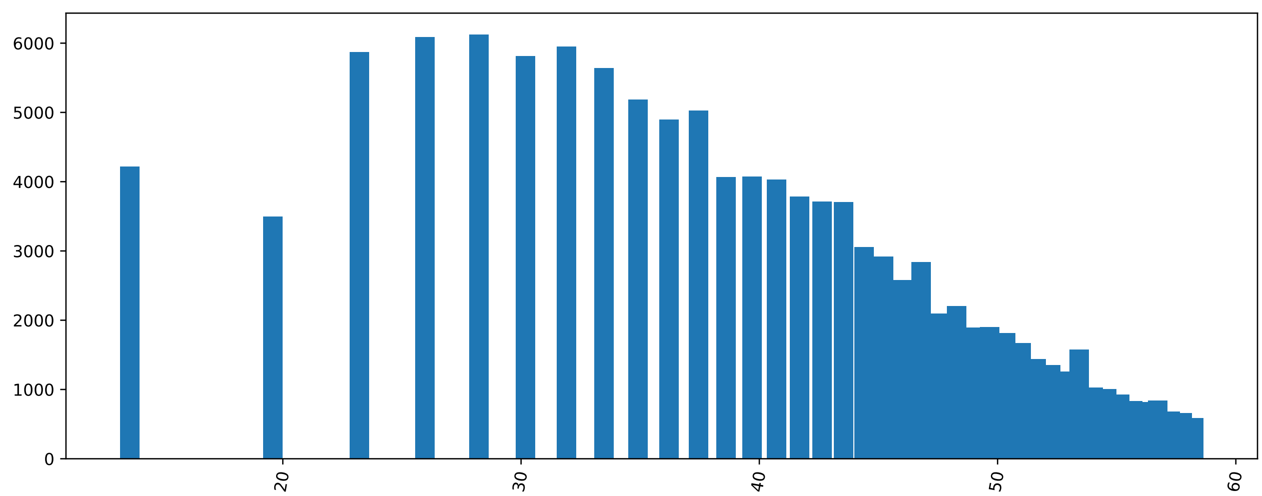
```
income_df = income_df.drop([40, 41])
print(income_df['Midpoints'].head())
```

```
0     2500
1     7500
2    12500
3    17500
4    22500
Name: Midpoints, dtype: int64
```

In [7]:

```
#Since the data is right skewed, we can use a square-root transformation, or a cube-roo

income_df['Transform'] = np.power(income_df['Midpoints'], (1/3))

plt.figure(figsize=(13,5))
plt.xticks(rotation=80)
plt.bar(income_df['Transform'], income_df['Households'])

#...and you'll see that the plot looks a lot closer to a normal distribution! Even thou
#effect of "compressing" some of our points along the right-hand side and creating "gap
#this is just because we were given the household counts at specific midpoint values in
#points, which would resemble something more continuous. We could also try using a hist
#will delve into in week 2.
```

