NC Final

Wajahat Ali
18k-0363
Section D

Q1. a) $P(x) = x^3 - 3x^2 + 3x - 1$

$P(2.19) = 2.19^3 - 3(2.19)^2 + 3(2.19) - 1$

$= 10.5 - 14.4 + 6.57 - 1$

$= 1.67$

Abs. Err $= |1.685159 - 1.67| = 0.015159$

Relative Err $= \frac{0.015159}{1.685159} = 8.99 \times 10^{-3}$

$Q(x) = ((x-3)x+3)x - 1$

$Q(2.19) = ((2.19-3) \times 2.19 + 3) \times 2.19 - 1$

$= ((-0.81 \times 2.19) + 3) \times 2.19 - 1$

$= 1.23 \times 2.19 - 1 = 1.69$

Abs. Err $= |1.685159 - 1.69| = 0.004841$

Relative Err $= \frac{0.004841}{1.685159} = 2.8727 \times 10^{-3}$

b) $F(x) = \cos x = 1 - x^2/2! + x^4/4! - x^6/6!$

True Value $\Rightarrow \cos(\pi/4) = \sqrt{2}/2 = 0.70711$

Approximate value $\Rightarrow 1 - \frac{(\pi/4)^2}{2!} + \frac{(\pi/4)^4}{4!} - \frac{(\pi/4)^6}{6!}$

$= 0.70709.$

Absolute error $= |0.70711 - 0.70709| = 0.00002$

Relative error $= \left|\frac{\text{True} - \text{Approx}}{\text{True}}\right| = \left|\frac{0.0002}{0.70711}\right| = 0.00003.$

c)

ii. $\Delta = \mu\delta + \frac{\delta^2}{2}$

Taking RHS

$$\Delta = \frac{1}{2}(E - E^{-1}) + \frac{(E^{1/2} - E^{-1/2})^2}{2}$$

$$= \frac{E - E^{-1}}{2} + \frac{E - 2 + E^{-1}}{2}$$

$$= \frac{2E - 2}{2} = \frac{2(E-1)}{2}$$

$\Delta = E - 1$ ∴ Hence proved.

iii. $\mu^2 = 1 + 1/4\ \delta^2$

Taking R.H.S

$$\mu^2 = 1 + \frac{(E^{1/2} - E^{-1/2})^2}{4}$$

$$= 1 + \frac{E - 2 + E^{-1}}{4}$$

$$= \frac{4 + E - 2 + E^{-1}}{4} = \frac{E + 2 + E^{-1}}{4}$$

$\mu^2 = \left(\frac{1}{2}(E + E^{-1})\right)^2$ ∴ Hence proved.

Q2.

a) $f(x) = 3x + \sin x - e^x$ $[0, 1]$

secant method $\Rightarrow$ $x = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$f(x_0) = -1$ $f(x_1) = 1.12319$.

Iteration	x_{n-1}	x_n	x_{n+1}	$f(x_{n+1})$	$x_{n+1} - x_n$
1	0	1	0.47098	0.50827	0.52902
2	1	0.47098	0.30750	-0.13484	0.16348
3	0.47098	0.30750	0.36261	5.9704×10^{-3}	0.05511
4	0.30750	0.36261	0.360461	9.83×10^{-5}	2.149×10^{-3}
5	0.36261	0.360461	0.360421	-1.75×10^{-6}	4×10^{-5}

$\boxed{x = 0.360421}$

b) $x = 7^{1/3}$

$x^3 = 7$

$x \cdot x^2 = 7$

$x^2 = 7/x$

$x = \sqrt{7/x}$

$x_0 = 2$

$g_1 = 1.870$

$g_2 = 1.9343$

$g_3 = 1.9023$

$g_4 = 1.9182$

$g_5 = 1.9102$

Q3 a) using interpolating formula for degree 2.

$$f_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$x_0 = 0$ $x_1 = 2$ $x_2 = 3$

