$2\ \ 12u^2 - 30u + 10$
$6u^2 - 15u + 5$

Chapter # 05 Date:

Lecture # 18 Numerical Differentiation And Numerical Integration.

Estimate the derivation of tabulated Function.

(1) Equal spaced data
(1) Newton's Forward Differenciation.
(2) Backward Differenciation.
(3) Central Difference Formula

* Newton's Forward Interpolation Formula.

$$\boxed{y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots}$$

$u = \frac{x - x_0}{h}$

$uh = x - x_0$

$$\boxed{y(x_0+uh) = y_0 + u\Delta y_0 + \frac{u^2-u}{2}\Delta^2 y_0 + \frac{u^3-3u^2+2u}{2}\Delta^3 y_0 + \cdots}$$

Diff w.r.t. x:

$$y'(0+h) = 0 + \Delta y_0 + \frac{2u-1}{2}\Delta^2 y_0 + \frac{3u^2-6u+2}{6}\Delta^3 y_0 \cdots$$

$du = \frac{dx}{h}$

$$y'(x) = \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{2}\Delta^2 y_0 + \frac{3u^2-6u+2}{6}\Delta^3 y_0 + \cdots\right.$$

For second derivative.

$$y''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 \cdots\cdots\right] \quad \Delta^4 y_0$$

$u(u-1)(u-2)(u-3)$ $\quad d/dx$

$(u^2-u)(u-2) \Rightarrow (u^3-2u^2-u+2)(u-3)$

$-3u^3+6u^2+3u-6$ PRODUCT OF $= u^4-2u^3-u^2+2u$

$y'(2) - y''$ Date:

Problem

x	1.5	20	2.5	3.0	3.5	4.0
y	3.375	7.0	13.0625	2.4	38.875	59.

To find $y'(1.5)$, $y''(1.5) = ?$

$$u = \frac{x - x_0}{h} = \frac{1.5 - 1.5}{0.5} = \frac{0}{0.5} = 0.$$

$$y'(1.5) = \frac{1}{0.5}\left[3.625 + \frac{2(0) - 1}{2}(3) + \frac{3(0)^2 - 6(0) + 2}{6}(0.75)\right.$$

$$+ 0 + 0$$

$$= \frac{1}{0.5}\left[3.625 - \frac{3}{2} + \frac{0.75}{3}\right]$$

$$y'(1.5) = 4.75$$

Difference Table.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.5	3.375					
		3.625				
2.0	7.0		3			
		6.625		0.75		
2.5	13.625		3.75		0	
		10.375		0.75		0.
3.0	2.4		4.5		0	
		14.875		0.75		
3.5	38.875		5.25			
		20.125				
4.0	59					

Date:

Problem # 02.

x	2	4	6	8	10
y	105	42.5	25.3	16.3	13

Ans -52.

Find (i) $y'(2) = ?$ (ii) $y''(2) = ?$

Practice Problem

x_i	1.10	1.30	1.50	1.70	1.90
$f(x)$	3.24075	2.68999	2.33333	1.99221	

Find $f'(x)$ at $x = 1.10$.

$f''(x)$ at $x = 1.10$.

Monday!

Date: 18/03/2019.

Lecture # 20

Numerical Differentiation And Numerical Integeration.

Central Difference Formula. For Differenciation (For Equal spaced data)

① Three point Mid-point Formula.

$$\boxed{f'(x) = \frac{1}{2h}\left[f(x_0+h) - f(x_0-h)\right]}$$

② Five point Mid-point Formula

$$\boxed{f'(x) = \frac{1}{12h}\left[f(x_0-2h) + 8f(x_0-h) - f(x_0+2h)\right]}$$

Problem 1.

x	$y = f(x)$	
1.8	10.889365	x_0
1.9	12.703199	x_1
2.0	14.778112	x_2
2.1	17.148957	x_3
2.2	19.855030	

Find $f'(2.0) = ?$ when $h = 0.1$ and $h = 0.2$.

using three Point Mid-point formula.

$$f'(x) = \frac{1}{2h}\left[f(x_0+h) - f(x_0-h)\right]$$

$h = 0.1 \quad x_0 = 2.0$

$$f'(2.0) = \frac{1}{2(0.1)}\left[f(2+0.1) - f(2-0.1)\right]$$

$$= \frac{1}{0.2}\left[f(2.1) - f(1.9)\right] \Rightarrow \frac{1}{0.2}\left[17.148957 - 12.703199\right]$$

