DEBER ASEGUNDO PARCIAL

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Sistemas de ecuaciones lineales y ajuste de curvas

1. **EJERCICIO:** Resolver el siguiente sistema lineal de ecuaciones Ax = B, a mano y con calculadora no programable, aplicando el método de Eliminación Gauss.

$$\begin{cases} 4x_1 + 8x_2 + 4x_3 & = 8 \\ x_1 + 5x_2 + 4x_3 - 3x_4 & = -4 \\ x_1 + 4x_2 + 7x_3 + 2x_4 & = 10 \\ x_1 + 3x_2 - 2x_4 & = -4 \end{cases}$$

Compruebe la solución encontrada con la obtenida con el programa computacional desarrollado en clases.

(1)
$$\begin{cases} 4x_1 + 8x_2 + 4x_3 = 8 \\ x_1 + 5x_2 + 4x_3 - 3x_4 = -4 \\ x_1 + 4x_2 + 7x_3 + 2x_4 = 10 \\ x_1 + 3x_2 - 2x_4 = -4 \end{cases}$$

$$A = \left(\begin{array}{cccc|c} 4 & 8 & 4 & 0 & 8 \\ 1 & 5 & 4 & -3 & -4 \\ 1 & 4 & 7 & 2 & 10 \\ 1 & 3 & 0 & -2 & -4 \end{array}\right)$$

1^{ra} Iteración

1) $a_{11} = 4 \neq 0$ ✓

2) $m_{21} = \frac{1}{4} = 0,25$

3) hacer ceros.

$F_2 = F_2 - m_{21}(F_1)$

$1 - (1/4)(4) = 0$
$5 - (1/4)(8) = 3$
$4 - (1/4)(4) = 3$
$-3 - (1/4)(0) = -3$
$-4 - (1/4)(8) = -6$

$F_3 = F_3 - m_{21}(F_1)$

$1 - (1/4)(4) = 0$
$4 - (1/4)(8) = 2$
$7 - (1/4)(4) = 6$
$2 - (1/4)(0) = 2$
$10 - (1/4)(8) = 8$

$F_4 = F_4 - m_{21}(F_1)$

$1 - (1/4)(4) = 0$
$3 - (1/4)(8) = 1$
$0 - (1/4)(4) = -1$
$-2 - (1/4)(0) = -2$
$-4 - (1/4)(8) = -6$

$$A = \left(\begin{array}{cccc|c} 4 & 8 & 4 & 0 & 8 \\ 0 & 3 & 3 & -3 & -6 \\ 0 & 2 & 6 & 2 & 8 \\ 0 & 1 & -1 & -2 & -6 \end{array}\right)$$

2^{da} Iteración

1) $a_{22} = 3 \neq 0$ ✓

2) $m_{32} = \frac{1}{3} = 0,33$

3) hacer ceros.

$F_3 = F_3 - m_{32}(F_2)$

$2 - (1/3)(3) = 0$
$6 - (1/3)(3) = 4$
$2 - (1/3)(-3) = 4$
$8 - (1/3)(-6) = 12$

$F_4 = F_4 - m_{32}(F_2)$

$1 - (1/4)(3) = 0$
$-1 - (1/4)(3) = -2$
$-2 - (1/4)(-3) = -1$
$-6 - (1/4)(-6) = -4$

3^{ra} Iteración

$$A = \left(\begin{array}{cccc|c} 4 & 8 & 4 & 0 & 8 \\ 0 & 3 & 3 & -3 & -6 \\ 0 & 0 & 4 & 4 & 12 \\ 0 & 0 & -2 & -1 & -4 \end{array}\right)$$

1) $a_{33} = 4 \neq 0$ ✓

2) $m_{43} = \frac{-\cancel{2}^{1}}{\cancel{4}_{2}} = -\frac{1}{2}$

3) hacer ceros

$$F_4 = F_4 - m_{43}(F_3)$$

$$-2 - (-1/2)(4) = 0$$
$$-1 - (-1/2)(4) = 1$$
$$-4 - (-1/2)(12) = 2$$

4^{ta} Iteración

$$A = \left(\begin{array}{cccc|c} 4 & 8 & 4 & 0 & 8 \\ 0 & 3 & 3 & -3 & -6 \\ 0 & 0 & 4 & 4 & -12 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right)$$

Calculamos x_4

$$x_4 = 2 \;//$$

Calculamos x_3

$$4x_3 + 4x_4 = -12$$
$$4x_3 + 4(2) = -12$$
$$4x_3 = -12 - 8$$
$$x_3 = 1 \;//$$

Calculamos x_2

$$3x_2 + 3x_3 - 3x_4 = -6$$
$$3x_2 + 3(-1) - 3(2) = -6$$
$$3x_2 = 15$$
$$x_2 = -1 \;//$$

