Tuesday
5/3/19

KhanAcademy
ME / Elasticity

Elasticity of Demand

→ Elasticity:

It is the percentage change in quantity demanded due to the percentage change in the price.

$$E_D = \% \Delta Q_D / \% \Delta P$$

(OR) It is the percentage change in the quantity demanded due to the variables affecting demand

$$E_d = \% \Delta Q_D / \% \Delta \text{ in variable affecting demand.}$$

∴ variable is generally "price"

→ Inelastic demand of goods:

- When quantity demanded of goods do not change much by price change
  - eg: Vegetables such as potatoes, onions, mineral, salts, wheat
- The consumption is not affected by price.
- Utility of inelastic goods are high.

→ Elastic Demand of Goods:

- Choices are there, as we can substitute goods.
- Slight price change can affect quantity demanded.

$$\Delta Q_D > \Delta P$$

- Consumption is not rigid.

* Baking, schools, educational institutes are inelastic
* For entertainment, mobile & tv are elastic goods

→ Factors Affecting Elasticity:

1. Price
2. Income
3. Substitute / Complements
4. Used Goods
5. Culture
6. Habits
7. Utility
8. Real income ⇒ (money earned = constant, price = changes)

① Price Factor: (Also consider types of goods)

- Normal goods
- Inferior goods
- Luxury / Superior goods

Normal Goods:

① Price $\propto$ 1/quantity demanded.

② real income $\propto$ quantity demanded

real income ↑ when price ↓

③ real income $\propto$ 1/price

Inferior Goods: (quality is less than normal goods)

① real income $\propto$ 1/quantity demanded

② price $\propto$ quantity demanded

③ real income $\propto$ 1/price.

Luxury Goods: (similar to normal goods)

① price $\propto$ 1/quantity demanded

② real income $\propto$ quantity demanded

③ real income $\propto$ 1/price.

② Income:

- Positive effect
- If income increases then you will reach to elasticity as choices increases.

③ Substitutes / Complements:

- Cross Elasticity:

; A & B are goods

$$\varepsilon_{cross} = \frac{\% \Delta Q_{DA}}{\% \Delta Q_{DB}}$$

④ Used Goods:

- cheaper to buy
- If used goods market ↑ then new goods market ↓
- If income is high the new goods market ↑

⑤ Culture:

eg: Rings in christian & muslim weddings.

- A/c to culture & consumption pattern demand changes.

(6) Habits:

eg: Jogging habit → track suit & shoes required.

eg: Fast food lovers → eating fast food required

(7) Utility:

- What is important to you?
- eg: specs are in demand.

Handout # 04

Q2 Point Elasticity:

% change at one point of the demand curve

$$\varepsilon_{PT} = \text{slope} \times P/Q \Rightarrow \text{slope} = \Delta Q/\Delta P$$

(a)

$Q = 20 - 2P$

slope $= -2$ (derivative)

$P = 5$

$Q = 20 - 2(5)$

$= 10$

$\varepsilon_{PT} = -2 \times 5/10$

$= -1$

(Unitary elastic)

$P = 9$

$Q = 20 - 2(9)$

$= 2$

$\varepsilon_{PT} = -2 \times 9/2$

$= -9$

(elastic demand b/c $\%\Delta Q > \%\Delta P$).

$\Rightarrow$ price $\propto 1/Q_D$

If $\% \Delta Q > \% \Delta P \rightarrow$ demand is elastic

(b) Arc Elasticity:

$$\mathcal{E}_{arc} = \left(\frac{Q_2 - Q_1}{(Q_2 + Q_1)/2} \right) \div \left(\frac{P_2 - P_1}{(P_2 + P_1)/2} \right) \rightarrow \text{mid point}$$

