1

1(a) $H^2 = (I - 2uu^T)^2 = I^2 - 4Iuu^T + 4uu^Tuu^T =$

$= I - 4Iuu^T + 4uu^T =$

I

WE KNOW THAT H IS NOT ONLY SYMETRIC BUT ALSO ORTHOGONAL

1(b) $(I - 2uu^T)u = \lambda u$

$u - 2uu^Tu = \lambda u$

$u - 2u = \lambda u$

$-u = \lambda u$

$\lambda = -1$

(c) $(I - 2uu^T)v =$

$v - 2uu^Tv =$

v

$\lambda = 1$

REPEATED N-1 TIMES

H IS DIAGONISABLE BECAUSE SYMETRIC

2

$H_{11} = 1-2u_1^2$

$H_{ii} = 1-2u_i^2$

$H_{11} + \dots + H_{nn} = \cancel{u_1^2 + \dots u_n^2 = \|u\|^2 = 1}$

$\cancel{1-2u_1}$

$= 1-2u_1^2 + \dots 1-2u_n^2 =$

$n - 2\cdot(u_1^2 + \dots u_n^2) =$

$n-2$

$\lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_n =$

$-1 + 1 + 1 + \dots + 1 =$

$n-2$

~~SUM OF~~ TRACE EQUALS SUM OF LAMBDAS.

2a) THE λs OF A^{-1} ARE $\frac{1}{\lambda}$.

SINCE $\lambda > 0$ THEN ALSO $1/\lambda > 0$

2b) QAQ^T IS POSITIVE IF AND ONLY IF A IS POSITIVE DEFINITE. ~~BECAUSE~~ THEY'RE SIMILAR. SINCE A IS POSITIVE DEFINITE, SO IS. SO WE CAN CONDUCT THE TEST ON EITHER A OR QAQ^T, DOESN'T MATTER.

THE EASIEST AND LESS COMPUTATIONAL INTENSIVE WOULD BE TO CHECK THAT ALL THE ~~EIGEN VECT~~ PIVOTS ARE POSITIVE.

QAQ^T SHARE THE SAME EIGEN VALUES AS A. SINCE A'S EIGENVALUES ARE POSITIVE, THEN ALSO ARE THE EIGEN VECTORS OF QAQ^T.

THE BEST TEST IS TO CHECK THAT ALL PIVOTS OF EITHER A OR QAQ^T ARE POSITIVE.

2c) WE SHALL CHECK THAT ENERGY ≥ 0

$$\begin{bmatrix} x_1^T, x_2^T \end{bmatrix} \begin{bmatrix} A & A \\ A & A \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$$

$$\begin{bmatrix} x_1^T, x_2^T \end{bmatrix} \begin{bmatrix} Ax_1 + Ax_2 \\ Ax_1 + Ax_2 \end{bmatrix} \overset{?}{\geq} 0$$

$$\begin{bmatrix} x_1 + x_2 \end{bmatrix}^T A \begin{bmatrix} x_1 + x_2 \end{bmatrix} \overset{?}{\geq} 0$$

FOR $x_1 + x_2 \neq 0$

INDEED $\begin{bmatrix} x_1 + x_2 \end{bmatrix}^T A \begin{bmatrix} x_1 + x_2 \end{bmatrix} > 0$

HOWEVER FOR THE CASE WHERE

$x_2 = -x_1$

WE GET 0

AND THAT'S HOW WE CAN DEMOSTRATE THAT B IS NOT POSITIVE DEFINITE.

~~Hence~~

ALSO SINCE WE HAVE ~~2~~ IDENTICAL ROWS IN B THEN WE KNOW B IS SINGULAR WHICH MEANS IT HAS $\lambda = 0$ $\Rightarrow$ SEMIDEFINITE.

5

3a) $\lambda_1 + \lambda_2 = 0$

$\lambda_1 \cdot \lambda_2 = \det(A) = 4$

$\lambda_1 = 2i$

$\lambda_2 = -2i$

$$X_1 = \begin{bmatrix} 1 \\ -2i \end{bmatrix} \quad X_2 = \begin{bmatrix} 1 \\ 2i \end{bmatrix}$$

$$U[0] = \begin{bmatrix} 2 \\ 0 \end{bmatrix} = X_1 + X_2 = \begin{bmatrix} 1 \\ -2i \end{bmatrix}\begin{bmatrix} 1 \\ 2i \end{bmatrix}$$

$$U(t) = e^{2it}\begin{bmatrix} 1 \\ -2i \end{bmatrix} + e^{-2it}\begin{bmatrix} 1 \\ 2i \end{bmatrix}$$

3b) STAGE 1

$$\begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 16 \end{bmatrix}$$

$\lambda_1 = 1 \quad \lambda_2 = 16$

$$\cancel{X} V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

STEP 2:

$$V_1 = \frac{1}{\sqrt{1}} \begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$u_2 = \frac{1}{\sqrt{16}} \begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{4} \\ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & -\frac{1}{4} \\ 4 & 0 \end{bmatrix}$$

AND INDEED

$$\begin{pmatrix} 0 & -1 \\ 4 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{4} \\ 4 & 0 \end{pmatrix} \begin{pmatrix} 1 & \\ & 4 \end{pmatrix} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$