CAIT
Mobile Robotics Lab
Perception in Robotics Course

Lecture 09: Data Association

Gonzalo Ferrer

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1 Euclidean nearest neighbours

The data association problem consist of finding for each observation z^i to which landmark m_j it corresponds. In other words:

$$c_t = \{c_t^i\},\ (z^i \to m_j)$$

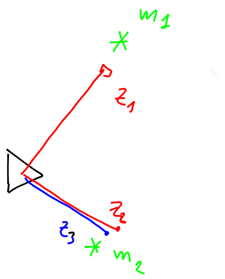


Figure 1: Euclidean nearest neighbours

$$c_t^i = \arg\min_j ||m_j - z_t^i||_2$$

Algorithm 1 Euclidean nearest neighbour

for $i = 1 : K$ **do**
  for $j = 1 : N$ **do**
    $c_t^i = \min(||m_j - z_t^i||_2)$
  end for
end for

Pros:

- Match each z^i to closest m_j
- Easy and Fast $\mathcal{O}(K \cdot N)$

Cons: Greedy Data association.

2 Mahalanobis nearest neighbour

$$d_{ij}^2 = ||m_j - z^i||_\Sigma^2 = (m_j - z^i)^T \Sigma^{-1} (m_j - z^i)$$

MH captures uncertainty
Pros:

- More robust to noise

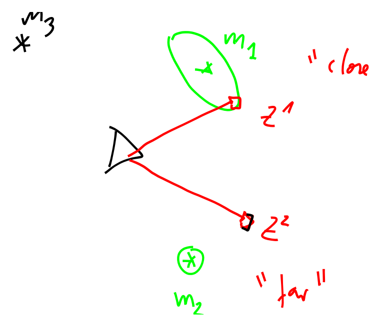


Figure 2: Mahalanobis nearest neighbours. The term “far” and “close” denote uncertainty for MH distances.

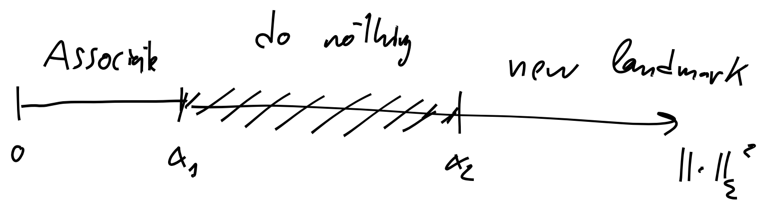


Figure 3: Decision Making for new observations. New landmarks might need conformation over several observations.

- Easy and fast $\mathcal{O}(K \cdot N)$

Cons: Still greedy.

3 Maximum Likelihood (ML) Data Association

From the Bayes Filter we should calculate the distribution of correspondences:

$$
\begin{aligned}
p(x_{0:t}, c_{1:t}|Z, U) &= \text{(product rule)} \\
&= p(c_{1:t}|x_{0:t}, Z, U)p(x_{0:t}|Z, U)
\end{aligned}
$$

Very complicated! Sequence of all correspondences should be re-calculated completely for all observations.

3.1 Assumption I: Solve DA incrementally

$p(c_t|z_t, y_t)$. The history of correspondences $c_{1:t}$ only depends on the last correspondence. Assume previous correspondences were correct.

$$p(c_t|z_t, y_t) = p(c_t|z_t, y_t, c_{1:t-1})$$

$$p(c_t|z_t, y_t) = \frac{p(z_t|c_t, y_t)p(c_t|y_t)}{p(z_t|y_t)} \propto p(z_t|c_t, y_t)$$

Posterior $\propto$ likelihood for a given c_t.

$$c_t^* = \arg\max_{c_t}\{p(z_t|c_t, y_t)\} \text{ ML estimator}$$

$$
\begin{aligned}
p(z_t|c_t, y_t) = p(z_t^1|z_t^{2:t}, y_t, c_t) \\
\cdot p(z_t^2|z_t^{3:t}, y_t, c_t) \\
\vdots \\
\cdot p(z_t^K|y_t, c_t)
\end{aligned}
$$

3.2 Assumption II: Independence

$$p(z_t|c_t, y_t) \simeq \prod_{i=1}^{K} p(z_t^i|c_t^i, y_t)$$

$$c_t^* = \arg\max_{c_t} \prod_{i=1}^{K} p(z_t^i|c_t^i, y_t),$$

where $c_t = \{c_t^1 = m_{j_1}, c_t^2 = m_{j_2}, \ldots, c_t^i = m_{j_i}, \ldots\}$
Since c_t are independent:

$$\max_{c_t}\left(\prod_{i=1}^{K} p(z_t^i|c_t^i, y_t)\right) = \prod_{i=1}^{K} \max_{c_t^i} p(z_t^i|c_t^i, y_t)$$

Evaluate c_t^i individually!

$$z_t^i \sim \mathcal{N}(z_t^i; h(\bar{\mu}_t, c^i), H_t^i \bar{\Sigma}_t (H_t^i)^T + Q_t)$$

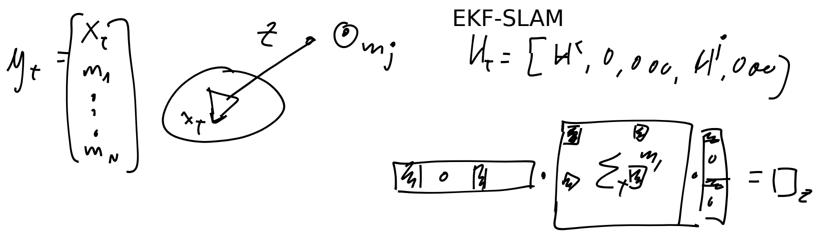
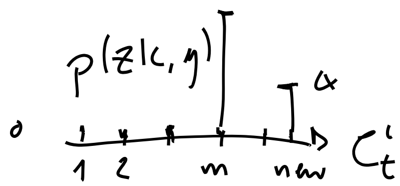


Figure 4: The Jacobian used to propagate he covariances of observations is the same as explained the the EKF-SLAM lecture.

New landmarks (ProbRob 322) under this method would never be detected. Is for that reason that the hypothesis of being a new landmark should be considered:

$$p(z_t^i|c_t^i = new, y_t) = \alpha$$

where α is a threshold value, hard to tune in practice.
We create a landmark only if distance to all other landmarks is higher than α.



4 Summary

$c^i = \arg\min_j \|m_j - z^i\|_z$ – Euclidean Nearest Neighbor

$c^i = \arg\min_j \|m_j - z^i\|_{\Sigma_j}$ – Mahalanobis Nearest Neighbor

$c_t^* = \arg\max_{c_t} \{p(z_t|c_t, y_t)\}$ – Maximum Likelihood

$z_t = \{z^1, z^2, \ldots, z^k\}$

Assumption: z independent

$\max(\prod p(\cdot)) = \prod(\max_{c^i} p(\cdot))$ – is equivalent to Mahalanobis Nearest Neighbor