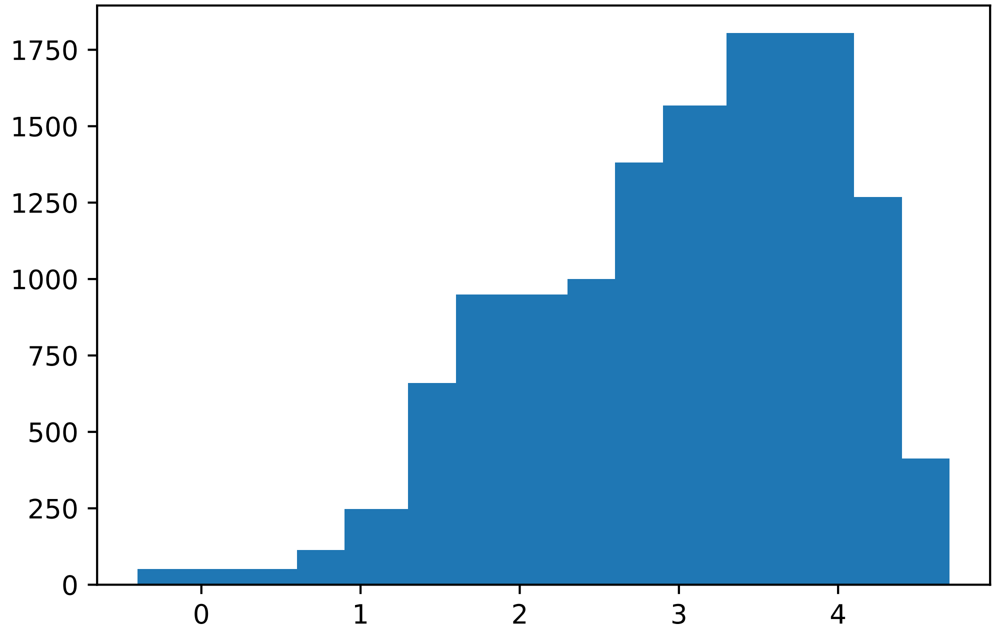
Out[7]: <BarContainer object of 40 artists>



In [8]:

```
#Now for a follow-up question: what if we have something that is left skewed?
#Here are data obtained from UMich's Acadmeic Reporting Tools (art.ai.umich.edu) for EE
#Data Structures and Algorithms. Note that we changed the final grades (A+, A, A- all t
#into their corresponding grade points instead, where we've assigned A+ as 4.3 to see a
#(even though both A+ and A's are typically counted as a 4.0).

N_students = 10312
grade_labels = [0, 0.7, 1.0, 1.3, 1.7, 2.0, 2.3, 2.7, 3.0, 3.3, 3.7, 4.0, 4.3]
percentages = [0.5, 0.5, 1.1, 2.4, 6.4, 9.2, 7.9, 9.7, 13.4, 15.2, 17.5, 12.3, 4.0]
counts = [(x / 100) * N_students for x in percentages]
plt.bar(grade_labels, counts)
```

