### **Benchmark functions and convergence analysis**

By selecting representative unimodal functions, multimodal functions, hybrid functions and partial combination functions in the CEC2021 and CEC2022 function sets. The HGAO model is compared to other state of the art models with sets of functions specifically designed to evaluate and compare the performance of different intelligent optimization algorithms. CEC2021 and CEC2022 reference functions are described in Table 1 and Table 2 respectively. Unimodal functions are usually used to test and compare the performance of optimization algorithms, especially those algorithms that need to find the global optimal solution and its convergence rate when searching for the optimal solution, and are used to verify the utilization ability of the algorithm. The multimodal function allows us to evaluate how well the algorithm performs in the face of problems with multiple local optima and whether they can find a global optimum, which is mainly used to validate the exploratory nature of the algorithm. Hybrid functions that combine features of unimodal and multimodal functions are commonly used to test and compare the performance of optimization algorithms in the face of problems with different characteristics. Composite functions are usually composed of several basic functions and are used to test and compare the robustness and tractability of optimization algorithms for complex problems.

**Table 1**. Summary of the CEC2021 Test Functions.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Type** | **No.** | **Description** | **Range** | **Dimension** | **Fmin** |
| Unimodal Function | F1 | Shifted and Rotated Bent Cigar Function | [-100,100] | 20 | 100 |
| Basic Functions | F2 | Shifted and Rotated Schwefel’s Function | [-100,100] | 20 | 1100 |
| F3 | Shifted and Rotated Lunacek Bi-Rastrigin Function | [-100,100] | 20 | 700 |
| F4 | Expanded Rosenbrock’s plus Girewangk’s Function | [-100,100] | 20 | 1900 |
| Hybrid Functions | F5 | Hybrid Function 1 (N=3) | [-100,100] | 20 | 1700 |
| F6 | Hybrid Function 1 (N=4) | [-100,100] | 20 | 1600 |
| F7 | Hybrid Function 1 (N=5) | [-100,100] | 20 | 2100 |
| Composition  Functions | F8 | Composition Function 1 (N=3) | [-100,100] | 20 | 2200 |
| F9 | Composition Function 1 (N=4) | [-100,100] | 20 | 2400 |
| F10 | Composition Function 1 (N=5) | [-100,100] | 20 | 2500 |

**Table 2**. Summary of the CEC2022 Test Functions.

| **Type** | **No.** | **Description** | **Range** | **Dimension** | **Fmin** |
| --- | --- | --- | --- | --- | --- |
| Unimodal Function | F1 | Shifted and full Rotated Zakharov Function | [-100,100] | 20 | 300 |
| Basic Functions | F2 | Shifted and full Rotated Rosenbrock’s Function | [-100,100] | 20 | 400 |
| F3 | Shifted and Rotated Expanded Schaffer’s f6 Function | [-100,100] | 20 | 600 |
| F4 | Shifted and Rotated Non-Continuous Rastrigin’s Function | [-100,100] | 20 | 800 |
| F5 | Shifted and Rotated Levy Function | [-100,100] | 20 | 900 |
| Hybrid Functions | F6 | Hybrid Function 1 (N=3) | [-100,100] | 20 | 1800 |
| F7 | Hybrid Function 2 (N=6) | [-100,100] | 20 | 2000 |
| F8 | Hybrid Function 3 (N=5) | [-100,100] | 20 | 2200 |
| Composition  Functions | F9 | Composition Function 1 (N=5) | [-100,100] | 20 | 2300 |
| F10 | Composition Function 2 (N=4) | [-100,100] | 20 | 2400 |
| F11 | Composition Function 3 (N=5) | [-100,100] | 20 | 2600 |
| F12 | Composition Function 4 (N=6) | [-100,100] | 20 | 2700 |

For each function, we measure its performance using the following metrics:

**1) Average value (*Ave*):** It represents the average of the solutions obtained by running the algorithm M times, which is computed according to;



where  is the solution obtained by the *i* th time.