$P_1 = 5$

$Q_1 = 20 - 2P_1$

$= 20 - 2(5)$

$Q_1 = 10$

$P_2 = 6$

$Q_2 = 20 - 2P_2$

$= 20 - 2(6)$

$Q_2 = 8$

$$\mathcal{E}_{arc} = \frac{-2}{9} \div \frac{1}{11/2}$$

$$= \frac{-2}{9} \times \frac{11/2}{1}$$

$$= -11/9$$

$$\mathcal{E}_{arc} = -1.22$$

(c) $\mathcal{E}_{PT} = 1$ b/c $\% \Delta Q = \% \Delta P$

see $\rightarrow$ pg#48

Q4 $Q = 30 - 2P$

(a) $P = 7$, $\varepsilon_{PT} = ?$

slope $= -2$

$Q = 30 - 2(7) = 16$

$\varepsilon_{PT} = -2 \times 7/16 = -7/8 = -0.875$

(b) $P_1 = 5$, $P_2 = 6$, $\varepsilon_{arc} = ?$

$Q_1 = 20$ $Q_2 = 18$

$$\varepsilon_{arc} = \frac{-2}{16} \div \frac{1}{11/2}$$

$$= \frac{-\cancel{2}}{19} \times \frac{11}{\cancel{2}}$$

$$= \frac{-11}{19}$$

(c) ε_{PT} & ε_{arc} will not change until demand curve changes.

Q6 $Q_1 = 3000$, monthly = \$25 ,
similar sell = \$28

(a) $\mathcal{E} = -3$
price lowered = \$22
AM/FM Clock Radio

$P_1 = 25$, $P_2 = 22$, $Q_1 = 3000$, $Q_2 = ?$
$\mathcal{E}_{arc} = -3$.

$$-3 = \frac{Q_2 - 3000}{(3000 + Q_2)/2} \div \frac{-3}{47/2}$$

$$\frac{-3 \times -3}{47/2} = \frac{Q_2 - 3000}{(3000 + Q_2)/2}$$

$$\frac{\cancel{18}}{47} = \frac{\cancel{2}(Q_2 - 3000)}{(3000 + Q_2)}$$

$$9(3000 + Q_2) = 47(Q_2 - 3000)$$
$$27000 + 9Q_2 = 47Q_2 - 141000$$
$$38Q_2 = 168000$$
$$Q_2 = 4421.0526$$

(b) A = ABC company & B = Competition

~~$P_1 = \$28$~~ $P_2 = \$24$

$\varepsilon_{cross} = \%\Delta Q_{DA} / \%P_B$

$0.3 = Q/24 \Rightarrow Q = 0.3(24)$

Q3 $Q = 100 - 10P + 0.5Y$

$P = 7, \; Y = 50$

$\Delta Y \propto Q_D \uparrow$

$\Delta P \propto 1/Q_D \downarrow$

(a) $Q = 100 - 10(7) + 0.5(50)$

$= 55$

(b) $\varepsilon_{PT} = ?$

$\varepsilon_{PT} = -10 \times 7/55$

$= -14/11$

$|\varepsilon_{PT}| > 1 \rightarrow$ elastic
$|\varepsilon_{PT}| < 1 \rightarrow$ inelastic
$|\varepsilon_{PT}| = 1 \rightarrow$ unitary elastic

(c) Point income elasticity.

$$\varepsilon_{PT} = \frac{\Delta Q}{\Delta Y} \times \frac{Y}{Q}$$

$$= 0.5 \times \frac{50}{55}$$

$$= 5/11$$

(d) $Y = 70$, $P = 8$, $\varepsilon_{PT} = ?$

$$Q = 100 - 10(8) + 0.5(70)$$
$$= 55$$

$$\varepsilon_{PT} = -10 \times \frac{8}{55}$$

$$= -16/11.$$

Q15 $Q = 2000 - 20P$

(a) How many units sold at $10?

$$Q = 2000 - 20(10)$$
$$Q = 1800 \text{ units.}$$

(b) $Q = 2000$, $P = ?$

$$2000 = 2000 - 20P$$
$$20P = 0$$
$$\boxed{P = \$0}$$

(c) Equation for total revenue & marginal revenue ?

