4.0 Amdahl's Law:

4.1 Suppose the sequential part of a program accounts for 40% of the program's execution time on a single processor. Find an expression for the overall speedup that can be achieved by running the program on an n-processor machine. (The expression should be a formula for speedup as a function of n.) Show all intermediate steps. What is the limit of the speed-up if there is no bound to how many processors your system can have?

Solution:

Given that, $S = 40\% = 0.4$, we know $P = 1 - S = 1 - 0.4 = 0.6$. By applying Amdahl's law, we have

$$\text{Speedup} = \frac{1}{S + \frac{P}{n}} = \frac{1}{0.4 + \frac{0.6}{n}}$$

Let's simplify it:

$$\text{Speedup} = \frac{1}{\frac{0.4n+0.6}{n}}$$

$$\text{Speedup} = \frac{n}{0.4n + 0.6}$$

$$\text{Speedup} = \frac{n}{0.2(2n + 3)}$$

$$\text{Speedup} = \frac{5n}{2n + 3}$$

Finally, we have

$$\text{Speedup}(n) = \frac{5n}{2n + 3}$$

If there is no bound to the number of processors we can have,

$$\lim_{n \to \infty} \frac{5n}{2n + 3} = \lim_{n \to \infty} \frac{5}{2 + \frac{3}{n}} = \frac{5}{2} = 2.5$$

4.2 Now suppose the sequential part accounts for 30% of the program's computation time. Let sn be the program's speedup on n processors, assuming the rest of the program is perfectly parallelizable. Your supervisor tells you to double this speedup: the revised program should have speedup $2s_n' \geq 2s_n$. You advertise for a programmer to replace the sequential part with an improved version that decreases the sequential time percentage by a factor of k. Find an expression for the value of k you should advertise for. Show all intermediate steps.

Solution:

Given that, $s_n' = 2s_n$ and $\frac{0.3}{k}$.

Since the original speedup is $s_n = \frac{1}{0.3 + \frac{0.7}{n}}$ and we want to double it $s_n' = 2s_n = \frac{2}{0.3 + \frac{0.7}{n}}$.

With the new sequential time percentage $s = \frac{0.3}{k}$, update parallel time percentage accordingly $p = 1 - \frac{0.3}{k}$.

Rewrite the speedup $s_n' = \frac{1}{\frac{0.3}{k} + \frac{1-\frac{0.3}{k}}{n}}$.

Want to solve k for this equation:

$$\frac{2}{0.3 + \frac{0.7}{n}} = \frac{1}{\frac{0.3}{k} + \frac{1-\frac{0.3}{k}}{n}}$$

$$0.3 + \frac{0.7}{n} = 2\left[\frac{0.3}{k} + \frac{1-\frac{0.3}{k}}{n}\right]$$

$$0.3 + \frac{0.7}{n} = \frac{0.6}{k} + \frac{2\left(1-\frac{0.3}{k}\right)}{n}$$

$$0.3 + \frac{0.7}{n} = \frac{0.6}{k} + \frac{2-\frac{0.6}{k}}{n}$$

$$k\,(0.3n + 0.7) = 0.6n + k\left(2 - \frac{0.6}{k}\right)$$

$$0.3kn + 0.7k = 0.6n + 2k - 0.6$$

$$0.3kn + 0.7k - 2k = 0.6n - 0.6$$

$$k(0.3n - 1.3) = 0.6(n - 1)$$

$$k = \frac{0.6(n-1)}{0.3n - 1.3}$$

$$k = \frac{0.6(n-1)}{0.1(3n - 13)}$$

$$k = \frac{6(n-1)}{3n - 13}$$

The expression for k is:

$$k = \frac{6(n-1)}{3n - 13} \text{ for } n > \frac{13}{3}$$

For n ≤ 4.33 processors, it's impossible to double the speedup, even with perfect parallelization.

4.3 Suppose the sequential time percentage could be decreased 3-fold, and when we do so, the modified program takes half the time of the original on n processors. What fraction of the overall execution time did the sequential part account for? Express your answer as a function of n. Show all intermediate steps.

Given information that the sequential time percentage is decreased 3-fold, then we have $S_{new} = \frac{S}{3}$, then the parallel time percentage is $P_{new} = \frac{2S}{3}$.

Let T be the original execution time, and T' be the new execution time. We want to find S as a function of n.

Original Speedup: $\dfrac{1}{S + \frac{1-S}{n}}$

New Speedup: $\dfrac{1}{\frac{S}{3} + \frac{1-\frac{S}{3}}{n}}$

Since the modified program takes half the time of the original, then:

$$T_{modified} = \frac{1}{2} T_{original}$$

Since execution time is inversely proportional to speedup, then:

$$\frac{T_{modified}}{T_{original}} = \frac{Speedup_{original}}{Speedup_{modified}}$$

$$\frac{Speedup_{original}}{Speedup_{modified}} = \frac{1}{2}$$

$$\frac{1}{\frac{S}{3} + \frac{1-\frac{S}{3}}{n}} = 2 \cdot \frac{1}{S + \frac{1-S}{n}}$$

$$\frac{1}{\frac{S}{3} + \frac{1-\frac{S}{3}}{n}} = \frac{2}{S + \frac{1-S}{n}}$$

Cross multiply:

$$S + \frac{1-S}{n} = 2\left(\frac{S}{3} + \frac{1-\frac{S}{3}}{n}\right)$$

$$S + \frac{1-S}{n} = \frac{2S}{3} + \frac{2-\frac{2S}{3}}{n}$$

Multiply both sides by 3n:

$$3n \cdot S + 3n \cdot \frac{1-S}{n} = 3n \cdot \frac{2S}{3} + 3n \cdot \frac{2-\frac{2S}{3}}{n}$$

$$3nS + 3(1-S) = 2nS + 3\left(2 - \frac{2S}{3}\right)$$

$$3nS + 3 - 3S = 2nS + 6 - 2S$$

$$3nS - 2nS - 3S + 2S = 6 - 3$$

$$S(n-1) = 3$$

$S = \dfrac{3}{n-1}$, where $n > 1$