Homework 3 - CSE 527 - Introduction to Computer Vision

Abhinandan Dubey — SBU ID : 110872892

1 Projective transformations of the plane

I have tried to prove it using two approaches.

Approach 1:
If there exists a matrix H such that $q \equiv Hp$, then

$$\begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = H \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$$

If

$$H = \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}$$

Thus, in homogeneous coordinates,

$$u'_2 = \frac{Au_1 + Bv_1 + C}{Gu_1 + Hv_1 + I}$$

similarly,

$$v'_2 = \frac{Du_1 + Ev_1 + F}{Gu_1 + Hv_1 + I}$$

Rearranging the above equations we obtain,

$$X.F^T = 0 \quad \text{and} \quad Y.F^T = 0$$

Such that

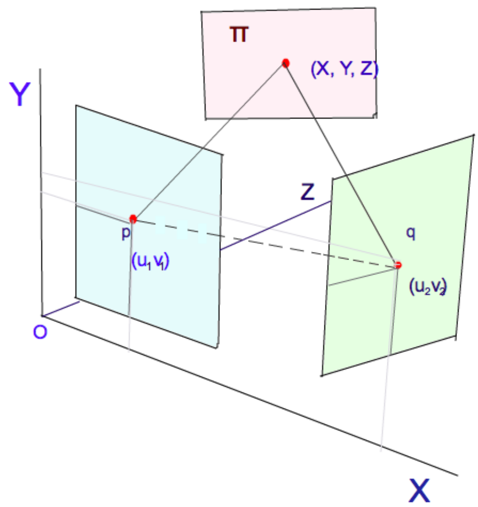
$$F = \begin{bmatrix} A & B & C & D & E & F & G & H & I \end{bmatrix}$$

$$X = \begin{bmatrix} -u_1 & -v_1 & -1 & 0 & 0 & 0 & u_1u'_2 & v_1u'_2 & u'_2 \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} 0 & 0 & 0 & -u_1 & -v_1 & -1 & u_1v'_2 & v_1v'_2 & v'_2 \end{bmatrix}$$

Thus, given a set of corresponding points, we can form a system of linear equations. This would lead us to three equations in 8 unknowns, which will always have at least one solution. Thus, we can always find an H such that

$$q \equiv Hp$$

Approach 2:



This approach is less generalized. We have two cameras looking at a point (X, Y, Z) on a plane π such that p (u_1, v_1) and q (u_2, v_2) are the two projections of the point (X, Y, Z) on the two image planes.

Now, let us assume camera 1 is located at world origin $(0,0,0)$ and camera 2 is located at (t_x, t_y, t_z) in world coordinates. Now, for camera 1,

$$\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = M_1 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

where M_1 is the projection matrix for camera 1. Similarly for camera 2,

$$\begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = M_2 \begin{bmatrix} 1 & 0 & 0 & -t_x \\ 0 & 1 & 0 & -t_y \\ 0 & 0 & 1 & -t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

where M_1 and M_2 are projection matrices for camera 1 and 2 respectively. Now from the above equations we obtain,

$$u_1 = f_1\left(\frac{X}{Z}\right), v_1 = f_1\left(\frac{Y}{Z}\right); u_2 = f_2\left(\frac{X - t_x}{Z - t_z}\right), v_2 = f_2\left(\frac{Y - t_y}{Z - t_z}\right)$$

From the last result, we can derive a matrix H such that

$$q \equiv Hp$$

Here,

$$H = \begin{bmatrix} \frac{X-t_x}{X} & 0 & 0 \\ 0 & \frac{Y-t_y}{Y} & 0 \\ 0 & 0 & \frac{f_1(Z-t_z)}{f_2(Z)} \end{bmatrix}$$

2 Estimating transformations from the image points

The algorithm starts by checking the number of arguments provided, if it is not two, then it initializes it with a preselected set of points. Then the algorithm computers the number of points provided, and other variables as described below and explained in Approach 1 of Section 1.
After that, the algorithm computes the matrix A, which is the matrix on which we will perform Singular Value Decomposition. Finally the algorithm returns the reshaped matrix U obtained from singular value decomposition.

Data: Corresponding sets of points in two images, $t1$ and $t2$
Result: Homography Matrix H
Initialization;
if *Number of Arguments != 2* **then**
  $t1 \leftarrow$ [Preselcted 2XN Matrix] ;
  $t2 \leftarrow$ [Preselcted Corresponding 2XN Matrix] ;
end
n := SIZE(t1,2);
u := t2(1,:);
v := t2(2,:);
x := t1(1,:);
y := t1(2,:);
A := A = [[-[x; y; ones(1,n)]; zeros(3, n); u.*x; u.*y; u] [zeros(3, n); -[x; y; ones(1,n)]; v.*x; v.*y; v]];
$[U, ,] \leftarrow COMPUTE - SVD(A)$;
Reshape A into a $3X3$ Matrix ;
return(A)

Algorithm 1: Computing the Homography Matrix

3 Image warping and mosaicing

Please check the file `warpImage.m`, to simulate the whole program, you may run `driver.m`

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