$$Q = 2000 - 20P$$

Total Revenue = $Q \times P$

Marginal Revenue = change in total revenue

* $Q = 2000 - 20P$

$$20P = 2000 - Q$$
$$P = \frac{200\cancel{0}}{2\cancel{0}} - \frac{Q}{20}$$
$$\boxed{P = 100 - 0.05\,Q}$$

Multiply "Q" on both sides

$$\boxed{Q \cdot P = 100Q - 0.05\,Q^2} \rightarrow (A)$$

Total revenue in terms of Q.

* $\frac{d}{dQ}(Q \cdot P) = \frac{d}{dQ}(100Q - 0.05Q^2)$

$$\boxed{(P \cdot Q) = 100 - 0.05(2)\,Q} \rightarrow (i)$$

Marginal Revenue in terms of Q.

* $Q = 2000 - 20P$

$\boxed{P.Q = 2000P - 20P^2} \rightarrow$ B

Total revenue in terms of price.

* $\frac{d}{dp}(P.Q) = \frac{d}{dp}(2000P - 20P^2)$

$\boxed{(P.Q)' = 2000 - 20(2)P.} \rightarrow$ (ii)

Marginal Revenue in terms of price.

(d) $P = \$70$ $TR = ?$ $MR = ?$

$T.R = 2000(70) - 20(70)^2 = 42000$

$M.R = 2000 - 20(2)(70) = -800.$

(e) Use point elasticity?

$\frac{d}{dQ}(Q) = \frac{d}{dQ}(2000 - 20P)$

$\varepsilon_{PT} = \boxed{\text{slope}} \times P/Q$

$\text{slope} = -20$

$Q = 2000 - 20(70) = 600$

$\varepsilon_{PT} = -2\cancel{0} \times 7\cancel{0}/60\cancel{0}$

$= -7/3$

$\varepsilon_{PT} = -2.33$

(f) $P = \$60$, $TR = ?$, $MR = ?$, $\varepsilon_{PT} = ?$

$$T.R = (60)2000 - 20(60)^2 = 48000$$

$$M.R = 2000 - 20(2)(60) = -400$$

$$Q = 2000 - 20(60) = 800$$

$$\text{slope} = -20$$

$$\varepsilon_{PT} = -2\cancel{0} \times \cancel{60}^{\,3} / \cancel{800}_{\,\cancel{4}\,2}$$

$$= -1.5$$

(g) Negative slope & unitary elastic

$$\varepsilon_{PT} = \text{slope} \times P/Q$$

$$-1 = -20 \times \frac{P}{2000 - 20P}$$

$$-1 = \frac{-20P}{2000 - 20P}$$

$$-2000 + 20P = -20P$$

$$4\cancel{0}P = 200\cancel{0}$$

$$\boxed{P = \$50}$$

Q14 $\varepsilon_{arc} = ?$ $\varepsilon_{cross} = ?$ $\varepsilon_{PT} = ?$ $\varepsilon_{Y} = ?$

(a) In month 1, 2 we apply cross elasticity

Price = same, Q = changes (200 - 210)
Income = same, P_{goods} = changes (130 - 135)

(b) In month 2, 3 we apply income elasticity.

Price = same, Q = changes
Income = increase, P_{goods} = same.

Income elasticity =
$$\left(\frac{Q_2 - Q_1}{(Q_2+Q_1)/2}\right) \div \left(\frac{Y_2 - Y_1}{(Y_2+Y_1)/2}\right)$$

Y_1 = Month 2 = 4000
Y_2 = Month 3 = 4200
$Q_1 = 210$
$Q_2 = 220$

(c) In month 3, 4 we use arc elasticity

Month 3 $\Rightarrow P_1 = 120$, $Q_1 = 220$
Month 4 $\Rightarrow P_2 = 110$, $Q_2 = 240$.

$$E_{arc} = \left(\frac{Q_2 - Q_1}{(Q_2+Q_1)/2}\right) \div \left(\frac{P_2 - P_1}{(P_2+P_1)/2}\right)$$

(d) In month 4, 5 we use arc elasticity

Month 4 $\Rightarrow P_1 = 110$, $Q_1 = 240$
Month 5 $\Rightarrow P_2 = 114$, $Q_2 = 230$.

