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Numerical solution of the coupled viscous Burgers' equation

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ABSTRACT

In the present paper, a numerical method is proposed for the numerical solution of a coupled system of viscous Burgers' equation with appropriate initial and boundary conditions, by using the cubic B-spline collocation scheme on the uniform mesh points. The scheme is based on Crank–Nicolson formulation for time integration and cubic B-spline functions for space integration. The method is shown to be unconditionally stable using von-Neumann method. The accuracy of the proposed method is demonstrated by applying it on three test problems. Computed results are depicted graphically and are compared with those already available in the literature. The obtained numerical solutions indicate that the method is reliable and yields results compatible with the exact solutions.

1. Introduction

The present study is concerned with the numerical solution of a coupled viscous Burgers' equation which was derived by Esipov [1] to study the model of polydispersive sedimentation. This system of coupled viscous Burgers' equation is a simple model of sedimentation or evolution of scaled volume concentrations of two kinds of particles in fluid suspensions or colloids under the effect of gravity.

The coupled viscous Burgers' equation is given by

$$u_t - u_{xx} + \eta u u_x + \alpha (uv)_x = 0, \quad x \in [a,b], \quad t \in [0,T], \tag{1}$$
$$v_t - v_{xx} + \eta v v_x + \beta (uv)_x = 0, \quad x \in [a,b], \quad t \in [0,T], \tag{2}$$

with the initial conditions

$$u(x,0) = \phi_1(x), \quad v(x,0) = \phi_2(x), \tag{3}$$

and the boundary conditions

$$\begin{aligned} u(a,t) &= f_1(a,t), \quad u(b,t) = f_2(b,t), \\ v(a,t) &= g_1(a,t), \quad v(b,t) = g_2(b,t), \end{aligned} \tag{4}$$

where η is a real constant, α and β are arbitrary constants depending on the system parameters such as Peclet number, Stokes velocity of particles due to gravity and Brownian diffusivity [2].

In recent years, several studies for the coupled linear and nonlinear initial/boundary value problems arises in the literature. A number of numerical algorithm such as Harmonic Differential Quadrature Finite differences coupled approach [3] and conjugate filter approach [4] are available for obtaining approximate solutions of coupled equations as well as nonlinear

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differential equations. Also an application of meshfree interpolation method for the numerical solution of the coupled nonlinear partial differential equation is proposed in [5]. Khater et al. [6] have obtained approximate solution of the viscous coupled Burgers' equation using cubic-spline collocation method. The equation has been solved by Deghan et al. [7] using a Pade technique and Rashid et al. [8] have used Fourier Pseudospectral method to find numerical solution of the equation. Variational iteration method has been presented for solving the coupled viscous Burgers' equation by Adbou and Soliman [9].

The exact solution of the equation has been obtained by Kaya [10] using Adomian Decomposition method and Soliman [11] presented modified extended tanh-function method to obtain its exact solution.

The aim of the paper is to investigate the solution of the coupled viscous Burgers' equation when cubic B-spline functions are used to express the approximate function in the collocation method. Numerical methods using B-spline functions have been successfully applied to solve various linear and nonlinear partial differential equations. The use of various degree of B-spline functions in getting the numerical solution of some partial differential equations are shown to provide easy and simple algorithms, as an example, cubic B-spline collocation method is used in [12,13]. Quintic B-spline collocation method is used to find numerical solution of some nonlinear equations in [14,15].

The brief outline of this paper is as follows. In Section 2, cubic B-spline collocation scheme is explained. In Sections 3 and 4, the method is described and applied to the coupled viscous Burgers' equation. In Section 5, stability of the method is discussed. In Section 6, numerical examples are included to establish the applicability and accuracy of the proposed method computationally. Conclusion is given in Section 7 that briefly summarizes the numerical outcomes.

2. Cubic B-spline functions

To construct numerical solution, consider nodal points (x_i, t_j) defined in the region $[a,b] \times [0,T]$ where

$$a = x_0 < x_1, \ldots, x_{N-1} < x_N = b, \quad x_{i+1} - x_i = h,$$
$$0 = t_0 < t_1, \ldots, t_n < \ldots < T, \quad t_{j+1} - t_j = \Delta t,$$

we can define

$$x_i = a + ih, \quad i = 0, 1, 2, \ldots, N$$

and $t_n = n\Delta t$, $n = 0, 1, 2, \ldots$

The cubic B-spline basis functions at knots are given by:

$$B_m(x) = \frac{1}{h^3}\begin{cases} (x - x_{m-2})^3 & x \in [x_{m-2}, x_{m-1}) \\ (x - x_{m-2})^3 - 4(x - x_{m-1})^3 & x \in [x_{m-1}, x_m) \\ (x_{m+2} - x)^3 - 4(x_{m+1} - x)^3 & x \in [x_m, x_{m+1}) \\ (x_{m+2} - x)^3 & x \in [x_{m+1}, x_{m+2}) \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Using cubic B-spline basis function (5) the values of $B_m(x)$ and its derivatives at the nodal points can be calculated, which are tabulated in Table 1.