Calculamos x_1

$$4x_1 + 8x_2 + 4x_3 = 8$$
$$4x_1 + 8(5) + 4(-5) = 8$$
$$4x_1 + 40 - 20 = 8$$
$$4x_1 = 8 + 20 - 40$$
$$x_1 = -3 \;//$$

$$X = \begin{pmatrix} 3 \\ -1 \\ 1 \\ 2 \end{pmatrix} //$$

```
## Author: PC <PC@ALI>
## Created: 2024-01-09

function x=sistemaGaus(A,B)
[n n]=size(A);
Ab=[A';B]';
%---SistemaTriangSuperior
for k=1:n
    [bb ll]=max(abs(Ab(k:n,k)));
    if bb==0
        error('La matriz es singular');
    end
    m=k+ll-1;
    Ab=intercambio_filas(Ab,k,m);
    for j=k+1:n
        Ab=combinar_filas(Ab,k,j,-Ab(j,k)/Ab(k,k));
    end
end
x=Sistem_T_sup(Ab(:,1:n),Ab(:,n+1));
disp('Triangular Superior')
Ab
end
```

```
>> A = [4 8 4 0; 1 5 4 -3; 1 4 7 2; 1 3 0 -2 ]
A =

   4   8   4   0
   1   5   4  -3
   1   4   7   2
   1   3   0  -2

>> B = [8 -4 10 -4 ]
B =

    8   -4   10   -4

>> x=sistemaGaus(A,B)
Triangular Superior
Ab =

    4    8    4    0    8
    0    3    3   -3   -6
    0    0    4    4   12
    0    0    0    1    2

x =

   3  -1   1   2
```

2. **EJERCICIO:** Considere el siguiente sistema lineal de ecuaciones $AX = B$, donde A y B están dados por:

$$A = \begin{pmatrix} 7 & 0 & -2 & 3 \\ 5 & 8 & -6 & 4 \\ 2 & 0 & 4 & -2 \\ 0 & 4 & -2 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} -2 \\ 4 \\ -5 \\ 7 \end{pmatrix};$$

Resolver el sistema de ecuaciones lineales aplicando el método de Gauss Jordan; calculado a mano y con calculadora no programable:

(2)

$$A=\begin{pmatrix} 7 & 0 & -2 & 3 \\ 5 & 8 & -6 & 4 \\ 2 & 0 & 4 & -2 \\ 0 & 4 & -2 & 5 \end{pmatrix} \qquad B=\begin{pmatrix} -2 \\ 4 \\ -5 \\ 7 \end{pmatrix}$$

Paso 1 Triangular Superior

$$C=\left(\begin{array}{cccc|c} 7 & 0 & -2 & 3 & -2 \\ 5 & 8 & -6 & 4 & 4 \\ 2 & 0 & 4 & -2 & -5 \\ 0 & 4 & -2 & 5 & 7 \end{array}\right)$$

$P1 = 7$
$P2 = 5$
$P3 = 2$

$$C=\left(\begin{array}{cccc|c} 1 & 0 & -2/7 & 3/7 & -2/7 \\ 0 & 3 & -11 & -1 & -1 \\ 0 & -2 & 2 & -4 & -7 \\ 0 & 4 & -2 & 5 & 7 \end{array}\right)$$

$F_3 - (-2/3)(F_2)$
$-2 - (-2/3)(3) = 0$
$2 - (-2/3)(-11) = -\frac{16}{3}$
$-4 - (-2/3)(-1) = -\frac{14}{3}$
$-7 - (-2/3)(-1) = -\frac{23}{3}$

$F_4 - (-4/3)(F_2)$
$4 - (-4/3)(3) = 0$
$-2 - (-4/3)(-11) = -28/3$
$5 - (-4/3)(-1) = -17/3$
$7 - (-4/3)(-1) = 19/3$

$$C=\left(\begin{array}{cccc|c} 1 & 0 & -2/7 & 3/7 & -2/7 \\ 0 & 3 & -11 & -1 & -1 \\ 0 & 0 & -16/3 & -14/3 & -23/3 \\ 0 & 0 & -28/3 & -17/3 & 19/3 \end{array}\right)$$

$F_4 - (7/4)(F_3)$
$-\frac{28}{3} - (7/4)(-16/3) = 0$
$-\frac{17}{3} - (7/4)(-14/3) = 5/2$
$\frac{19}{3} - (7/4)(-23/3) = 79/4$

$$C=\left(\begin{array}{cccc|c} 1 & 0 & -2/7 & 3/7 & -2/7 \\ 0 & 3 & -11 & -1 & -1 \\ 0 & 0 & -16/3 & -14/3 & -23/3 \\ 0 & 0 & 0 & 5/2 & 79/4 \end{array}\right)$$

Paso 2: Triangular Inferior

$$C=\left(\begin{array}{cccc|c} 1 & 0 & -2/7 & 0 & -257/70 \\ 0 & 3 & -11 & 0 & 69/10 \\ 0 & 0 & -16/3 & 0 & 146/5 \\ 0 & 0 & 0 & 5/2 & 79/4 \end{array}\right)$$

$F_3 - F_4$
$-14/3 - (-28/15)(5/2) = 0$
$-16/3 - (-28/15)(0) = -16/3$
$-23/3 - (-28/15)(79/4) = 146/5$

F_1
$3/7 - (6/35)(5/2) = 0$
$-2/7 - (6/35)(0) = -2/7$
$-2/7 - (6/35)(79/4) = -\frac{257}{70}$

