COLLEGE OF COMPUTING, INFORMATICS AND MATHEMATICS
DIPLOMA IN COMPUTER SCIENCE

STA116: INTRODUCTION TO PROBABILITY AND STATISTICS

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Question 1

A random variable X has the following probability distribution.

X	0	1	2	3	4
P(X = x)	k	0.25	0.15	0.14	0.34

a) Show that the value of k = 0.12.

(2 marks)

b) Find P(X < 2).

(2 marks)

c) Calculate E(3X − 1) and E(X^2 − 2X).

(6 marks)

Answer:

Question 1

A random variable X has the following probability distribution.

X	0	1	2	3	4
P(X = x)	k	0.25	0.15	0.14	0.34

a) Show that the value of k = 0.12.

$\Sigma P(X = x) = 1$

$k + 0.25 + 0.15 + 0.14 + 0.34 = 1$

$k + 0.88 = 1$

$k = 1 - 0.88$

$k = 0.12$

b) Find P(X < 2)

→ Less than 2 = 0, 1

X	0	1
P(X = x)	0.12	0.25

$P(X < 2) = 0.12 + 0.25$

$= 0.37$

c) Calculate E(3X − 1) and E(X^2 − 2X)

↳ E = Expected Value

$E(X) = 0(0.12) + 1(0.25) + 2(0.15) + 3(0.14) + 4(0.34)$

$= 2.33$

$E(X^2) = 0^2(0.12) + 1^2(0.25) + 2^2(0.15) + 3^2(0.14) + 4^2(0.34)$

$= 7.55$

$E(3X - 1) = 3(2.33) - 1$

$= 5.99$

$E(X^2 - 2X) = 7.55 - 2(2.33)$

$= 2.89$

Question 2

The number of printers that are repaired at Anas Hardware has a Poisson distribution with an average of five in a day. a) Find the probability that i) exactly three printers are repaired in a day. (2 marks) ii) less than four printers are repaired in two days. (3 marks) b) Using an appropriate approximation, calculate the probability that more than thirty printers are repaired in seven consecutive days. (5 marks)

Answer:

Question 2

The number of printers that are repaired at Anas Hardware has a Poisson distribution with an average of five in a day.

a) Find the probability that

i) exactly three printers are repaired in a day.

$P(X=k) = \frac{e^{-\mu}\mu^k}{k!}$, $k = 0, 1, 2, \ldots$

$\mu = 5 \quad k = 3$

$P(x=3) = \frac{e^{-5} \cdot 5^3}{3!}$

$e^{-5} \approx 0.006737947$, $5^3 = 125$

$P(x=3) = \frac{(0.006737947)(125)}{6}$

≈ 0.140374

ii) less than four printers are repaired in two days.

$P(Y=k) = \frac{e^{-10} \cdot 10^k}{k!}$ $\quad P(Y<4)$

$P(Y=0) = \frac{e^{-10} \cdot 10^0}{0!}$

$P(Y=1) = \frac{e^{-10} \cdot 10^1}{1!}$

$P(Y=2) = \frac{e^{-10} \cdot 10^2}{2!}$

$P(Y=3) = \frac{e^{-10} \cdot 10^3}{3!}$

$P(Y<4) = 0.007567 + 0.002270 + 0.000454 + 0.000454$

$= 0.0107$

b) using Normal distribution:

$\sigma^2 = \sqrt{35} = 5.9161$

$P(X \geq 30) = P\left(Z \geq \dfrac{29.5 - 35}{5.9161}\right)$

$= P(Z \geq 0.9297)$

$= 1 - 0.1762$

$= 0.8238$

Question 3

The probability density function of a random variable X is given by

$$f(x) = \begin{cases} k(5 - x^2), & 0 < x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$$

a) Show that $k = \frac{1}{6}$.

(3 marks)

b) Find $P(1 < X < 2)$.

(3 marks)

c) Calculate $E(2X + 1)$.

(4 marks)

Answer:

Question 3

The probability density function of a random variable x is given by

$$f(x) = \begin{cases} k(5-x^2), & 0 < x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$$

a) Show that $k = \frac{1}{6}$

$\int_0^3 f(x)\,dx = 1$

$k \int_0^3 (5 - x^2)\,dx = 1$

$k\left[5x - \frac{x^3}{3}\right]_0^3 = 1$

$k\left[5(3) - \frac{(3)^3}{3}\right] - \left[5(0) - \frac{(0)^3}{3}\right] = 1$

$k[15 - 9] - 0 = 1$

$k \cdot 6 = 1$

$6k = 1$

$k = \frac{1}{6}$

b) Find $P(1 < x < 2)$

$P(1 < x < 2) = \frac{1}{6}\int_1^2 (5 - x^2)\,dx$

$= \frac{1}{6}\left[5x - \frac{x^3}{3}\right]_1^2$

$= \frac{1}{6}\left[\left(5(2) - \frac{(2)^3}{3}\right) - \left(5(1) - \frac{(1)^3}{3}\right)\right]$

$= \frac{1}{6}\left[\left(10 - \frac{8}{3}\right) - \left(5 - \frac{1}{3}\right)\right]$

$= \frac{1}{6}\left(\frac{8}{3}\right)$

$= \frac{4}{9}$ #

$E(2x+1) = E(2x) + E(1)$

$= 2E[x] + 1$

c) Calculate $E(2x+1)$

$E(x) = \int_{-\infty}^{\infty} x\, f(x)\,dx$

$= \int_0^3 x\left[\frac{1}{6}(5 - x^2)\right]dx$

$= \frac{1}{6}\int_0^3 x(5 - x^2)\,dx$

$= \frac{1}{6}\int_0^3 5x - x^3\,dx$

$= \frac{1}{6}\left[\frac{5x^2}{2} - \frac{x^4}{4}\right]_0^3$

$= \frac{1}{6}\left[\frac{5(3)^2}{2} - \frac{(3)^4}{4}\right] - \left[\frac{5(0)^2}{2} - \frac{(0)^4}{4}\right]$

$= \frac{1}{6}\left[\frac{9}{4} - 0\right]$

$E(2x+1) = E(2x) + E(1)$

$= 2E[x] + 1$

$= 2\left(\frac{3}{8}\right) + 1$

$= \frac{7}{4}$ #

Question 4

The time taken by students to complete calculus test is normally distributed with a mean of 150 minutes and a standard deviation of 8 minutes.

a) Find the probability that a student will take more than 130 minutes to complete the test.

(5 marks)

b) If 5% of the students complete the test within k minutes, find the value of k.

(5 marks)

Answer:

Question 4

a) $x \sim N(150, 8)$

$$P(x > 130) = P\left(z > \frac{130-150}{8}\right)$$

$$= P(z > -2.5)$$

$$= 1 - P(z > -2.5)$$

$$= 1 - 0.00621$$

$$= 0.99379$$

b) mean = 150 minutes, standard deviation = 8 minutes

$$P(X \leqslant k) = 0.05$$

$$P(z \leqslant k) = 0.05$$

$$z = -1.64$$

$$\frac{k-150}{8} = -1.64$$

$$k - 150 = -13.12$$

$$\therefore k = 136.88$$