3. Solution of coupled viscous Burgers' equation

To apply the proposed method, discretizing the time derivative in the usual finite difference way and applying Crank–Nicolson scheme to Eqs. (1) and (2), we get

$$\left[\frac{u^{n+1} - u^n}{\Delta t}\right] - \left[\frac{u_{xx}^{n+1} + u_{xx}^n}{2}\right] + \eta\left[\frac{(uu_x)^{n+1} + (uu_x)^n}{2}\right] + \alpha\left[\frac{((uv)_x)^{n+1} + ((uv)_x)^n}{2}\right] = 0, \tag{6}$$

$$\left[\frac{v^{n+1} - v^n}{\Delta t}\right] - \left[\frac{v_{xx}^{n+1} + v_{xx}^n}{2}\right] + \eta\left[\frac{(vv_x)^{n+1} + (vv_x)^n}{2}\right] + \beta\left[\frac{((uv)_x)^{n+1} + ((uv)_x)^n}{2}\right] = 0, \tag{7}$$

where Δt is the time step.

Table 1
Value of $B_m(x)$ and its derivatives at the nodal points.

	x_{m-2}	x_{m-1}	x_m	x_{m+1}	x_{m+2}
$B_m(x)$	0	1	4	1	0
$B'_m(x)$	0	3/h	0	−3/h	0
$B''_m(x)$	0	$6/h^2$	$-12/h^2$	$6/h^2$	0

In the Crank–Nicolson scheme, the time stepping process is half explicit and half implicit. So the method is better than simple finite difference method.

The nonlinear terms in Eqs. (6) and (7) is linearized using the form given by Rubin and Graves [16] as:

$$(uu_x)^{n+1} = u^{n+1}u_x^n + u^n u_x^{n+1} - (uu_x)^n.$$

Similarly the linearized form for uv_x and $(uv)_x$ can be obtained. Expressing $u(x,t)$ and $v(x,t)$ by using cubic B-spline functions $B_m(x)$ and the time dependent parameters $\delta_m(t)$ and $\sigma_m(t)$, for $u(x,t)$ and $v(x,t)$ respectively, the approximate solution can be written as:

$$U_m(x,t) = \sum_{m=-1}^{N+1} \delta_m(t)B_m(x), \quad V_m(x,t) = \sum_{m=-1}^{N+1} \sigma_m(t)B_m(x). \tag{8}$$

Using approximate function (8) and cubic B-spline functions (5), the approximate values of u and v denoted by $U(x)$, $V(x)$ and their derivatives up to second order are determined in terms of the time parameters $\delta_m(t)$ and $\sigma_m(t)$, respectively, as

$$\begin{aligned} &U_m = \delta_{m-1} + 4\delta_m + \delta_{m+1}, \quad hU'_m = 3(\delta_{m+1} - \delta_{m-1}), \quad h^2U''_m(x) = 6(\delta_{m-1} - 2\delta_m + \delta_{m+1}),\\ &V_m = \sigma_{m-1} + 4\sigma_m + \sigma_{m+1}, \quad hV'_m = 3(\sigma_{m+1} - \sigma_{m-1}), \quad h^2V''_m(x) = 6(\sigma_{m-1} - 2\sigma_m + \sigma_{m+1}) \end{aligned} \tag{9}$$

On substituting the approximate solution for u and v and its derivatives from Eq. (9) at the knots in Eqs. (6) and (7) yields the following difference equation with the variables δ_m and σ_m. Here terms of left-hand side consists of terms at $(n+1)$th time level.

$$\begin{aligned} &a_1(\delta_{m-1} + 4\delta_m + \delta_{m+1}) + a_2(\delta_{m+1} - \delta_{m-1}) - a_3(\delta_{m-1} - 2\delta_m + \delta_{m+1}) + a_4(\sigma_{m-1} + 4\sigma_m + \sigma_{m+1}) + a_5(\sigma_{m+1} - \sigma_{m-1})\\ &\quad = U^n + U^n_{xx}(\Delta t/2), \end{aligned} \tag{10}$$

$$\begin{aligned} &a_6(\delta_{m-1} + 4\delta_m + \delta_{m+1}) + a_7(\delta_{m+1} - \delta_{m-1}) + a_8(\sigma_{m-1} + 4\sigma_m + \sigma_{m+1}) + a_9(\sigma_{m+1} - \sigma_{m-1}) - a_{10}(\sigma_{m-1} - 2\sigma_m + \sigma_{m+1})\\ &\quad = V^n + V^n_{xx}(\Delta t/2), \end{aligned} \tag{11}$$