**2) Statistical standard deviation (*Std*):** It is used to measure the dispersion of the obtained solutions and is used to illustrate the stability and performance of the algorithm, which is calculated as follows;



**3)** **Best value (*Best*):** It is the best value selected from the *M* solutions, which is calculated as follows;



**4) Rank:** These algorithms are ranked according to their Ave and Std. The smaller the Ave and Std, the smaller the ranking value.

**Table 3.** The Detailed Comparison of Various Algorithms on Dimensions 20 of CEC2021 Benchmark Functions.

| **Function** | | **Algorithms** | | | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **HGAO** | **PSO** | **BWO** | **WOA** | **TSO** | **GO** | **CFOA** | **ESOA** | **HLOA** | **GAO** |
| F1 | Best | **13852.78916** | 26339.8558 | 48969.4217 | 29366.04935 | 20560.4652 | 20803.15615 | 18268.149 | 75585.343 | 29111.4831 | 35644.2342 |
| Ave | **36697.61981** | 126432.4677 | 99155.3166 | 56286.11373 | 47474.26649 | 44427.67204 | 41909.14294 | 120637.0966 | 65766.84485 | 137119.4442 |
| Std | 45096.61742 | 125702.9699 | 112987.827 | 45507.05803 | **37675.75976** | 40372.30146 | 65824.48069 | 95768.78683 | 61019.74193 | 136748.398 |
| F2 | Best | **1282.2416** | 1895.6411 | 2829.4718 | 4506.9545 | 3248.7501 | 5064.3346 | 2344.4156 | 4789.0643 | 1712.5587 | 2686.3723 |
| Ave | **5010.0026** | 6372.4179 | 6055.8603 | 9949.495 | 10277.0686 | 15744.5016 | 5589.0697 | 9257.0981 | 16930.8445 | 6801.176 |
| Std | 8647.4228 | 5853.0683 | 7363.8727 | 8471.3051 | 13147.7087 | 17328.326 | 7895.3816 | 7934.5555 | 27796.5804 | **5425.5644** |
| F3 | Best | **4411.11626** | 22697.98457 | 5851.77509 | 20694.74733 | 24206.46871 | 15449.96913 | 8962.854891 | 14470.0823 | 6559.41345 | 5487.609947 |
| Ave | 17632.3317 | 38185.75744 | **15760.14793** | 52050.15953 | 52933.52645 | 47554.93596 | 18914.16736 | 24991.19818 | 22233.18199 | 23633.11353 |
| Std | 26157.32616 | 31844.75541 | 19598.29442 | 46942.06278 | 38931.69135 | 58477.51381 | 21510.38185 | 24574.9318 | **18035.89869** | 23402.35205 |
| F4 | Best | 1927.9186 | 2257.1202 | 6174.7883 | 6752.5652 | 7162.7826 | 3115.6258 | 2503.9543 | 6211.9353 | **1909.07494** | 3663.5585 |
| Ave | **119221.408** | 392100.8962 | 522396.631 | 225808.6083 | 453399.887 | 571317.6023 | 225538.2663 | 43879.6793 | 177180.8736 | 962775.9563 |
| Std | **103723.188** | 1441945.045 | 2995433.4 | 932913.6974 | 1307655.157 | 2181015.208 | 1330755.331 | 138700.6988 | 143905.3599 | 3165101.233 |
| F5 | Best | 1767.774 | 2603.3752 | 5885.95 | 2311.1568 | 3937.2774 | 5836.9867 | **1733.9429** | 9320.4148 | 2202.1771 | 3005.6515 |
| Ave | 9662.9181 | 35242.9697 | 12544.2852 | 19830.594 | 45175.1254 | 21515.0421 | **4344.5421** | 18816.1559 | 22459.5198 | 6867.2651 |
| Std | 32614.251 | 80223.1393 | 14413.3417 | 45718.186 | 154905.016 | 27125.8429 | **9628.0325** | 14133.4382 | 60086.6674 | 11882.0912 |
| F6 | Best | **7999.34167** | 66171.713 | 186163.251 | 110952.44 | 655087.322 | 283180.243 | 35355.331 | 194689.547 | 8522.9736 | 208706.018 |
| Ave | **1266470.016** | 67691710.65 | 3199097.271 | 10731320.03 | 55174876.35 | 37973001.23 | 18392914.83 | 8242130.229 | 38040609.11 | 3943907.162 |
| Std | 18427789.18 | 512776802.8 | 33964563.08 | 32769581.68 | 196516279.3 | 247704943.9 | 125247818.3 | 42815973.49 | 232619377.2 | **16634103.58** |
| F7 | Best | 2250.034 | 3221.002 | 5414.4313 | 4064.4621 | 2605.6924 | 4464.7093 | 2329.3602 | **2138.1574** | 2816.2139 | 2764.0987 |
| Ave | 17585.2372 | 79482.4109 | 17606.3499 | 63719.1413 | 44262.1497 | 58958.6555 | **4181.3823** | 18158.2132 | 59924.3661 | 45709.8569 |
| Std | 70866.6573 | 212784.7771 | 130933.2912 | 152487.9964 | 97310.9396 | 200456.6337 | **4451.3602** | 99496.1369 | 270043.2557 | 123973.1479 |
| F8 | Best | **2207.6708** | 2243.8836 | 2362.4068 | 2284.7949 | 2253.8382 | 2249.6248 | 2258.4344 | 2266.3183 | 2229.3719 | 2250.6077 |
| Ave | **2251.4849** | 2315.8305 | 2458.5094 | 2350.0679 | 2353.5391 | 2313.6909 | 2319.8822 | 2343.2949 | 2278.7515 | 2310.2507 |
| Std | 118.8789 | 144.6674 | 266.341 | 140.0124 | 229.1694 | **87.7824** | 208.379 | 253.2423 | 156.0297 | 138.8874 |
| F9 | Best | **2640.9249** | 3799.1011 | 3876.9853 | 4423.0846 | 3457.6276 | 4380.8366 | 3413.5276 | 4021.1284 | 2956.4572 | 3439.8909 |
| Ave | **4426.462** | 6117.088 | 4861.2945 | 6769.7421 | 6720.0529 | 6339.9553 | 4587.4056 | 5413.8603 | 5253.5832 | 5729.9579 |
| Std | 3286.1601 | 3640.5517 | 2422.0432 | 3278.0191 | 5145.1113 | 2702.403 | **2118.3725** | 2982.9524 | 3925.3042 | 3334.9736 |
| F10 | Best | **3198.1625** | 4313.4059 | 3774.9994 | 4023.243 | 4931.641 | 4795.2618 | 4277.1888 | 3628.8144 | 3311.114 | 3859.2302 |
| Ave | **3516.8352** | 5504.7417 | 5262.9689 | 5445.0945 | 5663.5656 | 6682.4878 | 5199.3145 | 4365.7766 | 3936.6216 | 5971.1633 |
| Std | 1658.8191 | 1643.5448 | 3503.5934 | 2942.0888 | 991.9852 | 3637.2369 | 2383.497 | 2551.4283 | **1464.7502** | 4594.4328 |
| **Rank** |  | 1 | 6 | 5 | 6 | 5 | 5 | 2 | 5 | 3 | 4 |

