

Subject4: ANNs in Practice



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Computational Intelligence - Ali Tourani - Fall 2020-2021

Agenda

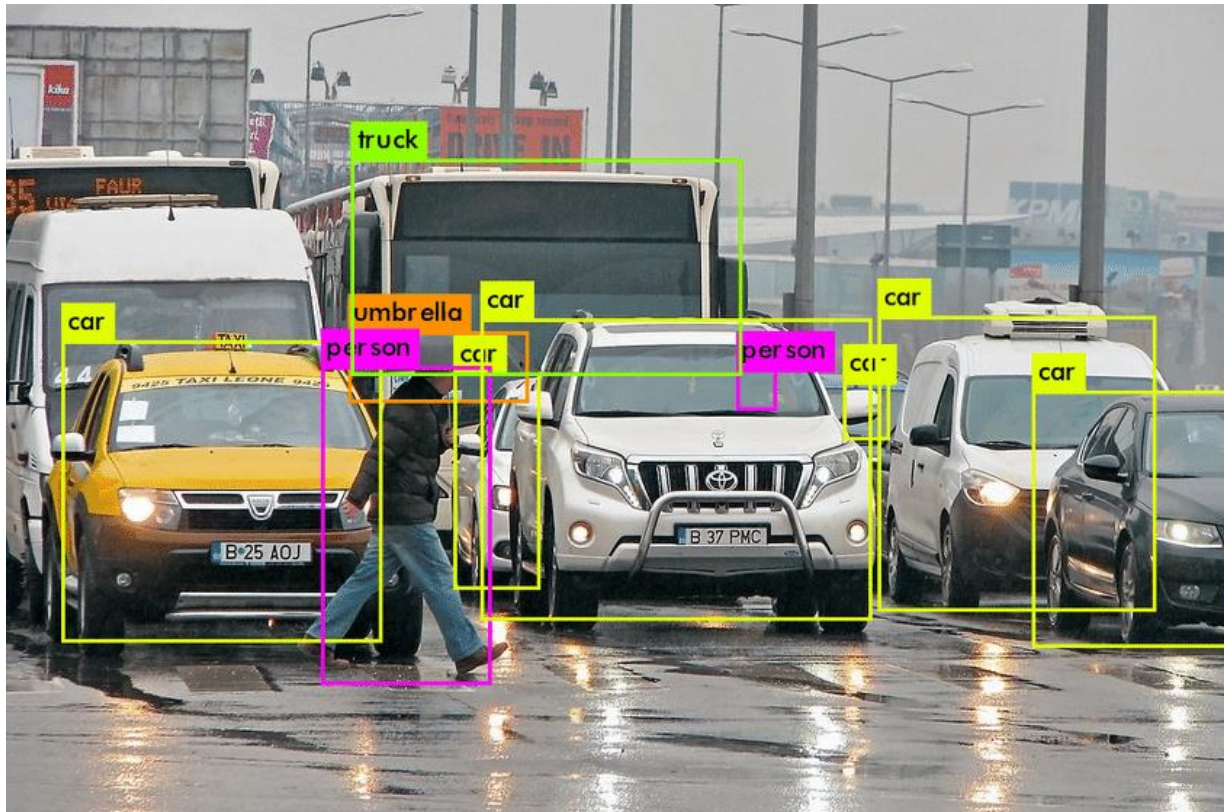
- ▶ How do ANNs learn?
- ▶ McCulloch-Pitts Neuron
- ▶ Hebb Neural Network
- ▶ Perceptron



How do ANNs learn?

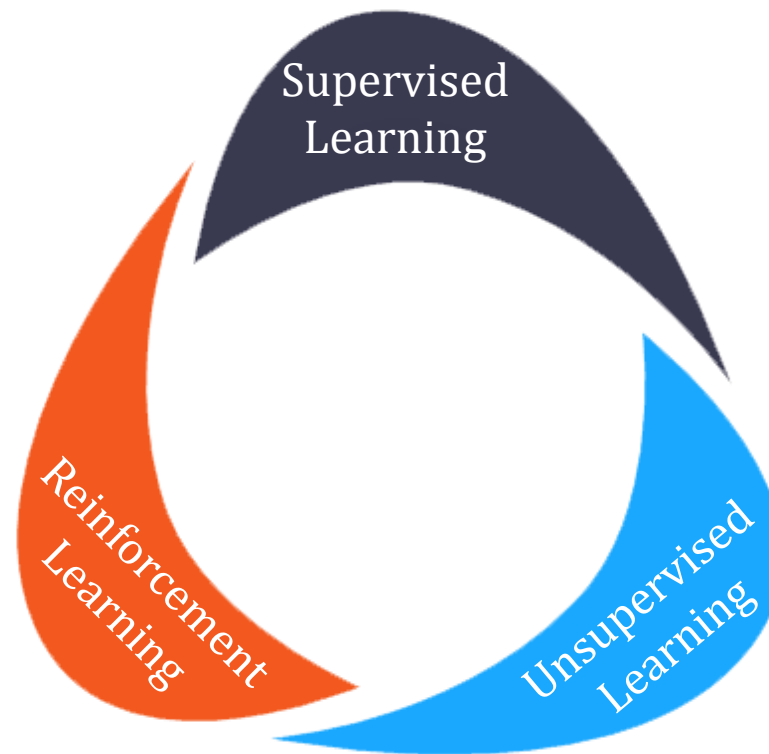
- ▶ Choosing a set of weight is easy to solve linear problems
- ▶ What about non-linear issue?
- ▶ How can we adjust weights to recognize human face in an image?
- ▶ **Solution:** providing feedback to the ANN on how to update weights to learn from its experience
 - ▶ Learning process: altering the network's weights using learning algorithms
 - ▶ Finding a set of weight matrices for mapping any input to a correct output

How do ANNs learn?



How do ANNs learn?

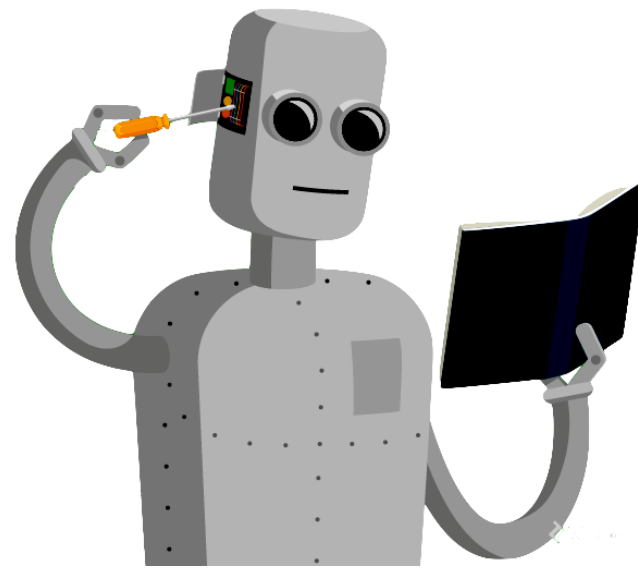
Learning Types



How do ANNs learn?