where $m = 0,\ldots,N$ and

$$\begin{aligned} &a_1 = 1 + \frac{\Delta t}{2}(\eta U_x^n + \alpha V_x^n), \quad a_2 = \frac{3\Delta t}{2h}(\eta U^n + \alpha V^n), \quad a_3 = \frac{3\Delta t}{h^2}, \quad a_4 = \frac{\Delta t}{2}(\alpha U_x^n), \quad a_5 = \frac{3\Delta t}{2h}(\alpha U^n),\\ &a_6 = \frac{\Delta t}{2}(\beta V_x^n), \quad a_7 = \frac{3\Delta t}{2h}(\beta V^n), \quad a_8 = 1 + \frac{\Delta t}{2}(\beta U_x^n + \eta V_x^n), \quad a_9 = \frac{3\Delta t}{2h}(\beta U^n + \eta V^n), \quad a_{10} = \frac{3\Delta t}{h^2}. \end{aligned}$$

The system thus obtained on simplifying Eqs. (10) and (11) consists of $(2N+2)$ linear equations in $(2N+6)$ unknowns

$$(\delta_{-1}, \delta_0, \delta_1, \ldots, \delta_N, \delta_{N+1}), \quad (\sigma_{-1}, \sigma_0, \sigma_1, \ldots, \sigma_N, \sigma_{N+1})$$

To obtain a unique solution to the resulting system four additional constraints are required. These are obtained by imposing boundary conditions. Eliminating δ_{-1}, δ_{N+1} and σ_{-1}, σ_{N+1} the system get reduced to a matrix system of dimension $(2N+2) \times (2N+2)$ which is a bi-tridiagonal system that can be solved by a modified form of Thomas algorithm considering elements of (2×2) matrices [17].

4. Initial values

At a particular time-level, the approximate solutions $U(x,t)$ and $V(x,t)$ can be determined repeatedly by solving the recurrence relation, once the initial vectors have been computed from the initial and boundary conditions.

From the initial condition

$$u(x_m, 0) = \phi_1(x_m), \quad m = 0,\ldots,N$$

we get $(N+1)$ equations in $(N+3)$ unknowns. The two unknowns δ_{-1} and δ_{N+1} can be obtained from the relation $u_x(x_0,0) = \phi'_1(x_0)$, and $u_x(x_N,0) = \phi'_1(x_N)$ at the knots. It leads to system of $(N+1)$ equations in $(N+1)$ unknowns which can be solved by Thomas algorithm.

Similarly, using initial condition $v(x_m,0) = \phi_2(x_m)$, the initial vectors for v can be computed.

5. Stability of the scheme

We have investigated stability of the proposed method by applying von-Neumann method. To apply this method, we have linearized the nonlinear terms uu_x and $(uv)_x$ in the Eq. (1) by considering u and v as a local constants γ_1 and γ_2, respectively.

Substituting the approximate solution for u and v and their derivatives at the knots in the modified equation after discretizing the time derivative in the usual finite difference way and applying Crank–Nicolson scheme yields a difference equation with the variables δ_m given as:

$$\delta_{m-1}^{n+1}\omega_1 + \delta_m^{n+1}\omega_2 + \delta_{m+1}^{n+1}\omega_3 + \omega(\sigma_{m+1}^{n+1} - \sigma_{m-1}^{n+1}) = \delta_{m-1}^n\omega_4 + \delta_m^n\omega_5 + \delta_{m+1}^n\omega_6 + \omega(\sigma_{m+1}^n - \sigma_{m-1}^n), \tag{12}$$

where

$$\omega_1 = 1 - \frac{3(\eta\gamma_1 + \alpha\gamma_2)\Delta t}{2h} - \frac{3\Delta t}{h^2}, \quad \omega_2 = 4 + \frac{6\Delta t}{h^2}, \quad \omega_3 = 1 + \frac{3(\eta\gamma_1 + \alpha\gamma_2)\Delta t}{2h} - \frac{3\Delta t}{h^2},$$
$$\omega_4 = 1 + \frac{3(\eta\gamma_1 + \alpha\gamma_2)\Delta t}{2h} + \frac{3\Delta t}{h^2}, \quad \omega_5 = 4 - \frac{6\Delta t}{h^2}, \quad \omega_6 = 1 - \frac{3(\eta\gamma_1 + \alpha\gamma_2)\Delta t}{2h} + \frac{3\Delta t}{h^2},$$
$$\omega = \frac{3\alpha\gamma_1\Delta t}{2h}.$$