**Table 4.** The Detailed Comparison of Various Algorithms on Dimensions 20 of CEC2022 Benchmark Functions.

| **Function** | | **Algorithms** | | | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **HGAO** | **PSO** | **BWO** | **WOA** | **TSO** | **GO** | **CFOA** | **ESOA** | **HLOA** | **GAO** |
| F1 | Best | **369.0318** | 500.0366 | 617.9226 | 769.3917 | 573.445 | 956.202 | 825.9869 | 491.3596 | 688.6789 | 520.1659 |
| Ave | **797.0857** | 886.529 | 846.9743 | 1071.7639 | 1667.8412 | 1725.8279 | 1139.7479 | 954.4837 | 1366.9273 | 1098.7961 |
| Std | 607.6057 | 443.4546 | 486.9884 | **315.2132** | 647.334 | 833.6243 | 696.1984 | 1019.5523 | 741.3974 | 541.3 |
| F2 | Best | **542.2326** | 910.8606 | 851.1526 | 815.8822 | 615.0549 | 688.485 | 891.5685 | 790.737 | 548.1169 | 647.0487 |
| Ave | **1019.2644** | 1546.8417 | 1223.8249 | 1482.5079 | 1435.1446 | 1321.1653 | 1118.4115 | 1072.6903 | 1089.1828 | 1184.3503 |
| Std | 2810.6119 | 1614.8347 | 1227.0122 | 1079.3636 | 2080.497 | 1094.763 | **814.6801** | 806.9396 | 4102.3534 | 1143.6701 |
| F3 | Best | **600.0029** | 600.2033 | 600.2759 | 600.3119 | 600.3801 | 600.2988 | 600.1663 | 600.1068 | 600.0071 | 600.4028 |
| Ave | **600.1957** | 600.7495 | 600.5156 | 600.7769 | 600.8302 | 600.8649 | 600.4945 | 600.304 | 600.2128 | 601.1167 |
| Std | 0.5599 | 0.6503 | 0.5107 | 0.6673 | **0.323** | 0.6967 | 0.5958 | 0.429 | 0.5633 | 0.5159 |
| F4 | Best | 800.683 | 802.9111 | 802.3881 | 801.4101 | 802.1792 | 801.8132 | **800.5049** | 800.5707 | 800.7476 | 802.1279 |
| Ave | 801.2486 | 803.0993 | 802.9762 | 801.9095 | 802.669 | 802.5263 | **800.924** | 801.1584 | 802.2163 | 802.8662 |
| Std | 0.7784 | 0.4709 | 0.787 | 0.6554 | 0.8529 | 0.7937 | 0.7376 | 0.9873 | 1.3869 | **0.4027** |
| F5 | Best | **901.3821** | 904.5788 | 905.9129 | 904.5707 | 908.4421 | 902.8804 | 904.4693 | 903.8558 | 900.4595 | 903.1322 |
| Ave | **902.6246** | 907.1998 | 906.82 | 909.8673 | 911.2512 | 905.2756 | 906.8116 | 905.4494 | 901.8452 | 906.7853 |
| Std | **2.3961** | 4.3551 | 2.7342 | 6.7916 | 5.8564 | 6.1825 | 6.3232 | 3.9704 | 2.9704 | 5.7163 |
| F6 | Best | **1847.2081** | 3015.3346 | 2841.7598 | 2887.8353 | 3168.407 | 2295.4532 | 2314.8695 | 2585.8242 | 2576.5709 | 3294.8975 |
| Ave | 46126.1496 | 30308.8839 | 87518.3663 | 30849.1959 | **18281.1387** | 58101.7565 | 356706.7475 | 143821.0199 | 43537.3714 | 87405.8717 |
| Std | 387681.8115 | **55004.8577** | 393282.5643 | 109040.1767 | 80175.2687 | 143967.5573 | 1664566.271 | 767043.9582 | 220959.1537 | 288853.4727 |