$f'(2.0) \Rightarrow 22.22879$

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using Five point Mid point Formula.

$$f'(x) = \frac{1}{12h}\left[f(x_0-2h) - 8f(x_0-h) + 8f(x_0+h) - f(x_0+2h)\right]$$

$$f'(2.0) = \frac{1}{12(0.1)}\left[f(1.8) - 8f(1.9) + 8f(2.1) - f(2.2)\right]$$

$$= \frac{1}{1.2}\left[10.889365 - 8(12.703199) + 8(17.148957) - 19.855030\right]$$

$$f'(2.0) = 22.166999.$$

when $h = 0.2$.

$$f'(2.0) = \frac{1}{2(0.2)}\left[f(2.2) - f(1.8)\right]$$

$$\boxed{f'(2.0) = 22.41416.}$$

Problem:-

x	1	2	3	4	5
$f(x)=y$	2.4142	2.673	2.8974	3.0974	3.2804

Find $f'(1)$ (ii) $f'(3)$ (iii) $f'(5)$ when $h = 1$.

Date: 25/03/2019

Lecture # 22 Numerical Differential and Numerical Integration

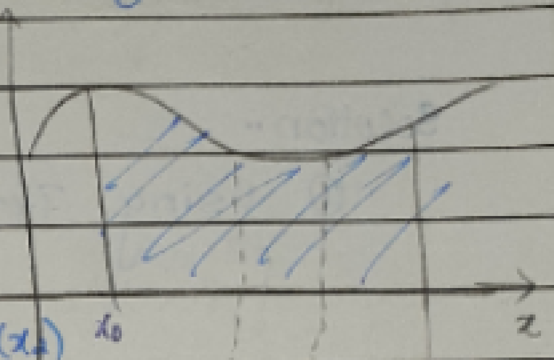
Numerical Integration.

To find the integrated value without analytically.

→ Integration process is the summing process.

$$\int_a^b f(x)\,dx \simeq \lim_{x\to n} \sum_{k=0}^{n} f(x_k)\,\Delta x_n.$$

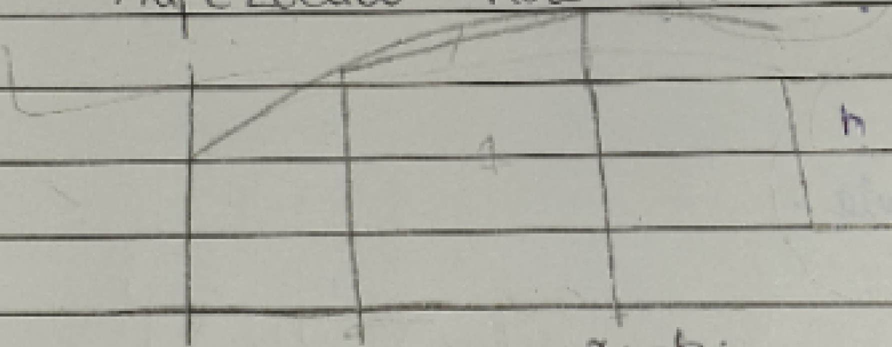
if $\Delta x_k = C_k$.



$$\int_a^b f(x)\,dx = C_0 f(x_0) + C_1 f(x_1) + C_2 f(x_2)$$

Methods Formulae :-

① Trapezodial Rule (n=1)



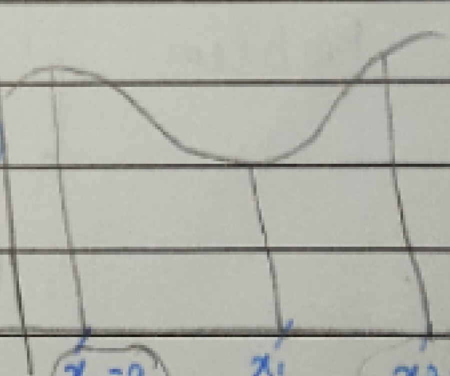
$$\boxed{h = \frac{b-a}{n}}$$

$$\int_{x_0}^{x_1} f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right] - \frac{h^3}{12} f''(\xi), \quad x_0 < \xi < x_1$$