$F_2 \quad F_4$
$-1 - (-2/5)(5/2) = 0$
$-11 - (-2/5)(0) = -11$
$-1 - (-2/5)(79/4) = 69/10$
$3 - (-2/5)(0) = 3$

$$C=\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -733/140 \\ 0 & 3 & 0 & 0 & -2133/40 \\ 0 & 0 & -16/3 & 0 & 146/5 \\ 0 & 0 & 0 & 5/2 & 79/4 \end{array}\right)$$

$F_2 \quad F_3$
$-11 - (33/16)(-16/3) = 0$
$69/10 - (33/16)(146/5) = -2133/40$

Dividimos para el pivote.

$$C=\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -733/140 \\ 0 & 1 & 0 & 0 & -711/40 \\ 0 & 0 & 1 & 0 & -219/40 \\ 0 & 0 & 0 & 1 & 79/10 \end{array}\right)$$

$F_1 \quad F_3$
$-2/7 - (3/56)(-16/3) = 0$
$-257/70 - (3/56)(146/5) = -733/140$

Soluciones del sistema

$$x=\begin{pmatrix} -733/140 \\ -711/40 \\ -219/40 \\ 79/10 \end{pmatrix} \approx \begin{pmatrix} -0,8309 \\ 0,2592 \\ -0,3978 \\ 1,0235 \end{pmatrix}$$

```
## Author: PC <PC@ALI>
## Created: 2024-01-09

function x=sistemaGaus(A,B)
[n n]=size(A);
Ab=[A';B]';
%---SistemaTriangSuperior
for k=1:n
    [bb ll]=max(abs(Ab(k:n,k)));
    if bb==0
        error('La matriz es singular');
    end
    m=k+ll-1;
    Ab=intercambio_filas(Ab,k,m);
    for j=k+1:n
        Ab=combinar_filas(Ab,k,j,-Ab(j,k)/Ab(k,k));
    end
end
x=Sistem_T_sup(Ab(:,1:n),Ab(:,n+1));
disp('Triangular Superior')
Ab
end
```

Ventana de comandos

```
   5   8  -6   4
   2   0   4  -2
   0   4  -2   5

>> B = [-2 4 -5 7]

B =

  -2   4  -5   7

>> x = GaussJordan(A,B)
Triangular Superior
Ab =

   7.0000        0  -2.0000   3.0000  -2.0000
        0   8.0000  -4.5714   1.8571   5.4286
        0        0   4.5714  -2.8571  -4.4286
        0        0        0   4.2500   4.5625

Triangular Inferior
Ab =

   7.0000        0        0        0  -5.8162
        0   8.0000        0        0   2.0735
        0        0   4.5714        0  -1.3613
        0        0        0   4.2500   4.5625

Diagonal de Unos
Ab =

   1.0000        0        0        0  -0.8309
        0   1.0000        0        0   0.2592
        0        0   1.0000        0  -0.2978
        0        0        0   1.0000   1.0735

Soluciones
x =

  -0.8309
   0.2592
  -0.2978
   1.0735
```

3. **EJERCICIO:** Resolver el siguiente sistema lineal, a mano y con calculadora no programable, aplicando el método de la inversa.

$$\begin{cases} x_1 + x_2 = 5 \\ 2x_1 - x_2 + 5x_3 = -9 \\ 3x_2 - 4x_3 + 2x_4 = 19 \\ 2x_3 + 6x_4 = 2 \end{cases}$$

$A \cdot X = B$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & -1 & 5 & 0 \\ 0 & 3 & -4 & 2 \\ 0 & 0 & 2 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 5 \\ -9 \\ 19 \\ 2 \end{pmatrix}$$

$X = A^{-1} \cdot B$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & -1 & 5 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & -4 & 2 & 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 6 & 0 & 0 & 0 & 0 \end{array}\right)$$

$F_2 \rightarrow \dfrac{F_2 - 2F_1}{-3}$

$2 - 2(1)/3 = 0$

$-1 - 2(1)/3 = 1/3$

$5 - 2(5)/3 = -5/3$

$0 - 2(0)/3 = 0$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1/3 & -5/3 & 0 & 2/3 & -1/3 & 0 & 0 \\ 0 & 3 & -4 & 2 & 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 6 & 0 & 0 & 0 & 1 \end{array}\right)$$

$F_1 \rightarrow F_1 - F_2$

$F_3 \rightarrow F_3 - 3F_2$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 5/3 & 0 & 1/3 & 1/3 & 0 & 0 \\ 0 & 1/3 & -5/3 & 0 & 2/3 & -1/3 & 0 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 & 1 & 0 \\ 0 & 0 & 2 & 6 & 0 & 0 & 0 & 1 \end{array}\right)$$

$F_4 \rightarrow \dfrac{F_4 - 2F_3}{2}$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 5/3 & 0 & 1/3 & 1/3 & 0 & 0 \\ 0 & 1 & -5/3 & 0 & 2/3 & -1/3 & 0 & 0 \\ 0 & 0 & 1 & 2 & -6 & 3 & 3 & -1 \\ 0 & 0 & 0 & 1 & 2 & -1 & -1 & 1/2 \end{array}\right)$$