| F7 | Best | 2185.5481 | 2606.0216 | 3176.951451 | 2316.5153 | 2185.9424 | 2596.2323 | 2385.288 | 2909.3861 | 2189.1243 | **2057.5161** |
| Ave | **2535.5928** | 3118.7842 | 3912.8281 | 3520.9915 | 2619.6284 | 3080.6132 | 2618.3507 | 3110.8591 | 2794.8437 | 3086.2368 |
| Std | 1071.6656 | 1419.4497 | 1697.6808 | 1767.8507 | 1434.25 | 870.4415 | 792.2933 | **753.4285** | 1063.4874 | 2594.6448 |
| F8 | Best | 2614.286 | 8020.7125 | 6259.0106 | 8849.3868 | 10223.3608 | 9598.0148 | 6725.9777 | **2040.6869** | 4135.4037 | 8046.2587 |
| Ave | 1.85342E+12 | 53203769960 | 1.19099E+11 | 5.81385E+11 | 6190405249 | 1.44968E+13 | 4.30256E+11 | **6412379743** | 2.4885E+12 | 2.06296E+12 |
| Std | 2.58018E+13 | 3.84935E+11 | 1.68381E+12 | 4.04506E+12 | 87537853778 | 6.53404E+13 | 6.04672E+12 | **7991344589** | 2.73668E+13 | 2.84525E+13 |
| F9 | Best | **2677.0831** | 2930.0587 | 3068.4821 | 2954.6123 | 3168.55 | 3012.1634 | 3225.277 | 3562.8452 | 2710.8956 | 3003.9643 |
| Ave | 2979.8276 | 3699.6435 | 3581.0679 | 3489.3485 | 3505.2054 | 3514.2433 | 3936.4604 | 3893.4096 | **2977.6482** | 3625.069 |
| Std | 833.7877 | 1387.3854 | 1285.6887 | 1310.7595 | **692.9948** | 1114.7127 | 1431.6391 | 841.952 | 792.9494 | 1178.0733 |
| F10 | Best | **2521.3534** | 2852.5839 | 2737.8428 | 2668.8573 | 2847.3183 | 2832.8642 | 2985.6435 | 2694.1382 | 3083.3855 | 2576.2201 |
| Ave | **2795.4636** | 2995.024 | 2883.5283 | 3749.5014 | 3526.1141 | 3678.3678 | 3088.5099 | 2886.9787 | 3495.981 | 2852.0715 |
| Std | 861.9277 | 604.5074 | 470.3284 | 1712.2703 | 1485.5473 | 1654.8461 | **448.2186** | 529.9895 | 1345.5544 | 948.1432 |
| F11 |  | 2613.9512 | 2830.2465 | 3452.8051 | 2736.133 | 2813.2215 | 2705.9042 | 2830.4716 | 2634.0217 | **2609.1754** | 3066.0086 |
|  | **2930.7088** | 4066.7254 | 4036.0251 | 3253.5948 | 3231.2321 | 3418.7072 | 3265.2529 | 3090.4013 | 3137.3156 | 3506.556 |
|  | 1018.8042 | 2094.9264 | 1373.6488 | 1538.485 | 1283.3028 | 1704.3599 | 1209.2856 | 1481.4202 | 1709.1529 | **907.1321** |
| F12 |  | **2973.3274** | 3071.0697 | 3041.4966 | 3150.431 | 3096.925 | 3130.8294 | 3446.0244 | 3073.9976 | 3000.9503 | 3075.3691 |
|  | **3034.7718** | 3346.1911 | 3082.7864 | 3321.9718 | 3351.5337 | 3241.6303 | 3499.6458 | 3143.6883 | 3076.2928 | 3270.5949 |
|  | **110.9656** | 256.4213 | 184.7445 | 206.3759 | 319.5615 | 227.7702 | 152.5059 | 202.3844 | 140.5571 | 273.2897 |
| **Rank** |  | 1 | 5 | 6 | 5 | 3 | 6 | 2 | 2 | 4 | 3 |

