| [x] <b>7.1</b> | (2pt) Sort the sequence (3, 1, 4, 1, 5, 9, 2, 6, 5) using insertion sort |
|----------------|--|
|                | Unsorted Array Problem 7.1   |

- [0]: 3
- [1]: 1
- [2]: 4
- [3]: 1
- [4]: 5
- [5]: 9
- [6]: 2
- [7]: 6
- [8]: 5

# **Sorted Array Problem 7.1:**

- [0]: 1
- [1]: 1
- [2]: 2
- [3]: 3
- [4]: 4
- [5]: 5
- [6]: 5
- [7]: 6
- [8]: 9

# $[x] \ \mbox{7.2 (1pt)}$ What is the running time of insertion Sort if all elements are equal?

O(N)

[x] 7.11 (4pt: 1pt for building heap, 2pt for sort, 1pt for memory use) Show how heapsort processes the input (142, 543, 123, 65, 453, 879, 572, 434, 111, 242, 811, 102)

# **Unsorted Array Problem 7.11**

| $\Gamma \cap \Gamma$ |    | • | •  | _ |
|----------------------|----|---|----|---|
| 111                  | •  | 1 | /1 | , |
| [0]                  | ١. |   | 4  | _ |

- [1]: 543
- [2]: 123
- [3]: 65
- [4]: 453
- [5]: 879
- [6]: 572
- [7]: 434
- [8]: 111
- [9]: 242
- [10]: 811
- [11]: 102

# Enqueued Element: 142

- [0]: 0
- [1]: 142
- [2]: 0
- [3]: 0
- [4]: 0
- [5]: 0
- [6]: 65
- [7]: 0
- [8]: 0
- [9]: 0
- [10]: 0
- [11]: 0
- [12]: 0

# Enqueued Element: 543

# Binary Heap Data

- [0]: 0
- [1]: 142
- [2]: 543
- [3]: 0
- [4]: 0
- [5]: 0
- [6]: 65
- [7]: 0
- [8]: 0
- [9]: 0
- [10]: 0
- [11]: 0
- [12]: 0

# Enqueued Element: 123

- [0]: 0
- [1]: 123
- [2]: 543
- [3]: 142
- [4]: 0
- [5]: 0
- [6]: 65
- [7]: 0
- [8]: 0
- [9]: 0
- [10]: 0
- [11]: 0
- [12]: 0

# Enqueued Element: 65

| Binary | Heap | Data |
|--------|------|------|
|--------|------|------|

- [0]: 0
- [1]: 65
- [2]: 123
- [3]: 142
- [4]: 543
- [5]: 0
- [6]: 65
- [7]: 0
- [8]: 0
- [9]: 0
- [10]: 0
- [11]: 0
- [12]: 0

# Enqueued Element: 453

- [0]: 0
- [1]: 65
- [2]: 123
- [3]: 142
- [4]: 543
- [5]: 453
- [6]: 65
- [7]: 0
- [8]: 0
- [9]: 0
- [10]: 0
- [11]: 0
- [12]: 0

# Enqueued Element: 879

# Binary Heap Data

- [0]: 0
- [1]: 65
- [2]: 123
- [3]: 142
- [4]: 543
- [5]: 453
- [6]: 879
- [7]: 0
- [8]: 0
- [9]: 0
- [10]: 0
- [11]: 0
- [12]: 0

# Enqueued Element: 572

- [0]: 0
- [1]: 65
- [2]: 123
- [3]: 142
- [4]: 543
- [5]: 453
- [6]: 879
- [7]: 572
- [8]: 0
- [9]: 0
- [10]: 0
- [11]: 0
- [12]: 0

# Enqueued Element: 434

# Binary Heap Data

- [0]: 0
- [1]: 65
- [2]: 123
- [3]: 142
- [4]: 434
- [5]: 453
- [6]: 879
- [7]: 572
- [8]: 543
- [9]: 0
- [10]: 0
- [11]: 0
- [12]: 0

# Enqueued Element: 111

- [0]: 0
- [1]: 65
- [2]: 111
- [3]: 142
- [4]: 123
- [5]: 453
- [6]: 879
- [7]: 572
- [8]: 543
- [9]: 434
- [10]: 0
- [11]: 0
- [12]: 0

