$$\begin{pmatrix}1 & a & b\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}X\end{pmatrix}$$

$2+a=0$

$a=-2$

$5+b=0$

$b=-5$

(a) $R=\begin{pmatrix}1 & -2 & -5\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}4\\0\\0\end{pmatrix}=\begin{pmatrix}4\\0\\0\end{pmatrix}$

$d=\begin{pmatrix}4\\0\\0\end{pmatrix}$

(b) $\Rightarrow \left[\begin{array}{ccc|c}1 & -2 & -5 & 4\\ 0 & 0 & 0 & 0\\ 5 & -10 & -25 & 20\end{array}\right]$

$\Rightarrow \left[\begin{array}{ccc|c}1 & -2 & -5 & 4\\ 3 & -6 & -15 & 12\\ 5 & -10 & -25 & 20\end{array}\right]$

-1320

-360

-600

Null:

$$\begin{bmatrix}4+2c_1+5c_2\\ c_1\\ c_2\end{bmatrix}$$

$4+2c_1+5c_2-2c_1-5c_2=4$

$12+6c_1+15c_2-6c_1-15c_2=12$

$20+10c_1+25c_2-10c_1-25c_2=20$

YES BUT WHERE IS THE MATRIX ?

2) (a) $\begin{bmatrix} 0 & 1 & 2 & 2 \\ 0 & 3 & 8 & 7 \\ 0 & 0 & 4 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 4 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & -2 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

WRONG

$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 2 \\ 1 \\ -2 \end{bmatrix}$

A PLANE IN $\mathbb{R}^4$ THAT IS SPAN BY BOTH VECTORS

(b) THE COLUMN SPACE IS A PLANE IN $\mathbb{R}^3$ THAT IS SPAN BY THE VECTORS IN THE PIVOT COLUMS: $\begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 7 \\ 2 \end{pmatrix}$

$X + ay + bz = 0$

$(1, a, b) \cdot (1, 3, 0) = 0$

$(1, a, b)(2, 7, 2) = 0$

$1 + 3a = 0$

$a = -\frac{1}{3}$

$2 - \frac{7}{3} + 2b = 0$

$6 - 7 + 6b = 0$

$b = -\frac{1}{6}$

$X - \frac{1}{3}y - \frac{1}{6}z = 0$

$6X - 2y + z = 0$

$$\begin{matrix} 0 & 1 & -2 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \qquad \begin{matrix} 0 & 1 & -2 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$$

(3) (a) ALWAY HAS MANY SOLUTIONS

(b) THE COLUMN SPACE OF A IS R^3

THE NULL SPACE OF A IS $\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

(4) $\left[\begin{array}{cc|c} 2 & 1 & b_1 \\ 6 & 5 & b_2 \\ 2 & 4 & b_3 \end{array}\right]$

$\Downarrow$

$$\begin{matrix} 2 & 1 & b_1 \\ 0 & 2 & b_2 - 3b_1 \\ 0 & 3 & b_3 - b_1 \end{matrix}$$

$\Downarrow$

$$\left.\begin{matrix} 1 & \frac{1}{2} \\ 0 & 1 \\ 0 & 0 \end{matrix}\,\right|\, \begin{matrix} \frac{b_1}{2} \\ \frac{b_2 - 3b_1}{2} \\ b_3 - b_1 - \frac{3b_2}{2} + \frac{9b_1}{2} \end{matrix}$$

$\Downarrow$

$$\left.\begin{matrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix}\,\right|\, \begin{matrix} \frac{b_1}{2} - \frac{b_2}{4} + \frac{3b_1}{4} \\ -\frac{3b_1}{2} + \frac{b_2}{2} \\ \frac{7}{2}b_1 - \frac{3}{2}b_2 + b_3 \end{matrix}$$

ALL SOLUTION MUST MEET

$7b_1 - 3b_2 + 2b_3 = 0$

~~AS SOLUTION~~

IN ORDER FOR b TO BE A SOLUTION IT HAS TO BE IN THE COLUMN SPACE OF A WHICH REQUIRES

$7b_1 - 3b_2 + 2b_3 = \ldots$

(4b) $7b_1 - 3b_2 + 2b_3 =$

$56 - 84 + 28 =$

0

YES, $\begin{bmatrix} 8 \\ 28 \\ 14 \end{bmatrix}$ IS IN THE COLUMN SPACE OF A