The dimension is set to 20 for each benchmark function. Tables 3 and 4 show the execution results of CEC2021 and CEC2022, respectively. We came to the following conclusions:

1) In addition to the fact that the three indicators of F5 and F7 of CEC2021 and F8 and F4 of CEC2022 perform worse than other models, the HGAO model achieves the optimal performance in at least one indicator on the remaining functions.

2) For unimodal functions (CEC2021 F1; CEC2022 F1), HGAO model shows obvious advantages over other models, ranking first in the best value and average value of these two functions. This indicates that the HGAO model has excellent local search capability.

3) On the multimodal function, the HGAO model also performs better than the other models. The HGAO model found five optimal solutions obtained for the seven functions, except for F4 for CEC2021 and F4 for CEC2022. This shows that the model has certain advantages in global search ability.

4) For hybrid functions, the HGAO model only achieves the optimal solution on 3 out of the 6 functions, and the performance is not as good as expected. This result shows that the model is less able to capture and process complex features, which explains the slightly worse performance of the model in the first few epochs of training on the dataset.

5) We also conducted experiments on the combination functions, and the HGAO model all achieved the optimal solution. This indicates that the HGAO model is robust in dealing with complex problems.

Based on the CEC2021 and CEC2022 function sets, the convergence of different models is compared, and the results are shown in Figures. 1 and 2. After our analysis, we get the following conclusions.

