1 Main Equations

$$\frac{\partial A}{\partial t} = p(I,q)A - l_{bg}A - v\frac{\partial A}{\partial z} + d\frac{\partial^2 A}{\partial z^2}$$
$$\frac{\partial R_b}{\partial t} = \rho(q,R_d)A - l_{bg}Rb - v\frac{\partial R_b}{\partial z} + d\frac{\partial^2 R_b}{\partial z^2}$$
$$\frac{\partial R_d}{\partial t} = -\rho(q,R_d)A + l_{bg}Rb + d\frac{\partial^2 R_d}{\partial z^2}$$
$$I(z) = I_0 exp - \left(\int_0^z kAdz + k_{bg}z\right)$$
$$\frac{\partial R_s}{\partial t} = vR_b(z_m ax) - rR_s$$

2 Other Equations

Algal nutrient quota: $q = \frac{R_b}{A}$
Note: this calculation for q is correct as long as v = d in equation $\frac{\partial A}{\partial t}$?
Specific algal growth rate: $p(I,q) = \mu_{max}\left(\frac{q-q_{min}}{q}\right)\frac{I}{h+I}$
Specific algal nutrient uptake rate: $\rho(q,R_d) = \rho_{max}\left(\frac{q_{max}-q}{q_{max}-q_{min}}\right)\frac{R_d}{m+R_d}$

3 First Order Equations

$$A_1' = A_2$$
$$A_2' = \frac{1}{d}\left(vA_2 - p(I,q)A_1 + l_{bg}A_1\right)$$
$$R_{b1}' = R_{b2}$$
$$R_{b2}' = \frac{1}{d}\left(vR_{b2} - \rho(q,R_{d1})A_1 + l_{bg}R_{b1}\right)$$
$$R_{d1}' = R_{d2}$$
$$R_{d2}' = \frac{1}{d}\left(\rho(q,R_{d1})A_1 - l_{bg}R_{b1}\right)$$
$$I' = -(kA_1 + k_{bg}z)I$$

4 Boundary Conditions

$$vA(0) - dA'(0) = 0 \qquad A'(z_{max}) = 0$$
$$vR_b(0) - dR_b'(0) = 0 \qquad R_b'(z_{max}) = 0$$
$$R_d'(0) = 0 \qquad dR_d'(z_{max}) - vR_b(z_{max}) = 0$$
$$I(0) = I_0$$

5 Values

Initial guesses for shooting method taken from the *Standard Model*:

$$A_1 = 100\, mg\, C\, m^{-3}$$
$$R_{b1} = 2.2\, mg\, P\, m^{-3}$$
$$R_{d1} = 30\, mg\, P\, m^{-3}$$

But currently using $R_{b1} = 5 * (q_{min} A_1)$ to keep uptake function positive and $R_{d1} = 89.333$.
Following the *Standard Model*

$d = 0.01 - 1,000$	$v = 0.25$
$z_{max} = 10 - 60$	$I_0 = 300$
$h = 120$	$l_{bg} = 0.1$
$k = 0.0003$	$k_{bg} = 0.4$
$\mu_{max} = 1.2$	$\rho_{max} = 0.2$
$q_{min} = 0.004$	$q_{max} = 0.04$
$m = 15$	

Redfield Ratio: 0.022 mg P mg C - 1

6 Finite Difference Approximations

$$A_z^{t+1} = \Delta t \left(\frac{d}{\Delta z^2} - \frac{v}{2\Delta z} \right) A_{z+1}^t + \Delta t \left(\frac{1}{\Delta t} + p(I_z^t, q_z^t) - l_{bg} - \frac{2d}{\Delta z^2} \right) A_z^t + \Delta t \left(\frac{d}{\Delta z^2} - \frac{v}{2\Delta z} \right) A_{z-1}^t$$

$$Rb_z^{t+1} = \Delta t \left(\frac{d}{\Delta z^2} - \frac{v}{2\Delta z} \right) Rb_{z+1}^t + \Delta t \left(\frac{1}{\Delta t} - l_{bg} - \frac{2d}{\Delta z^2} \right) Rb_z^t + \Delta t \left(\frac{d}{\Delta z^2} - \frac{v}{2\Delta z} \right) Rb_{z-1}^t + \Delta t \rho(q_z^t, Rd_z^t$$

$$Rd_z^{t+1} = \Delta t \frac{d}{\Delta z^2} Rd_{z+1}^t + \Delta t \left(\frac{1}{\Delta t} - \frac{2d}{\Delta z^2} \right) Rd_z^t + \Delta t \frac{d}{\Delta z^2} Rd_{z-1}^t - \Delta t \rho(q_z^t, Rd_z^t) A_z^t + \Delta t l_{bg} Rb_z^t$$