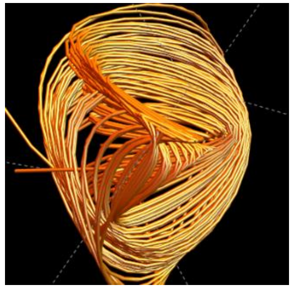
Forecasting in Mineral Systems

Alison Ord & Bruce Hobbs

Centre for Exploration Targeting, The University of Western Australia
CSIRO



AIM

to provide interpretable dynamical models that can aid in understanding the complex interactions of geological processes.

Data issues include

Multiple resolutions

Noise

Incompleteness

Uncertainty

The Solution

Is nonlinear

Has non-stationary characteristics

Honours the rare but significant events or ‘outliers’

Sprott 1993

Is represented by an ATTRACTOR - a set of numerical values toward which a system tends to evolve, for a wide variety of starting conditions of the system.

The attractor is a region in n-dimensional space.

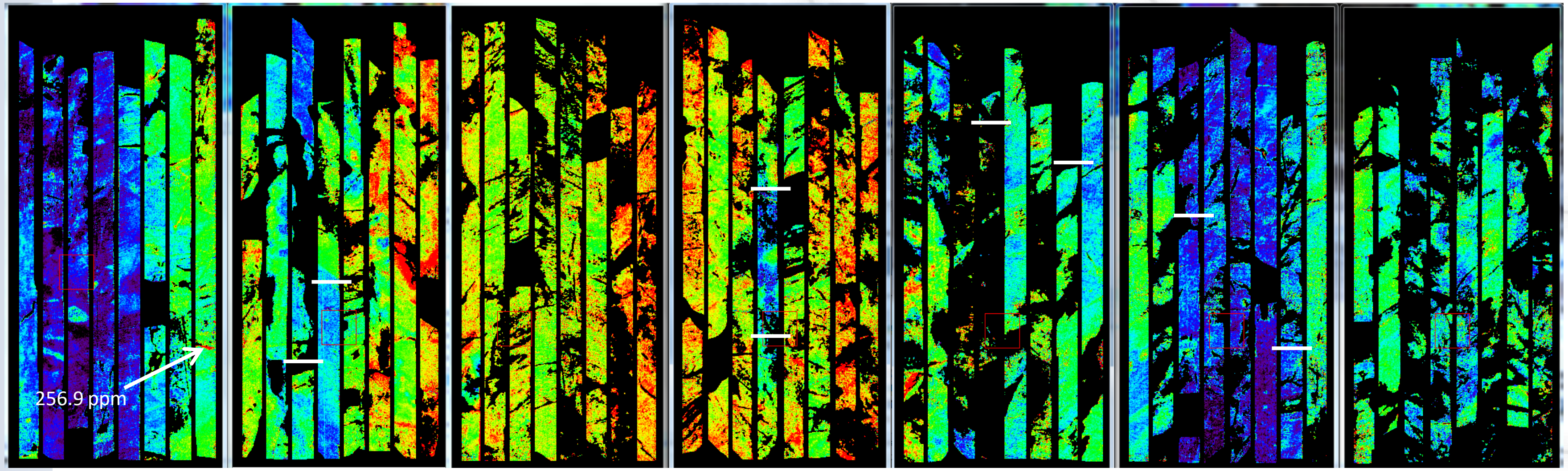
The Solution

Emerging data-driven methods are allowing for the discovery of physical and engineering principles directly from time-series recordings.

Update on nonlinear analyses

The Data

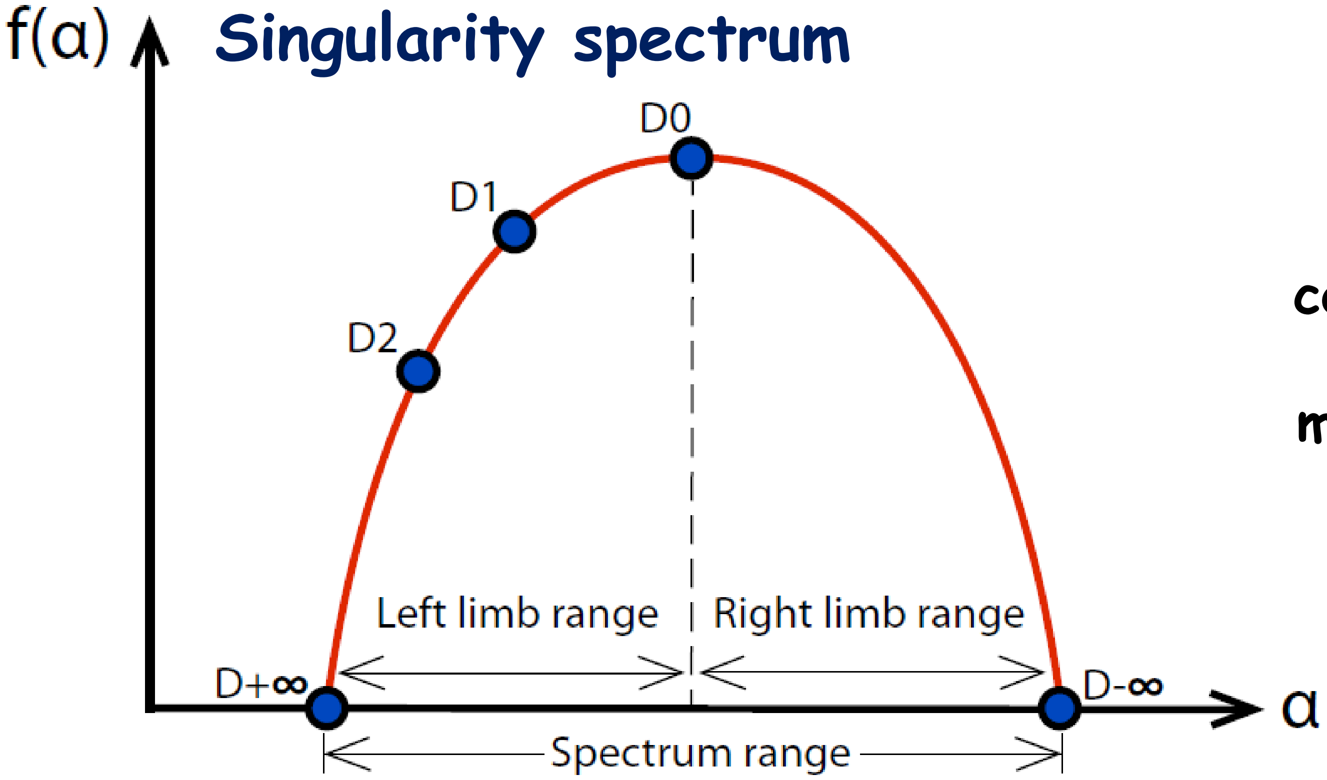
Near infra-red reflectance spectra of many kms of diamond drill core
→ detailed mineralogy + chemistry at mm / micron resolution



