Lecture 11. Graph SLAM II

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1 Chi squared error

$$\chi^2 = \sum \|\cdot\|^2_{\Sigma_i} + \|\cdot\|^2_{\Sigma_k} = \|A\delta - b\|^2_2$$

If χ^2 has converged ($\delta = 0$), then

$$\chi^2\big|_{\delta=0} = b^T b$$

1.1 Information matrix ($A^T A = \Lambda$)

SAM is a MAP estimator of $x_{0:t_i}$

$$\arg\max_\theta P(\mathcal{X}, \mathcal{M}, \mathcal{Z}, \mathcal{U}) \xrightarrow{(-\log\,,\ \text{linearisation})} \arg\min_\delta \|A\delta - b\|^2_2$$

In fact, all these factors express a distribution as well.

$$\begin{aligned}\|A\delta - b\|^2_2 = (A\delta - b)^T(A\delta - b) = \delta^T A^T A\delta - \delta^T A^T b - b^T A\delta + b^T b &\overset{\text{if } b = A\mu}{=} \\ \delta^T A^T A\delta - 2\delta^T A^T A\mu + \mu^T A^T A\mu &= \\ (\delta - \mu)^T \underbrace{A^T A}_{\Lambda}(\delta - \mu)&\end{aligned}$$

1.2 Normal equation

$$\begin{aligned}A\delta &= b \Rightarrow (\delta = A^{-1} \cdot b) \\ A^T A\delta &= A^T b \\ \delta &= (A^T A)^{-1} A^T b. \qquad \text{complexity of this: } \mathcal{O}(n^3)\end{aligned}$$

A is space $\rightarrow$ exploit by SOTA Linear algebra.

1.3 Cholesky factorisation

$\Lambda = A^T A = L \cdot L^T = R^T R$, where R is an upper-triangular matrix (can be a lower-triang.)

$$\begin{aligned}A^T A\delta &= A^T b \\ &\Downarrow \text{(cholesky)} \\ R^T R\delta &= A^T b \qquad \text{(Squared root method)}\end{aligned}$$

$$\left[\begin{aligned}R^T \cdot y &= A^T b \\ R \cdot \delta &= y\end{aligned}\right.$$

Equations above solved efficiency by back-substitution

Example:

$$\begin{bmatrix} 1 & 0 & 0 \\ 5 & 7 & 0 \\ 6 & -7 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 5 \end{bmatrix}$$

Solving:

1. $y_1 = 2$
2. $5 \cdot 2 + 7y_2 = 5 \Rightarrow y_2 = -\frac{5}{7}$
3. $6y_1 - 7y_2 + 3y_3 = 5$
   $6 \cdot 2 - 7(-\frac{5}{7}) + 3y_3 \Rightarrow y_3 = \dfrac{5 - 12 - 5}{3} = 4$

Cholesky factorisation regiven to solve 2 systems by back-substitution

Other efficient factorizations include:

- QR factorization
- Schur complement (to marginalize landmarks)
- Exploiting sparsity of A, so calling sparse methods
- Ordering of nodes factorizations resulting in more zero elements.

2 Pose SLAM

Only poses are estimated.

Example: 2D

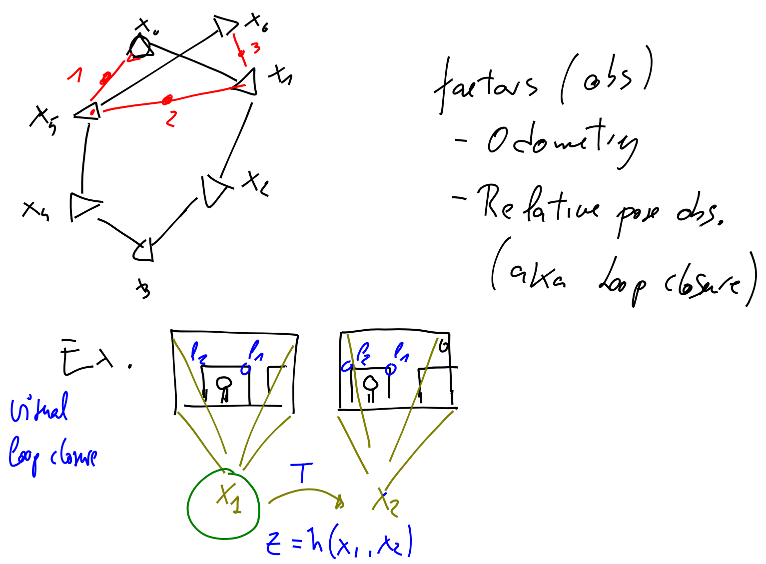

Figure 1: Poses are observed from different poses, for instance, from x_5 we observe the initial pose x_0. In the bottom part there is an example for visual loop closure, relating a pair of poses from visual information.

