Q3) Assumed $x_i \sim N(\mu, \sigma^2)$
We need UMVUE for μ and σ^2
also MME estimator for μ and σ^2.

$$f(x;\mu) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{\frac{-(x-\mu)^2}{2\sigma^2}\right\}$$

Check whether it belongs to exponential family or not.

$$f(x;\mu) = e^{\ln f(x;\mu)}$$

$$= e^{\ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{\frac{-(x-\mu)^2}{2\sigma^2}\right\}\right)}$$

$$= e^{-\frac{1}{2}\ln(2\pi\sigma^2) - \frac{(x-\mu)^2}{2\sigma^2}}$$

$$f(x;\mu) = e^{-\frac{1}{2}\ln(2\pi\sigma^2) - \frac{x^2}{2\sigma^2} + \frac{x\mu}{\sigma^2} - \frac{\mu^2}{2\sigma^2}}$$

$p(\mu) \neq c$ ✓ $\quad \sum k(x_i) = \sum x_i$ is CSS.

$k(x) = x$ ✓

$S(x) = \frac{-x^2}{2\sigma^2}$ cont ✓

$q(\mu) = \frac{-\mu^2}{2\sigma^2}$ ✓

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to find UMVUE, we need to find ue for $y = \Sigma x_i$

$X \sim N(\mu, \sigma^2)$

$\Sigma x_i \sim N(n\mu, n\sigma^2)$

$E(\Sigma x_i) = n\mu \neq \mu$

$E\left(\frac{\Sigma x_i}{n}\right) = \mu \rightarrow \frac{\Sigma x_i}{n}$ is ue.

Since $\frac{\Sigma x_i}{n} = \bar{x}$ is 1-1 fnc of CSS Σx_i, $\bar{x}$ is UMVUE By L-S theorem.

Also MME of μ is $\bar{x}$ for Normal dist.

So, $\hat{\mu}_{UMVUE} = \hat{\mu}_{MME} = \bar{x} = 5.13$

For variance σ^2

$$\hat{\sigma}^2_{UMVU} = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

$$\hat{\sigma}^2_{MME} = \frac{1}{n} \sum (x_i - \bar{x})^2$$

For dataset 3:

$\hat{\sigma}^2_{UMVU} = 5.16$ $\hat{\sigma}_{UMVU} = 2.27$

$\hat{\sigma}^2_{MME} = 5.17$ $\hat{\sigma}_{MME} = 2.27$

Let $a = 5$

$$P(X > a) = 1 - F(X < a)$$

$$\text{CDF of Normal} = \phi\left(\frac{a - \hat{\mu}}{\hat{\sigma}}\right)$$

$$z = \frac{a - \hat{\mu}}{\sigma}$$

$$z_{umvu} = \frac{5 - 5.13}{2.27} = -0.06$$

$$z_{MME} = \frac{5 - 5.13}{2.27} = -0.06$$

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$z_{UMVU} = -0.06 \quad \phi(-0.06) \cong 0.48$

$z_{MME} = -0.06 \quad \phi(-0.06) \cong 0.48$

So, $P(x<5) = 0.48$ and
$P(x>5) = 0.52$
for dataset 3.

Since $\hat{\mu}$ and $\hat{\sigma}$ are almost same for UMVU and MME estimators, probabilities are equal.