$f(x_0) = 1$ $f(x_1) = 2$ $f(x_2) = 4$

$$f_2(x) = \frac{(x-2)(x-3)}{(0-2)(0-3)}(1) + \frac{(x-0)(x-3)}{(2-0)(2-3)}(2) + \frac{(x-0)(x-2)}{(3-0)(3-2)}(4)$$

$$= \frac{x^2-5x+6}{6} - (x^2-3x) + \frac{4x^2-8x}{3}$$

$$= \frac{3x^2-15x+18-18x^2+54x+24x^2-48x}{18}$$

$$= \frac{9x^2-9x+18}{18} = \boxed{\frac{x^2}{2} - \frac{x}{2} + 1}$$

b)

X	Y	ΔY	$\Delta^2 Y$	$\Delta^3 Y$
15500	124.499	0.0402		
15510	124.5392	0.0401	-0.0001	
15520	124.5793	0.0401	0	0.0001
15530	124.6193			

c) $f(0.05)$.

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1				
		0.3498			
0.1	1.3498		0.12251		
		0.47231		0.04267	
0.2	1.82211		0.16518		0.01517
		0.63749		0.05784	
0.3	2.4596		0.22302		
		0.86051			
0.4	3.32011				

$$p = \frac{0.05 - 0}{0.1} = 0.5$$

$$y(x) = 1 + 0.5(0.3498) + \frac{(0.5)(0.5-1)(0.12251)}{2!} + \frac{0.5(0.5-1)(0.5-2)(0.04267)}{3!}$$

$$+ \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)(0.01517)}{4!}$$

$$\boxed{y(0.05) = 1.161067}$$

Q4. a) Using 5 point end point formula.

$u_0 = 0.2$

$$f'(0.2) = \frac{1}{12\times 0.2}\left[\begin{matrix}-25(0.978652) + 48(0.9177718) - 36(0.8080348) \\ +16(0.6386093) - 3(0.3843735)\end{matrix}\right]$$

$\boxed{f'(0.2) = -0.19510}$ ⇒ Velocity at $t = 0.2$

$u_0 = 1.0$

$$f'(1.0) = \frac{1}{12\times 0.2}\left[\begin{matrix}-25(0.3843735) + 48(0.6386093) - 36(0.8080348 \\ +16(0.9177718) - 3(0.978852).\end{matrix}\right.$$

$\boxed{f'(1.0) = -1.5414}$ ⇒ Velocity at $t = 1.0$

b) $x_0 = 0.6$

Using 5 point mid point.

$$f'(0.6) = \frac{1}{12(0.2)}\left[0.9798652 - 8(0.9177718) + 8(0.638093) - 0.3848735\right]$$

$$\boxed{f'(0.6) = -0.68241} \Rightarrow \text{Velocity at } t = 0.6$$

Time (T)	0.2	0.6	1.0
Velocity (V)	-0.1951	-0.68241	-1.5414

Using 3 point mid point formula

$x_0 = 0.6$

$$f'(0.6) = \frac{1}{2\times 0.4}\left[(-1.5141) - (-0.1951)\right]$$

$$\boxed{f'(0.6) = -1.6828} \Rightarrow \text{Acceleration at } t = 0.6$$

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Q5. $\int_0^{12} 1/(1+x^2)\,dx$. $n=6$.

x	0	2	4	6	8	10	12
y	1	1/5	1/17	1/37	1/65	1/101	1/145

$h=2$.

a) Composite Trapizoidal Rule:

$$\Rightarrow \frac{h}{2}\left(y_0 + 2(y_1+y_2 \cdots y_{n-1}) + y_n\right)$$

$$= \frac{2}{2}\left(1 + 2\left(\tfrac{1}{5} + \tfrac{1}{17} + \tfrac{1}{37} + \tfrac{1}{65} + \tfrac{1}{101}\right) + \tfrac{1}{145}\right)$$

$$= 1.62917$$

b) i Simpsons 1/3rd rule

$$\Rightarrow \frac{h}{3}\left(y_0 + 4(y_1+y_3\cdots) + 2(y_2+y_4\cdots) + y_{2n}\right)$$