(e) In month 5, 6 we use arc & cross elasticity

Practically we use cross elasticity because P change by 1 by which P goods decreases by 20 and Q change decreases so this decline in demand is due to price decline ie unit change in price affect so much to Q.

$Q_1 = 230$, $P_1 = 145$

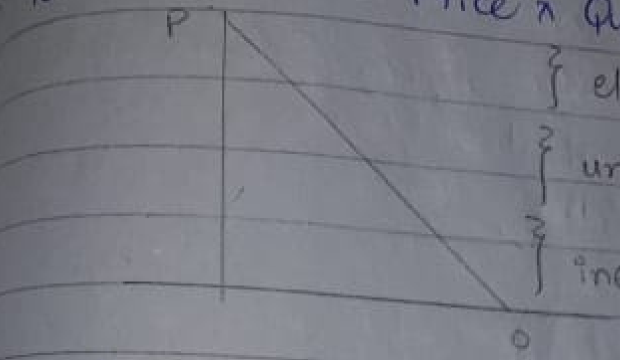
$Q_2 = 215$, $P_2 = 125$

(F) In month 6, 7 we apply income elasticity

13/Mar/19

Total Revenue & Demand Elasticity

Total Revenue = Price x Quantity

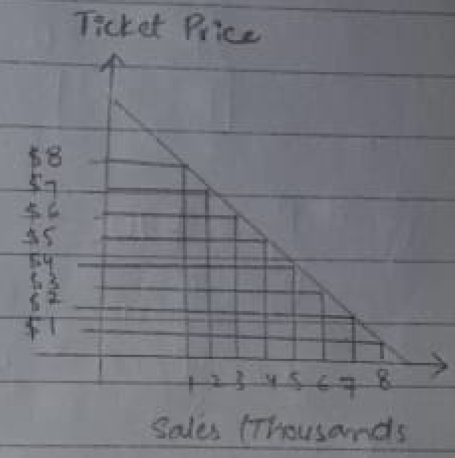


elastic $\varepsilon > -1$

unielastic $\varepsilon = -1$

inelastic $\varepsilon < -1$

→ Movie tickets & its sales:



P×Q

$TR_1 = 8 \times 1 = 8000$

$TR_2 = 7 \times 2 = 14000$

$TR_3 = 6 \times 3 = 18000$

$TR_4 = 5 \times 4 = 20000$

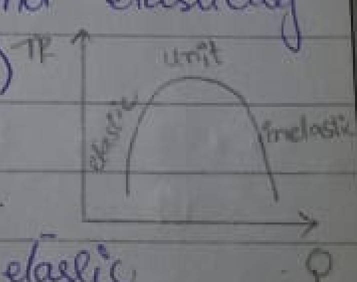
$TR_5 = 4 \times 5 = 20000$

$TR_6 = 3 \times 6 = 18000$

$TR_7 = 2 \times 7 = 14000$

$TR_8 = 1 \times 8 = 8000$

- In Unit elastic region:
  both total revenue & demand elasticity
  both are same (constant)



TR ↑ → $\varepsilon > -1$ → elastic.