Now on substituting $\delta_m^n = A\xi^n \exp(im\varphi h)$ and $\sigma_m^n = B\xi^n \exp(im\varphi h)$ into the Eq. (12) and simplifying, where A and B are the harmonics amplitude, φ is the mode number, h is the element size and $i = \sqrt{-1}$ we obtain

$$[X_2 + iY]\xi^{n+1} = [X_1 - iY]\xi^n, \tag{13}$$

where

$$X_1 = A\left[2(\cos\varphi h + 2) - 6\frac{\Delta t}{h^2}(1 - \cos\varphi h)\right], \quad X_2 = A\left[2(\cos\varphi h + 2) + 6\frac{\Delta t}{h^2}(1 - \cos\varphi h)\right], \tag{14}$$

$$Y = \sin\varphi h\left\{A\left[\frac{3(\eta\gamma_1 + \alpha\gamma_2)\Delta t}{h}\right] + B\left[\frac{3\alpha\gamma_1\Delta t}{h}\right]\right\}. \tag{15}$$

Table 2
Errors at different time for $u(x,t)$ Example 1 with $\Delta t = 0.001$.

t	Number of partitions = 200		Number of partitions = 400		Rashid [5]	
	L_2	L_∞	L_2	L_∞	L_2	L_∞
0.1	8.21E−06	7.45E−06	2.05E−06	1.86E−06	–	–
0.5	2.49E−05	4.10E−05	1.02E−05	6.22E−06	–	–
1	3.00E−05	8.21E−05	2.04E−05	7. 56E−06	2.88E−05	1.16E−05

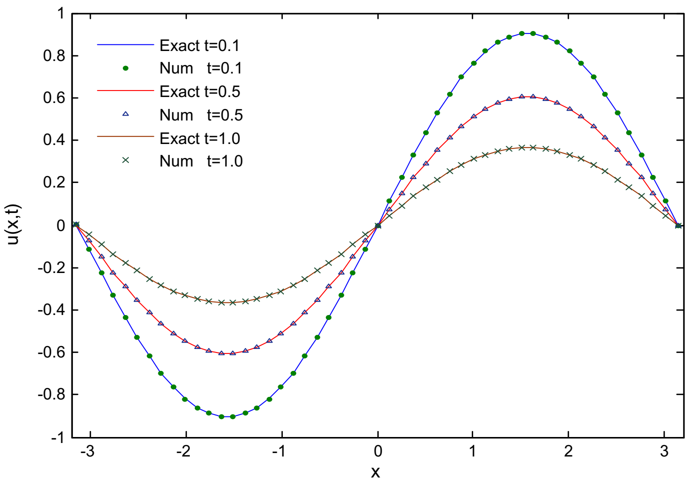

Fig. 1. A comparison between numerical and analytical results for Example 1.

Table 3
Maximum error and computed order of convergence of the present method for u in Example 1.

	$t = 0.1$			$t = 0.5$		
N	L_∞	Ratio	Order of conv.	L_∞	Ratio	Order of conv.
32	2.9104E−04	–	–	9.7478E−04	–	–
64	7.2704E−05	4.0030	2.001	2.4361E−04	4.0014	2.005
128	1.8178E−05	3.9996	1.999	6.0896E−05	4.0004	2.001
256	4.5497E−05	3.9953	1.998	1.5223E−05	4.0003	2.001
512	1.1430E−06	3.9806	1.993	3.8052E−05	4.0006	2.002

Table 4
Comparisons of errors at different time for $u(x,t)$ for Example 2.

t	α	β	Khater [6]		Rashid [8]		Present method	
			L_2	L_∞	L_2	L_∞	L_2	L_∞
0.5	0.1	0.3	1.44E−3	4.38E−5	3.245E−5	9.619E−4	6.736E−4	4.167E−5
	0.3	0.03	6.68E−4	4.58E−5	2.733E−5	4.310E−4	7.326E−4	4.590E−5
1.0	0.1	0.3	1.27E−3	8.66E−5	2.405E−5	1.153E−3	1.325E−3	8.258E−5
	0.3	0.03	1.30E−3	9.16E−5	2.832E−5	1.268E−3	1.452E−3	9.182E−5

Table 5
Comparisons of errors at different time for $v(x,t)$ for Example 2.

t	α	β	Khater [6]		Rashid [8]		Present method	
			L_2	L_∞	L_2	L_∞	L_2	L_∞
0.5	0.1	0.3	5.42E−4	4.99E−5	2.746E−5	3.332E−4	9. 057E−4	1.480E−4
	0.3	0.03	1.20E−3	1.81E−4	2.454E−4	1.148E−3	1.591E−3	5.729E−4
1.0	0.1	0.3	1.29E−3	9.92E−5	3.745E−5	1.162E−3	1. 251E−3	4.770E−5
	0.3	0.03	2.35E−3	3.62E−4	4.525E−4	1.638E−3	2.250E−3	3.617E−4

Table 6
Maximum values of u and v at different time for $\alpha = \beta = 10$.

t	Max value of u	At point	Max value of v	At point
0.1	0.14456	0.58	0.14306	0.66
0.2	0.05237	0.54	0.04697	0.56
0.3	0.01932	0.52	0.01725	0.52
0.4	0.00718	0.50	0.00641	0.50