176m 183m 190m 198m 205m 212m 219m

The colour → chemical composition of white micas from K-rich to Fe-rich

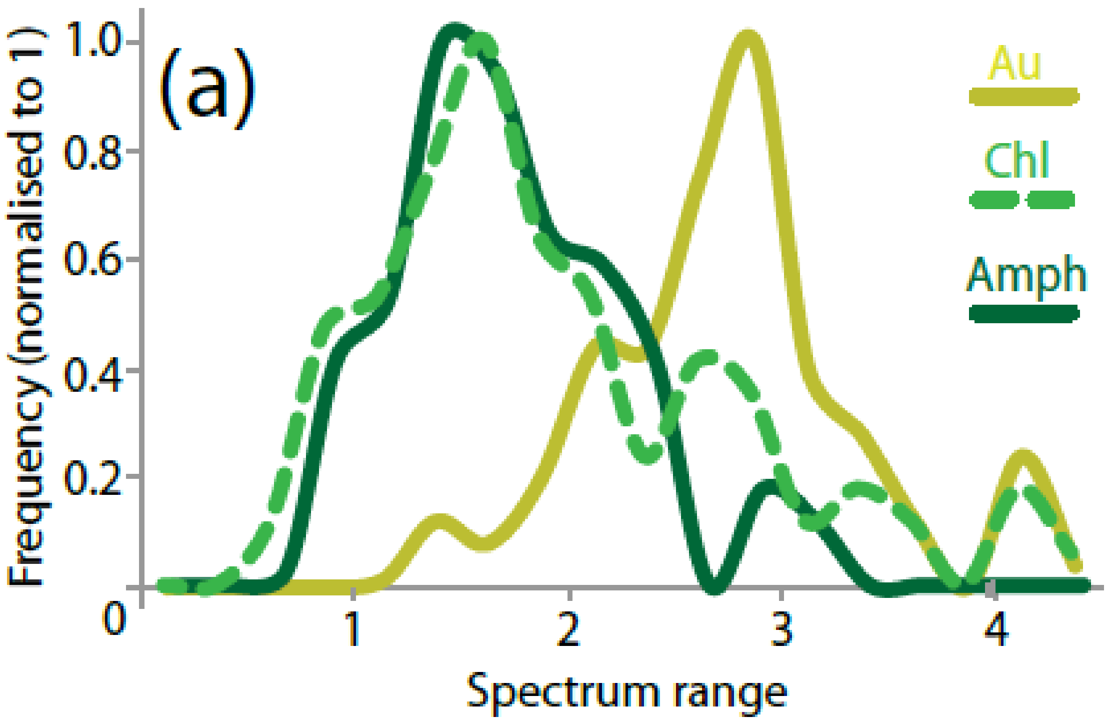
Multifractal



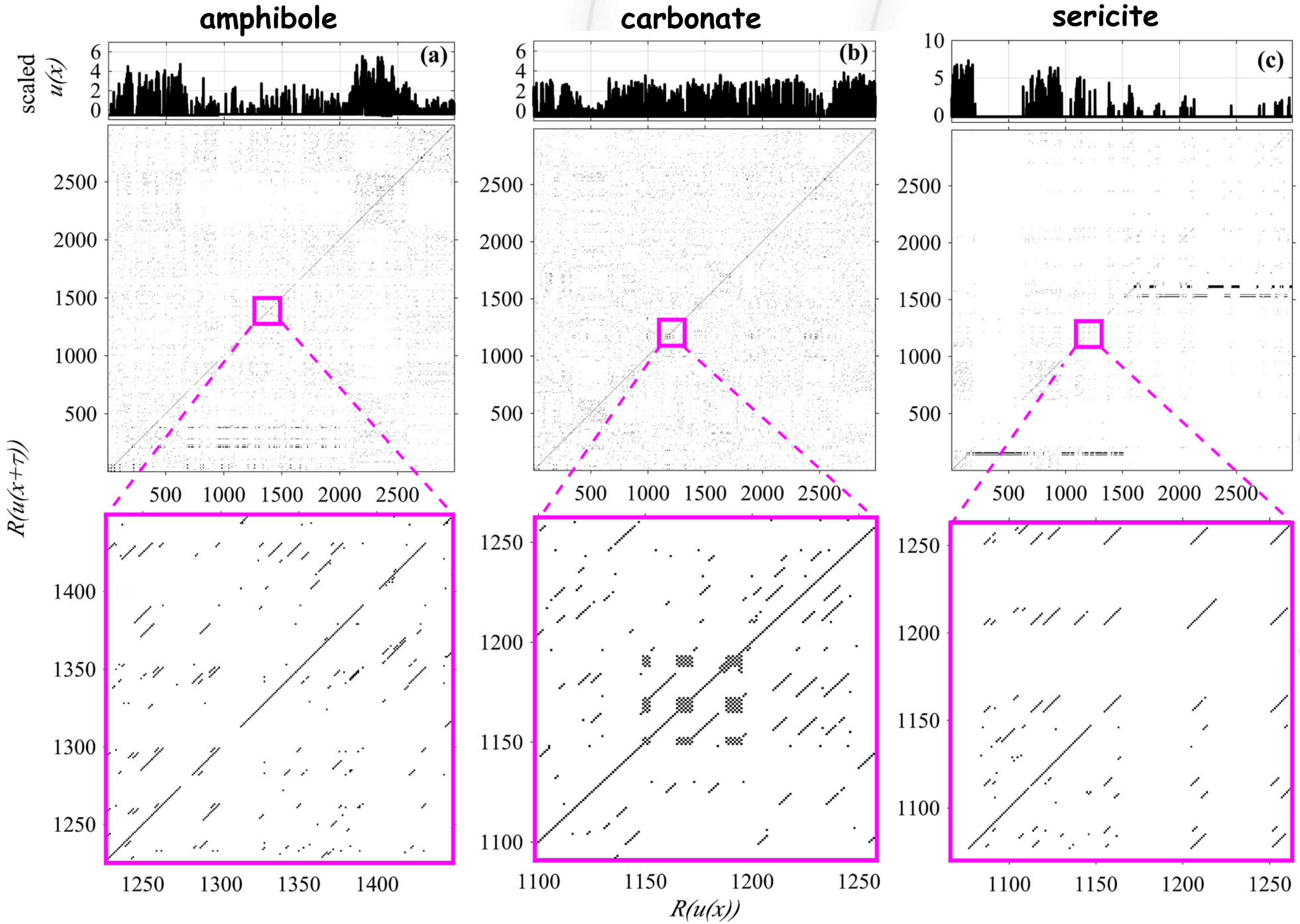
All drill core analysed throughout the Sunrise Dam ore body displayed such a multifractal form.

Munro et al. 2018

Nonlinear analysis between the drill cores throughout the ore body demonstrates considerable variation in singularity spectrum range, and therefore in the strength of multifractality and hierarchical organization.



Deterministic



Recurrence plots

Oberst et al. 2018

Deterministic

Nonlinear analysis reveals episodic & recurring low-dimensional dynamic structures with depth

embedded in & originating from higher-dimensional processes of the complex mineralising system

Oberst et al. 2018

Sprott 1993

May be represented by an ATTRACTOR - a set of numerical values toward which a system tends to evolve, for a wide variety of starting conditions of the system.

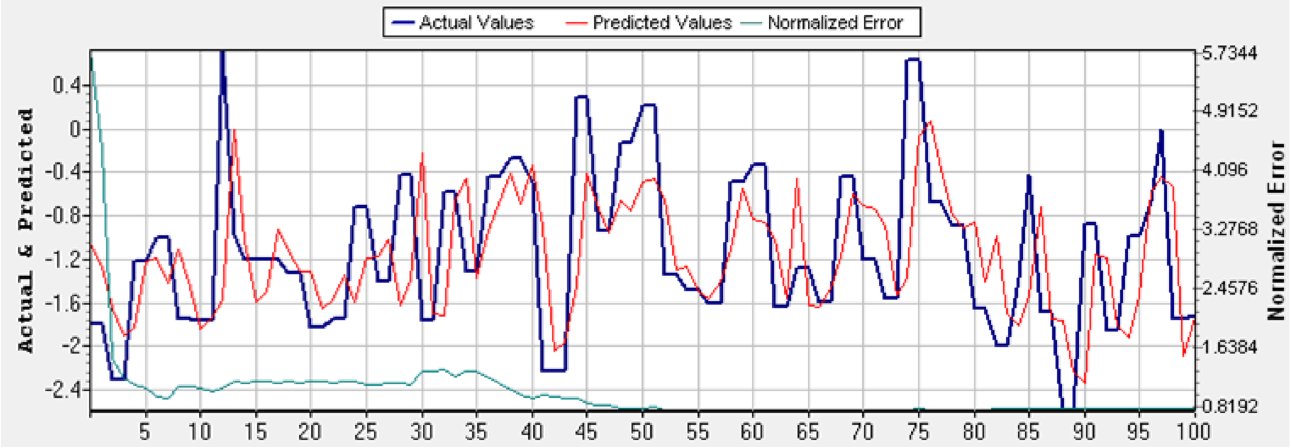
The attractor is a region in n-dimensional space.

Training& Prediction

Non-parametric nonlinear prediction of gold grade

Raw data
Log
(Au ppm)

We use the first 1000 points to predict over the next 200 points.



The normalised error is ≈0.8 which is surprisingly good.

Prediction

The readily observable manifestations of the attractor of a system are its multifractal spectrum and recurrence derived from observable data.

With enough investigation one can arrive at a good estimate of the attractor but this is not possible yet for mineralising systems.

Multifractal spectra and recurrence plots can however be established with very little effort and are readily quantified.

The Grand Challenge

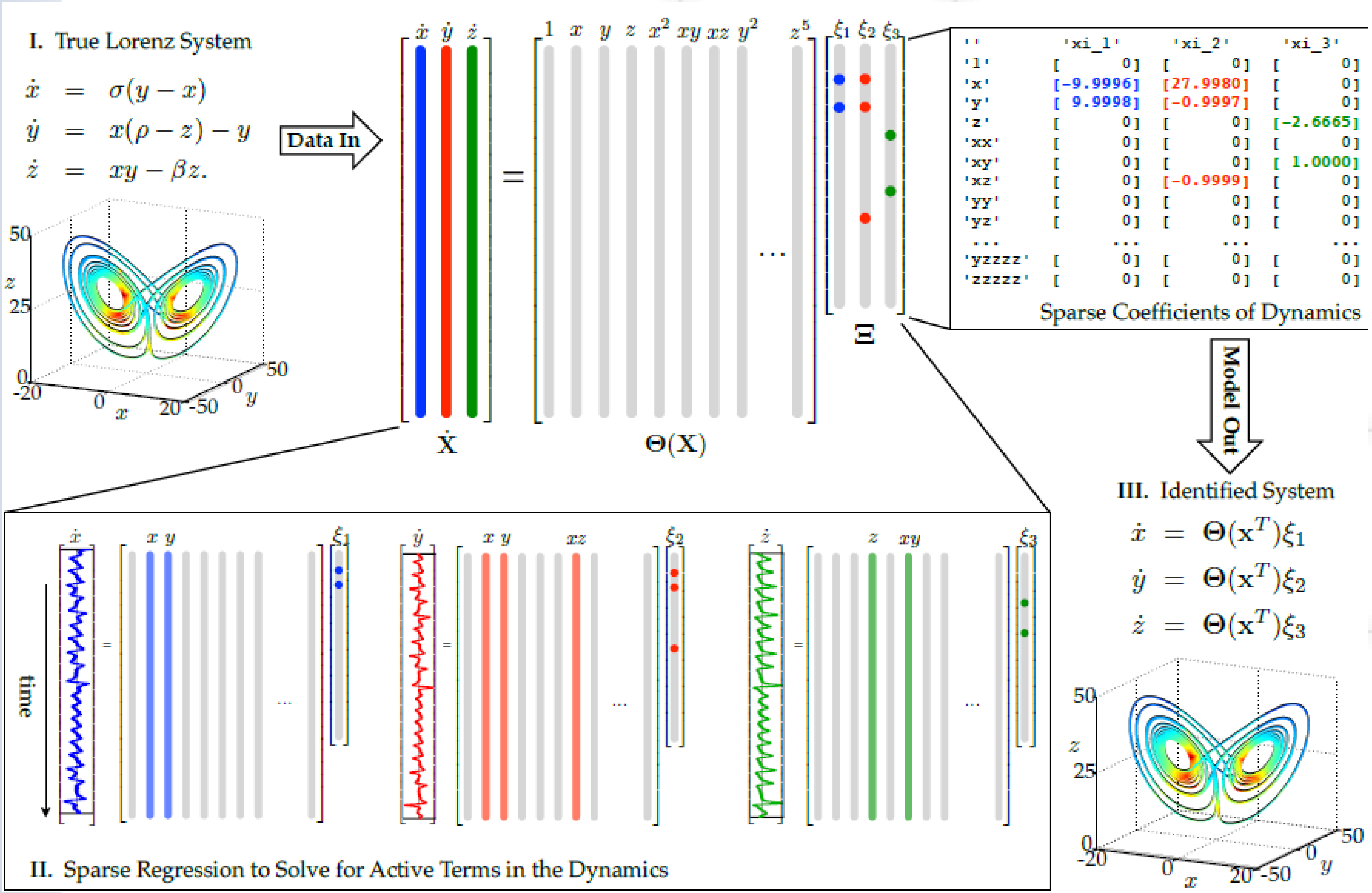
Nonlinear and complex systems identification aims at inferring, from data, the mathematical equations that govern the dynamical evolution and the complex interaction patterns, or topology, among the various components of the system.

Brunton et al. 2015 use sparse regression to determine the fewest terms in the dynamic governing equations required to represent the data accurately.