Out[8]: <BarContainer object of 13 artists>

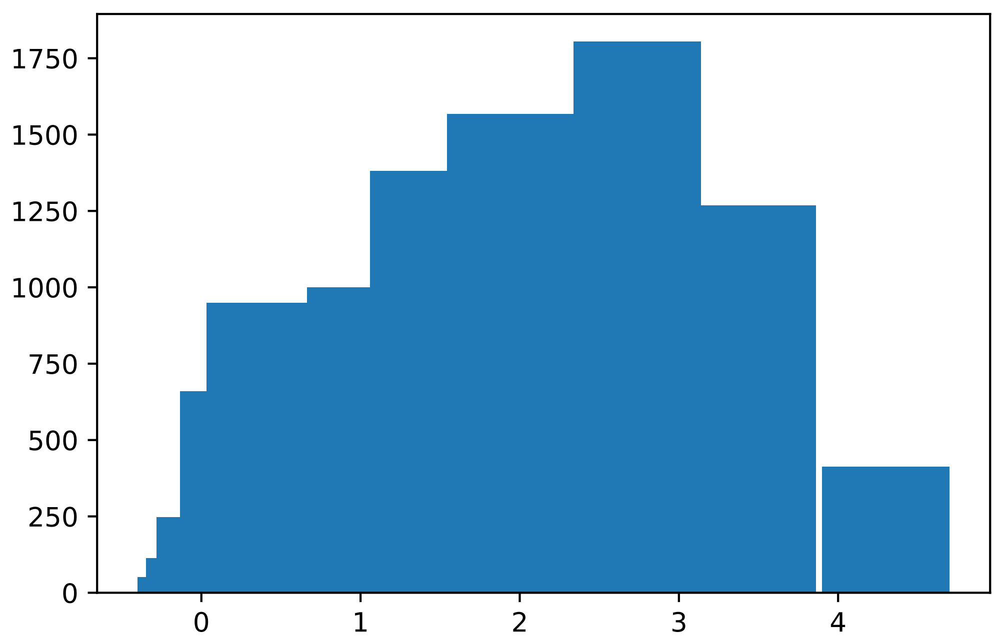


In [9]:

```
#Since we took roots for positively-skewed data, it might not be surprising that for ne
#we just perform the inverse operation. So, instead of taking a cube root, we'll raise
#the third power.

transformed_grades = np.power(grade_labels, 3)
transformed_grades = (transformed_grades/max(transformed_grades)) * 4.3
print(transformed_grades)
plt.bar(transformed_grades, counts)

#See how we've "curved" the grades? Rest assured, there won't be any curving in this co
#there were, we certainly wouldn't be dragging scores downwards!
```

```
[0.         0.01855057 0.05408329 0.11882098 0.2657112  0.43266631
 0.65803137 1.06452136 1.46024878 1.94359113 2.7394808  3.46133045
 4.3       ]
```

Out[9]: <BarContainer object of 13 artists>



Linearity

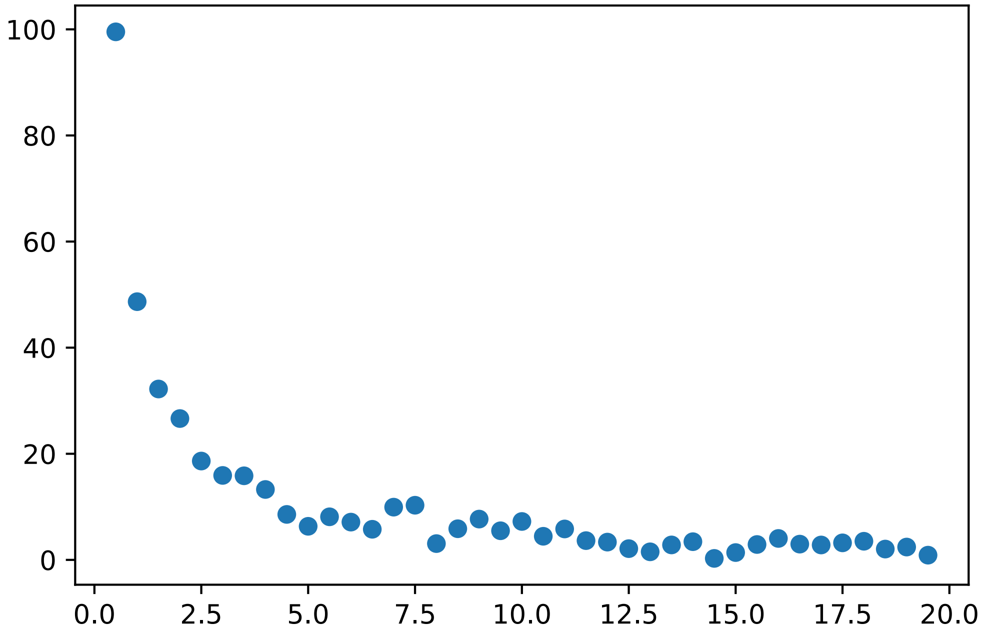
In [10]:
```
#Let's look at another reason for why we might want to transform data. Before we dive i
#give you a bit of background knowledge about Newton's second law of motion, which stat
#to mass times acceleration. Rearranging, we see that a = F / m. In other words, the fa
#to accelerate, the more force you'll need to apply. And, if you're pushing or pulling
#then the more mass an object has, the more it will resist motion (or have a smaller ac
```

$$F = ma \implies a = \frac{F}{m}$$

In [11]:
```
#Now, what I've mentioned probably aligns with your intuition and isn't very shocking,
#insights we can gain from this relatively simple equation. To help us out, let's turn
#sport: curling.
```