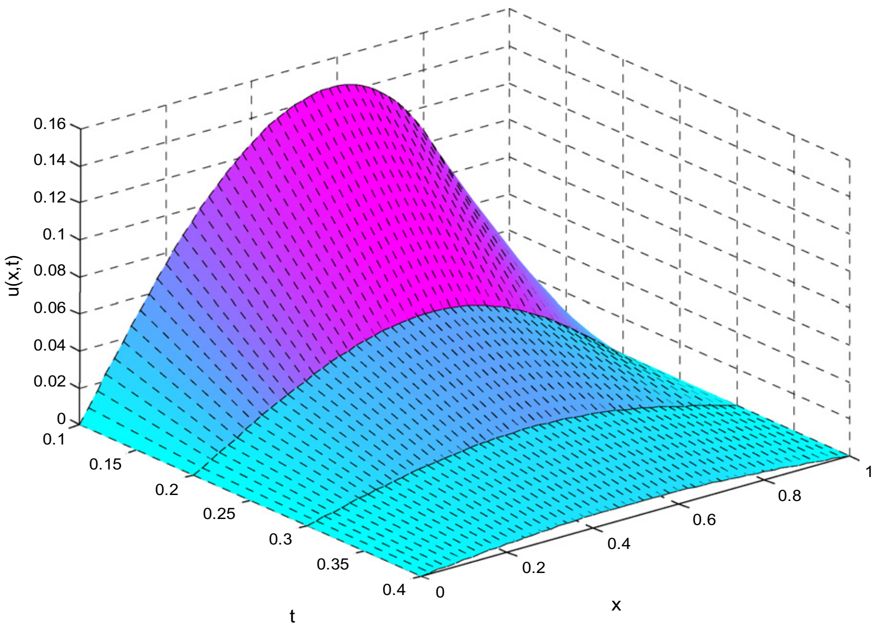

Fig. 2. The numerical solution $u(x,t)$ of Example 3 at different time for $\alpha = \beta = 10$.

The von-Neumann stability condition for the system (13) is that the maximum modulus of the eigen-values of the matrix A is to be less than or equal to one. On direct calculation of these eigen-values we obtain

$$\xi = \frac{X_1 - iY}{X_2 + iY}. \tag{16}$$

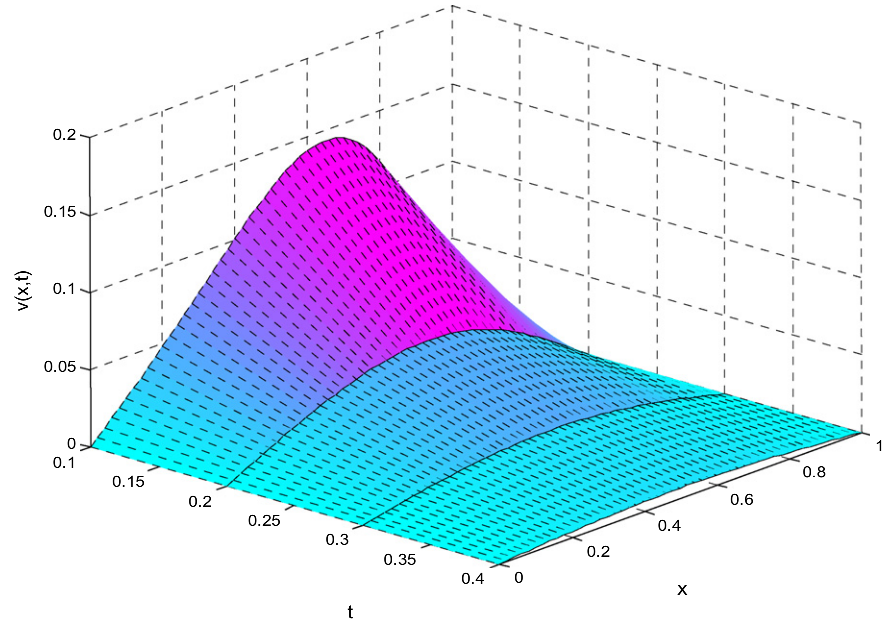

Fig. 3. The numerical solution $v(x,t)$ of Example 3 at different time for $\alpha = \beta = 10$.