$F_3 \rightarrow F_3 - 2F_4$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 5/3 & 0 & 31/3 & -14/3 & -5 & 5/3 \\ 0 & 1 & -5/3 & 0 & -28/3 & 14/3 & 5 & -5/3 \\ 0 & 0 & 1 & 0 & -6 & 3 & 3 & -1 \\ 0 & 0 & 0 & 1 & 2 & -1 & 1 & -1/2 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 31/3 & -14/3 & -5 & 5/3 \\ -28/3 & 14/3 & 5 & -5/3 \\ -6 & 3 & 3 & -1 \\ 2 & 1 & 1 & 1/2 \end{pmatrix}$$

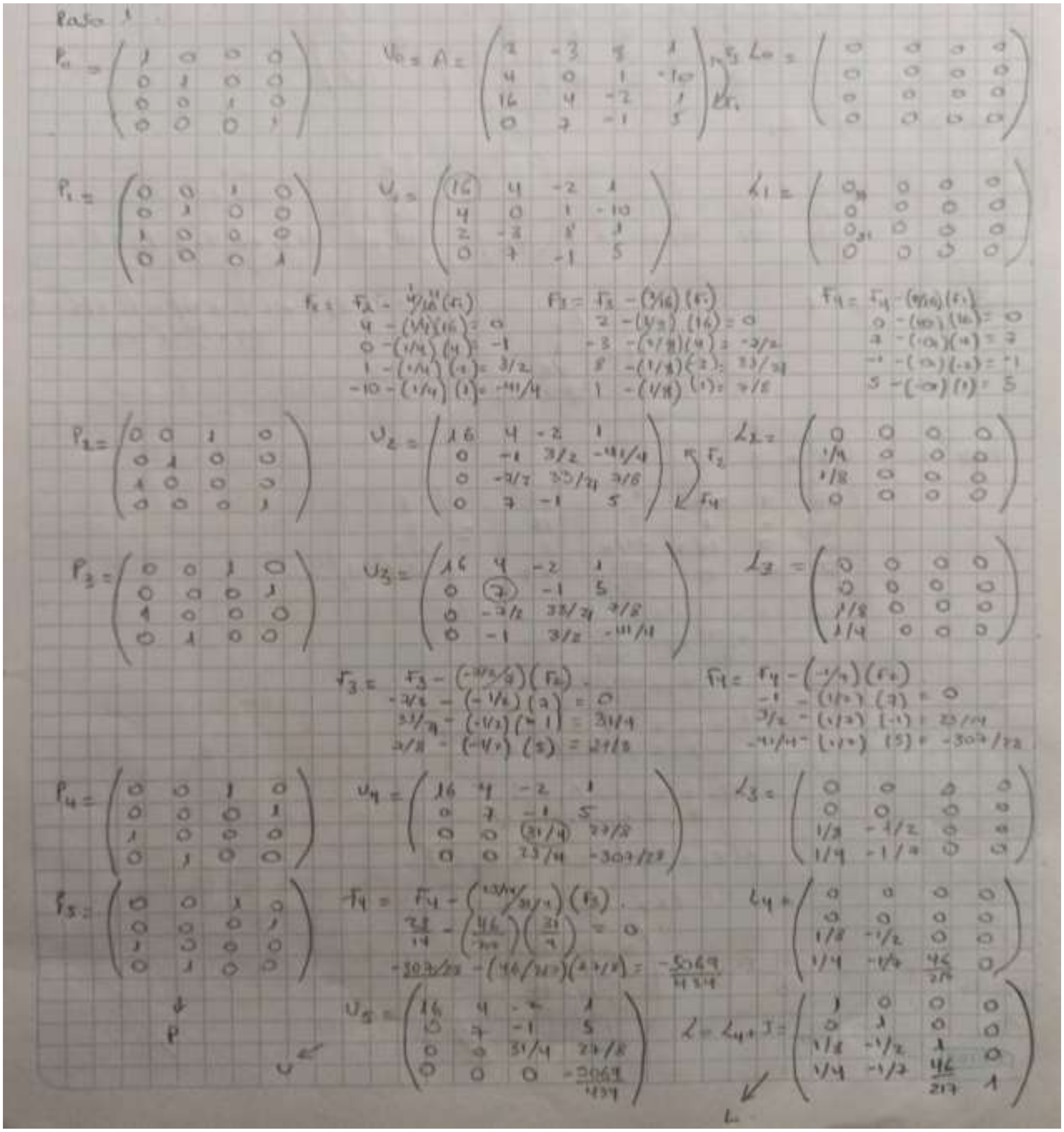
$X = A^{-1} \cdot B$

$$\begin{pmatrix} 31/3(5) & -14/3(-9) & -5(19) & 5/3(2) \\ -28/3(5) & 14/3(-9) & 5(19) & -5/3(2) \\ -6(5) & 3(-9) & 3(19) & -1(2) \\ 2(5) & -1(-9) & 1(19) & 1/2(2) \end{pmatrix}$$

$$X = \begin{pmatrix} 2 \\ 3 \\ -2 \\ 1 \end{pmatrix}$$

4. **EJERCICIO:** Resolver el siguiente sistema lineal de ecuaciones $Ax = B$, a mano y con calculadora no programable, aplicando solo la factorización $PA = LU$.

