CmpE 545 Artificial Neural Networks

Homework 3 – Nonlinear Logistic Regression

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NonLinear Logistic Regression

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In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
SIZE_TRAIN=40
SIZE_VAL=30
SIZE_TEST=30
```

In [2]:
```
def shuffle_dataset(x,y):
    """
    Shuffles two arrays in the same order.
    """
    randomize = np.arange(len(x))
    randomize=np.random.shuffle(randomize)
    return np.squeeze(x[randomize]),np.squeeze(y[randomize])
def read_data(file_name):
    df=pd.read_csv("data.txt",sep="  ",names=["X","y"],engine="python")
    X=df.X.values
    y=df.y.values
    return X[0:SIZE_TRAIN],y[0:SIZE_TRAIN],X[SIZE_TRAIN:SIZE_VAL+SIZE_TRAIN],y[SIZE_TRAIN:SIZE_VAL+S
def sigmoid_function( X ):
    return np.exp(-np.logaddexp(0, -X))
def calculate_Z(x,number_of_z,weights_W):
    """
    Calculates values for hidden layers
    Bias term should be added two instance before calling this function
    """
    z=np.zeros(number_of_z,dtype=np.float64)

    for h in range(number_of_z):
        sum_of_wx=0
        for j in range(len(x)):
            sum_of_wx+=(weights_W[h][j]*x[j])
        z[h]=sigmoid_function(sum_of_wx)
    return z
def calculate_Y(z,weights_T):
    """
    Calculates values for output layers
```

```
        Bias term should be added two instance before calling this function
        """
        sum_of_ih=0
        for h in range(len(z)):
             sum_of_ih+=weights_T[h]*z[h]
        return sum_of_ih

In [3]: def predict_one_instance(x,number_of_hidden_nodes,w_t,w_w):
        x=np.array([1,x])# +1 bias
        z=calculate_Z(x,number_of_hidden_nodes,w_w)
        z=np.insert(z,0,1)# +1 bias
        return calculate_Y(z,w_t)

    def predict(testX,number_of_hidden_nodes,w_t,w_w):
        """
        Predicts given dataset inputs.
        """
        return [predict_one_instance(x,number_of_hidden_nodes,w_t,w_w) for x in testX]
    def loss_func(predictions,targets):
        """
        Computes binary cross entropy
        """
        #predictions = np.clip(predictions, epsilon, 1. - epsilon)
        total=0
        for y,r in zip(predictions,targets):
            total+=np.square(r-y)
        return total/2

In [4]: X_train,y_train,X_val,y_val,X_test,y_test=read_data("data.txt")
    epochs=250

    number_of_hidden_nodes=50
    metrics=[]
    weights_W_all_h_nodes=[]
    weights_T_all_h_nodes=[]
    z_s=[]
    def fit(trainX,trainY,valX,valY,h_nodes,epochs):
        learning_rate=0.001
        momentum_rate=0.01
        np.random.seed(1000)
        weights_W=np.random.uniform(-0.001,0.001,(h_nodes,2)) #initialize W ,2(input+bias)-#hidden_laye
        weights_T=np.random.uniform(-0.001,0.001,h_nodes+1)#initialize T
        for m in range(epochs):
            shuffled_X,shuffled_Y=shuffle_dataset(trainX,trainY)
            grad_Th=np.zeros(h_nodes+1,dtype=np.float64)
            grad_Whj=np.zeros((h_nodes,2),dtype=np.float64)
            for x,r in zip(shuffled_X,shuffled_Y):
```

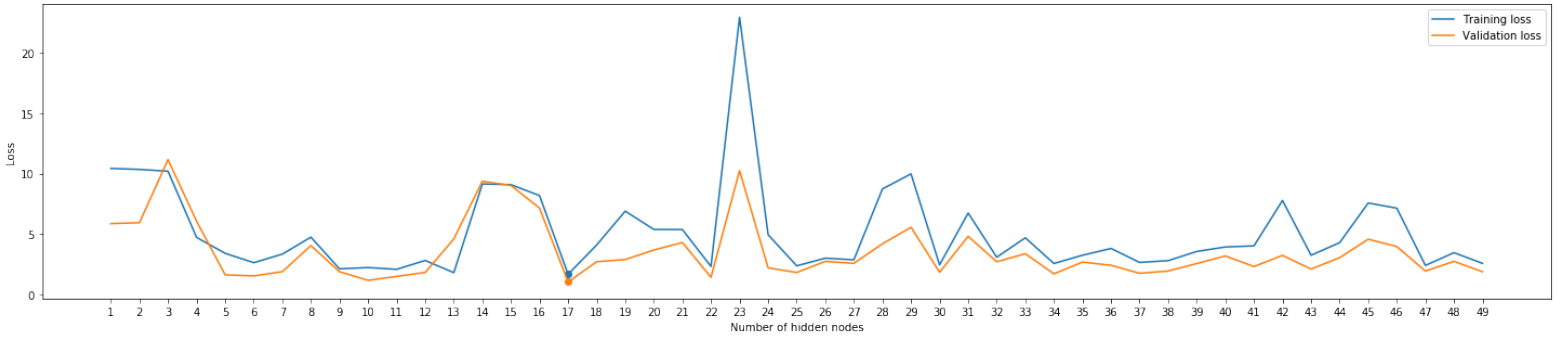
```
                """storing previous gradients for momentum"""
                prev_grad_Th=np.copy(grad_Th)
                prev_grad_Whj=np.copy(grad_Whj)

                x=np.array([1,x])# +1 bias
                z=calculate_Z(x,h_nodes,weights_W)
                z=np.insert(z,0,1)# +1 bias
                y=calculate_Y(z,weights_T)

                """Backpropagation"""
                #1-Calculate gradients
                for h in range(len(grad_Th)):
                    grad_Th[h]=learning_rate*(r-y)*z[h]

                for h in range(h_nodes):
                    for j in range(len(x)):
                        grad_Whj[h][j]=learning_rate*(r-y)*weights_T[h]*z[h]*(1-z[h])*x[j]