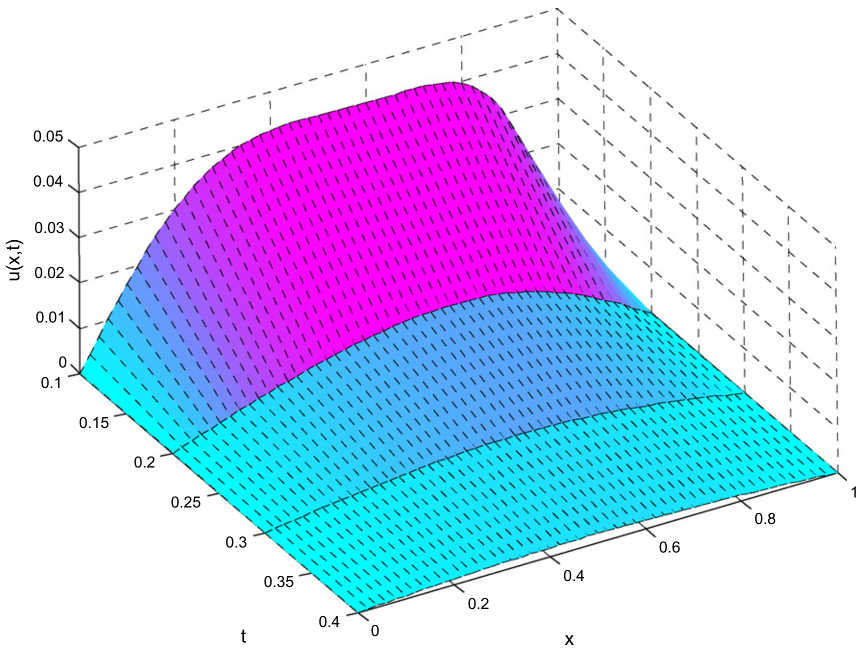

Fig. 4. The numerical solution $u(x,t)$ of Example 3 at different time for $\alpha = \beta = 100$.

From (16) it is evident that the modulus of these eigen-values is less than one and hence the scheme is unconditionally stable. It means that there is no restriction on the grid size, i.e. on h and Δt, but we should choose them in such a way that the accuracy of the scheme is not degraded.

Similar results can be obtained from Eq. (2) due to symmetric u and v.

6. Numerical results and discussion

To gain insight into the performance of the suggested method, three numerical examples are given in this section with L_∞ and relative L_2 errors obtained by formula given by:

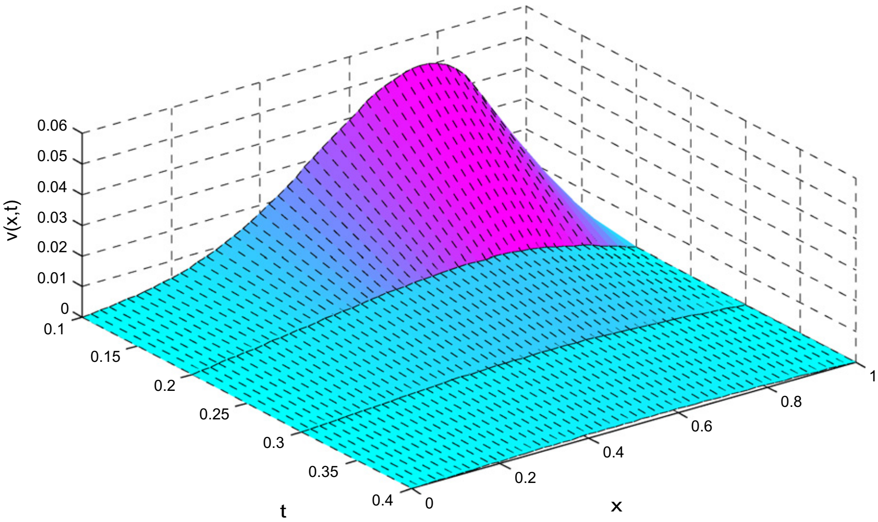

Fig. 5. The numerical solution $v(x,t)$ of Example 3 at different time for $\alpha = \beta = 100$.