Brunton, Proctor & Kutz arXiv:1509.03580v1 2015

30 years on

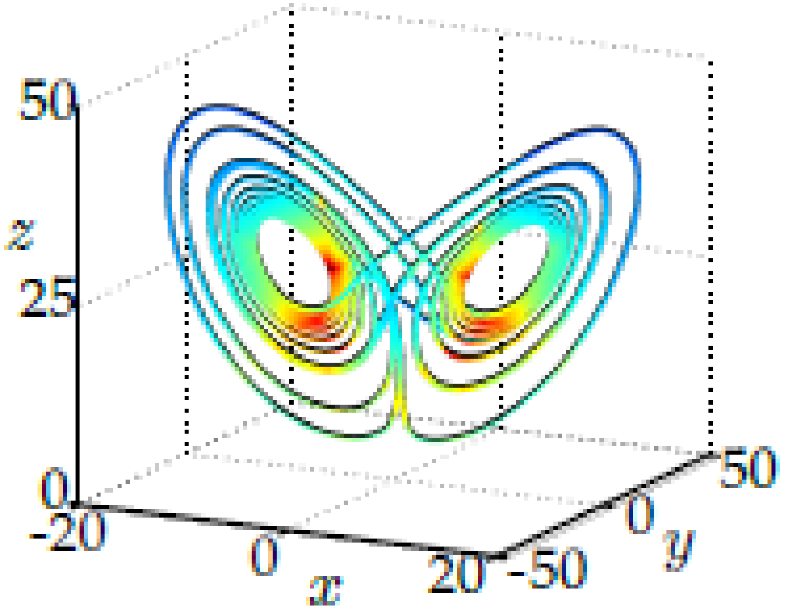
Sparse identification of nonlinear dynamics (SINDy)



Brunton et al. 2015

I. True Lorenz System

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = x(\rho - z) - y$$
$$\dot{z} = xy - \beta z$$



Lorenz's parameters

σ = 10
β = 8/3
ρ = 28

$$\dot{x} = 10(y - x)$$
$$\dot{y} = x(28 - z) - y$$
$$\dot{z} = xy - \frac{8}{3}z$$

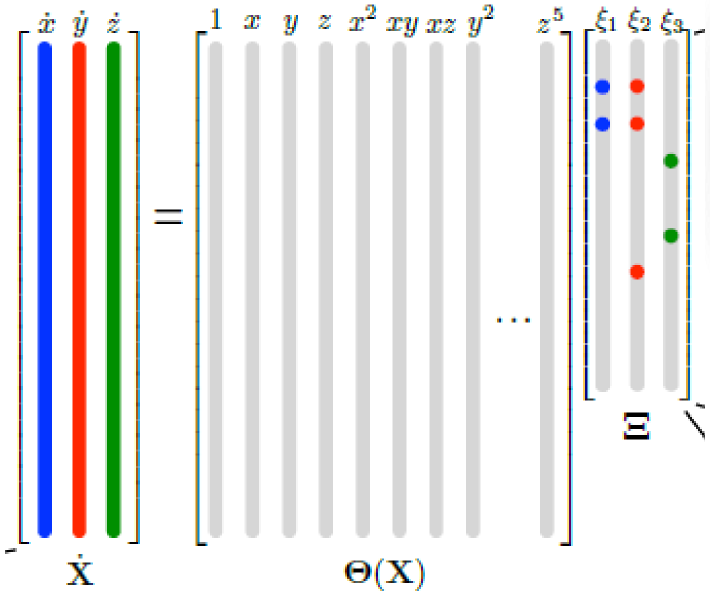
Initial conditions

x = -8
y = 8
z = 27

II. Sparse regression to solve for active terms in the dynamics

Compute derivative

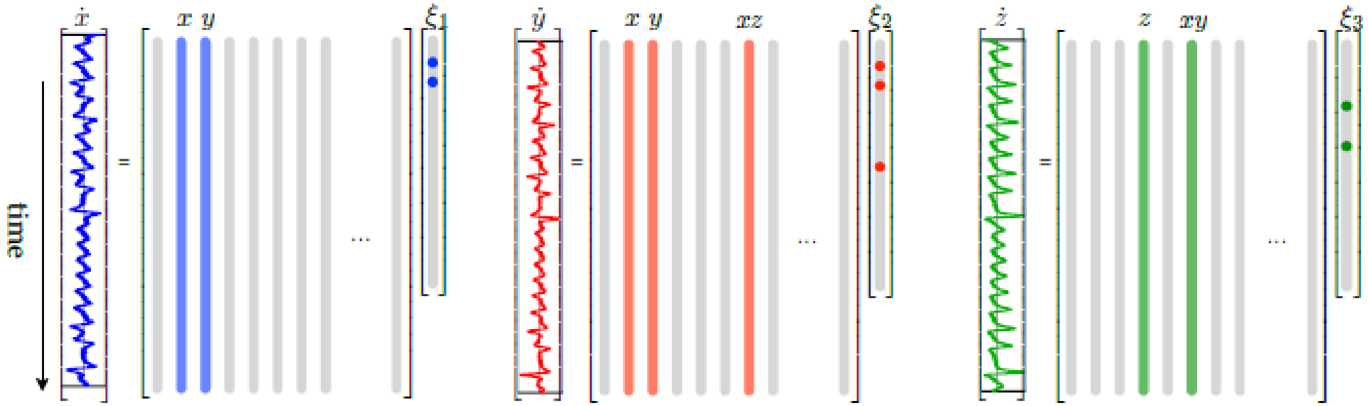
Data in



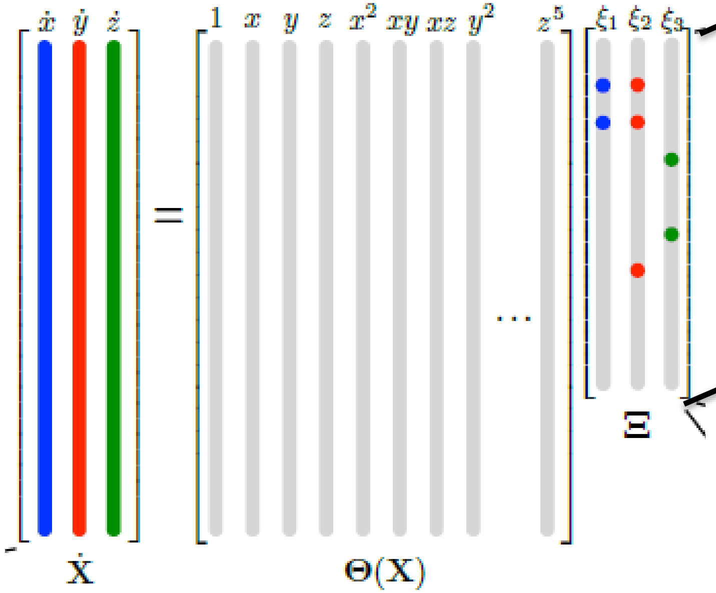
$$\dot{\mathbf{X}} = \Theta(\mathbf{X})\Xi.$$

Compute sparse regression

Build library of nonlinear time series



Compute sparse regression



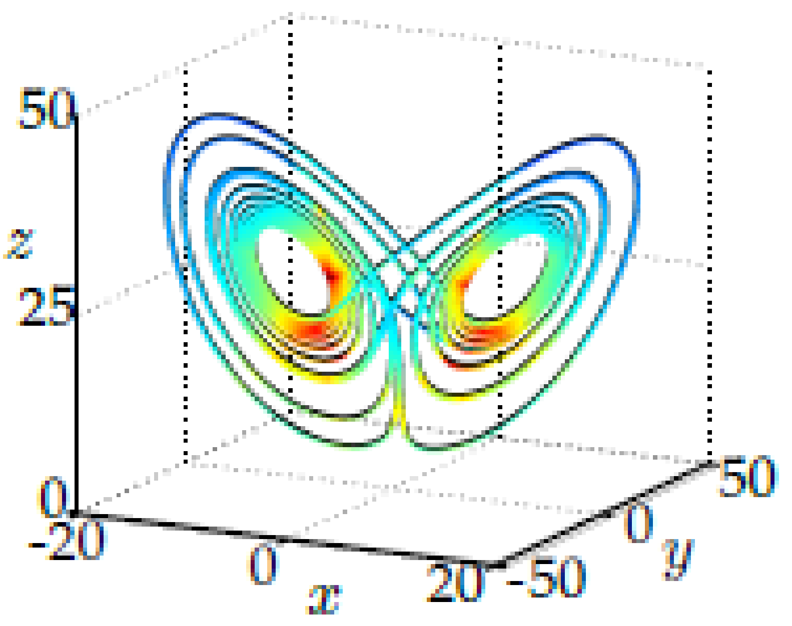
''	'xi_1'	'xi_2'	'xi_3'
'1'	[0]	[0]	[0]
'x'	[-9.9996]	[27.9980]	[0]
'y'	[9.9998]	[-0.9997]	[0]
'z'	[0]	[0]	[-2.6665]
'xx'	[0]	[0]	[0]
'xy'	[0]	[0]	[1.0000]
'xz'	[0]	[-0.9999]	[0]
'yy'	[0]	[0]	[0]
'yz'	[0]	[0]	[0]
...	...	...	...
'yzzzz'	[0]	[0]	[0]
'zzzzz'	[0]	[0]	[0]

Sparse coefficients of dynamics

	Xi_1	Xi_2	Xi_3
1	0.0000	0.0000	0.0000
x	-9.9996	27.9980	0.0000
y	9.9998	-0.9997	0.0000
z	0.0000	0.0000	-2.6665
xx	0.0000	0.0000	0.0000
xy	0.0000	0.0000	1.0000
xz	0.0000	-0.9999	0.0000