$$= \frac{2}{3}\left(1 + 4\left(\tfrac{1}{5} + \tfrac{1}{37} + \tfrac{1}{101}\right) + 2\left(\tfrac{1}{17} + \tfrac{1}{65}\right) + \tfrac{1}{145}\right)$$

$$= 1.40202$$

ii Simpsons 3/8 Rule

$$\Rightarrow \frac{3h}{8}(y_0 + 3y_1 + 3y_2 + 2y_3 \cdots 2y_{n-3} + 3y_{n-2} 3y_{n-1} + y(5))$$

$$= \frac{3(2)}{8}\left[1 + 3(1/5 + 1/17 + 1/65 + 1/101) + 2(1/37) + 1/145)\right]$$

$$= 1.43496.$$

Q6. a) Modified Eulers Method.

$$\frac{dy}{dt} = \frac{1+t}{1+y} \qquad 1 \le t \le 2 \qquad y(1) = 2$$

$$h = 0.5$$

given $y_0 = 2$ $t_0 = 1$

$t_1 = 1 + 0.5 = 1.5$

$t_2 = 1.5 + 0.5 = 2.0$

Using formula $y_1 = y_0 + h/2[f(t_0, y_0) + f(t_1, P_1)]$

$$f(t_0, y_0) = \frac{1+t}{1+y} = \frac{1+1}{1+2} = 0.666$$

$P_1 = y_0 + hf(t_0, y_0) = 2 + 0.5(0$

$P_1 = 2.333$

$$f(1.5, 2.333) = \frac{1+1.5}{1+2.333} = 0.750$$

$y_1 = y_0 + h/2[f(t_0, y_0) + f(t_1, P_1)] = 2 + 0.5/2[0.666 + 0.750$

$y(1.5) = 2.354.$

$f(t_1, y_1) = \frac{1+1.5}{1+2.354} = 0.745$

$P_2 = y_1 + hf(t_1, y_1)$
$= 2.354 + 0.5(0.745)$
$= 2.7265$

$f(2, 2.765) = \frac{1+2}{1+2.726} = 0.805$

$y_2 = 2.354 + \frac{0.5}{2}[0.745 + 0.805]$

$\boxed{y(2) = 2.7415.}$

Exact Solution

t_i	Modified	Exact
1.5	2.354	2.354109 66
2	2.741	2.741657387

R-K Method (4).

$y_{i+1} = y_i + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$K_1 = hf(t_i, f_i)$
$K_2 = hf(t_i + \frac{h}{2}, f_i + \frac{1}{2}K_1)$
$K_3 = hf(t_i + h/2, f_i + \frac{1}{2}K_2)$
$K_4 = hf(t_i + 1, f_i + K_3)$.

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given $y_0 = 2 \quad t_0 = 1$

$$y(1.5) = y_0 + 1/6 (K_1 + 2K_2 + 2K_3 + K_4)$$

$K_1 = 0.5 f(2/3) = 0.333$

$K_2 = 0.5 f\left(1 + \frac{0.5}{2}, 2 + \frac{0.33}{2}\right) = 0.355$

$K_3 = 0.5 f(1.25, 2.355) = 0.3353$

$K_4 = 0.5 f(2, 2.35) = 0.449$

$$\boxed{y(1.5) = 2.360333}$$

$$y(2) = y_1 + 1/6 (K_1 + 2K_2 + 2K_3 + K_4)$$

$K_1 = hf(t_1, y_1) = 0.371$

$K_2 = 0.5 f(1.5 + 0.5/2, 2.360 + 0.5/2) = 0.387$

$K_3 = 0.5 f(1.75, 2.360 + 0.387/2) = 0.386$

~~$K_4 = 0.5 f(1.75, 2.553) = 0$~~

$K_4 = 0.5 f(2.5, 2.746) = 0.467$

$$y(2) = 2.360333 + 1/6 (0.371 + 2(0.387 + 0.386) + 0.467)$$

$$\boxed{y(2) = 2.757666}$$

t_i	exact	modified	RK	Error
1.5	2.35401966	2.354	2.36033	$+6.23 \times 10^{-3}$
2	2.7416572	2.741	2.7566	0.0160092