1) The convergence speed of the HGAO model is faster, but the performance is slightly worse than that of the CFOA model on the F5 and F7 functions of CEC2021 and the F4 function of CEC2022. The F5 and F8 functions in CEC2022 underperform the HLOA and ESOA models, respectively.

2) On multi-modal functions, the HGAO model can converge more quickly and find the global optimal solution. In CEC2021, the F4 function achieves fast convergence within 25 iterations, and the F2 function converges within 100 iterations. In CEC2022, the F2 function also converges rapidly within 25 iterations, and the other multimodal functions also converge within 100 iterations. The results show that the HGAO model has strong exploration ability, and can effectively optimize the learning rate and Dropout rate of DenseNet121 to obtain better solutions.

3) The HGAO model in the hybrid function, except for CEC2021 F5 and F7, which converge in 150 iterations, all others converge rapidly within 100.

4) In the combination functions, the HGAO model can achieve fast convergence after 75 iterations, especially on the F8 and F10 functions in CEC2021, which complete convergence after 25 iterations respectively. This further shows that the HGAO model can find the globally optimal DenseNet121 learning rate and Dropout rate faster when dealing with complex problems, thus improving the accuracy of classification tasks.

**Figure 1.** Convergence curves for some typical functions with CEC2021.

 

**Figure 2.** Convergence curves for some typical functions with CEC2022.

   

To clarify the results of the algorithm comparison, this section counts the number of winners, losers, scores as well as ranks for each algorithm on different test functions. The calculation formula is below.

where denotes the score of the *i* th algorithm and denotes the best value of the ith algorithm in the *j* th problem. is the minimum of all algorithms in the *j* th problem, while is the maximum. is the number of wins of the *i* th algorithm in all problems.

CEC2021 and CEC2022 as shown in Table 5 and the results, HGAO ranks first among all algorithms, while HLOA and ESOA rank second, respectively. HGAO performs particularly well on CEC2021 and CEC2022, scoring 7.3797 and 7.0422 respectively, especially nearly 0.7 points higher than the second-ranked algorithm in CEC2021. The results for different functions of CEC2021 and 2022 are shown in Table 6, where HGAO is the winner for multimodal problems and sum combination functions, in particular, the combination function scores nearly 0.6 and 0.9 points higher than the second-ranked HLOA. For hybrid functions, HGAO performs slightly worse than ESOA, with 2.0386 and 2.2885, and 2.4712 and 2.7339, respectively. Table 5 and Table 6 data show that HGAO has the ability to quickly converge in simple problems while maintaining excellent generalization and robustness to complex problems.

**Table 5.** The overall rank in CEC2021(a) and CEC2022(b) test functions, the bold numbers means the best performance among whole competitors.

(a) The overall rank in CEC2021 test functions

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **CEC2021** | | | |
| **Winner** | **Loser** | **Score** | **Rank** |
| HGAO | **7** | 0 | **7.3797** | **1** |
| PSO | 0 | 0 | 6.6117 | 3 |
| BWO | 0 | 2 | 5.8842 | 9 |
| WOA | 0 | 1 | 5.9713 | 8 |
| TSO | 0 | 4 | 5.7503 | 10 |
| GO | 0 | 1 | 6.0785 | 6 |
| CFOA | 1 | 0 | 6.5325 | 4 |
| ESOA | 1 | 2 | 6.0161 | 7 |
| HLOA | 1 | 0 | 6.6457 | 2 |
| GAO | 0 | 0 | 6.509 | 5 |

(b) The overall rank in CEC2022 test functions

|  | **CEC2022** | | | |
| --- | --- | --- | --- | --- |
| **Winner** | **Loser** | **Score** | **Rank** |
| HGAO | **7** | 0 | **7.0422** | **1** |
| PSO | 0 | 2 | 3.4168 | 10 |
| BWO | 0 | 2 | 5.7116 | 8 |
| WOA | 0 | 2 | 5.646 | 9 |
| TSO | 0 | 2 | 5.8184 | 6 |
| GO | 0 | 1 | 5.7449 | 7 |
| CFOA | 1 | 0 | 6.5802 | 5 |
| ESOA | 1 | 1 | 6.9448 | 2 |
| HLOA | 2 | 0 | 6.8214 | 3 |
| GAO | 1 | 2 | 6.7886 | 4 |

**Table 6.** The overall rank in CEC2021(a) and CEC2022(b) test functions, the bold numbers means the best performance among whole competitors.