                #2-Update weights
                for h in range(len(grad_Th)):
                    weights_T[h]+=(grad_Th[h]+(momentum_rate*prev_grad_Th[h]))
                for h in range(h_nodes):
                    for j in range(len(x)):
                        weights_W[h][j]+=(grad_Whj[h][j]+(momentum_rate*prev_grad_Whj[h][j]))

            train_loss=loss_func(predict(shuffled_X,h_nodes,weights_T,weights_W),shuffled_Y)
            val_loss=loss_func(predict(valX,h_nodes,weights_T,weights_W),valY)
            learning_rate+=0.001
        return train_loss,val_loss,weights_W,weights_T,z
    for h_nodes in range(1,number_of_hidden_nodes):
        train_loss,val_loss,weights_W,weights_T,z=fit(X_train,y_train,X_val,y_val,h_nodes,epochs)
        print("Number of hidden nodes(without bias) "+str(h_nodes)+" Training Error: %.4f"%train_loss +"
        metrics.append([train_loss,val_loss])
        z_s.append(z)
        weights_W_all_h_nodes.append(weights_W)
        weights_T_all_h_nodes.append(weights_T)
    metrics=np.asarray(metrics)
```

```
Number of hidden nodes(without bias) 1 Training Error: 10.4292 Validation Error: 5.8629
Number of hidden nodes(without bias) 2 Training Error: 10.3492 Validation Error: 5.9395
Number of hidden nodes(without bias) 3 Training Error: 10.1978 Validation Error: 11.1586
Number of hidden nodes(without bias) 4 Training Error: 4.7195 Validation Error: 6.0576
Number of hidden nodes(without bias) 5 Training Error: 3.4046 Validation Error: 1.6226
Number of hidden nodes(without bias) 6 Training Error: 2.6302 Validation Error: 1.5363
Number of hidden nodes(without bias) 7 Training Error: 3.3387 Validation Error: 1.8868
Number of hidden nodes(without bias) 8 Training Error: 4.7421 Validation Error: 4.0462
Number of hidden nodes(without bias) 9 Training Error: 2.1209 Validation Error: 1.8855
Number of hidden nodes(without bias) 10 Training Error: 2.2276 Validation Error: 1.1680
```

```
Number of hidden nodes(without bias) 11 Training Error: 2.0825 Validation Error: 1.4954
Number of hidden nodes(without bias) 12 Training Error: 2.8142 Validation Error: 1.8245
Number of hidden nodes(without bias) 13 Training Error: 1.8046 Validation Error: 4.5911
Number of hidden nodes(without bias) 14 Training Error: 9.1418 Validation Error: 9.3676
Number of hidden nodes(without bias) 15 Training Error: 9.0892 Validation Error: 9.0235
Number of hidden nodes(without bias) 16 Training Error: 8.1850 Validation Error: 7.1506
Number of hidden nodes(without bias) 17 Training Error: 1.6725 Validation Error: 1.0348
Number of hidden nodes(without bias) 18 Training Error: 4.0973 Validation Error: 2.7090
Number of hidden nodes(without bias) 19 Training Error: 6.8970 Validation Error: 2.8910
Number of hidden nodes(without bias) 20 Training Error: 5.3856 Validation Error: 3.6780
Number of hidden nodes(without bias) 21 Training Error: 5.3711 Validation Error: 4.3019
Number of hidden nodes(without bias) 22 Training Error: 2.3212 Validation Error: 1.4216
Number of hidden nodes(without bias) 23 Training Error: 22.9431 Validation Error: 10.2520
Number of hidden nodes(without bias) 24 Training Error: 4.9439 Validation Error: 2.2018
Number of hidden nodes(without bias) 25 Training Error: 2.3702 Validation Error: 1.8191
Number of hidden nodes(without bias) 26 Training Error: 3.0038 Validation Error: 2.7371
Number of hidden nodes(without bias) 27 Training Error: 2.8667 Validation Error: 2.5754
Number of hidden nodes(without bias) 28 Training Error: 8.7330 Validation Error: 4.1918
Number of hidden nodes(without bias) 29 Training Error: 9.9797 Validation Error: 5.5620
Number of hidden nodes(without bias) 30 Training Error: 2.4510 Validation Error: 1.8361
Number of hidden nodes(without bias) 31 Training Error: 6.7426 Validation Error: 4.8116
Number of hidden nodes(without bias) 32 Training Error: 3.0822 Validation Error: 2.7066
Number of hidden nodes(without bias) 33 Training Error: 4.6909 Validation Error: 3.3735
Number of hidden nodes(without bias) 34 Training Error: 2.5615 Validation Error: 1.7085
Number of hidden nodes(without bias) 35 Training Error: 3.2472 Validation Error: 2.6772
Number of hidden nodes(without bias) 36 Training Error: 3.8069 Validation Error: 2.4167
Number of hidden nodes(without bias) 37 Training Error: 2.6502 Validation Error: 1.7523
Number of hidden nodes(without bias) 38 Training Error: 2.8004 Validation Error: 1.9424
Number of hidden nodes(without bias) 39 Training Error: 3.5599 Validation Error: 2.5577
Number of hidden nodes(without bias) 40 Training Error: 3.9307 Validation Error: 3.1904
Number of hidden nodes(without bias) 41 Training Error: 4.0190 Validation Error: 2.3113
Number of hidden nodes(without bias) 42 Training Error: 7.7795 Validation Error: 3.2349
Number of hidden nodes(without bias) 43 Training Error: 3.2461 Validation Error: 2.1023
Number of hidden nodes(without bias) 44 Training Error: 4.2894 Validation Error: 3.0476
Number of hidden nodes(without bias) 45 Training Error: 7.5711 Validation Error: 4.5741
Number of hidden nodes(without bias) 46 Training Error: 7.1411 Validation Error: 3.9664
Number of hidden nodes(without bias) 47 Training Error: 2.4039 Validation Error: 1.9382
Number of hidden nodes(without bias) 48 Training Error: 3.4583 Validation Error: 2.7403
Number of hidden nodes(without bias) 49 Training Error: 2.5756 Validation Error: 1.8799
```

In [5]:
```
fig=plt.figure(figsize=(25,5))
plt.xlabel("Number of hidden nodes")
plt.ylabel("Loss")
plt.plot(range(1,number_of_hidden_nodes),metrics[:,0],label="Training loss")
plt.plot(range(1,number_of_hidden_nodes),metrics[:,1],label="Validation loss")
plt.xticks(np.arange(1, number_of_hidden_nodes, 1.0))
```

