

1 Main Equations

$$\begin{aligned}
\frac{\partial A}{\partial t} &= p(I, q)A - l_{bg}A - v\frac{\partial A}{\partial z} + d\frac{\partial^2 A}{\partial z^2} \\
\frac{\partial R_b}{\partial t} &= \rho(q, R_d)A - l_{bg}R_b - v\frac{\partial R_b}{\partial z} + d\frac{\partial^2 R_b}{\partial z^2} \\
\frac{\partial R_d}{\partial t} &= -\rho(q, R_d)A + l_{bg}R_b + d\frac{\partial^2 R_d}{\partial z^2} \\
I(z) &= I_0 \exp - \left(\int_0^z kAdz + k_{bg}z \right) \\
\frac{\partial R_s}{\partial t} &= vR_b(z_{max}) - rR_s
\end{aligned}$$

2 Other Equations

Algal nutrient quota: $q = \frac{R_b}{A}$

Note: this calculation for q is correct as long as $v = d$ in equation $\frac{\partial A}{\partial t}$?

Specific algal growth rate: $p(I, q) = \mu_{max} \left(\frac{q - q_{min}}{q} \right) \frac{I}{h + I}$

Specific algal nutrient uptake rate: $\rho(q, R_d) = \rho_{max} \left(\frac{q_{max} - q}{q_{max} - q_{min}} \right) \frac{R_d}{m + R_d}$

3 First Order Equations

$$\begin{aligned}
A'_1 &= -A_2 \\
A'_2 &= \frac{1}{d} (-vA_2 - p(I, q)A_1 + l_{bg}A_1) \\
R'_{b1} &= -R_{b2} \\
R'_{b2} &= \frac{1}{d} (-vR_{b2} - \rho(q, R_{d1})A_1 - l_{bg}R_{b1}) \\
R'_{d1} &= -R_{d2} \\
R'_{d2} &= \frac{1}{d} (\rho(q, R_{d1})A_1 - l_{bg}R_{b1}) \\
I' &= -(kA_1 + k_{bg}z)I
\end{aligned}$$

4 Boundary Conditions

$$\begin{aligned}
vA_1(0) - dA_2(0) &= 0 & A_2(z_{max}) &= 0 \\
vR_{b1}(0) - dR_{b2}(0) &= 0 & R_{b2}(z_{max}) &= 0 \\
R_{d2}(0) &= 0 & dR_{d2}(z_{max}) - vR_{b1}(z_{max}) &= 0 \\
I(0) &= I_0
\end{aligned}$$

5 Values

Initial guesses for shooting method taken from the *Standard Model*:

$$\begin{aligned}A_1 &= 100 \text{ mg C m}^{-3} \\R_{b1} &= 2.2 \text{ mg P m}^{-3} \\R_{d1} &= 30 \text{ mg P m}^{-3}\end{aligned}$$

But currently using $R_{b1} = 5 * (q_{min}A_1)$ to keep uptake function positive and $R_{d1} = 89.333$.

Following the *Standard Model*

$$\begin{aligned}d &= 0.01 - 1,000 & v &= 0.25 \\z_{max} &= 10 - 60 & I_0 &= 300 \\h &= 120 & l_{bg} &= 0.1 \\k &= 0.0003 & k_{bg} &= 0.4 \\\mu_{max} &= 1.2 & \rho_{max} &= 0.2 \\q_{min} &= 0.004 & q_{max} &= 0.04 \\m &= 15\end{aligned}$$

Redfield Ratio: 0.022 mg P mg C - 1