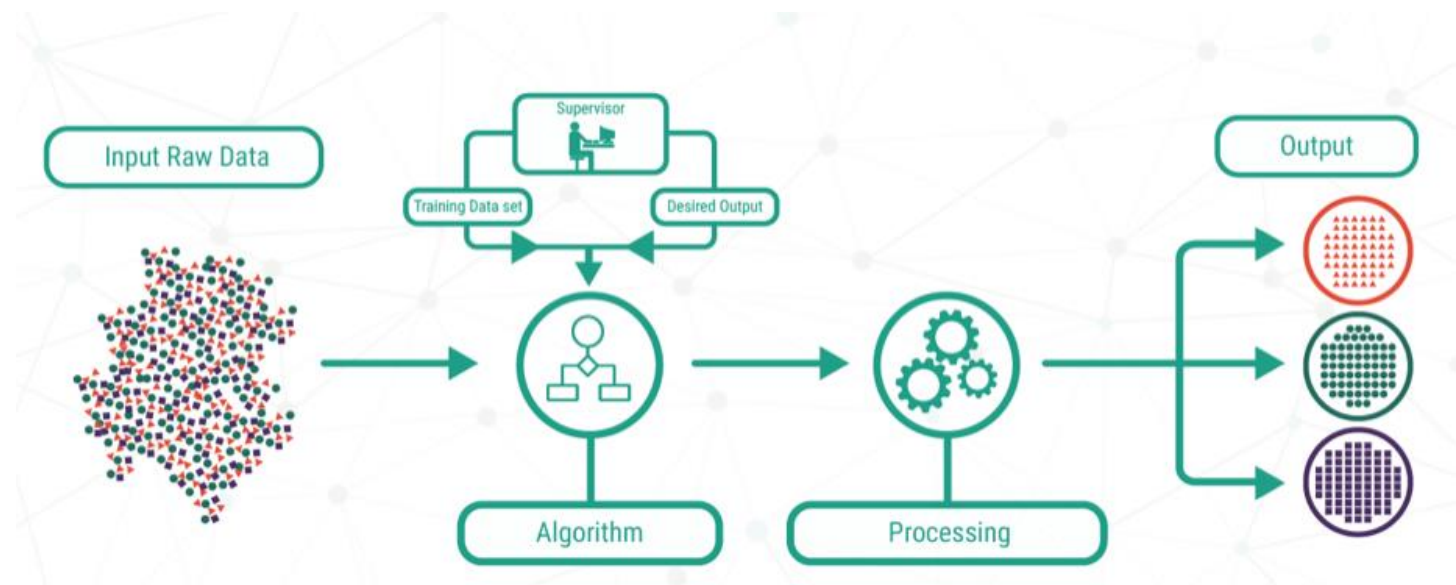
Learning Types - Supervised Learning

- ▶ While training, the desired output is also provided with the input
- ▶ Possible to calculate an error based on its real and calculated outputs
 - ▶ Why? to make corrections to the network by updating its weights
 - ▶ Basically, feedforward networks
- ▶ There is a target class for any inputs
- ▶ Related to: Classification problems



How do ANNs learn?

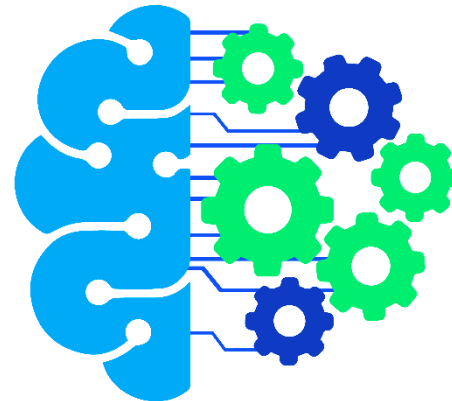
Learning Types - Supervised Learning



How do ANNs learn?

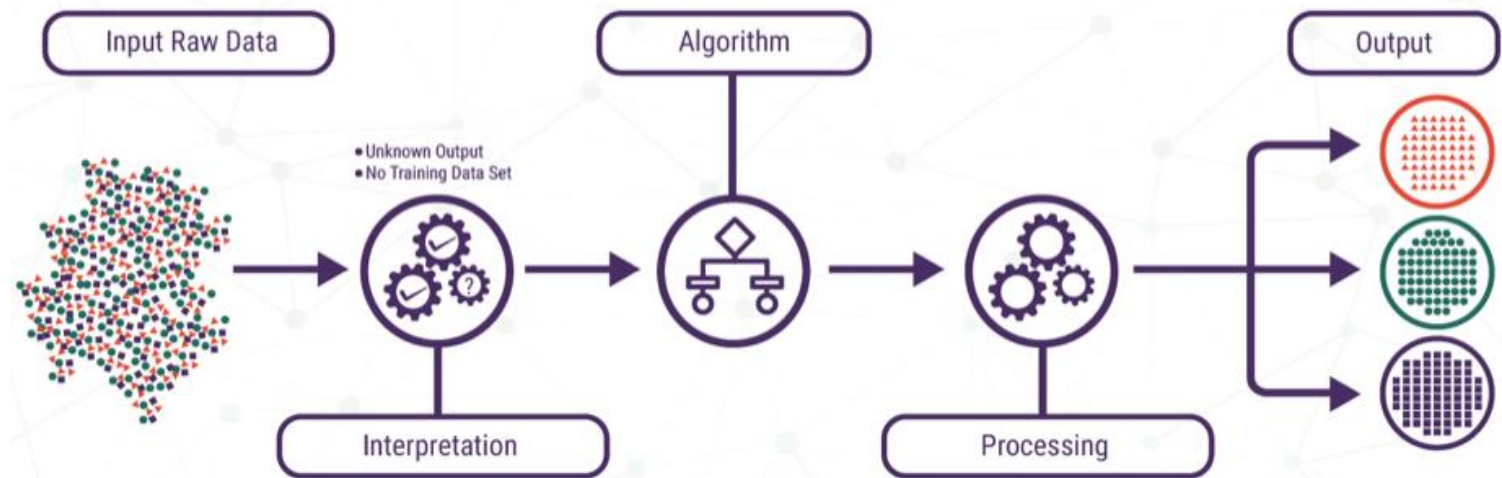
Learning Types - Unsupervised Learning

- ▶ The ANN should find a pattern within the provided inputs
 - ▶ No external aid!
 - ▶ The patterns are recognized based on similarities
 - ▶ For instance, predicting a user's preferences based on the preferences of other similar users
- ▶ Related to: Clustering (data mining)



How do ANNs learn?

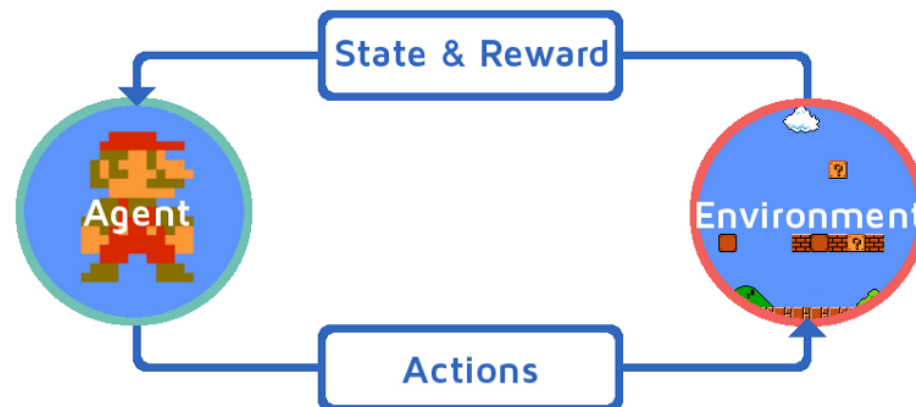
Learning Types - Unsupervised Learning



How do ANNs learn?

Learning Types - Reinforcement Learning

- ▶ Similar to Supervised Learning, a feedback is provided
- ▶ But we just inform the ANN about how good it was predicted
- ▶ Goal: maximizing the reward through trial-and-error



How do ANNs learn?