```
min_training_error_idx=np.argmin(metrics[:,0])
min_val_error_idx=np.argmin(metrics[:,1])

plt.scatter(min_training_error_idx+1,metrics[min_training_error_idx,0])
plt.scatter(min_val_error_idx+1,metrics[min_val_error_idx,1])

plt.legend()

plt.show()
```

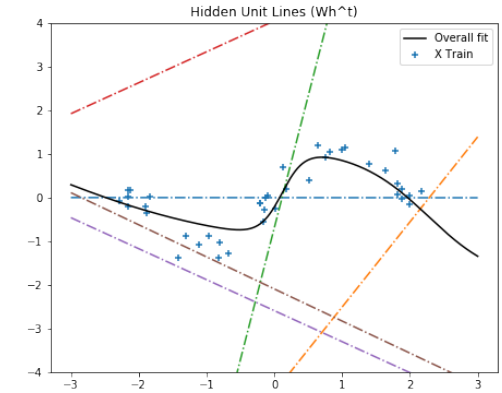


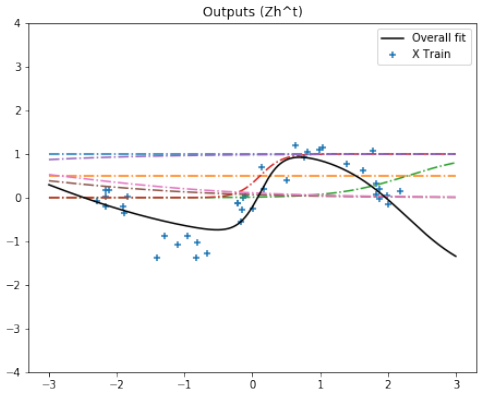
1 Choosing number of hidden nodes

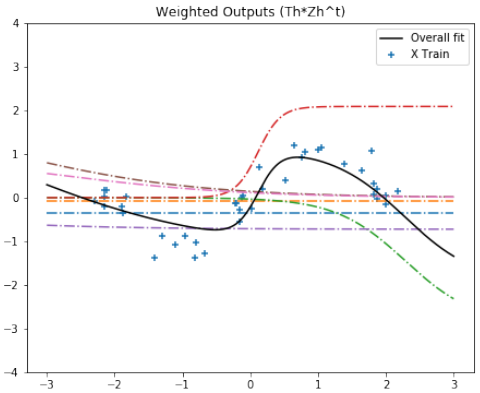
Each training was done in 100 epochs As we can see from the plot and results, 5 number of hidden nodes (6 with bias) seems a good fit.
Number of hidden nodes(without bias) 6 Training Error: 2.6302 Validation Error: 1.5363
Good fit = 6 Hidden Nodes
Error has been reducing till that point, so 6 hidden nodes seems a good fit.
Underfit = 2 Hidden Nodes
2 hidden nodes shows a good example of underfitting.
Overfit = 40 Hidden Nodes
40 hidden nodes shows a good example of overfitting. As we can see from the plot, training loss have been reducing but, validation loss have been started increasing.

```
In [6]: def plot_props_of_hidden_layer(h_node,w_T,w_w):
            linspace=np.linspace(-3,3,num=500)

            fig=plt.figure(figsize=(25,20))

            ax=fig.add_subplot(331)
            ax.scatter(X_train,y_train,label="X Train",marker="+")

            y_ax=np.dot(w_w,np.array([np.ones(len(linspace)),linspace]))
            for line in y_ax:
```

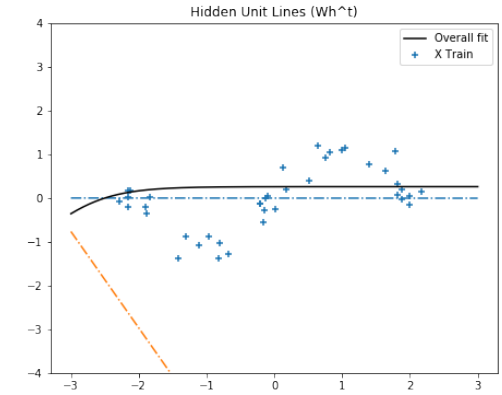
```
        plt.plot(linspace,line,'-.')
    plt.plot(linspace,predict(linspace,h_node,w_T,w_w),c="black",label="Overall fit")
    ax.set_ybound(lower=-4,upper=4)
    ax.set_title("Hidden Unit Lines (Wh^t)")

    plt.legend()

    ax=fig.add_subplot(332)
    ax.scatter(X_train,y_train,label="X Train",marker="+")
    z=[]
    for x in linspace:
        x=np.array([1,x])# +1 bias
        z_i=calculate_Z(x,h_node,w_w)
        z.append(np.insert(z_i,0,1))# +1 bias

    for line in np.array(z).T:
        plt.plot(linspace,line,'-.')
    plt.plot(linspace,predict(linspace,h_node,w_T,w_w),c="black",label="Overall fit")
    ax.set_ybound(lower=-4,upper=4)
    ax.set_title("Outputs (Zh^t)")

    plt.legend()

    ax=fig.add_subplot(333)
    ax.scatter(X_train,y_train,label="X Train",marker="+")
    for line in np.array(w_T*z).T:
        plt.plot(linspace,line,'-.')
    plt.plot(linspace,predict(linspace,h_node,w_T,w_w),c="black",label="Overall fit")
    ax.set_ybound(lower=-4,upper=4)
    ax.set_title("Weighted Outputs (Th*Zh^t)")
    plt.legend()