$$A = \begin{pmatrix} 2 & -3 & 8 & 1 \\ 4 & 0 & 1 & -10 \\ 16 & 4 & -2 & 1 \\ 0 & 7 & -1 & 5 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$



Paso 1

$$P_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad U_0 = A = \begin{pmatrix} 2 & -3 & 8 & 1 \\ 4 & 0 & 1 & -10 \\ 16 & 4 & -2 & 1 \\ 0 & 7 & -1 & 5 \end{pmatrix} \quad L_0 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad U_1 = \begin{pmatrix} 16 & 4 & -2 & 1 \\ 4 & 0 & 1 & -10 \\ 2 & -3 & 8 & 1 \\ 0 & 7 & -1 & 5 \end{pmatrix} \quad L_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$F_2 = F_2 - (4/16)(F_1)$: $4 - (1/4)(16) = 0$; $0 - (1/4)(4) = -1$; $1 - (1/4)(-2) = 3/2$; $-10 - (1/4)(1) = -41/4$

$F_3 = F_3 - (2/16)(F_1)$: $2 - (1/8)(16) = 0$; $-3 - (1/8)(4) = -7/2$; $8 - (1/8)(-2) = 33/4$; $1 - (1/8)(1) = 7/8$

$F_4 = F_4 - (0/16)(F_1)$: $0 - (0)(16) = 0$; $7 - (0)(4) = 7$; $-1 - (0)(-2) = -1$; $5 - (0)(1) = 5$

$$P_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad U_2 = \begin{pmatrix} 16 & 4 & -2 & 1 \\ 0 & -1 & 3/2 & -41/4 \\ 0 & -7/2 & 33/4 & 7/8 \\ 0 & 7 & -1 & 5 \end{pmatrix} \quad L_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1/4 & 0 & 0 & 0 \\ 1/8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad U_3 = \begin{pmatrix} 16 & 4 & -2 & 1 \\ 0 & 7 & -1 & 5 \\ 0 & -7/2 & 33/4 & 7/8 \\ 0 & -1 & 3/2 & -41/4 \end{pmatrix} \quad L_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/8 & 0 & 0 & 0 \\ 1/4 & 0 & 0 & 0 \end{pmatrix}$$

$F_3 = F_3 - (-7/2 / 7)(F_2)$: $-7/2 - (-1/2)(7) = 0$; $33/4 - (-1/2)(-1) = 31/4$; $7/8 - (-1/2)(5) = 27/8$

$F_4 = F_4 - (-1/7)(F_2)$: $-1 - (-1/7)(7) = 0$; $3/2 - (-1/7)(-1) = 23/14$; $-41/4 - (-1/7)(5) = -307/28$

$$P_4 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad U_4 = \begin{pmatrix} 16 & 4 & -2 & 1 \\ 0 & 7 & -1 & 5 \\ 0 & 0 & 31/4 & 27/8 \\ 0 & 0 & 23/14 & -307/28 \end{pmatrix} \quad L_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/8 & -1/2 & 0 & 0 \\ 1/4 & -1/7 & 0 & 0 \end{pmatrix}$$

$F_4 = F_4 - \left(\frac{23/14}{31/4}\right)(F_3)$: $\frac{23}{14} - \left(\frac{46}{217}\right)\left(\frac{31}{4}\right) = 0$; $-307/28 - (46/217)(27/8) = -\frac{5069}{434}$

$$P_5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad L_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/8 & -1/2 & 0 & 0 \\ 1/4 & -1/7 & \frac{46}{217} & 0 \end{pmatrix}$$

P

$$U_5 = \begin{pmatrix} 16 & 4 & -2 & 1 \\ 0 & 7 & -1 & 5 \\ 0 & 0 & 31/4 & 27/8 \\ 0 & 0 & 0 & -\frac{5069}{434} \end{pmatrix} \quad L = L_4 + I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1/8 & -1/2 & 1 & 0 \\ 1/4 & -1/7 & \frac{46}{217} & 1 \end{pmatrix}$$

U

L

Paso 2: Calcular B_1

$$B_1 = P * B$$

$$B_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Paso 3: $Ly = B_1$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1/8 & -1/2 & 1 & 0 \\ 1/4 & -1/7 & 46/217 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$y_1 = 1$