In [12]:
```
#Say we've given some curlers custom curling stones with various weights ranging from h
#20 kilos, have them apply a (roughly) constant force, and then measure the acceleratio
#photosensors. Let's plot the results below:

mass = np.arange(0.5, 20, 0.5)
acceleration = [99.53, 48.67, 32.21, 26.64, 18.63, 15.92, 15.85, 13.27, 8.57, 6.33, 8.1
                5.87, 7.69, 5.49, 7.24, 4.45, 5.83, 3.63, 3.35, 2.12, 1.52, 2.82, 3.43,
                3.21, 3.51, 2.04, 2.42, 0.89]

plt.scatter(mass, acceleration)
```

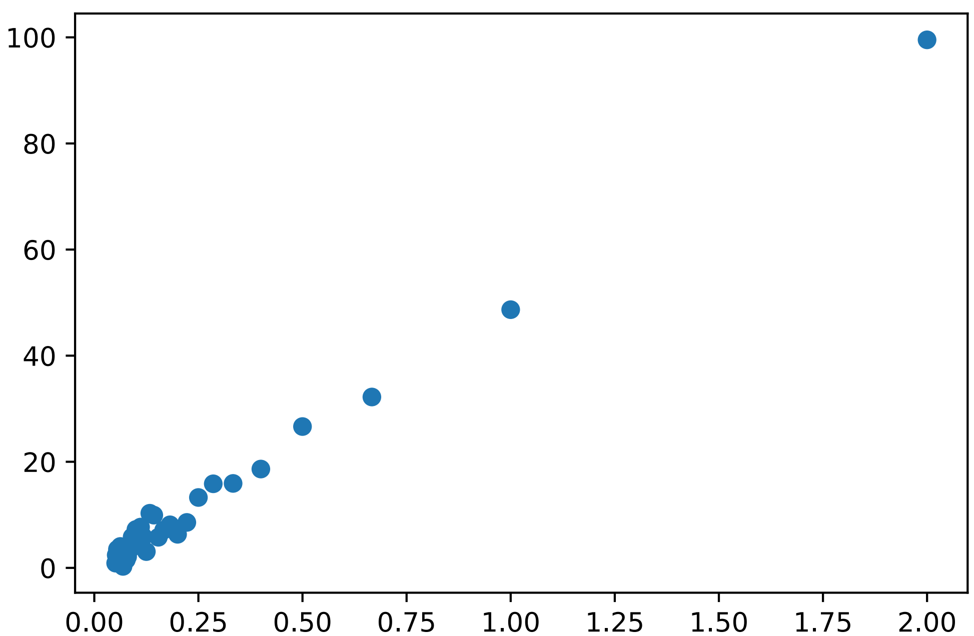
Out[12]: <matplotlib.collections.PathCollection at 0x1eb1cba9490>



In [13]:

```
#We want to find out how much force the curler is applying, but it's kind of difficult
#looking at the curve above. Can we transform one of the variables to get something tha
#to work with? Well, if we take the reciprical of mass (1/m) and call it x, we'll get
#awfully like the equation of a line! Let's try it out:

plt.scatter(1 / mass, acceleration)
```