TR constant → $\varepsilon = -1$ → unit elastic

TR ↓ → $\varepsilon < -1$ → inelastic

Q2 (c) ε_{PT} should be -1

$Q = 20 - 2P$

(b/c of unit elastic region of demand curve)

sol

$$\varepsilon_{PT} = \text{slope} \times P/Q$$

$$-1 = -2 \times \frac{P}{20-2P}$$

$$-20 + 2P = -2P$$

$$4P = 20$$

$$\boxed{P = \$5}$$

Q5 $P_1 = \$70$, $Q_1 = Q_{old} = 4000$

$P_2 = P_{new} = \$63$, $\varepsilon_{arc} = -2.5$

$Q_2 = Q_{new} = ?$

sol

$$\varepsilon_{arc} = \left(\frac{Q_2 - Q_1}{(Q_2+Q_1)/2}\right) \div \left(\frac{P_2 - P_1}{(P_2+P_1)/2}\right)$$

$$\Rightarrow -2.5 = \left(\frac{Q_2 - 4000}{(Q_2+4000)/2}\right) \div \left(\frac{63 - 70}{(63+70)/2}\right)$$

$$\Rightarrow \frac{-2.5 \times -2}{19} = \frac{2Q_2 - 8000}{Q_2 + 4000}$$

&

$$\Rightarrow \frac{5}{19} = \frac{2Q_2 - 8000}{Q_2 + 4000}$$

$$\Rightarrow 5(Q_2 + 4000) = 19(2Q_2 - 8000)$$

$$\Rightarrow 5Q_2 + 20000 = 38Q_2 - 152000$$

$$\Rightarrow 38Q_2 - 5Q_2 = 20000 + 152000$$

$$\Rightarrow 33Q_2 = 172000$$

$$\Rightarrow Q_2 = 5212.1212$$

TR ↑ b/c $\varepsilon > -1$

Q_{12} Formula of ε_{cross}

Q_{13} (a) $\varepsilon_{cross} = ?$

(b)

(c)

Q 16 (a)

(b)

(c) Factors other than price that affect elasticity

Q_9

19/03/19

Utility Analysis.

① Budget line approach.
② Indifference curve approach.

→ Objective Approach.

① Budget Line Approach (Budget Constraint)

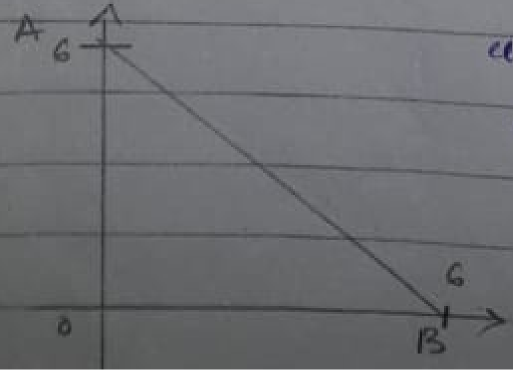
Budget constraint is a Linear curve showing fixed income and individual can have various combinations of (two) goods within the fixed income.
(also called "Price Line")

Equation : $M = P_a \cdot A + P_B \cdot B$

where

M = income
P_a = Price of X
A = Unit of X
P_B = Price of B
B = Unit of B.

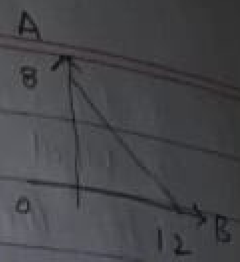
Limitation : o- Only two goods.
o- Fixed income.



"Area under the curve is attainable, beyond is not attainable.

Eg#. M = income = \$12

P_A = \$1.50 $\qquad P_B$ = \$1



For Good A : if A is not consumed then 12 units of B can be consumed

12/1 (0, 12).