I. True Lorenz System

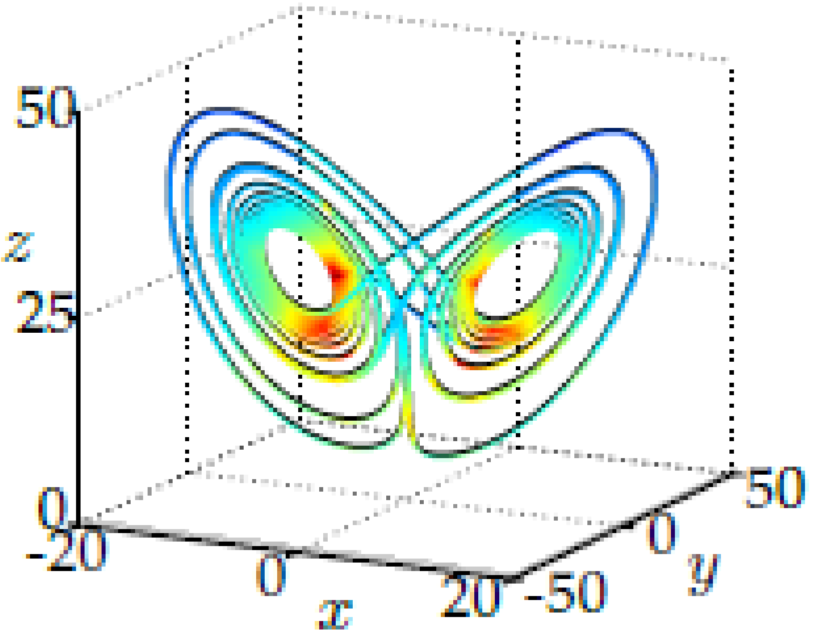
$$\dot{x} = 10(y - x)$$
$$\dot{y} = x(28 - z) - y$$
$$\dot{z} = xy - \frac{8}{3}z$$

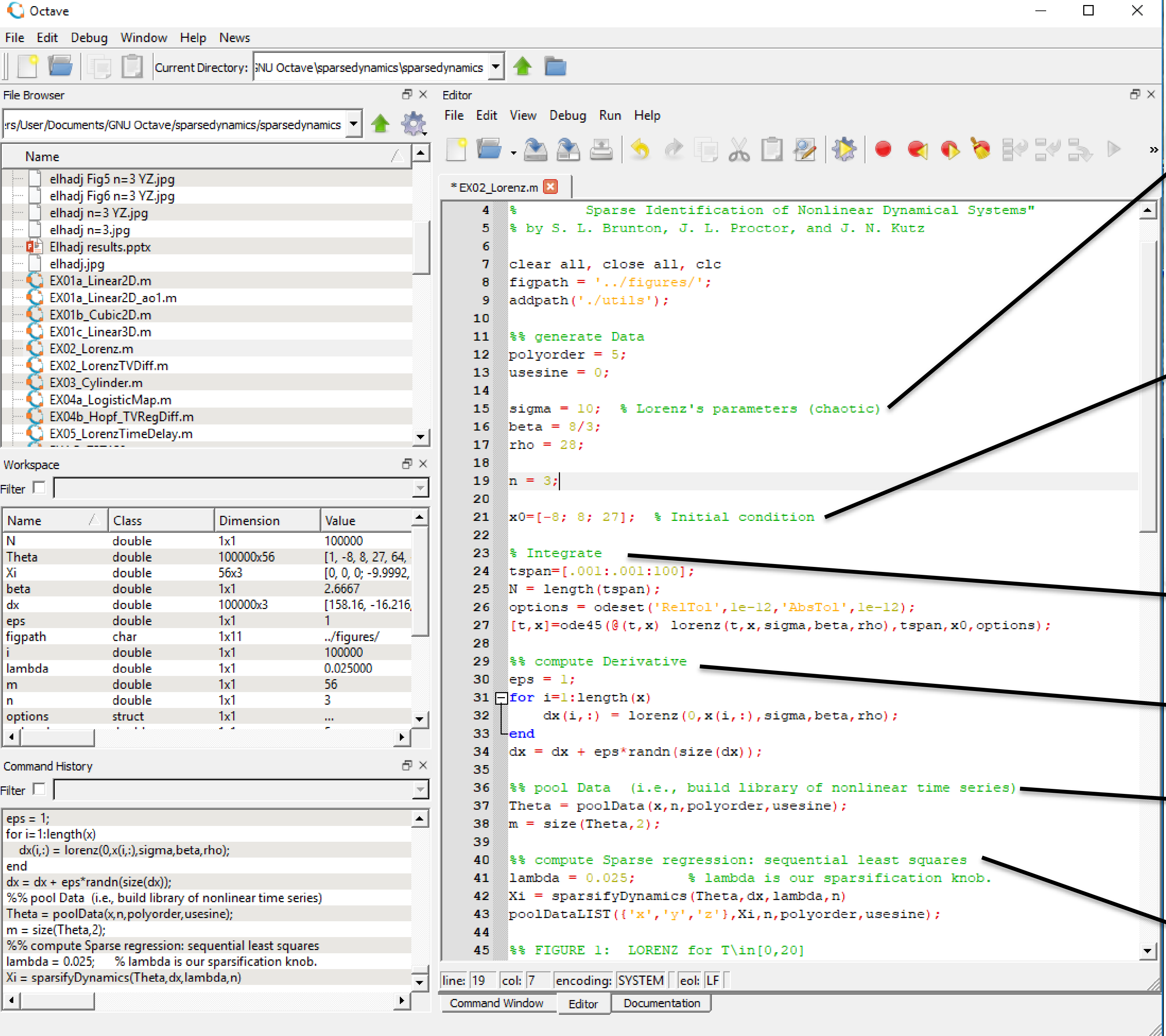


III. Identified System

Model out

$$\dot{x} = 0 - 10x + 10y = 10(y - x)$$
$$\dot{y} = 0 + 28x - y - xz = x(28 - z) - y$$
$$\dot{z} = 0 - 2.7z + xy = xy - \frac{8}{3}z$$