$y_2 = 1$

$1/8\, y_1 - 1/2\, y_2 + y_3 = 1$

$y_3 = 1/8$

$1/4\, y_1 - 1/7\, y_2 + 46/217\, y_3 + y_4 = 1$

$y_4 = 283/434$

$$y = \begin{pmatrix} 1 \\ 1 \\ 1/8 \\ 283/434 \end{pmatrix}$$

Paso 4: $Ux = y$

$$\begin{pmatrix} 16 & 4 & -2 & 1 \\ 0 & 7 & 1 & 5 \\ 0 & 0 & 31/4 & 27/2 \\ 0 & 0 & 0 & -\frac{3069}{434} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1/8 \\ 283/434 \end{pmatrix}$$

$$-\frac{3069}{434} x_4 = \frac{283}{434}$$

$$x_4 = -0,0556$$

$$31/4\, x_3 + 27/2 = 1/8$$

$$x_3 = 0,20546$$

$$7x_2 + x_3 + 5x_4 = 1$$

$$x_2 = 0,2182$$

$$16x_1 + 4x_2 - 2x_3 + x_4 = 1$$

$$x_1 = 0,03765$$

$$x = \begin{pmatrix} 0,03765 \\ 0,2182 \\ 0,20546 \\ -0,0556 \end{pmatrix}$$

6. **EJERCICIO:** Considere la función $f(x) = x^2 e^{-x^2}$. Se pide calcular un valor aproximado para $\int_{-1}^{2} f(x)$; usando el polinomio de Lagrange, calculado a mano, que interpola $f(x)$ en los puntos $x_0 = -1,\ x_1 = 0,\ x_2 = 1$, y $x_4 = 2$.

	X_i	$f(x)$	
x_0	$\to -2$	0,073	$f(x_0)$
x_1	$\to -1$	0,368	$f(x_1)$
x_2	$\to 0$	0	$f(x_2)$
x_3	$\to 2$	0,073	$f(x_3)$

$n=4$ $P(x)$ grado 3

$$P(x) = \sum_{k=0}^{3} f(x_k)\, L_{3,k}(x)$$

$$P(x) = f(x_0) L_{3,0}(x) + f(x_1) L_{3,1}(x) + f(x_2) L_{3,2}(x) + f(x_3) L_{3,3}(x)$$

$k=0$

$$L_{3,0} = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} = \frac{(x+1)(x+0)(x-2)}{(-2+1)(-2-0)(-2-2)} = -\frac{1}{8} x(x+1)(x-2)$$

$k=1$

$$L_{3,1} = \frac{(x+2)(x+0)(x-2)}{(-1+2)(-1+0)(-1-2)} = \frac{1}{3} x(x+2)(x-2)$$

$k=2$

$$L_{3,2} = \frac{(x+2)(x+1)(x-2)}{(0+2)(0+1)(0+2)} = \frac{1}{4}(x+2)(x+1)(x-2)$$

$K=2$

$$L_{3,3} = \frac{(x+2)(x+1)(x-0)}{(2+2)(2+1)(2-0)} = \frac{1}{24}(x+2)(x+1)x$$

$$P(x) = 0{,}073\left(-\frac{1}{8}\right) x(x+1)(x+2) + 0{,}368\left(\frac{1}{3}\right) x(x+2)(x-2) + 0 + 0{,}073\left(\frac{1}{24}\right) x(x+2)(x+1)$$

$$P(x) = 0{,}117x^3 - 0{,}018x^2 - 0{,}466x$$

$$\int_{-2}^{2} P(x)\,dx = 0{,}117\int_{-2}^{2} x^3\,dx + 0{,}018\int_{-2}^{2} x^2\,dx - 0{,}466\int_{-2}^{2} x\,dx$$

$$I = 0{,}117 \frac{x^4}{4}\Big|_{-2}^{2} + 0{,}018\frac{x^3}{3}\Big|_{-2}^{2} - 0{,}466\frac{x^2}{2}\Big|_{-2}^{2}$$

$$I = 0{,}117(0) + 0{,}018\left(\frac{16}{3}\right) - 0{,}466(0)$$

$$I = 0{,}096$$

7. **EJERCICIO:** Con el siguiente conjunto de nodos:

x_i	40	60	80	100
y_i	1	2	5	9

A mano, encontrar:

a) El polinomio interpolador de Newton.

b) El polinomio interpolador de Lagrange.

7

x_i	40	60	80	100
y_i	1	2	5	9

a) Polinomio Interpolador de Newton

x_k	$f(x_k)$	$f[\,]$	$f[\,,\,]$	$f[\,,\,,\,]$
40	1			
60	2	0,05		
80	5	0,15	$2,5\times10^{-3}$	
100	9	0,2	$-1,5\times10^{-3}$	$-2,1\times10^{-5}$