    plt.text(-10,-5,"Number of hidden units: "+str(h_node),fontsize=20)
    plt.show()
h_node=6
plot_props_of_hidden_layer(h_node,weights_T_all_h_nodes[h_node-1],weights_W_all_h_nodes[h_node-1])
h_node=2
plot_props_of_hidden_layer(h_node,weights_T_all_h_nodes[h_node-1],weights_W_all_h_nodes[h_node-1])
h_node=40
plot_props_of_hidden_layer(h_node,weights_T_all_h_nodes[h_node-1],weights_W_all_h_nodes[h_node-1])
```

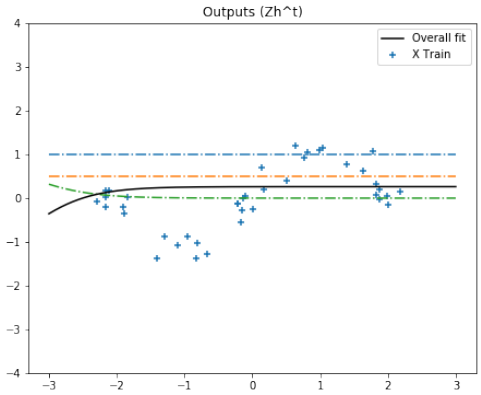


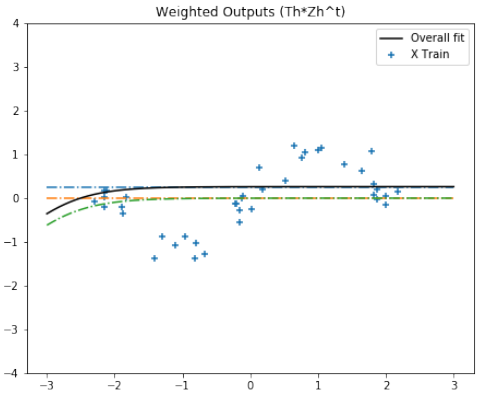




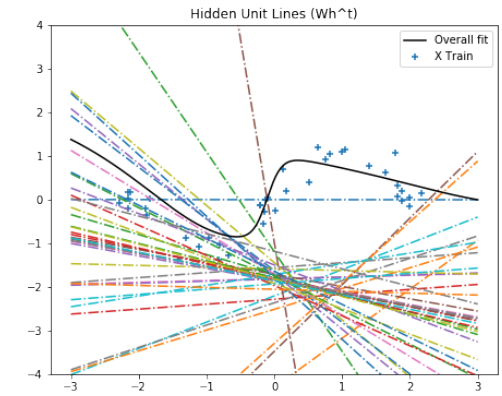
Number of hidden units: 6

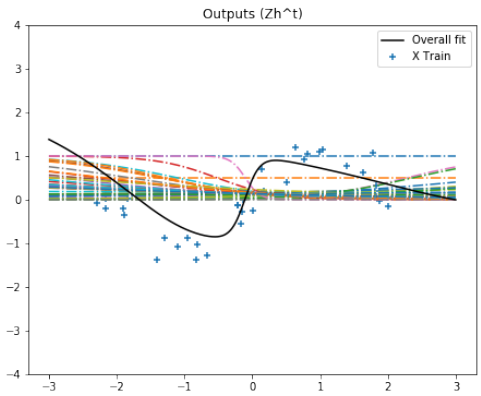


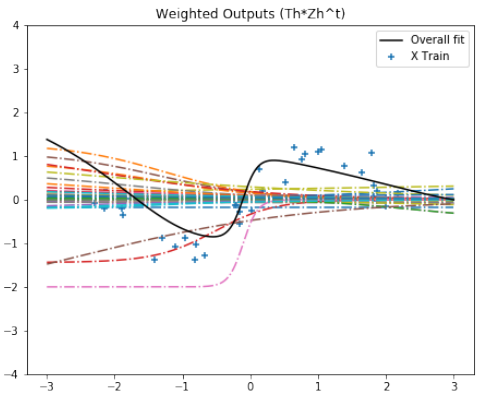




Number of hidden units: 2





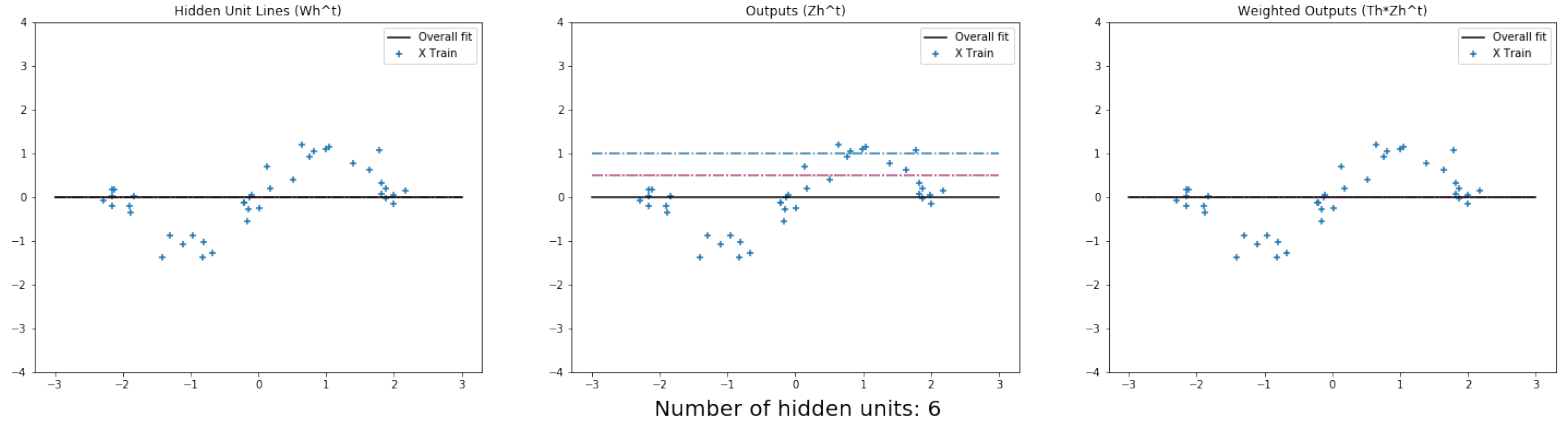


Number of hidden units: 40

In [7]:
```
learning_rate=0.001
momentum_rate=0.01
h_nodes=6
weights_W=np.random.uniform(-0.001,0.001,(h_nodes,2)) #initialize W ,2(input+bias)-#hidden_layers
weights_T=np.random.uniform(-0.001,0.001,h_nodes+1)#initialize T
print("Beginning...")
plot_props_of_hidden_layer(h_nodes,weights_T,weights_W)


epochs=250
for m in range(epochs):
    shuffled_X,shuffled_Y=shuffle_dataset(X_train,y_train)
    grad_Th=np.zeros(h_nodes+1,dtype=np.float64)
    grad_Whj=np.zeros((h_nodes,2),dtype=np.float64)
    if (m==124):
        print("Middle...")
        plot_props_of_hidden_layer(h_nodes,weights_T,weights_W)
    for x,r in zip(shuffled_X,shuffled_Y):