Lorenz's parameters

$\sigma = 10$

$\beta = 8/3$

$\rho = 28$

Initial conditions

$x = -8$

$y = 8$

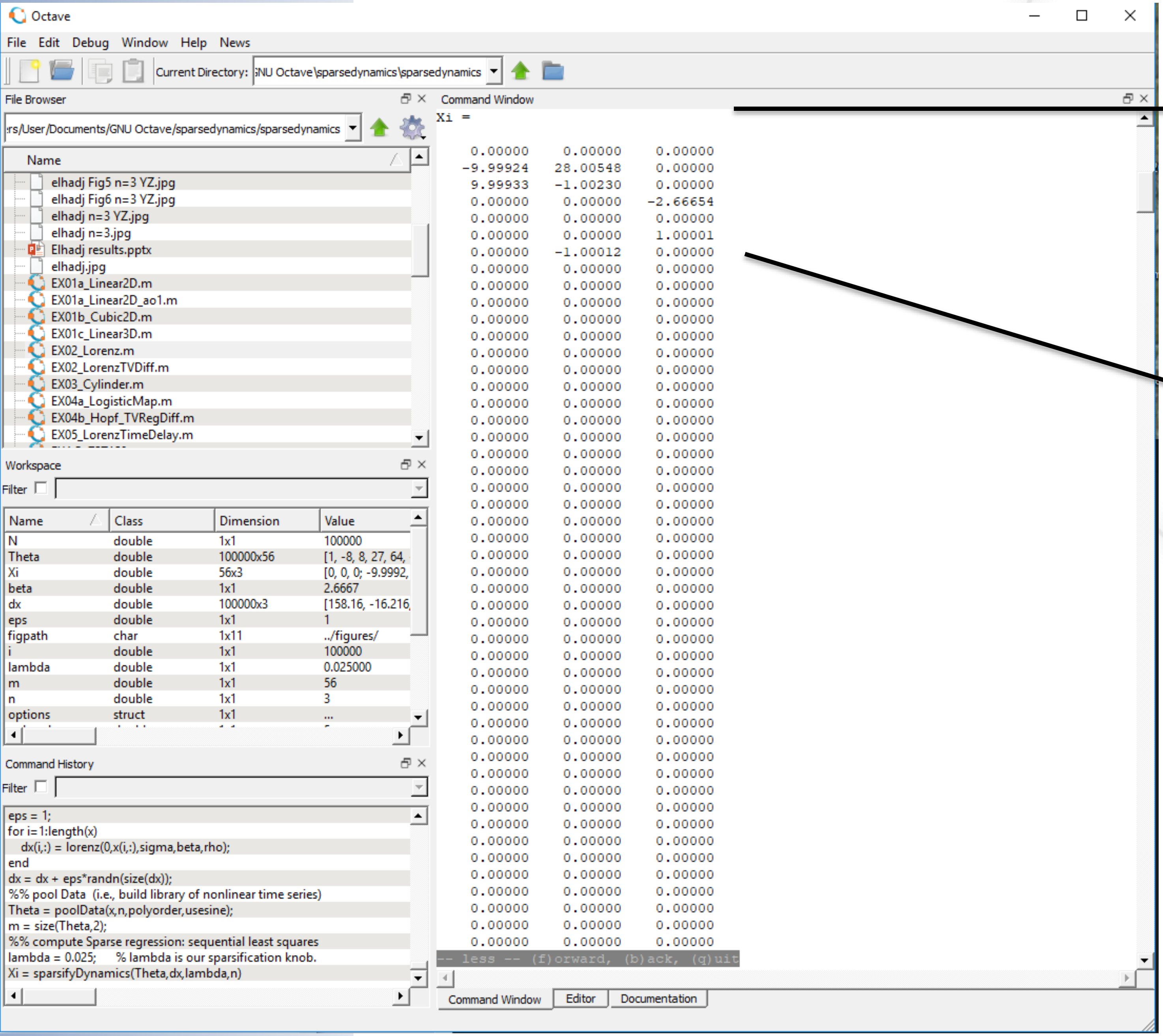
$z = 27$

Integrate to create input data

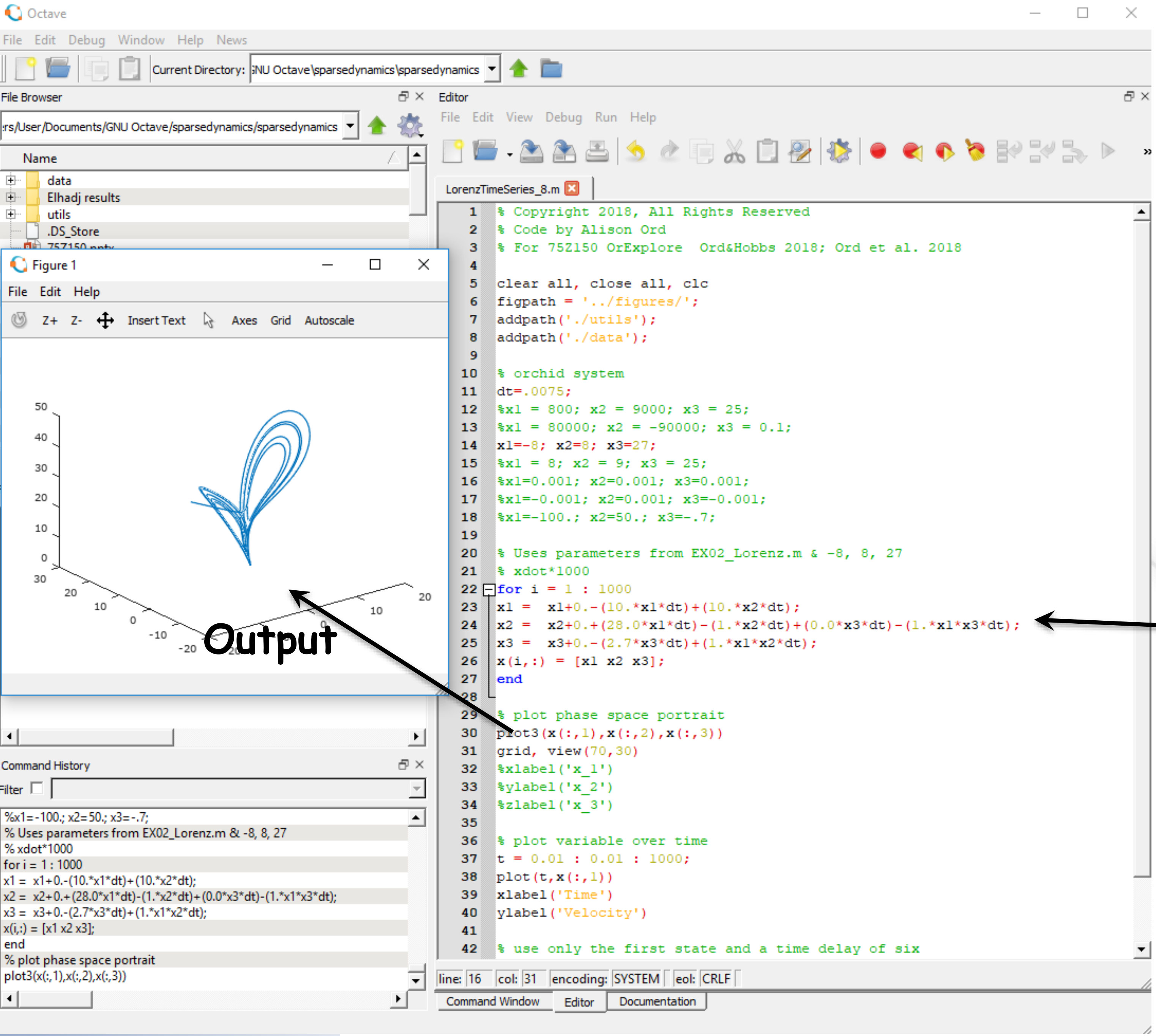
Compute derivative

Build library of nonlinear time series

Compute sparse regression



Xi_1	Xi_2	Xi_3	
0.00000	0.00000	0.00000	1
-9.99924	28.00548	0.00000	x
9.99933	-1.00239	0.00000	y
0.00000	0.00000	-2.66654	z
0.00000	0.00000	0.00000	xx
0.00000	0.00000	1.00001	xy
0.00000	-1.00012	0.00000	xz

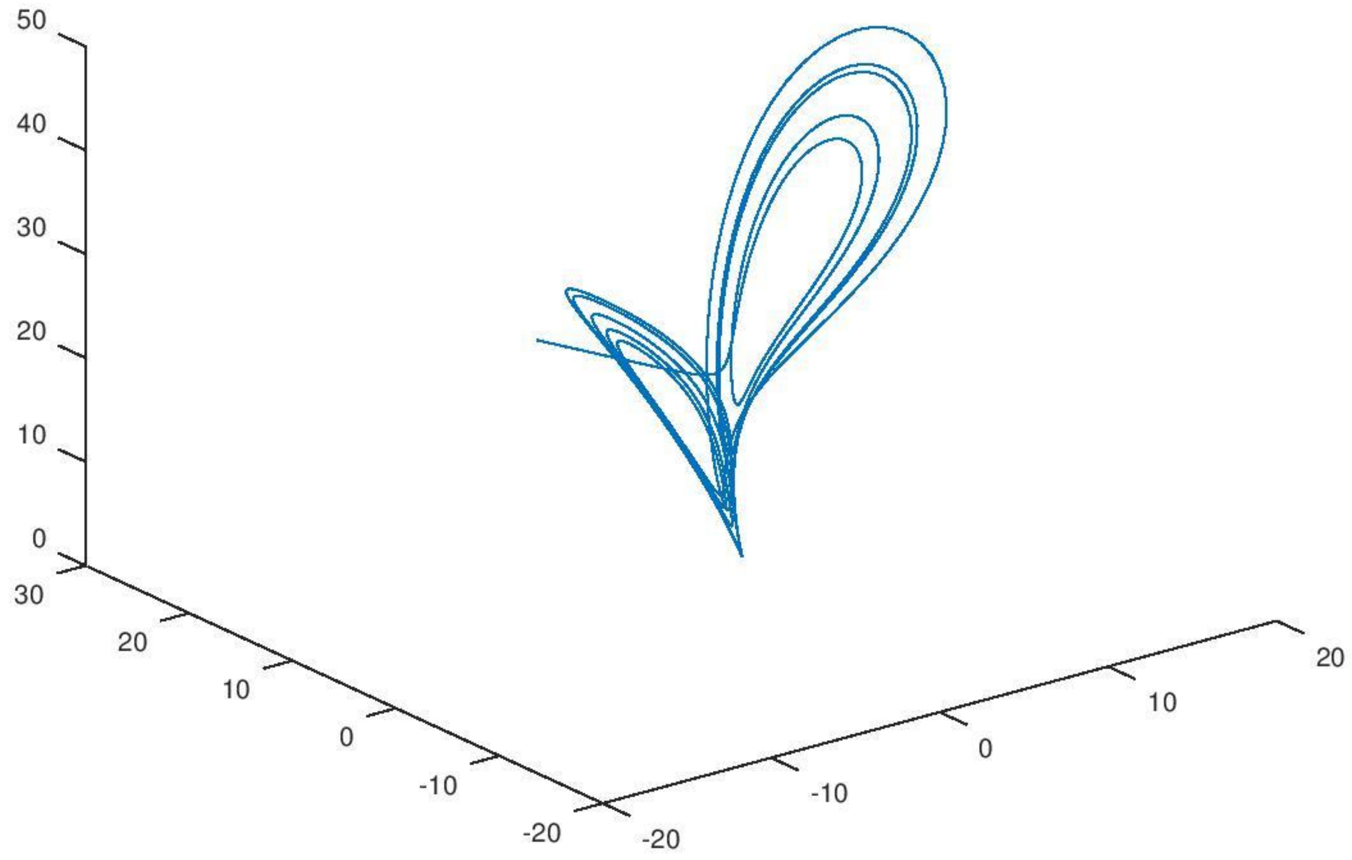


Output

Input

Xi_1	Xi_2	Xi_3	
0.00000	0.00000	0.00000	1
-9.99924	28.00548	0.00000	x
9.99933	-1.00239	0.00000	y
0.00000	0.00000	-2.66654	z
0.00000	0.00000	0.00000	xx
0.00000	0.00000	1.00001	xy
0.00000	-1.00012	0.00000	xz

$$\dot{x} = 0 - 10x + 10y$$
$$\dot{y} = 0 + 28x - y - xz$$
$$\dot{z} = 0 - 2.7z + xy$$