Learning Types - Reinforcement Learning

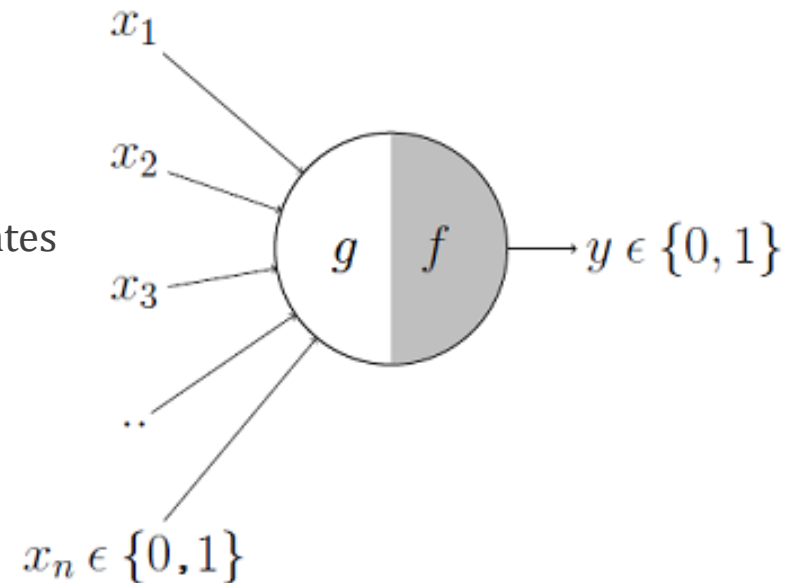


McCulloch-Pitts Neuron

- ▶ Introduced in 1943 by McCulloch (neuroscientist) and Pitts (logician)

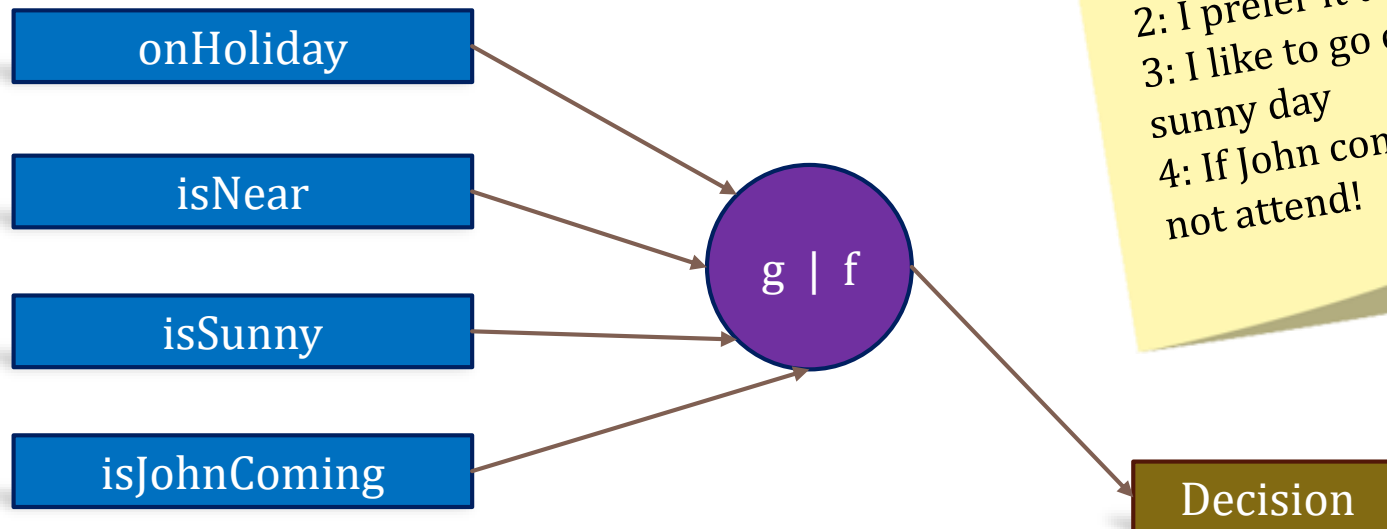
- ▶ Properties:

- ▶ The inputs and outputs are binary
- ▶ The AF is binary
- ▶ The first part of the network (***g***) aggregates the input data
- ▶ The second part (***f***) makes a decision



McCulloch-Pitts Neuron

Sample application – going out with friends:



1: I prefer to go on holiday
2: I prefer it to be near
3: I like to go on a sunny day
4: If John comes, I will not attend!

McCulloch-Pitts Neuron

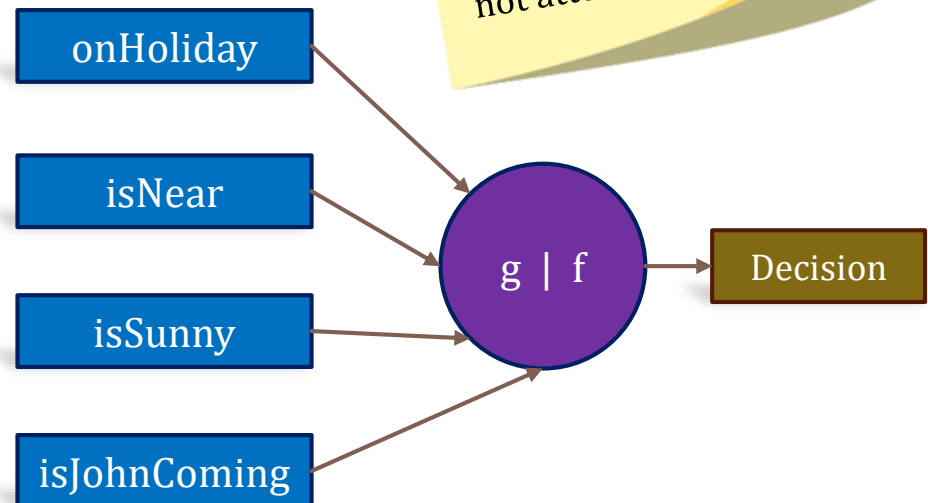
Sample application – going out with friends:

- ▶ Two types of inputs:
 - ▶ **Inhibitory**: inputs with maximum effects on the decision making
 - ▶ Note: irrespective of others
 - ▶ Sample: if x_4 (isJohnComing) is 1, my output will always be 0
 - ▶ **Excitatory**: will make the neuron fire when combined to others
- ▶ Thresholding Parameter (θ):
 - ▶ Sample: I attend the journey when the sum turns out to be more than 2 or more
 - ▶ How to calculate?!

McCulloch-Pitts Neuron

Sample application – going out with friends:

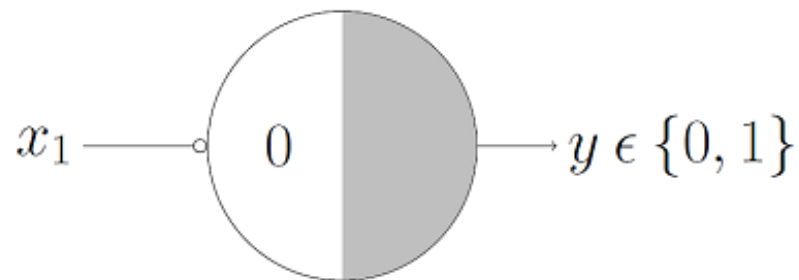
- ▶ Inhibitory nodes: *isJohnComing*
- ▶ Threshold: 3 (at least two conditions)
- ▶ So, if John comes → 0 (not attend)
- ▶ If it is sunny, and holiday and John doesn't come → 1 (attend)
- ▶ If John doesn't come and it is sunny → 0 (not attend, lower than threshold)