# Enqueued Element: 242

## Binary Heap Data

- [0]: 0
- [1]: 65
- [2]: 111
- [3]: 142
- [4]: 123
- [5]: 242
- [6]: 879
- [7]: 572
- [8]: 543
- [9]: 434
- [10]: 453
- [11]: 0
- [12]: 0

# Enqueued Element: 811

- [0]: 0
- [1]: 65
- [2]: 111
- [3]: 142
- [4]: 123
- [5]: 242
- [6]: 879
- [7]: 572
- [8]: 543
- [9]: 434
- [10]: 453
- [11]: 811
- [12]: 0

# Enqueued Element: 102

## Binary Heap Data

- [0]: 0
- [1]: 65
- [2]: 111
- [3]: 102
- [4]: 123
- [5]: 242
- [6]: 142
- [7]: 572
- [8]: 543
- [9]: 434
- [10]: 453
- [11]: 811
- [12]: 879

# The heap is empty.

# **Sorted Array Problem 7.11**

- [0]: 65
- [1]: 102
- [2]: 111
- [3]: 123
- [4]: 142
- [5]: 242
- [6]: 434
- [7]: 453
- [8]: 543
- [9]: 572
- [10]: 811
- [11]: 879

## [X] 7.12 (1pt) What is the running time of heapsort for presorted input?

O(log(N)) to build heap \* O(N) to evaluate each item.

So, the time complexity is O(Nlog(N))

## [x] 7.15 (2pt) Sort (3, 1, 4, 1, 5, 9, 2, 6) using mergesort

## **Unsorted Array problem 7.15**

- [0]: 3
- [1]: 1
- [2]: 4
- [3]: 1
- [4]: 5
- [5]: 9
- [6]: 2
- [7]: 6

## **Sorted Array Problem 7.15:**

- [0]: 1
- [1]: 1
- [2]: 2
- [3]: 3
- [4]: 4
- [5]: 5
- [6]: 6
- [7]: 9

## [X] 7.17 (3pt) Determine the running time of merge sort for

## [X] a. Sorted input:

O(Nlog(N)) this is because it still divides the array log(N) times and makes N comparisons whether or not its sorted so O(log(N)) \* O(N)

## [X] b. Reverse-Ordered Input:

O(Nlog(N)) this is because it splits the Array log(N) times, makes N comparisons and N copies so O(log(N)) \* O(N) + O(N)

#### [X] c. Random input:

O(Nlog(N)) is the average case, so this is for random input, but the same method for reverse order applies here as the comparisons and copying are still done

#### [x] Chapter 7 Bonus Homework

# [x] 7.3 (1pt) Suppose we exchange elements a[i] and a[i+k] which were originally out of order. Prove that at least 1 and at most 2k-1 inversions are removed.

We know that an inversion occurs only if an object is out of order.

The best-case scenario for this problem is if a[i] and a[i+k] are the only objects out of order.

Average case is N(N-1)/4 inversions.

If a[i] and a[i+k] are the only objects out of order this removes every inversion but the initial Implying N(N-1)/4 - N-1((N-1)-1)/4 = -(N-1) = N-1 inversions removed.

Therefore, there will be N-1 inversions removed.

Now if Every object is out of order in the worst case

There will be N(N-1)/4 - N(N-1)/4 + 1 = -1 = 1 inversion removed

Therefore, there will be at least 1 and at most 2k-1 inversions removed

# [x] 7.19 (1pt) Sort (3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5) using quicksort with median-of-three partitioning and a cutoff of 3.

#### **Unsorted Array Problem 7.19:**

- [0]: 3
- [1]: 1
- [2]: 4
- [3]: 1
- [4]: 5

- [5]: 9
- [6]: 2
- [7]: 6
- [8]: 5
- [9]: 3
- [10]: 5

# **Sorted Array Problem 7.19:**

- [0]: 1
- [1]: 1
- [2]: 2
- [3]: 3
- [4]: 3
- [5]: 4
- [6]: 5
- [7]: 5
- [8]: 5
- [9]: 6
- [10]: 9