$P_1(x) = a_0 + a_1(x-x_0) = 1 + 0,05(x-40) = 0,05 - 1.$

$P_2(x) = P_1 + a_2(x-x_0)(x-x_1)$

$= 2,5\times10^{-3}\, x^3 (x^2 - 100x + 2400) + P_1$

$= 0,05x - 1 + 2,5\times10^{-3}x^2 - 0,25x + 6$

$= 2,5\times10^{-3}x^2 - 0,2x + 5$

$P_3(x) = P_2 + a_3(x-x_0)(x-x_1)(x-x_2)$

$= P_2 + (-2,1\times10^{-5})(x-40)(x-60)(x-80)$

$= P_2 + (-2,1\times10^{-5})(x^3 - 180x^2 + 10400x - 19200)$

$= P_2 + (-2,1\times10^{-5})x^3 - (3,78\times10^{-3})x^2 - 0,22x + 0,9$

$= -2,1\times10^{-5}x^3 - 1,3\times10^{-3}x^2 - 0,42x + 5,4$

$P(x) = -2,1\times10^{-5}x^3 - 1,3\times10^{-3}x^2 - 6,42x + 5,4.$

5) Polinomio Interpolador de Lagrange $(n=4,\ P_3(x))$

$k=0$

$$L_{3,0}(x)=\frac{(x-2)(x-5)(x-9)}{(1-2)(1-5)(1-9)}$$

$$=-\frac{1}{32}(x-2)(x-5)(x-9)$$

$k=1$

$$L_{3,1}(x)=\frac{(x-1)(x-5)(x-9)}{(2-1)(2-5)(2-9)}$$

$$=\frac{1}{21}(x-1)(x-5)(x-9)$$

$k=2$

$$L_{3,2}(x)=\frac{(x-1)(x-2)(x-9)}{(5-1)(5-2)(5-9)}$$

$$=-\frac{1}{48}(x-1)(x-2)(x-9)$$

$k=3$

$$L_{3,3}(x)=\frac{(x-1)(x-2)(x-5)}{(9-1)(9-2)(9-5)}$$

$$=\frac{1}{224}(x-1)(x-2)(x-5)$$

$$P(x)=40\left(-\frac{1}{32}\right)(x-2)(x-5)(x-9)+60\left(\frac{1}{21}\right)(x-1)(x-5)(x-9)$$
$$+80\left(-\frac{1}{48}\right)(x-1)(x-2)(x-9)+100\left(\frac{1}{224}\right)(x-1)(x-2)(x-5)$$

$$P(x)=-\frac{5}{6}(x-2)(x-5)(x-9)+\frac{20}{7}(x-1)(x-5)(x-9)$$
$$-\frac{5}{3}(x-1)(x-2)(x-9)+\frac{25}{26}(x-1)(x-2)(x-5)$$

$$P(x)=-2,1\times10^{-2}x^3-1,3\times10^{-3}x^2-6,42x+5,4$$

Tienen el mismo Resultado

8. **EJERCICIO:** Al medir la velocidad (con un tubo Pitot) en una tubería circular de diámetro interior de 20 cm, se encontró la siguiente información:

$r(cm)$	0	3	5	7	8
$v(cm/s)$	600	550	450	312	240

Donde r es la distancia en cm medida a partir del centro del tubo.

a) Calcule la velocidad cuando $r = 7{,}5$ cm, con un polinomio interpolador de Newton de grado 2.

8

r (cm)	0	3	5	7	8
v (cm/s)	600	550	450	312	240

x	$f(x_k)$	$f[,]$	$f[,,]$	$f[,,,]$	$f[,,,,]$
0	600				
3	550	-16,67			
5	450	-50	-6,67		
7	312	-69	-4,75	0,273	
8	240	-72	1	0,75	0,060

$a_0 = 600$ $\quad a_1 = (550-600)/3 = -16{,}67$

$P_1(x) = a_0 + a_1(x - x_0) = 600 - 16{,}67(x-0) = -16{,}67x + 600$

$a_2 = (-50 + 16{,}67)/(5-0) = -6{,}67$

$P_2(x) = P_1 + a_2(x-x_0)(x-x_1) = P_1 - 6{,}67(x)(x-3)$

$= -6{,}67x^2 + 3{,}34x + 600$

$a_3 = (-4{,}75 + 6{,}67)/7 = 0{,}273$

$P_3(x) = P_2 + a_3(x-x_0)(x-x_1)(x-x_2) = P_2 + 0{,}273(x)(x-3)(x-5)$

$= 0{,}273x^3 - 8{,}854x^2 + 7{,}435 + 600$

$P(x) = 0{,}273x^3 - 8{,}854x^2 + 7{,}35x + 600$

$P(7{,}5) = 0{,}273(7{,}5)^3 - 8{,}854(7{,}5)^2 + 7{,}435(7{,}5) + 600$

$v = 272{,}897$ cm/s

9. Encontrar el polinomio de ajuste exponencial y el polinomio grado 2, por mínimos cuadrados, para el siguiente conjunto de nodos:

x	-1	0	1	2
y	2	1	3	6

Indicar el valor de $P(1.5)$ en cada polinomio encontrado.

(9)

x	-1	0	1	2
y	2	1	3	6

Polinomio grado 2.

$$\hat{y} = b_0 + b_1 x + b_2 x^2$$

$$\sum_{i=1}^{5} (1)\, b_0 + \left(\sum_{i=1}^{5} x_i\right) b_1 + \left(\sum_{i=1}^{5} x_i^2\right) b_2 = \left(\sum_{i=1}^{5} y_i\right)$$

$$\sum_{i=1}^{5} x_i b_0 + \sum_{i=1}^{5} x_i^2 b_1 + \sum_{i=1}^{5} x_i^3 b_2 = \sum_{i=1}^{5} x_i y_i$$

$$\sum_{i=1}^{5} x_i^2 b_0 + \sum_{i=1}^{5} x_i^3 b_1 + \sum_{i=1}^{5} x_i^4 b_2 = \sum_{i=1}^{5} x_i^2 y_i$$

x_i	y_i	x_i^2	$x_i y_i$	x_i^3	x_i^4	$x_i^2 y_i$
-1	2	1	-2	-1	1	2
0	1	0	0	0	0	0
1	3	1	2	1	1	2
2	6	4	12	8	16	24
2	11	6	12	8	18	28

$$\begin{cases} 1 b_0 + 2 b_1 + 6 b_2 = 11 \\ 2 b_0 + 6 b_1 + 8 b_2 = 12 \\ 6 b_0 + 8 b_1 + 18 b_2 = 28 \end{cases}$$