50
40
30
20
10
0
30
20
10
0
-10
-20
-20
-10
0
10
20

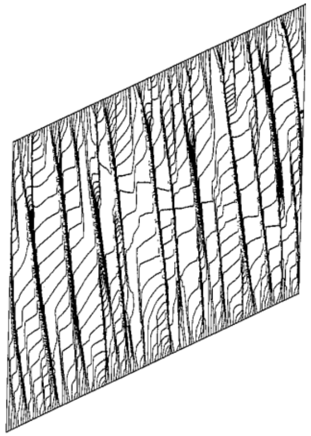
New Results

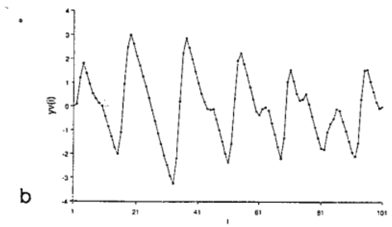
Computational shearing

Folded rock

Drill core

I. Original System

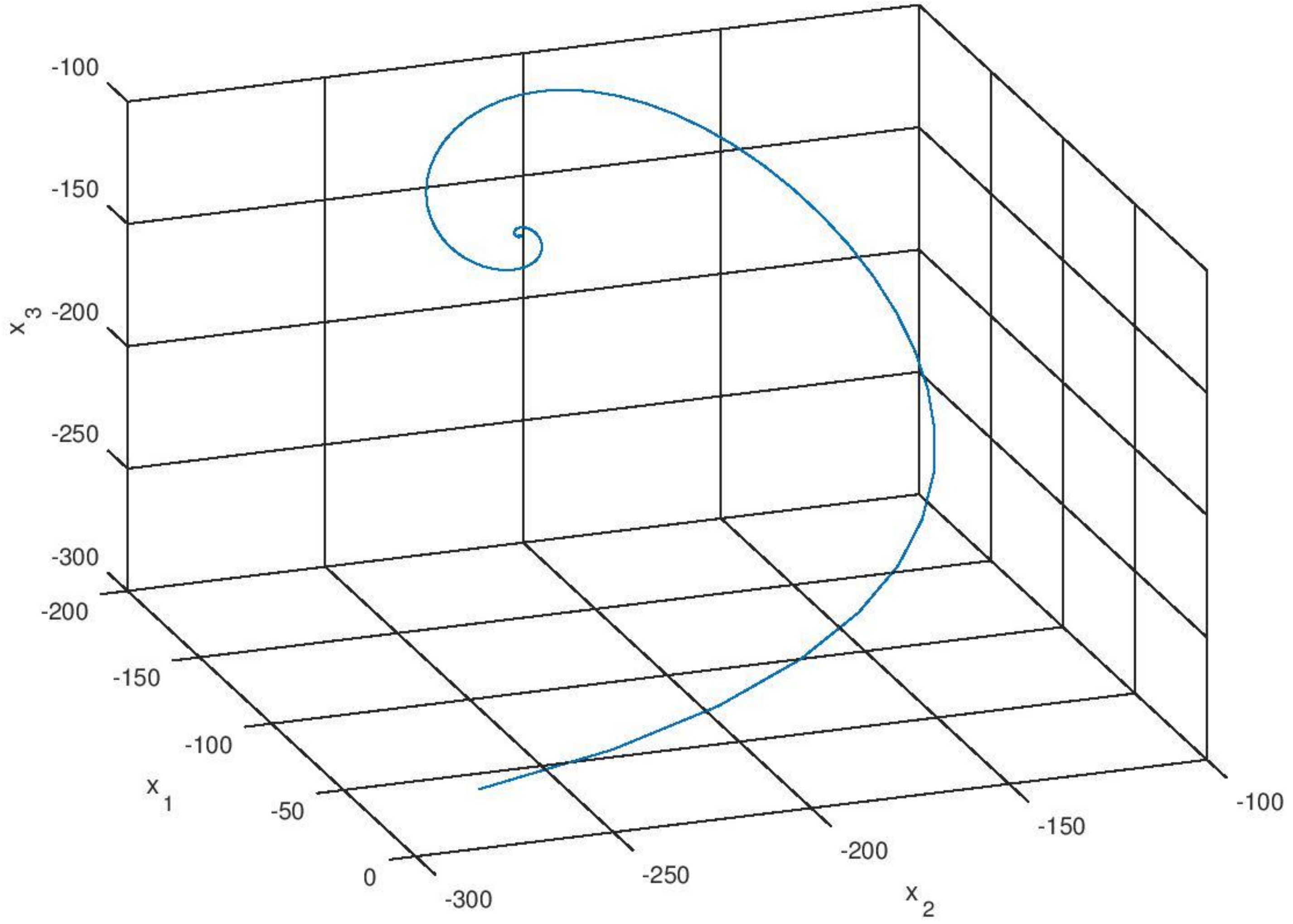




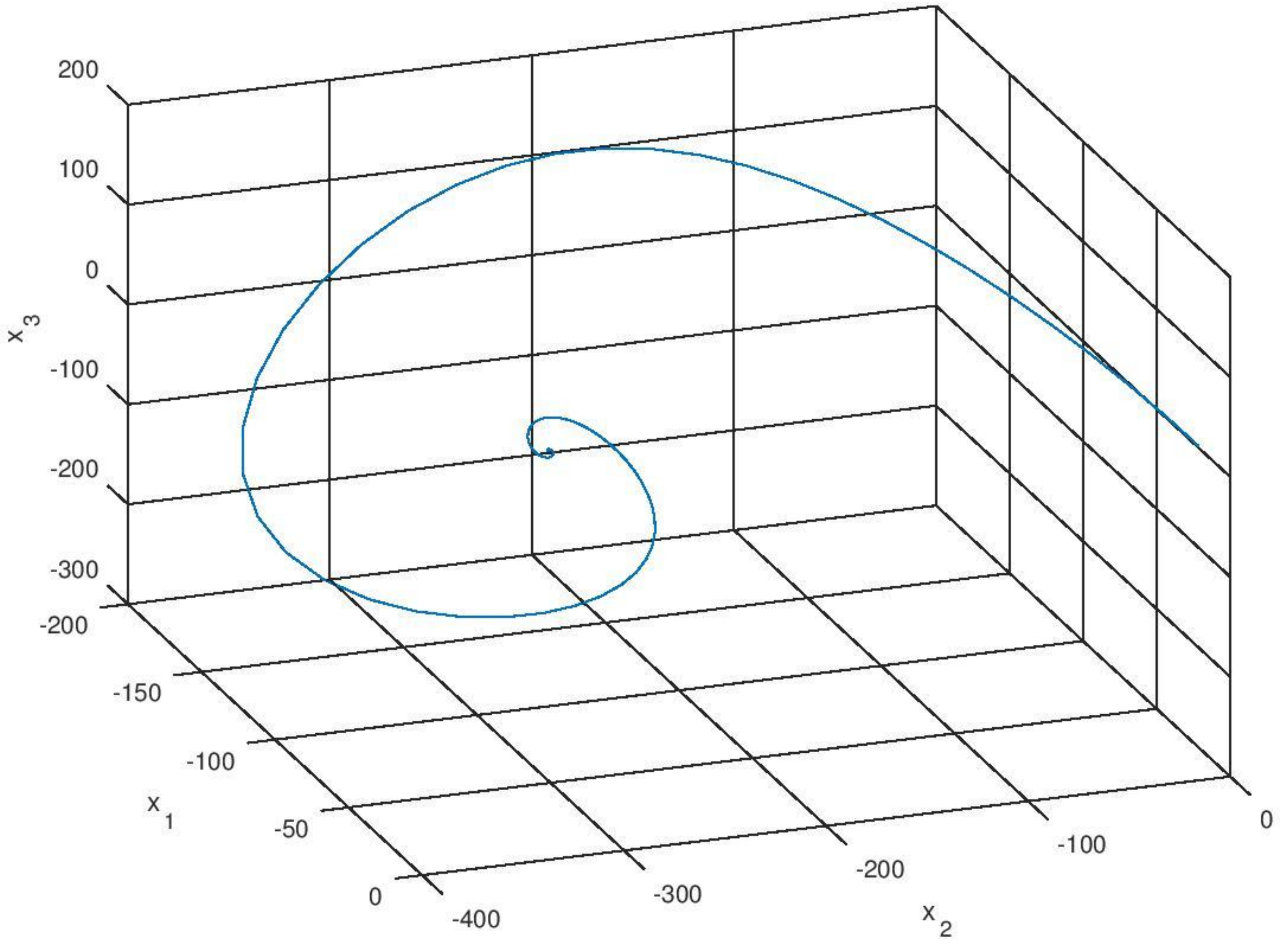
III. Identified System

x1 = -13.+2.*x1+3.*x2-0.5*x3+0.06*x1*x2
x2 = -6.+2.*x1+2.*x2-3.*x3-
0.04*x1*x3+0.06*x2*x3
x3 = 15.+0.4*x1+2.*x2-2.*x3-0.03*x2*x2

Ord 1994

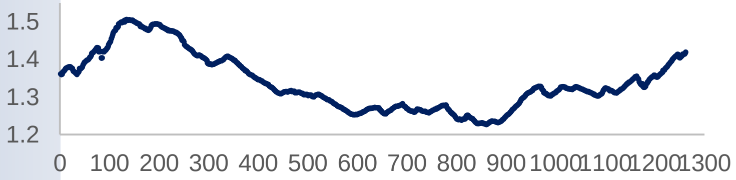


x1 = 4.38; x2 = -325.5; x3 = -278.5;



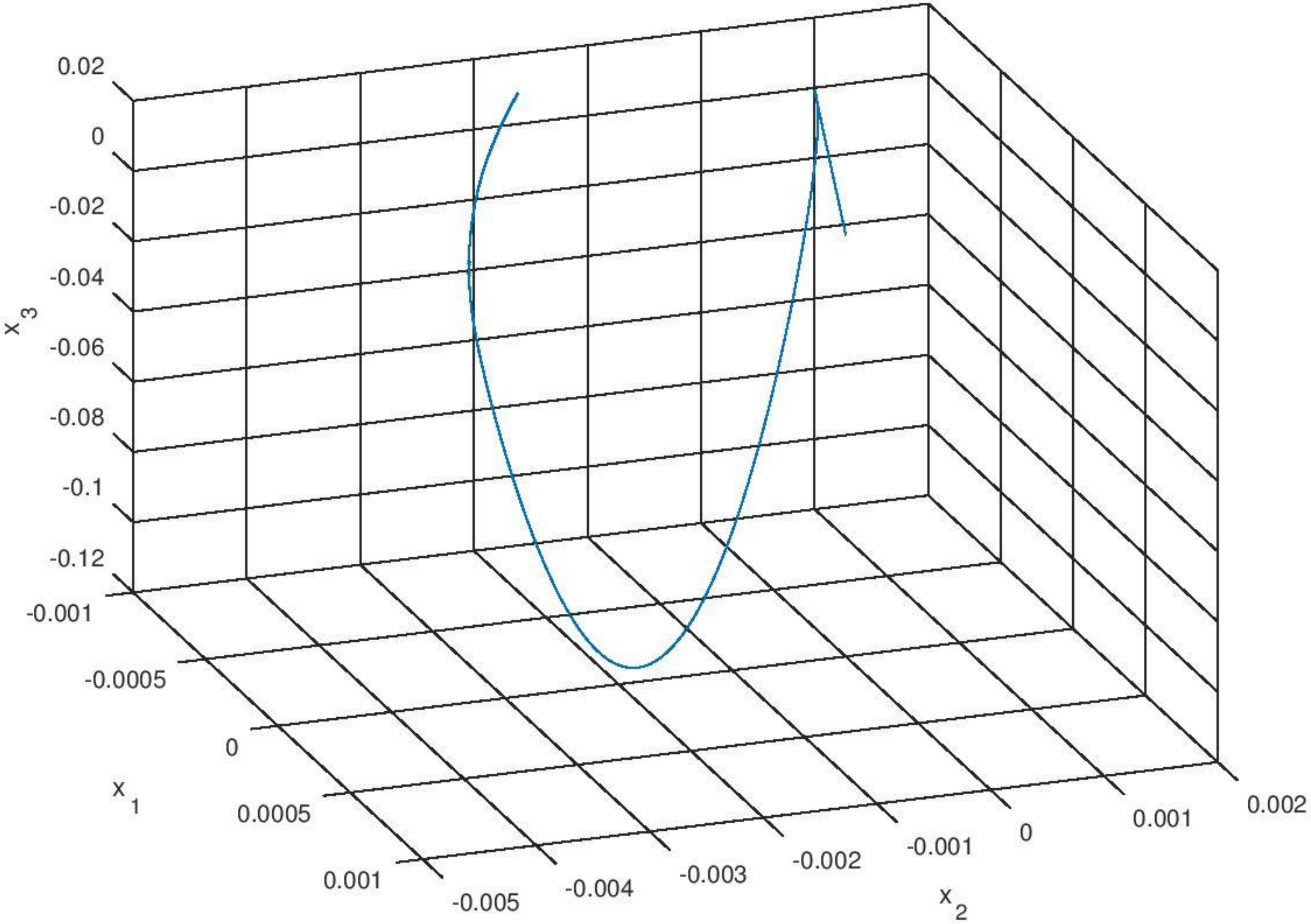
x1=-0.01; x2=0.005; x3=0.05;