Out[13]: <matplotlib.collections.PathCollection at 0x1eb1b8a09d0>

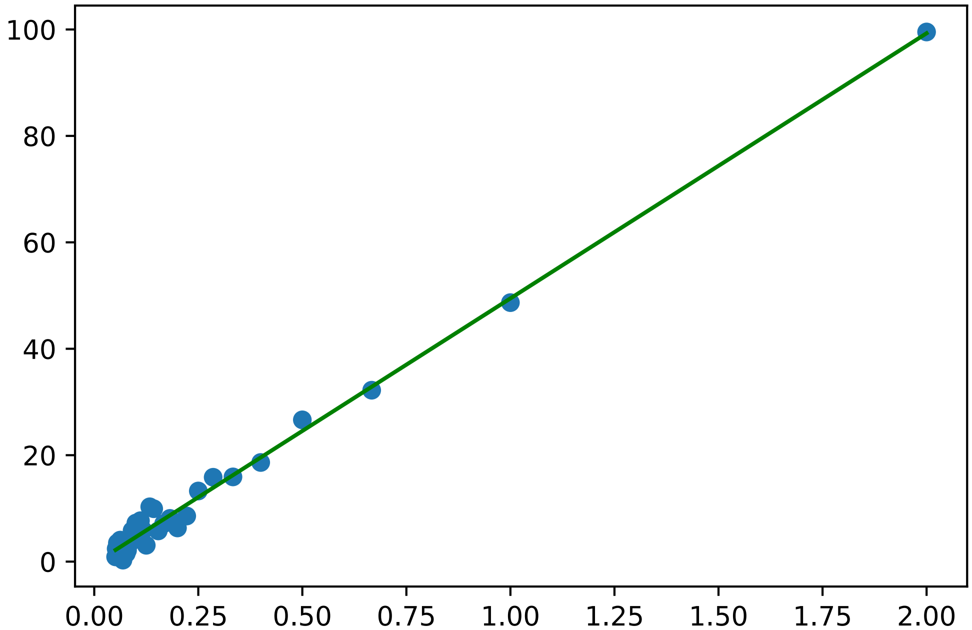


In [14]:

```
#Linearizing a function comes with benefits aside from just looking easier to work with
#we'll save the details of this for the next lecture, we can plot a trendline and get t
#of this line.

plt.scatter(1 / mass, acceleration)

F, b = np.polyfit(1 / mass, acceleration, 1)
abline_values = [F * x + b for x in 1 / mass]
```

```
plt.plot(1 / mass, abline_values, 'g')

print("Force:", F, "Newtons")

#The slope of the line corresponds to approximately 50 Newtons or 11 pounds of force. P
#you're interested, we've posted an optional link to a discussion on other fun facts an
#regarding curling (http://www.madsci.org/posts/archives/2007-09/1190770482.Ph.r.html).
#transformations allow us to linarize functions, which may be easier to manipulate, or
#extrapolate other details of interest.
```

```
Force: 49.82320955370261 Newtons
```



Transformations: Making life easier (or harder)?

In [15]:
```
#Now, at this point, you might be thinking that transformations feel a bit "unnatural"
#clear what technique to use, when. But, transformations do often work well in practice
#intuition as we work through more examples throughout this course. In fact, I'll bet y
#familiar with this concept even before this lecture. For instance, many real-life scal
#transformations, such as: the pH scale (used for measuring acidity levels), the Richte
#measuring the magnitude of an earthquake (although the modern standard is actually the
#and the decibel scale used for measuring sound levels.
```

Decibel Scale:

Source of Sound	Sound Pressure (μPa)	Decibels
Launching of a space shuttle	2,000,000,000	180
Full symphony orchestra	2,000,000	100
High speed diesel freight train at 25 m	200,000	80
Normal conversation	20,000	60
Soft whispering at 2m in a library	2,000	40
Unoccupied broadcast studio	200	20
Threshold of hearing	20	0