        """storing previous gradients for momentum"""
        prev_grad_Th=np.copy(grad_Th)
        prev_grad_Whj=np.copy(grad_Whj)

        x=np.array([1,x])# +1 bias
        z=calculate_Z(x,h_nodes,weights_W)
        z=np.insert(z,0,1)# +1 bias
        y=calculate_Y(z,weights_T)

        """Backpropagation"""
        #1-Calculate gradients
        for h in range(len(grad_Th)):
            grad_Th[h]=learning_rate*(r-y)*z[h]

        for h in range(h_nodes):
            for j in range(len(x)):
                grad_Whj[h][j]=learning_rate*(r-y)*weights_T[h]*z[h]*(1-z[h])*x[j]

        #2-Update weights
        for h in range(len(grad_Th)):
            weights_T[h]+=(grad_Th[h]+(momentum_rate*prev_grad_Th[h]))
        for h in range(h_nodes):
            for j in range(len(x)):
                weights_W[h][j]+=(grad_Whj[h][j]+(momentum_rate*prev_grad_Whj[h][j]))

    train_loss=loss_func(predict(shuffled_X,h_nodes,weights_T,weights_W),shuffled_Y)
    val_loss=loss_func(predict(X_val,h_nodes,weights_T,weights_W),y_val)
    print("Epoch: "+str(m+1)+" Training Error: %.4f"%train_loss +" Validation Error: %.4f"%val_loss
```

```
            learning_rate+=0.001
        print("End")
        plot_props_of_hidden_layer(h_nodes,weights_T,weights_W)
        print("Test set loss: %.4f"%loss_func(predict(X_test,h_nodes,weights_T,weights_W),y_test))
```

```
Beginning...
```




```
Epoch: 1 Training Error: 9.3913 Validation Error: 5.1536
Epoch: 2 Training Error: 9.3906 Validation Error: 5.1523
Epoch: 3 Training Error: 9.3902 Validation Error: 5.1508
Epoch: 4 Training Error: 9.3904 Validation Error: 5.1496
Epoch: 5 Training Error: 9.3918 Validation Error: 5.1491
Epoch: 6 Training Error: 9.3943 Validation Error: 5.1494
Epoch: 7 Training Error: 9.3978 Validation Error: 5.1505
Epoch: 8 Training Error: 9.4020 Validation Error: 5.1523
Epoch: 9 Training Error: 9.4069 Validation Error: 5.1546
Epoch: 10 Training Error: 9.4124 Validation Error: 5.1575
Epoch: 11 Training Error: 9.4186 Validation Error: 5.1608
Epoch: 12 Training Error: 9.4253 Validation Error: 5.1646
Epoch: 13 Training Error: 9.4327 Validation Error: 5.1689
Epoch: 14 Training Error: 9.4407 Validation Error: 5.1738
Epoch: 15 Training Error: 9.4494 Validation Error: 5.1791
Epoch: 16 Training Error: 9.4587 Validation Error: 5.1849
Epoch: 17 Training Error: 9.4686 Validation Error: 5.1913
Epoch: 18 Training Error: 9.4792 Validation Error: 5.1981
Epoch: 19 Training Error: 9.4904 Validation Error: 5.2054
Epoch: 20 Training Error: 9.5022 Validation Error: 5.2131
Epoch: 21 Training Error: 9.5145 Validation Error: 5.2213
Epoch: 22 Training Error: 9.5274 Validation Error: 5.2300
Epoch: 23 Training Error: 9.5407 Validation Error: 5.2390
Epoch: 24 Training Error: 9.5545 Validation Error: 5.2485
```