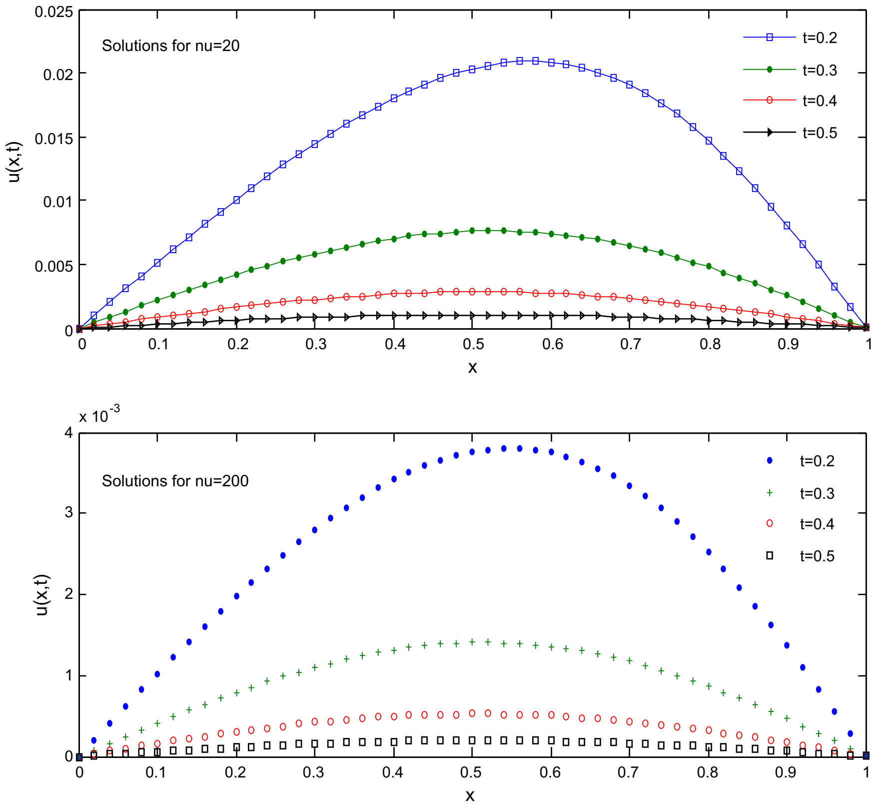

Fig. 6. Solution profile at different time levels for $\eta = 20$ and $\eta = 200$.

$$L_\infty = \max_i |U_i^{exact} - U_i^{num}|, \quad L_2 = \sqrt{\sum_{i=0}^{N} |U_i^{exact} - U_i^{num}|^2} \Bigg/ \sqrt{\sum_{i=0}^{N} |U_i^{exact}|^2}.$$

The numerical order of convergence R of the scheme is calculated by using the formula,

$$\mathrm{R} = \frac{\log(Error(N_1)/Error(N_2))}{\log(N_2/N_1)} \tag{17}$$

where $Error(N_1)$ and $Error(N_2)$ are the L_∞ errors at number of partitions N and $2N$, respectively.

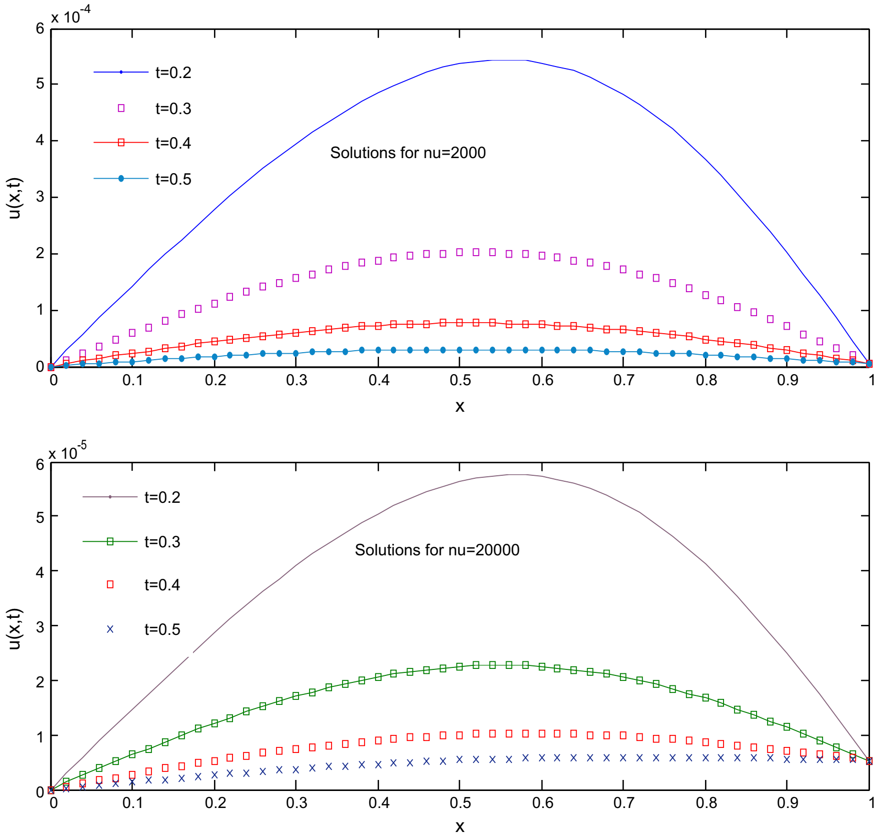

Fig. 7. Solution profile at different time levels for $\eta = 2000$ and $\eta = 20{,}000$.

Example 1. Numerical solution of coupled viscous Burgers' Eqs. (1) and (2) is obtained for $\alpha = 1$, $\beta = 1$, $\eta = -2$ which leads Eqs. (1) and (2) as

$$u_t - u_{xx} - 2uu_x + (uv)_x = 0, \quad v_t - v_{xx} - 2vv_x + (uv)_x = 0,$$

with the initial condition given by

$$u(x,0) = v(x,0) = \sin(x)$$

and boundary conditions taken from exact solution.