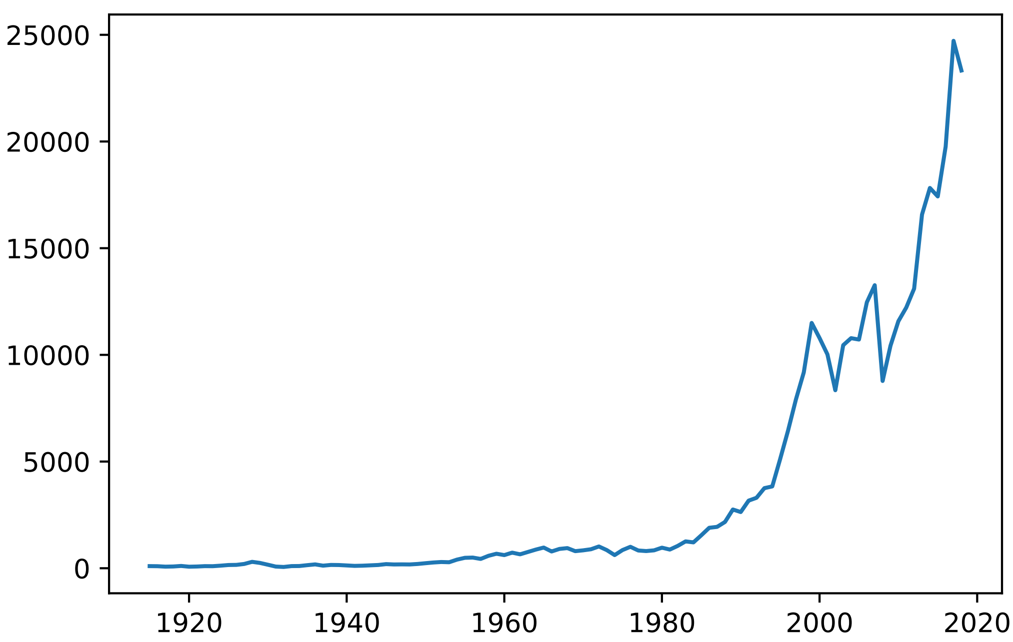
Adapted from: https://www.epd.gov.hk/epd/noise_education/web/ENG_EPD_HTML/m1/intro_5.html

In [16]:
```
#Note that the extreme range of values, like sound pressure in this case (measured in m
#rather inconvinient to talk about, because it spans several orders of magnitude. That'
#loud sound is in terms of decibels. With that said, it's important to note that going
#80 decibels does NOT mean that the source becomes twice as loud. Since the scale is lo
#difference is actually 100-fold! That's equivalent to going from a soft whisper to hav
#train chugging away nearby.
```

Case Study: New York Stock Exchange

In [17]:
```
#We'll end this lecture by going through a case study. The following data are the closi
#Jones Industrial Average (DJIA), a stock market index of 30 large, publicly owned comp
#from 1915 to 2018. Note that the most recent global 2008 recession is clearly depicted
#other recessions? How we would identify periods of economic downturn or stagnation in

#Data obtained from https://www.macrotrends.net/1319/dow-jones-100-year-historical-char
years = np.arange(1915, 2019, 1)
closing_values = [99.15, 95.00, 74.38, 82.20, 107.23, 71.95, 80.80, 98.17, 95.52, 120.5
                  248.48, 164.58, 77.90, 59.93, 99.90, 104.04, 144.13, 179.90, 120.85,
                  135.89, 152.32, 192.91, 177.20, 181.16, 177.30, 200.13, 235.41, 269.2
                  499.47, 435.69, 583.65, 679.36, 615.89, 731.14, 652.10, 762.95, 874.1
                  800.36, 838.92, 890.20, 1020.02, 850.86, 616.24, 852.41, 1004.65, 831
                  1046.54, 1258.64, 1211.57, 1546.67, 1895.95, 1938.83, 2168.57, 2753.2
                  3834.44, 5117.12, 6448.27, 7908.30, 9181.43, 11497.12, 10787.99, 1002
                  10717.50, 12463.15, 13264.82, 8776.39, 10428.05, 11577.51, 12217.56,
                  19762.60, 24719.22, 23327.46]

plt.plot(years, closing_values)
```

Out[17]: [<matplotlib.lines.Line2D at 0x1eb1ce6a6a0>]