I. Original System



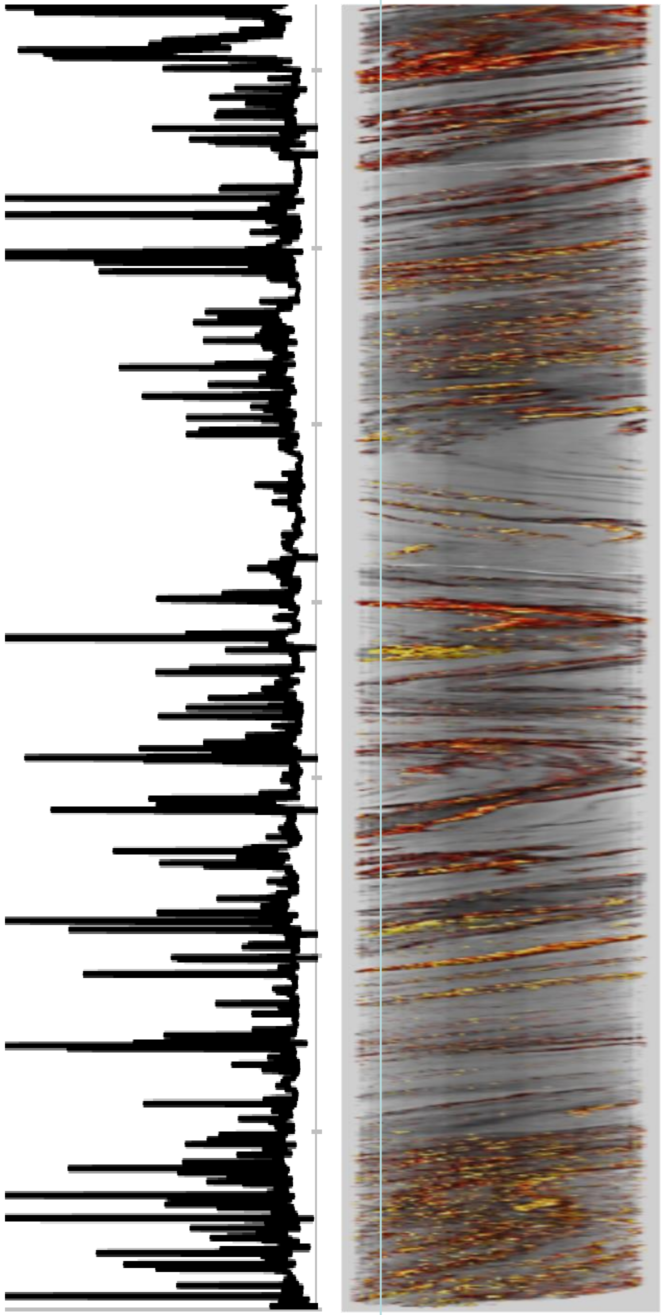
III. Identified System

$$\dot{x}=0-0.12x+0.15y-0.03z$$
$$\dot{y}=0-0.04x+0.05y-0.09z$$
$$\dot{z}=0+2.5x-7.2y+4.8z$$
$$+0.05xx-0.07xy-0.05xz$$
$$+0.03yy+0.03yz$$



0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
x 3
-0.001
-0.0005
0
0.0005
0.001
x 1
-0.005
-0.004
-0.003
-0.002
-0.001
0
0.001
0.002
x 2

I. Original System

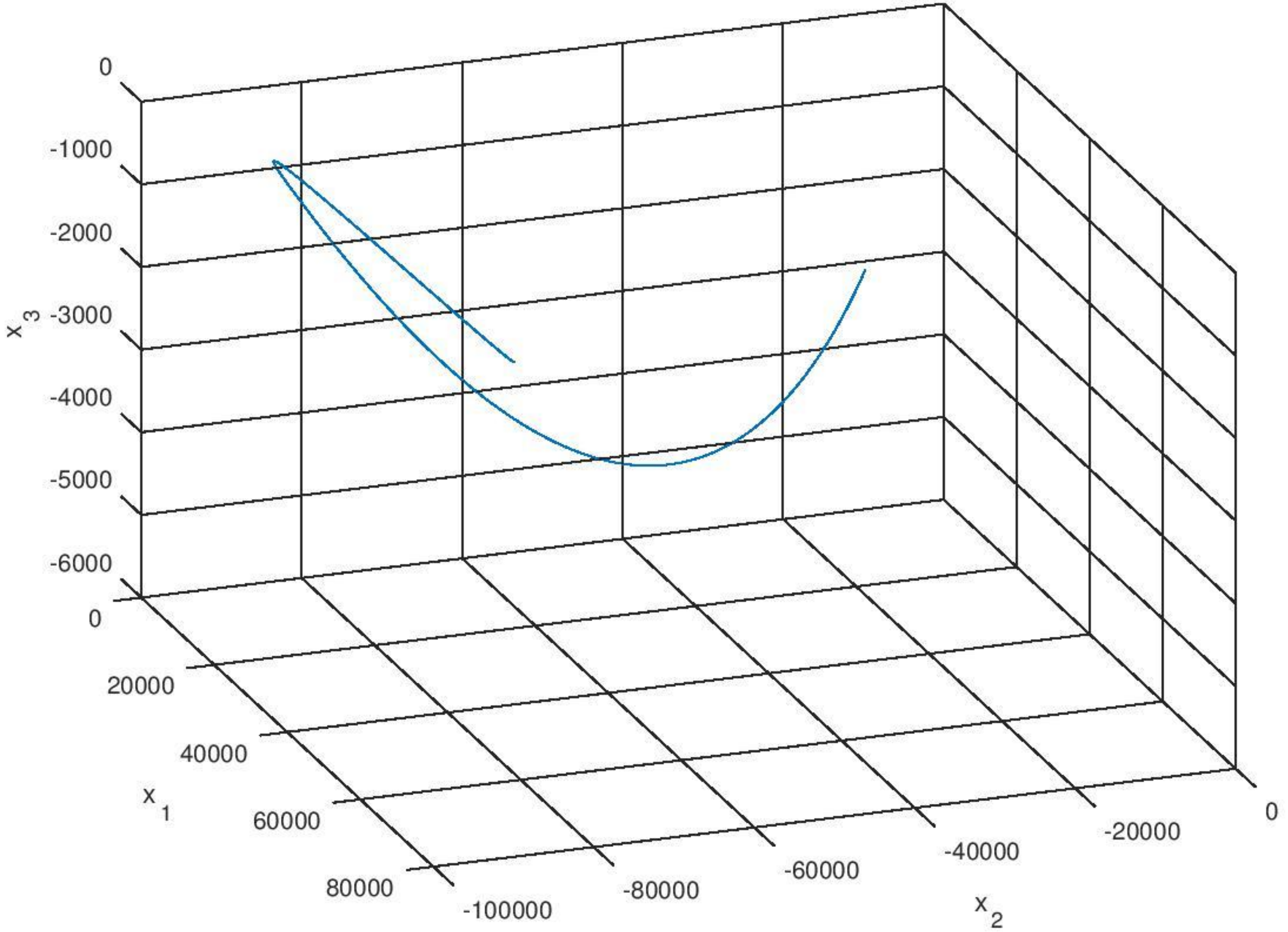


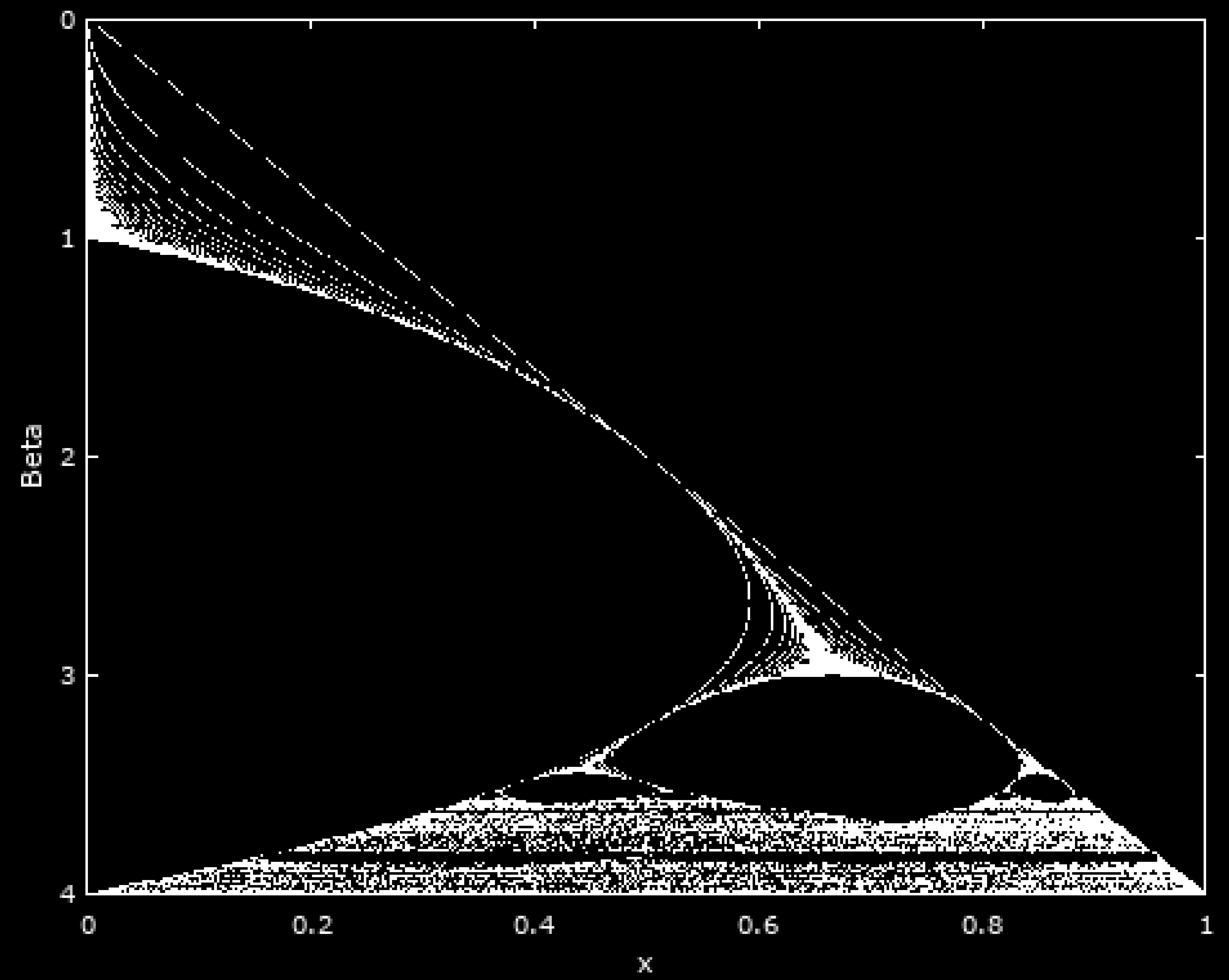
III. Identified System

$$\dot{x}=2.0-0.51x+0y-0.15z +0.04xz$$

$$\dot{y}=1.0+0.04x-0.3y+0.06z$$

$$\dot{z}=1.2+0x+0.05y-0.3z$$


0
-1000
-2000
x_3 -3000
-4000
-5000
-6000
0
20000
40000
x_1
60000
80000
-100000
-80000
-60000
-40000
-20000
0
x_2