McCulloch-Pitts Neuron

Boolean Functions using M-P Neuron

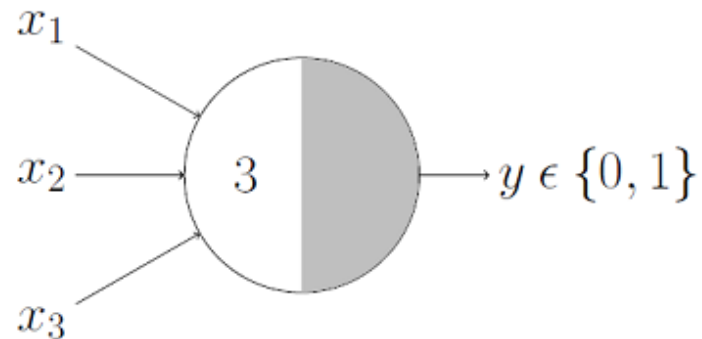
- ▶ NOT function
 - ▶ Fires with no conditions (takes the input as an inhibitory input)
 - ▶ Here, if $g(x) \geq 0$



McCulloch-Pitts Neuron

Boolean Functions using M-P Neuron

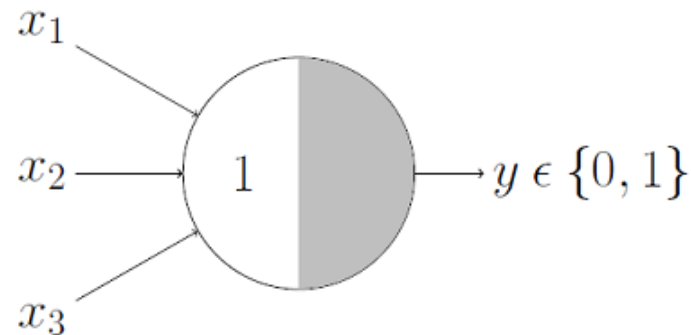
- ▶ AND function
 - ▶ Fires when ALL inputs are ON (or 1)
 - ▶ Here, if $g(x) \geq 3$



McCulloch-Pitts Neuron

Boolean Functions using M-P Neuron

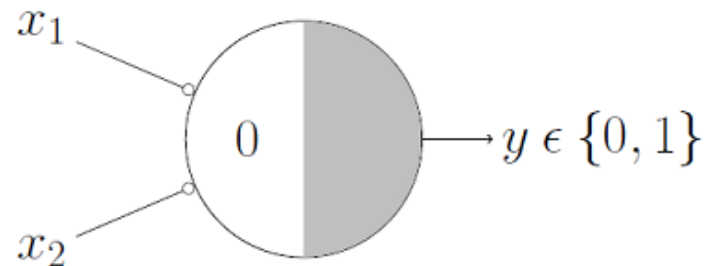
- ▶ OR function
 - ▶ Fires when AT LEAST ONE OF the inputs are ON (or 1)
 - ▶ Here, if $g(x) \geq 1$



McCulloch-Pitts Neuron

Boolean Functions using M-P Neuron

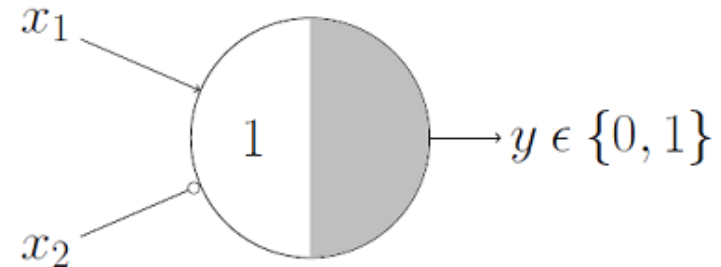
- ▶ NOR function
 - ▶ Fires when ALL of the inputs are ZERO
 - ▶ Here, if $g(x) \geq 0$



McCulloch-Pitts Neuron

Boolean Functions using M-P Neuron

- ▶ A more complex function
 - ▶ Here, x_2 is inhibitory
 - ▶ So if $x_2=1 \rightarrow y=0$
 - ▶ It triggers when:
 - ▶ x_1 is 1 and x_2 is 0



$$x_1 \text{ AND } !x_2^*$$

McCulloch-Pitts Neuron

Limitations of M-P Neuron

- ▶ Inability to have non-Boolean inputs
- ▶ Sometimes, we do not know about the threshold
- ▶ There is no priorities in choosing nodes (no weights)
- ▶ Inability to solve non-linear problems (e.g. XOR)



McCulloch-Pitts Neuron

Extension – Weighted M-P Neuron

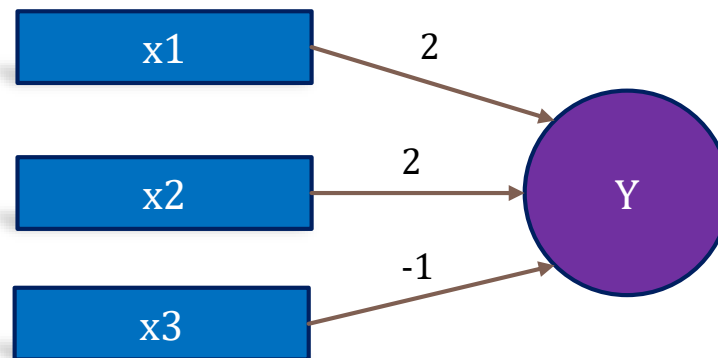
- ▶ All input signals arrive at the same time
- ▶ Each input signal can have a weight w_i to show its strength
 - ▶ The value of positive weights are equal
- ▶ Each neuron has a thresholding parameter θ
 - ▶ Neuron is activated if $net \geq \theta$
- ▶ To calculate the threshold, if n is the number of signals with positive weights, w is the value of the positive weights, and p is the number of signals with negative weights :

$$\theta > nw - p$$

McCulloch-Pitts Neuron

Extension – Weighted M-P Neuron

► Sample:



$$\theta > nw - p \quad \rightarrow \quad \theta > (2 * 2) - 1$$

Hebb Neural Network

- ▶ The major drawback of M-P: the capability of learning
 - ▶ They cannot develop capabilities for classification or recognition
- ▶ What is Hebbian Learning?
 - ▶ A mechanism to update weights between neurons
 - ▶ The process of repeatedly activating weakly connected neurons
 - ▶ Result: **providing stronger connections**

Hebb Neural Network

- ▶ The simplest Learning Rule for ANNs, AKA **Hebbian Learning Rule**
- ▶ Learning process is equal to changing the weights of the network

- ▶ Initialization of weights

$$w_i = 0 \quad (i = 1 \dots n)$$

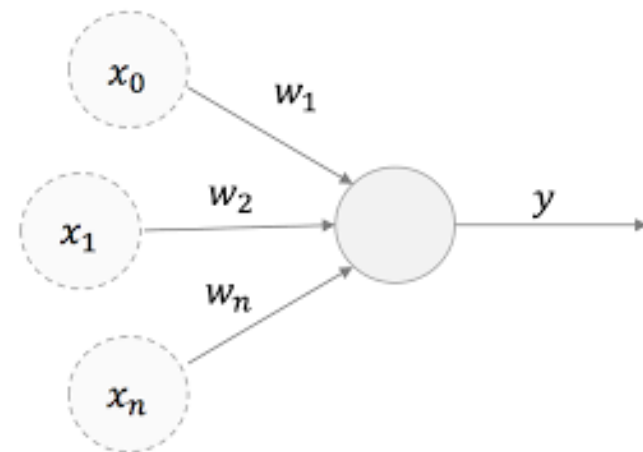
- ▶ Calculation of the *net*
- ▶ Calculation of AF's output

$$y = t \quad (\text{AKA target})$$

- ▶ Updating weights

$$w_i (\text{new}) = w_i (\text{old}) + x_i y$$

$$b(\text{new}) = b(\text{old}) + y$$



Hebb Neural Network

Sample: AND function with different inputs

- We know that:

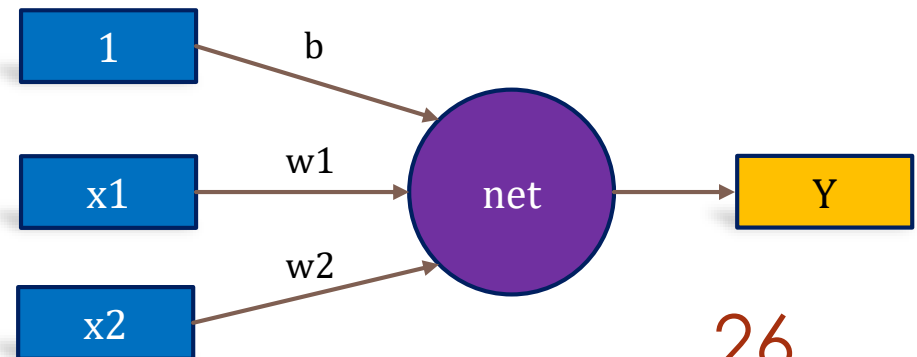
$$y_{in} = 1.b + w_1x_1 + w_2x_2$$

Input			Target
X1	X2	bias	t
1	1	1	1
1	0	1	0
0	1	1	0
0	0	1	0

- Now, lets define Δw and Δb :

$$w(new) = w(old) + \Delta w$$

$$\Delta w_i = x_i t \quad \text{and} \quad \Delta b = t$$



Hebb Neural Network

Sample: AND function with **binary** inputs

Initialize weights: (0,0,0)

			$\Delta w_i = x_i t$			$\Delta b = t$	$w(\text{new}) = w(\text{old}) + \Delta w$		
Input			Target	Weight Changes			Weights		
X1	X2	1	t	$\Delta w1$	$\Delta w2$	Δb	W1	W2	b
1	1	1	1						
1	0	1	0						
0	1	1	0						
0	0	1	0						

We know the
values of t

Hebb Neural Network

Sample: AND function with **binary** inputs

Initialize weights: (0,0,0)

			$\Delta w_i = x_i t$		$\Delta b = t$	$w(new) = w(old) + \Delta w$			
Input			Target	Weight Changes			Weights		
X1	X2	1	t	$\Delta w1$	$\Delta w2$	Δb	W1	W2	b
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1	0	1	0						
0	1	1	0						
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Sample: AND function with **binary** inputs

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1	0	1	0						
0	1	1	0						
0	0	1	0						

We know the
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Hebb Neural Network

Sample: AND function with **binary** inputs

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1	0	1	0	0	0	0			
0	1	1	0						
0	0	1	0						

We know the
values of t

Hebb Neural Network

Sample: AND function with **binary** inputs

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1	1	1	1	1	1	1	1	1	1
1	0	1	0	0	0	0	1	1	1
0	1	1	0						
0	0	1	0						

We know the
values of t

Hebb Neural Network

Sample: AND function with **binary** inputs

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X1	X2	1	t	$\Delta w1$	$\Delta w2$	Δb	W1	W2	b
1	1	1	1	1	1	1	1	1	1
1	0	1	0	0	0	0	1	1	1
0	1	1	0	0	0	0	1	1	1
0	0	1	0	0	0	0	1	1	1

We know the
values of t

Hebb Neural Network

Sample: AND function with **binary** inputs

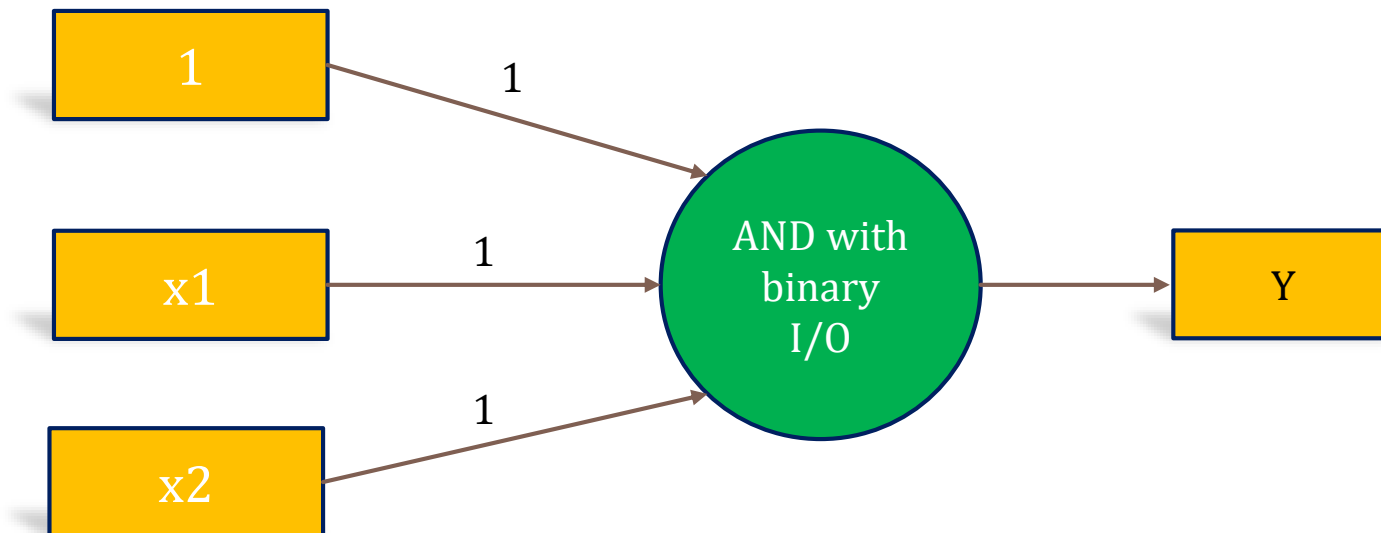
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X1	X2	1	t	$\Delta w1$	$\Delta w2$	Δb	W1	W2	b
1	1	1	1	1	1	1	1	1	1
1	0	1	0	0	0	0	1	1	1
0	1	1	0	0	0	0	1	1	1
0	0	1	0	0	0	0	1	1	1

The output weights do not change! So, we cannot solve binary AND with Hebb

Hebb Neural Network

Sample: AND function with **binary** inputs



Hebb Neural Network

Sample: AND function with **binary** inputs and **bipolar** outputs

Initialize weights: (0,0,0)

			$\Delta w_i = x_i t$			$\Delta b = t$	$w(new) = w(old) + \Delta w$		
Input			Target	Weight Changes			Weights		
X1	X2	1	t	$\Delta w1$	$\Delta w2$	Δb	W1	W2	b
1	1	1	1	1	1	1	1	1	1
1	0	1	-1	-1	0	-1	0	1	0
0	1	1	-1	0	-1	-1	0	0	-1
0	0	1	-1	0	0	-1	0	0	-2