② Simpson's 1/3 Rule (n=2).

$$\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right] - \frac{h^5}{90} f^{(4)}(\xi)$$

where $x_0 < \xi < x_2$



$$\boxed{h = \frac{b-a}{n}}$$

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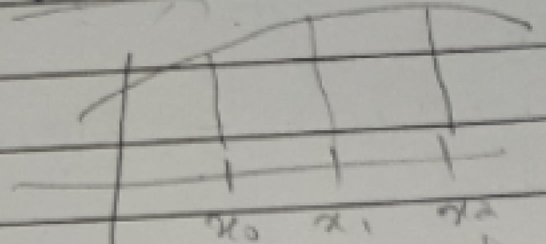
Problem :-

Consider the function $f(x) = 1 + e^{-x}\sin(4x)$.
and the equally spaced quadrature nodes.
$x_0 = 0.0$, $f(x_0) = 1$, $x_1 = 0.5$, $f(x_1) = 1.55152$.
$x_2 = 1.0$, $f(x_2) = 0.72159$.
use (i) Trapezodial Rule with $h = 0.5$ (2 point)
(ii) Simpson's $\frac{1}{3}$ Rule



Solution ..

(i) Using Trapezodial Rule.

$$\int_{x_0}^{x_1} f(x)\,dx \simeq \frac{0.5}{2}\left[f(x_0) + f(x_1)\right]$$

$b - a$

$$\int_0^{0.5} f(x) = \frac{0.5}{2}\left[1 + 1.55152\right]$$

$= 0.63788$

$a_0 =$

using Simpson's $\frac{1}{3}$ Rule .. $n = 2$. $h = \frac{b-a}{n}$

$$\int_{x_0}^{x_2} f(x)\,dx \approx \frac{0.5}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

$$\int_0^{1.0} f(x)\,dx = 1.32128$$

Problem $\int_0^1 x^2 e^{-x}$ use Trapezodial Rule.

Date: 26/03/2019

Lecture #23 Numerical Integration

Formula:

Simpson's $\frac{1}{3}$ Rule $(n=2)$

$$\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right] - \frac{h^5}{90} f^{4}(\xi)$$

Q5 Problem: Approximate the following integral using Simpson's $\frac{1}{3}$ Rule.

$$\int_{0}^{\pi/4} e^{3x}\sin 2x\,dx$$

$a = x_0 = 0$

$b = x_2 = \pi/4$

Given:

$a = x_0 = 0$, $b = x_2 = \pi/4$

$$h = \frac{b-a}{n} = \frac{\frac{\pi}{4} - 0}{2} = \frac{\pi}{8}$$

$x_1 = x_0 + h$

$= 0 + \pi/8$

$x_1 = \pi/8$

$f(x) = e^{3x}\sin 2x$

$f(0) = 0$

$f(\pi/8) = 2.29682$

$f(\pi/4) = 10.55072$

using formula:-

$$\int_0^{\pi/4} e^{3x}\sin 2x\,dx$$

$\frac{h}{3}$ $f(x_0) + 4f(x_1) + f(x_2)$

$$\approx \frac{\pi/8}{3}\left[0 + 4(2.29682) + 10.55072\right]$$

$$\approx \boxed{2.58369}$$

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Q3 (d) $\int_0^1 x^2 e^{-x} \, dx$

$f(x) = x^2 e^{-x}$

$f'(x) = e^{-x}(2x) + x^2 e^{-x}(-1)$

$f''(x) = e^{-x}(2-2x) + (2x - x^2) e^{-x}(-1)$

$= e^{x}(2-2x - 2x + x^2)$

$= e^{-x}(2-4x+x^2)$

$f'''(x) = e^{-x}(-4+2x) + (2-4x+x^2) e^{-x}(-1)$

$= e^{-x}(-4+2x - 2 + 4x - x^2)$

$= e^{-x}(-6+6x-x^2$

$f^{(4)} = e^{-x}(6-2x) + (-6+6x-x^2) e^{-x}(-1)$

$= e^{-x}(6-2x+6-6x+x^2)$

$f^{(4)} = e^{-x}(12-8x+x^2)$

let $\xi = 1$ (for max value only)