2.1 Observations in 2D poses

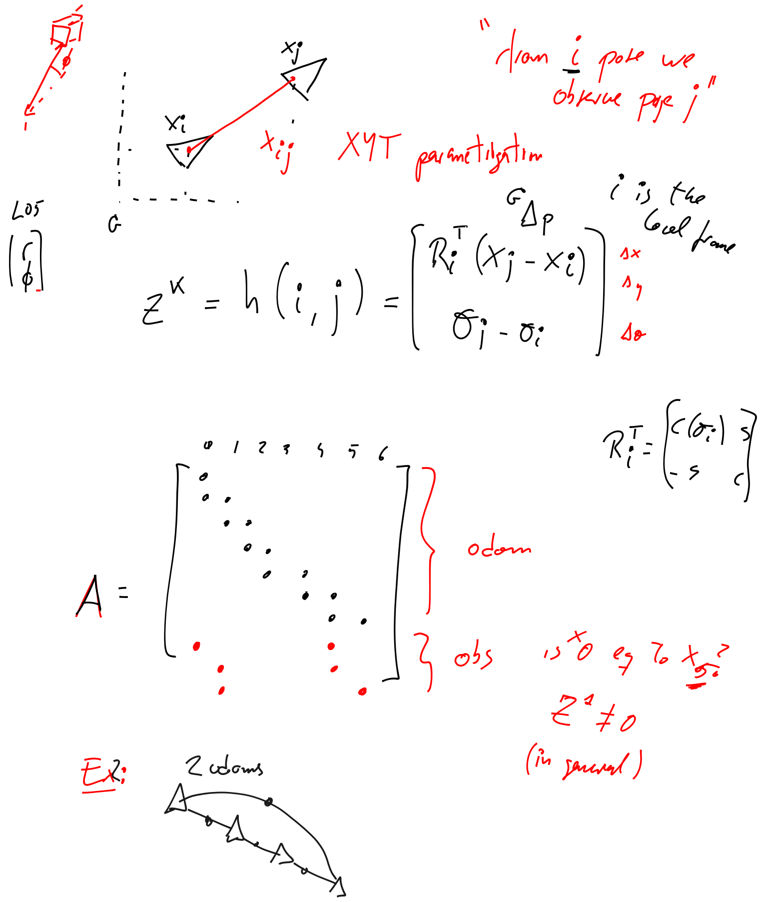

Figure 2: Observations in the adjacency matrix indicate relation between poses.

$$2DPoseJacobian \qquad H_k^j = \begin{bmatrix} R_i^T & 0 \\ 0 & 1 \end{bmatrix} \qquad H_k^i = \begin{bmatrix} -R_i^T & \begin{matrix} -s\Delta x + c\Delta x \\ -c\Delta x - s\Delta x \end{matrix} \\ 0 & -1 \end{bmatrix}$$

3 Covariance in graph SLAM

$\Sigma = \Lambda^{-1}$ (Λ is sparse but inversion is not efficient $\rightarrow$ dense)

<u>Idea:</u> No need to invert Λ, we have R

$$\begin{aligned} \Lambda = A^T A = R^T R \quad &= \Sigma^{-1} \\ &\Downarrow \\ R^T R \Sigma \quad &= I \end{aligned}$$

$$\begin{cases} R^T \cdot Y &= I \\ R \cdot \Sigma &= Y \end{cases} \qquad \text{2 back-substitution, now of a matrix (set of vectors)}$$

3.1 Landmarks elimination

Eliminating m_1 will add more factors to substitute the previous factors to m_1.

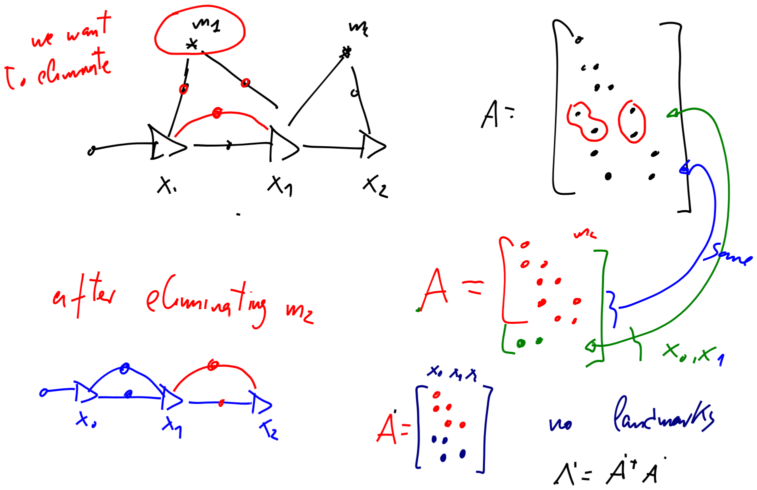

Figure 3: Example of landmark elimination in the graph and adjacency matrix.

All landmarks have been eliminated, but the new factors have appeared to express the equivalent graph.

4 Relation to the Schur complement

$$\Lambda = A^T A = \begin{bmatrix} \Lambda_x & \Lambda_{xm} \\ \Lambda_{mx} & \Lambda_m \end{bmatrix}, \delta = \begin{bmatrix} \delta_x \\ \delta_m \end{bmatrix}, A^T b = \begin{bmatrix} b_x \\ b_m \end{bmatrix}$$

$$(\Lambda_x - \Lambda_{xm}\Lambda_m^{-1}\Lambda_{mx})\delta_x = b_x - \Lambda_{xm}\Lambda_m^{-1}$$

The Schur complement is equivalent to eliminate (marginalize) all landmarks in the information matrix. These new information blocks are the result of the marginalization of landmarks, and they maintain the same relations as in the original problem.

	marginalize	condition
Σ	Lookup	Schur complement
Λ	Schur complement	Summation