$b_0 = -1,307$

$b_1 = -0,538$

$b_2 = 2,230$

$$\hat{y} = -1,307 + (-0,538)\, x + 2,230\, x^2$$

$$P(1,5) = -1,307 + (-0,538)(1,5) + 2,30\,(1,5)^2$$

$$= 2,903$$

10. **EJERCICIO:** Con el siguiente conjunto de nodos, a mano y con calculadora no programable:

x_i	1.35	1.70	1.90	3
y_i	3	5	6	10

a) Construir una tabla de diferencias divididas, a mano, para aproximar la función en estos puntos.

b) Solo aplicando la tabla del literal anterior, encontrar una aproximación para $x = 2$, con un polinomio interpolador de grado 2. Explique el procedimiento.

(10)

x_i	1,35	1,70	1,90	3
y_i	3	5	6	10

Ajuste exponencial

$$\hat{y} = e^{b_0 + b_1 x}$$

$$\ln \hat{y} = \ln\left(e^{b_0 + b_1 x}\right)$$

Se tiene

$$E = \sum_{i=1}^{5} \left(\ln(y_i) - \ln(\hat{y}_i)\right)^2$$

$$E = \sum_{i=1}^{5} \left(\ln(y_i) - b_0 + b_1 x_i\right)^2$$

$$\frac{dE}{d_0} = \sum_{i=1}^{5} b_0 + \sum_{i=1}^{5} x_i b_1 = \sum_{i=1}^{5} \ln(y_i)$$

$$\frac{dE}{d_0} = \sum_{i=1}^{5} x_i b_0 + \sum_{i=1}^{5} x_i^2 b_1 = \sum_{i=1}^{5} x_i \ln(y_i)$$

x_i	y_i	x_i^2	$\ln(y_i)$	$x_i \ln(y_i)$
-1	2	1	0,69315	-0,6932
0	1	0	0	0
1	2	1	0,69315	0,6932
2	6	4	1,79176	3,5835
3	10	9	2,3026	6,90776
5	21	15	5,48064	10,49177

$$\begin{pmatrix} 5 & 5 \\ 5 & 15 \end{pmatrix}\begin{pmatrix} b_0 \\ b_1 \end{pmatrix} = \begin{pmatrix} 5,48064 \\ 10,44127 \end{pmatrix}$$

$$\begin{pmatrix} b_0 \\ b_1 \end{pmatrix} = \begin{pmatrix} 0,5951 \\ 0,5011 \end{pmatrix}$$

$$\hat{y} = e^{0,5951 + 0,5611x}$$

$$\hat{y} = 1,8132\, e^{0,5011x}$$

$$P(2,5) \rightarrow \hat{y}(2,5) = 1,812\, e^{0,5011(2,5)}$$

$$= 6,3461$$

b) Polinomio Grado 2

Ajuste $\hat{y} = b_0 + b_1 x + b_2 x^2$

$$\sum_{i=1}^{5} b_0 + \sum_{i=1}^{5} x_i b_1 + \sum_{i=1}^{5} x_i^2 b_2 = \sum_{i=1}^{5} y_i$$

$$\sum_{i=1}^{5} x_i b_0 + \sum_{i=1}^{5} x_i^2 b_1 + \sum_{i=1}^{5} x_i^3 b_2 = \sum_{i=1}^{5} x_i y_i$$

$$\sum_{i=1}^{5} x_i^2 b_0 + \sum_{i=1}^{5} x_i^3 b_1 + \sum_{i=1}^{5} x_i^4 b_2 = \sum_{i=1}^{5} x_i^2 y_i$$

x_i	y_i	x_i^2	x_i^3	$x_i y_i$	$x_i^2 y_i$
-1	2	1	-1	-2	2
0	1	0	0	0	0
1	2	1	1	2	2
2	6	4	8	12	24
3	10	9	27	30	90
5	21	15	35	42	118

$$\begin{pmatrix} 5 & 5 & 15 \\ 5 & 15 & 35 \\ 15 & 35 & 99 \end{pmatrix}\begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 21 \\ 42 \\ 118 \end{pmatrix}$$

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 1,17 \\ 0,2429 \\ 0,9286 \end{pmatrix}$$

$$\hat{y} = 1,17 + 0,2429x + 0,9286x^2$$

$$P(2,5) = \hat{y}(2,5) = 1,17 + 0,2429(2,5) + 0,9286(2,5)^2$$

$$\hat{y}(2,5) = 7,58$$