AMERICAN UNIVERSITY OF ARMENIA
College of Science and Engineering
CS 140 Mechanics
PROJECT - Billiards

PROGRESS EVALUATION: Tuesday, May 16 2023
DISCUSSION SESSION: Wednesday, May 17 2023, from 16:00 till 18:00 in room# 314W
FINAL SUBMISSION DEADLINE: Sunday, May 21 2023, not later than 23:59

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Task 1 (reading: **D. Kleppner, R. Kolenkow. "Introduction to_Mechanics"**, 2nd ed., 2014 – see Kleppner_Kolenkow_Introduction_to_Mechanics_2ed_2014.pdf in References.zip archive, **Chapter 4**. *Momentum*)

Consider a 2D horizontal planar geometry in which a particle moves with constant velocity along straight lines until it elastically reflects off the boundary. This straight line motion occurs in various "billiard" systems. A simple example of such a system is a particle moving with fixed speed within a circle. For this geometry the angle between the particle's momentum and the tangent to the boundary at a reflection is the same for all points.

1. Write a program to simulate the horizontal circular billiard model of unit radius. The algorithm for determining the path of the particle is as follows:
   1.1 Begin with a randomly selected initial position (x_0, y_0) inside the circle and momentum (p_{x0}, p_{y0}) of the particle such that $|\vec{p}_0| = 1$.
   1.2 Calculate the next position of the particle from the intersection of the straight line defined by the current position and momentum, and the equation for the unit circle where the next reflection occurs.
   1.3 Determine the new momentum, (p'_x, p'_y), of the particle after reflection such that the angle of incidence equals the angle of reflection. For reflection off a circle we have
   $$p'_x = (y^2 - x^2)p_x - 2xyp_y$$
   $$p'_y = -2xyp_x + (x^2 - y^2)p_y$$
   where (0, 0) is the center of the circle.

2. Run the program for different number of reflections n. In each run save the coordinates of all n reflection points.

3. Test the reversibility of the motion. Reverse the momentum after the particle has made n reflections and check if the reversed path coincides with the straight one. If not, determine after how many reflections the paths deviate more than a specified small δ.

Task 2 (reading: **D. Kleppner, R. Kolenkow. "Introduction to_Mechanics"**, 2nd ed., 2014 – see Kleppner_Kolenkow_Introduction_to_Mechanics_2ed_2014.pdf in References.zip archive, **Chapter 5**. *Energy*)

Now consider a vertical circular billiard where a unit mass particle is moving within a vertical circle under the action of the constant gravitational acceleration g = 10 (in

dimensionless units) directed along the vertical y axis. As in **Task 1**, it elastically reflects off the boundary.

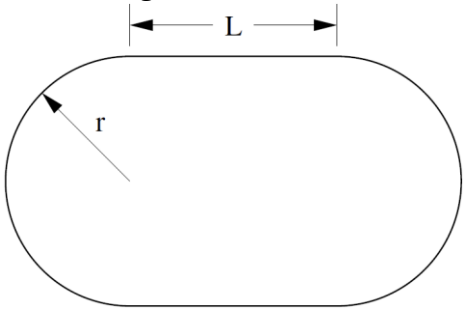
1. Write a program to simulate the vertical circular billiard model of unit radius. The algorithm for determining the path of the particle is as follows:
   1.1 Begin with a randomly selected initial position (x_0, y_0) inside the circle and momentum (p_{x0}, p_{y0}) of the particle such that $5 \leq |\vec{p}_0| \leq 10$.
   1.2 Calculate the next position of the particle from the intersection of the parabolic path defined by the current position and momentum, and the equation for the unit circle where the next reflection occurs.
   1.3 Determine the new momentum, (p'_x, p'_y), of the particle after reflection such that the angle of incidence equals the angle of reflection. For reflection off a circle we have
   $$p'_x = (y^2 - x^2)p_x - 2xyp_y$$
   $$p'_y = -2xyp_x + (x^2 - y^2)p_y$$
   where (0, 0) is the center of the circle.

2. Run the program for the same number of reflections n and the initial conditions (x_0, y_0) and (p_{x0}, p_{y0}) as in **Task 1**. In each run save the coordinates of all n reflection points.

3. Test the reversibility of the motion. Reverse the momentum after the particle has made n reflections and check if the reversed path coincides with the straight one. If not, determine after how many reflections the paths deviate more than a specified small δ.

Task 3

Consider a horizontal stadium billiard by dividing the circle into two equal parts and connecting them by straight lines of length L as shown.:



1. Write a program to simulate the horizontal stadium billiard model. Take $r = 1$. The algorithm for determining the path of the particle is as follows:
   1.1 Begin with a randomly selected initial position (x_0, y_0) inside the circle and momentum (p_{x0}, p_{y0}) of the particle such that $|\vec{p}_0| = 1$.
   1.2 Determine which of the four sides the particle will hit. The possibilities are the top and bottom line segments and the right and left semicircles.
   1.3 Calculate the next position of the particle from the intersection of the straight line defined by the current position and momentum, and the equation for the segment where the next reflection occurs.
   1.4 Determine the new momentum, (p'_x, p'_y), of the particle after reflection such that the angle of incidence equals the angle of reflection. For reflection off the line segments we have $(p'_x, p'_y) = (p_x, -p_y)$ .For reflection off a circle we have
   $$p'_x = [y^2 - (x - x_c)^2]p_x - 2(x - x_c)yp_y$$
   $$p'_y = -2(x - x_c)yp_x + [(x - x_c)^2 - y^2]p_y$$

where (x_c, 0) is the center of the circle.

2. Run the program for different number of reflections n and different values of L. Assume the x coordinate of the center of the left semicircle is $x_c = 0$, and the x coordinate of the center of the right semicircle is $x_c = L$. In each run save the coordinates of all n reflection points.

3. Test the stadium billiard as a pseudo-random number generator by considering reflections from the straight segments. Take the x/L values as a sequence of random numbers from the unit interval and test its uniformity. Divide the unit interval into M equal bins and count the number of entries in each bin. Check if they are approximately equal. Compute the mean and variance of the number of entries in the bins.

Bonus

Supplement the implemented programs with GUI that draws the path of the moving particle. Use two different foreground colors for the regular and reversed motion.

Submission Format and Project Requirements

1. Create a new subfolder **\Billiards** in the existing repository of the first **Springs** project. Collect all project deliverables in this subfolder.
2. The project may be implemented in any OOP environment.
3. The project deadline is Sunday May 21, not later than 23:59. No special submission is needed – the project will be evaluated based on the repository status at the moment of the deadline.
4. The progress evaluations will be conducted on Tuesday May 16. Again, no special submissions are needed – the progress will be evaluated based on the ongoing repository status and a short free-format status report. The reports list all project-related activities, including understanding, study, reading, design, development, testing, analysis, etc.
5. A required presentation and discussion session will be conducted on Wednesday May 17 from 16:00 till 18:00 in room# 314W PAB. The session format is live discussion – slides or other special presentation tools are not needed.
6. This is an individual project. Therefore, the individual contributions must be explicitly stated in the project docs.
7. You are welcome to use / reuse / consult external (re)sources, including open-source code, electronic and hard-copy texts, videos, group discussions, instructor / TA assistance, etc. All such sources must be explicitly acknowledged / referenced. Any unreferenced use of external sources will violate the rules of the academic integrity.