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LSSC

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Abstract

1 Overview

2 Theoritical Description

2.1 Notations

In order to keep coherence with [3], the notations used are the same. A picture of n pixels is seen as a column vector in $\mathbb{R}^n$. The noisy picture is noted y and the denoised one x. The i-th pixel of x is noted $x[i]$ and the patch centered in $x[i]$ and of size m is noted x_i.

2.2 Learned Sparse Coding

The idea behind this method is to assume that the denoised picture is a signal that can be approximated by a sparse linear combinations of elements from a basis set. The basis set is called a dictionary $\mathbf{D} \in \mathbb{R}^{m \times k}$ and is composed of k elements. The denoised patches are then computed from $\mathbf{D}$ with:

$$min_{\boldsymbol{\alpha} \in \mathbb{R}^k} ||\boldsymbol{\alpha}||_p \quad s.t. \quad ||y_i - \mathbf{D}\boldsymbol{\alpha}||_2^2 \leqslant \epsilon \tag{1}$$

$\mathbf{D}\boldsymbol{\alpha}$ is the estimate of the denoised patch and $||\boldsymbol{\alpha}||_p$ is a regularization term that impose sparsity for $\boldsymbol{\alpha}$.
p is usually 0 or 1. Eq. 1 becomes NP-hard to solve when $p = 0$ and a greedy algorithm such as Orthogonal Matching Pursuit [5] can give an approximation. With $p = 1$, the problem is convex and solved efficiently with the LARS algorithm [6]. Experimental observations [7] have shown that the learning part is better with $p = 1$ and the recomposition part with $p = 0$.
ϵ can be chosen according to the value of the estimated standard deviation of the noise.

2.3 Simultaneous Sparse Coding

3 Algorithm Description

3.1 Algorithm Overview

The first part consists in initializing a dictionnary that will denoise roughly the picture.
Once the picture is denoised a first time, a clustering is made in order to regroup similar patches for further treatment.
Then, iteratively for each clusters, the dictionnary is updated using simultaneous sparse coding and the cluster is denoised.

3.2 Dictionnary Initialization

The initial dictionnary is first learned offline on the 10 000 images of the PASCAL VOC'07 database using the online dictionnary learning procedure of [2]. This procedure is then used on the noisy picture in order to improve the dictionnary efficiency.
In fact, only a fixed number T of patches in the picture are used to update the dictionnary. However they are chosen so that they are independently and identically distributed in the picture.
The algorithm corresponds then to the minimization of eq.1 on the T patches with the l_1 norm using the LARS [6] algorithm.

Input : number of iterations $\mathbf{T}$, i.i.d. sampling of $\mathbf{T}$ patches of the noisy picture $\mathbf{Y}_T$, initial dictionnary $\mathbf{D}^0 \in \mathbb{R}^{m\times k}$, regularization parameter λ
Output : learned dictionnary $\mathbf{D}$
Initialization : $\mathbf{B}^0 \in \mathbb{R}^{k\times k} \leftarrow 0$, $\mathbf{C}^0 \in \mathbb{R}^{m\times k} \leftarrow 0$
for $t = 1..\boldsymbol{T}$ **do**

$\mathbf{y}_t = \mathbf{Y}_T[\mathrm{t}]$
Sparse coding: compute with LARS algorithm:

$$\boldsymbol{\alpha}^t = \mathrm{argmin}_{\boldsymbol{\alpha}\in\mathbb{R}^k} \frac{1}{2}||\mathbf{y}^t - \mathbf{D}^{t-1}\boldsymbol{\alpha}||_2^2 + \lambda||\boldsymbol{\alpha}||_1$$

$\mathbf{B}^t \leftarrow \mathbf{B}^{t-1} + \boldsymbol{\alpha}^t\boldsymbol{\alpha}^{tT}$
$\mathbf{C}^t \leftarrow \mathbf{C}^{t-1} + \mathbf{y}^t\boldsymbol{\alpha}^{tT}$
Update dictionnary from $\mathbf{D}^{t-1}$ to $\mathbf{D}^t$ so that:

$$\mathbf{D}^t = \mathrm{argmin}_{\mathbf{D}\in\mathbb{R}^{m\times k}} \frac{1}{t}\sum_{i=1}^{t}(\frac{1}{2}|\mathbf{y}^i - \mathbf{D}\boldsymbol{\alpha}^i||_2^2 + \lambda||\boldsymbol{\alpha}^i||_1)$$

end
return $\mathbf{D}^T$

Algorithm 1: Online Dictionnary Learning

Input : dictionary $\mathbf{D}= [\mathbf{d}^1, \ldots, \mathbf{d}^k] \in \mathbb{R}^{m\times k}$, noisy patch $\mathbf{y}$, regularization parameter λ
Output : minimal argument $\boldsymbol{\alpha} \in \mathbb{R}^k$
Initialization : $\boldsymbol{\mu}_{\mathcal{A}} \in \mathbb{R}^k \leftarrow 0$, $\boldsymbol{\alpha} \in \mathbb{R}^k \leftarrow 0$
for $i = 1..m$ **do**

$\mathbf{c} \in \mathbb{R}^k \leftarrow \mathbf{D}^T(y - \boldsymbol{\mu}_{\mathcal{A}})$
$C \in \mathbb{R}^+ \leftarrow \max_{j=1..m}(|\mathbf{c}_j|)$
$\mathcal{A} \leftarrow \{j \text{ s.t. } |\mathbf{c}_j| = C\}$
$\boldsymbol{D}_{\mathcal{A}} \in \mathbb{R}^{m\times|\mathcal{A}|} \leftarrow (...s_j\mathbf{d}_j...)$ where $s_j = \mathrm{sgn}(\mathbf{c}_j)$
$\boldsymbol{u}_{\mathcal{A}} \in \mathbb{R}^{|\mathcal{A}|} \leftarrow A_{\mathcal{A}}\boldsymbol{D}_{\mathcal{A}}\left(\boldsymbol{D}_{\mathcal{A}}^T\boldsymbol{D}_{\mathcal{A}}\right)^{-1}\mathbb{1}_{\mathcal{A}}$ where $A_{\mathcal{A}} = \frac{1}{\sqrt{\mathbb{1}_{\mathcal{A}}^T\left(\boldsymbol{D}_{\mathcal{A}}^T\boldsymbol{D}_{\mathcal{A}}\right)^{-1}\mathbb{1}_{\mathcal{A}}}}$ and $\mathbb{1}_{\mathcal{A}}^T = (1, ..., 1) \in \mathbb{R}^{|\mathcal{A}|}$

end

Algorithm 2: LARS algorithm

Input :input dictionnary $\mathbf{D}= [\mathbf{d}^1, \ldots, \mathbf{d}^k] \in \mathbb{R}^{m\times k}$,
$\mathbf{B} = [\mathbf{b}^1, \ldots, \mathbf{b}^k] \in \mathbb{R}^{k\times k}$, $\mathbf{C} = [\mathbf{c}^1, \ldots, \mathbf{c}^k] \in \mathbb{R}^{m\times k}$
Output : updated dictionnary $\mathbf{D}$
repeat

for $i = 1..k$ **do**

update the j^{th} column:

$$\mathbf{u}^j \leftarrow \frac{1}{\mathbf{B}(j,j)}(\mathbf{c}^j - \mathbf{D}\mathbf{b}^j) + \mathbf{d}^j$$

$$\mathbf{d}^j \leftarrow \frac{1}{\max(||\mathbf{u}^j||_2, 1)}\mathbf{u}^j$$

end

until *convergence* ;
return $\mathbf{D}$

Algorithm 3: Dictionnary Update

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4 References

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