Initial value x=0.5

The Grand Challenge

With successful reconstruction of the system equations and the connecting topology, it may be possible to address challenging and significant problems such as identification of causal relations among interacting components and the detection of hidden nodes.

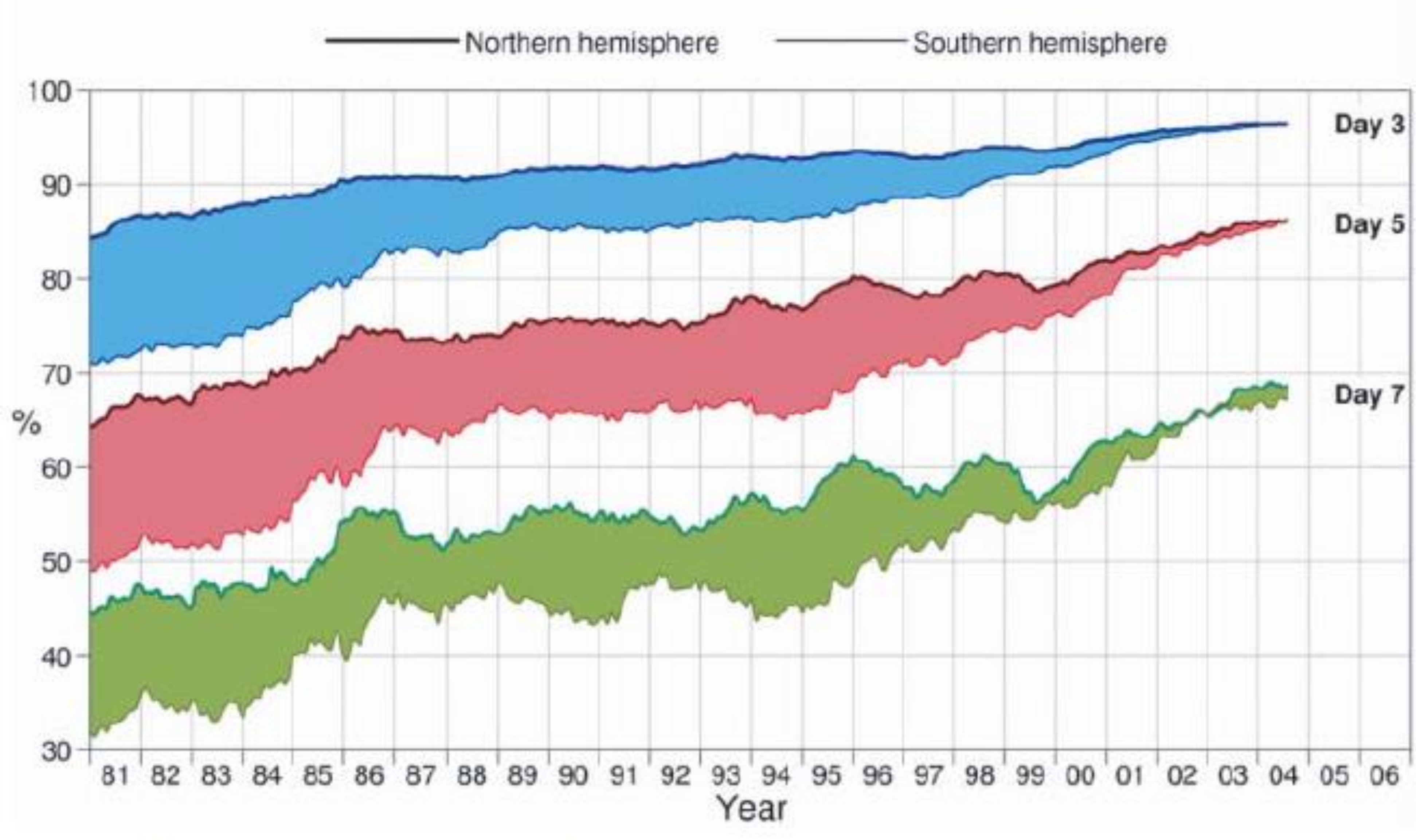
The Grand Challenge

The 'inverse' problem thus presents a grand challenge, requiring new paradigms beyond the traditional delay coordinate embedding methodology

Which we now have.

Such a process provides critical insight for potential new mineralising systems -

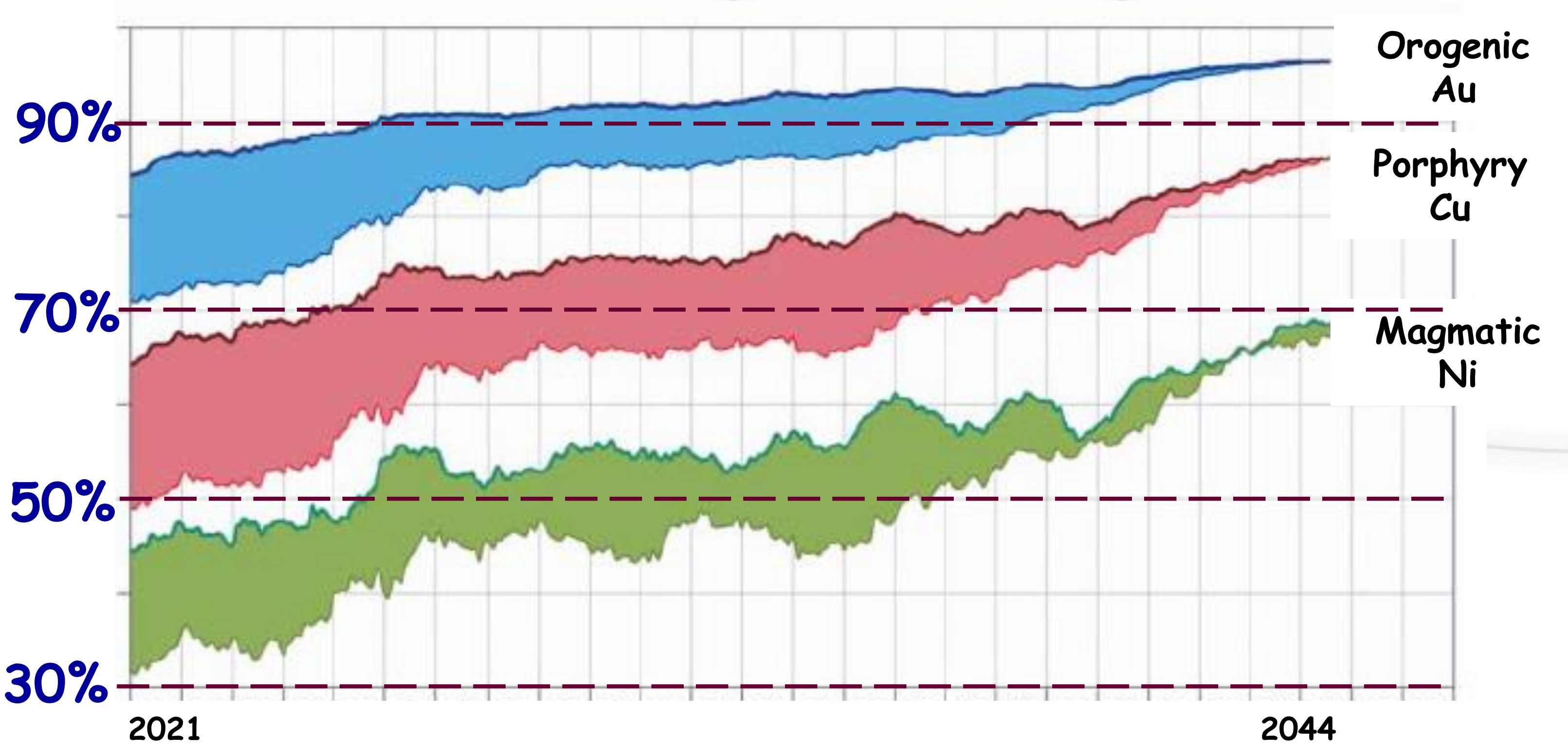
Evolution of mean forecast skill for the extratropical northern and southern hemispheres, 1981-2004.



Yoden, S. 2007. Atmospheric predictability. J. Met. Soc. Japan,, 85, 77-102.

AIM

Evolution of mean forecast skill for mineralising systems 2021-2044.



After Yoden, S. 2007. Atmospheric predictability. J. Met. Soc. Japan, 85, 77-102.

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Centre for EXPLORATION TARGETING

CSIRO

Thank you

Hobbs & Ord Nonlinear dynamical analysis of GNSS data: Quantification, precursors and synchronisation. Progress in Earth and Planetary Science. 5:36, 2018

Oberst et al. Detection of unstable periodic orbits in mineralising geological systems. Chaos 28, 085711, 2018.

Ord & Hobbs Quantitative measures of deformed rocks: The links to dynamics. Journal of Structural Geology. https://doi.org/10.1016/j.jsg.2018.05.027 2018.

Ord et al. Nonlinear analysis of natural folds using wavelet transforms and recurrence plots. Phil Trans A. 376, 20170257. 2018