$f^{4}(1) = e^{-1}(12-8(1)+(1)^2)$

$= 1.83939.$

$f''(1) = e^{-1}[2-4(1)+(1)^2] = -0.367879$

$\frac{h^3}{12} f''(1) = \frac{(1)^3}{12}(-0.367879) = -0.03065$

$\frac{h^5}{90} f^{(4)}(1) = \frac{(0.5)^5}{90}(1.83939) = 0.0006.$

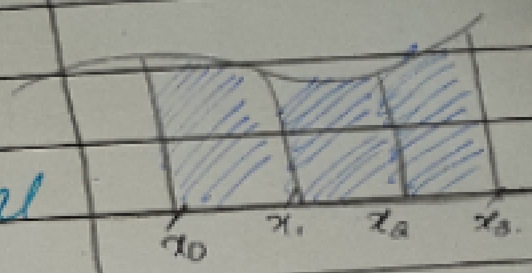
Date:

Lecture # 24 Numerical Integration

Formula Simpson's Three-eight Rule $(n=3)$

$$\int_{x_0}^{x_3} f(x)\,dx = \frac{3h}{8}\left[f(x_0)+3f(x_1)+3f(x_2)+f(x_3)\right]$$

$$\boxed{-\frac{3h^5}{80} f^4(\xi)}$$



Problem:- Approximate the following integral using simpson's $\frac{3}{8}$ Rule also find error bounds from error formula

$$\int_{1}^{1.6} \frac{2x}{x^2-4}\,dx$$

x_0 x_3

$$\boxed{h = \frac{b-a}{n}}$$

Solution:-

$a = 1$

$b = 1.6$

$$h = \frac{b-a}{n}$$

$$= \frac{1.6-1}{3} = \frac{0.6}{3}$$

$h = 0.2$

$b = x_3 = 1.6$

$x_0 = a = 1$ $\qquad f(1) = \frac{2(1)}{(1)^2-4} = -0.6666$

$x_1 = x_0 + h$

$= 1 + 0.2$ $\qquad f(1.2) = -0.9375$

$= 1.2$ $\qquad h+h$ $\qquad f(1.4) = -1.3725$

$x_2 = x_0 + 2h$ $\qquad f(1.6) = -2.2222$

$= 1 + 0.4$

$= 1.4$

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using formula:-

$$\int_1^{1.6} \frac{2x}{x^2-4}\,dx \approx \frac{3(0.2)}{8}\left[f(1)+3f(1.2)+3f(1.4)+f(1.6)\right]$$