Q7.

a) $x+y+5z = 21.5$
$-3x-6y+2z = -61.5$
$10x+2y-z = 27$

$x = -21.5-y-5z$
$y = (-61.5+3x-2z)/-6$
$z = -(27-10x-2y)$

$x_0 = 0 \quad y_0 = 0 \quad z_0 = 0$

iteration 1

$x_1 = -21.5-0-5(0) = -21.5$
$y_1 = -61.5+3(-21.5)-2(0) = 21$
$z_1 = -27+10(-21.5)+2(21) = -200$

iteration 2

$x_2 = -21.5-21-5(-200) = 957.5$
$y_2 = (-61.5+3(957.5)-2(-200)/-6 = -535.16$
$z_3 = -(27-10(957.5)-2(-535.16)) = 8477.68$

iteration 3

$x_3 = -21.5+535.16-5(8477.68) = -41874.74$
$y_3 = (-61.5+3(-41874.74)-2(8477.68) = 23773.51$
$z_3 = -(27-10(-41874.74)-2(23773.51) = -371227.38$

b) $\begin{bmatrix} 2 & 4 \\ 4 & 5 \end{bmatrix}$

It is positive definite because
$A = A^T$

$$\begin{bmatrix} 2 & 4 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 5 \end{bmatrix}^T.$$

$i=2$, i not ≥ 4
4 not ≥ 5.

Q8. a) $\begin{bmatrix} 4 & -1 & 1 \\ 1 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ .. Find LDL^{-1}

$$A = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} \begin{bmatrix} 1 & L_{21} & L_{31} \\ 0 & 1 & L_{32} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} d_1 & 0 & 0 \\ L_{21}d_1 & d_2 & 0 \\ L_{31}d_1 & L_{32}d_2 & d_3 \end{bmatrix} \begin{bmatrix} 1 & L_{21} & L_{31} \\ 0 & 1 & L_{32} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} d_1 & d_1L_{21} & d_1L_{31} \\ L_{21}d_1 & L_{21}^2 d_1 + d_2 & L_{31}L_{21}d_1 + d_2L_{32} \\ L_{31}d_1 & L_{31}d_1 + L_{21}L_{31}d_1 & L_{31}d_1 + L_{32}d_2 + d_3 \end{bmatrix}$$

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$d_1 = 4$

$d_1 l_{21} = -1$

$l_{21} = -1/4$

$d_1 l_{31} = 1$

$l_{31} = 1/4$

$l_{21}^2 d_1 + d_2 = 3$

$d_2 = 11/4$

$l_{21} l_{31} d_1 + d_2 l_{32} = 0$

$l_{32} = -1/11$

$l_{31}^2 d_1 + l_{32}^2 d_2 + d_3 = 2$

$d_3 = 19/11$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -0.25 & 1 & 0 \\ 0.25 & 0.909 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2.75 & 0 \\ 0 & 0 & 1.727 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -0.25 & 1 & 0 \\ 0.25 & 0.909 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2.75 & 0 \\ 0 & 0 & 1.727 \end{bmatrix} \begin{bmatrix} 1 & -0.25 & 0.25 \\ 0 & 1 & 0.909 \\ 0 & 0 & 1 \end{bmatrix}$$

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b) $\begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & c & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 2 & 2 \\ -1 & 0 & 1 \\ 3 & 5 & 6 \end{bmatrix}$

Doolittles formula

$$\begin{bmatrix} 2 & 2 & 2 \\ -1 & 0 & 1 \\ 3 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$$

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$d=2$ $e=2$ $f=2$ $a=0.5$ $h=2$

$b=1.5$ $g=1$ $c=2$ $i=6$

$LY = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 1.5 & 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$$

$y_1 = 1$

$y_2 = 2.5$

$y_3 = 3.5$

Weekend

$Ux = Y$

$$\begin{bmatrix} 2 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2.5 \\ -3.5 \end{bmatrix}$$

$x = 0.5$
$y = -4.5$
$z = 3.5$