Hebb Neural Network

Sample: AND function with **binary** inputs and **bipolar** outputs

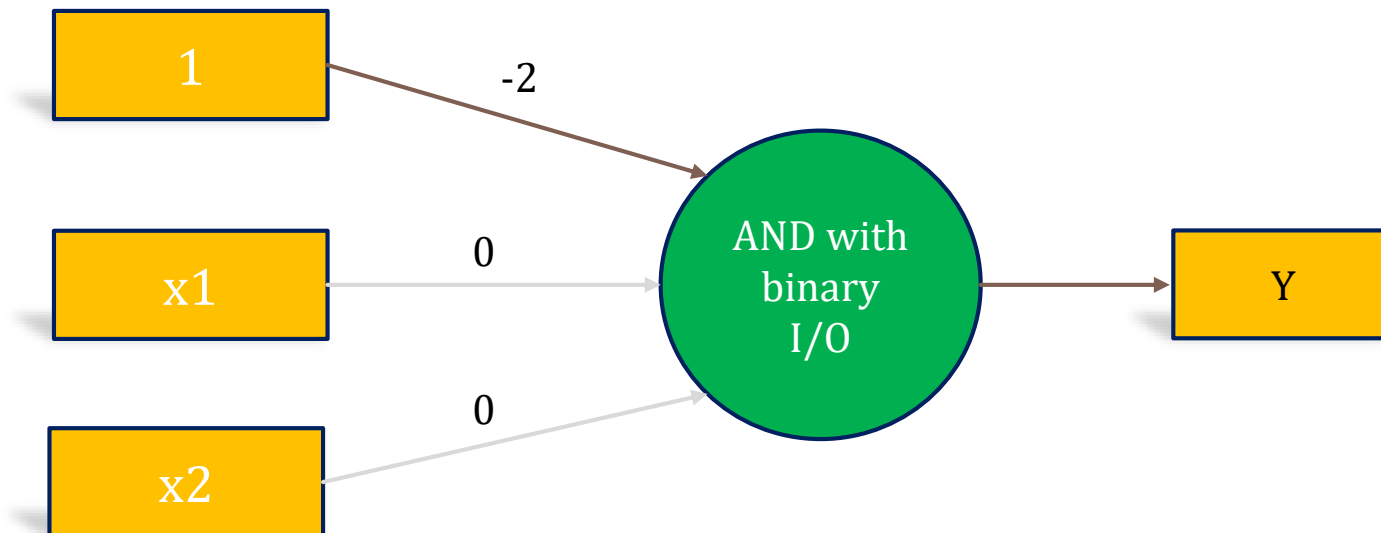
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Input			Target	Weight Changes			Weights		
X1	X2	1	t	$\Delta w1$	$\Delta w2$	Δb	W1	W2	b
1	1	1	1	1	1	1	1	1	1
1	0	1	-1	-1	0	-1	0	1	0
0	1	1	-1	0	-1	-1	0	0	-1
0	0	1	-1	0	0	-1	0	0	-2

The two signal nodes become inactive ...

Hebb Neural Network

Sample: AND function with **binary** inputs and **bipolar** outputs



Hebb Neural Network

Sample: AND function with **bipolar** inputs and outputs

Initialize weights: (0,0,0)

			$\Delta w_i = x_i t$		$\Delta b = t$	$w(new) = w(old) + \Delta w$			
Input			Target	Weight Changes			Weights		
X1	X2	1	t	$\Delta w1$	$\Delta w2$	Δb	W1	W2	b
1	1	1	1	1	1	1	1	1	1
1	-1	1	-1	-1	1	-1	0	2	0
-1	1	1	-1	1	-1	-1	1	1	-1
-1	-1	1	-1	1	1	-1	2	2	-2

Hebb Neural Network

Sample: AND function with **bipolar** inputs and outputs

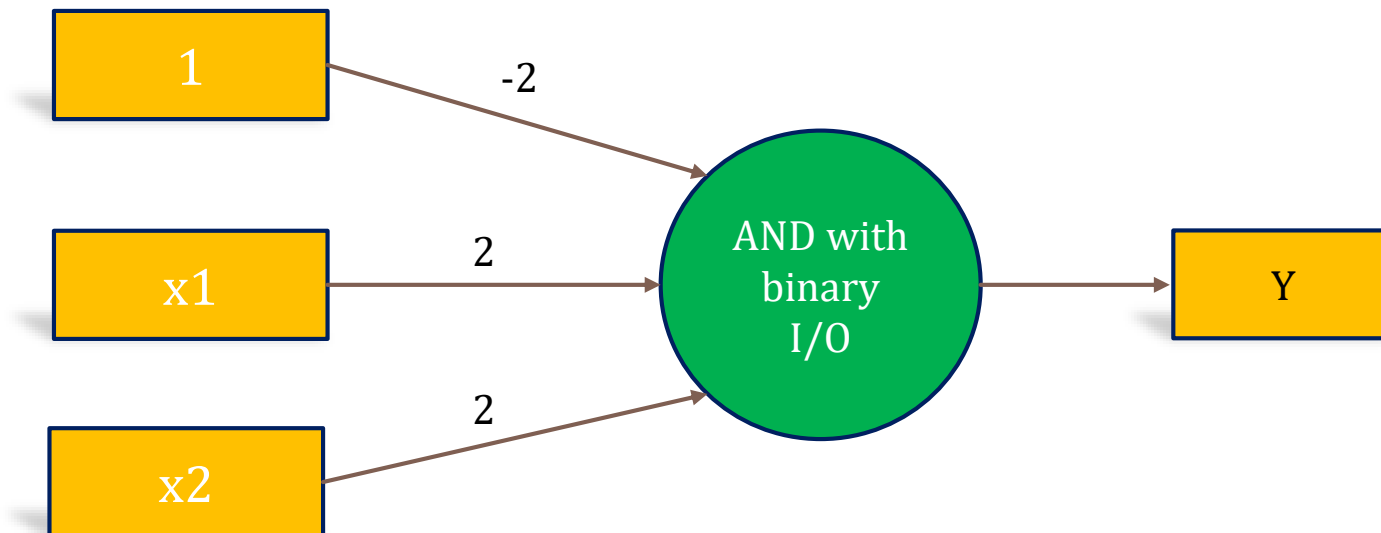
Initialize weights: (0,0,0)

			$\Delta w_i = x_i t$		$\Delta b = t$	$w(\text{new}) = w(\text{old}) + \Delta w$			
Input			Target	Weight Changes			Weights		
X1	X2	1	t	$\Delta w1$	$\Delta w2$	Δb	W1	W2	b
1	1	1	1	1	1	1	1	1	1
1	-1	1	-1	-1	1	-1	0	2	0
-1	1	1	-1	1	-1	-1	1	1	-1
-1	-1	1	-1	1	1	-1	2	2	-2

Now it learns!

Hebb Neural Network

Sample: AND function with **bipolar** inputs and outputs



Hebb Neural Network

Important conclusion

- ▶ The type of the inputs and outputs can make a problem solvable or not
- ▶ A very important hint in Hebbian Learning Rule:
 - ▶ This network is suitable for **bipolar** data
 - ▶ It can help us to recognize missing from wrong data