```
Epoch: 25 Training Error: 9.5688 Validation Error: 5.2584
Epoch: 26 Training Error: 9.5834 Validation Error: 5.2686
Epoch: 27 Training Error: 9.5984 Validation Error: 5.2791
Epoch: 28 Training Error: 9.6137 Validation Error: 5.2900
Epoch: 29 Training Error: 9.6292 Validation Error: 5.3012
Epoch: 30 Training Error: 9.6448 Validation Error: 5.3126
Epoch: 31 Training Error: 9.6606 Validation Error: 5.3243
Epoch: 32 Training Error: 9.6763 Validation Error: 5.3362
Epoch: 33 Training Error: 9.6920 Validation Error: 5.3483
Epoch: 34 Training Error: 9.7075 Validation Error: 5.3606
Epoch: 35 Training Error: 9.7226 Validation Error: 5.3730
Epoch: 36 Training Error: 9.7371 Validation Error: 5.3856
Epoch: 37 Training Error: 9.7508 Validation Error: 5.3983
Epoch: 38 Training Error: 9.7634 Validation Error: 5.4110
Epoch: 39 Training Error: 9.7743 Validation Error: 5.4237
Epoch: 40 Training Error: 9.7829 Validation Error: 5.4365
Epoch: 41 Training Error: 9.7883 Validation Error: 5.4491
Epoch: 42 Training Error: 9.7894 Validation Error: 5.4615
Epoch: 43 Training Error: 9.7842 Validation Error: 5.4737
Epoch: 44 Training Error: 9.7706 Validation Error: 5.4856
Epoch: 45 Training Error: 9.7453 Validation Error: 5.4971
Epoch: 46 Training Error: 9.7041 Validation Error: 5.5085
Epoch: 47 Training Error: 9.6421 Validation Error: 5.5204
Epoch: 48 Training Error: 9.5536 Validation Error: 5.5341
Epoch: 49 Training Error: 9.4338 Validation Error: 5.5525
Epoch: 50 Training Error: 9.2805 Validation Error: 5.5806
Epoch: 51 Training Error: 9.0970 Validation Error: 5.6261
Epoch: 52 Training Error: 8.8944 Validation Error: 5.6980
Epoch: 53 Training Error: 8.6909 Validation Error: 5.8042
Epoch: 54 Training Error: 8.5070 Validation Error: 5.9474
Epoch: 55 Training Error: 8.3591 Validation Error: 6.1216
Epoch: 56 Training Error: 8.2537 Validation Error: 6.3135
Epoch: 57 Training Error: 8.1879 Validation Error: 6.5064
Epoch: 58 Training Error: 8.1530 Validation Error: 6.6858
Epoch: 59 Training Error: 8.1391 Validation Error: 6.8426
Epoch: 60 Training Error: 8.1376 Validation Error: 6.9735
Epoch: 61 Training Error: 8.1427 Validation Error: 7.0794
Epoch: 62 Training Error: 8.1508 Validation Error: 7.1638
Epoch: 63 Training Error: 8.1600 Validation Error: 7.2309
Epoch: 64 Training Error: 8.1692 Validation Error: 7.2846
Epoch: 65 Training Error: 8.1780 Validation Error: 7.3284
Epoch: 66 Training Error: 8.1863 Validation Error: 7.3650
Epoch: 67 Training Error: 8.1940 Validation Error: 7.3966
Epoch: 68 Training Error: 8.2012 Validation Error: 7.4246
Epoch: 69 Training Error: 8.2079 Validation Error: 7.4502
Epoch: 70 Training Error: 8.2142 Validation Error: 7.4741
Epoch: 71 Training Error: 8.2201 Validation Error: 7.4968
Epoch: 72 Training Error: 8.2257 Validation Error: 7.5189
```

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Epoch: 73 Training Error: 8.2310 Validation Error: 7.5404
Epoch: 74 Training Error: 8.2361 Validation Error: 7.5617
Epoch: 75 Training Error: 8.2410 Validation Error: 7.5827
Epoch: 76 Training Error: 8.2457 Validation Error: 7.6037
Epoch: 77 Training Error: 8.2504 Validation Error: 7.6247
Epoch: 78 Training Error: 8.2550 Validation Error: 7.6456
Epoch: 79 Training Error: 8.2596 Validation Error: 7.6666
Epoch: 80 Training Error: 8.2642 Validation Error: 7.6876
Epoch: 81 Training Error: 8.2688 Validation Error: 7.7086
Epoch: 82 Training Error: 8.2735 Validation Error: 7.7296
Epoch: 83 Training Error: 8.2783 Validation Error: 7.7507
Epoch: 84 Training Error: 8.2831 Validation Error: 7.7717
Epoch: 85 Training Error: 8.2881 Validation Error: 7.7927
Epoch: 86 Training Error: 8.2931 Validation Error: 7.8137
Epoch: 87 Training Error: 8.2983 Validation Error: 7.8346
Epoch: 88 Training Error: 8.3036 Validation Error: 7.8553
Epoch: 89 Training Error: 8.3090 Validation Error: 7.8759
Epoch: 90 Training Error: 8.3145 Validation Error: 7.8963
Epoch: 91 Training Error: 8.3200 Validation Error: 7.9165
Epoch: 92 Training Error: 8.3257 Validation Error: 7.9365
Epoch: 93 Training Error: 8.3315 Validation Error: 7.9562
Epoch: 94 Training Error: 8.3373 Validation Error: 7.9756
Epoch: 95 Training Error: 8.3432 Validation Error: 7.9947
Epoch: 96 Training Error: 8.3492 Validation Error: 8.0135
Epoch: 97 Training Error: 8.3552 Validation Error: 8.0320
Epoch: 98 Training Error: 8.3613 Validation Error: 8.0501
Epoch: 99 Training Error: 8.3675 Validation Error: 8.0680
Epoch: 100 Training Error: 8.3737 Validation Error: 8.0855
Epoch: 101 Training Error: 8.3799 Validation Error: 8.1028
Epoch: 102 Training Error: 8.3862 Validation Error: 8.1198
Epoch: 103 Training Error: 8.3927 Validation Error: 8.1366
Epoch: 104 Training Error: 8.3992 Validation Error: 8.1532
Epoch: 105 Training Error: 8.4058 Validation Error: 8.1696
Epoch: 106 Training Error: 8.4125 Validation Error: 8.1859
Epoch: 107 Training Error: 8.4194 Validation Error: 8.2021
Epoch: 108 Training Error: 8.4265 Validation Error: 8.2183
Epoch: 109 Training Error: 8.4337 Validation Error: 8.2344
Epoch: 110 Training Error: 8.4411 Validation Error: 8.2506
Epoch: 111 Training Error: 8.4488 Validation Error: 8.2668
Epoch: 112 Training Error: 8.4566 Validation Error: 8.2831
Epoch: 113 Training Error: 8.4647 Validation Error: 8.2994
Epoch: 114 Training Error: 8.4731 Validation Error: 8.3159
Epoch: 115 Training Error: 8.4817 Validation Error: 8.3325
Epoch: 116 Training Error: 8.4906 Validation Error: 8.3492
Epoch: 117 Training Error: 8.4997 Validation Error: 8.3661
Epoch: 118 Training Error: 8.5092 Validation Error: 8.3831
Epoch: 119 Training Error: 8.5189 Validation Error: 8.4003
Epoch: 120 Training Error: 8.5289 Validation Error: 8.4176
```