Crowds outside NYSE after crash

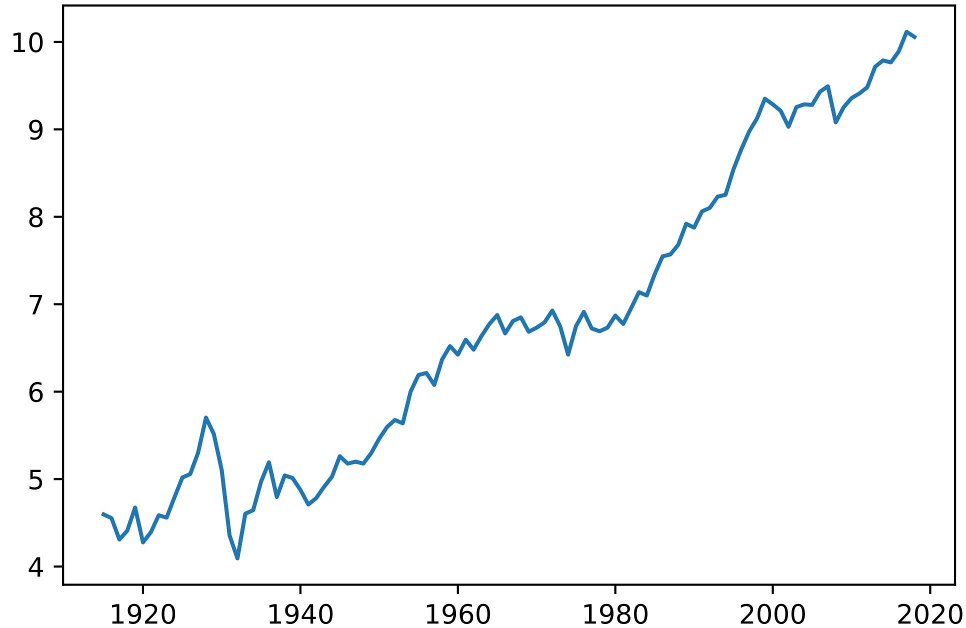
"Bank runs"

Crowds outside NYSE after crash	"Bank runs"
	

In [18]:

```
#For those of you who are familiar with US history, you may know that there was a sever
#downturn in the 1930s, after WWI and before the start of WWII. The unemployment rate r
#began to fail, and a swaths of people lined up to withdraw whatever savings they had l
#above. The "Great Depression", as it's called, doesn't seem to appear in our plot thou

#For starters, the Dow Jones used to be measured in the hundereds, whereas in modern ti
#about tens of thousands of points. Just the daily fluctuations might exceed a hundred 
#obscuring all the details in the left-hand region of the plot. Since the rates of chan
#proportional to the index's current value, it might be worth exploring a logarithmic t
#Let's see what happens when we do that:

log_closing_values = np.log(closing_values)
plt.plot(years, log_closing_values)

#Voila! Now the Great Depression is clearly visible. Even though a change of 30 points 
#nowadays, on October 29, 1929, or "Black Tuesday", this was a 12% decrease, which acco
#portion of that giant dropoff in the left-hand side of the graph. Note that there are 
#where the market seems to stagnate, and while it's still difficult to precisely pinpoi
#recession, we are able to make out a lot more intricacies of the data, whereas this wa
#we applied the transformations.
```

Out[18]: [<matplotlib.lines.Line2D at 0x1eb1ceb59a0>]



Which transformation should I choose?

- Not a "one size fits all" process, should start by exploring your data
- **Normalizing** data is a common and sometimes necessary transformation for applying later steps in a statistical pipeline
- Can sometimes reduce skewness by applying square/square root transformations (Nia will come back to this in the context of histograms in week 2!)
- Reciprocal and logarithmic transformation are other useful transformations to know
- These transformations have visual effects: the right choice might make analysis easier or emphasize different features of the data
- Very useful resource for much of the information in this lecture http://fmwww.bc.edu/repec/bocode/t/transint.html
- In the next section, we'll look at some ways to spruce up our graphs (previewed here), such as drawing trendlines, highlighting regions, etc.

```
In [ ]:
```