For Good B : if B is not consumed then 8 units of A can be consumed

12/1.50 (8, 0)

Units of A	Units of B	$M = P_a \cdot A + P_B \cdot B$
8	0	$12 = 8 \times 1.5 + 0(1)$
6	3	$12 = 6(1.5) + 3(1)$
4	6	$12 = 4(1.5) + 1(6)$
2	9	$12 = 2(1.5) + 9(1)$
0	12	$12 = 0(1.5) + 12(1)$

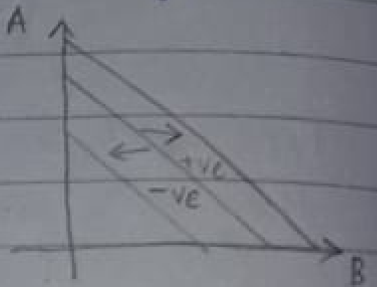
$$\text{Slope} = \frac{P_B}{P_a} = \frac{1}{1.5} = \frac{2}{3}$$

$$\frac{P_B}{P_a} = \frac{2}{3}$$

$$3P_B = 2P_a$$

→ What would be the budget constraint if there is a price or income change.

→ Shift And Movement In Budget Constraints:



① If income improves budget constraints will shift rightwards

② If income decreases budget constraints will shift leftwards

eg. If income is double

$M = \$12 \Rightarrow M = \$24$

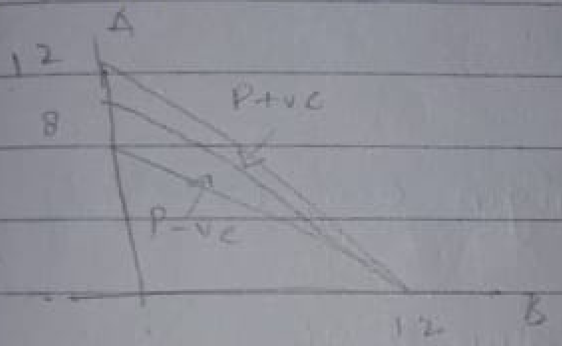
$P_A = \$1.50$, $P_B = \$1$

Units of A	Units of B	$M = P_A \cdot A + P_B \cdot B$
16	0	$24 = 16(1.50) + 0(1)$
14	3	$24 = 14(1.50) + 3(1)$
12	6	$24 = 12(1.50) + 6(1)$
10	9	$24 = 10(1.50) + 9(1)$
8	12	$24 = 8(1.50) + 12(1)$
6	15	$24 = 6(1.50) + 15(1)$
4	18	$24 = 4(1.50) + 18(1)$
2	21	$24 = 2(1.50) + 21(1)$
0	24	$24 = 0(1.50) + 24(1)$

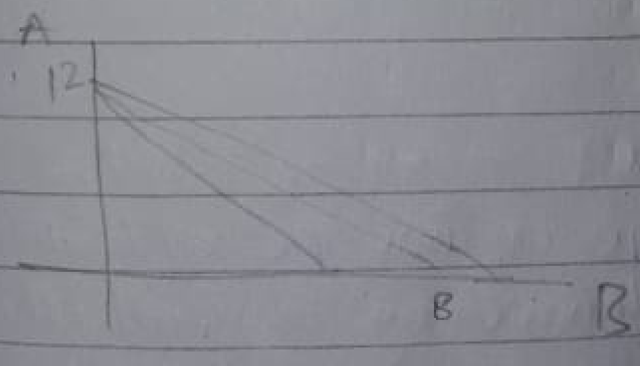
(3) If price increases, budget line rotates inward (horizontal axis) -ve