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Epoch: 121 Training Error: 8.5392 Validation Error: 8.4350
Epoch: 122 Training Error: 8.5497 Validation Error: 8.4526
Epoch: 123 Training Error: 8.5605 Validation Error: 8.4703
Epoch: 124 Training Error: 8.5715 Validation Error: 8.4881
Middle...
```

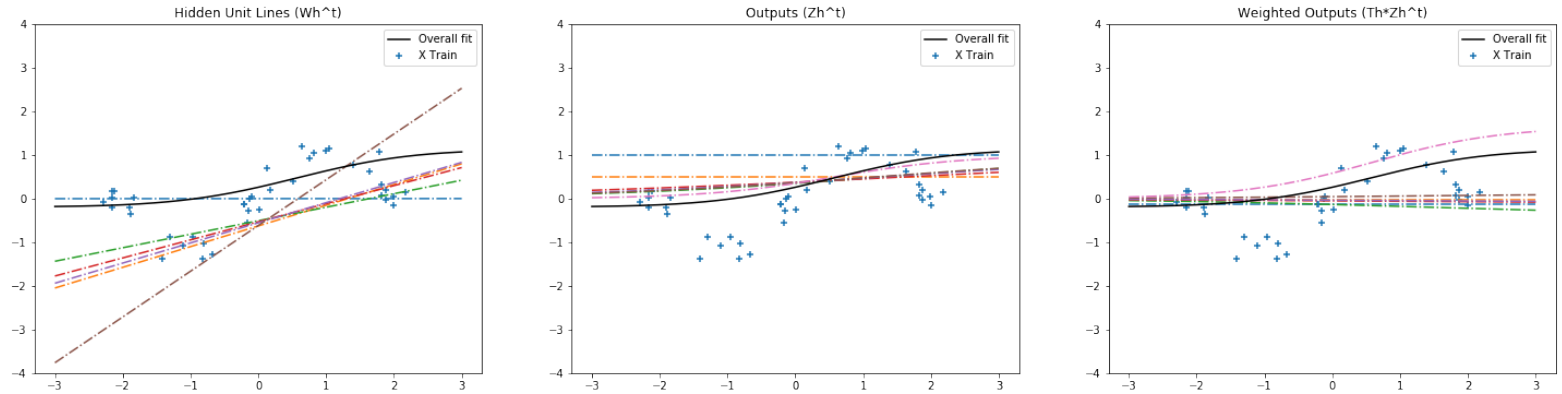


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Epoch: 125 Training Error: 8.5827 Validation Error: 8.5059
Epoch: 126 Training Error: 8.5942 Validation Error: 8.5237
Epoch: 127 Training Error: 8.6058 Validation Error: 8.5416
Epoch: 128 Training Error: 8.6176 Validation Error: 8.5593
Epoch: 129 Training Error: 8.6295 Validation Error: 8.5770
Epoch: 130 Training Error: 8.6414 Validation Error: 8.5945
Epoch: 131 Training Error: 8.6534 Validation Error: 8.6118
Epoch: 132 Training Error: 8.6654 Validation Error: 8.6287
Epoch: 133 Training Error: 8.6774 Validation Error: 8.6453
Epoch: 134 Training Error: 8.6893 Validation Error: 8.6615
Epoch: 135 Training Error: 8.7010 Validation Error: 8.6771
Epoch: 136 Training Error: 8.7125 Validation Error: 8.6921
Epoch: 137 Training Error: 8.7238 Validation Error: 8.7063
Epoch: 138 Training Error: 8.7348 Validation Error: 8.7198
Epoch: 139 Training Error: 8.7454 Validation Error: 8.7324
Epoch: 140 Training Error: 8.7556 Validation Error: 8.7440
Epoch: 141 Training Error: 8.7653 Validation Error: 8.7547
Epoch: 142 Training Error: 8.7746 Validation Error: 8.7643
Epoch: 143 Training Error: 8.7833 Validation Error: 8.7728
Epoch: 144 Training Error: 8.7915 Validation Error: 8.7803
Epoch: 145 Training Error: 8.7991 Validation Error: 8.7867
Epoch: 146 Training Error: 8.8062 Validation Error: 8.7922
Epoch: 147 Training Error: 8.8127 Validation Error: 8.7967
Epoch: 148 Training Error: 8.8186 Validation Error: 8.8005
```

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Epoch: 149 Training Error: 8.8241 Validation Error: 8.8037
Epoch: 150 Training Error: 8.8291 Validation Error: 8.8064
Epoch: 151 Training Error: 8.8336 Validation Error: 8.8087
Epoch: 152 Training Error: 8.8379 Validation Error: 8.8109
Epoch: 153 Training Error: 8.8418 Validation Error: 8.8131
Epoch: 154 Training Error: 8.8454 Validation Error: 8.8155
Epoch: 155 Training Error: 8.8488 Validation Error: 8.8181
Epoch: 156 Training Error: 8.8521 Validation Error: 8.8211
Epoch: 157 Training Error: 8.8552 Validation Error: 8.8246
Epoch: 158 Training Error: 8.8582 Validation Error: 8.8286
Epoch: 159 Training Error: 8.8611 Validation Error: 8.8330
Epoch: 160 Training Error: 8.8639 Validation Error: 8.8379
Epoch: 161 Training Error: 8.8665 Validation Error: 8.8433
Epoch: 162 Training Error: 8.8690 Validation Error: 8.8489
Epoch: 163 Training Error: 8.8714 Validation Error: 8.8548
Epoch: 164 Training Error: 8.8735 Validation Error: 8.8607
Epoch: 165 Training Error: 8.8753 Validation Error: 8.8666
Epoch: 166 Training Error: 8.8768 Validation Error: 8.8721
Epoch: 167 Training Error: 8.8779 Validation Error: 8.8772
Epoch: 168 Training Error: 8.8785 Validation Error: 8.8817
Epoch: 169 Training Error: 8.8785 Validation Error: 8.8853
Epoch: 170 Training Error: 8.8778 Validation Error: 8.8878
Epoch: 171 Training Error: 8.8764 Validation Error: 8.8890
Epoch: 172 Training Error: 8.8742 Validation Error: 8.8885
Epoch: 173 Training Error: 8.8710 Validation Error: 8.8862
Epoch: 174 Training Error: 8.8667 Validation Error: 8.8817
Epoch: 175 Training Error: 8.8613 Validation Error: 8.8747
Epoch: 176 Training Error: 8.8546 Validation Error: 8.8649
Epoch: 177 Training Error: 8.8466 Validation Error: 8.8518
Epoch: 178 Training Error: 8.8371 Validation Error: 8.8352
Epoch: 179 Training Error: 8.8260 Validation Error: 8.8146
Epoch: 180 Training Error: 8.8134 Validation Error: 8.7896
Epoch: 181 Training Error: 8.7990 Validation Error: 8.7598
Epoch: 182 Training Error: 8.7830 Validation Error: 8.7250
Epoch: 183 Training Error: 8.7651 Validation Error: 8.6849
Epoch: 184 Training Error: 8.7456 Validation Error: 8.6393
Epoch: 185 Training Error: 8.7244 Validation Error: 8.5883
Epoch: 186 Training Error: 8.7016 Validation Error: 8.5320
Epoch: 187 Training Error: 8.6775 Validation Error: 8.4707
Epoch: 188 Training Error: 8.6522 Validation Error: 8.4050
Epoch: 189 Training Error: 8.6261 Validation Error: 8.3360
Epoch: 190 Training Error: 8.5994 Validation Error: 8.2646
Epoch: 191 Training Error: 8.5727 Validation Error: 8.1923
Epoch: 192 Training Error: 8.5463 Validation Error: 8.1205
Epoch: 193 Training Error: 8.5208 Validation Error: 8.0508
Epoch: 194 Training Error: 8.4966 Validation Error: 7.9850
Epoch: 195 Training Error: 8.4745 Validation Error: 7.9247
Epoch: 196 Training Error: 8.4551 Validation Error: 7.8719
```

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Epoch: 197 Training Error: 8.4391 Validation Error: 7.8285
Epoch: 198 Training Error: 8.4278 Validation Error: 7.7975
Epoch: 199 Training Error: 8.4227 Validation Error: 7.7829
Epoch: 200 Training Error: 8.4264 Validation Error: 7.7904
Epoch: 201 Training Error: 8.4426 Validation Error: 7.8281
Epoch: 202 Training Error: 8.4759 Validation Error: 7.9036
Epoch: 203 Training Error: 8.5301 Validation Error: 8.0136
Epoch: 204 Training Error: 8.6013 Validation Error: 8.1212
Epoch: 205 Training Error: 8.6689 Validation Error: 8.1495
Epoch: 206 Training Error: 8.7086 Validation Error: 8.0645
Epoch: 207 Training Error: 8.7257 Validation Error: 7.9493
Epoch: 208 Training Error: 8.7473 Validation Error: 7.8999
Epoch: 209 Training Error: 8.7956 Validation Error: 7.9489
Epoch: 210 Training Error: 8.8827 Validation Error: 8.0933
Epoch: 211 Training Error: 9.0084 Validation Error: 8.3144
Epoch: 212 Training Error: 9.1475 Validation Error: 8.5684
Epoch: 213 Training Error: 9.2327 Validation Error: 8.7674
Epoch: 214 Training Error: 9.1571 Validation Error: 8.7797
Epoch: 215 Training Error: 8.8289 Validation Error: 8.4854
Epoch: 216 Training Error: 8.2538 Validation Error: 7.8727
Epoch: 217 Training Error: 7.5419 Validation Error: 7.0569
Epoch: 218 Training Error: 6.8202 Validation Error: 6.1894
Epoch: 219 Training Error: 6.1646 Validation Error: 5.3736
Epoch: 220 Training Error: 5.5993 Validation Error: 4.6551
Epoch: 221 Training Error: 5.1226 Validation Error: 4.0455
Epoch: 222 Training Error: 4.7252 Validation Error: 3.5412
Epoch: 223 Training Error: 4.3975 Validation Error: 3.1332
Epoch: 224 Training Error: 4.1307 Validation Error: 2.8102
Epoch: 225 Training Error: 3.9170 Validation Error: 2.5602
Epoch: 226 Training Error: 3.7486 Validation Error: 2.3711
Epoch: 227 Training Error: 3.6183 Validation Error: 2.2313
Epoch: 228 Training Error: 3.5193 Validation Error: 2.1305
Epoch: 229 Training Error: 3.4455 Validation Error: 2.0596
Epoch: 230 Training Error: 3.3915 Validation Error: 2.0111
Epoch: 231 Training Error: 3.3527 Validation Error: 1.9790
Epoch: 232 Training Error: 3.3251 Validation Error: 1.9582
Epoch: 233 Training Error: 3.3056 Validation Error: 1.9453
Epoch: 234 Training Error: 3.2917 Validation Error: 1.9373
Epoch: 235 Training Error: 3.2814 Validation Error: 1.9325
Epoch: 236 Training Error: 3.2733 Validation Error: 1.9295
Epoch: 237 Training Error: 3.2664 Validation Error: 1.9276
Epoch: 238 Training Error: 3.2603 Validation Error: 1.9262
Epoch: 239 Training Error: 3.2546 Validation Error: 1.9252
Epoch: 240 Training Error: 3.2491 Validation Error: 1.9246
Epoch: 241 Training Error: 3.2441 Validation Error: 1.9243
Epoch: 242 Training Error: 3.2397 Validation Error: 1.9246
Epoch: 243 Training Error: 3.2363 Validation Error: 1.9257
Epoch: 244 Training Error: 3.2343 Validation Error: 1.9280
```