The exact solution of the equation is given by [10] as $u(x,t) = v(x,t) = \exp(-t)\sin(x)$.

We have obtained the numerical solution by taking domain $x \in [-\pi, \pi]$ with $\Delta t = 0.001$. The solution is tabulated in Table 2 with different number of partitions at different time-level. Results also depicted graphically for $u(x,t)$ in Fig. 1 for $t \in (0,1]$. Due to symmetric initial and boundary conditions, results are similar for $v(x,t)$. The order of convergence of this example is calculated by formula given by (17) and is tabulated in Table 3.

Example 2. Numerical solution of coupled Burgers' Eqs. (1) and (2) is obtained with $\eta = 2$ for different values of α and β at $t = 0.5$ and 1.0. The exact solution of the equation is given by [11] as

$$u(x,t) = a_0(1 - \tanh(A(x - 2At)),$$
$$v(x,t) = a_0\left(\left(\frac{2\beta - 1}{2\alpha - 1}\right) - \tanh(A(x - 2At))\right),$$

where

$$a_0 = 0.05 \quad \text{and} \quad A = \frac{1}{2}a_0\left(\frac{4\alpha\beta - 1}{2\alpha - 1}\right).$$

Results are calculated by considering domain as $x \in [-10, 10]$ with $\Delta t = 0.01$ and number of partitions as 100. The initial and boundary conditions are taken from the exact solution. The L_2 and L_∞ errors are calculated and compared in Tables 4 and 5 with those available in the literature.

Table 7
Maximum values of u and v at different time for $\alpha = \beta = 100$.

t	Max value of u	At point	Max value of v	At point
0.1	0.04175	0.46	0.05065	0.76
0.2	0.01479	0.58	0.01033	0.64
0.3	0.00534	0.54	0.00350	0.56
0.4	0.00198	0.52	0.00129	0.52

Table 8
Maximum absolute error and order of convergence for u and v for Example 3 at $t = 0.1$.

No. of partitions	u			v		
	L_∞	Ratio	Order of convergence	L_∞	Ratio	Order of convergence
$\alpha = \beta = 100$						
50	0.018812	–	–	0.0131387	–	–
100	0.005508	3.4154	1.772	0.0042798	3.0699	1.618
200	0.001649	3.3402	1.740	0.0013143	3.2563	1.703
$\alpha = \beta = 10$						
50	0.016182	–	–	0.015818	–	–
100	0.004935	3.2790	1.713	0.004873	3.2460	1.699
200	0.001493	3.3045	1.724	0.001477	3.2992	1.722

Example 3. Numerical solution of coupled Burgers' Eqs. (1) and (2) is obtained with the initial condition given by

$$u(x,0) = \begin{cases} \sin(2\pi x), & 0 \leqslant x \leqslant 0.5, \\ 0, & 0.5 < x \leqslant 1, \end{cases}$$
$$v(x,0) = \begin{cases} 0, & 0 \leqslant x \leqslant 0.5, \\ -\sin(2\pi x), & 0.5 < x \leqslant 1, \end{cases}$$

and zero boundary conditions. The solution is computed for domain $x \in [0,1]$ with $\Delta t = 0.01$ and number of partitions as 50. The numerical solution is obtained for different time-levels $t \in [0,1]$ with different values of α and β and is shown in Tables 6 and 7 for $\eta = 2$. The computed results are depicted graphically in Fig. 2–5 for $\alpha = \beta = 10$ and $\alpha = \beta = 100$ for u and v, respectively. A sharp decay is noticed in the solution for the higher values of α and β.

In authors knowledge, there is no exact solution available in the literature for a general value of η. Hence, experiments are conducted to discuss the behavior of solution at higher values of η. For Example 3 the numerical solutions are obtained for $\eta = 20, 200, 2000, 20{,}000$ with $\alpha = \beta = 10$. The solutions profile for u are plotted to show the effect of increasing value of η in Figs. 6 and 7. From the figures it can be concluded that the solution decays to zero with increasing time levels and with increase in value of η. It can also be observed that the method is capable of finding the solution at higher values of η.

Since for this example, the exact solution is not known, hence the numerical solutions at different numbers of partitions is compared with the solution obtained with number of partitions taken as 400, considering it as exact solution and is tabulated in Table 8.

From the order of convergence calculated in Tables 3 and 8 the method is shown to have a second order of convergence.

7. Conclusions

In this paper, a numerical treatment for the coupled viscous Burgers' equation is proposed using collocation method with the cubic B-spline functions. It is noted that sometimes the accuracy of solution reduces as time increases due to the time truncation errors of time derivative term. But, by the application point of view the cubic B-spline method considered in this work is simple and straight forward. The algorithm described above works for a large class of linear and nonlinear problems. The solution obtained is presented graphically at various time intervals and are compared with the exact solution by finding the L_2 and L_∞ errors.

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