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$\hat{\theta}_{MLE} = X_n$ $\quad L(\theta) = \frac{1}{\theta^n}$ - decreasing function ✱

iii) $n \nearrow$ Variance $\searrow$ (See graphs)
As $n \nearrow$, distribution approaches normal.
Thus $\hat{\theta}_{MLE} = X_{(n)}$ is BAN

iv) $E(\hat{\theta}) = E(X_{(n)}) = \int_0^\theta x \cdot n \cdot \left(\frac{x}{\theta}\right)^{n-1} \cdot \frac{1}{\theta}\, dx = \frac{n}{\theta^n}\left[\frac{x^{n+1}}{n+1}\Big|_0^\theta\right]$

$\Rightarrow E(\hat{\theta}) = \frac{n\theta}{n+1} \neq \theta$ Not unbiased. Can be adjusted and but asymptotically unbiased. $E(\hat{\theta}) \xrightarrow[n\to\infty]{} \theta$

Consistency: $P(\hat{\theta} - \theta > \varepsilon)$ must approach 0 as $n \to \infty$

$\theta = 3$ ; $\hat{\theta} \xrightarrow[n\to\infty]{} 3 \Rightarrow \hat{\theta}_{MLE} = X_{(n)}$ is consistent