(a) The overall rank in CEC2021 test functions

|  | **Simple Multimodal** | | | | **Hybrid Functions** | | | | **Composition Functions** | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Winner** | **Loser** | **Score** | **Rank** | **Winner** | **Loser** | **Score** | **Rank** | **Winner** | **Loser** | **Score** | **Rank** |
| HGAO | **2** | 0 | **2.0036** | **1** | 1 | 0 | 2.0386 | 2 | **3** | 0 | **3** | **1** |
| PSO | 0 | 0 | 1.1522 | 6 | 0 | 0 | 0.535 | 10 | 0 | 0 | 1.5273 | 7 |
| BWO | 0 | 0 | 1.2938 | 4 | 0 | 1 | 1.8226 | 3 | 0 | 1 | 1.1263 | 8 |
| WOA | 0 | 0 | 1.5971 | 3 | 0 | 0 | 0.8231 | 7 | 0 | 1 | 1.0744 | 9 |
| TSO | 0 | 2 | 0.52 | 10 | 0 | 1 | 1.4331 | 5 | 0 | 1 | 1.0566 | 10 |
| GO | 0 | 1 | 0.7873 | 7 | 0 | 0 | 1.6762 | 4 | 0 | 0 | 2.1688 | 5 |
| CFOA | 0 | 0 | 0.624 | 9 | 1 | 0 | 1.1006 | 7 | 0 | 0 | 1.884 | 6 |
| ESOA | 0 | 0 | 1.2544 | 5 | 1 | 0 | **2.2885** | **1** | 0 | 0 | 2.4019 | 3 |
| HLOA | 1 | 0 | 1.8223 | 2 | 0 | 0 | 1.2695 | 6 | 0 | 0 | 2.4825 | 2 |
| GAO | 0 | 0 | 0.7596 | 8 | 0 | 0 | 0.6689 | 9 | 0 | 0 | 2.3072 | 4 |

(b) The overall rank in CEC2022 test functions

|  | **Simple Multimodal** | | | | **Hybrid Functions** | | | | **Composition Functions** | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Winner** | **Loser** | **Score** | **Rank** | **Winner** | **Loser** | **Score** | **Rank** | **Winner** | **Loser** | **Score** | **Rank** |
| HGAO | **2** | 0 | **3.0172** | **1** | 1 | 0 | 2.4712 | 2 | **3** | 0 | **3.0057** | **1** |
| PSO | 0 | 2 | 1.3868 | 10 | 0 | 0 | 1.7823 | 7 | 0 | 0 | 1.3438 | 7 |
| BWO | 0 | 0 | 2.2586 | 7 | 0 | 1 | 0.8774 | 10 | 0 | 1 | 1.4713 | 6 |
| WOA | 0 | 0 | 2.4063 | 4 | 0 | 0 | 1.7144 | 8 | 0 | 0 | 1.1009 | 10 |
| TSO | 0 | 1 | 1.9837 | 9 | 0 | 1 | 1.1884 | 9 | 0 | 0 | 1.6382 | 4 |
| GO | 0 | 0 | 2.1896 | 8 | 0 | 0 | 1.9563 | 6 | 0 | 0 | 1.3804 | 9 |
| CFOA | 1 | 0 | 2.9865 | 2 | 0 | 0 | 2.0274 | 5 | 0 | 1 | 1.7073 | 3 |
| ESOA | 0 | 0 | 2.3569 | 5 | 1 | 0 | **2.7339** | **1** | 0 | 1 | 1.5499 | 5 |
| HLOA | 1 | 0 | 2.8366 | 3 | 0 | 0 | 2.0277 | 4 | 1 | 1 | 2.0966 | 2 |
| GAO | 0 | 1 | 2.2937 | 6 | 1 | 1 | 2.2025 | 3 | 0 | 0 | 1.224 | 8 |