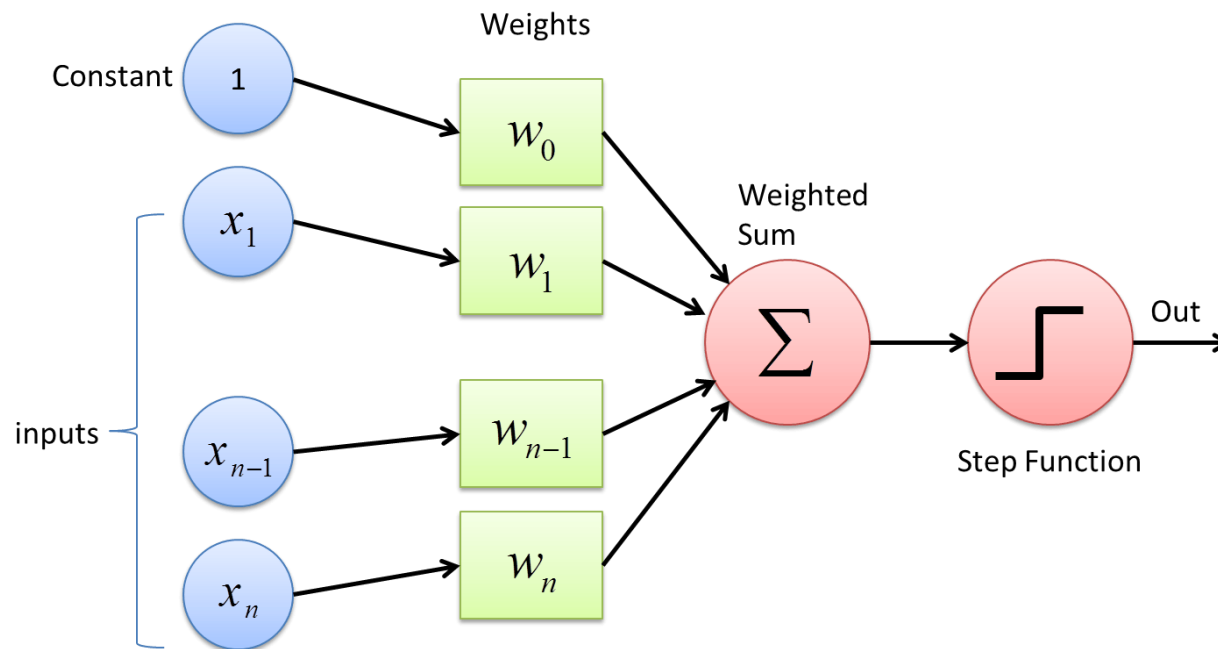
Perceptron

Single Perceptron Networks

- ▶ In their basic type, single-layer feed-forward networks
- ▶ Here, weights can be **inhibitory**, **excitatory** or **zero** (-1, +1 or 0)
- ▶ The activation function is a binary step function
- ▶ A very common type of ANNs for Supervised Learning
- ▶ Better learning process comparing to Hebb NN
- ▶ We may need several **epochs** to finish the learning process

Perceptron

Single Perceptron Networks



Perceptron

Perceptron Learning Algorithm

- ▶ Initialize weights, bias, and a learning rate $0 \leq \alpha \leq 1$

- ▶ Calculate the net value y_{in}
$$y_{in} = b + \sum x_i w_i$$

- ▶ Apply the AF over the net input to obtain an output
 - ▶ We might need a threshold θ

$$y = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } -\theta < y_{in} < \theta \\ -1 & \text{if } y_{in} < -\theta \end{cases}$$

Perceptron

Perceptron Learning Algorithm

- ▶ Now, compare the desired target value t and the calculated output y
- ▶ If $y \neq t$ then update the weights and bias (as below):
$$w_i(new) = w_i(old) + \alpha x_i t \qquad b(new) = b(old) + \alpha t$$
- ▶ Else, no need to update:
$$w_i(new) = w_i(old) \qquad b(new) = b(old)$$
- ▶ Continue the iteration until there is no weight change

Perceptron

Sample: AND function with **binary** inputs and **bipolar** outputs

► Parameters (initialization): $weights(0,0,0)$ $\alpha = 1$ $\theta = 0.2$

Input			NET	Out	Target	Weight Changes			Weights		
X1	X2	1	y _{in}	y	t	Δw_1	Δw_2	Δb	W1	W2	b
1	1	1			1						
1	0	1			-1						
0	1	1			-1						
0	0	1			-1						

Perceptron

Epoch #1

Sample: AND function with **binary** inputs and **bipolar** outputs

► Parameters (initialization): $weights(0,0,0)$ $\alpha = 1$ $\theta = 0.2$

$$\sum x_i w_i$$

Input			NET	Out	Target	Weight Changes			Weights		
X1	X2	1	y _{in}	y	t	Δw_1	Δw_2	Δb	W1	W2	b
1	1	1	0		1						
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Perceptron

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$$y = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } -\theta < y_{in} < \theta \\ -1 & \text{if } y_{in} < -\theta \end{cases}$$

Perceptron

Epoch #1

Sample: AND function with **binary** inputs and **bipolar** outputs

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Input			NET	Out	Target	Weight Changes			Weights		
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1	1	1	0	0	1						
1	0	1			-1						
0	1	1			-1						
0	0	1			-1						

Not equal! So
we need to
update weights

$$y = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } -\theta < y_{in} < \theta \\ -1 & \text{if } y_{in} < -\theta \end{cases}$$

Perceptron

Epoch #1

Sample: AND function with **binary** inputs and **bipolar** outputs

► Parameters (initialization): $weights(0,0,0)$ $\alpha = 1$ $\theta = 0.2$

$$\sum x_i w_i$$

$$\Delta w_i = \alpha x_i t$$

Input			NET	Out	Target	Weight Changes			Weights		
X1	X2	1	y _{in}	y	t	Δw1	Δw2	Δb	W1	W2	b
1	1	1	0	0	1	1	1	1			
1	0	1			-1						
0	1	1			-1						
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Perceptron

Epoch #1

Sample: AND function with **binary** inputs and **bipolar** outputs

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$\sum x_i w_i$			$\Delta w_i = \alpha x_i t$						$w_i(new) = w_i(old) + \alpha x_i t$		
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X1	X2	1	y _{in}	y	t	Δw1	Δw2	Δb	W1	W2	b
1	1	1	0	0	1	1	1	1	1	1	1
1	0	1			-1						
0	1	1			-1						
0	0	1			-1						

$$y = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } -\theta < y_{in} < \theta \\ -1 & \text{if } y_{in} < -\theta \end{cases}$$

Perceptron

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$\sum x_i w_i$			$\Delta w_i = \alpha x_i t$						$w_i(new) = w_i(old) + \alpha x_i t$		
Input			NET	Out	Target	Weight Changes			Weights		
X1	X2	1	y _{in}	y	t	Δw1	Δw2	Δb	W1	W2	b
1	1	1	0	0	1	1	1	1	1	1	1
1	0	1	2	1	-1	-1	0	-1	0	1	0
0	1	1			-1						
0	0	1			-1						

$$y = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } -\theta < y_{in} < \theta \\ -1 & \text{if } y_{in} < -\theta \end{cases}$$

Perceptron

Epoch #1

Sample: AND function with **binary** inputs and **bipolar** outputs

► Parameters (initialization): $weights(0,0,0)$ $\alpha = 1$ $\theta = 0.2$

$$\sum x_i w_i$$

$$\Delta w_i = \alpha x_i t$$

$$w_i(new) = w_i(old) + \alpha x_i t$$

Input			NET	Out	Target	Weight Changes			Weights		
X1	X2		y _{in}	y	t	Δw1	Δw2	Δb	W1	W2	b
1	1	1	0	0	1	1	1	1	1	1	1
1	0	1	2	1	-1	-1	0	-1	0	1	0
0	1	1	1	1	-1	0	-1	-1	0	0	-1
0	0	1			-1						

$$y = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } -\theta < y_{in} < \theta \\ -1 & \text{if } y_{in} < -\theta \end{cases}$$

Perceptron

Epoch #1

Sample: AND function with **binary** inputs and **bipolar** outputs

► Parameters (initialization): $w_i(0,0,0)$ $\alpha = 1$ $\theta = 0.2$

$\sum x_i w_i$						$\Delta w_i = \alpha x_i t$			$w_i(new) = w_i(old) + \alpha x_i t$		
Input			NET	Out	Target	Weight Changes			Weights		
X1	X2	1	y _{in}	y	t	Δw1	Δw2	Δb	W1	W2	b
1	1	1	0	0	1	1	1	1	1	1	1
1	0	1	2	1	-1	-1	Equal! So no need to update weights		0	1	0
0	1	1	1	1	-1	0		0	0	0	-1
0	0	1	-1	-1	-1						

Equal! So no
need to update
weights

$$y = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } -\theta < y_{in} < \theta \\ -1 & \text{if } y_{in} < -\theta \end{cases}$$