(4) If price decreases, budget line rotates outward (vertical axis). +ve

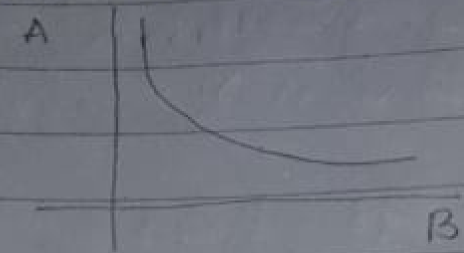
eg $P_A = 1.5 \downarrow$ to $P_A = 1$



eg $P_B = 1 \uparrow$ to $P_B = 1$



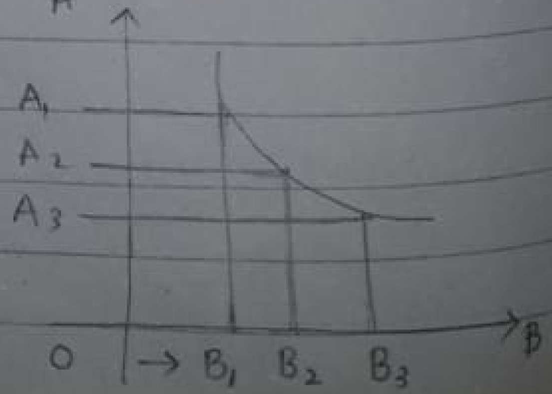
② Indifference Curve Approach:
Indifference is the locus of point indicating various combinations of two goods in a subjective manner.



Properties:
① Convex to origin and negatively sloped.
② Indifference map of the curves.
③ Consumer equilibrium on the indifference curve.

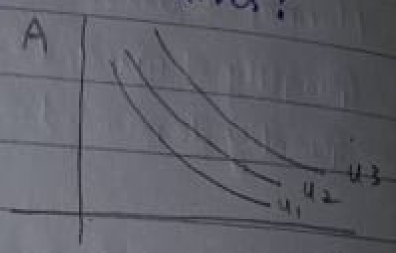
Q Why convex? because of perfect application
Convex shape explains the application of Law of diminishing marginal utility and Marginal rate of substitution.

Law of Diminishing MU.
At $(A_1, 0)$
$TuA \downarrow \rightarrow MuA \uparrow$
$TuB \uparrow \rightarrow MuB \downarrow$



Marginal Rate of Substitution
Unit A ↓ ⟶ Unit B ↑

Q Indifference map of the curves?



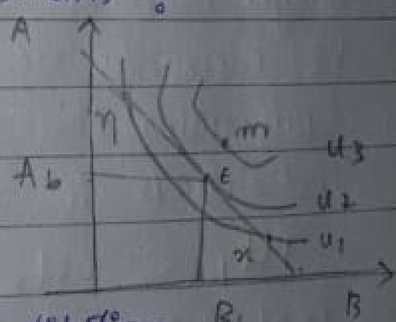
u_1 is closer to origin has min TU

u_3 is away from origin has max TU

- It explains utility level.

$TU_3 > TU_2 > TU_1$

Q Consumer Equilibrium?



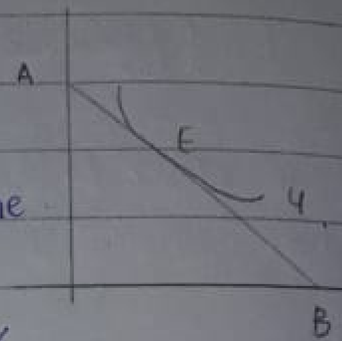
Optimum point at E
b/c
slope of budget constraint
= Marginal Rate of substitution.

Refers to the tangency and equilibrium of the consumer

$MRS = P_A/P_B$

25/Mar/19

* Consumer's Equilibrium & Deriving Demand Curve:



Normal Goods:

- Quantity demanded α 1/price.
- Quantity demanded α 1/real income

Real income = income earned/price of the good.

Real income = income earned − inflation.

* Utility At the Indifference Curve:

