Problem Set 10

MIT CW Linear Algebra (18.06)

Aviel Livay

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Former Section 6.6, question 12

$$JM = \begin{bmatrix} m_{2,1} & m_{2,2}, & m_{2,3} & m_{2,4} \\ 0 & 0, & 0 & 0 \\ m_{4,1} & m_{4,2}, & m_{4,3} & m_{4,4} \\ 0 & 0, & 0 & 0 \end{bmatrix}$$

$$MK = \begin{bmatrix} 0 & m_{1,1}, & m_{1,2} & 0 \\ 0 & m_{2,1}, & m_{2,2} & 0 \\ 0 & m_{3,1}, & m_{3,2} & 0 \\ 0 & m_{4,1}, & m_{4,2} & 0 \end{bmatrix}$$

In order for the matrices to be the same we need:

1. $m_{2,1} = 0$
2. $m_{2,2} = m_{1,1} = a,$
3. $m_{2,3} = m_{1,2} = b,$
4. $m_{2,4} = 0,$
5. $m_{4,1} = 0,$
6. $m_{4,2} = m_{3,1} = c,$
7. $m_{4,3} = m_{3,2} = d$
8. $m_{4,4} = 0$

So we end up with

$$M = \begin{bmatrix} a & b & m_{1,3} & m_{1,4} \\ 0 & a & b & 0 \\ c & d & m_{3,3} & m_{3,4} \\ 0 & c & d & 0 \end{bmatrix}$$

Former Section 6.6, question 14

We know that

$$A = X^{-1}\Lambda X$$

Where X contains the eigenvectors of A So we can write it as

$$\Lambda = XAX^{-1}$$

If we multiply both sides by Y^{-1} and Y where Y is the matrix containing the eigenvectors of A then we get

$$\begin{aligned} Y^{-1}\Lambda Y &= Y^{-1}XAX^{-1}Y \\ A^T &= (Y^{-1})XA(X^{-1}Y) \\ A^T &= (X^{-1}Y)^{-1}A(X^{-1}Y) \end{aligned}$$

Former Section 6.6, question 20

a If A is similar to B then there exist M such that

$$A = M^{-1}BM$$

Now if we square both sides then we get -

$$\begin{aligned} A^2 &= M^{-1}BMM^{-1}BM \\ A^2 &= M^{-1}B^2M \end{aligned}$$

b Suppose A is similar to $-B$. So there's M such that $M^{-1}AM = -B$. In that case A cannot be similar to B because their eigenvalues are not the same, they are negative to each other. So if we square both sides, still we can get that A^2 is similar to B^2

c There's a matrix M_A such that $M_A^{-1}AM_A = \Lambda$ and then there's a matrix M_B such that $B = M_B\Lambda {M_B}^{-1}$ so $B = M_BM_A^{-1}AM_A{M_B}^{-1}$ And we get that $B = (M_A{M_B}^{-1})^{-1}A(M_A{M_B}^{-1})$

Section 7.3, question 11 (Former Section 6.7, question 4)

$$A^TA = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

The