(3c)

$$A_3 = \begin{matrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ 0 & 0 & -1 \end{matrix}$$

FOR $\lambda = 1$ WE GET

$$\det \begin{vmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{vmatrix} = 0$$

AND THE VECTORS THAT ARE SENDING THIS MATRIX TO ITS NULL SPACE ARE

$-1 \quad -1 \quad -1$

$$\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

(3d) THE SUM ACCORDING TO THE TRACE:

$$\lambda_1 + \lambda_2 + \lambda_3 = 0$$

$$1 + 1 + \lambda_3 = 0$$

$$\lambda_3 = -2$$

$$X_3 = (1,1,1)$$

QUIZ #2 (A)

(1a) $P = \frac{aa^T}{a^Ta} = \frac{1}{14}\begin{bmatrix} 4 & 2 & 6 \\ 2 & 1 & 3 \\ 6 & 3 & 9 \end{bmatrix}$

(1b) The column space are all the vectors on the line which the vector (2, 1, 3) is part of. The basis is of course the vector (2, 1, 3).

The null space is the plane:

$2x + y + 3z = 0$

A basis for this has two vectors:

$\begin{bmatrix} -1\frac{1}{2} \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

(1c) Since this is a projection matrix then given x that's on the column space, it projects it on it self.

Which means $Px = \lambda x = 1x$

and $\lambda_1 = 1$.

The trace here is 1 which means $\lambda_2 \lambda_3 = 0$

Given that the matrix is singular

WE GET THAT $\lambda_2 \cdot \lambda_3 = 0$ (2)

SO $\lambda_2 = \lambda_3 = 0$

THE EIGN VECTOR IS FOR EXAMPLE $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

(2a) $A\hat{x} = b$

SINCE A IS INVERTIBLE THEN WE CAN WRITE

$\hat{x} = A^{-1}b$

THE ERROR IS ACTUALLY 0 PROVIDED THAT B IS IN THE COLUMN SPACE OF A.

THE VECTORS THAT ARE PERPENDICULAR TO THE ERROR ARE ACTULLY ALL THE VECTORS IN THE COLUMN SPACE.

(2b) We said $A\hat{x} = b$

BUT NOW $A = QR$

SO $QR\hat{x} = b$

$R\hat{x} = Q^{-1}b$

$R\hat{x} = Q^T b$

$\hat{x} = R^{-1}Q^T b$

AND AS FOR P:

$P = A\hat{x} = QRR^{-1}Q^T b =$

$= QQ^T b =$

b

AND INDEED, EVERY b THAT'S ALREADY IN C(A) PROJECTS TO ITSELF

(3)

(2c) IF A IS A 5X2 MATRIX WHOSE COLUMNS ARE q_1 AND q_2

THEN WE CAN'T SIMPLY SOLVE

$$A\hat{x} = b$$

MULTIPLYING BY A^T GIVES

$$A^T A \hat{x} = A^T b$$

~~SINCE $A^T A$~~

IT'S OBVIOUS THAT $A^T A = I$ ($A^T A$ IS 2X2)

$$\hat{x} = A^T b = \begin{bmatrix} q_1^T b \\ q_2^T b \end{bmatrix}$$

$$p = A\hat{x} = \begin{bmatrix} q_1 & q_2 \end{bmatrix} \begin{bmatrix} q_1^T b \\ q_2^T b \end{bmatrix} = (q_1^T b) q_1 + (q_2^T b) q_2$$

3a) IF WE TAKE $n=5$ FOR DEMONSTRATION

$$\begin{matrix} 0 & -1 & -1 & -1 & -1 \\ -1 & 0 & -1 & -1 & -1 \\ -1 & -1 & 0 & -1 & -1 \\ -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & -1 & 0 \end{matrix}$$

SUM ALL BUT LAST AND ADD TO LAST:

$$\begin{matrix} 0 & -1 & -1 & -1 & -1 \\ -1 & 0 & -1 & -1 & -1 \\ -1 & -1 & 0 & -1 & -1 \\ -1 & -1 & -1 & 0 & -1 \\ -4 & -4 & -4 & -4 & -4 \end{matrix}$$

FACTOR OUT (-1) (4)

$$\begin{matrix} 0 & -1 & -1 & -1 & -1 \\ -1 & 0 & -1 & -1 & -1 \\ -1 & -1 & 0 & -1 & -1 \\ -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & -1 & -1 \end{matrix}$$

SUM ALL BUT LAST, DIVIDE BY 3 AND SUBTRACT FROM LAST

$$\begin{matrix} 0 & -1 & -1 & -1 & -1 \\ -1 & 0 & -1 & -1 & -1 \\ -1 & -1 & 0 & -1 & -1 \\ -1 & -1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & \frac{1}{3} \end{matrix}$$

So

$$\det(A_n) = (n-1) \cdot \frac{1}{n-2} \cdot \det(A_{n-1})$$

STARTING WITH A_3:

$$\det(A_3) = 2 \cdot \frac{1}{1} \cdot \det(A_2) = 2 \cdot \frac{1}{1} \cdot -1 = 2$$

$$\det(A_4) = 3 \cdot \frac{1}{2} \cdot 2 \cdot \frac{1}{1} \cdot (-1)$$

$$\det(A_5) = 4 \cdot \frac{1}{3} \cdot 3 \cdot \frac{1}{2} \cdot 2 \cdot \frac{1}{1} \cdot (-1)$$

$$\det(A_n) = (n-1) \cdot -1 = 1-n$$

(5)

3(b) ACCORDING TO ~~CRAMER~~

~~$x = A^{-1}b$~~ $$X_1 = \frac{\det B_1}{\det A}$$

WHERE B_1 IS A BUT WITH FIRST COLUMN REPLACED BY FIRST COLUMN IN B

HOWEVER IN THE CASE OF $B = I$ (WHEN TRYING TO FIND THE INVERSE) WE GET

$$X_1 = \frac{\det\begin{pmatrix} 1 & x\,x\,x \\ 0 & \\ \vdots & \det A' \\ 0 & \end{pmatrix}}{\det A}$$

WERE A' CONTAINS COLUMNS AND ROWS OF A STARTING WITH INDEX 2.

SO IN OUR CASE WE KNOW THAT $\det(A_4) = -3$

AND WHATS LEFT OF A_4 IS $A' = \begin{bmatrix} 1 & x\,x\,x \\ 0 & \\ 0 & A_3 \\ 0 & \end{bmatrix}$

$\det(A_3) = -2$

SO THE FIRST ELEMENT (1,1) OF $A_4^{-1} =$

$$= \frac{\det(A_3)}{\det(A_4)} = \frac{-2}{-3} = \frac{2}{3}$$