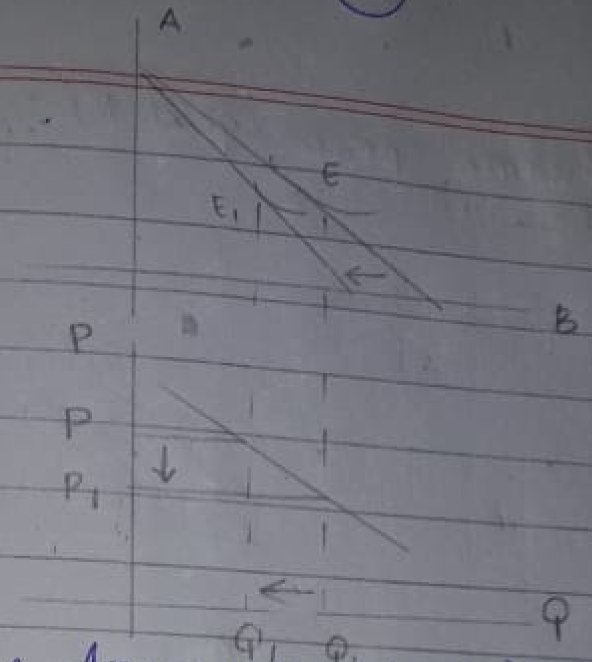
Case 1: Assume price (good B) increases.

- Budget constraint will rotate inwards on the horizontal axis
- Equilibrium changes to E_1 and now the individual is on U_1.

$$TU_1 < TU_2$$

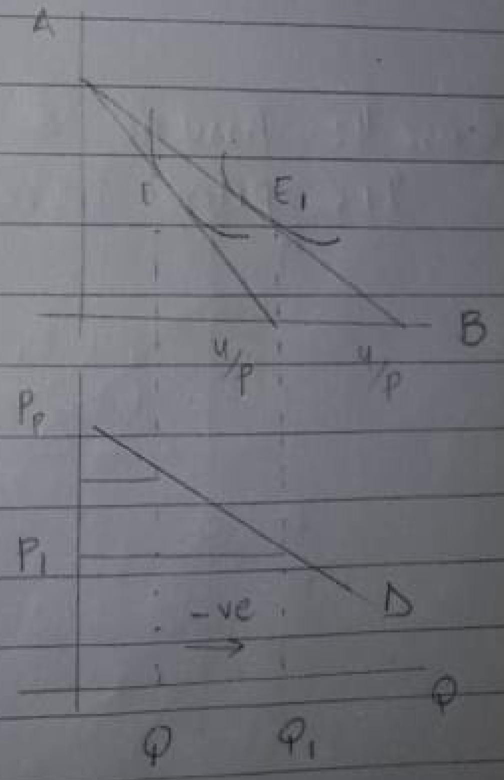
- Real income declined because of increase in price

$$M/P_1 > M/P.$$

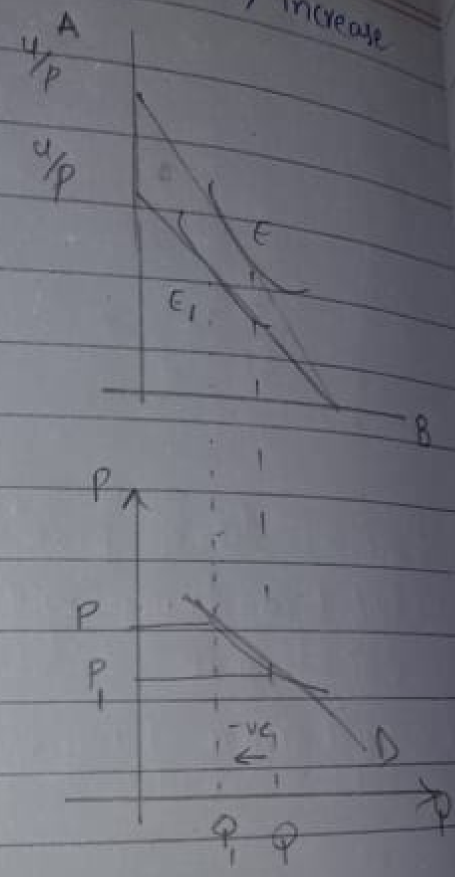


Case 2: Assume price (good B) decreases

- Budget constraint will rotate outwards on vertical axis
- Equilibrium changes to



Case 3: Assume price (good A) increase



Case 4: Assume a good B a giffen good
Its price increases.