$$= \frac{0.6}{8}\left[-0.6666+3(-0.9375)+3(-1.37325)+(-2.2222)\right]$$

$$\boxed{= -0.73364.}$$

Error:-

$$f(x) = \frac{2x}{x^2-4}$$

$$f'(x) = \frac{(x^2-4)(2)-(2x)(2x)}{(x^2-4)^2}$$

$$= \frac{2x^2-8-4x^2}{(x^2-4)^2}$$

$$= \frac{-2x^2-8}{(x^2-4)^2}$$

$$f''(x) = \frac{(x^2-4)^2(-4x)-(-2x^2-8)\,2(x^2-4)(2x)}{(x^2-4)^2}$$

$$= \cancel{(x^2-4)^2}\ \frac{x^4-8x^2+16-(-2x^2-8)(4x^3-16x)}{(x^2-4)^2}$$

$$= \frac{x^4-8x^2+16-(-8x^5+\cancel{32x^3}-\cancel{32x^3}+128x)}{(x^2-4)^2}$$

$$= \frac{x^4-8x^2+16+8x^5-128x}{(x^2-4)^2}$$

$$= \frac{8x^5+x^4-8x^2-128x+16}{(x^2-4)^2}$$

$$\Rightarrow -39.43.$$

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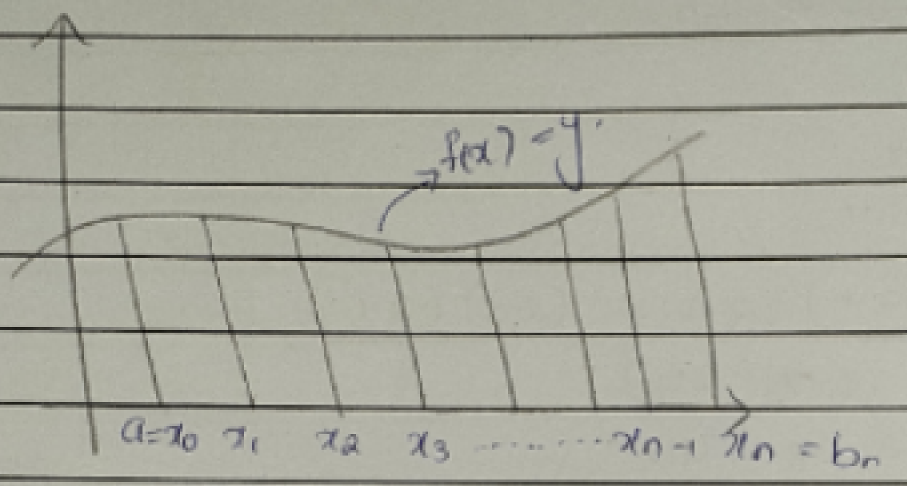
lecture # 26 Composite Integration.

Methods (Integration for Large interval

1. Composite Trapezodial Rule $n = 1, 2, 3, 4, 5$ ----
2. " Simpson's $\frac{1}{3}$ Rule $n = 2, 4, 6, 8, 10$ ---
3. " " $\frac{3}{8}$ Rule $n = 3$ $n = 3, 6, 9, 12$ ----



① Composite Trapezodial Rule:-

$$\int_{a=x_0}^{b=x_n} f(x)\,dx = \frac{h}{2}\Big[f(x_0) + 2\{f(x_1) + f(x_2) + f(x_3) + \dots + f(x_{n-1})\} + f(x_n)\Big]$$

② Composite Simpson's $\frac{1}{3}$ Rule:-

$$\int_{x_0=a}^{b=x_n} f(x)\,dx = \frac{h}{3}\Big[f(x_0) + 4\{f(x_1) + (x_3) \dots f(x_{n-2})\} + f(x_n)\Big]$$

③ Composite Simpson's 3/8 Rule:-

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{3h}{8}\Big[f(x_0) + 3\{f(x_1) + f(x_2) + f(x_4) + f(x_5) + f(x_7) + \dots f(x_{n-1})\} + 2\{f(x_3) + f(x_6) + \dots f(x_{n-3})\} + f(x_n)\Big]$$

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Q. Use composite Trapezodial, Simpson's $\frac{1}{3}$ Rule.

$f(x) = \int_1^3 \frac{x}{x^2+4}\,dx$, $n=8$

$x_0 = a = 1$ $\quad b = x = 3$

$h = \frac{b-a}{n} \Rightarrow \frac{3-1}{8} = \frac{1}{4}$

$= 0.25$

$x_0 = 1$

$x_1 = 1 + 0.25 = 1.25$

$x_2 = 1.25 + 0.25 = 1.50$

$x_3 = 1.50 + 0.25 = 1.75$

$x_4 = 1.75 + 0.25 = 2$

$x_5 = 2 + 0.25 = 2.25$

$x_6 = 2.25 + 0.25 = 2.50$

$x_7 = 2.50 + 0.25 = 2.75$

$x_8 = 2.75 + 0.25 = 3$.

$f(x_0) = 0.2$

$f(x_1) = 0.22$

$f(x_2) = 0.24$

$f(x_3) = 0.247$

$f(x_4) = 0.25$

$f(x_5) = 0.248$

$f(x_6) = 0.244$

$f(x_7) = 0.238$

$f(x_8) = 0.23$.

Trapezodial Rule:-

$\int_1^3 \frac{x}{x^2+4}\,dx$

$= \frac{0.25}{2}\left[f(x_0) + 2\{f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7)\} + f(x_8)\right]$

$\Rightarrow \frac{0.25}{2}\left[0.2 + 2\{0.22 + 0.24 + 0.247 + 0.25 + 0.248 + 0.244 + 0.238\} + 0.23\right]$

$\Rightarrow 0.476977$.

Simpson Rule:-

$\int_1^3 \frac{x}{x^2+4}\,dx = \frac{0.25}{3}\left[0.2 + 4\{0.22 + 0.247 + 0.248 + 0.238\} + 2\{0.24 + 0.25) + 0.244\} + 0.23\right]$

$\Rightarrow 0.4777547$.