Perceptron

Epoch #1

Sample: AND function with **binary** inputs and **bipolar** outputs

► Parameters (initialization): $weights(0,0,0)$ $\alpha = 1$ $\theta = 0.2$

$\sum x_i w_i$						$\Delta w_i = \alpha x_i t$			$w_i(new) = w_i(old) + \alpha x_i t$		
Input			NET	Out	Target	Weight Changes			Weights		
X1	X2	1	y _{in}	y	t	Δw1	Δw2	Δb	W1	W2	b
1	1	1	0	0	1	1	1	1	1	1	1
1	0	1	2	1	-1	-1	0	-1	0	1	0
0	1	1	1	1	-1	0	-1	-1	0	0	-1
0	0	1	-1	-1	-1	0	0	0	0	0	-1

$$y = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } -\theta < y_{in} < \theta \\ -1 & \text{if } y_{in} < -\theta \end{cases}$$

Perceptron

Epoch #1

Sample: AND function with **binary** inputs and **bipolar** outputs

► Parameters (initialization): $weights(0,0,0)$ $\alpha = 1$ $\theta = 0.2$

$\sum x_i w_i$						$\Delta w_i = \alpha x_i t$			$w_i(new) = w_i(old) + \alpha x_i t$		
Input			NET	Out	Target	Weight Changes			Weights		
X1	X2	1	y _{in}	y	t	Δw1	Δw2	Δb	W1	W2	b
1	1	1	0	0	1	1	1	1	1	1	1
1	0	1	2	1	-1	-1	0	-1	0	1	0
0	1	1	1	1	-1	0	-1	-1	0	0	-1
0	0	1	-1	-1	-1	0	0	0	0	0	-1

$$y = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } -\theta < y_{in} < \theta \\ -1 & \text{if } y_{in} < -\theta \end{cases}$$

Init weights of the next epoch

56

Perceptron



Epoch #2

Sample: AND function with **binary** inputs and **bipolar** outputs

► Parameters (initialization):  weights(0,0,-1) $\alpha = 1$ $\theta = 0.2$

$\sum x_i w_i$						$\Delta w_i = \alpha x_i t$			$w_i(\text{new}) = w_i(\text{old}) + \alpha x_i t$		
Input			NET	Out	Target	Weight Changes			Weights		
X1	X2		y _{in}	y	t	Δw_1	Δw_2	Δb	W1	W2	b
1	1	1			1						
1	0	1			-1						
0	1	1			-1						
0	0	1			-1						

$$y = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } -\theta < y_{in} < \theta \\ -1 & \text{if } y_{in} < -\theta \end{cases}$$

Perceptron

Epoch #2

Sample: AND function with **binary** inputs and **bipolar** outputs

► Parameters (initialization): $weights(0,0,-1)$ $\alpha = 1$ $\theta = 0.2$

$\sum x_i w_i$						$\Delta w_i = \alpha x_i t$			$w_i(new) = w_i(old) + \alpha x_i t$		
Input			NET	Out	Target	Weight Changes			Weights		
X1	X2		y_{in}	y	t	$\Delta w1$	$\Delta w2$	Δb	W1	W2	b
1	1	1	-1	-1	1	1	1	1	1	1	0
1	0	1	1	1	-1	-1	0	-1	0	1	-1
0	1	1	0	0	-1	0	-1	-1	0	0	-2
0	0	1	-2	-1	-1	0	0	0	0	0	-2

$$y = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } -\theta < y_{in} < \theta \\ -1 & \text{if } y_{in} < -\theta \end{cases}$$

Init weights of the next epoch

58

Perceptron



Epoch #3

Sample: AND function with **binary** inputs and **bipolar** outputs

► Parameters (initialization):  $weights(0,0,-2)$ $\alpha = 1$ $\theta = 0.2$

$\sum x_i w_i$			$\Delta w_i = \alpha x_i t$						$w_i(new) = w_i(old) + \alpha x_i t$		
Input			NET	Out	Target	Weight Changes			Weights		
X1	X2	1	y _{in}	y	t	Δw1	Δw2	Δb	W1	W2	b
1	1	1			1						
1	0	1			-1						
0	1	1			-1						
0	0	1			-1						

$$y = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } -\theta < y_{in} < \theta \\ -1 & \text{if } y_{in} < -\theta \end{cases}$$

Perceptron

Epoch #3

Sample: AND function with **binary** inputs and **bipolar** outputs

► Parameters (initialization): $weights(0,0,-2)$ $\alpha = 1$ $\theta = 0.2$

$$\sum x_i w_i$$

$$\Delta w_i = \alpha x_i t$$

$$w_i(new) = w_i(old) + \alpha x_i t$$

Input			NET	Out	Target	Weight Changes			Weights		
X1	X2	1	y _{in}	y	t	Δw1	Δw2	Δb	W1	W2	b
1	1	1	-2	-1	1	1	1	1	1	1	-1
1	0	1	0	0	-1	-1	0	-1	0	1	-2
0	1	1	-1	-1	-1	0	0	0	0	1	-2
0	0	1	-2	-1	-1	0	0	0	0	1	-2

$$y = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } -\theta < y_{in} < \theta \\ -1 & \text{if } y_{in} < -\theta \end{cases}$$

Perceptron

**FIFTEEN
— YEARS
LATER:**

Perceptron



Epoch #10

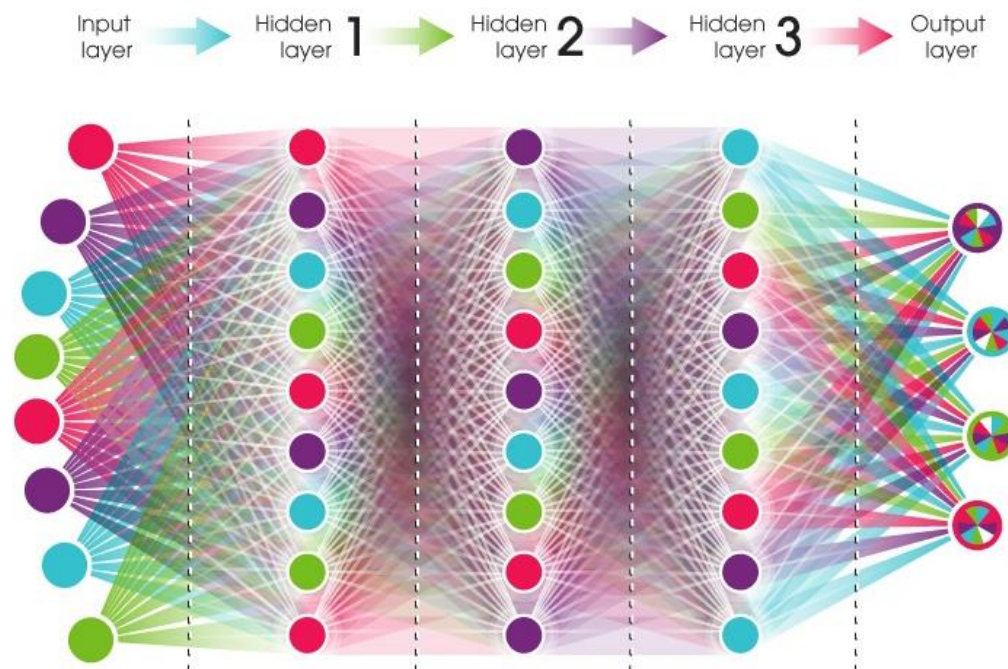
Sample: AND function with **binary** inputs and **bipolar** outputs

Input			NET	Out	Target	Weight Changes			Weights		
X1	X2	1	y _{in}	y	t	Δw_1	Δw_2	Δb	W1	W2	b
1	1	1	1	1	1	0	0	0	2	3	-4
1	0	1	-2	-1	-1	0	0	0	2	3	-4
0	1	1	-1	-1	-1	0	0	0	2	3	-4
0	0	1	-4	-1	-1	0	0	0	2	3	-4

Termination Condition 😊

What's Next?

► Deep Neural Networks (DNNs)



Questions?