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Epoch: 245 Training Error: 3.2341 Validation Error: 1.9321
Epoch: 246 Training Error: 3.2363 Validation Error: 1.9388
Epoch: 247 Training Error: 3.2414 Validation Error: 1.9487
Epoch: 248 Training Error: 3.2496 Validation Error: 1.9626
Epoch: 249 Training Error: 3.2612 Validation Error: 1.9805
Epoch: 250 Training Error: 3.2757 Validation Error: 2.0020
End
```

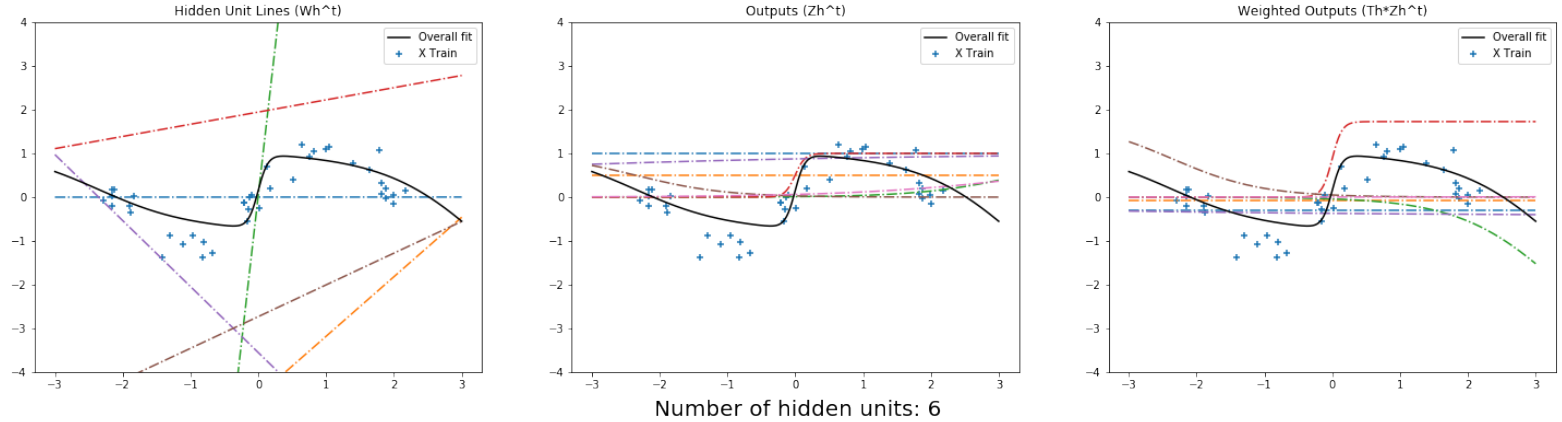


```
Test set loss: 2.1177
```